A Triangular Periodic Table of Elementary Circuit Elements

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***Abstract*—Chua proposed an Elementary Circuit Element Quadrangle including the three classic elements (resistor, inductor, and capacitor) and his formulated, named memristor as the fourth element. Based on an observation that this quadrangle may not be symmetric, I proposed an Elementary Circuit Element Triangle, in which memristor as well as mem-capacitor and mem-inductor lead three basic element classes, respectively. An intrinsic mathematical relationship is found to support this new classification. It is believed that this triangle is concise, mathematically sound and aesthetically beautiful, compared with Chua's quadrangle. The importance of finding a correct circuit element table is similar to that of Mendeleev's periodic table of chemical elements in chemistry and the table of 61 elementary particles in physics, in terms of categorizing the existing elements and predicting new elements. A correct circuit element table would also request to rewrite the 20th century textbooks.**

***Index Terms*—Memristor, circuit elements, circuit theory, future computing paradigm, modern information technology.**

I. Chua's Elementary Circuit Element Quadrangle

In 1971 Leon Chua presented an elementary circuit element quadrangle including the three classic elements (resistor, inductor, and capacitor) and his formulated, named memristor (short for memory resistor) as the fourth element [1]. As shown in Fig. 1, Chua noted that there are six different mathematical relations connecting pairs of the four fundamental circuit attributes: electric current i, voltage v, charge q, and magnetic flux φ. One of these relations, the charge as the time integral of the current, is determined from the definitions of two of the attributes, and another, the flux as the time integral of the electromotive force, or voltage, is determined from Faraday's Law of Induction. Thus, there should be four elementary circuit elements described by the remaining relations between the attributes [1]–[4]. The memristor, with memristance M, provides a constitutive relation between charge q and flux φ as given under $d\varphi/dq = M(q)$ memristance.

What was still missing in 1971 was an engineering realization of the memristor as a new "electronic element." In 2008, a team from HP linked the memory behavior of thin films of titanium dioxide with the theory of memristors [5].

In Chua's Elementary Circuit Element Quadrangle (Fig. 1), the memristor is not simply one more electronic element among others but the closure of the system of electronics as we know

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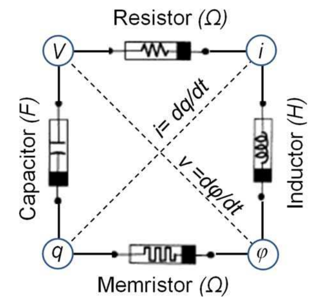


Fig. 1. Chua's Elementary Circuit Element Quadrangle (redrawn based on [3]–[5]). Chua thought a fourth element existed to provide conceptual symmetry and this quadrangle historically helped Chua in inventing his named memristor (memory resistor).

it as a whole. Before the invention of the memristor there was still a systematical gap in the table of elements [6]. In 2003, Chua presented a "Four-Element Torus," which is different from his circuit element quadrangle and includes resistor, inductor, capacitor and negative resistor ([3, Fig. 32]). Using the above torus as a seed, Chua generated a 5×5 periodic table of circuit elements that is periodic modulo ± 4 ([3, Fig. 31] and [4, Fig. 11]).

II. Is Chua's Quadrangle Basic and Symmetric?

If you observe carefully Chua's quadrangle as well as its variant/enlargement, you may find the following asymmetries and anomalies:

1) Unit: Memristance is measured in the same units (ohms) as resistors, whereas the standard 3 circuit elements each have their own units of measure.

2) Nonlinearity: It is clear that the four elements in Chua's quadrangle naturally fall into two groups. One group is the classic resistor, capacitor, and inductor. This group contains only "linear" elements. The other group is the new memristor only, which has to be nonlinear otherwise it degenerates to a resistor [1], [2].

3) Internal Links: There are two internal links within the quadrangle: one is $i = dq/dt$, the other is $v = d\varphi/dt$. I think these two internal links completely destroy the fairness and the symmetry. One should not introduce any internal link in such an elementary element table. A table of elementary elements should only link basic physical attributes, not derived attributes. If φ and q are basic, v and i should be viewed as "derived attributes" that are not independent. A table linking mixed basic and derived attributes may result in asymmetries.

4) Negative Elements: Chua's periodic table and "Four-Element Torus" include an element category of negative resistor and its counterparts along a diagonal line [3], [4]. Although some elements under this category have found useful applications in active circuit design (an amplifier, an oscillator, or a computer), a table linking mixed passive and active elements may not appear harmonic and consistent.

From the above observations, it appears that Chua's quadrangle may not be pedagogically appealing as originally thought to be. Consequently we argue against that a memristor is the 4th elementary circuit element. A dilemma of claiming memristor as the 4th element is that mem-capacitor, mem-inductor, and other high-order elements may have to be claimed as the 5th, 6th, and so on [3], [6], which would be an endless game. As will be elaborated in the following sections, memristor will actually have a more important role to play than being only the fourth element.

III. My Elementary Element Triangle

As mentioned above, within a certain context, φ and q should be more basic than v and i. Physically, magnetic flux, φ, and electric charge, q, are fundamental features to describe an object. In other words, φ and q are internal features associated with the device material and its physical operating mechanism. Contrarily, voltage, v, and current, i, could be derived from φ and q via $v = d\varphi/dt$ (Faraday's Law) and $i = dq/dt$ (by definition). Although conveniently used in practice, voltage and current only exhibit external measures of an object. Voltage and current should not be chosen to generate new elements as they are not physically basic (e.g., voltage v is always a "difference" when you measure it; current i is always the movement of electric charge q).

We can find at least three evidences to support the claim that φ, q are physically intrinsic and basic.

The first evidence is the fact that φ or q exhibits a memory feature in a memristor, mem-capacitor, or a mem-inductor [4]. This is why they are called ("mem" is short for "memory," mathematically the time integral of i or v). For example, in a memristor, when the power is switched off, both voltage v and current i become zero instantaneously but the memristor does not lose its value of φ and q (holding the value unchanged forever) [5]. Actually, φ or q is viewed as an ideal memristor's "state," which remembers what has happened in the past.

Another evidence is our work on a so-called "Delayed Switching" phenomenon, i.e., the switching of a memristor takes place with a time delay [7]. The physical interpretation is that charge q or flux φ possesses certain inertia with the tendency to remain unchanged (settle to some equilibrium state). It cannot respond as rapidly as the fast variation in the excitation waveform v or i and always takes a finite but small time interval for the memristor to change its resistance value. Needless to say, this natural feature of φ, q could be very useful in future computing paradigms and modern information technology such as memristor-based computer memories [7], memristor-based neural networks, and memristor-based neuro-mophic engineering [8].

The third evidence is the v-i loci's frequency dependence. As frequency ω of an excitation v or i moves towards ∞, the hysteresis loop v-i shrinks and eventually collapses into a straight line through the origin [5], [4]. In contrast, an intrinsic attribute should not vary from the measurement.

In summary, I think that "φ" and "q" are not only defined mathematically but also with physical interpretations. They are basic and intrinsic. It is unreasonable to define a new element based on φ, q, v and i, the latter two of which are only defined mathematically. Hence, I will choose only two attributes (φ, q) to define a new element.

Although there are no rigorous ways to identify what a new circuit element would have to look like, I think an elementary element represents a "relation" between a pair of physical attributes. This relation should be unique and not synthesized. My definition of a new (two-terminal) circuit element is as follows:

An elementary electronic circuit element should link two physical attributes, at least one of which should be basic.

This seemingly simple definition eliminates the possibility to synthesize an elementary element from other basic elements. In other words, even an element links one basic attribute and one derived attribute but the relation is still unique. Such an elementary element is irreducible, similar to those chemical elements in Mendeleev's periodic table. Note that, while defining a new element, I am not considering a differentiation, d/dt, as it may transform a non-linear function to a linear one (e.g., $dt^2/dt = 2t$) and hence lose the generality (i.e., a generic case should be nonlinear). Therefore there will be the following three basic element classes:

1. A basic element class linking φ and q with an unit of Ω or *Ohm* (This is a basic class because both φ and q are basic. I will mathematically prove that this class includes memristor and its higher order or lower order counterparts);
2. A basic element class linking $\int q$ and φ with an unit of F or *Farad* (Although $\int q$ is derived, φ is basic so this is still an independent class. I will mathematically prove that this class includes mem-capacitor and its higher order or lower order counterparts);
3. A basic element class linking $\int \varphi$ and q with an unit of H or *Henry* (Although $\int \varphi$ is derived, q is basic so this is still an independent class. I will mathematically prove that this class includes mem-inductor and its higher order or lower order counterparts).

My basic element triangle is depicted in Fig. 2. φ and q are thought to be two basic physical attributes that generate elementary circuit elements. Sharing the same SI unit, each apex represents a class of basic elements, which is equivalent to a "Group" in Mendeleev's Periodic Table of the Chemical Elements. Furthermore each apex includes more subclasses (equivalent to "Period" in Mendeleev's Table). Instead of being only the fourth element, memristor has a more important role to play: leading a basic element class. Similarly, mem-capacitor and mem-inductor will lead another two classes, as shown in Fig. 2.

This triangle can also be infinitely expanded inwards and outwards to have the apexes' higher order or lower order counterparts ($\int\int, \int\int\int, \ldots d^2/dt^2, d^3/dt^3 \ldots$).

Note that the element linking $\int \varphi$ and $\int q$ is not elementary because both $\int \varphi$ and $\int q$ are derived. The exclusion of $\int \varphi / \int q$ is decisive otherwise the Chua's quadrangle will not collapse to a triangle.

Also note that negative resistor is not given a place in my triangle because my triangular periodic table is purely a collection of passive elementary circuit elements. The exclusion of the negative elements is important in terms of reducing the periodicity from "four" in Chua's periodic table to "three" in mine.

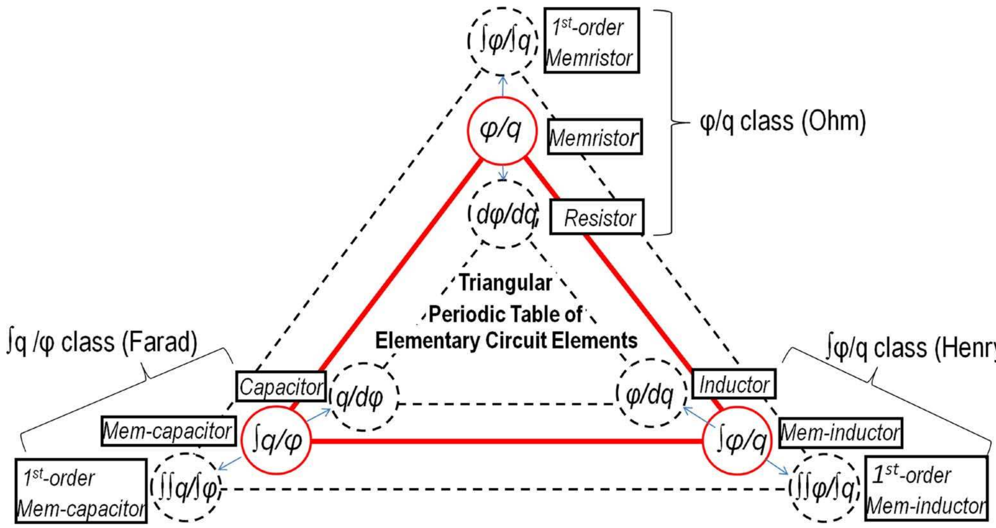


Fig. 2. My Triangular Periodic Table of Elementary Circuit Elements. φ and q are thought to be a pair of complementary basic attributes that generate elementary circuit elements. Sharing the same SI unit, each apex represents an exclusive class of elements. The periodicity of the layout (it is periodic modulo ± 3 if you count all the apexes triangle by triangle spirally) inspires me to henceforth call it the triangular periodic table of elementary circuit elements.

To cover all higher order or lower order counterparts, Chua introduced $v^{(\alpha)}$-$i^{(\beta)}$ to conveniently define an (α, β) element, where α and β can be any positive integer (representing αth-order or βth-order differential w.r.t. time), any negative integer (representing αth-order or βth-order integral w.r.t. time), or zero [3]. In Chua's system, every $(0,0)$ element is a resistor, $(-1,0)$ is an inductor, $(0,-1)$ is a capacitor and $(-1,-1)$ is a memristor.

As mentioned above, I use only (φ, q) to define a new element. The shift from the pair v, i (that is the "origin" of Chua's Periodic Table) to the pair φ, q (that is the "origin" of mine) may be analogous to that from "Earth-centered Universe" to "Sun-centered Universe." I use $\varphi^{(\alpha)}$-$q^{(\beta)}$ to define an (α, β) element. For example, in my periodic table, every $(0,0)$ element is a (0th-order) mem-resistor, $(1,1)$ is a resistor, $(-1,-1)$ is a 1st-order memristor, $(-1,0)$ is a mem-inductor and $(0,-1)$ is a mem-capacitor.

It is easy to prove that any class shares the same unit (elements are assumed to be linear, which does not affect the deduction on units) because

$$M(q) = \frac{\varphi}{q} = \frac{d\int\varphi}{d\int q} = \frac{d\varphi}{dq} = \frac{d\varphi(t)/dt}{dq(t)/dt} = \frac{v(t)}{i(t)} = R(\Omega) \tag{1}$$

Similarly

$$\begin{aligned} C(q) &= \frac{\int q}{\varphi} = \frac{d\int\int q}{d\int\varphi} = \frac{d\int q}{d\varphi} = \frac{d\int q(t)dt/dt}{d\varphi(t)/dt} \\ &= \frac{q(t)}{v(t)} = C(F) \end{aligned} \tag{2}$$

$$\begin{aligned} L(q) &= \frac{\int\varphi}{q} = \frac{d\int\int\varphi}{d\int q} = \frac{d\int\varphi}{dq} = \frac{d\int\varphi(t)dt/dt}{dq(t)/dt} \\ &= \frac{\varphi(t)}{i(t)} = L(H) \end{aligned} \tag{3}$$

For convenience, division is being used in Formulae (1), (2), and (3), the flux and charge may go through 0 at certain times, which may not be mathematically sound but does not affect the generic characterization of the element categories. The above charge-controlled cases could be extended to flux-controlled cases.

IV. Mathematic Linkage Within Each Class

Based on the above observation that φ and q are basic physical attributes, I will focus my study on the φ-q plane and its transformation to the v-i plane. Although v and i are thought to be "mathematically derived" rather than "physically basic," they are conveniently used in daily life and the features described in the v-i plane is easily understood (e.g., Ohm's Law).

As shown in Fig. 3, an arbitrary $\varphi = \overset{\frown}{\varphi}(q)$ curve in the φ-q plane represents a generic memristor. This curve should be origin-crossing as $\overset{\frown}{\varphi}(q) = M(q) \cdot q$. Note that the memristor shown in Fig. 3 is charge-controlled but principles found here should be applicable to another type of the flux-controlled memristor. In Fig. 3, $\overset{\frown}{\varphi}(q)$ and $\overset{\frown}{v}(i)$ are continuous and piecewise-differentiable functions with bounded slopes [4]. The notation $\overset{\frown}{v}(i)$ may be misleading as it implies an algebraic (memory-less) relation between voltage and current. Note that the voltage v is not only a function of i but also of q and thereby $\overset{\frown}{v}(i)$ should be a double-valued function of the current i displaying dependence on history.

It is convenient to assume a sinusoidal charge function to cover the full operating range $\{-A, +A\}$ of this memristor.

Note that a full range scanning is necessary to expose a distinctive "fingerprint" otherwise the obtained fingerprint is incomplete. This function is defined by

$$\begin{cases} q(t) = A\sin\omega t, & t \geq 0 \\ \quad = 0, & t < 0 \end{cases} \tag{4}$$

where the initial charge $q(0) = 0$.

Its corresponding current, $i(t)$, as a testing signal across the memristor, is

$$\begin{cases} i(t) = A\omega\cos\omega t, & t \geq 0 \\ \quad = 0, & t < 0 \end{cases} \tag{5}$$

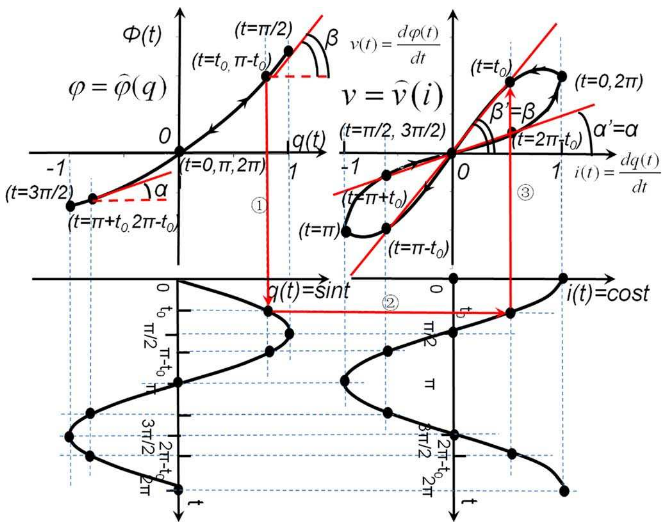


Fig. 3. Based on a "differential conformal transformation" that preserves angles between $\varphi = \widehat{\varphi}\ (q)$ and $v = \widehat{v}\ (i)$, Projection Line 1, 2, and 3 denotes a simple graphic method to project a given $\widehat{\varphi}\ (q)$ curve in the φ-q plane onto the v-i plane.

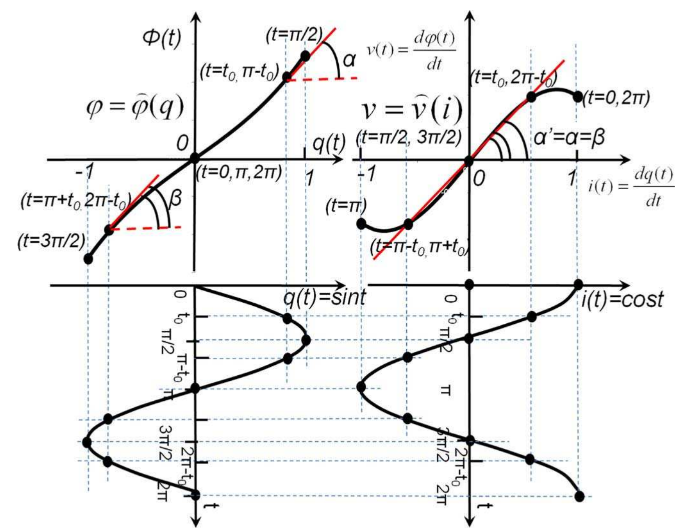


Fig. 4. If a $\varphi = \widehat{\varphi}\ (q)$ curve is an anti-symmetric function with respect to its mid-point, its corresponding $v = \widehat{v}\ (i)$ loci becomes a single-valued function. The element behaves like a nonlinear resistor $v(t) = R(i) \cdot i(t)$.

For convenience, it is assumed that $A = 1$ and $\omega = 1$ so the full operating range of the memristor is $i(t) \in \{-1, 1\}$ and $q(t) \in \{-1, 1\}$, as plotted in Fig. 3.

There must be a correspondence between the φ-q plane and the v-i plane because the coordinates of the latter is the differential (with respect to time) of the coordinates of the former [4], [9]. It is easy to have

$$\begin{aligned}\alpha &= \mathrm{arctg}\left(\frac{d\,\widehat{\varphi}(q)}{dq}\right) = \mathrm{arctg}\left(\frac{d\varphi(t)}{dq(t)}\right)\\ &= \mathrm{arctg}\left(\frac{\frac{d\varphi(t)}{dt}}{\frac{dq(t)}{dt}}\right)\\ &= \mathrm{arctg}\left(\frac{v(t)}{i(t)}\right) = \mathrm{arctg}\left(\frac{\widehat{v}\ (i)}{i}\right) = \alpha' \qquad (6)\end{aligned}$$

That is to say, the slope (α) of the line tangent to the $\varphi = \widehat{\varphi}\ (q)$ curve at an operating point $(t = t_0)$ in the φ-q plane is equal to the slope (α') of a straight line connecting the corresponding point $(t = t_0)$ to the origin in the $v = \widehat{v}\ (i)$ loci (on the same scale as the φ-q plane) [4]. Similar to (elementary-functions-based) Conformal Transformation [9], this transformation from a φ-q space to its differential v-i space preserves angles. The resemblance inspires me to name it "Differential Conformal Transformation," in which I think a lot of interesting studies remain about its mathematical features, e.g., its frequency (ω) dependency and irreversibility.

The above "Differential Conformal Transformation" can be simplified as follows: Linearizing $\varphi = \widehat{\varphi}\ (q)$ at the operating point (φ_0, q_0) corresponding to t_0 (Fig. 3) via series expansion, $\varphi = \varphi_0 + (d\hat{\varphi})/(dq)|_{q_0}(q - q_0)$ is obtained which is the equation for the tangent at (φ_0, q_0). Differentiating the above equation for the tangent w.r.t. time $v(t) = (d\hat{\varphi})/(dq)|_{t_0} i(t)$ is obtained which is the equation of the line joining the origin to the operating point at t_0 in the v-i plane. So the slopes match.

I found a simple graphic method to draw the voltage-current loci $v = \widehat{v}\ (i)$ corresponding to the above given $\varphi = \widehat{\varphi}\ (q)$ curve in Fig. 3: 1) Getting α at an operating point $(t = t_0)$ in the $\varphi = \widehat{\varphi}\ (q)$ curve, one would draw a straight line through the origin in the v-i plane whose slope is $\beta' = \beta$; 2) Projecting the point $(t = t_0)$ from the φ-q plane onto the v-i plane by following Projection Line ①, ② and ③, one would eventually end up with the same time point $(t = t_0)$ in the v-i plane by meeting Projection Line ③ with the drawn line in the first step.

As shown in Fig. 3, an arbitrary $\varphi = \widehat{\varphi}\ (q)$ curve (that should be continuous and differentiable) results in a generic memristor meeting the distinctive "fingerprint" defined by Chua [4]: 1. Zero-crossing or pinched; 2. (Double-valued) Lissajous figure $v = \widehat{v}\ (i)$.

Observing Fig. 3, it is found that the chord (a straight line connecting a point to the origin) sweeps in the first quadrant during the first half cycle $(0 \le t \le \pi)$, and then reverses the sweep in asymmetric manner in the third quadrant during the second half cycle $(\pi \le t \le 2\pi)$, resulting in an anti-symmetric pinched hysteresis loop. Because the outgoing path $(0 \le t \le \pi)$ overlaps the returning path $(\pi \le t \le 2\pi)$ in $\varphi = \widehat{\varphi}\ (q)$, the corresponding pinched Lissajous figure $v = \widehat{v}\ (i)$ is anti-symmetric with respect to the origin.

If the $\varphi = \widehat{\varphi}\ (q)$ curve is split into two branches (the outgoing path doesn't overlap the returning path), the pinched figure $v = \widehat{v}\ (i)$ will become asymmetric with respect to the origin. This actually represents a wide range of practical memristors with an asymmetric bi-polar or uni-polar pinched Lissajous figure [10]–[14].

If a $\varphi = \widehat{\varphi}\ (q)$ curve is an anti-symmetric function with respect to its mid-point of the operating range (in this case it is the origin) one could imagine that its corresponding $v = \widehat{v}\ (i)$ loci will collapse from a (double-valued) pinched Lissajous figure (as shown in Fig. 3) to a single-valued function, as shown in Fig. 4. The element with such an anti-symmetric $\widehat{\varphi}\ (q)$ curve behaves like a nonlinear resistor $v(t) = R(i) \cdot i(t)$ (current-controlled in this case).

Its mathematical proof is as below.

By definition of anti-symmetry $[f(-y,-x) = -f(x,y)]$ and $q(t_0) = \sin t_0 = -\sin(2\pi - t_0) = -q(2\pi - t_0)$, $i(t_0) = \cos t_0 = \cos(2\pi - t_0) = i(2\pi - t_0)$, we have

$$\varphi(t_0) = -\varphi(2\pi - t_0) \tag{7}$$

for a pair of points $(t_0, 2\pi - t_0)$ that are anti-symmetric against the mid-point (the origin in Fig. 4). We then have

$$\left.\frac{d\,\widehat{\varphi}\,(q)}{dq}\right|_{t=t_0} = \left.\frac{d\,\widehat{\varphi}\,(q)}{dq}\right|_{t=2\pi-t_0} \tag{8}$$

and

$$\begin{aligned} v(t_0) &= \left.\frac{d\varphi(t)}{dt}\right|_{t=t_0} = \left.\frac{d\,\widehat{\varphi}\,(q)}{dq(t)}\frac{dq(t)}{dt}\right|_{t=t_0} \\ &= \left.\frac{d\,\widehat{\varphi}\,(q)}{dq}\right|_{t=t_0} \cdot i(t_0) \\ &= \left.\frac{d\,\widehat{\varphi}\,(q)}{dq(t)}\right|_{t=2\pi-t_0} \cdot i(2\pi - t_0) \\ &= \left.\frac{d\varphi(t)}{dq(t)}\frac{dq(t)}{dt}\right|_{t=2\pi-t_0} \\ &= \left.\frac{d\varphi(t)}{dt}\right|_{t=2\pi-t_0} = v(2\pi - t_0) \end{aligned} \tag{9}$$

The above two anti-symmetric points $(t_0, 2\pi - t_0)$ in the φ-q plane overlap in the v-i plane and therefore $v = \widehat{v}\,(i)$ becomes a single-valued function.

For a non-linear resistor, the slope at $v = \widehat{v}\,(i)$ in general varies with the time evolution. However, one can keep the slope approximately constant over time by choosing a sufficiently small amplitude A while fixing the frequency ω, assuming the φ-q curve is continuous at that time point. Under this small-signal condition, the slope at any operating point on the single-valued curve $v = \widehat{v}\,(i)$ in the v-i plane is called small-signal resistance. It may become negative due to the phase lag between the peak of the voltage waveform and the peak of the current waveform [4], as shown in Fig. 4.

Different from the above small-signal resistance, the "chord resistance" represents large-signal resistance. In the v-i plane, the chord resistance at $t = t_0$ equals the slope of the chord connecting an operating point $(i(t_0), v(t_0))$ to the origin (0,0). In the φ-q plane, the chord resistance is equal to the slope of the line drawn tangent to the $\varphi = \widehat{\varphi}\,(q)$ curve at the corresponding point $(q(t_0), \varphi(t_0))$ (differential conformal transformation).

In many occasions, the large-signal chord resistance and the small-signal resistance are complementary and equally important. The chord resistance could be used to design a biasing circuit to fix a (static) working point whereas the small-signal resistance reflects a dynamic behavior under the condition that a small signal is applied on top of that biasing voltage or current. The chord resistance is also widely used by neuro-biologists, including Hodgkin and Huxley [15].

Although the φ-q plane reserves the chord resistance info (the slope of the chord), it should be noted that the φ-q plane cannot replace the v-i plane to describe a non-linear resistor comprehensively, e.g., the small-signal resistance info has been lost in the φ-q plane.

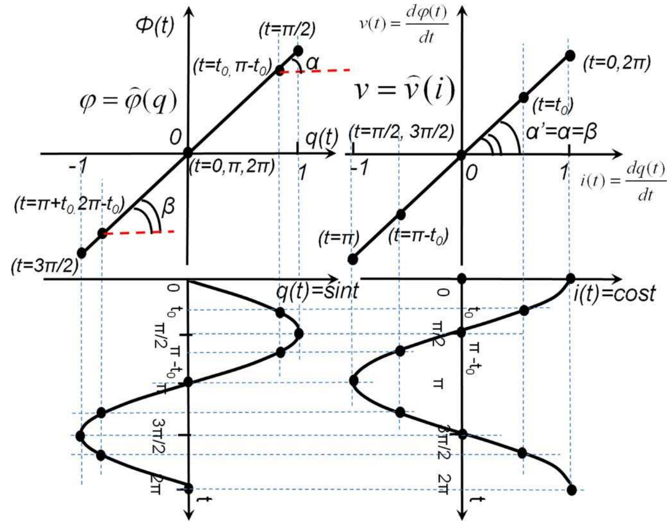


Fig. 5. If the $\varphi(q)$ curve becomes a straight line through the origin, the φ-q element degenerates to a classic (linear) resistor, which represents the simplest case.

Note that the above deduction is not intended to challenge or replace the strict definition of nonlinear elements in nonlinear circuit theory. Nonlinear circuit theory is a well-developed non-trivial discipline with a solid mathematical foundation [3]. The definition of a "nonlinear resistor" is: A 2-terminal element is called a nonlinear resistor if and only if, for "any" excitation voltage $v(t)$ (resp. current $i(t)$), and corresponding response $i(t)$ (res. $v(t)$), the graph of $(v(t), i(t))$ (resp., $(i(t), v(t))$ is a "fixed" single-valued function $i = g(v)$ (resp., $v = f(i)$). The key concept here is that one must "always" get the "same" single-valued function for "all" possible excitation waveforms. The reader should use (4)–(5) to verify that changing the parameter A, or ω, or changing the waveforms of $i(t)$ would result in different v-i curves.

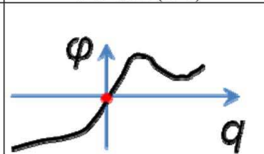
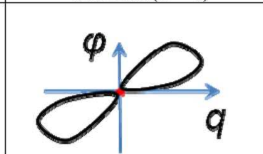
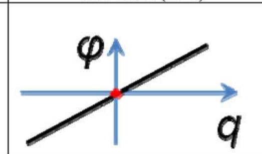
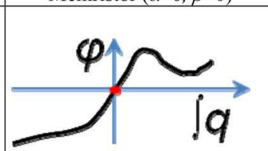
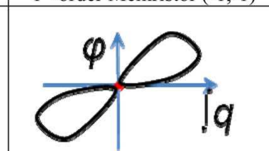
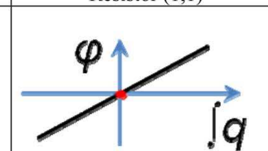
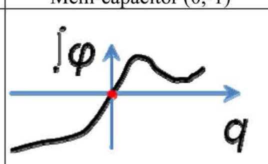
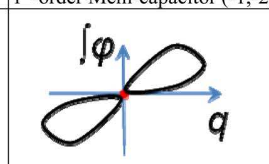
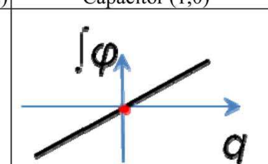
The reason the definition requires that one must have the "same" graph for all input waveforms is that a model, by definition, must be able to predict the response for any given input waveform. This will not be possible if one's graph changes with the input waveform. This is a fundamental concept—called a "constitutive relation"—in system theory [3], [4]. It follows that the "anti-symmetric" $\widehat{\varphi}\,(q)$ curve in Fig. 4 just "behaves like" a nonlinear resistor.

If the $\varphi = \widehat{\varphi}\,(q)$ curve becomes a straight line through the origin, we have

$$\begin{aligned} M(q) &= \frac{\widehat{\varphi}\,(q)}{q(t)} = \frac{d\,\widehat{\varphi}\,(q)}{dq(t)} = \frac{\frac{d\varphi(t)}{dt}}{\frac{dq(t)}{dt}} = \frac{v(t)}{i(t)} \\ &= \frac{\widehat{v}\,(i)}{i} = R \end{aligned} \tag{10}$$

that represents the simplest degenerate case of a classic (linear) resistor, as shown in Fig. 5. Because it is impossible to distinguish a linear resistor from a linear memristor, it does not matter to study its feature in the φ-q plane or v-i plane although, in principle, a two-terminal resistor should be defined by a constitutive relation in the v-i plane.

TABLE I
A LAYOUT TO ORGANIZE ELEMENTARY CIRCUIT ELEMENTS BASED ON AN INTRINSIC MATHEMATICAL RELATION. THE SHAPE OF THE CORRESPONDING CURVE RETROGRESSES MATHEMATICALLY, WHICH DETERMINES WHICH SUBCLASS AN ELEMENT SHOULD GO TO. A POSITIVE INTEGER n REPRESENTS nTH-ORDER DIFFERENTIAL (W.R.T. TIME) WHEREAS A NEGATIVE n REPRESENTS nTH-ORDER INTEGRAL (W.R.T. TIME)

Class $\varphi^{(\alpha)}$-$q^{(\beta)}$	Unit	The shape of the curve		
		An arbitrary curve Subclass (n=0)	A hysteresis loop Subclass (n= -1)	A straight line Subclass (n=1)
φ/q or $\varphi^{(n)}$-$q^{(n)}$	Ω			
		Memristor (α=0, β=0)	1st-order Memristor (-1,-1)	Resistor (1,1)
$\int q/\varphi$ or $\varphi^{(n)}$-$q^{(n-1)}$	F			
		Mem-capacitor (0,-1)	1st-order Mem-capacitor (-1,-2)	Capacitor (1,0)
$\int\varphi/q$ or $\varphi^{(n-1)}$-$q^{(n)}$	H			
		Mem-inductor (-1,0)	1st-order Mem-inductor (-2,-1)	Inductor (0,1)

The above principles summarized for the φ-q class should be applicable to the $\int q - \varphi$ class and the $\int \varphi - q$ class. A detailed discussion of all the relevant classes is outside the scope of this paper.

V. CONCLUSIONS AND DISCUSSIONS

Twenty-five hundred years ago, a mystic philosopher of ancient China, Lao Zi, postulated "Three gives birth to all things" [16]. Today we can find many applications of his Taoism. Television and other computer/video displays use three primary colors (the RGB model: red, green, and blue) to reproduce a broad array of colors. In finite element method (FEM), a mesh of triangles is used to form a piecewise linear approximation of an arbitrary geometric shape. In gastrulation (an early phase in the embryonic development of most animals), the single-layered blastula is reorganized into a trilaminar ("three-layered") structure known as the gastrula.

Inspired by the postulation, I presented a Triangular Periodic Table of Elementary Circuit Elements (Fig. 2) based on an intrinsic mathematical relationship. The three apexes represent three different element classes. Furthermore each apex of the triangle includes more subclasses. Playing a more important role than that in Chua's quadrangle, memristor, as well as mem-capacitor and mem-inductor, lead three basic element classes, respectively. Starting from "flux" and "charge," I achieve a classification of the elements which is quite general, reasonable, coherent and can include all the circuit elements.

To further elaborate the above triangle, Table I is another way to organize information based on an intrinsic mathematical relation. This table is an arrangement of the circuit elements ordered by a combination of two basic physical attributes in rows (Class "$\varphi^{(\alpha)}$-$q^{(\beta)}$," equivalent to "Group" in Mendeleev's Table) and columns (Subclass "n," equivalent to "Period" in Mendeleev's Table) presented so as to show their periodicity. The proof that a hysteresis loop in the φ-q plane represents a 1st-order memristor is omitted here. The reader should repeat a deduction, similar to that used in Fig. 3, to verify that an arbitrary curve (the constitutive relation of a 1st-order memristor) in the $\int \varphi$-$\int q$ plane would lead to a hysteresis loop in the φ-q plane.

The importance of finding a correct circuit element table in electrical/electronic engineering and physics is similar to that of Mendeleev's Periodic Table of Chemical Elements in Chemistry. A correct circuit element table would help us understand the complex world of electric circuitries/systems and also request to rewrite the physics and electrical/electronic engineering textbooks. Unfortunately, Chua's Basic Element Quadrangle is thought to have introduced some intrinsic asymmetries and anomalies. I believe that my triangle is more reasonable in the sense that it is concise, mathematically sound and aesthetically beautiful. On the other hand, I think my triangle reserves well "holy trinity" of resistors, capacitor and inductors, which are the roots of many system theories. For example, in Laplace-transformation-based Linear Circuit Theory, a resistor is expressed by "$R = Rs^0$," an inductor is "sL," and a capacitor is "$s^{-1}C$," in which s is the Laplace operator. As can be seen, the "holy trinity" is already mathematically complete without any room for a fourth element (on the other hand, a memristor has to be strictly non-linear).

It is worth mentioning that Mendeleev's original table was almost circular (Fig. 6) even though most commonly it is not drawn so [17], which reflects the layout of the periodic table has evolved over time, as new elements have been discovered, and new theoretical models have been developed to explain chemical behaviour. Although the present periodic table is almost a standard now, it may happen in the future that some additional considerations, such quantum-mechanical, or entirely new

Fig. 6. Dmitri Mendeleev and his Periodic Table in circular layout.

perspectives arising from the super-symmetry "string" theory might dictate yet another table.

Another analogue is the table of 61 elementary particles or fundamental particles in the Standard Model of particle physics. An elementary particle has no substructure, thus it is one of the basic building blocks of the universe from which all other particles are made. This table includes the famous Higgs boson (often referred to as "the God particle"), named for Peter Higgs who proposed the mechanism that suggested such a particle and explained the origin of mass in 1964. On July 4, 2012, after many years of experimentally searching for evidence of its existence, the Higgs boson was announced to have been observed at CERN's Large Hadron Collider [18]. The discovery of the last-found Higgs boson is likely to greatly affect human understanding of the universe and there may be lessons of history for us to learn.

ACKNOWLEDGMENT

The author wishes to thank Prof. Leon Chua (the father of memristor, an emeritus professor of University of California, an honorary professor of the University of Kent) for giving several comments that are included in the present version while he was a Leverhulme Trust Visiting Professor in the U.K. last year. Leon agreed that "three gives birth to all things" is beautiful in philosophy. His other comments include: 1. "Linearity" and "Nonlinearity" should not be used as a criterion for grouping circuit elements; 2. In circuit theory, there are two complementary approaches to defining basic circuit elements, namely, a "physical approach" used in this paper and an "axiomatic approach" [3], [4], which do not contradict, but instead they complement each other; 3. From a pedagogical perspective, the triangle table is simpler and more elegant than the axiomatic table.

Prof. Chua also stresses that the proposed element triangle is for the "ideal" memristor/mem-inductor/mem-capacitor and their counterparts with single state variable [1]. However in reality, we believe that most real-world memristors, memcapacitors, and meminductors are not ideal, but have additional state variables (called generalized memristor etc.) [2]. For example, the two time-varying conductances in the classic Hodgkin-Huxley model are first-order and second-order memristors, where the state variables are ionic gate probability variables [18]. Even in the HP memristor, the state variable is the width "w" of the active TiO_2 domain, and is not related to charge or flux in a natural way [5]. Nevertheless, there is a common fingerprint in the case of the memristor; namely, their v-i curves are pinched at the origin [19].

The author is particularly thankful to Prof. Chua for his professionalism to encourage him to submit this paper although the author is challenging some of his opinions.

The author wishes to thank an anonymous reviewer for his/her two questions: "Why are flux and charge more basic?" and "Is it really important (to achieve the triangle-based classification) to make this assumption?"

Regarding the first question, the reviewer thinks "flux," "charge," "voltage," and "current" may be all basic, if "basic" can mean something in this context. The reviewer also mentioned that there are many phenomena directly connected with voltage and current, e.g., the Joule effect is connected to current, some devices operate because a current flows and not because there is a charge. The author agrees that "basic" means something within a certain context. In Section III, the author lists three evidences to support the claim that φ, q are physically intrinsic and basic: 1. φ or q exhibits a memory feature in a memristor, mem-capacitor or a mem-inductor; 2. Our discovered "Delayed Switching" phenomenon; 3. The v-i loci's frequency dependence. The author thinks the above constitutes the context of this work.

Regarding the second question, the reviewer thinks that, although he/she does not agree with the starting point, the final point is very interesting. His/her suggestion is to say that the author is proposing a new classification of circuit components starting from flux and charge. The reviewer further comments that this leads to a classification which has several advantages, such as being coherent in terms of measurement units and treatment of linear and nonlinear cases.

The author also wishes to thank another anonymous reviewer for providing a simplified proof of "Differential Conformal Transformation." This simplified proof is recorded in Section IV.

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