When Is Utilitarian Welfare Higher Under Insurance Risk Pooling?

Indradeb Chatterjee, Angus S. Macdonald, Pradip Tapadar and R. Guy Thomas

Abstract This paper focuses on the effects of bans on insurance risk classification on utilitarian social welfare. We consider two regimes: full risk classification, where insurers charge the actuarially fair premium for each risk, and pooling, where risk classification is banned and for institutional or regulatory reasons, insurers do not attempt to separate risk classes, but charge a common premium for all risks. For the case of iso-elastic insurance demand, we derive sufficient conditions on higher and lower risks' demand elasticities which ensure that utilitarian social welfare is higher under pooling than under full risk classification. Empirical evidence suggests that these conditions may be realistic for some insurance markets.

1 Outline Of Our Approach

We consider two alternative regimes: full risk classification, where insurers charge the actuarially fair premium for each risk, and pooling, where risk classification is banned and insurers charge a common premium for all risks. Pooling implies a redistribution from lower risks towards higher risks. The outcome in terms of utilitarian social welfare depends on how we evaluate the trade-off between the utility gains and losses of the two types.

Such evaluations are typically made with models which assume that all individuals share a common utility function. Given an offered premium, individuals with the same probabilities of loss (i.e. individuals from the same *risk-group*) then all make the same purchasing decision. However, this does not correspond well to the observable reality of many insurance markets, where individuals with similar probabilities of loss often appear to make different decisions, and many individuals do not purchase insurance at all.

To reproduce observable reality, we instead introduce *heterogeneity* of utility functions (not necessarily all risk-averse) across individuals from any given risk-group. Individual utility functions then determine individual purchasing decisions, which (when aggregated) determine the insurance demand curve, and hence the equilibrium price of insurance when all risks are pooled.

Indradeb Chatterjee, Pradip Tapadar, R. Guy Thomas
University of Kent, Canterbury, CT2 7FS, UK. e-mail: ic252@kent.ac.uk

Angus S. Macdonald
Heriot-Watt University, Edinburgh EH14 4AS, UK. e-mail: A.S. Macdonald@hw.ac.uk

Individual utility functions also determine the expected utilities which individuals assign to their outcomes *given* an insurance price. The measure of social welfare is expected utility *given* the distributions of loss probabilities and risk preferences.

2 Model Set-up

2.1 Insurance Demand from the Individual Viewpoint

Suppose that an individual has wealth W and risks losing an amount L with probability μ. The individual's utility of wealth is given by $u(\cdot)$, where $u'(\cdot) > 0$. The individual is offered insurance against the potential loss amount L at premium π (per unit of loss), i.e. for a payment of πL. He will purchase insurance if:

$$u(W - \pi L) > (1-\mu)\,u(W) + \mu\,u(W-L). \tag{1}$$

Since certainty-equivalent decisions do not depend on the origin and scale of a utility function, standardising $u(W) = 1$ and $u(W-L) = 0$, simplifies the decision rule to:

$$u(W - \pi L) > (1-\mu). \tag{2}$$

Assuming small premiums (such that the second and higher-order terms in the Taylor series of expansion of $u(W - \pi L)$ are negligible), we can then write:

$$u(W - \pi L) \approx u(W) - \pi L u'(W) = 1 - \pi L u'(W), \quad \text{as } u(W) = 1 \tag{3}$$

and hence the decision rule becomes:

$$\gamma < \nu. \tag{4}$$

where $\gamma = Lu'(W)$ is the *risk preferences index* and $\nu = \mu/\pi$ is *risk-premium ratio*.

2.2 Insurance Demand from the Insurer's Viewpoint

From an insurer's perspective, it cannot observe individual utility functions; it observes only the *proportion* of each risk-group who choose to buy insurance. We call this a (proportional) demand function and define it as:

$$d(\pi) = \mathrm{P}\,[\,\gamma < \nu\,]\,. \tag{5}$$

It can be shown that if the underlying random variable Γ from which individual realisations of γ are generated has a particular distribution, this implies the *iso-elastic* demand function:

$$d(\pi) = \tau \left(\frac{\mu}{\pi}\right)^{\lambda} \tag{6}$$

where λ is the constant *elasticity of demand* and τ is the *fair-premium demand*.

2.3 Market Equilibrium and Social Welfare

We assume a market with n risk-groups, where competition between insurers leads to zero expected profits in equilibrium. We define a *risk classification regime* as a vector of premiums $(\pi_1, \pi_2, \ldots, \pi_n)$ charged to the risk-groups. Social welfare, $S(\underline{\pi})$, under that regime is the expected utility of an individual selected at random from the population. For the special case of iso-elastic demand, it can be shown that:

$$S(\underline{\pi}) = \sum_{i=1}^{n} p_i \, \tau_i \, \frac{1}{(\lambda_i + 1)} \left(\frac{\mu_i}{\pi_i}\right)^{\lambda_i + 1} \pi_i + K \tag{7}$$

where K is a constant, and the premium regime $\underline{\pi}$ satisfies the equilibrium condition:

$$\sum_{i=1}^{n} p_i \, \tau_i \left(\frac{\mu_i}{\pi_i}\right)^{\lambda} (\pi_i - \mu_i) = 0. \tag{8}$$

3 Results For Iso-Elastic Demand

Result 3.1 *Suppose there are n risk-groups with risks $\mu_1 < \mu_2 < \cdots < \mu_n$ and the same iso-elastic demand elasticity $\lambda > 0$. Then $\lambda \leq 1 \Rightarrow S(\pi_0) \geq S(\underline{\mu})$.*

Result 3.1 says that if the common demand elasticity for all risk-groups is less than 1, pooling gives higher social welfare than full risk classification.

Result 3.1 assumes constant iso-elastic demand elasticity for all individuals. However, for most goods and services, we expect demand elasticity to rise with price, because of the income effect on demand: at higher prices, the good forms a larger part of the consumer's total budget constraint, and so the effect of a small percentage change in its price might be larger. For insurance this suggests that demand elasticity for higher risks might be higher. This motivates the following Result 3.2:

Result 3.2 *Suppose there are n risk-groups with risks $\mu_1 < \mu_2 < \cdots < \mu_n$ with iso-elastic demand elasticities $\lambda_1, \lambda_2, \ldots, \lambda_n$ respectively. Define $\lambda_{lo} = \max\{\lambda_i : \mu_i \leq \pi_0\}$ and $\lambda_{hi} = \min\{\lambda_i : \mu_i > \pi_0\}$ where π_0 is the pooled equilibrium premium. If $\lambda_i < 1$ for all $i = 1, 2, \ldots, n$ and $\lambda_{lo} \leq \lambda_{hi}$, then $S(\pi_0) \geq S(\underline{\mu})$.*

Roughly speaking, Result 3.2 says that if all higher risk-groups' (iso-elastic) demand elasticities are higher than all lower risk-groups' (iso-elastic) demand elasticities, *and* all demand elasticities are less than 1, then social welfare is higher under pooling than under full risk classification.

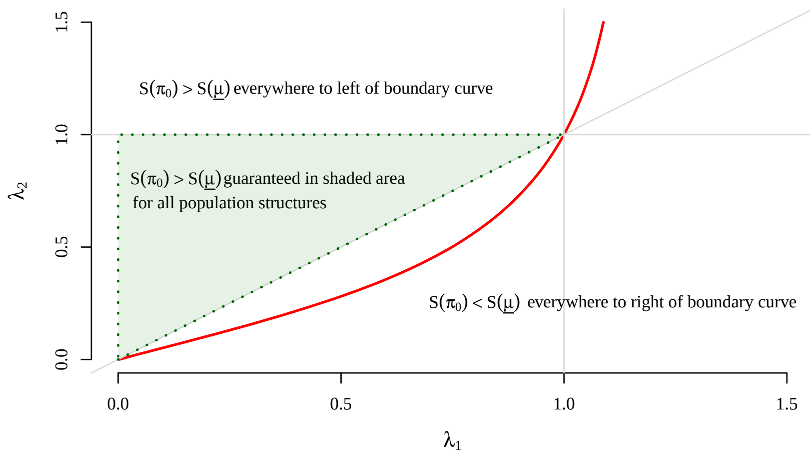

Fig. 1 Social welfare is higher under pooling to the left of the curve (guaranteed for *any* population structure in green triangle)

For the two risk-groups case, Result 3.2 references the green triangle in Figure 1. The two axes represent demand elasticities for lower and higher risk-groups, λ_1 and λ_2. Social welfare under pooling is higher than under full risk classification everywhere on the left of the boundary curve, and lower everywhere on the right. The exact position of the boundary curve depends on the population structure and relative risks; the curve shown is for $\mu_2/\mu_1 = 4$ and 80% of the population are low risks. The sufficient conditions in Result 3.2 specify that in the shaded triangle where $\lambda_1 \leq \lambda_2 < 1$, social welfare under pooling is *always* higher than that under full risk classification, *irrespective* of the population structure and relative risks.

4 Discussion

The conditions in the above results encompass many plausible combinations of higher and lower risks' demand elasticities. The conditions are stringent because they are sufficient for *any* population structures and relative risks, but they are *not* necessary (as shown by the white areas to the left of the boundary in Figure 1).

A condition common to both results is that all demand elasticities should be less than 1. Most relevant empirical estimates found in literature are of magnitude significantly less than 1. Whilst the various contexts in which these estimates were made may not correspond closely to the set-up in this paper, it is at least suggestive of the possibility that insurance demand elasticities may typically be less than 1.