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The use of dynamical networks to detect the hierarchical organization of financial market sectors

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Abstract. Two kinds of filtered networks: minimum spanning trees (MSTs) and planar maximally filtered graphs (PMFGs) are constructed from dynamical correlations computed over a moving window. We study the evolution over time of both hierarchical and topological properties of these graphs in relation to market fluctuations. We verify that the dynamical PMFG preserves the same hierarchical structure as the dynamical MST, providing in addition a more significant and richer structure, a stronger robustness and dynamical stability. Central and peripheral stocks are differentiated by using a combination of different topological measures. We find stocks well connected and central; stocks well connected but peripheral; stocks poorly connected but central; stocks poorly connected and peripheral. It results that the Financial sector plays a central role in the entire system. The robustness, stability and persistence of these findings are verified by changing the time window and by performing the computations on different time periods. We discuss these results and the economic meaning of this hierarchical positioning.

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1 Introduction

In the last few years different methods have been proposed for filtering relevant information in financial data by extracting a structure of interactions from cross-correlation matrices where only a subset of relevant entries are selected by means of criteria borrowed from network theory [1–14]. In particular, two methods that have been proved to be very effective are the Minimum Spanning Tree (MST) [1,15] and the Planar Maximally Filtered Graph ($PMFG$) [7,8]. Both methods are based on an iterative construction of a constrained graph (a tree or a planar [16] graph) which retains the largest correlations between connected nodes.

In this paper we analyze daily time series of the $n = 300$ most capitalized $NYSE$ stocks from 2001 to 2003, for a total of $T = 748$ days [9]. Return time series are computed as logarithmic differences of daily prices $Y_s(t) = \log(P_s(t+1)) - \log(P_s(t))$ ($s = 1...300$), and daily prices are computed as averages of daily quotations. Closing quotations are excluded from the computation. Stocks are classified into 12 economic sectors and 77 economic subsectors, according to the classification of Forbes magazine. Names of sectors, the labels used in this paper and the number of stocks in each sector are reported in Table 1.

We have considered moving windows from time (t) to time ($t + \Delta t - 1$), where $t = 1,\ 2\ ,\ ..., T - \Delta t + 1$ and $\Delta t = 21,\ 42,\ 63,\ 84,\ 126,\ 251$ market days, corresponding approximately to $\Delta t = 1,\ 2,\ 3,\ 4,\ 6,\ 12$ months.

For each of the resulting time series $Y_s(t)$, we have computed the correlation matrix $C(t, \Delta t)$ whose coefficients are given by the following formula:

$$c_{i,j}(t,\Delta t) = \frac{\langle Y_i Y_j\rangle_{(t,\Delta t)} - \langle Y_i\rangle_{(t,\Delta t)}\langle Y_j\rangle_{(t,\Delta t)}}{\sqrt{\left(\langle Y_i^2\rangle_{(t,\Delta t)} - \langle Y_i\rangle^2_{(t,\Delta t)}\right)\left(\langle Y_j^2\rangle_{(t,\Delta t)} - \langle Y_j\rangle^2_{(t,\Delta t)}\right)}}, \quad (1)$$

where $\langle f\rangle_{(t,\Delta t)} = \frac{1}{\Delta t}\sum_{\tau=0}^{\Delta t-1} f(t+\tau)$ is the time average of a given time series $f(\tau)$ over the window Δt from time t to time $t + \Delta t - 1$. From these correlation coefficients $c_{i,j}$, we compute distances between stocks i and j: $d_{i,j} = \sqrt{2(1 - c_{i,j})}$ [1,17–19]. The resulting matrix $D(t, \Delta t) = \sqrt{2(1 - C(t, \Delta t))}$ is the dynamical distance matrix of the weighted complete graph which has $n(n-1)/2$ edges connecting all pairs of nodes. Different methods exist in literature in order to filter the information contained in such a huge amount of data, otherwise hardly readable and usable.

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Table 1. Name of sectors, Labels and corresponding Number of Stocks.

Sector	Label	Number of Stocks
Basic Materials	$S01$	24
Capital Good	$S02$	12
Conglomerates	$S03$	8
Consumer Cyclical	$S04$	22
Consumer Non Cyclical	$S05$	25
Energy	$S06$	17
Financial	$S07$	53
Healthcare	$S08$	19
Services	$S09$	69
Technology	$S10$	34
Transportation	$S11$	5
Utilities	$S12$	12

In this paper we filter this information by using minimal graphs, namely MST (a connected graph with no cycles and $n-1$ edges, [20]) and $PMFG$ (a connected planar graph [16] in which the number of edges is $3n-6$ [8,21]). Such graphs are generated dynamically from correlations computed over a moving window. Dynamics adds a quantification of stability/variability over time which is very important in systems such as financial markets that are constantly evolving. In previous papers [8,21], it has been proved that the MST is always a subgraph of the $PMFG$ and the dynamical $PMFG$ preserves the same hierarchical structure as the dynamical MST, providing also a more significant and richer structure, a stronger robustness and a better dynamical stability. In this paper we investigate the hierarchical positioning of stocks in MST and $PMFG$ graphs by computing the Degree, Betweenness, Eccentricity and Closeness and we obtain a clear differentiation of stocks in terms of centrality and peripherality.

In Section 2 we report the results concerning the Degree, Betweenness, Eccentricity and Closeness of MST and $PMFG$ graphs; we also discuss the changes in the graph organization with respect to the window size Δt. In Section 3 a principal component analysis and a cluster analysis are performed. The stability over time of the results is also analyzed. Conclusions are given in Section 4.

2 Centrality and peripherality measures

Our aim in this work is to classify each stock in terms of its relative position in the network by quantitatively distinguishing between nodes that are more or less central. To this aim, for each node of both dynamical MSTs and $PMFG$s and for each Δt we have computed the time average of Degree, Betweenness, Eccentricity and Closeness [22,23], which are defined as follows[1]:

- the *Degree* of a node is the number of edges connected to that node;
- the *Betweenness* of node i is the total number of shortest paths between all possible pairs of vertices that pass through node i. The *Shortest Path* (or *geodesic*) between node k and node j is the shortest chain of connected pairs of vertices joining vertices k and j. The length of a path from node k to node j is the number of edges included in the path;
- the *Eccentricity* of node i is the maximum length of the shortest paths that connect i to any other node j;
- the *Closeness* of node i is the average length of all Shortest Paths that connect i and any other node j.

[1] We consider unweighted, undirected connected graphs only.

It is clear from the definition that Degree and Betweenness are 'centrality' measures which return larger values for more central, better connected, nodes; conversely Eccentricity and Closeness are 'peripherality' measures returning larger values for less central nodes.

In order to assess the relevance of sectors from a centrality/peripherality point of view we made several rankings of the stocks sorted respectively in *descending* order for Degree and Betweenness and *ascending* order for Eccentricity and Closeness. For each sector we then counted the number of stocks present in the top fifty positions and in the bottom fifty positions of the rankings. Results are reported in Table 2 for $\Delta t = 1$ and 12.

We find that the Financial sector ($S07$) is always strongly predominant among the central nodes of the system. This predominant role is retrieved in all the four measures, for all Δt, both for the dynamical MSTs and $PMFG$s. Such predominance slightly decreases as Δt increases. Though proportions are very similar for all the four measures, Eccentricity and Closeness show higher figures than Degree and Betweenness. Other strong presences among the central nodes can also be attributed to Basic Materials (S01), Capital Goods (S02), Conglomerates (S03), Consumer Cyclical (S04). We note, in particular, that the sector Basic Materials appears very central when $\Delta t = 1$ month but its centrality gradually fades away when Δt increases. Technology (S10) and Services (S09) have a more mixed positioning, indeed they occupy many of the lower fifty most peripheral positions, according to the Degree and the Betweenness but they do not have high Eccentricity and Closeness. Conversely, very high relative values of Eccentricity and Closeness are found in nodes belonging to Utilities (S12), Energy (S06), Consumer Non Cyclical (S05). In particular, we find absolutely outstanding the relevance of Utilities sector where most of the 12 Utilities stocks are counted among the fifty stocks with largest Eccentricity and largest Closeness. Other sectors, such as Healthcare (S08) and Transportation (S11) rank low in all the measures resulting therefore nor central neither peripheral. We find particularly noteworthy the similarity in behaviors of MSTs and $PMFG$s which show a remarkable correspondence of sectorial structures.

2.1 Effect of the time-window

From Table 2 it emerges that changes in the ranking occur as the time window Δt changes. In order to check and quantify the robustness of these results with respect to Δt, we have computed for all measures the average over time t at different Δt values for each stock.

Table 2. Number of stocks, for each sector, among the fifty top and the fifty bottom values for Degree, Betweenness, Eccentricity and Closeness for both MST and $PMFG$, for $\Delta t = 1$ and 12. In boldface are highlighted the most central or peripheral sectors accordingly with each measure.

Degree (descending order)																								
	(*a*)					*MST*'s Top Fifty						(*a*)	(*b*)					*PMFG*'s Top Fifty						(*b*)
	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	**S07**	*S*08	*S*09	*S*10	*S*11	*S*12	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	**S07**	*S*08	*S*09	*S*10	*S*11	*S*12
$\Delta t = 1$	8	3	2	3	1	1	**25**	0	2	5	0	0	8	3	3	3	1	2	**26**	0	1	3	0	0
$\Delta t = 12$	3	3	3	2	5	4	**15**	2	8	3	0	2	4	3	3	2	5	2	**17**	2	5	6	0	1
	(*c*)					*MST*'s Bottom Fifty						(*c*)	(*d*)					*PMFG*'s Bottom Fifty						(*d*)
	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	*S*07	*S*08	*S*09	**S10**	*S*11	*S*12	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	*S*07	*S*08	*S*09	**S10**	*S*11	*S*12
$\Delta t = 1$	3	1	0	1	9	2	3	7	14	**9**	0	1	3	2	0	1	8	2	2	7	15	**7**	1	2
$\Delta t = 12$	3	0	1	2	7	1	4	2	14	**14**	0	2	4	0	0	3	5	2	3	2	14	**15**	1	1
Betweenness (descending order)																								
	(*e*)					*MST*'s Top Fifty						(*e*)	(*f*)					*PMFG*'s Top Fifty						(*f*)
	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	**S07**	*S*08	*S*09	*S*10	*S*11	*S*12	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	**S07**	*S*08	*S*09	*S*10	*S*11	*S*12
$\Delta t = 1$	10	3	3	2	0	1	**27**	0	0	4	0	0	8	3	4	2	0	2	**27**	0	1	3	0	0
$\Delta t = 12$	3	3	3	5	2	5	**17**	2	7	3	0	0	4	3	3	4	2	5	**18**	2	5	4	0	0
	(*g*)					*MST*'s Bottom Fifty						(*g*)	(*h*)					*PMFG*'s Bottom Fifty						(*h*)
	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	*S*07	*S*08	*S*09	**S10**	*S*11	*S*12	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	*S*07	*S*08	*S*09	**S10**	*S*11	*S*12
$\Delta t = 1$	2	1	0	1	10	2	4	8	17	**4**	1	0	2	1	0	1	9	2	7	7	16	**5**	0	0
$\Delta t = 12$	4	1	1	2	7	1	5	2	11	**15**	0	1	2	0	0	2	6	3	6	2	15	**13**	1	0
Eccentricity (ascending order)																								
	(*i*)					*MST*'s Top Fifty						(*i*)	(*l*)					*PMFG*'s Top Fifty						(*l*)
	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	**S07**	*S*08	*S*09	*S*10	*S*11	*S*12	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	**S07**	*S*08	*S*09	*S*10	*S*11	*S*12
$\Delta t = 1$	11	4	4	3	0	0	**28**	0	0	0	0	0	11	3	4	2	0	0	**29**	0	0	1	0	0
$\Delta t = 12$	2	3	4	4	0	1	**22**	0	8	6	0	0	1	3	2	3	0	2	**21**	0	14	4	0	0
	(*m*)					*MST*'s Bottom Fifty						(*m*)	(*n*)					*PMFG*'s Bottom Fifty						(*n*)
	*S*01	*S*02	*S*03	*S*04	**S05**	**S06**	*S*07	*S*08	*S*09	*S*10	*S*11	**S12**	*S*01	*S*02	*S*03	*S*04	**S05**	**S06**	*S*07	*S*08	*S*09	*S*10	*S*11	**S12**
$\Delta t = 1$	2	0	0	0	**7**	**13**	2	6	7	1	0	**12**	2	0	0	0	**12**	**11**	1	5	7	1	0	**11**
$\Delta t = 12$	4	1	0	1	**14**	**10**	0	2	8	1	0	**9**	5	3	0	2	**13**	**10**	0	2	6	1	0	**8**
Closeness (ascending order)																								
	(*o*)					*MST*'s Top Fifty						(*o*)	(*p*)					*PMFG*'s Top Fifty						(*p*)
	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	**S07**	*S*08	*S*09	*S*10	*S*11	*S*12	*S*01	*S*02	*S*03	*S*04	*S*05	*S*06	**S07**	*S*08	*S*09	*S*10	*S*11	*S*12
$\Delta t = 1$	10	3	4	4	0	0	**29**	0	0	0	0	0	9	3	5	3	0	0	**29**	0	0	1	0	0
$\Delta t = 12$	3	3	6	7	0	0	**23**	0	5	3	0	0	2	3	4	6	0	1	**20**	1	10	3	0	0
	(*q*)					*MST*'s Bottom Fifty						(*q*)	(*r*)					*PMFG*'s Bottom Fifty						(*r*)
	*S*01	*S*02	*S*03	*S*04	**S05**	**S06**	*S*07	*S*08	*S*09	*S*10	*S*11	**S12**	*S*01	*S*02	*S*03	*S*04	**S05**	**S06**	*S*07	*S*08	*S*09	*S*10	*S*11	**S12**
$\Delta t = 1$	2	0	0	0	**8**	**13**	2	6	6	1	0	**12**	2	1	0	0	**9**	**12**	1	5	7	1	0	**12**
$\Delta t = 12$	3	1	0	1	**10**	**14**	1	2	7	1	0	**10**	4	1	0	1	**11**	**12**	0	2	6	2	0	**11**

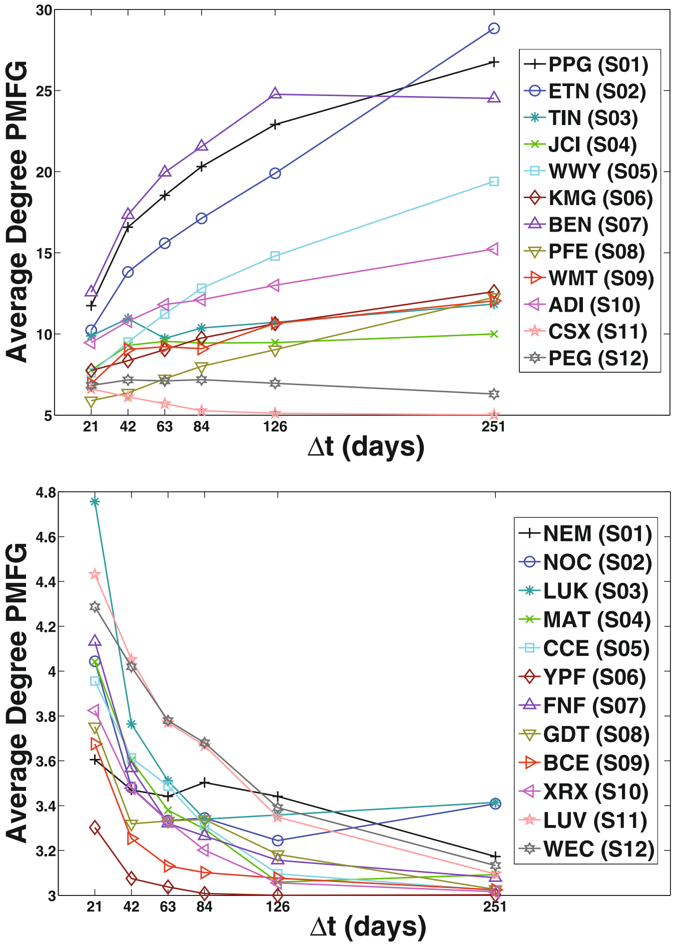

Fig. 1. (Color online) (*Above*) Average Degree as function of Δt for the stocks having the *highest* relative Degree within their own sector. (*Below*) Average Degree as function of Δt for the stocks having the *lowest* relative Degree within their own sector.

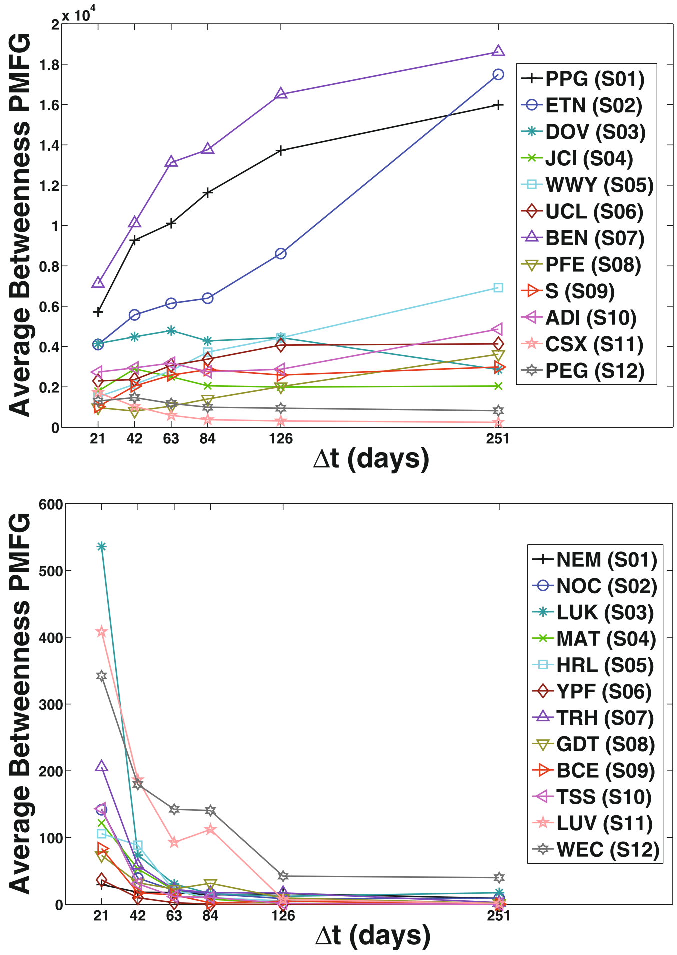

Fig. 2. (Color online) (*Above*) Average Betweenness as function of Δt for the stocks having the *highest* relative Degree within their own sector. (*Below*) Average Betweenness as function of Δt for the stocks having the *lowest* relative Degree within their own sector.

In particular, we have computed how the average Degree in the $PMFG$ for each of the 300 stocks changes when Δt increases. We found a decrease for 67% of stocks and an increase for the remaining 33%. Since the average Degree for a $PMFG$ is a constant, the sum of negative and positive values must be zero. Therefore, there are only few stocks in the graph which become more central but those stocks acquire in proportion a larger number of links. We observe a similar behavior for the Betweenness. On the other hand, Closeness and Eccentricity reveal a different behavior showing a clear decreasing trend in the lowest values but rather stationary or even decreasing trends for the highest values. This is due to the fact that the overall average for both measures is decreasing with Δt.

In order to better visualize such different behaviors in Figures 1–4 we report respectively the average Degree, average Betweenness, average Eccentricity and average Closeness as function of Δt for a selection of stocks having respectively the highest relative values and lowest relative values. The data refer to PMFG graphs only but MSTs have very similar properties and trends. We observe that –as a general trend– the differentiation between central and peripheral nodes in terms of Degree and Betweenness becomes stronger with increasing trends for high values and decreasing trends for low values. This is a indication that the graphs become increasingly structured when the window-size increases.

Let us note that most of the stocks reported in Figure 1 are also present in Figure 2 and most of the stocks reported in Figure 3 are also in Figure 4.

3 Principal component analysis and cluster analysis

The analysis of Degree, Betweenness, Eccentricity and Closeness introduced in the previous section, suggests that there could be two different gatherings of the variables which fully explain the different classifications obtained from Degree and Betweenness on one side and Eccentricity and Closeness on the other. In order to increase robustness, we have calculated all the quantities for all

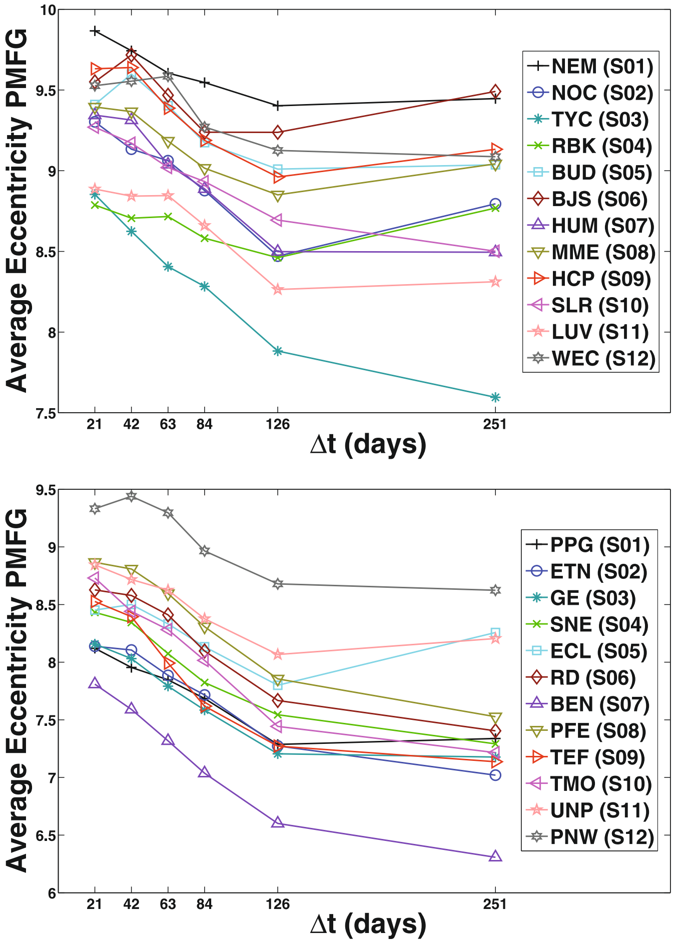

Fig. 3. (Color online) (*Above*) Average Eccentricity as function of Δt for the stocks having the *highest* relative Degree within their own sector. (*Below*) Average Eccentricity as function of Δt for the stocks having the *lowest* relative Degree within their own sector.

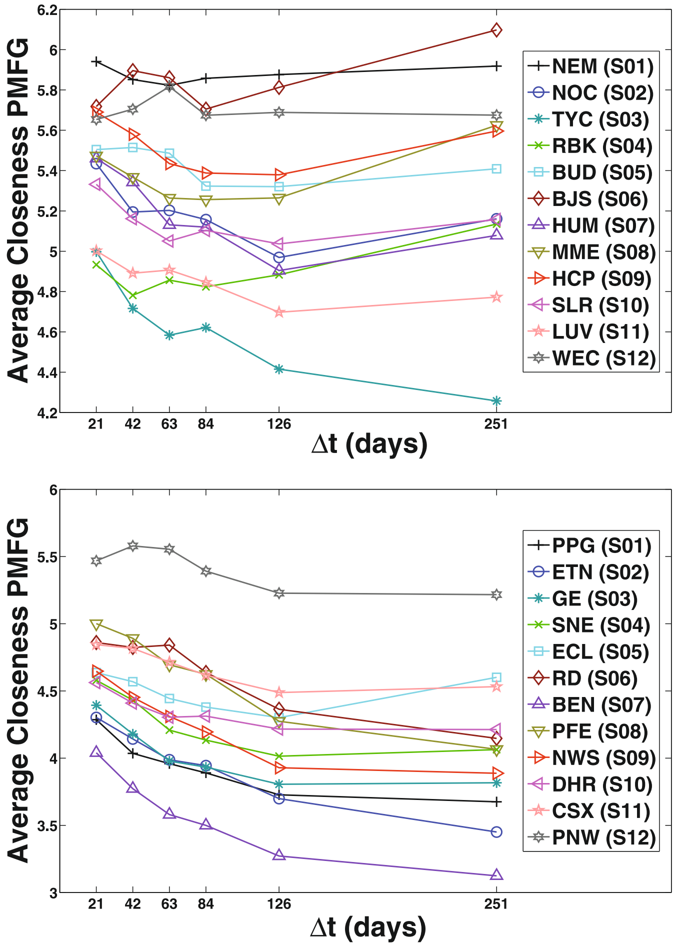

Fig. 4. (Color online) (*Above*) Average Closeness as function of Δt for the stocks having the *highest* relative Degree within their own sector. (*Below*) Average Closeness as function of Δt for the stocks having the *lowest* relative Degree within their own sector.

Dynamical MSTs and for all Dynamical PMFGs, for all time periods and all the running windows. In order to compare all these different measures we have transformed them in Fractional Rankings[2]. Then, for each measure, for each running window and for each stock, for both MSTs and PMFGs, we compute the time average of rankings. Afterwards, for each measure, for each stock and for both MSTs and PMFGs, we compute the average with respect to the running windows. This average ranking procedure provides four synthetic measures for Degree, Betweenness, (–)Eccentricity and (–)Closeness and for both MST and $PMFG$ (eight variables in total).

Principal Components analysis of these 8 variables yields to two significant principal factors. The first principal factor is essentially an average of all the eight variables and it explains the 74% of the total variance with 82–88% correlation between all the measures. Nodes that are highly connected and in central positions in the network have high relative scores for this measure. The second principal factor is also statistically significant, explaining 24% of the total variance and it is essentially the sum of the first two measures (Degree, Betweenness) minus the second two ((–)Eccentricity and (–)Closeness). This component gives high scores to nodes that are highly connected but that do not reside in central regions of the network.

Let us here report some stocks selected in terms of the two principal components:

1) nodes with the largest values of the first component (highly connected and central): A.G. Edwards (AGE), Franklin Resources (BEN) and Merrill Lynch (MER) belonging to the Financial sector (S07);
2) nodes with the largest values of the second component (highly connected but eccentric): Eastman Kodak (EK), Leucadia National (LUK), Golden West Financial (GDW) and Unisys (UIS);
3) nodes with the smallest values of the first component (poorly connected and peripheral): Health Care Property Invs (HCP), Valero Energy (VLO), Sara Lee (SLE) and Sociedad Anonima ADS (YPF);

[2] In Fractional Ranking, the same mean rank is assigned to entries with the same score.

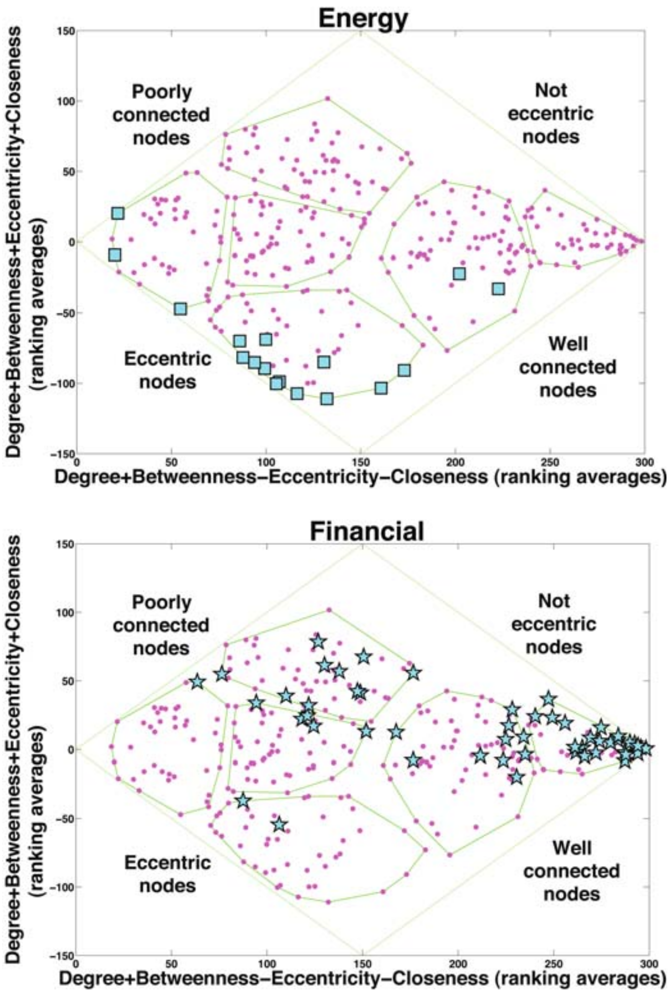

Fig. 5. (Color online) Stocks belonging to two different sectors are clearly differentiated in terms of their positions within the network: the energy sector (large squares, figure above) is in the periphery of the graphs whereas the financial sector (large stars, figure below) resides mostly in the central regions of the graphs.

4) nodes with the smallest values of the second component (non peripheral but poorly connected): Apache (APA), Kerr-Mcgee (KMG), Colgate-Palmolive (CL) and Smith International (SII);
5) nodes neutral from both points of view (nor particularly well connected and neither especially central): Best Buy (BBY), Jones Apparel (JNY) and Meredith (MDP).

Within this classification we can recognize regions dominated by particular sectors. For instance, in Figure 5 we can see that stocks belonging to the Energy sector are all clearly placed in the region of large Eccentricity whereas stocks belonging to the Financial sector stay mostly in the region of high connectivity and high centrality. We note however that in Figure 5 there are two stocks belonging to the Financial sector that appear to stay in rather eccentric regions, separated from the other Financial stocks. A careful scrutiny reveals that these stocks are belonging to the sub-sector "Insurance accidental & health". Interestingly, the region where they are confined is highly populated by stocks belonging to Healthcare.

3.1 Cluster analysis

In order to better differentiate the relative positioning of the stocks and their gathering in central or peripheral regions, we have performed a cluster analysis [18,24] using the 8 synthetic measures from Fractional Rankings described in the previous section. This leads to the following six clusters which are enclosed inside the polyhedral lines in Figure 5. The analysis has been performed by using SPAD software where a hierarchical tree (dendrogram) has been constructed based on all the axes obtained by Principal Components analysis and Wards aggregation criterion with a cut performed using a consolidation procedure [24]. We verified that the qualitative gathering of the stocks is robust and independent on the details of the clustering procedure with exception for some stocks at the cluster boundaries which might be swapped between neighboring clusters.

In one cluster we find 44 stocks that are both extremely central and well connected. This cluster contains BEN and MER, it is dominated by 25 financial stocks, with a much lighter presence of stocks belonging to Basic Materials (6), Conglomerates (4) and Capital Good (3).

In a second cluster we find 66 stocks that are central and connected, but less than the first cluster and with some stocks that tend to be slightly eccentric in certain occasions but still remain well connected. There are again many stocks belonging to the Financial sector (10). Significant are also the stocks belonging to Consumer Cyclical (12), Basic Materials (7), Conglomerates (2) and Capital Good (4). There are also 15 stocks from Services and 7 from Technology.

A third cluster is characterized by 45 stocks that are eccentric but well connected. This cluster contains APA (Apache corp, S06) and it is dominated by Utilities (9), Energy (12), Consumer Non Cyclical (10), with a moderate presence of Services as well (8).

Scarcely connected but neither eccentric are the 53 stocks belonging to a fourth cluster. This cluster contains LUK (Leucadia National Corp, S03) and it is mainly composed by stocks belonging to Technology (13), Services (19), and Financial (10) sectors.

A fifth cluster is characterized by 51 stocks which are poorly connected and peripheral and it isn't clearly characterized from a sectorial point of view, apart from 14 stocks belonging to Services.

In a sixth cluster we find 41 stocks that are always peripheral and poorly connected. There are 10 stocks of Consumer Non Cyclical, 8 from Healthcare and 9 from Services. This cluster contains SLE (Sara Lee corp, S05).

3.2 Persistence over time

It is now quite clear that the above measures can distinguish well between stocks in central or peripheral positions

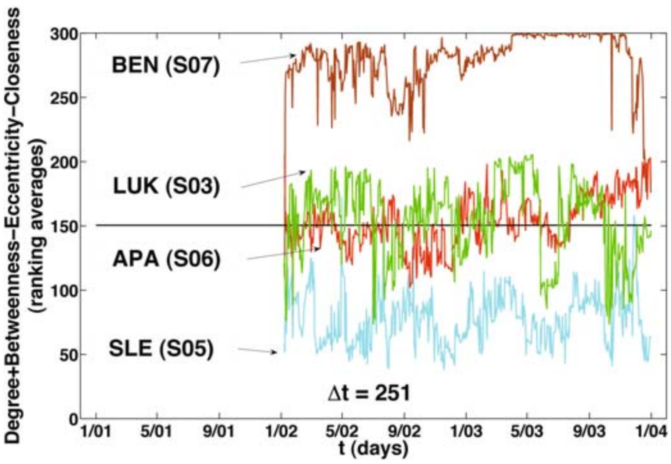

Fig. 6. (Color online) Average rankings ordered by Degree, Betweenness, (–)Eccentricity and (–)Closeness, for $\Delta t = 12$ months as function of time. Data are for BEN (S07), LUK(S03), APA (S06) and SLE (S05).

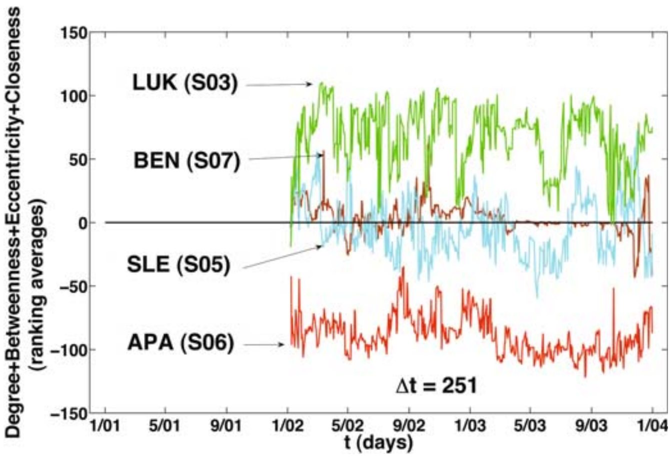

Fig. 7. (Color online) Difference between the average rankings of Degree and Betweenness and the average rankings of (–)Eccentricity and (–)Closeness as function of time for BEN (S07), LUK(S03), APA (S06) and SLE (S05) at $\Delta t = 12$.

in the network and that this differentiation is consistent with the independent classification of the stocks accordingly to their economic sector. We now want to verify if such classification is persistent with time. To this end we follow the measure associated with the x-axis in Figure 5 over time for four stocks belonging to the first, fourth, third and sixth cluster, namely BEN (S07), LUK (S03), APA (S06) and SLE (S05). Accordingly to the previous analysis BEN and SLE are respectively very central and very peripheral, whereas APA and LUK have a more mixed location. Indeed, in Figure 6 we observe that LUK and APA are fluctuating around the mean value (150) while BEN is well above and SLE is well below the average.

An opposite scenario is shown in Figure 7 where we plot the measure associated to y-axis in Figure 5 as function of time. We observe that, in this case, BEN and SLE are now varying around zero whereas LUK and APA clearly differentiate assuming values respectively above and below zero.

We have also investigated how these results are modified by changing Δt from 1, 2, 3, 4, 6 to 12 months. In Figure 8 we show the results for BEN and SLE for the first measure. We can see that BEN is always above SLE and as Δt increases the two sectors are separating indicating an increase of robustness of the measure. The central stocks are becoming more central and the peripheral stocks are becoming more peripheral.

Figure 9 shows the measure associated to y-axis in Figure 5 as function of time for LUK and APA. Also in this case we observe an increased differentiation with Δt: the two stocks gradually separate from zero becoming respectively more positive and more negative when Δt increases from 1 to 12 months.

4 Conclusions

We have applied different graph measures on MST and $PMFG$ filtered graphs retrieving a well defined and robust hierarchical classification among stocks. Our approach introduces two measures which differentiate between stocks well connected and central; stocks well connected but at the same time peripheral; stocks poorly connected but central; stocks poorly connected and peripheral. We have observed that the differentiation provided by these measures is in agreement with the independent classification of stocks in economic sectors. In particular, it is clearly revealed that stocks belonging to the Financial sector play a crucial role in the entire system resulting both well connected and central. This is not surprising, indeed, all companies involved in a production activity need funds before they start their business. Funds are provided directly and primarily by investors (self-financing), then conspicuously by the financial system and only at the end, for the residual part, by private lenders. It is therefore straightforward that we find banks and stocks belonging to the Financial sector at the center of the network. But companies need also raw materials (such as steel, aluminium or copper) and other intermediate goods (all those goods used as inputs in the production of other goods) and capital goods or physical capital (such as factories, machinery, tools, and various buildings): these are all "specific inputs" of the production. No surprise then that, after the Financial sector, we find that Basic Materials, Capital Goods and Conglomerates also share the central part of the filtered graphs. Sectors specialized in final products, such as Consumer Non Cyclical and Healthcare are concentrated in the periphery instead. Similarly sectors like Transportation, Energy and Utilities, which are general inputs and serve indistinctly all other activities, are rather peripheral as well. We have also verified the robustness of these measures by building dynamical graphs over moving windows.

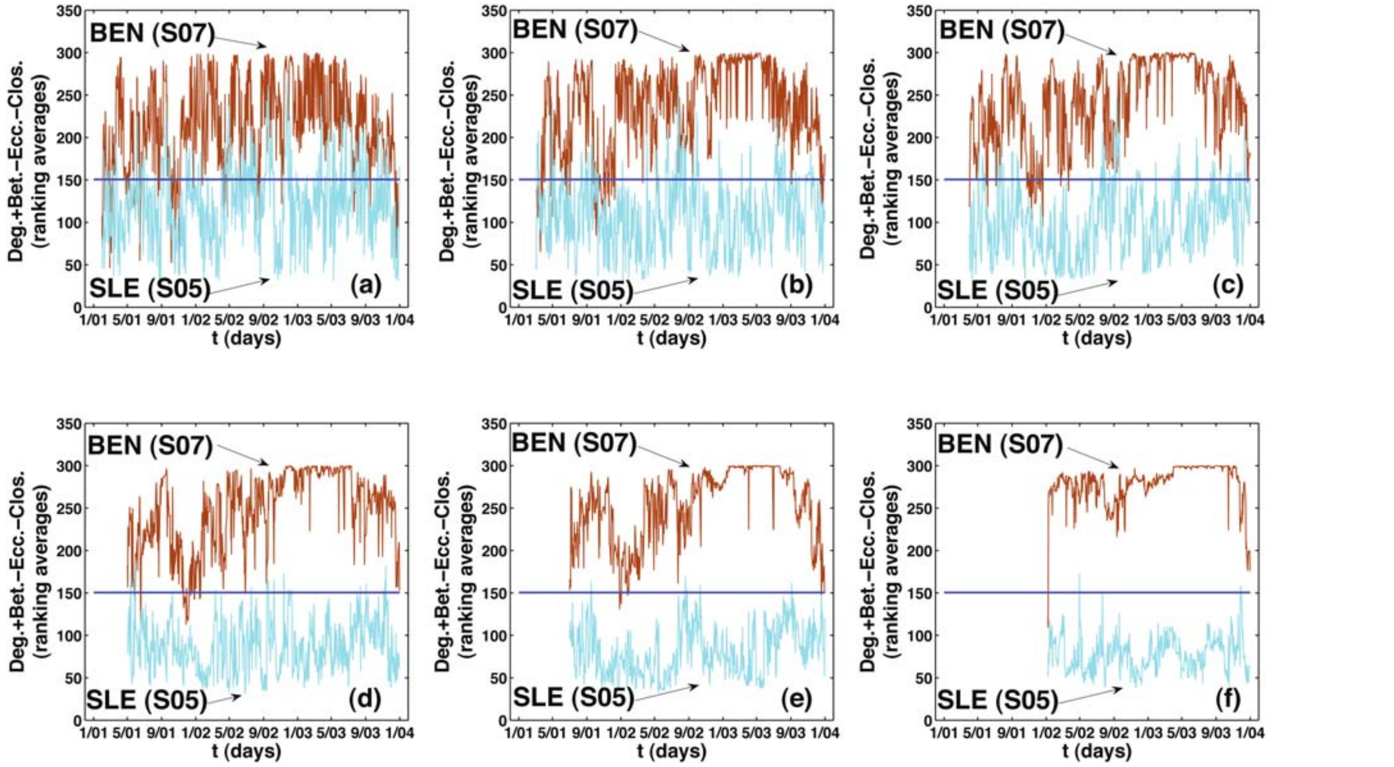

Fig. 8. (Color online) Average rankings ordered by Degree, Betweenness, (–)Eccentricity and (–)Closeness as function of time for BEN (S07) and SLE (S05) for $\Delta t = 1$ month (a), 2 months (b), 3 months (c), 4 months (d), 6 months (e) and 12 months (f).

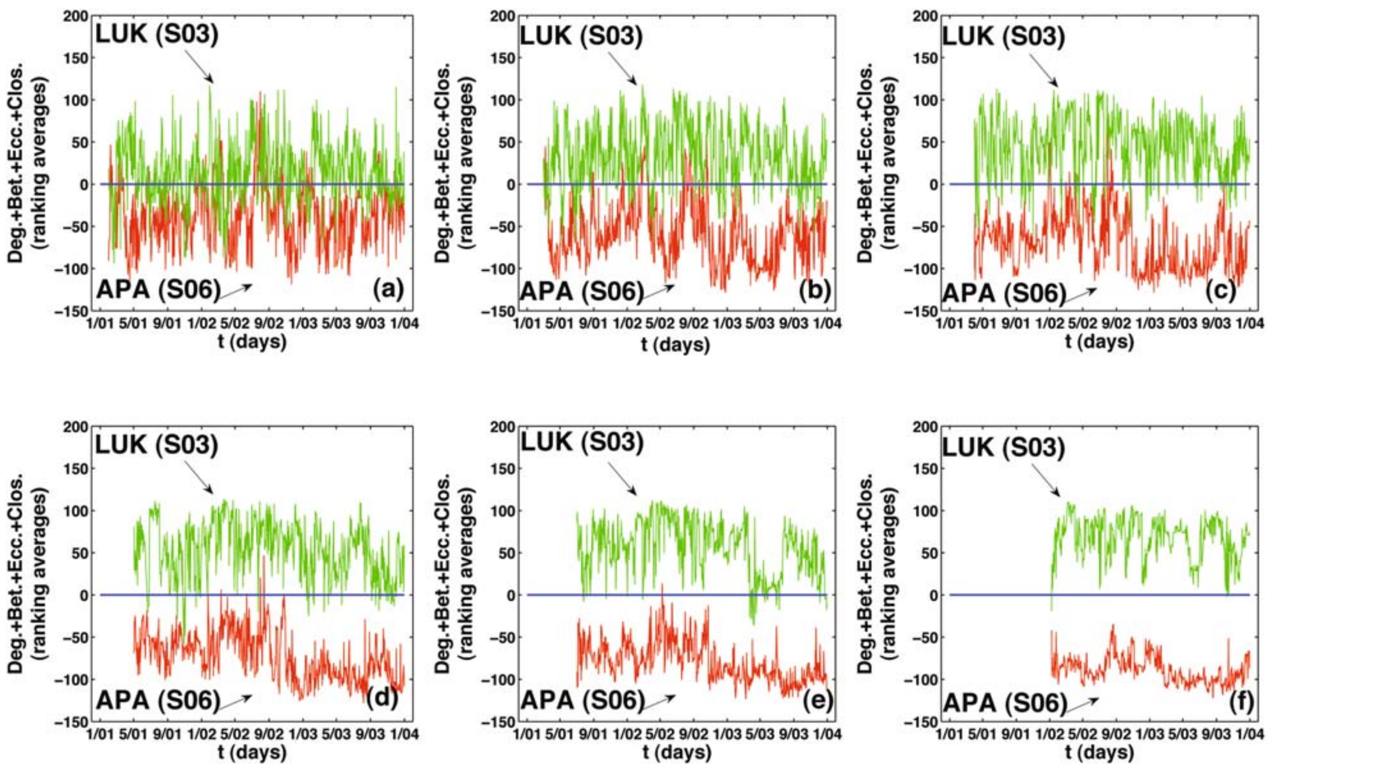

Fig. 9. (Color online) Difference between the average of Degree and Betweenness rankings and the average of (–)Eccentricity and (–)Closeness rankings as function of time for APA (S06) and LUK(S03) for $\Delta t = 1$ month (a), 2 months (b), 3 months (c), 4 months (d), 6 months (e) and 12 months (f).

The results show that this classification is robust over time and the differentiation becomes stronger when the window size increases. The dynamical changes in such hierarchical structuring associated with market turbulences, will be the topic of future studies.

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