Model Risk in Financial Modelling

PhD in Finance

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List of Notation

Y	dependent variable in linear regression
X	independent variables in linear regression
z_t	the vector of predictors at t-1
β	coefficient of independent variable in linear regression
ε	residual error in linear regression / model error
θ_i	a single parameter of a model
Θ	parameter vector
M_i	a single model
$\boldsymbol{\mathcal{M}}$	a set of candidate model
D_t	observed data set at time t
S_t	security price at time t
V_t	model option price at time t
C_t	observed market price of a European option at time t
P_t	realised option trading price at time t
$\hat{E}[]$	Expectation operator
$f(.)$	option payoff function
μ	expected returns of securities/portfolio
σ	volatility of security returns/hedge fund returns
δ	dividend yield of securities
λ	intensity of jump events per unit time interval
I_t	a Poisson process with intensity λ
W_t	a standard Wiener Process
μ_t^J	jump size of security price at time t
a	mean of log jump size $ln(\mu_t^J)$

ζ^2	variance of log jump size $ln(\mu_t^J)$
$N(.)$	normal distribution probability density function
Φ	normal distribution cumulative density function
R_t	returns of security/hedge fund at time t
K	strike price of European option
r	risk free rate
S_i	security price when i jumps occur
σ_i	security volatility when i jumps occur
g_i	mean of samples from the posterior distribution of parameter θ
$T(.)$	general notation of a test statistics
$p(.)$	probability density function
Dev	posterior mean deviance, $Dev = -2\,log\,p(D_{t-1} \mid \theta)$
$Loss(.)$	loss function
VaR	value-at-risk
Var	variance
F_t	total value of a firm at t
L	notional value of debt at maturity
η_t	residual of predictive coefficients at time t
H_t	variance of ε_t
Q_t	variance of η_t
Σ_t	variance of parameter θ_t
υ	forgetting factor in the prediction of $\Sigma_{t\|t-1}$
κ	decay factor of the EWMA estimator
$\pi(.)$	probability mass; $\pi_{t,k}$ of model k being the correct model at time t
k	model indicator
α	forgetting factor in the prediction equation of $\pi_{t\|t-1,k}$

w_t	proportion of wealth invested at time t
γ	relative risk aversion coefficient of investor

Chapter 1

Introduction

Since the development of modern financial markets, various mathematical models have played an important role in all financial activities, including asset pricing, investment appraisal, portfolio selection and risk management. However, models are only imperfect solutions to real world puzzles (Diamond, 1967). Although developments of computational techniques enable implementation of more sophisticated mathematical models, model risk remains a key issue that should not be neglected and should be treated carefully. Risk that stems from financial modelling can have a substantial impact on financial quantities such as option prices, hedging ratios, expected returns of assets, asset volatilities and default probabilities.

Model risk has been linked to a long series of significant events in the financial markets, see Jacque (2015). In 1987, Merrill Lynch reported losses of $300 million on stripped mortgage-backed securities caused by an incorrect pricing model. In 1992, J.P. Morgan lost about $200 million in the mortgage-backed securities market due to the inadequate modelling of prepayments. Later in 1997, the New York subsidiary of the Bank of Tokyo/Mitsubishi lost $83 million because their internal pricing model overvalued OTM Bermudan swaptions by using a one-factor BDT model calibrated to ATM Bermudan swaptions (Dowd, 2003). A Deutsche Bank subsidiary in Japan used some smart models to trade electronically. The model lost control in June 2010 and went into an infinite loop, taking out a $183 billion stock position (Tunaru, 2015). Most recently in 2013, J.P. Morgan revealed a trading loss of more than $6.2 billion, which was indirectly caused by the underestimation of risk level by their value-at-risk (VaR) model. If the company had used the model they implemented recently, the company's risk estimation in 2012 would have doubled the original reported figure. These are only some of the many significant losses incurred by banks and financial institutions due to model risk.

Model risk has also been acknowledged by regulators. The Basel Committee has paid particular attention to internal modelling methods of exposure at default (EAD) and counterparty credit risk exposure models. Detailed guidelines and model validation requirements are documented in a separate section in the Basel II agreement, which has become more comprehensive in the Basel III version. Financial institutions that are using internal modelling methods need to backtest and validate their models on an ongoing basis, so that *"the models are, and continue to be, appropriate ... model assumptions are not violated and known limitations are, and remain, appropriate"* (Basel Committee on Banking Supervision, 2010, p.1). Model risk has indeed become one of the principle risks which financial institutions need to take into account in their risk assessment. Regulators require banks to establish internal independent model validation processes to improve the integrity in using all types of financial mathematical models. Model validation is an important component of the Pillar 1 Minimum Capital Requirements and Pillar 2 Supervisory Review Process; see Basel Committee on Banking Supervision (2006) and Basel Committee on Banking Supervision (2011). Development and improvement in more comprehensive approaches in managing model risk is clearly in demand from both a market and a regulatory perspective.

Therefore, being aware of the existence of model risk; knowing how to identify, measure, and account for it, constitutes a central tenet of risk management development. However, conventional stress testing and sensitivity analysis remain the mainstream practice of model risk management in financial institutions. In particular, sensitivity analysis consists of tests against sensitivity of model ouput to data, model parameter value and model design are commonly in use. Design and subjects of these tests are dependent on each model validator's knowledge of the model and related financial area. In other words, there is no industry consensus or standardised approach in managing and assessing model risk. Although prudence and reliability of current model validation practices are clearly in need of improvement, academic research in the field is yet to provide a comprehensive and thorough answer to the issue of model risk (Henaff and Martini, 2011; Boucher et al., 2014). More in depth research and investigation is expected by the market sector, regulators and academia. This gap in the research provides the motivation to conduct the present study.

1.1 Definition of Model Risk

Model risk or model uncertainty has frequently been related back to the Knightian uncertainty in literature. The Knightian uncertainty, named after Frank Knight, refers to uncertainty that is not measurable. Knight (1921) defined risk as a type of uncertainty with known probability distribution, and defined uncertainty as a type of uncertainty with unknown probability and outcome states. Financial activities involve human intervention and other random events, and hence induce unpredictable outcomes and possibilities. Such elements in a way determine the immeasurable nature of model uncertainty in Finance as stringent statistic science can never fully capture the unpredictable randomness. Therefore, financial literature frequently relates model uncertainty to the Knightian uncertainty for it contains unknown and immeasurable information (Cont, 2006; Kogan and Wang, 2002; Ellsberg, 1961). Nevertheless, researchers have begun to gauge the measurable part of model uncertainty in the finance subject field for it brings significant impacts to financial activities. Ellsberg (1961) raises the idea of (Knightian) uncertainty aversion, and proves that model risk does play an important role in decision making. From late 1980s, model uncertainty started to attract more attention in the financial field (See for example: Miller 1977, Draper et al. 1987, Draper, 1995). Draper et al. (1987) and Hodges (1987) explicitly define three main sources of model uncertainty: uncertainty in prediction (unexplained stochastic fluctuation), uncertainty in parameter estimation and uncertainty in model structure. Tunaru (2015) further expands the scope of model risk to include five different categories: Parameter estimation risk; Model selection risk; Model identification risk; Computational implementation risk; Model protocol risk. In this research, we will focus on investigating the first two sources of model risk in Tunaru's list. The phrases "model risk" and "model uncertainty" will be used interchangeably.

1.1.1 Parameter Estimation Risk

Parameter estimation risk refers to the uncertainty of the true parameter value given a model structure. In general practice, only estimation mean values of parameters are inserted into a model with standard estimation errors being ignored completely. For example, in a very simple linear regression:

$$Y = \beta X + \varepsilon \tag{1.1}$$

There are two sources of parameter estimation risk when using the estimated result of β; firstly, risk of using inappropriate estimation methods (e.g. Ordinary Least Square (OLS), Maximum Likelihood (MLE), Generalised Method of Moments (GMM)); secondly, risk of using point estimation result of β without consideration of estimation errors given the chosen estimation method. We will focus on the second type of parameter estimation risk.

1.1.2 Model Selection Risk

Model selection risk refers to the uncertainty of the correct model structure given a set of candidate models. Model structure consists of three elements (Chatfield, 1995; Draper et. al. 1987):

- Selection of explanatory variable: what explanatory variables should be included in the model as predictors of the variable of interest;
- Assumptions on how these explanatory variables would behave in the prediction period: marginal distribution, joint distribution, and any stochastic specification of the variables;
- Assumptions on the algebraic relationship between explanatory variables and the variable of interest.

Given a data set, a set of candidate models may all seem reasonable, even when different candidate models provide completely different arguments toward the underlying subject (Raftery, 1995). Therefore, relying on one particular model without considering the risk and difficulty in model selection may result in misleading research conclusions.

An increasing amount of literature about model risk has been published since the late 1990s in Finance and related disciplines.

1.2 Model Risk in Financial literature

Model risk is not trivial either in magnitude or in its impact to financial activities. In the following literature review, we show how model risk was found to be significant and important in various financial modelling areas.

In security pricing and forecasting, Draper (1995) first documents an example of future oil price forecast. His result shows that forecasting without taking into account model uncertainty delivers a result with narrow interval ($27, $51) that does not contain the real future outcome ($13). Conversely, it was found that forecasting with consideration of model uncertainty yields an even wider result interval that contains the real outcome. Draper argues that if prediction was carried out with consideration of model uncertainty, corresponding parties would have better awareness of potential outcome, and would be able to carry out better hedging strategy. In the subsequent development of related literature, Avramov (2002) and Schrimf (2010) investigate model uncertainty in stock excess return prediction model, and identify variables which have their prediction power turned insignificant after consideration of model uncertainty. Avramov (2002, p.424) concludes that *'ignoring model uncertainty could lead to erroneous inferences about the relevance of predictive variables'*. Kogan and Wang (2002) investigate model uncertainty in asset pricing, and find that model uncertainty in financial markets can be distinguished from market risk and bring significant impact to asset pricing. Chung et al. (2013) account for parameter estimation risk in equity pricing models by calculating the Bayesian posterior standard deviation of parameters, concluding that parameter uncertainty is sufficient to explain the price discrepancy between Chinese A- and H-share prices. Naser and Alaali (2017) show that mitigating model selection risk by a dynamic model averaging approach has significantly improved model forecast performance of crude oil price.

Particularly in option pricing, Avellaneda et al. (1995) investigates methods to deal with parameter uncertainty of stock volatility in option-pricing models, and shows that using stock volatility in extreme boundary values could help market makers to better hedge their position than using stock volatility in a point estimation manner. Cont (2006) and Detering and Packham (2016) propose mathematical frameworks to measure model risk in money units and demonstrate their application in option pricing. Detering and Packham (2013) further develop a loss function for option-pricing model risk to be used in VaR models and applied in calculating capital requirement. Jacquier and Jarrow (2000) and Bunnin et al. (2002) further incorporate both parameter and model structure uncertainty into option pricing using a Bayesian approach. Jarrow (2012) evaluates the preference of hedging instruments with consideration of model uncertainty in option trading.

Regarding investment decision making and portfolio construction, Barberis (2000) and Xia (2001) show that taking consideration of parameter uncertainty in stock return prediction model influences investment decision significantly. Overall, investors who consider parameter uncertainty tend to hold a smaller position in stock investment than investors who neglect parameter uncertainty. Uppal and Wang (2003) conclude that after accounting for uncertainty in joint distribution and marginal distribution of risky stock return, portfolio choices would be different from the standard mean-variance portfolio. Johannes et al. (2014) present strong portfolio benefits when investors adopt return forecast models that accounts for time-varying volatility and estimation risk. They suggest that a lack of economic value when applying return forecast models as documented in prior research is largely due to neglecting time-varying volatility and estimation risk.

In risk management, Butler and Schachter (1997), Christoffersen and Gonçalves (2005), and Embrechts et al. (2013) argue for reporting Value-at-Risk (VaR) with upper and lower bounds to account for parameter uncertainty. Tarashev (2010) accounts for parameter risk in credit risk management by applying Bayesian inference, and shows that ignoring parameter uncertainty would lead to substantial underestimation of risk level. Tarashev and Zhu (2008) and Wu (2010) provide quantitative frameworks to effectively incorporate both parameter and model selection uncertainty in portfolio credit risk modelling. Kerkhof et al. (2010) and Escanciano and Olmo (2010) incorporate model risk in calculating capital requirements through Value-at-Risk (VaR) models to meet regulatory requests in risk management. Rodríguez et al. (2015) present the advantage of applying the Bayesian estimation method to structural credit risk models to capture parameter estimation risk.

There are various methods that have been developed in financial literature to deal with model risk. We can classify them into two main types.

One type is to seek a quantitative solution that quantifies the model risk. Cont (2006) develops a quantitative framework to measure model uncertainty in option pricing and express it in monetary units. He provides two ways to achieve this: 1)calculate the difference between model prices of a set of models and treat the value as a representation of model risk; 2)penalise the model price by its pricing error on the benchmark instruments. Bannör and Scherer (2013) later adopts a similar method to capture parameter risk. Branger et al. (2012), Elices and Giménez (2013) and Detering and Packham (2016) quantify model risk by calculating hedging errors under various models. The quantify model risk is generally used as a reference to impose overlay to model output in order to mitigate the underlying model risk.

Another type of methods are data driven methods, which accounts for most of the developed methods in literature. Regarding the parameter estimation problem, the most fundamental methods include simple t-test, tests for outliers, and structural break methods which involve estimating parameter values separately for each subgroup data (Wooldridge, 2012). Regarding the model structure and selection issue, F-test and coefficient of determination R^2 or adjusted R^2 are the most fundamental methods to use in selecting explanatory variables and models. Further development of goodness of fit measures also include the large family of "Information Criterion" methods (also known as panalised model selection criteria), within which AIC (Akaike information criterion) and BIC (Bayesian information criterion, also known as SIC Schwarz information criterion) are the most widely used and discussed ones. These measures compare different models using statistics of likelihood $p(D_t \mid M_k)$ of a model M_k given a set of observed data D_t with penalty on the increase in the number of parameters. These methods can be used in comparing both nested or non-nested models (Kuha, 2004). BIC involves use of Baysian econometrics in computing the likelihood value of models. Furthermore, out-of-sample tests are also a type of method which compares models by assessing their prediction performance (see for example: Bakshi, Cao and Chen, 1997; Dahlbokum, 2010). When data are limited, the bootstrapping technique is frequently used to carry out such kinds of validation test (Draper, 1995). Worst scenario tests and stress tests are also frequently adopted in the risk management field and treated as a way to mitigate model uncertainty (Cont, 2006).

Bayesian Econometrics, I contend, stands out from various methods and provides a more comprehensive way to solve both parameter and model selection uncertainty. This technique has advantages in delivering the parameter estimation result in a complete posterior distribution form instead of point estimation values. Posterior probability mass of each candidate model can also be computed in order to carry out model comparison or Bayesian model averaging. Section 1.3 further explains the Bayesian estimation framework.

To conclude, model risk is an important subject area that requires more comprehensive study particularly after the lesson learnt in the 2008 financial crisis. Both historical events and literature studies demonstrate its impact to various financial activities. My research will seek solutions to better account for and tackle model risk when using financial models and, in particular, to fill literature gaps in option pricing, risk management and hedge fund investments.

1.3 Baysian Econometrics Framework

The idea of applying Bayesian econometrics in solving parameter estimation risk was raised in the 1960s-1970s, but its implementation and investigation only started in the mid-1990s enabled by the development in relevant computational techniques (Draper, 1995; Hoeting et al., 1999).

Bayes theorem implies that for parameter θ of model M given observed data D_{t-1}:

$$p(\theta \mid D_{t-1}) = \frac{p(D_{t-1} \mid \theta)p(\theta)}{p(D_{t-1})} \tag{1.2}$$

$p(\theta \mid D_{t-1})$ - posterior distribution of θ given data D_{t-1}
$p(D_{t-1} \mid \theta)$ - the likelihood of θ given the observed data
$p(\theta)$ - prior distribution of θ
$p(D_{t-1})$ - distribution of data D_{t-1}

Therefore, the model outcome under the Bayesian estimation framework delivers not only point estimation values, but also the entire posterior distributions of parameters by the inference of observed evidence and prior beliefs about parameters. It shows a shift in philosophy of the traditional estimation methods. Gupta and Reisinger (2014) assert that finding a best-fit solution in model calibration is ill-posed as all estimation errors are simply neglected in this manner. Any best-fit point value is not good enough to underpin the correct model form. Focus should be shifted onto exploring a distribution of solutions, which sufficiently captures all possible values that are suggested by the observed data.

In the case when one estimation value is required for model usage purpose, model users can deal with parameter estimation risk by integrating it out in the model output. For instance, in option pricing, where option price $V(S_t, t)$ of an option with a payoff function $f(S_T, T)$ at time t is:

$$V(S_t, t) = e^{-(r-\delta)(T-t)} E[f(S_T, T)] \tag{1.3}$$

Given θ is the parameter that drives stock price process, the option price $V(S_t, t)$ after integrating out parameter estimation risk is (Bunnin et al., 2002):

$$
\begin{aligned}
V(S_t, t) &= e^{-(r-\delta)(T-t)}\hat{E}[f(S_T, T)] \\
&= e^{-(r-\delta)(T-t)} \int_{-\infty}^{\infty} f(S_T, T) \int_{-\infty}^{\infty} p(S_T, T \mid S_t, t, \theta) \times p(\theta \mid D_{t-1}) d\theta dS_T
\end{aligned} \tag{1.4}
$$

When there is a set of candidate models to choose from, Baysian Econometrics is able to calculate the posterior probability of each model (Lancaster, 2004; Bunnin et al., 2002). For model $M_i \in \boldsymbol{\mathcal{M}}$ given observed data D_{t-1}, the Bayesian theorem implies that:

$$
p(M_i \mid D_{t-1}) = \frac{p(D_{t-1} \mid M_i)p(M_i)}{p(D_{t-1})} \tag{1.5}
$$

$p(M_i \mid D_{t-1})$ - posterior probability of M_i being true
$p(D_{t-1} \mid M_i)$ - the likelihood of M_i given observed data D_{t-1}
$p(M_i)$ - prior probability of M_i

For instance, a model with posterior probability of 30% indicates that, given the observed data, the probability of the model being the correct model is 30%. Model users may select to implement the model with the highest posterior probability mass. Or, alternatively, the posterior probabilities can be used naturally as weights of each model in a model averaging practice. This technique is also known as Bayesian model averaging.

We continue on the example of option pricing to further elaborate Bayesian model averaging. If we have a set of candidate models $\boldsymbol{\mathcal{M}} = \{M_i\}_{i=1,2,\ldots,n}$, and are not sure which is the correct model to use, integrating out parameter estimation and applying Bayesian model averaging is equivalent to (Bunnin et al., 2002):

$$
\begin{aligned}
V(S_t, t, \boldsymbol{\mathcal{M}}) &= e^{-(r-\delta)(T-t)} \hat{E}[f(S_T, T) \mid \boldsymbol{\mathcal{M}}] \\
&= e^{-(r-\delta)(T-t)} \sum_{i=1}^{n} \int_{-\infty}^{\infty} \left\{ f(S_T, T) \int_{-\infty}^{\infty} \left[p(S_T, T \mid S_t, t, \theta_i) \right. \right. \\
&\left. \left. \times\, p(\theta_i \mid D_{t-1}) \right] d\theta_i \right\} dS_T p(M_i \mid D_{t-1})
\end{aligned} \tag{1.6}
$$

Further innovation has been developed to dynamically predict model probability in the subsequent period before new information arrives. This innovation relaxes the assumption that each model will merit the same goodness-of-fit in the future period compared to the past. Both conventional Bayesian model averaging and the extended dynamic model averaging techniques are applied in the topic of hedge funds returns forecasting and portfolio construction in Chapter 4.

In the Bayesian formula (equation (1.2) and (1.5)), a likelihood represents your beliefs about the value of data conditioned on parameter θ or model estimation M_i. A likelihood can be selected based on relevant economic theories or evidenced by observed data. It should be able to represent the economic model within it and enable you to discredit the model when it is inconsistent with the observed evidence (Lancaster, 2004). In general, the selection of likelihood is equivalent to a model selection problem. A falsely imposed restriction on likelihood may distort the conclusion of the model. For example, normality may distort the conclusion of tail event probabilities when data exhibits fat tails. However, this does not imply that a likelihood must be a more general with less/no restrictions, instead it highlights the necessity of exploring variations in the selection of likelihood and assessing the merit of a chosen likelihood given the observed data.

The prior is your beliefs about the nature of parameters in the form of a probability distribution or discrete probability (e.g. in the case of model prior probability). Since the prior together with selected likelihood determines the kernel probability density of the posterior distribution, the selection of a prior could also distort the conclusion of model outcome. For example, if the selected prior imposed zero probability at values with none-zero likelihood, the posterior would show zero probability to those values regardless of the likelihood as evidenced from the data. Overall, non-informative priors are deemed relative objective priors as they impose less restriction on parameters and hence bring minimal impact to the posterior distributions. In contrast, informative priors are deemed subjective: this is equivalent to imposing expert opinion upon the posterior results.

Congdon (2014) summarised that non-informative priors (e.g. uniform distribution between $-\infty$ to $+\infty$) can be applied in the situation when existing knowledge is insufficient or difficult to summarise in the form of an informative prior. However, such a non-informative prior could be improper (i.e. does not integrate to 1 over its range), for example, the uniform distribution between $-\infty$ to $+\infty$. This may result in an identifiability problem to the posterior (Congdon, 2014; Gelfand and Sahu, 1999)[1]. An alternative solution is to adopt vague priors with minimal restrictions but a proper form, for example, a normal distribution with zero mean and large variance. Spiegelhalter et al. (1996) suggest that a prior shall be expected to have minimal impact on the posterior results if the prior standard deviations are sufficiently greater than the corresponding posterior standard deviations. In this thesis, a non-informative prior indicates the type of vague priors as defined here. Although I tried to keep the influence of model hypothesis at a minimum level via imposing non-informative priors to parameters, this is not always a feasible strategy. In the case, where the likelihood is complex (e.g. unbounded likelihood as in the Merton's Jump-Diffusion model), non-informative priors would result in failure of convergence in the Bayesian inference. In these circumstances, informative priors that are consistent with the existing literature are adopted. More details will be provided in Chapter

[1]details of exceptions where proper posterior can be derived from improper priors can be found in Lancaster (2004) Chaper 1.

2 and 3.

In practice, prior beliefs can adopt historical information on the underlying parameters. Imperfect models often lead to time varying parameter values and require periodic re-estimations (Jacquier and Jarrow, 2000). When the re-estimation frequency is high, the amount of new arrival data will be small. In this case, parameter posterior distributions of the last period can serve as the prior distributions to improve estimation accuracy with a smaller amount of data, and to avoid the time-consuming recursive re-estimation approach.

Posterior probability distributions of parameters and posterior probability of each candidate model can be updated with new information, and parameter and model selection risk can be easily integrated out. Due to mathematical complexity, it could be difficult or impossible to find the closed form posterior probability distribution function of parameters. Markov Chain Monte Carlo (MCMC) simulation methods are commonly used to implement the Bayesian algorithm by simulating observation sequence from the posterior distributions, thereby enabling us to assess various statistical features of the posterior distributions. On the other hand, the conventional Kalman Filter can be applied to the estimation of latent dynamics under linear quadratic Gaussian circumstances (Chen et al., 2003; Julier and Uhlmann, 1997) with much less computational burden compared with the MCMC methods. The MCMC methods are applied in Chapter 2 and 3, whereas the Kalman Filter method is applied in Chapter 4.

1.4 Structure of Content

Chapter 2 assesses parameter estimation risk in asset pricing and risk management. Asset pricing, derivatives in particular, is highly dependent on the chosen financial model. Even when the chosen model is realistic, any unknown parameters can only be estimated with limited precision from empirical data. Therefore, parameter uncertainty constitutes a significant part of model risk in asset pricing. However, this fact is commonly neglected in practice, and point estimation value of unknown parameters are usually adopted to underpin the selected model. Focused on option pricing, earlier literature has described the Bayesian techniques but paid little attention to empirical practices or has only employed unrealistically simple models. Some other literature, such as Eraker et al. (2003); Yu et al. (2011); Kaeck and Alexander (2013), has applied the Bayesian estimation framework to more sophisticated models. However, this literature focuses more on comparing performance of the estimated mean value of model output. Application of the advocated methodology in dealing with parameter estimation risk has not been discussed. In Chapter 2, we focus on parameter estimation risk, and carry out extensive empirical study in pricing European options and determining the Greeks parameters in hedging. We have employed the Merton's jump-diffusion model in comparison to the Black-Scholes model. We also advocate a VaR-type parameter estimation risk measure. Finally, we apply the Bayesian method to the Merton's credit risk model in the computation of default probabilities to gauge the impact of parameter uncertainty. This chapter has been published in the International Review of Financial Analysis (please see Tunaru and Zheng (2017)).

In Chapter 3, we trace the volatility and skewness evolution of the S&P 500 index from 01/01/1980 to 30/12/2015. We compare and contrast the market dynamic among three significant financial crises during the study period by assessing the movement of estimated volatility and skewness together with associated parameter estimation risk. In Chapter 2, we observed that while estimation mean of parameters are time-varying, estimation uncertainty also changes throughout time. Therefore, we review the evolution of both quantities throughout time with focus on the different behaviour around market stress periods. Volatility and skewness are estimated using Merton's jump-diffusion model. The analysis contributes to the crisis literature from another angle, in which we show that different behaviour of parameter estimation risk in volatility and skewness can help to identify the different nature of market crisis. Another finding of the study is that significant model selection risk is identified in the pre-2008 Global Financial crisis period when the equity market returns reverted back to a Gaussian distribution indicating a very calm period without extreme events. We further investigate the inter-relationship of such abnormal market return behaviour and the subsequent market crash. We emphasise from these empirical findings the importance of gauging model risk in financial modelling as they may give a further signal to market vulnerability.

In Chapter 4, we focused on investigating the value of incorporating both parameter estimation risk and model selection risk in hedge fund return forecasting and fund of funds construction. Similar to the general asset pricing literature, existing literature does not provide a correct model form (i.e. a fixed set of risk factors) in pricing or forecasting hedge funds returns. Instead, the literature constantly reveals the problem of model selection uncertainty since different 'best' models are identified in each study. In addition, each hedge fund has its unique characteristics; specifically the fund components may change from time to time subject to fund manager's trading tactics. As a result, risk factor loadings in determining hedge fund returns can change significantly throughout time, which introduce extra model risk in modelling hedge funds returns time-series. Nevertheless, studies on the topic of model risk in hedge funds returns modelling are rather scarce. Among the limited amount of relevant literature, Vrontos et al. (2008) and Vrontos (2012) apply Bayesian model averaging, but empirical analysis is either limited to hedge fund indices or forecasting ability without further analysis on portfolio construction. To fill the gap in literature, we investigate the statistical value and economic value of incorporating heteroscedasticity, non-normality, time-varying parameter value, model selection risk and parameter estimation risk in hedge fund return forecasts and portfolio construction. We adopt a dynamic model averaging method introduced by Koop and Korobilis (2012) in economic literature, which is an extended method of the conventional Bayesian averaging technique. The dynamic model averaging techniques dynamically update model weights when new data arrives rather than assuming constant model posterior probability as in conventional Bayesian model averaging method.

Details of methodology and a literature review of each study are presented in the corresponding chapter to facilitate the ease of reading. Chapter 5 summarises key findings and provides further future research directions.

Chapter 2

Parameter Estimation Risk in Option Pricing and Risk Management[1]

2.1 Introduction

Focusing on parameter estimation risk, in this chapter, we adopt a Bayesian approach with applications to option pricing, including hedging ratios, and to default probabilities calculations. We advocate using distributions of any quantity of interest to the investor. The distributions are generated by the Markov Chain Monte Carlo (MCMC) inferential process. Existing literature eloquently explains how to implement the Bayesian estimation framework to option pricing models, yet empirical applications are very limited. We advocate an improved methodology for investigating the impact of parameter estimation risk in option pricing, hedging and risk management activities. Moreover, we propose a VaR-type of method to measure parameter estimation risk in option pricing. Interestingly, our results indicate that model risk

[1]This chapter has been published in the International Review of Financial Analysis (please see Tunaru and Zheng (2017))

may not be symmetric for the buyer and the seller in a derivative contract.

Parameter estimation risk is an important source of model risk (Glasserman and Xu, 2014; Tunaru, 2015). It refers to the uncertainty of estimating the correct parameter values given a model structure. Any estimation method would induce a certain level of parameter estimation risk. However, in practice, it is very rare that standard estimation errors would feature in the final projection of financial quantities (e.g. asset price, economic capital, value-at-risk). There is no justification for neglecting any of these errors. Furthermore, standard estimation error measure has not taken into account the distribution of estimation risk.

While most market makers and researchers agree that liquidly traded option prices of major stocks and indices are determined by market supply and demand, less liquid options such as exotic options do not have available market prices and depend heavily on models to determine their values (Cont, 2006; Jacquier and Jarrow, 2000; Dahlbokum, 2010). Therefore, parameter estimation risk in option pricing is of great interest to many market participants. Jacquier and Jarrow (2000) carried out a study to incorporate parameter estimation risk of the Black-Scholes (BS) model and its non-parametric extensions into option pricing using the Bayesian estimation approach. They found that even upon consideration of parameter estimation risk, these models cannot deliver promising results in forecasting due to rigid model assumptions. They suggest that further study should be extended to use models with parameters capturing missing time varying dynamics (e.g. jump process). Later studies also confirm the capability of Bayesian econometrics in capturing model uncertainty as well as the feasibility of implementing it in financial practices, but most of this literature focuses on describing the methodology; see for example Bunnin et al. (2002), Jacquier and Polson (2010), Johannes and Polson (2010). Other contributions to the literature emphasise the advantage of extracting latent parameters using the Bayesian estimation approach, but paying little attention to its application in dealing with parameter estimation risk in practice; see for example Eraker et al. (2003), Yu et al. (2011), Kaeck and Alexander (2013).

The Bayesian method advances in its ability of delivering the joint posterior distribution of parameters, which contains all possible value of the parameters given the model and the observed data, so that shapes of the distributions as well as credibility intervals can be obtained easily (Laurini and Hotta, 2010). Therefore, under the Bayesian framework, all parameters are stochastic, accounting for the uncertainty in their estimation. We highlight our proposed methodology using two well-known models as vehicles of research, the Black-Scholes (BS) model and the Merton Jump-Diffusion model (MJD). The MJD model was developed by Merton (1976) as a key alternative model to the BS model, capable of generating kurtosis and skewness in line with empirical literature on stock returns; see Bakshi et al. (1997) and Dahlbokum (2010). Nevertheless, this model has been omitted from most of the related literature which provides empirical tests (Jacquier and Jarrow, 2000; Eraker et al., 2003; Gupta and Reisinger, 2014; Kaeck and Alexander, 2013; Yu et al., 2011), except for Frey (2013) which adopts the model in pricing CO_2 options.

In this chapter, we apply the Bayesian methodology and demonstrate how to construct the posterior distributions of various financial quantities in interest. The key contribution of the study is that we show how to measure parameter estimation risk, and how significant it is in empirical practices, including European option pricing (BS vs. MJD model), Greek parameters, and probability of default calculation. Furthermore, we are also the first to show the application of Tunaru's VaR-type parameter estimation risk measure in option pricing. Our results reveal that model risk is asymmetric to buyers and sellers of options.

The rest of the chapter presents as follow: Section 2.2 provides a summary of literature review; Section 2.3 reviews briefly the MJD model; Section 2.4 introduces the Bayesian econometrics and MCMC simulation techniques;

Section 2.6 shows the empirical application results of both the BS and MJD models under the Bayesian estimation approach and a VaR-type measure for parameter estimation risk in option pricing; Section 2.7 demonstrates the application of Bayesian econometrics in the Merton's Credit Risk model; and Section 2.8 provides summary conclusions.

2.2 Literature Review

There has been an array of evidence that the BS model is not consistent with empirical data (Das and Sundaram, 1999; Merton, 1976; Jorion, 1988; Drost, Nijman and Werker, 1998; Backus, Foresi and Wu, 2004; Batten and Ellis, 2005). The model suggests a normal distribution of stock return, whereas empirical evidence, as we know it, generally shows excessive kurtosis and negative skewness.

The MJD model developed by Merton (1976) is a key extension of the BS model. Several studies suggest that the anomalies of market return could be a result of jump events, and large price jumps are observed in market return data; see Das and Sundaram (1999), Drost et al. (1998), Jarrow and Rosenfeld (1984), Kim et al. (1994) and Maekawa et al. (2008). Burger and Kliaris (2013) argue that while the diffusion process captures the volatility generated by trading activities, the jump component captures more significant changes of stock prices generated by new information. The jump component also generates skewness and kurtosis to the stock return distribution as revealed by Das and Sundaram (1999), Gardon (2011) and Bates (1996).

Estimating the parameters of the MJD model is not a straightforward exercise because under this model the stock return distribution is an infinite mixture of normal distributions. Even under the simplified Bernoulli-Jump Diffusion setting proposed by Ball and Torous (1985), in which a maximum of one jump can occur during one unit time interval, the likelihood function is still unbounded and may have many local modes. This leads to the difficulty in estimating the parameter values using the maximum likelihood method (Kostrzewski, 2014). Due to this issue, many empirical studies of the MJD model show unreasonable large number of jumps: for example, 162 per annum (Hanson and Westman, 2002), 179 per annum (Ramezani and Zeng, 1998), 142 per annum (Honore, 1998). Honore (1998) suggests that this issue can be circumvented by treating the jump magnitude as a constant input to the model. However, the option pricing results under such strict constraints can hardly show any improvement compared to the BS model pricing results. Frühwirth-Schnatter (2006) and Kostrzewski (2014) show that MCMC Bayesian econometrics framework can provide a better solution to this calibration problem.

Parameter estimation risk is never a trivial problem in financial modelling. Focused on asset pricing and risk management, Chung et al. (2013) account for parameter estimation risk in equity pricing models by calculating the Bayesian posterior standard deviation of parameters, and they conclude that parameter uncertainty is sufficient to explain the price discrepancy between Chinese A- and H-share prices. Jacquier et al. (1994), Bunnin et al. (2002) and Gupta and Reisinger (2014) also emphasise the importance of parameter estimation risk in option pricing and suggest the Bayesian estimation approach through MCMC computational techniques as a solution. Butler and Schachter (1997), Christoffersen and Gonçalves (2005), and Kerkhof et al. (2010) report value-at-risk (VaR) with upper and lower bounds to account for parameter uncertainty. Tarashev (2010) shows that ignoring parameter uncertainty would lead to substantial underestimation of risk level. Rodríguez et al. (2015) present the advantage of the Bayesian estimation method in capturing parameter estimation risk in structural credit risk models.

The distinct advantage of MCMC methods is the ability of delivering not only point estimation values, but also the entire posterior distributions of parameters by the inference of the observed data and prior beliefs about the parameters (Stanescu et al., 2014). Therefore, the final option price can be seen as the expectation of the model price over the distributions of unknown parameters (Jacquier and Polson, 2010). This argument shows a shift from the philosophy of the traditional estimation approach. Gupta and Reisinger (2014) assert that finding a best-fit solution in model calibration makes the problem ill-posed as all estimation errors are simply neglected in this manner. Any best-fit parameter point value cannot be good enough to underpin the correct model form and inference should be shifted onto exploring a distribution of solutions, which sufficiently captures all possible parameter values given the observed data. Bunnin et al. (2002) were among the first to show how Bayesian posterior distributions of parameters can capture and reflect parameter estimation risk in option pricing. However, they only apply the method on a single European at-the-money call option. For simplicity, they also ignore any dividend impact on the underlying asset and fix the stock return rate at 10% without any justification. Therefore, the paper provides little information about the empirical performance and feasibility of the underlying method in capturing model risk.

Jacquier and Jarrow (2000) calibrate model parameters using the Bayesian approach on call option data of TOYSR US between 4-Dec- and 15-Dec-1989. They found that the standard BS model leads to a narrow model price distribution, whereas static non-parametric extension models tend to produce more dispersed model prices. Extended models show improvements over the BS model in in-sample performance but the BS model is superior in out-of-sample tests. Therefore, the non-parametric extended models do not fully overcome the shortcomings of the BS model. On the other hand, the authors show that parameter uncertainty is well projected on the Bayesian posterior distributions. They concluded that failure of considering such uncertainty would result in underestimate of price variations. Furthermore, the authors suggest that models with additional parameters capturing missing time varying dynamics (e.g. jump process) might improve the pricing performance. We based the research on this suggestion.

Eraker, Johannes and Polson (2003) apply the Bayesian technique in option pricing with a stochastic volatility model, a stochastic volatility with jumps in stock return process model, and a master model with jumps in both the stock return process and the stochastic volatility process. Models are tested on both the S&P 500 and Nasdaq equity indices. Results suggest that the master model with jumps in both the stock return process and the stochastic volatility process provides the best fit to market return data. However, regarding on the impacts on option pricing, they have only evaluated how model prices are influenced under different models. Therefore, we are still unclear about the practical effectiveness of using Bayesian method in capturing model risk in option pricing. Yu et al. (2011) and Kaeck and Alexander (2013) have also used the Bayesian estimation approach through MCMC techniques to extract inference on parameters and latent volatility/jump variables of the Lévy jump models. However, all literature listed above do not take into consideration the application of Bayesian method in dealing with parameter estimation risk.

Gupta and Reisinger (2014) and Johannes and Polson (2010) provide a step-by-step instruction of how to calibrate the BS and MJD models using MCMC techniques. Prior distributions of parameters are carefully derived to ensure appropriate posterior distributions of all parameters. Gupta and Reisinger (2014) also suggest the application of the Bayesian approach in option hedging activities as a potential future research direction. We follow this methodology in our paper and carry out an extensive option pricing exercise to see whether the Bayesian approach can effectively generate posterior distributions of option prices that contain the realised future market values under the two models investigated.

2.3 Merton's Jump-Diffusion Model

The Merton's Jump-Diffusion model (Merton, 1976) is a continuous-time model developed utilising a Poisson arrival distributed stochastic jump component capable to produce skewness and kurtosis similar to observed return data. The model is described by the stochastic differential equation[2]:

$$dS_t = (\mu - \delta - \lambda\varphi)S_t dt + \sigma S_t dW_t + (\mu_t^J - 1)S_t dI_t \tag{2.1}$$

where μ is the expected rate of return; δ is the dividend yield, σ is the volatility of stock return; $\{W_t\}_{t\geq 0}$ is a standard Wiener Process; λ is the intensity of jump events per unit time interval; $\{I_t\}_{t\geq 0}$ is a Poisson process with intensity λ; μ_t^J is the jump size of stock price; and $\varphi = E[\mu_t^J - 1]$.

Under the assumption that $\ln(\mu_t^J)$ is normally distributed $\ln(\mu_t^J) \sim N(a, \zeta^2)$, the probability density of log stock return $R_{t+\Delta} = ln\left(\frac{S_{t+\Delta}}{S_t}\right)$ is the weighted average of normal densities by the probability that i jumps would occur:

$$p(R_{t+\Delta}) = \sum_{i=0}^{\infty} \frac{e^{-\lambda\Delta}[\lambda\Delta]^i}{i!} N(R_{t+\Delta}; (\mu - \delta - \frac{\sigma^2}{2} - \lambda\varphi)\Delta + ia, \sigma^2\Delta + i\zeta^2) \tag{2.2}$$

Under a risk-neutral measure, Merton (1976) proved that the European option pricing formula (for both call and put options) under the MJD model is a weighted average of the BS option prices V_t^{BS} by the probability that i jumps would occur (Matsuda, 2004):

[2] Please refer to Matsuda (2004); McDonald et al. (2006); Merton (1976) for more details of the MJD model.

$$V_t^{Merton} = \sum_{i=0}^{\infty} \frac{e^{-\lambda(T-t)}[\lambda(T-t)]^i}{i!} V_t^{BS}(S_i, \sigma_i, \delta, T-t, r, K)$$

$$S_i \equiv S_t \exp\left\{ ia + \frac{i\zeta^2}{2} - \lambda(e^{a+\frac{\zeta^2}{2}} - 1)(T-t) \right\} \tag{2.3}$$

$$\sigma_i \equiv \sqrt{\sigma^2 + \frac{i\zeta^2}{T-t}}$$

where r is the risk-free return, K is the strike price of the option, and $T-t$ is the time-to-maturity. To obtain values of the model parameters, one can either estimate them using the historical return data with equation (2.2) in an *estimation* exercise or imply them from market options data using equation (2.3) in a *calibration* exercise. The distinction is important since risk managers use the former while traders employ the later, *both* ignoring the error caused by using a specific dataset.

2.4 Integrating Parameter Estimation Risk under Bayesian Econometrics

Recall the Bayesian formula in Chapter 1 Section 1.3 equation (1.2). For a model M with a parameter θ and given the observed data D_{t-1}, Bayes' formula gives

$$p(\theta \mid D_{t-1}) = \frac{p(D_{t-1} \mid \theta)p(\theta)}{p(D_{t-1})} \tag{2.4}$$

where $p(\theta \mid D_{t-1})$ is the posterior distribution of θ given D_{t-1}; $p(D_{t-1} \mid \theta)$ is the conditional likelihood under the model given θ; $p(\theta)$ is the prior marginal distribution of θ and $p(D_{t-1})$ is the marginal distribution of D_{t-1}.

We estimate parameters from the time series of returns under the risk neutral measure using an MCMC approach pioneered in finance by Jacquier et al. (1994); Eraker (2001); Eraker et al. (2003); Jacquier and Jarrow (2000); Jacquier and Polson (2010); Johannes et al. (2009). The MCMC techniques provide very accurate results in the presence of jumps as demonstrated by Eraker et al. (2003), Yu et al. (2011) and Kaeck (2013).

In the MJD model, with the parameter vector $\Theta = \{\lambda, \sigma, a, \zeta, \delta\}$ [3], using the joint posterior distribution of all parameters $p(\Theta \mid D_{t-1})$ is the key way to consider parameter estimation risk in all asset pricing and risk management calculations. From a computational point of view, the option price V_t^{Merton} is just a function of S_t, t and Θ, hence the ergodic results underpinning the MCMC inference provide a direct mechanism to extract the posterior distribution of model price $p(V_t^{Merton} \mid S_t, t, \Theta)$. MCMC simulation methods can efficiently sample from the posterior distribution of $p(V_t^{Merton} \mid S_t, t, \Theta)$ without knowing its mathematical analytical form; hence the mean, standard deviation, quantiles and shape of the distribution can be easily assessed. In each MCMC iteration, a joint draw of parameter values is obtained from $p(\Theta \mid D_{t-1})$. Then for each joint draw of parameters, option price V_t^{Merton} can be calculated following formula (2.3). This yields a sample of draw from $p(V_t^{Merton} \mid S_t, t, \Theta)$.

2.5 MCMC Algorithm

Gibbs sampling is the simplest MCMC algorithm, which is a special case of the Metropolis-Hastings algorithm. It is applicable when the conditional distribution of each variable is known. For a model with parameter vector $\Theta = \theta_1, \theta_2$ and data D_{t-1}. The target distribution we seek to obtain is $p(\Theta \mid D_{t-1}) \propto p(D_{t-1} \mid \Theta)p(\Theta)$. When the complete conditional distributions

[3] Parameter $\lambda, \sigma, a, \zeta$ are estimated from historical return data or calibrated from option prices, whereas parameter δ is estimated separately from historical dividend yield data. More details are presented in section 2.6.2.

of all parameters are in proper form, and it is possible to directly sample from them, the Gibbs sampler can be easily implemented following below steps:

1. Set up initial value $\theta_1^{(0)}$, $\theta_2^{(0)}$for all parameter
2. Draw $\theta_1^{(1)}$ from $p(\theta_1 \mid \theta_2^0, D_{t-1})$
3. Draw θ_2^1 from $p(\theta_2 \mid \theta_1^1, D_{t-1})$
4. Repeat step 2 and 3 for k iterations

Gelman et al. (2014) show that after a sufficient number of iterations, a sample from the joint posterior distribution $p(\Theta \mid D_{t-1})$ can be obtained.

When one or more conditional distributions are not in proper form, and it is not possible to use Gibbs sampler, Metropolis-Hastings algorithm can be adopted to perform the MCMC simulation. This algorithm requires an additional specification of a proposal density $q(\Theta^{k+1} \mid \Theta^k)$ (i.e. the transition kernel). Metropolis-Hastings algorithm follows the procedure below:

1. Set up initial value $\theta_1^{(0)}$, $\theta_2^{(0)}$for all parameter
2. Draw Θ^* from the proposal density $q(\Theta^1 \mid \Theta^0)$
3. let
$$\Theta^{(1)} = \begin{cases} \Theta^* & \text{if Unif(0,1)} \leqslant \alpha(\Theta^0, \Theta^*) \\ \Theta^0 & \text{otherwise} \end{cases}$$
where
$$\alpha(\Theta^0, \Theta^*) = min\{\frac{\pi(\Theta^*)/q(\Theta^* \mid \Theta^0)}{\pi(\Theta^0)/q(\Theta^0 \mid \Theta^*)}, 1\}$$

Gibbs sampler is often the first choice of MCMC algorithm due to its simplicity and the Metropolis-Hastings method is used when it is difficult to sample from some conditional distributions. This would create a hybrid simulation chain in practice. Hybrid simulation induces more complex algorithm and computing skill. The reader is referred to Lancaster (2004), Gelman et al. (2014), Johannes and Polson (2010) and Tunaru (2015) for more details of Bayesian inference and MCMC simulation methods. The open source software OpenBUGS provides a readily written stable programme to run the MCMC simulation even on complex models. Conveniently, the OpenBUGS programme will automatically choose the most feasible updater according to model setting (Lunn et al., 2012).

2.5.1 Convergence

Detecting convergence of Markov chains is crucial to the success of Bayesian inference. It helps to ensure that chains reach the target (stationary) distribution, so that the samples we draw are reliable to use. The literature has made it clear that it is impossible to fully diagnose convergence from the simulated data of chains. However, simulated data still has some information for us to assess convergence, and various convergence tests are proposed by the literature (Johannes and Polson, 2010; Lunn et al., 2012). We introduce two main ways to assess the convergence of Markov chains.

The first method is to detect convergence by eyeballing. Plotting historical traces of several simulation chains is the most straightforward way to assess convergence. Converged Markov chain trace plots as shown in Figure 2.1 has three key features: 1) chains are moving around within a stable value span; 2) chains that started from different initial values overlap with each after a certain amount of iterations; 3) trace plots look like a "fat hairy caterpillar". The period when the two chains do not overlap with each other is called a "burn-in" period. During this period, convergence (target distribution) has not yet been achieved, and simulated output of these iterations shall be discarded to avoid biases.

[Figure 2.1 about here.]

The Gelman-Rubin ratio $GR = B/G$ (Brooks and Gelman, 1998) is another way to detect convergence. The basic idea of this method is as follows:

G denotes within-chain variability and B denotes between-chain variability. When good convergence is achieved, W and B should converge to stability, and GR should converge to 1. Figure 2.2 Shows an example of the Gelman-Rubin ratio diagram, GR is depicted in red, G in blue and B in Green. The diagram shows that convergence is achieved after 2000 iterations.

[Figure 2.2 about here.]

2.5.2 Accuracy and Efficiency

The question remains as to how many iterations after achieving convergence we should run to obtain a sample which fully represents the posterior distributions of parameters. More iterations certainly give higher level of accuracy. However, Monte Carlo (MC) standard errors can be used as a way to assess the level of accuracy. The MC standard error is an estimation of the difference between the simulated sample mean and the true posterior mean. Suppose we have n independent [4] posterior samples drawn from the posterior distribution of parameter θ, each sample has their sample mean g_i. According to the Central Limit Theorem, the MC error can be estimated by $\sqrt{Var(g_i)/n}$. The calculated MC error can then be compared with the estimated posterior standard deviation of θ. In general, an acceptable accuracy level is achieved when the MC error is less than 5% of the estimated posterior standard deviation (Lunn et al., 2012).

Existence of autocorrelation among iterations will decrease the efficiency of simulation. However, autocorrelation in MCMC simulations are inevitable due to the nature of the sampling algorithm of Gibbs sampler or Metropolis-Hasting (Lunn et al., 2012). Johannes and Polson (2010) also pointed out that chains with very low level of autocorrelation may never achieve convergence (see the Wiches hat distribution in Geyer and Thompson, 1995). However, higher level of autocorrelation will increase the number of itera-

[4] see Jones (2004) for the case of dependent samples

tions required to achieve convergence as well as posterior sampling accuracy, and hence reduce the efficiency of simulation. Higher level of autocorrelation results in a "snake" shape trace plot (Figure 2.3(a)). It does not necessarily indicate a failure of convergence. After increasing the number of iteration, the trace plot may return to a "fat hairy caterpillar" shape. Alternatively, a method known as "thinning" can also be used to reduce the level of autocorrelation and increase efficiency. Thinning of 10 means that only one value is retained as simulation output in every ten consecutive iterations, the rest are discarded. Figure 2.3(b) shows the trace plot of the same simulation in Figure 2.3(a) with more iterations and "thin 10".

[Figure 2.3 about here.]

All MCMC simulation results in this thesis are checked against convergence, accuracy and efficiency using the methodology described in Section 2.5.1 and 2.5.2.

2.6 Option Pricing with Parameter Estimation Risk: BS vs MJD

2.6.1 Data

All empirical data used in this chapter is obtained from Bloomberg. Empirical tests are carried out using daily S&P 500 index log-return data from 31/07/2012 to 29/08/2014, and S&P 500 index European call and put option data in August 2014. This is a randomly selected period when the market was in general conditions. The option data set excludes[5] options with $\mid K/S_t - 1 \mid > 0.2$, options with time-to-maturities shorter than 7 days or longer than 500 days, options with market prices less than \$0.5, options

[5] We follow standard empirical option pricing filtering methodology described in Bakshi et al. (1997); Dahlbokum (2010).

with zero trading volume, and options which do not fulfil the non-arbitrage profit conditions. Furthermore, on a trading day, if there are less than 5 options for a maturity date or the available option data set for a maturity date does not cover all types of moneyness (i.e. ITM, ATM and OTM), options of this maturity date will be dropped out from the data set of that particular trading day. This is to ensure that there are enough data points to calibrate the implied parameter values in the calibration exercise of Section 2.6.4. Our final option sample contains 2871 pairs of option contracts. The option data is classified as in-the-money (ITM) for call option and out-of-the-money (OTM) for put option if $K/S_t < 0.99$; at-the-money (ATM) if $0.99 \leqslant K/S_t \leqslant 1.01$; OTM for call option and ITM for put option if $K/S_t > 1.01$.

2.6.2 Parameter Inference

Under the MJD model, the log stock return distribution is an infinite mixture of normal distributions. To circumvent this problem, we follow Ball and Torous (1985) [6] which approximated the MJD model with the Bernoulli-Jump Diffusion model, using the assumption that only one jump is allowed to happen per unit time interval. When the time interval Δ is short (e.g.daily), the jump intensity $\lambda\Delta$ will be very small, and the Bernoulli-Jump Diffusion model will be a good proxy for the Merton's model. The parameters are estimated using daily historical returns using the likelihood:

[6] Another widely used proxy is the M-Jump Diffusion model described in (Kostrzewski, 2014; Burger and Kliaris, 2013). This model refers to a specification of a cutting point M, so that the model takes into account a maximum of M jumps. Researchers need to ensure that the probability of observing more than M jumps is extremely small, hence neglecting it would not bring any significant impact to the results.

$$
\begin{aligned}
p(R_{t+\Delta}) =& Pr(i=0)N(R_{t+\Delta}; (\mu - \delta - \frac{\sigma^2}{2} - \lambda k)\Delta, \sigma^2\Delta) \\
&+ Pr(i=1)N(R_{t+\Delta}; (\mu - \delta - \frac{\sigma^2}{2} - \lambda k)\Delta + a, \sigma^2\Delta + \zeta^2)
\end{aligned} \tag{2.5}
$$

$$
Pr(i=0) = e^{-\lambda}, \quad Pr(i=1) \approx 1 - Pr(i=0)
$$

The equity index return is impacted by dividend payments. Among all studies on index option pricing, many of them ignore dividend impacts; see Eraker et al. (2003), Johannes and Polson (2010), Jacquier and Jarrow (2000), Maekawa et al. (2008) and Bauwens and Lubrano (2002). However, this may bias the results. Ferreira and Gama (2005) and Bakshi et al. (1997) stress the importance of considering dividend impact in calculating option prices. In our study, a posterior predictive dividend yield value $\widehat{\delta}$ is estimated separately from historical dividend yield data and used in option pricing as an estimate of δ in equation (2.3). If the historical dividend yield data follows a gamma distribution:

$$
\delta \sim Gamma(c, \vartheta)
$$

The posterior predictive distribution of dividend yield can be computed by:

$$
\begin{aligned}
p(\widehat{\delta} \mid \delta) &= \int p(\widehat{\delta}, c, \vartheta) dc d\vartheta \\
&= \int p(\widehat{\delta} \mid \delta, c, \vartheta) p(c, \vartheta \mid \delta) dc d\vartheta
\end{aligned}
$$

Statistical results of the posterior predictive dividend yield are tabulated in Table 2.1:

[Table 2.1 about here.]

For the MJD model, prior distributions of parameters should be selected carefully. Uninformative prior distributions are not suggested as they could make convergence difficult to achieve (Eraker et al., 2003; Johannes and Polson, 2010). The selection of prior distributions is also critical to control for the size of jump intensity (Kostrzewski, 2014). Similarly, Jacquier and Polson (2010) emphasise that uninformative priors could result in an unbounded likelihood for the jump-diffusion model and useful priors should be given to "impose constraints on the parameter space". Following Jacquier and Polson (2010) and Eraker et al. (2003), informative priors $Beta(2,\ 40)$ and $Gamma(10,\ 0.01)$ are assigned to the daily jump intensity λ and jump size precision $1/\zeta^2$. Uninformative normal distribution N (0,100) is assigned to the stock return drift μ' ($\mu' = \mu - \delta$) and the jump size drift a, whereas the prior of stock precision $1/\sigma^2$ takes the form of $Gamma(0.0001,\ 0.0001)$.

Parameters of both the BS and MJD models are estimated from the S&P 500 index daily log-return data. After checking for the convergence of the MCMC chains, inferential results are presented in Table 2.2.

[Table 2.2 about here.]

The results indicate that, under the BS model, the annual return drift $\mu' = \mu - \delta$ can range from 1.79% to 32.91% at the 95% credibility level with a mean value of 17.29%, and the annual stock return volatility reports at a mean value of 11.13% with a 95% credibility interval of $[10.47\%, 11.85\%]$. With the impact of jumps, the annual return drift μ' of the MJD model is estimated at a mean value of 16.95%. The 95% credibility interval of μ' is enlarged to $[-1.32\%, 34.85\%]$. The jump magnitude a is estimated at an average level of -0.84% with a 95% credibility interval of $[-2.82\%, 0.75\%]$, meaning that both positive and negative jumps are feasible. Results of jump intensity λ show an average of 8.38 jumps per annum, but the amount may vary between 2.48 jumps to 16.55 jumps. With the introduction of jumps, a part of the stock return volatility is captured by the jump process, thus the estimated stock volatility σ of the MJD model is 10.05%, which is smaller than the σ under the BS model 11.13%. The volatility of jump size ζ has an estimated posterior mean of 2.72%, and a 95% credibility interval of $[2.05\%, 3.59\%]$.

QQ-plot

Goodness-of-fit tests are often ignored in options pricing literature. Here we carried out a goodness-of-fit analysis for both the BS and MJD models, as this is an important step before proceeding on drawing conclusions on empirical exercises. Standardised Pearson residuals are computed for each model using the posterior mean of parameters. The QQ-plots of both models are displayed in Figure 2.4, and they indicate that the MJD model provides better data fitting performance than the BS model. The MJD model exhibits a smaller deviation from the standard normal quantiles compared with the BS model. Results indicate that the MJD model better captures the leptokurtic of the data.

[Figure 2.4 about here.]

DIC comparison of the two models

The Deviance Information Criterion (DIC), introduced by Spiegelhalter et al. (2002), provides a robust yardstick for Bayesian model comparison:

$$DIC = \overline{Dev} + pDev \tag{2.6}$$

where $\overline{Dev}$ is the posterior mean deviance (a measure of fit), and $Dev = -2\,log\,p(D \mid \theta)$; and $pDev$ is the the effective number of parameters (a measure of model complexity). The smaller the DIC value, the better the model.[7] DIC results of the two models are tabulated in Table 2.3. Model comparison and selection are based on the difference of DICs between the two models, not the absolute values of DICs. There is no universal standard on what should be treated as an important difference in DIC values (Spiegelhalter et al., 2002). In this study, we follow Lunn et al. (2012): a DIC difference of 10 can definitely rule out the higher DIC model; a DIC difference of 5 is still

[7] In the case when the likelihood $p(D \mid \theta)$ is greater than 1, DIC could be legitimately negative, and this does not affect its validity in model comparison (Spiegelhalter, 2006).

substantial; a DIC difference of less than 5 is trivial, and neither model shall be ruled out.

[Table 2.3 about here.]

The DIC results again suggest a better in-sample fitting of the MJD model compared with the BS model. The DIC^{MJD} is less than the DIC^{BS} by 17, indicating superiority of the MJD model over the BS model.

Bayesian p-value

The Bayesian p-value measures the discrepancy between the model replicative simulation data and the observed data via selected test statistics (Gelman et al., 2014). The chosen test statistics can be some summary statistics of the observed data. Denoting generically a test statistic by $T(D)$, it is clear that $T(D)$ can be easily calculated given the observed data set. On the other hand, when a series of replicated data D^{rep} is simulated using the posterior predictive distribution, the test statistics $T(D^{rep})$ can also be calculated consequently. If the model fits the observed data well, the distribution of $T(D^{rep})$ will be concentrated around $T(D)$. This is assessed by the Bayesian p-value: $Pr(T(D^{rep}) \geqslant T(D) \mid D)$. A p-value of 0.5 means the probability of obtaining $T(D^{rep}) \geqslant T(D)$ is 50%, so that the model is likely to generate the observed data series. In contrast, obtaining an extreme p-value of $\leqslant 0.01$ or $\geqslant 0.99$ indicates a likely misfit between the model and the observed data (Gelman et al., 2014).

Four test statistics, including sample mean, variance, skewness and kurtosis (excess kurtosis), are chosen to test the ability of the two models in reproducing the observed sample data series. Parameter posterior distributions of each model are used to simulate the replicated data. Mean(D^{rep}), Variance(D^{rep}), Skewness(D^{rep}) and Kurtosis(D^{rep}) are computed consequently. The Bayesian p-values of test statistics are reported in Table 2.4.

[Table 2.4 about here.]

While the BS model tends to successfully replicate the distribution location and dispersion of the observed log-return data, its p-values of Skewness and Kurtosis reject the assumption of normal distributed stock return explicitly. Consistent with past empirical findings (Das and Sundaram, 1999; Merton, 1976; Jorion, 1988; Drost et al., 1998; Backus et al., 2004), the observed S&P 500 log-return data shows a negative skewness of -0.3824 and a excessive kurtosis of 1.4176. The p-values of $Skewness(D_{BS}^{rep})$ and $Kurtosis(D_{BS}^{rep})$ are strictly equal to 1 and 0, indicating that the skewness of the replicated data of the BS model is always higher than the sample skewness and the kurtosis of the replicated data is always smaller than the observed kurtosis.

According to the p-value results, the MJD model replicates the location of the observed log-return distribution successfully. It also seems to correctly reproduce the negative skewness of the observed data. Nevertheless, its ability in reproducing the observed variance and kurtosis is not as competent as expected. With p-values of 0.9267 and 0.9330 for variance and kurtosis respectively, the MJD model seems to produce too much variance and kurtosis in general. Although these results do not exceed the critical boundary of 0.99, they suggest that, over 90% of time, the model is generating higher variance and kurtosis than the empirical data.

The Bayesian p-values show that, although the MJD model is superior to the BS model, it also has some new problems. Even though the p-values indicate a certain level of imperfect fitting for both models, any single p-value shall not act as the evidence to reject a model (Gelman et al., 2014). The main purpose of the Bayesian p-values is to check and understand the limitation of a model's ability in replicating the observed data. Practical feasibility of a model shall also be evaluated based on its performance in applications. An out-of-sample option pricing performance analysis of the two models is presented next.

2.6.3 Out-of-sample Option Pricing

Out-of-sample option pricing tests are carried out using a rolling window approach. The estimation window contains 2 years of daily return data, estimated parameter results are used to price options one trading day ahead, and the estimation window rolls one day forward each time to price all option contracts in August 2014. An example of posterior density plots of an OTM S&P 500 European call option on 01/08/2014 with strike price $2000 and time-to-maturity 141 days under both of the models are shown in Figure 2.5. As reflected in the plots, MCMC methods successfully draws samples from the posterior distributions of model prices. In this way, all parameter estimation errors are incorporated.

[Figure 2.5 about here.]

The posterior price distribution range is wider under the MJD model than under the BS model, indicating that the MJD model, having more parameters, leads to a higher level of uncertainty in computing the final option price. However, this does not necessarily mean that the model with less parameter estimation risk is the better one to use. A narrower price range might in fact give a false security. For the data analysed here, the posterior distribution of the MJD model option price captures the realised market option value very close to the centre of the distribution. The BS model price, on the other hand, has the realised future market price positioned at the very end of its right tail area. It seems that the BS model is still far away from predicting the underlying option price successfully even when taking the parameter estimation risk into consideration. However, this is just one instance of option pricing, a more informed view of the overall out-of-sample option pricing performance of the two models is presented below.

In addition to paying attention to traditional performance evaluation statistics, such as pricing errors or absolute pricing errors, we are particularly interested in whether the posterior distributions, which incorporate parameter estimation risk, can cover the realised observations; if not, how far are the realised prices from the model price distributions. The overview of coverage performance is tabulated in Table 2.5, and pricing error performance results are shown in Table 2.6 and 2.7.

[Table 2.5 about here.]

[Table 2.6 about here.]

[Table 2.7 about here.]

Overall, posterior distribution coverage results of the MJD model are better than the results of the BS model for both call and put options as it can be seen in Table 3.2. In the tested sample period, the MJD model price distributions (i.e. 95% credibility intervals) cover an extra of 5.63% call option mid-quotes and an extra of 13.72% put option mid-quotes compared with the BS model results. The mid-quote coverage rates of the MJD model are also higher than the BS model across different moneyness and time-to-maturities, except for ITM call options and ATM put options. More importantly, the MJD model shows great improvements in both call and put OTM option pricing performance compared with the BS model (with improvements of 26% for OTM call options and 22.33% for OTM put options respectively). The bid/ask-quote coverage statistics show the percentages of options with both bid- and ask-quotes lying within the 95% credibility intervals of the posterior model price distributions. The MJD model outperforms the BS model on this measure across all moneyness and time-to-maturities in both call and put options. These evidences reflect that the narrow model price distributions produced by the BS model have more difficulty in capturing the bid-ask spreads of market prices.

Pricing Error (PE), Absolute Pricing Error (APE), and Root-mean-square Pricing Error (RMSPE) are calculated respectively to measure the differences between the posterior mean of model prices and the market prices (mid-quote). On the other hand, Outside Pricing Error (OPE), Absolute Outside Pricing Error (AOPE) and Root-mean-square Outside Pricing Error (RMSOPE) statistics aim to evaluate the distances between the closest 95% credibility interval bound values and the market prices (mid-quote) when the mid-quotes are not covered by the intervals. The BS model outperforms the MJD model with respect to these six statistical performance measures in general. The MJD model seems to produce results that can match the BS model in OTM options and longer term options (i.e. time-to-maturity > 180 days). However, for either model, options with time-to-maturity greater than 180 days remain the most challenging type of options to price.

The BS model price distributions capture market prices of OTM call options more effectively than ITM call options, and the advantage in pricing OTM call options is also reflected in the PE, APE, RMSPE, OPE, AOPE and RMSOPE results, which is consistent with the finding of Jacquier and Jarrow (2000). This pattern is reversed in its put option pricing performance. According to Backus et al. (2004), the anomalies in stock return distribution decrease as time interval increases. Hence, one could expect the BS model to perform better in longer-term options. However, our results of the S&P 500 option pricing do not support this statement. Both of the APE and RMSPE statistics increase with time-to-maturities, and the coverage performance measures also exhibit no advantage for the BS model in pricing longer-term options.

Regarding the MJD model results, Das and Sundaram (1999) argue that the MJD model can effectively reproduce skewness and kurtosis to stock return distribution in short period, but the ability declines as time length increases. They suggest that although the decline of anomalies is consistent with empirical stock return evidence, these anomalies generated by the MJD model disappear much quicker than they should be as suggested by empirical data. As a result, the MJD model is expected to perform worse in longer term option pricing but better in shorter term option pricing. The APE and

RMSPE statistics of the MJD model for both call and put options confirm this conclusion as they increase with time-to-maturities.

Over-pricing is defined as the case when the market price (mid-quote) is not covered by the 95% credibility interval of model price and located to the left of the distribution (positive OPE); Under-pricing is defined as the case when the market price is not covered by the 95% credibility interval of model price and located to the right of the distribution (negative OPE). Focused on the BS model results, consistent with Batten and Ellis (2005), under-pricing seems to have the dominant effect across all types of moneyness and maturities in both call and put option pricing. The only two exceptions are found in OTM call and ITM put options. It can be observed that the mis-pricing tendency moves from under-pricing to over-pricing with increases in the strike price. Moreover, the under-pricing magnitude tends to increase with time-to-maturities (see AOPE and RMSOPE results).

The MJD model, on the other hand, tends to under-price call options, but over-price put options. For both types of options, we find that the magnitude of mis-pricing decreases when option moneyness moves from ITM to OTM. Furthermore, the increase in time-to-maturities also tends to decrease the magnitude of mis-pricing (see AOPE and RMSOPE results). These results contradict with the trends we observed in the BS model, and indicate that when the model price distributions failed to capture the realised market data, the shortest distances between the market prices and the MJD model price distributions are found in OTM and longer-term options.

Yun (2014) argues that when parameter values are implied from market options prices (i.e. calibrated), the option pricing performance is better than when parameter values are estimated from historical stock return data. In the next section, we further investigate the out-of-sample option pricing performance of the BS and MJD models using implied parameter values.

2.6.4 Out-of-sample Option Pricing with Implied Parameter Values

Implied parameter values are calibrated under the Bayesian framework using option market prices (mid-quotes). Options with different strike prices but the same maturity date are used to calibrate posterior parameter distributions $p(\lambda, \sigma, a, \zeta \mid D_{t-1})$, which are then used to price the options with the same maturity date in the next trading day. Posterior distributions of parameters estimated using the historical daily log-return data in Section 2.6.3 are used as prior distributions of parameters in calibrating the implied parameter values.

Jacquier and Jarrow (2000) provide a thorough discussion on the necessity of allowing model error ε_t when calibrating implied parameter values from option price data; see also Jacquier and Polson (2010) and Johannes and Polson (2010). Ideally, observed market prices shall coincide with model prices. Nevertheless, this requires perfect synchronisation when recording the option price C_t and the underlying stock price S_t, which is empirically difficult to achieve. Even small non-synchronisation errors could result in the deviation between market and model option prices. Secondly, market prices could sometimes depart from their equilibrium values due to trading noises and market imperfection. The introduced error term could effectively capture the impact of such market errors. Finally, the deviation between market and model option prices could also stem from model structure uncertainty. Models are only approximations of the underlying asset pricing problem. Ignorance of the model imperfection issue could result in over-fitting, and consequently lead to the deterioration in model out-of-sample pricing performance as indicated by Dumas et al. (1998). Therefore, the model error term ε_t can also act as a vehicle to deal with model structure risk. Following Jacquier and Jarrow (2000), a Gaussian model error term is introduced in a multiplicative manner to ensure the non-negativity of option prices:

$$log(C_t) = log(V_t) + \varepsilon_t \tag{2.7}$$

where C_t is the market price of a European option, V_t is the model option price, and ε_t is a normally distributed error term with distribution $N(0, \sigma_\varepsilon^2)$.

[Table 2.8 about here.]

[Table 2.9 about here.]

[Table 2.10 about here.]

The statistics of the out-of-sample option pricing performance are tabulated in Table 2.8, 2.9 and 2.10. Comparing with the results in Section 2.6.3, the results by calibration show improvements in both Mid-quote coverage and Bid/Ask-quote coverage for both of the models overall. A remarkable change is that, for call options, the advantage of the MJD model is now more significant in pricing ITM options, whereas the advantage in ATM and OTM options remains for put options. Among the three different time-to-maturities, the MJD results improved most significantly over the BS results in pricing longer term options. The highest coverage rate of the longer term options is observed in the MJD put option pricing.

Moreover, the PE, APE, RMSPE, OPE, AOPE and RMSOPE statistics indicate significant improvements for the MJD model, suggesting that the MJD model works better with calibration. Unlike the results shown in Section 2.6.3, the MJD model now has a similar or even better performance indicated by these measures in all option categories. In addition, the mis-pricing tendencies across different option categories of the BS model remain the same as in Section 2.6.3. On the other hand, the mis-pricing tendencies of the MJD model has changed to follow the similar trends as the BS model. Both under- and over-pricing are observed in different option categories, and the tendency moves from under-pricing to over-pricing with increases in strike prices. Furthermore, options with the longest time-to-maturities now have the largest mis-pricing magnitude.

Overall, after incorporating parameter estimation risk, the MJD model outperforms the BS model in all aspects of the out-of-sample option pricing tests when implied parameters are used. In the next section, we will discuss the measure of parameter estimation risk exposure.

2.6.5 Measuring Parameter Estimation Risk

With the ability to obtain the entire posterior distributions of option prices, parameter estimation risk exposure to both long and short positions can be easily investigated. A simple VaR-type of measure to quantify parameter estimation risk is recently introduced by Tunaru (2015).

Regarding the MJD model, the posterior density of an option price V_t^{Merton} is obtained using MCMC techniques. The distributions of profit & loss for both long and short positions, given the trading price P_t, can be computed by simple linear loss functions shown below:

$$
\begin{aligned}
Loss^{long}(V_t^{Merton}) &= V_t^{Merton} - P_t \\
Loss^{short}(V_t^{Merton}) &= P_t - V_t^{Merton}
\end{aligned}
\tag{2.8}
$$

[Figure 2.6 about here.]

Denoting by $PER-VaR_\eta$, the measure of parameter estimation risk tells that the probability of having a loss beyond $PER-VaR_\eta$ due to parameter estimation risk is $\eta\%$. An example of the $PER-VaR_{1\%}$ for both long and short positions of a European call option computed using the posterior MJD model price distribution is given in Figure 2.6. It is important to notice that buyers and sellers do not have symmetric exposure towards parameter estimation risk. As illustrated in Figure 2.6, the magnitude of the $PER-VaR_{1\%}$ is smaller to the call option contract buyers than to the sellers. Furthermore, with the ability to visualise the entire distribution of potential loss caused by parameter estimation risk, market participants can easily assess the entire tail distribution beyond $PER - VaR_\eta$. In our example, a fatter tail is found in the parameter estimation risk exposure of short position holders, indicating that the potential loss which may be encountered by the sellers in extreme cases could be more tremendous. This is not a surprise as long position holders of European call options do not face downside variation in their final payoff.

Finally, it is worth highlighting that, in this one particular example, the $PER - VaR_{1\%}$ accounts for 48.86% and 26.85% of the trading price for option writers and buyers respectively. Therefore, parameter estimation risk can be substantial to all market participants. Neglecting parameter estimation risk when using either model in option pricing may place a blind spot in risk management analysis and result in great losses.

2.6.6 Greeks

Parameter estimation risk can also bring significant impact to option hedging activities. Following a similar approach as formula (2.3), the Greeks under the MJD model can be derived as the weighted average of the BS Greeks by the probability that i jumps would occur before the maturity date (Merton, 1976).

$$Delta_t^{Merton} = \sum_{i=0}^{\infty} \frac{e^{-\lambda(T-t)}[\lambda(T-t)]^i}{i!} Delta^{BS}(S_i, \sigma_i, \delta, T-t, r, K)$$
$$Gamma_t^{Merton} = \sum_{i=0}^{\infty} \frac{e^{-\lambda(T-t)}[\lambda(T-t)]^i}{i!} Gamma^{BS}(S_i, \sigma_i, \delta, T-t, r, K)$$
$$Theta_t^{Merton} = \sum_{i=0}^{\infty} \frac{e^{-\lambda(T-t)}[\lambda(T-t)]^i}{i!} Theta^{BS}(S_i, \sigma_i, \delta, T-t, r, K)$$
$$Rho_t^{Merton} = \sum_{i=0}^{\infty} \frac{e^{-\lambda(T-t)}[\lambda(T-t)]^i}{i!} Rho^{BS}(S_i, \sigma_i, \delta, T-t, r, K)$$
$$Vega_t^{Merton} = \sum_{i=0}^{\infty} \frac{e^{-\lambda(T-t)}[\lambda(T-t)]^i}{i!} Vega^{BS}(S_i, \sigma_i, \delta, T-t, r, K)$$

$$S_i \equiv S_t exp\left\{ ia + \frac{i\zeta^2}{2} - \lambda(e^{a+\frac{\zeta^2}{2}} - 1)(T-t) \right\}, \quad \sigma_i \equiv \sqrt{\sigma^2 + \frac{i\zeta^2}{T-t}} \tag{2.9}$$

Given the estimated $p(\Theta \mid D_{t-1})$, the posterior distributions of Greeks can be easily obtained following the simulation techniques described in Section 2.4. Examples of Greeks' density plots of a S&P 500 European OTM call option are presented in Figure 2.7. Density plots and the ability of assessing the quantile values of Greeks provide very rich information to practitioners in hedging activities. While the posterior mean value of Delta could be adopted, practitioners shall also be aware that they are subject to parameter estimation risk when setting up their hedging strategy. The Bayesian posterior distribution of Delta depicts such risk exposure in detail. In our example a fatter right tail is observed showing a higher probability of realising higher Delta value. In the case when extra caution is needed towards the trading of this option, short position holders may hedge at the 75% or even the 97.5%

upper quantile value of Delta to implement super hedging according to their capital availability. Similar discussions also apply to other Greeks hedging activities.

[Figure 2.7 about here.]

[Figure 2.8 about here.]

For risk management purposes, one can track the evolutions of the posterior means of Greeks as well as the upper and lower quantiles as illustrated in Figure 2.8. The potential risk or hedging errors generated by parameter estimation risk can be easily gauged with our approach. The 95% credibility interval widths are relatively stable for Delta and Rho throughout the test period. However, the Gamma, Theta and Vega plots all exhibit an increased uncertainty at the second estimation point. Moreover, the 95% credibility interval of Gamma is widened towards the end of the test period, while the distribution of Vega gradually become negative skewed. If only mean estimation value is considered under traditional estimation practices, traders might lose sight on such potential uncertainty and underestimate the movement of the Greeks.

2.7 Credit Risk Management with Parameter Estimation Risk

When using a model for risk management, adopting point estimation of parameters could result in under-estimation or over-estimation of risk. Lönnbark (2013) remarks that biases caused by parameter estimation risk could affect backtesting results of the model, and consequently affect regulation compliance. The Bayesian approach provides a direct way to deal with this issue. The probability of default (PD), the main concept in credit risk, can be reported with its full posterior distribution. We will demonstrate how to achieve this for the Merton's Credit Risk model.

In the Merton's Credit Risk model, the firm value F_t follows a Geometric Brownian Motion. If L is the notional of debt at maturity, the probability of default is given by $\Phi(-d_2)$

$$d_2 = \frac{ln(F_t/L) + (r - \sigma_f^2/2)(T-t)}{\sigma_f\sqrt{T-t}} \tag{2.10}$$

where r is the risk free rate and σ_f is the volatility of the firm value. While equity value is a function of the asset value, the equity volatility σ_e can be derived from the asset volatility σ_f using Itô's lemma. Consequently, F_t and σ_f can be calculated given the equity value and volatility; see Hull et al. (2004) and Merton (1974) for more details.

$$\begin{aligned} \sigma_e &= \frac{\sigma_f\Phi(d_1)}{\Phi(d_1) - A_t\Phi(d_2)} \\ A_t &= \frac{Le^{-r(T-t)}}{F_t} \end{aligned} \tag{2.11}$$

In the following example, the PD of Apple Inc. on 21/08/2014 is computed following the Bayesian approach. The equity value of Apple Inc. on the date is reported at \$589.40 billions. The parameters of the Merton's Credit Risk model are estimated using daily log-return data of Apple Inc. from 21/08/2012 to 21/08/2014. Figure 2.9 shows the PD of Apple Inc. for a time horizon of 5 years with L = \$100, \$200, \$300 or \$400 billions respectively. The density plots show that parameter estimation risk is well captured and incorporated into the computation of the PDs. The impact of parameter estimation risk becomes more critical when the company has its debt principal repayment equals to \$300 or \$400 billions. The PD of the company could vary from 0.25% to 1.5% when $L = \$300$ billions, and from 0.5% to 2.3% when $L = \$400$ billions. When practitioners only adopt point estimation values of the PDs (e.g. posterior means), they fail to gauge the uncertainty of PD values. This can result in biases and errors in counterparty credit risk calculations, regulatory capital computations, risk management and investment decision making towards the company.

[Figure 2.9 about here.]

2.8 Conclusion

In this paper, we demonstrate how parameter estimation risk can be incorporated into option pricing and credit risk models via the application of Bayesian econometrics using MCMC techniques. The option pricing performance of the MJD and BS models are also investigated carefully when parameters are estimated or calibrated.

Parameter estimation risk is non-trivial, and adopting point estimations in both asset pricing and risk management can result in a narrow view about the underlying issue with a huge amount of information being neglected. It also prevents practitioners to receive early signal of market variation as reflected in the parameter uncertainty. As a result, actions towards trading, hedging, regulatory requirement and risk management can be delayed, which may trigger the loss one may suffer in a later stage.

Regarding the European option pricing performance of the MJD and BS models, it is found that the MJD model with posterior parameter values implied from market option prices outperforms the BS model in all performance measures, especially for longer term options. Nevertheless, the MJD model does generate more parameter estimation risk due to the increase in the number of parameters. This is reflected in the widened intervals of the MJD model price distributions compared with the BS model.

On the other hand, even when accounting for parameter estimation risk, neither of the models is able to provide full coverage of realised future market values. Such deviation could stem from model misspecification, or in other words, model structure risk. Furthermore, our Bayesian MCMC approach allows us to easily calculate Tunaru's measure of parameter estimation risk for European option pricing. Sellers of options have more exposure to this type of risk than buyers. In addition, we highlight the potential biases when ignoring parameter estimation risk in the calculation of default probabilities under the Merton's Credit Risk model.

Figure 2.1: MCMC Historical Simulation Trace Plot - Example of Good Convergence

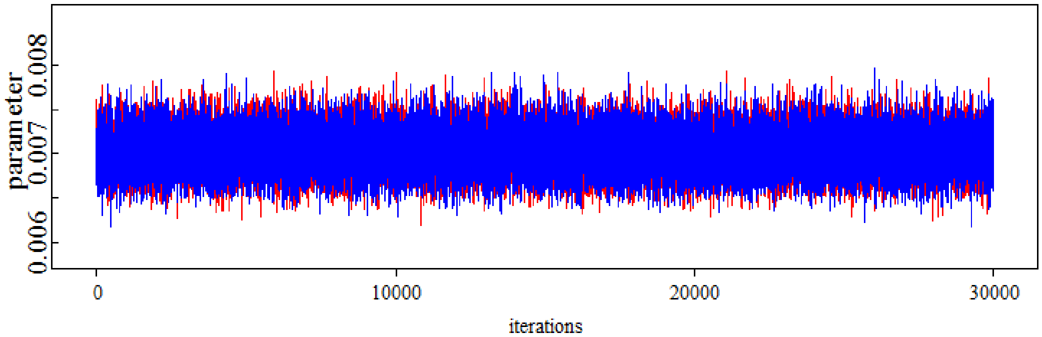


Figure 2.2: Example of Gelman-Rubin ratio diagram

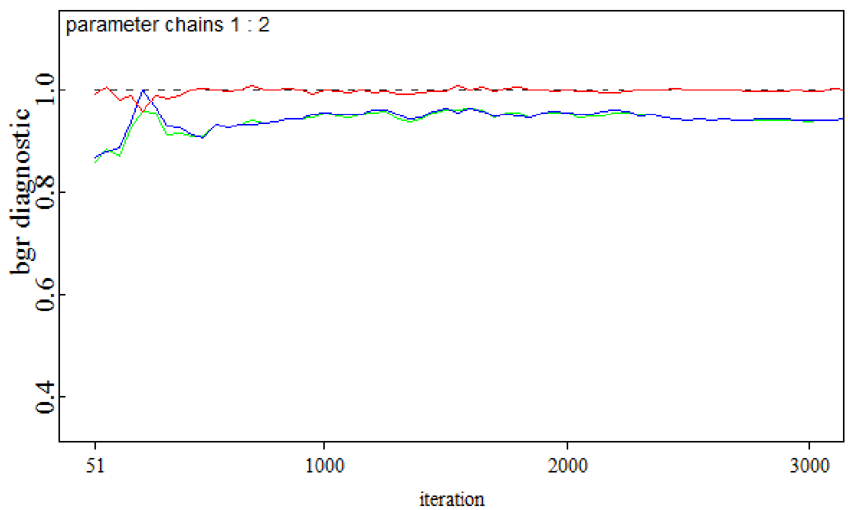
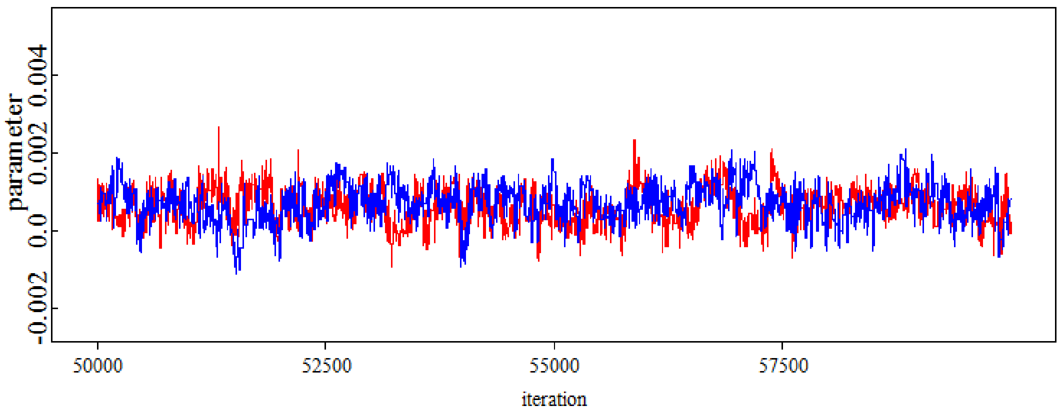


Figure 2.3: MCMC Historical Simulation Trace Plot - Example of high level of autocorrelation

(a) 10000 iterations



(b) 40000 iterations with “thin 10”

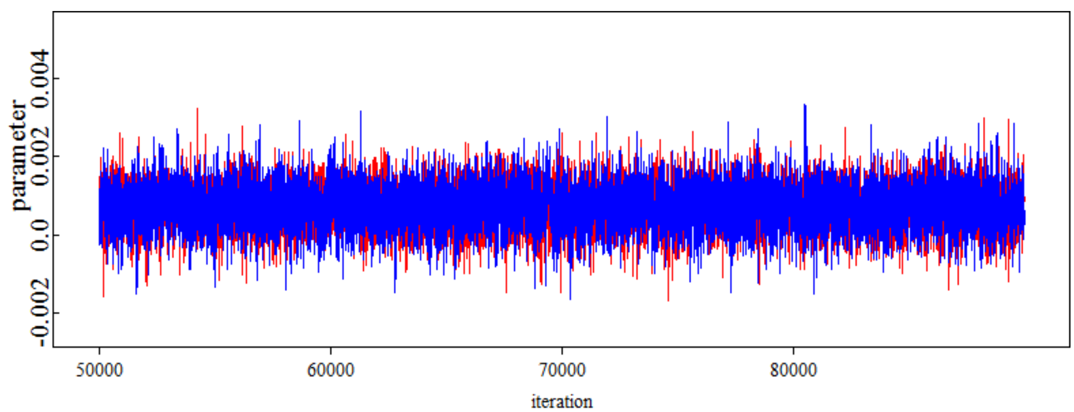
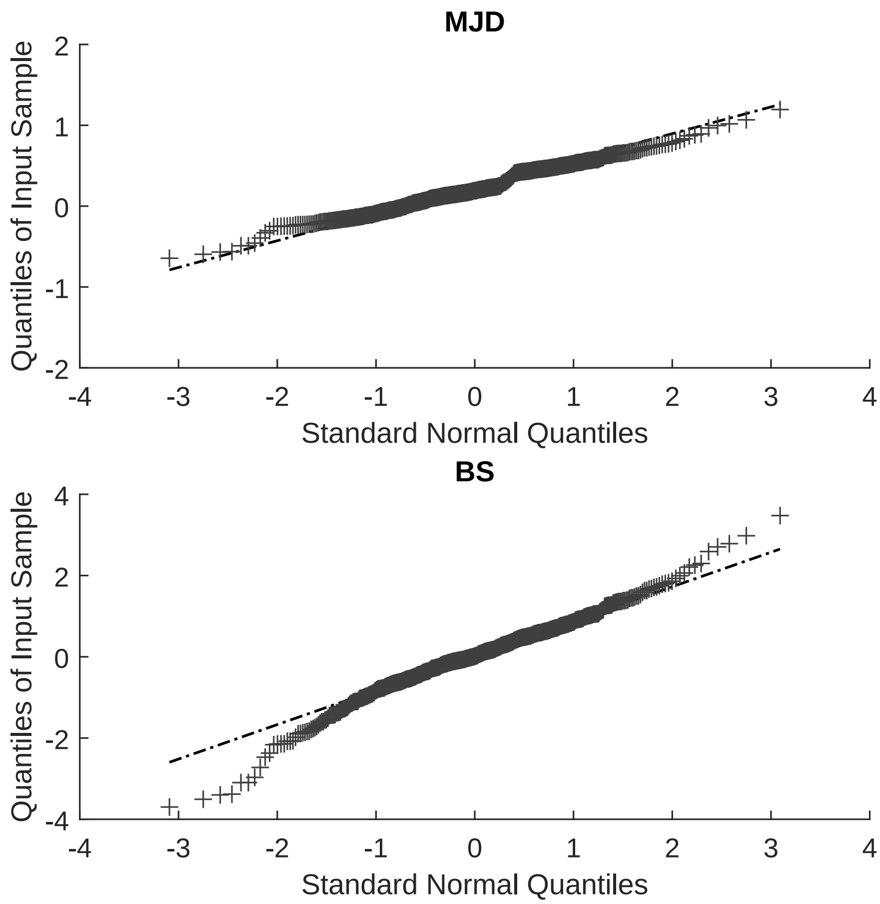
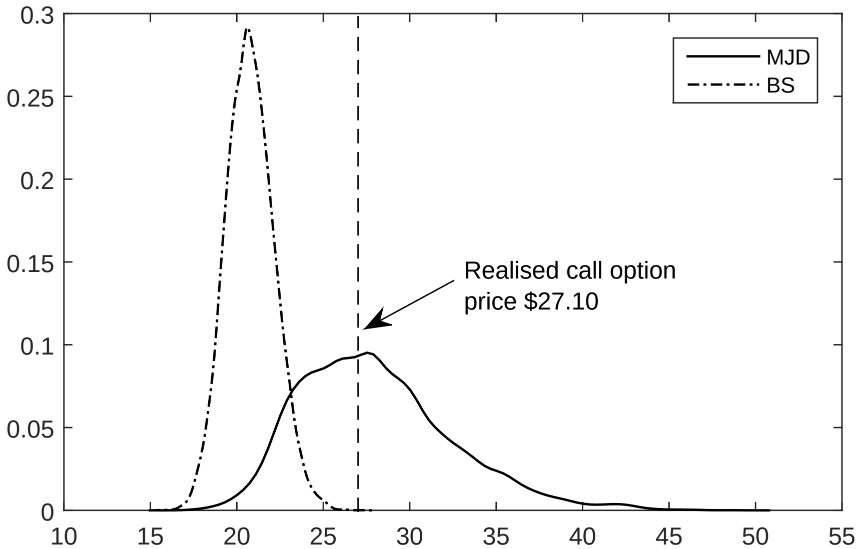


Figure 2.4: QQ-plots of the Standardised Pearson Residuals of the Black-Scholes and Merton's Jump-Diffusion Models



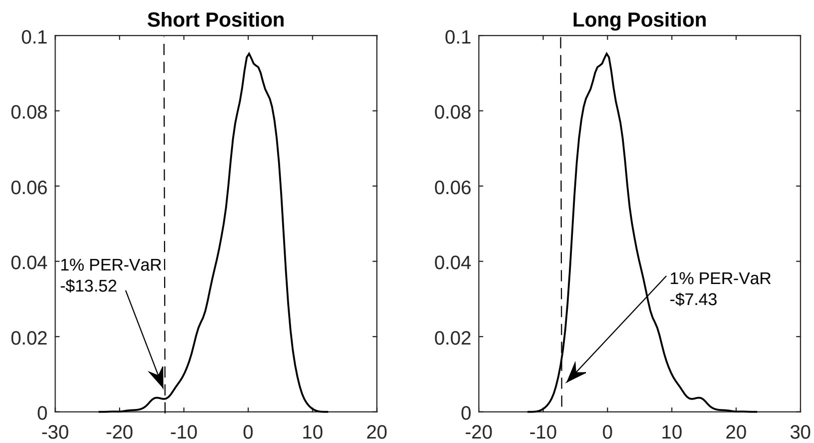
Note: the Standardised Pearson residuals are calculated based on the estimated parameter posterior means of both the BS and MJD models. Parameter estimation data: S&P 500 index daily log-return data 31/07/2012 - 31/07/2014.

Figure 2.5: Posterior Density Plots of a European Call Option under the Black-Scholes and Merton's Jump-Diffusion Models



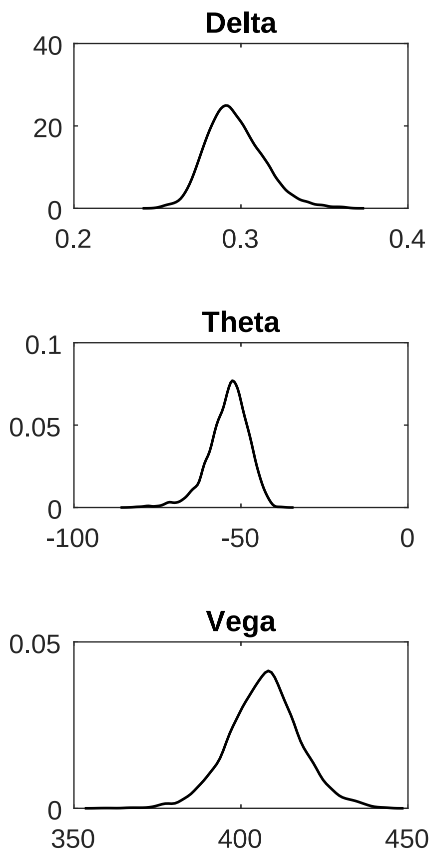
Note: the figure shows posterior density plots of an OTM S&P 500 index European call option contract on 01/08/2014 with strike price $2000 and time-to-maturity 141 days. Parameter estimation data: S&P 500 index daily log-return data 31/07/2012-31/07/2014.

Figure 2.6: An Example of PER-VaR of Parameter Estimation Risk Exposure under the Merton's Jump-Diffusion Option Pricing Model



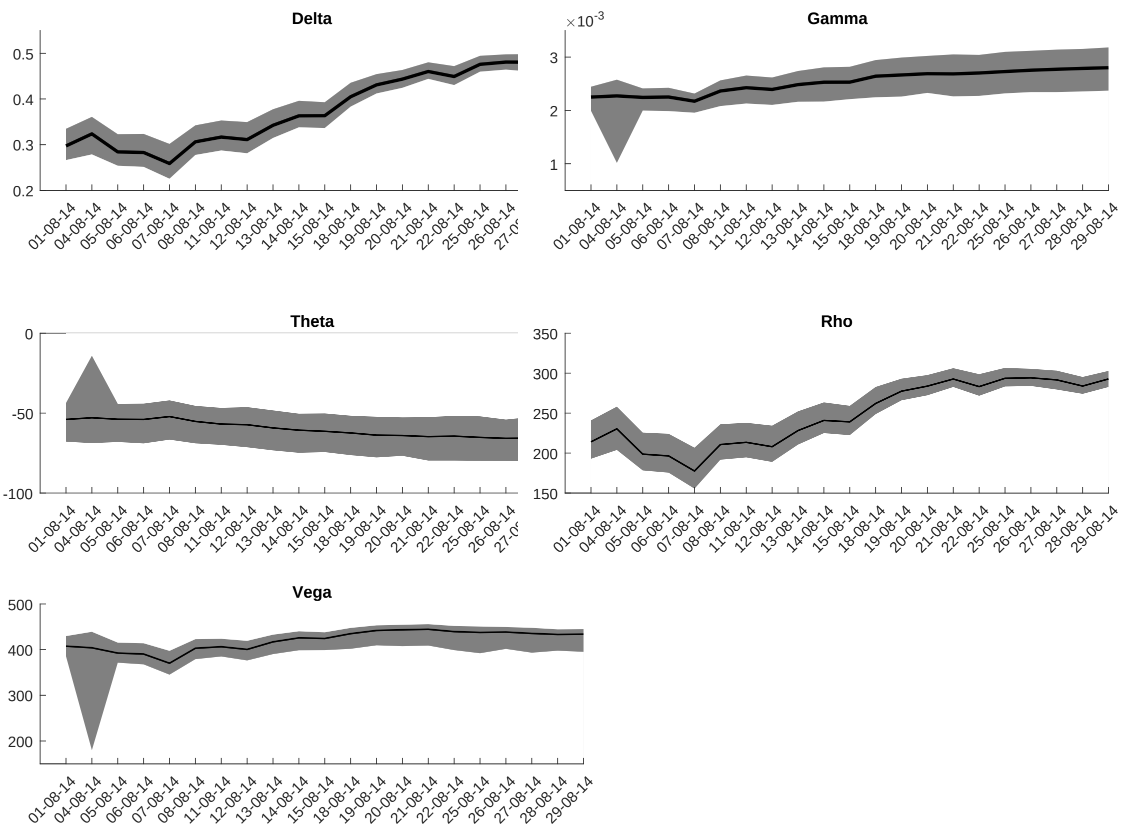
Note: the figure shows 1% PER-VaR values for the parameter estimation risk exposure of both the short and long positions of a European Call option of the S&P 500 index under the MJD option pricing model, assuming that the mean of the posterior option price \$27.67 is set as the trading price. The call option is an OTM option on 01/08/2014 with strike price 2000 and time-to-maturity 141 days. Parameter estimation data: S&P 500 index daily log-return data 31/07/2012 - 31/07/2014.

Figure 2.7: Posterior Distributions of Greeks of a European Call Option under the Merton's Jump-Diffusion Model



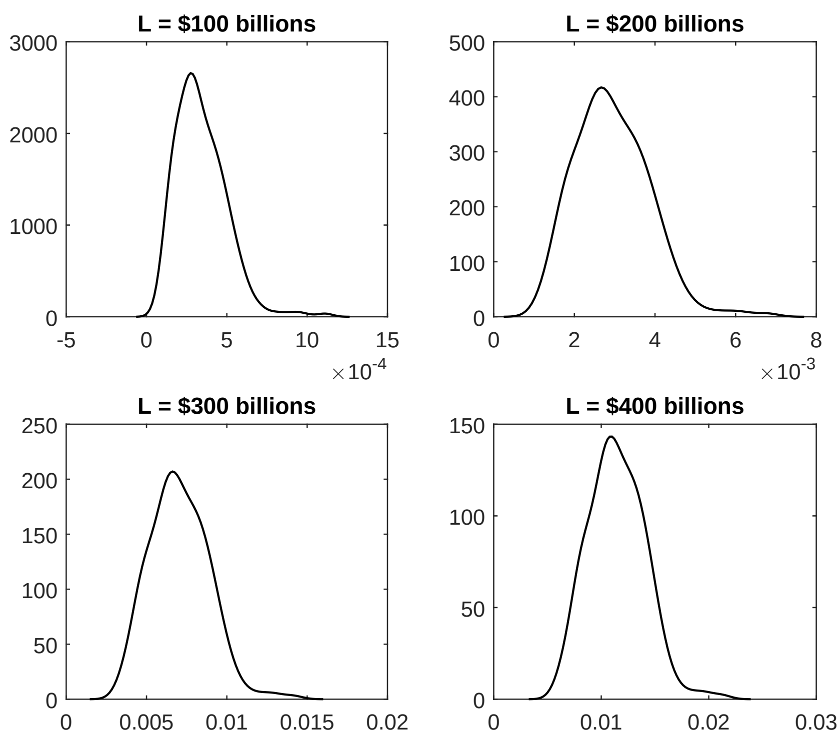
Note: the figure shows the posterior distributions of Delta, Gamma, Theta, Rho and Vega of a European Call option of the S&P 500 index given the estimated posterior distributions of parameters under the MJD model . The call option is an OTM option on 01/08/2014 with strike price 2000 and time-to-maturity 141 days. Parameter estimation data: S&P 500 index daily log-return data 31/07/2012 - 31/07/2014.

Figure 2.8: Movements of a European Call Option Greeks under the Merton's Jump-Diffusion Model during August 2014



Note: the figure shows the movements of posterior means, 95% credibility intervals of Delta, Gamma, Theta, Rho and Vega of a European Call option of the S&P 500 index given the estimated posterior distributions of parameters under the MJD model. The call option with strike price 2000 and maturity date 20/12/2014 is an OTM option on 01/08/2014, but becomes an ATM option on 19/08/2014 due to the increase in security spot price and remains ATM till the end of the test period. Parameter estimation data: S&P 500 index daily log-return data 31/07/2012 - 29/08/2014.

Figure 2.9: Probability of Default Posterior Distributions of Apple Inc.



Note: the figure shows the posterior distributions of default probabilities for Apple Inc. given different notional values of debt L on 21/08/2014. The equity value of Apple Inc. on the date is $589.40 billions. Debt is assumed to have a time-to-maturity of 5 years. Parameter estimation data: Apple Inc. daily log-return data 21/08/2012 - 21/08/2014.

Table 2.1: Statistical Results of the Posterior Predictive Distribution of Dividend Yield

	in %
	2.0490
$\widehat{\delta}$	(0.0954)
	[1.8650 - 2.2410]

Note: statistics shown above are posterior predictive mean of dividend yield with the standard deviation in ( ) and 95% credibility interval in [ ]. Parameter estimation data: S&P 500 index daily annualised dividend yield data 31/07/2012 - 31/07/2014

Table 2.2: Parameter Estimation Results of the Black-Scholes and Merton's Jump-Diffusion Models

Parameters	Models	
	BS	MJD
μ' (in %)	17.2872	16.9495
	(7.9128)	(9.2207)
	[1.7892-32.9112]	[-1.3159-34.8516]
σ (in %)	11.1300	10.0500
	(0.3533)	(0.4406)
	[10.4700-11.8500]	[9.2510-10.9300]
λ	-	8.3790
	-	(3.6263)
	-	[2.4792-16.5514]
a (in %)	-	-0.8350
	-	(0.9035)
	-	[-2.8170-0.7450]
ζ (in %)	-	2.7160
	-	(0.4009)
	-	[2.0450-3.5930]

Note: statistics shown above are posterior mean results of parameters with the standard deviation in ( ) and 95% confidence interval in [ ]. Statistics of μ', σ and λ are annualised results. $\mu' = \mu - \delta$. Parameter estimation data: S&P 500 index daily log-return data 31/07/2012 - 31/07/2014.

Table 2.3: DIC Results for the Black-Scholes and Merton's Jump-Diffusion Models

	DIC
BS	-3563
MJD	-3580
Difference	17

Note: Difference is calculated as $DIC^{BS} - DIC^{MJD}$. DICs are computed based on the models' fitting results of the S&P 500 index daily log-return data 31/07/2012 - 31/07/2014.

Table 2.4: Bayesian P-values of the Black-Scholes and Merton's Jump-Diffusion Models

	Sample	BS		MJD	
Test Statistics	$T(D)$	$T(D_{BS}^{rep})$	p-value	$T(D_{MJD}^{rep})$	p-value
Mean	0.1663	0.1648	0.4585	0.1628	0.4618
	-	(0.1119)	-	(0.1320)	-
Variance	0.0123	0.0124	0.6019	0.0170	0.9267
	-	(0.0011)	-	(0.0043)	-
Skewness	-0.3824	0.0005	1.0000	-0.8165	0.3936
	-	(0.1077)	-	(1.4121)	-
Kurtosis	1.4176	-0.02411	0.0000	11.7800	0.9330
	-	(0.2145)	-	(9.1570)	-

Note: Bayesian p-values account for $Pr(T(D^{rep}) \geqslant T(D))$. $T(D)$: test statistics of the observed S&P 500 index daily log-return data; $T(D^{rep})$: test statistics of replicated data simulated using the posterior results of parameters, standard deviation is reported in (). Parameter estimation data: S&P 500 index daily log-return data 31/07/2012 - 31/07/2014.

Table 2.5: Out-of-sample S&P 500 Index European Option Pricing Coverage Performance of the Black-Scholes and Merton's Jump-Diffusion Models

	Call Option		Put Option	
	Mid-quote Coverage Difference	Bid/Ask-quote Coverage Difference	Mid-quote Coverage Difference	Bid/Ask-quote Coverage Difference
All Options	5.63%	7.91%	13.72%	16.27%
ITM	-1.33%	0.49%	13.72%	11.16%
ATM	5.28%	7.13%	-2.07%	0.23%
OTM	26.00%	29.83%	22.33%	21.89%
<60 days	2.70%	5.77%	13.07%	14.79%
60 - 180 days	8.87%	10.32%	12.85%	16.92%
$>$ 180 days	14.6%	13.87%	28.47%	28.47%

Note: out-of-sample option pricing period: 01/08/2014 - 31/08/2014; rolling window estimation method with S&P 500 index daily log-return data 31/07/2012 - 29/08/2014; estimation window rolls one day ahead after each estimation. Estimated posterior parameter results are used to price options one day ahead. Mid-quote Coverage and Bid/Ask-quote Coverage account for the % of realised market option prices covered by the 95% credibility intervals of posterior model price distributions. The table reports the differences of coverage rates between the MJD and the BS model (i.e. MJD coverage rate - BS coverage rate).

Table 2.6: Out-of-sample S&P 500 Index European Call Option Pricing Error Performance of the Black-Scholes and Merton's Jump-Diffusion Models

Panel A. BS - S&P 500 Index European Call Option

	PE	APE	RMSPE	OPE	AOPE	RMSOPE
All Options	-5.32	6.68	9.76	-4.45	5.25	8.29
ITM	-7.82	7.95	11.21	-6.57	6.66	9.74
ATM	-2.81	5.19	7.38	-2.04	3.31	5.59
OTM	0.13	4.04	5.98	0.02	2.52	4.42
<60 days	-1.85	3.66	4.49	-1.61	2.76	3.66
60-180 days	-7.92	8.77	10.61	-6.47	6.85	8.88
>180 days	-25.60	25.72	29.14	-21.89	21.91	25.40

Panel B. MJD - S&P 500 Index European Call Option

	PE	APE	RMSPE	OPE	AOPE	RMSOPE
All Options	-26.82	27.82	37.60	-22.64	23.06	33.70
ITM	-39.46	39.99	46.73	-34.27	34.68	42.20
ATM	-11.08	11.65	14.15	-6.25	6.46	9.19
OTM	-1.26	3.94	5.75	-0.47	1.02	2.33
<60 days	-25.83	27.12	37.55	-23.26	23.92	34.91
60-180 days	-27.80	28.46	37.46	-22.16	22.26	32.40
>180 days	-30.62	30.99	39.17	-19.19	19.20	29.09

Note: out-of-sample option pricing period: 01/08/2014 - 31/08/2014; rolling window estimation method with S&P 500 index daily log-return data 31/07/2012 - 29/08/2014; estimation window rolls one day ahead after each estimation. Estimated posterior parameter results are used to price options one day ahead. PE (Pricing Error): the average of (posterior mean price - market mid-quote); APE: Absolute Pricing Error; RMSPE: Root-mean-square Pricing Error; OPE (Outside Pricing Error): the average of differences between the posterior 2.5% or 97.5% credibility interval bound values and market prices (mid-quote) when the market prices are not covered by the 95% intervals of model prices; AOPE: Absolute Outside Pricing Error; RMSOPE: Root-mean-square Outside Pricing Error.

Table 2.7: Out-of-sample S&P 500 Index European Put Option Pricing Error Performance of the Black-Scholes and Merton's Jump-Diffusion Models

Panel A. BS - S&P 500 Index European Put Option

	PE	APE	RMSPE	OPE	AOPE	RMSOPE
All Options	-4.21	6.16	8.89	-3.61	4.86	7.58
ITM	1.12	4.98	6.38	0.92	3.28	4.64
ATM	-1.86	5.51	7.20	-1.56	3.62	5.40
OTM	-6.63	6.72	9.95	-5.68	5.71	8.76
<60 days	-1.43	3.69	4.65	-1.31	2.87	3.85
60-180 days	-6.13	7.72	9.46	-5.13	6.01	7.97
>180 days	-21.85	22.88	26.03	-18.80	19.20	22.70

Panel B. MJD - S&P 500 Index European Put Option

	PE	APE	RMSPE	OPE	AOPE	RMSOPE
All Options	14.39	18.82	23.13	13.62	15.30	21.19
ITM	29.46	32.07	42.90	28.35	28.92	40.92
ATM	27.48	30.75	38.31	26.05	26.70	35.53
OTM	6.02	11.36	16.88	5.53	7.84	13.67
<60 days	15.62	18.13	28.59	14.75	15.79	27.08
60-180 days	14.32	19.41	27.92	13.11	14.84	24.68
>180 days	0.31	22.25	28.85	4.33	13.14	22.52

Note: out-of-sample option pricing period: 01/08/2014 - 31/08/2014; rolling window estimation method with S&P 500 index daily log-return data 31/07/2012 - 31/07/2014; estimation window rolls one day ahead after each estimation. Estimated posterior parameter results are used to price options one day ahead. PE (Pricing Error): the average of (posterior mean price - market mid-quote); APE: Absolute Pricing Error; RMSPE: Root-mean-square Pricing Error; OPE (Outside Pricing Error): the average of differences between the posterior 2.5% or 97.5% credibility interval bound values and market prices (mid-quote) when the market prices are not covered by the 95% intervals of model prices; AOPE: Absolute Outside Pricing Error; RMSOPE: Root-mean-square Outside Pricing Error.

Table 2.8: Implied Parameters - Out-of-sample S&P 500 Index European Option Pricing Coverage Performance Difference of the Black-Scholes and Merton's Jump-Diffusion Models

	Call Option		Put Option	
	Mid-quote Coverage Difference	Bid/Ask-quote Coverage Difference	Mid-quote Coverage Difference	Bid/Ask-quote Coverage Difference
All Options	14.98%	10.38%	10.38%	8.05%
ITM	27.42%	18.24%	-8.29%	-9.09%
ATM	0.46%	3.22%	21.61%	17.01%
OTM	-10.85%	-7.34%	14.15%	11.83%
<60 days	12.09%	6.45%	9.09%	7.06%
60 - 180 days	18.10%	15.38%	10.59%	8.05%
> 180 days	24.09%	16.79%	24.09%	19.71%

Note: out-of-sample test period: 01/08/2014 - 31/08/2014; Option market price (mid-quote) data: S&P 500 index European call and put options 01/08/2014 - 31/08/2014. Parameters are calibrated using the option data with same maturity date; calibrated posterior parameter results are used to price options with the same maturity date one day ahead. Mid-quote Coverage and Bid/Ask-quote Coverage account for the % of realised market option prices covered by the 95% credibility intervals of posterior model price distributions. The table reports the differences of coverage rates between the MJD and the BS model (i.e. MJD coverage rate - BS coverage rate).

Table 2.9: Implied Parameters - Out-of-sample S&P 500 Index European Call Option Pricing Error Performance of the Black-Scholes and Merton's Jump-Diffusion Models

Panel A. BS - S&P 500 Index European Call Option

	PE	APE	RMSPE	OPE	AOPE	RMSOPE
All Options	-4.80	6.21	9.05	-3.42	4.28	6.80
ITM	-7.48	7.58	10.55	-5.38	5.41	8.02
ATM	-2.12	3.93	6.03	-1.32	2.29	4.23
OTM	1.08	3.84	5.32	0.79	2.38	3.68
<60 days	-1.76	3.46	4.16	-1.22	2.33	3.16
60-180 days	-6.97	8.11	9.79	-4.94	5.54	7.32
>180 days	-23.45	23.71	27.17	-17.28	17.34	20.44

Panel B. MJD - S&P 500 Index European Call Option

	PE	APE	RMSPE	OPE	AOPE	RMSOPE
All Options	0.38	4.68	5.91	0.56	2.28	3.46
ITM	-2.70	4.07	5.38	-1.10	1.62	2.76
ATM	4.55	4.77	5.77	2.49	2.50	3.67
OTM	6.37	6.38	7.30	4.00	4.00	4.84
<60 days	1.23	3.43	4.34	1.07	1.91	2.94
60-180 days	-0.61	5.89	6.87	-0.04	2.59	3.74
>180 days	-1.88	9.80	11.30	-0.77	4.05	5.89

Note: out-of-sample test period: 01/08/2014 - 31/08/2014; Option market price (mid-quote) data: S&P 500 index European call options 01/08/2014 - 31/08/2014. Parameters are calibrated using the option data with same maturity date; calibrated posterior parameter results are used to price options with the same maturity date one day ahead. PE (Pricing Error): the average of (posterior mean price - market mid-quote); APE: Absolute Pricing Error; RMSPE: Root-mean-square Pricing Error; OPE (Outside Pricing Error): the average of differences between the posterior 2.5% or 97.5% credibility interval bound values and market prices (mid-quote) when the market prices are not covered by the 95% intervals of model prices; AOPE: Absolute Outside Pricing Error; RMSOPE: Root-mean-square Outside Pricing Error.

Table 2.10: Implied Parameters - Out-of-sample S&P 500 Index European Put Option Pricing Error Performance of the Black-Scholes and Merton's Jump-Diffusion Models

Panel A. BS - S&P 500 Index European Put Option

	PE	APE	RMSPE	OPE	AOPE	RMSOPE
All Options	-4.03	5.76	8.36	-3.44	4.43	6.93
ITM	1.62	4.51	5.76	1.14	2.43	3.55
ATM	-1.74	4.29	6.08	-1.19	2.53	4.15
OTM	-6.53	6.55	9.51	-5.57	5.58	8.23
<60 days	-1.56	3.34	4.09	-1.45	2.63	3.41
60-180 days	-5.62	7.35	8.96	-4.75	5.57	7.41
>180 days	-20.52	21.76	24.85	-16.60	16.64	20.62

Panel B. MJD - S&P 500 Index European Put Option

	PE	APE	RMSPE	OPE	AOPE	RMSOPE
All Options	-1.05	3.98	5.18	-0.88	2.24	3.29
ITM	4.82	5.22	6.34	2.58	2.64	3.53
ATM	1.56	2.68	3.23	0.50	0.85	1.44
OTM	-3.72	3.87	5.11	-2.41	2.43	3.52
<60 days	-0.60	2.50	3.03	-0.46	1.57	2.11
60-180 days	-1.59	5.47	6.37	-1.37	3.00	4.10
>180 days	-2.11	9.61	11.23	-1.95	4.00	6.17

Note: out-of-sample test period: 01/08/2014 - 31/08/2014; Option market price (mid-quote) data: S&P 500 index European put options 01/08/2014 - 31/08/2014. Parameters are calibrated using the option data with same maturity date; calibrated posterior parameter results are used to price options with the same maturity date one day ahead. PE (Pricing Error): the average of (posterior mean price - market mid-quote); APE: Absolute Pricing Error; RMSPE: Root-mean-square Pricing Error; OPE (Outside Pricing Error): the average of differences between the posterior 2.5% or 97.5% credibility interval bound values and market prices (mid-quote) when the market prices are not covered by the 95% intervals of model prices; AOPE: Absolute Outside Pricing Error; RMSOPE: Root-mean-square Outside Pricing Error.

Chapter 3

The Volatility and Skewness Crystal Ball: Estimation Risk and Structural Change around Crises

3.1 Introduction

Do equity markets signal forthcoming financial crises? If yes, what piece of information does provide that signal? The moments of the distribution of security returns contain rich information of market dynamics. Nevertheless, does estimation risk of the moments also play a role in signalling market turbulence? The normal or Gaussian distribution assumption is rejected by many studies (Arditti, 1967; Merton, 1976; Jorion, 1988; Hsieh, 1989; Drost, Nijman and Werker, 1998; Das and Sundaram, 1999; Backus, Foresi and Wu, 2004, for example). But is this finding consistently true throughout time, or does the evolution of return distribution involve structural changes indicating a model selection issue? Furthermore, how does the evolution of return distribution reflects different market dynamics and interact with investor sentiment, especially around a stress episode?

Many studies investigated the causation of crises through the analysis of pre-crisis events and factors that trigger the event (Lybeck, 2011; Acharya and Viswanathan, 2011; Baig and Goldfajn, 1999; Mun and Brooks, 2012; Schwert, 2011). Others identify general variables which have predictive or indicative power towards the events through quantitative models (Adrian and Brunnermeier, 2011; Kim et al., 2004; Oh et al., 2006; Rose and Spiegel, 2012). Popular crisis theories, including the argument of endogenous risk (Danielsson and Shin, 2003) and the Minsky theory (Minsky, 1982, 1992; Minsky and Kaufman, 2008), point out that some crises may occur for reasons external to financial markets while others may be constructed from within.

In this chapter we employ Merton's jump-diffusion model to analyse the evolution of the S&P 500 index returns volatility and skewness between 1980 and 2015, accounting for parameter estimation risk. Markov chain Monte Carlo (MCMC) methods are used for inferential purposes. We compare and contrast the market dynamics among the significant financial crises during our study period: the Black Monday Crash in 1987, the "dot-com" crisis in early 2000s and the global financial crisis in 2008.

Volatility is a major driver in gauging and explaining market performance around a distress episode (Schwert, 1990, 2011; Mun and Brooks, 2012; In et al., 2001, for example). Through volatility changes, Schwert (1990) concludes that the 1987 market crash exhibits different features compared to all previous crises, and Schwert (2011) shows that the 2008 crisis is different from the 1929 Great Depression. Jones (2003), through the constant elasticity of variance (CEV) model, shows that volatility of volatility (vol of vol) becomes higher when stock volatility increases while Corsi et al. (2008) find the vol of vol to evolve countercyclically. Danielsson et al. (2010), however, indicate that vol of vol tends to lead stock volatility. In other words, the vol of vol starts to increase before the surge of stock volatility, and it decreases when the stock volatility reaches its maximum level. In the recent finance literature, volatility is considered to be stochastic in the sense of evolving with a pre-specified stochastic process, for example Heston (1993). In this study we prefer to consider a more flexible route and consider volatility to be a random variable with a drawn from a probability distribution that is changing with the ebbs and flows of equity returns. Similar research in this direction is rather sparse. While in most literature vol of vol is estimated using a stochastic volatility model, under the Bayesian framework applied here it is estimated as the standard deviation of the posterior distribution of stock volatility. Evolution of skewness is also investigated carefully alongside with volatility. Estimation risk of skewness is captured by the same technique under the Bayesian estimation framework.

While the causes of the 'dot-com' crisis and the 2008 Global Financial Crisis are widely accepted, the financial cause of the 1987 'Black Monday' crash is not of the same type, at least, with possible explanations being proposed such as the implementation of portfolio insurance and dynamic hedging techniques; the US trade deficit and the announcement of the intention of de-valuing the US dollar by the Treasury Secretary during the weekend (Bernhardt and Eckblad, 1987); and the "triple witching" event, which describes a circumstance when the expiration dates of monthly options and futures coincides. Through the analysis we reflect on two important financial crisis theories: endogenous risk theory (Danielsson and Shin, 2003) and Minsky's theory (Minsky, 1992).

The concept of "endogenous risk" (Danielsson and Shin, 2003) states that the movement of asset prices has two components: "a portion due to the incorporation of initial exogenous shock, the second part due to a feedback effect from the market participants after the initial shock." The endogenous risk is the second component. Danielsson and Shin (2003) define endogenous risk as the risk from shocks that are generated and amplified within the system. They claim that the 1987 market crash is a classic example of endogenous risk, but the source of the initial shock remains unclear.

The economic theory of Minsky and his Financial Instability Hypothesis[1] has been ignored for decades, but returned to the stage and became popular recently after the 2008 global financial crisis (Wray, 2015). While not all crises episodes are associated with lending, the idea of "stability is destabilising" advocated by Minsky should not be limited to the 2008 global financial crisis. Low volatility and steady growth change market participants' behaviour and encourage risk taking, which could later become the causation of crises (Wray, 2015; Wolfson, 2002; Minsky, 1977, 1992; Minsky and Kaufman, 2008). Consistent with Minsky's idea, many studies found that lags of low volatility have predictive power towards both crises and asset bubbles (Liechty, 2013; Adrian and Brunnermeier, 2011; Riedle, 2016), or systemic risk is built during a "great moderation" before a crisis (Acharya and Viswanathan, 2011; Porras, 2016; Keen, 2013; Lybeck, 2011).

We identify a period before the 2008 Global Financial Crisis when the equity market in the U.S. exhibits normal returns, in contrast to general knowledge about stylized features of equity returns in the U.S.. Market conditions during this period of normal equity returns are investigated carefully and we found that this special period is associated with low volatility and steady returns. Minsky (1992) states that *"over periods of prolonged prosperity, the economy transits from financial relations that make for a stable system to financial relations that make for an instable system"(p. 7-8).* From this end, we test whether the gaussian or calm market periods may be indicative of imminent crises, and hence whether the Minsky's theory is supported. Moreover, we conjecture that the normality could also interact with

[1]The Financial Instability Hypothesis focuses on lending behaviour, and summarises 3 different phases of lending (Wray, 2015; Bernstein and Fridson, 2016; Minsky, 1992): the Hedge, the Speculative and the Ponzi phases. In the hedge stage, prudent loan is issued with both interest and principle healthily repayable. In the speculative stage, borrowers would be able to repay only the interest of their loan. This stage is associated with rising asset prices. The final stage is the Ponzi stage, in which loans are given out even though borrowers can neither pay the interest nor the principle, under the belief that asset prices will continue to rise. It is not difficult to relate the Ponzi stage to the period of late 2006 and early 2007, when sub-prime mortgages increased rapidly in the US market.

investor sentiment to affect subsequent market performance or to construct an instable financial system.

Investor sentiment is generally found to have intermediate to long horizon predictive power on future stock returns (Brown and Cliff, 2005; Lemmon and Portniaguina, 2006; Baker and Wurgler, 2007; Schmeling, 2009; Hengelbrock et al., 2013). While both positive and negative impacts are evidenced in different markets during different periods with different sentiment proxies, the average effect found in the U.S. market is negative. Moreover, either the sentiment is related to a missing fundamental pricing factor or it is related to irrational trading behaviour (Baker and Wurgler, 2007; Brown and Cliff, 2005). Hengelbrock et al. (2013) rule out the rational explanation of sentiment effects in the U.S. and German markets, and concluded that the intermediate and long horizon predictability is most likely stemming from under-reaction and mis-pricing. We are interested first to see whether macroeconomic variables have any explanatory power with future returns and what is the role played by market sentiment.[2] Then, for the first time in the literature, we show that the investor sentiment variables interact with the type of period as defined by the normality of the distribution of returns. This novel finding may also indicate a mechanism for bubble formation in equity markets. When the distribution of returns switches from non-gaussian to gaussian, rising levels of investor sentiment may lead to overvaluation and cumulative unidirectional equity price pressure that generates bubbles.

The remainder of the chapter is structured as follows. In Section 3.2 we revise the Merton's jump-diffusion model and describe the Bayesian set-up for our analysis. In addition, the estimation results of parameters of the MJD model are also reported. Section 3.3 contains the analysis of market dynamics revealed by volatility and its Bayesian measure of estimation risk while

[2] Baker and Wurgler (2007) advocate that the low return following a period of high sentiment level could be a correction of the overvaluation in the earlier period. Scheinkman and Xiong (2003); Meier (2015) suggest that overconfident investors play an important role in cumulating speculative bubbles, which latter on could lead to market crashes.

Section 3.4 presents the analysis of market dynamics revealed by skewness and its associated estimation risk. In Section 3.5 we report the investigation of the 'market returns to normal' impact. Section 3.6 concludes.

3.2 General Setup and Assumptions

Equity markets are characterised by significant time variation in volatility coupled with sudden jumps (see Andersen et al., 2015). Jump-diffusion models have been introduced as a feasible solution to the fat tails feature of empirical asset returns. Bakshi et al. (1997) analyse the importance of adding a price-jump component for hedging S&P 500 options and their evidence suggests that including jumps will not enhance hedging performance. However, more recently Kaeck (2013) finds that even misspecified jump-diffusion models can be very useful in improving hedging performance and risk assessment and provides ample support for the inclusion of jumps in the data generating process of the underlying process. Broadie et al. (2007) recommends that structural parameters of jump-diffusion models to be consistent with the dynamics of the underlying process in order to obtain robust empirical results Furthermore, they find strong evidence for jumps in prices.

3.2.1 Merton's Jump-Diffusion Model

Recall equation (2.1), the MJD model is described by the stochastic differential equation:

$$dS_t = (\mu - \delta - \lambda\varphi)S_t dt + \sigma S_t dW_t + (\mu_t^J - 1)S_t dI_t \tag{3.1}$$

where μ is the expected rate of return; δ is the dividend yield, σ is the volatility of stock return; $\{W_t\}_{t\geq 0}$ is a standard Wiener Process; λ is the intensity of jump events per unit time interval; $\{I_t\}_{t\geq 0}$ is a Poisson process with intensity λ; μ_t^J is the jump size of stock price; and $\varphi = E[\mu_t^J - 1]$.

As described in Chapter 2 Section 2.3 equation (2.2), under the assumption that $\ln(\mu_t^J)$ is normally distributed $\ln(\mu_t^J) \sim N(a, \zeta^2)$, the probability distribution of log stock return $R_{t+\Delta} = ln\left(\frac{S_{t+\Delta}}{S_t}\right)$ is the weighted average of normal distributions by the probability that i jumps would occur. Similar to Chapter 2, we follow Ball and Torous (1985)[3] which approximated the MJD model with the Bernoulli-Jump Diffusion model, using the assumption that only one jump is allowed to happen per unit time interval. This assumption is realistic when the time interval is short (i.e. daily as in this study). The parameters are estimated following the likelihood function presented in Section 2.6.2 equation (2.5)

Under our Bayesian setup, the total stock volatility and skewness are deterministic functions of the parameters and with MCMC it becomes straightforward to collect inference on these quantities. Navas (2003) derives the correct formula of the MJD total volatility as [4]:

$$\sigma_{Merton} = \sqrt{(\sigma^2 + \lambda(a^2 + \zeta^2))\Delta t} \tag{3.2}$$

The MJD skewness formula is derived by Matsuda (2004) as

$$Skewness_{Merton} = \frac{\lambda \Delta t(3\zeta^2 a + a^3)}{((\sigma^2 + \lambda\zeta^2 + \lambda a^2)\Delta t)^{3/2}} \tag{3.3}$$

3.2.2 Estimation of Stock Volatility and Skewness with Parameter Estimation Risk

We estimate parameters from the time series of daily S&P 500 index returns under the risk neutral measure using an MCMC approach. Since computationally the total stock volatility and skewness can be seen as statistics over the sample space of returns and parameters $\Theta = \{\mu', \sigma, \lambda, a, \zeta\}$ ($\mu' = \mu - \delta$),

[3]Another widely used proxy is the M-Jump Diffusion model described in (Kostrzewski, 2014; Burger and Kliaris, 2013).This model refers to a specification of a cutting point M, so that the model takes into account a maximum of M jumps.

[4]Δt is the time interval; in this study, $\Delta t = 1$ for daily time intervals

the ergodic results underpinning the MCMC inference provide a direct mechanism to extract the posterior distributions of these variables $p(\sigma_{Merton} \mid \Theta)$ and $p(Skewness_{Merton} \mid \Theta)$. Initial settings of parameter prior distributions of parameters are identical to Chapter 2 Section 2.6.2 following Jacquier and Polson (2010) and Eraker et al. (2003). More details of MCMC algorithm are introduced in Chapter 2 Section 2.5.

Parameters are estimated based on the historical daily log-return data of the S&P 500 index between 01/01/1980 and 30/12/2015. The estimation window contains two years of data, and the window moves each week (five data points), resulting in 1,715 estimation points throughout the sample period. After each estimation, prior distributions are updated according to posterior results to ensure more informative for the next estimation window. Figure 3.1 plots the posterior mean, median and 95% credibility interval of the 5 paramters. Estimated trends of μ' ($\mu' = \mu - \delta$), σ and ζ tend to follow the historical events: stock return decreases during distress periods, while volatilities of both the diffusion component and jump component increase. Paramter estimation risk also increases during the crisis periods as indicated by the 95% credibility interval of the three parameters. Jump intensity λ was around or below 20 in most of the time prior to the 2008 global financial crisis. It increased significantly during the 2008 crisis and restored slowly after the crisis. Jump size a, on the other hand, is more stable during the entire sample period except a very significant drop at the 'Black Monday' event. Noticeable noises of the estimated results of a are shown during the estimation windows ending around 2005-2006. This is due to a structural change in underlying sample data: the return data has returned to normal distribution. We will discuss this issue and the correction in more detail in the later sections (statistics in Table 3.1 excludes these periods to avoid misleading bias).

[Figure 3.1 about here.]

Table 3.1 reports the main statistics[5] of the posterior means of the five parameters. Our results indicate that throughout the periods, the average of estimated mean returns of the S&P 500 index is 9.79% (daily value 0.04% with s.d. 0.04%, min -0.12% and max 0.14%), with a minimum value of -31.05% resulted from the estimation window of 02/03/2007 - 03/03/2009 covering the 2008 crisis, and a maximum value of 34.28% resulting from the estimation window of 04/03/2009 - 03/03/2011 which is a period in the aftermath of the 2008 crisis! The stock volatility σ yields an average value of 13.44% (daily value 0.85% with s.d. 0.25%, min 0.43% and max 1.52%), with the maximum value resulting from the estimation window of 25/07/2007 - 24/07/2009, and the minimum value coming out the window of 20/10/1993 - 18/10/1995. Jump intensity λ averages at 22.67 jumps per year (daily value 0.09 with s.d. 0.06, min 0.006 and max 0.32)[6]. The highest jump intensity is found again in the estimation window of 21/09/2006 - 23/09/2008 which covers the beginning of the 2008 crisis. The jump size a is negative on average, indicating that most of the time jumps are downward[7], the largest negative size posterior mean estimate of -22.28% is found from the first estimation window that covers the 1987 crash event 24/10/1985 - 22/10/1987. The estimation window which gives the largest jump size volatility ζ is also associated with the 1987 crash event 11/11/1986 - 08/11/1988.

[Table 3.1 about here.]

[5] Other relevant statistics of pathwise fitted values are not reported due to limited space, but available upon request.

[6] Johannes and Polson (2010) applies the Bayesian method, and found 20.13 jumps per year in S&P 500 during 1986-2000; Honore (1998) estimates the Bernoulli-Jump Diffusion model and found an average of 40 jumps in S&P 500 during 1973-1997; Eraker et al. (2003) and Kou et al. (2016) apply the Bayesian method to the stochastic volatility with MJD model and found 1-2 jumps per year in S&P 500; Kou et al. (2016) applies the Bayesian method to stochastic volatility with double exponential jump models and found 6-26 jumps per year in S&P 500.

[7] This is in line with ample empirical evidence on the asymmetry in equity market returns and negative skewness. See Hansen (1994), Bekaert and Wu (2000), Harvey and Siddique (2000), Brandt and Kang (2004).

3.3 Historical Evolution of Stock Volatility

3.3.1 Parameter Estimation Risk of Stock Volatility

An overview of the posterior statistical inference of the MJD total volatility in different time periods is tabulated in Table 3.2 and plotted in Figure 3.2.

[Table 3.2 about here.]

[Figure 3.2 about here.]

The empirical results reveal that volatilities of equity returns in the U.S. in different time periods vary not only in their estimated mean values, but also in the associated posterior standard deviations. Therefore, relying on point estimation values would induce biases and deteriorate the applicability of the model. Under the MJD model, the estimated total volatilities tend to have positive skewness and longer right tail as reflected by the longer upper whiskers of the box-plots. It is important to notice that when the estimation window covers any of the three main crisis periods (i.e. 1987 crisis, 2001 dot-com bubble bust and 2008 sub-prime mortgage crisis), not only the estimated posterior means of volatilities increase significantly to reflect the increased risk during the periods, but also the widths between the upper and lower bounds of estimated values. Therefore, parameter estimation risk tends to coincide with market uncertainties as reflected in our results, supporting the results in Jones (2003). This is also consistent with empirical practices that, during crisis periods, forecasting with any model could be very difficult, because too much parameter estimation risk is embedded in the estimated value. Models find it hard to pin down the "correct" value of parameters. In our case, the proposed Bayesian MCMC method successfully capture the parameter estimation risk.

The highest volatility is observed during the global financial crisis in 2008. The parameter estimation uncertainty during this period is also the highest among all estimation windows, reflected by the widest 95% credibility interval[8] of [31.71% - 39.92%]. Moreover, the financial markets behave differently pre- and post-crisis in these three events as reflected by the parameter estimation uncertainties. In 1987 and 2008, the posterior mean values of volatilities jump suddenly by more than 10% and 20% respectively compared to their previous periods, the estimation standard deviations of volatilities also doubled their size. It seems that magnitude of both the volatility and its estimation error tend to react more significantly when extreme events with lager negative returns are observed. Furthermore, in the 1987 crisis, market uncertainty seems to decline quickly as the post-crisis estimation standard deviation decreases quickly to its pre-crisis level. On the other hand, in the 2008 crisis, market uncertainty does not seem to restore as quickly as in the 1987, post-crisis volatility is about 7% higher than its pre-crisis level, estimation standard deviation is also at a higher level of 1.15%. The "dot-com" crisis also has its own feature compared to the other two. Instead of having a sudden jump, the volatility and the estimation standard deviation already exhibit an increasing tendency before the crisis, possible because of the 1997 Asian financial crisis, the 1998 Russian financial crisis and the Long Term Capital Management crisis in the U.S. and Europe. On the other hand, the dot-come bubble started to accumulate in late 1990s, and the first mini-crash was observed on October 27, 1997, in the wake of the Asian crisis. The "dot-com" crash last for a much longer time than the other two, from March 2000 to October 2002 (Goldfarb et al., 2007), and there was also the September 11th event in 2001 (Danielsson and Shin, 2003).

Moreover, plots of posterior means of MJD volatilities and sample standard deviations in Figure 3.2 show that the Bayesian estimation method accurately estimates the volatility of return in-sample. In the periods covering the 1987 crash and the 2008 crisis, the MJD model volatility estimate

[8]The credibility interval is the equivalent of the confidence interval under a Bayesian paradigm. It is constructed as a coverage interval of the specific posterior estimate from the posterior distribution of the quantity of interest Gelman et al. (2014).

tends to be slightly higher than the realised volatility. This is due to the contribution of large jumps and increased parameter estimation uncertainty.

3.3.2 Volatility Evolution and the Three Financial Crises

The full volatility evolution during the sample period is illustrated by plotting the MJD volatilities of the 1715 estimation points on a time-series plot in Figure 3.3 that displays also posterior means and 95% credibility intervals.

[Figure 3.3 about here.]

Before analysing the market behaviour during the three crises periods, we spot an unusual uncertainty of the 95% credibility intervals between 2005 and late 2006. Since each estimation window contains two years of data, this "uncertain" period covers data from 2003 to late 2006. The intervals are abnormally widened while at the same time the sample volatility is very low and there is no known market uncertainty associated with that period. Furthermore, the realised volatility is very close to the low boundary of the credibility interval for return volatility resulting from the fitted MJD model.

We carry out in-depth investigation about this newly detected abnormality and we conjecture that the main reason behind it is due to the fact that equity stock returns in the U.S. during these estimation windows are gaussian. According to the test results of the Jarque-Bera (JB) and Shapiro-Wilk (SW) tests, the null hypothesis that the underlying return distribution is normal cannot be rejected during the consecutive 2-years periods from March 2003 to December 2006, which covers 90 estimation points (detail test results are presented in Section 3.5). Gauging into the detail results during the period, we find that the mean of jump size (parameter a), in particular, contributes the most to the widened 95% credibility intervals observed. Thus, when the data-generating process is gaussian, the MJD model finds it very hard to identify jump events.

We are the first to find such abnormal behaviour of the U.S. stock market in this pre-2008 crisis period. The importance of this finding is three folded. Firstly, it is in contradiction to common empirical findings that stock returns deviate from normal distribution and exhibit negative skewness and excess kurtosis. Our test results show that stock returns did return to gaussian, for a long period. Secondly, it indicates the necessity of considering model uncertainty in stock return parameter estimation. Throughout the changes of market performance, return distributions can deviate significantly from one model and switching onto another. Finally, the gaussian period happened in parallel with the period of very low market volatility before the 2008 crisis. We elaborate further on these ideas in Section 3.5.

In order to deal with the model volatility estimation uncertainty issue prior to the subprime crisis, we set the jump intensity λ to zero, thus falling back onto the Geometric Brownian Motion (GBM) setting, see equation (3.1). After this correction, Figure 3.4 depicts the re-fitted time-series plot:

[Figure 3.4 about here.]

The three crisis periods can be easily spotted on Figure 3.4, we can see that the market has actually reacted and behaved differently in the three main crises periods. The first important difference among the three crises is whether the market uncertainty continued to increase after the initial collapse or not? Recall the definition of endogenous risk: it is the risk from shocks that are generated and amplified within the system; it is a second part of the price movement due to a feedback effect from the market participants after the initial shock (Danielsson and Shin, 2003). It seems that the endogenous risk can be further classified into two types: feedback effect which roots on and reinforces a substantial valuation correction or feedback effect that are pure psychological panic. In the "dot-com" and 2008 global crises, the endogenous risk had revealed a problem rooted on the pricing fundamentals which had led to significant valuation bias (i.e. bubbles), and hence we see the problem persisting after the initial shocks to correct the market prices. During the "dot-com" crisis period, we do not observe a sudden dramatic change in volatility as in 2008, nevertheless, it fluctuates with an upward trend during the period. In 2008, the volatility started to surge pre-crisis indicating an increase in market uncertainty followed by the revelation of subprime mortgage crash in 2007, a huge rapid jump is observed right after the fall of Lehman Brothers. The market volatility continue to surge in the next 6-9 months.

The 1987 crash, on the other hand, shows a different pattern. This is an example of a crash caused by feedback effects that are psychological panic without a fundamental need on market prices correction. The financial markets in 1987 have no genuine problem, and the harm was mostly due to the overreaction of market participants such that the crisis did not last long and the market restored very quickly to the original level. The historical evolution of volatility in Figure 3.4 reveals that, after the 19th Oct 1987 data was included into the estimation window, the volatility increases immediately from 15.33% to 30.10%. When the Black Monday data is removed from the estimation window, the volatility graph exhibits a vertical drop.

By plotting the posterior mean estimate of equity index return in Figure 3.5, we further confirm the unique nature of the 1987 crash compared to the other two. In the "dot-com" and 2008 global crises, when the feedback effect are rooted on and reinforces the substantial valuation issue, the market experiences a significant period of negative performance and it takes a while for the market to restore to positive returns. After the initial crash, the market returns continue to fall at least for a period of six months to one year. Before those crises started, we observe a decreasing trend of the market performance, whereas for the 1987 crisis, although the market dropped sharply on the crash day, the returns fluctuate around zero and jump back up as soon as the Black Monday data is excluded from the estimation window. After the market realised that this was an overreaction, it restored very quickly[9]. The posterior mean of the Bayesian Sharpe ratio plotted in Figure 3.6 exhibits similar story as the posterior mean of stock return. The 95% credibility interval reveals that it is very rare for the inference results to yield a 2.5% percentile beyond 0 in all periods. On the other hand, the plotted sample Sharpe ratio almost perfectly coincides with the estimated posterior mean of inference results.

[Figure 3.5 about here.]

[Figure 3.6 about here.]

In order to verify that indeed there has been a change in model we also employ a simple Markov-switching dynamic regression model:

$$\bar{R}_t = \begin{cases} h_1 + \varepsilon_t, & \text{w.p. } P_1 \\ h_2 + \varepsilon_t, & \text{w.p. } P_2 = 1 - P_1 \end{cases} \tag{3.4}$$

where $\bar{R}_t$ denotes the posterior mean of 2-years moving averaging return estimated from each estimation window under the MJD model; h_1 and h_2 are different means of $\bar{R}_t$ in the two regimes and ε_t is the error term follows a gaussian distribution $N(0, b)$. P_1 denotes the probability of regime 1 and P_2 denotes the probability of regime 2. The regime switching process is characterised by the Markov chain probability transition matrix H:

$$H = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

The estimation results of the regime switching model are reported in Table 3.3. The model successfully distinguishes the periods of market distress,

[9] Lybeck (2011) claims that the "dot-com" crisis and the 1987 crash shall not be treated as financial crises as they did not affect other markets or the real economy, therefore shall be treated as pure market crashes. However, from our results of the historical evolution of both volatility and the market returns show that the "dot-com" crisis shares the same market dynamics as the 2008 crisis, while different features of the 1987 market crash is observed.

which is captured by regime 1. The two regimes are quite persistent according to $P_{11} = 0.9884$ and $P_{22} = 1-P_{21} = 0.9980$. Regime 1 identified by the model is shaded grey in Figure 3.5. The crisis period of the "dot-com" bubble burst and the 2008 Global Financial crisis are highlighted, whereas the 1987 market crash is not recognised as a market distress period by the model.

[Table 3.3 about here.]

Therefore, the different patterns of the three crises periods reveal that while endogenous risk can result in a tremendous crash through feedback effect of market participants, when it does not revealed a substantial need of continue adjustment to market prices, the crash will be rather short-lived. On the other hand, when the need of severe valuation correction is revealed, the volatility continue to surge and market returns continue to fall after the initial shock. Our finding reveals an improvement that can be made to the endogenous risk theory. While the shifts in market participants' beliefs and their subsequent reactions can result in market crashes, whether these beliefs are rooted in a substantial over-valuation problem and reinforces that problem or not would make a big difference to the final outcome. In the 1987 crash, when the beliefs are directly associated with psychological panic, the market restored quickly to correct the error of the dramatic price drop. On the other hand, in early 2000 and 2008, when the substantial over-valuation problem was revealed by the initial shocks and subsequent feedback effect of market participants reinforces this issue of financial markets, real crises occurred to clear the asset bubbles.

Minsky theory is based on the hypothesis that "stability is destabilising", meaning that when volatility in the market is at a low level, people in the market have increased risk appetite and tend to take more risk, which results in the increased likelihood of potential large shock. Danielsson and Shin (2003) and Altunbas et al. (2010) also suggest that in good times the perceived risk is low, complex financial networks are built up and the amount of imprudent leverage is likely to increase. As a result, the unseen hidden endogenous risk is cumulated during these periods and this may lead to a higher probability of triggering a crisis. During the two pre-crisis period in late 1990s and mid 2000s, we do observe such quiet market periods, when the market volatility was close to and even below 10%, and the parameter estimation risk is also at a minimal level, indicating less uncertainty in the financial markets. The gaussian distributed stock return fitted well during the 2003-2006 period further supports these theories as tail events vanished. The Minsky theory is further tested with the discovered calm period in Section 3.5.3. Overall, the results do show an abnormal quiet period pre-crisis, however, it was not the case for the 1987 crash.

3.4 Historical Evolution of Stock Return Skewness

The plot of the evolution of S&P 500 index skewness is shown in Figure 3.7. The gap in mid-2000s is the period with normally distributed stock returns. When using the GBM model instead of the MJD model, the estimated skewness equals to zero without any estimation risk due to the setting of the GBM model. The dash line plots the sample skewness, which almost straightly equals to zero as well during the period.

[Figure 3.7 about here.]

Throughout the test periods, the estimated posterior mean of the MJD skewness may slightly over-/under-estimate the sample skewness. Nevertheless, the sample skewness is well captured by the 95% credibility interval, except for the 2-years periods during which the “Black Monday” data is included in the estimation windows. Unlike the analysis for volatility, for the “dot-com” crisis and the 2008 global financial crisis neither the point estimate of skewness nor the parameter estimation risk has changed remarkably.

However, the disturbance of the 1987 market crash is quite significant as displayed on the figure, confirming the different natures between the 1987 crash and the other two. On the other hand, the crisis has impacted the skewness of stock distribution heavily in 1987. The estimated posterior mean has dropped to -3, while the sample skewness hitted a record of -8. The width of the posterior 95% credibility interval is almost five times wider than the previous period, but still failed to capture the realised skewness. This is also the only period when the credibility intervals fail to cover the realised skewness.

3.5 Normality of Stock Return Distribution

3.5.1 Normality Tests in Major Equity Markets

In the past four decades, scholars have highlighted their concerns about the unrealistic assumption of normal stock return distribution as asset returns were found skewed with excessive kurtosis from time to time (Aït-Sahalia and Brandt, 2001; Cont, 2001; Jondeau and Rockinger, 2003; Wen and Yang, 2009; Bakshi et al., 2003; Chung et al., 2006; Amaya et al., 2015; Chang et al., 2013). In contrast to this widely accepted view, in Section 3.3.2 we show an interesting finding that for a long period before the 2008 global financial crisis, the data-generating process for the S&P 500 index is gaussian. Figure 3.8 illustrates[10] the evolution of realised skewness and kurtosis over the entire study period, suggesting that, during the MJD model uncertain and low volatility period (the grey shaded area), the realised skewness and kurtosis are very close to zero jointly and this is the only period that exhibits such pattern. The normality of return distribution is tested using both the

[10] In order to have a clearer look at the model uncertainty period, the y-axis range is restricted, so part of plots during the 1987 crash is not shown due to the greatly inflated skewness and kurtosis by the crash event.

Jarque-Bera (JB) and Shapiro-Wilk (SW) tests[11].

In our case the results of the two tests are identical. Among the 1715 estimation windows covering the period from 1980-2015, the data-generating process in 106 estimation windows seems to be gaussian, 16 of them being periods between 1980-1982 (a US recession period defined by NBER) and the remainder covers the period prior to the 2008 crisis from March 2003 to December 2006 consecutively.

In order to see whether the 'return to normal' feature is a global event prior to the 2008 global crisis, we further test the normality of the NASDAQ Composite index and Dow Jones Industrial Average index in the U.S., and another six equity indices of the world's major markets using daily data between 03/2003 and 12/2006. The results at 5% critical level are reported in Table 3.4.

[Figure 3.8 about here.]

[Table 3.4 about here.]

The results show that the gaussian returns hypothesis seem to occur only in the US market, where the global financial crisis originated. The three key equity indices of the US exhibit normal distribution during the similar periods. The equity indices of EURO STOXX 50, FTSE 100, DAX 30, NIKKEI 225, Hang Seng 50, Shanghai Shenzhen CSI300 do not exhibit a similar normal feature, excepting a few occasional cases for the FTSE 100, which is negligible. While the 'return to normal' finding is robust in the U.S. market, it is essential to ask whether this matters, and whether it does have any impact to the subsequent market distress.

[11] The JB test is the most cited normality test in financial literature. However, it was criticised for its weaknesses when distributions have short tails (Thadewald and Büning, 2007). The SW test, on the other hand, has been shown as the one with better testing power (Chunhachinda et al., 1997; Shapiro et al., 1968; Razali et al., 2011).

3.5.2 Normal Distributed Return Period and Its Impact to Subsequent Market Return

Table 3.5 provides descriptive statistics of the general equity market main drivers and sentiment proxies during the normal distribution period prior to the 2008 crisis. Compared to the full sample statistics, this period is characterised by moderate market return, low volatility (below 30% percentile of full sample volatilities), low expected future volatility (indicated by the VIX). These conditions characterised a stable financial system as described by Minsky's theory. On the other hand, the period also associated with slightly higher than average survey based confidence level (indicated by the UOM index), and higher than average investor risk appetite (indicated by the Credit Spread). The number of IPOs during the period of normal distributed return is lower than average level, due to the overall reduction in IPO volume after the IPO booms in mid 1980s and 1990s. Changes in sentiment proxies are used in the regression analysis below.

The standard deviation of each variable during the normality periods are remarkably smaller than the full sample period, indicating a steady and calm period. With normal distributed return, investors shall expect symmetrical distribution with little worries of extreme tail events. Together with the low volatility (close or below 10%), much less estimation error as highlighted in Figure 3.4 and a regime of steady expansion (positive excessive return), observing normal distributed returns in the recent past may give more confidence to investors that the future returns are more likely to lie within a narrower range around the expected value situated on an increasing trend.

[Table 3.5 about here.]

3.5.3 Regression Analysis of the Impact of Gaussian Distributed Returns

To focus on the impact of gaussian distributed return and whether the Minsky theory is supported, we adopt similar parsimonious regression models as in most of the sentiment effect studies (Baker and Wurgler, 2007; Brown and Cliff, 2005; Hengelbrock et al., 2013; Schmeling, 2009; Lemmon and Portniaguina, 2006):

$$R_{t+1} = \beta_0 + \beta_{1,i}\Delta Sentiment_{i,t} + \beta_2 Normality_t + \beta_{3,j} Control\,Variable_{j,t} + \varepsilon_{t+1} \tag{3.5}$$

$$\begin{aligned} R_{t+1} = &\beta_0 + \beta_{1,i}\Delta Sentiment_{i,t} + \beta_2 Normality_t + \beta_{3,j} Control\,Variable_{j,t} \\ &+ \beta_{4,i}\Delta Sentiment_{i,t} * Normality_t + \varepsilon_{t+1} \end{aligned} \tag{3.6}$$

where $Normality_t$ is a dummy variable taking the value 1 if the daily returns of past two years are normally distributed, and 0 otherwise; $Sentiment_{i,t}$ are investor sentiment proxies, including the University of Michigan index of consumer sentiment (UOM index) also used by Figlewski (2016); Lemmon and Portniaguina (2006); Baker and Wurgler (2007); Schmeling (2009), the sum of IPO volume in the past 12 months (NIPO), and the credit spread series as the average yield spread of Moody's Baa and AAA rated bonds as used by Figlewski (2016); Baker and Wurgler (2007). Sentiment measures are subject to correlation with economic fundamentals. Therefore, it could be subject to the critique that the impact of sentiment is in fact a result of omitting relevant variables. Baker and Wurgler (2007) suggest that this critique is less of a concern in regressions of future returns. Fundamentals should only affect contemporaneous stock returns, predictability from fundamentals shall not exist if stocks are priced fairly. Nevertheless, macroeconomic variables are included to control such impacts if any. The final model contains only two[12] control variables: the annual percentage change in industrial production and the first lag of return.

The regression analysis is carried out for five different length of horizons of future returns: 1-, 2-, 4-, 6- and 8-quarters. While the regression with 1-quarter ahead returns uses non-overlapping quarterly observations, the quarterly observations of 2-, 4-, 6- and 8-quarters returns contain overlapping periods. The overlapping issue artificially creates strong autocorrelation in the response variables, which would result in severe downward bias in the estimation errors.[13] In this paper, we adopt the method developed by Britten-Jones, Neuberger and Nolte (2011) to circumvent the overlapping issue by transforming the regression, so that autocorrelation induced by overlapping data is cleared out while the OLS coefficients are identical to the original regression.

Consider r as a $T \times 1$ vector of single period log returns, and A as the $(T-k+1) \times T$ transformation matrix with 1s on the main diagonal and the first $k-1$ right off-diagonals and 0s otherwise. Then Ar is the vector of k-period log returns. Let X be the matrix of explanatory variables, the original regression with overlapping data is set up as:

$$Ar = X\beta + \epsilon \tag{3.7}$$

[12]Various macroeconomic variables as suggested by literature are included in the preliminary runs of the regression models, including: the annual percentage change in industrial production, the annual CPI inflation rate, the term spread, the percentage change in GDP, the NBER recession dummy variable and the first lag of return. Consistent with Baker and Wurgler (2007)'s argument, little of them show significant impact to future returns.

[13]Britten-Jones, Neuberger and Nolte (2011) pointed out that the commonly used White or Newey-West standard errors can result in misleading estimates of confidence interval and the Hansen-Hodrick standard error is also complex and unreliable.

and β can be efficiently estimated from the transformed regression:

$$\begin{aligned} r =& \tilde{X}\beta + \tilde{\epsilon} \\ \tilde{X} =& A'X(X'AA'X)^{-1}X'X \end{aligned} \tag{3.8}$$

Since the estimation period is based on windows of 2-years return data, the dummy variable 'Normality' only starts in 1982. Therefore, our testing is based on data from 1982-2015 rather than 1980-2015. The correlations of the three market sentiment proxies and the one quarter ahead market return are tabulated in Table 3.6. Our empirical evidence suggests that while positive changes in the UOM Index and NIPO are negatively correlated with future return, an increase in the risk appetite as indicated by credit spreads seems to increase future return. The correlations among sentiment proxies are within acceptable range. Multicollinearity is also tested properly via the variance inflation factor and no multicollinearity problem was found in all of our regression models.

[Table 3.6 about here.]

Table 3.7 presents the results regarding the relationship between Sentiment proxies and the dummy variables of normality. Consistent with the argument of Baker and Wurgler (2007), macroeconomic control variables have no explanatory power in general. Sentiment proxies tend to show the same relationship with future returns as in the correlation matrix. A positive shift in UOM index is found to negatively affect future returns overall, but the impact is statistically significant to future returns in 2-quarters horizon. This result is consistent with past findings in the U.S. market returns (Hengelbrock et al., 2013; Schmeling, 2009). The change in NIPO has a consistently negative but very weak association with future returns. The impact of the credit spread is statistically significant throughout different horizons of future returns while the results of changes in Credit Spread indicate that a positive shift of risk appetite in the current period associates with higher future return. These are similar to Hengelbrock et al. (2013)'s findings on the German market and in the U.S. market during 2001-2008, but contrast to the general findings of sentiment effects in the U.S. market. Two possible reasons for such different findings are, firstly, most of the literature focuses on cross-sectional stock return rather than market return; and the credit spread is not used as a sentiment proxy in Hengelbrock et al. (2013) or Schmeling (2009). Nevertheless, the significance of the credit spread does not support the hypothesis of correction of overvaluation in later period returns. When the normal distribution of past market return is observed, subsequent market returns tend to move upward. But the relationship tend to reverse, when the horizon of future returns increases to two years, indicating potential correction of overvaluation. However, the impact of Normality is not statistically significant as shown in the results.

[Table 3.7 about here.]

Table 3.8 reports the results when we consider that normality interacts with the investor sentiment. The interactive terms exhibit significant impact of normality to future market returns. Although changes in NIPO do not impact future return significantly originally, when interacted with gaussian past return, a positive shift in NIPO would significantly decrease the future returns by 0.17-0.45% (sum of coefficients of $\Delta NIPO$ and $\Delta NIPO * Normality$) depending on the length of future return horizons. When the future return horizon increases to six quarters, the interactive terms of UOM index and the credit spread also turns significant. The impact of credit spread is dramatic: a negative shift of 1% in credit spread (low credit spread indicate high sentiment) would result in about 25% to 30% (sum of coefficients of Δ credit spread and Δ credit spread*Normality) drop in market returns. These statistical evidence supports the hypothesis that during the period when market returns are normally distributed an increase in investor sentiment is likely to cause overvaluation and cumulate bubbles, followed by a correction of mispricing in the later periods. The correction of cumulative asset price bubbles is most significant for a long horizon of six to eight quarters.

[Table 3.8 about here.]

Therefore, the regression results show support evidences towards the Minsky's theory of "stability is destabilising". However, our results suggest that it is not the calm/stable market conditions on its own causes the destabilising in the later periods. We highlight the matter of how the investors interact with the calm conditions. When the calm gaussian period interacts with high investor sentiment and risk appetite, the economy may transit from a stable system into an instable system and result in market destruction in the later periods. This aligns with the Minsky theory.

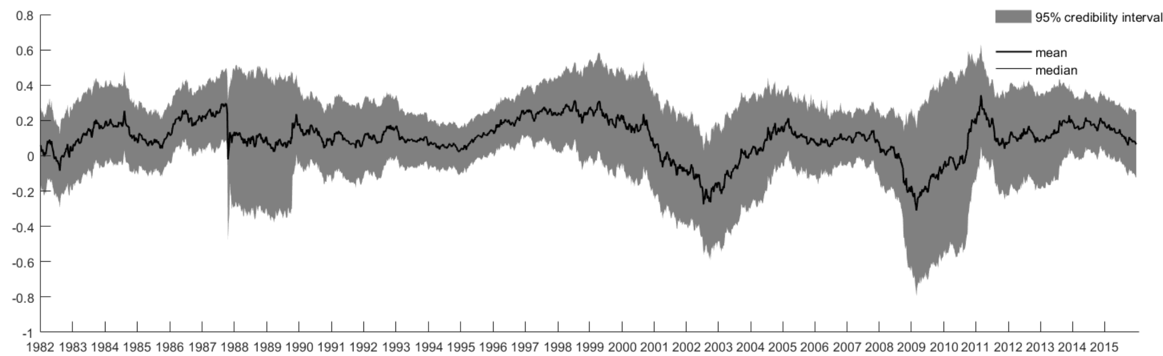
3.6 Conclusion

Analysing the volatility and skewness of the S&P 500 Index between 1980 and 2015 using the Merton's Jump Diffusion Model, we disentangled the different nature of the three main financial crises in the past three decades in equity markets, echoing key discussions in the literature. In particular, the 1987 market crash is found to have distinct market dynamics compared to the "dot-com" crisis and the 2008 global financial crisis. The empirical results show significant and consistent evidence towards the Minsky theory, whereas the theory of endogenous risk is also largely confirmed.

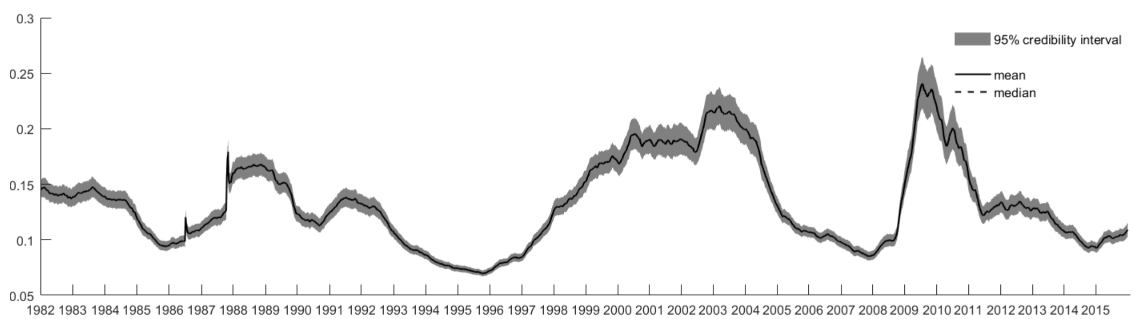
We provide the first-ever investigation of the parameter estimation risk associated throughout the test period, and find that parameter estimation risk of volatility captures well market uncertainty, whereas the estimation risk of skewness is more sensitive to outlier events rather than general market uncertainty. Furthermore, to the best of our knowledge, we are the first to discover a long consecutive period prior to the 2008 crisis when the market return distribution returned to normal, which only happened in the US market, where the 2008 crisis originated. Our results point out that when a calm period is observed, a high level of sentiment leads to an extra negative impact to the subsequent market returns, which is aligned with the Minsky theory of "stability is destabilising". The explanation is that when normality interacted with high levels of sentiment, it encourages extra risk taking and over expectation of future growth, and results in overvaluation or bubbles of assets prices which requires correction in a later period. When the magnitude of bubble is remarkable, the consequential correction would be a market crash or crisis similar to what we observed in the 2008 crisis episode.

Figure 3.1: Posterior Parameter Estimation Results of the Merton's Jump-Diffusion Models

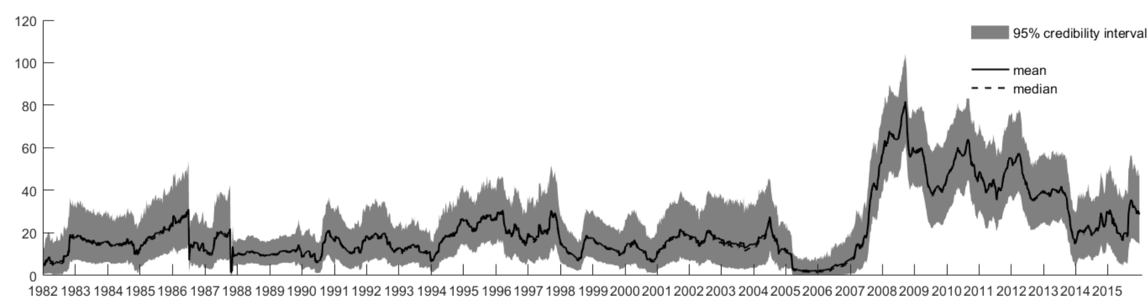
(a) μ'



(b) σ



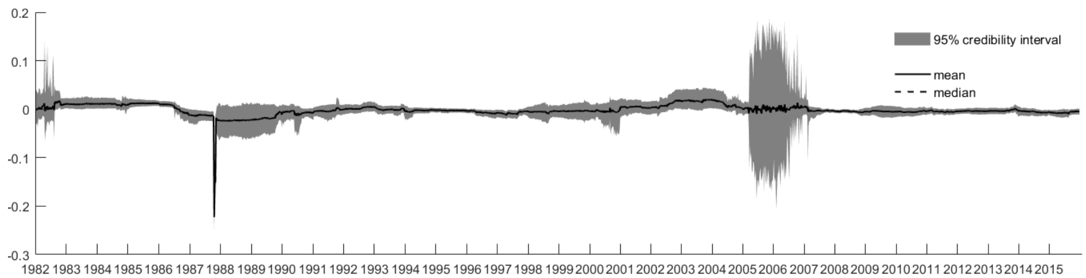
(c) λ



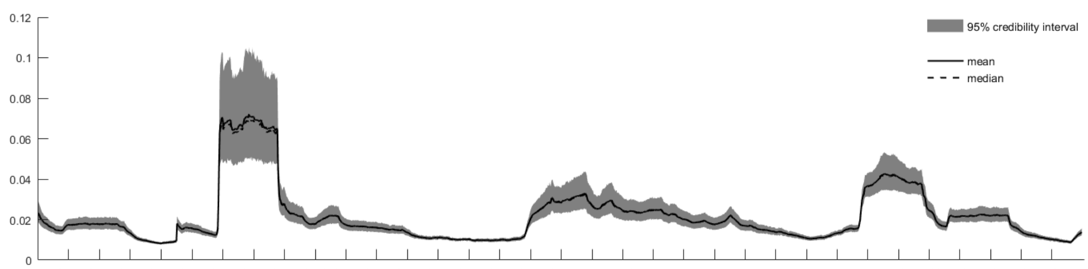
(continued)

Figure 3.1: Posterior Parameter Estimation Results of the Merton's Jump-Diffusion Models, Continued

(d) a

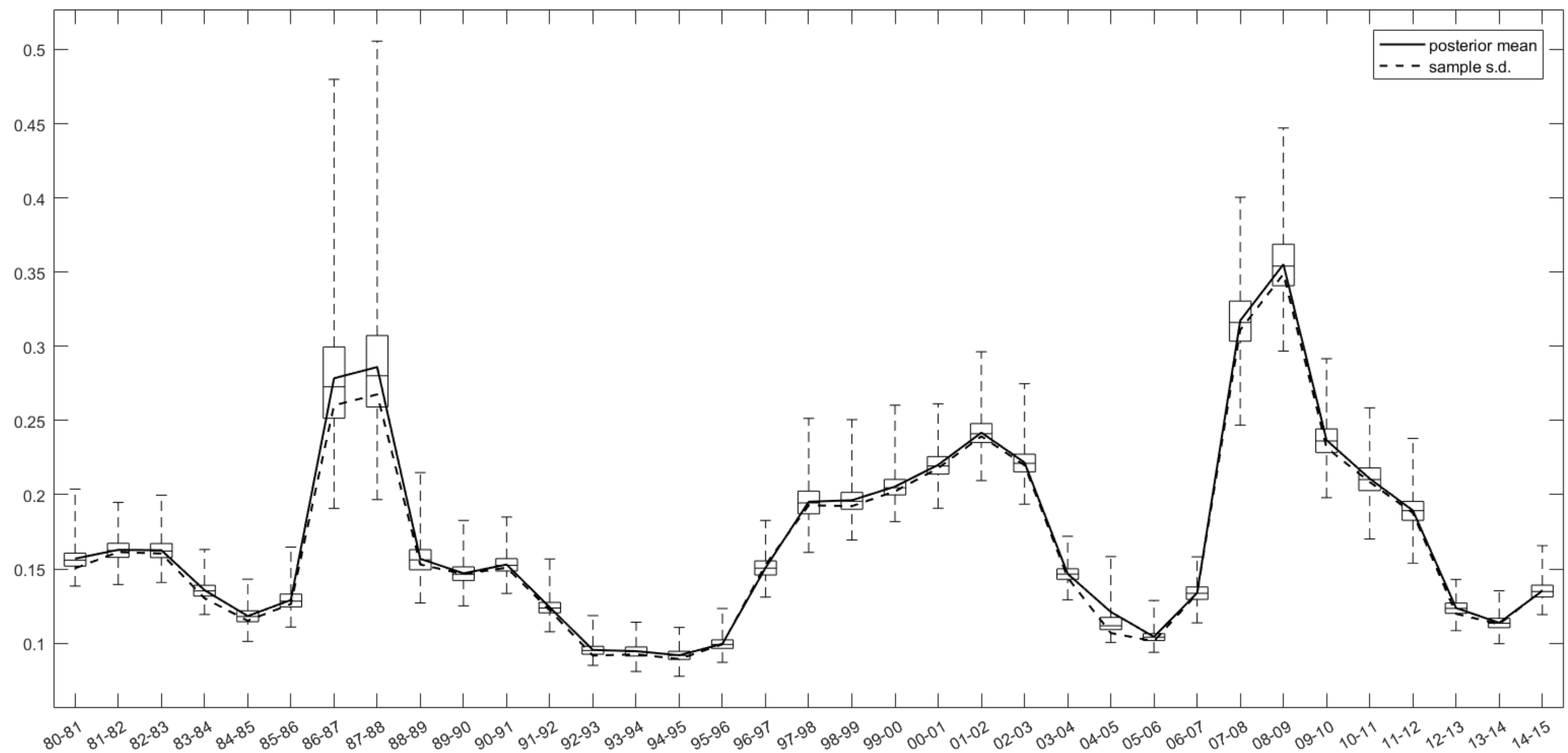


(e) ζ



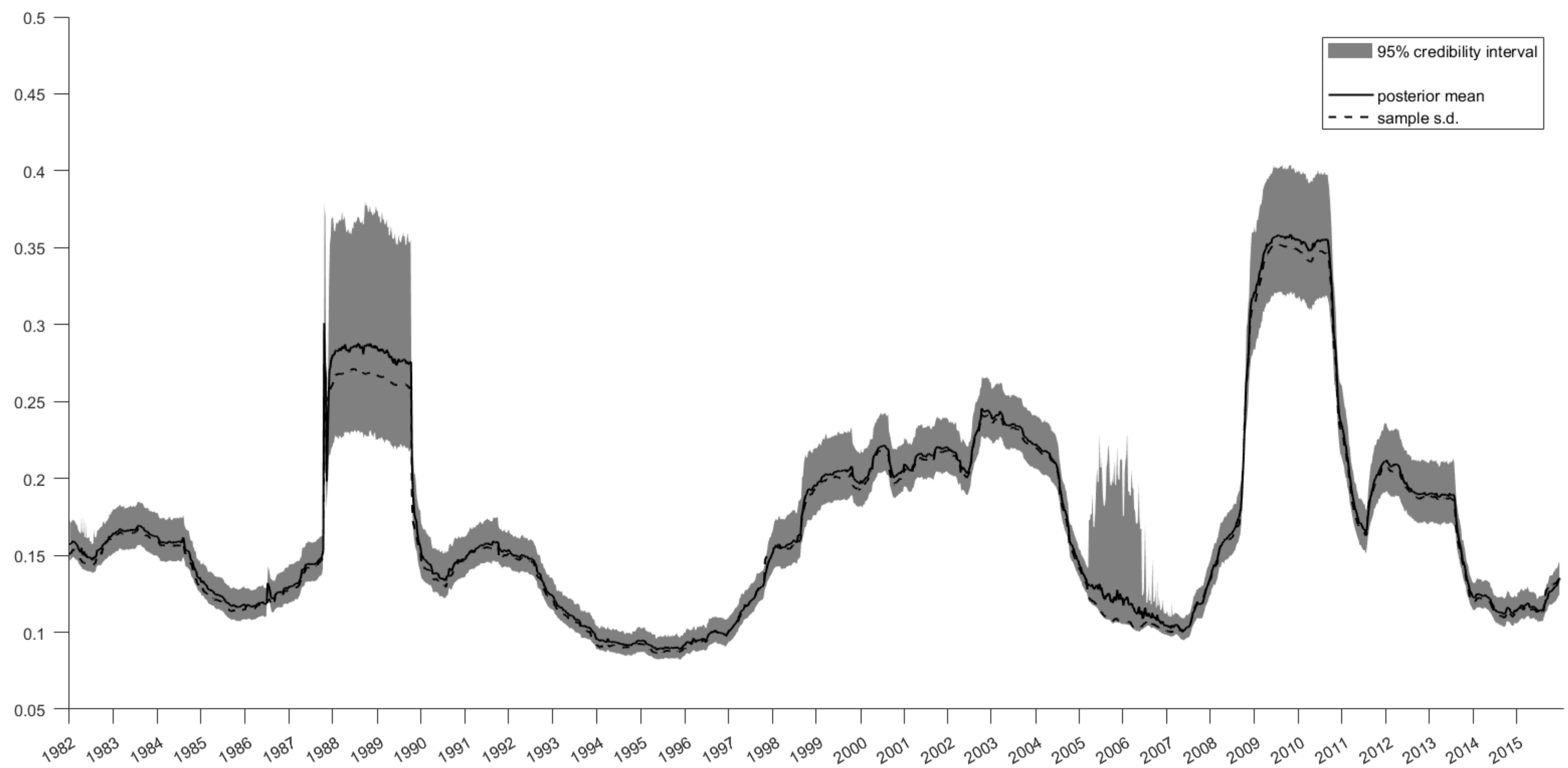
Note: The figure plots the 95% credibility interval, posterior mean and median of the parameters of the MJD model. Values of μ', σ and λ are annualised. $\mu' = \mu - \delta$. Parameters are estimated using 2-years moving window approach; Parameter estimation data: S&P 500 index daily log-return data 1980-2015.

Figure 3.2: S&P 500 Index Historical Volatility 1980-2015 Boxplots



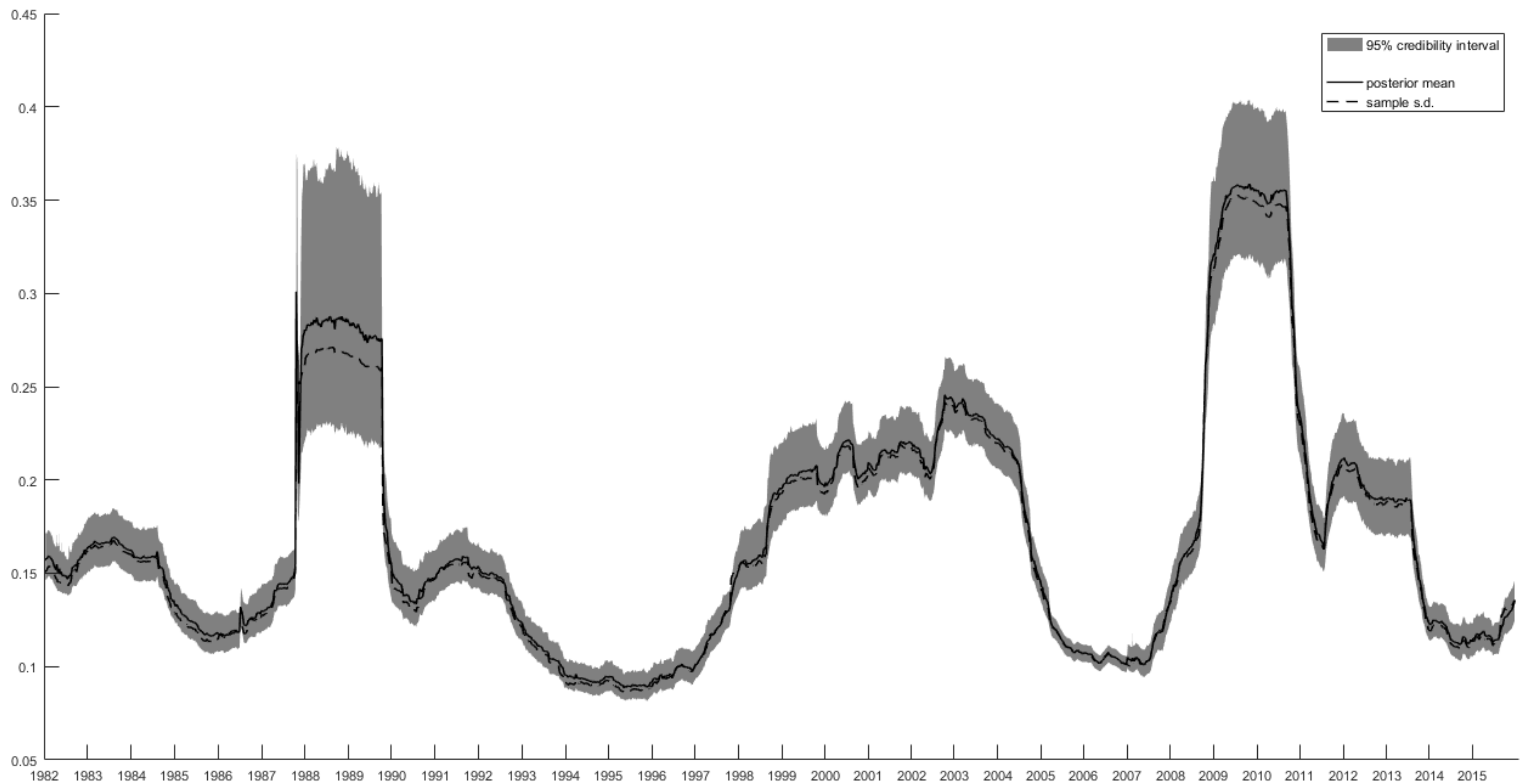
Note: the figure shows box-plots of the posterior total return volatilities under the MJD model during different periods. Parameter estimation data: S&P 500 index daily log-return data 1980-2015.

Figure 3.3: S&P 500 Index Historical Volatility 1982-2015 Time-series Plot



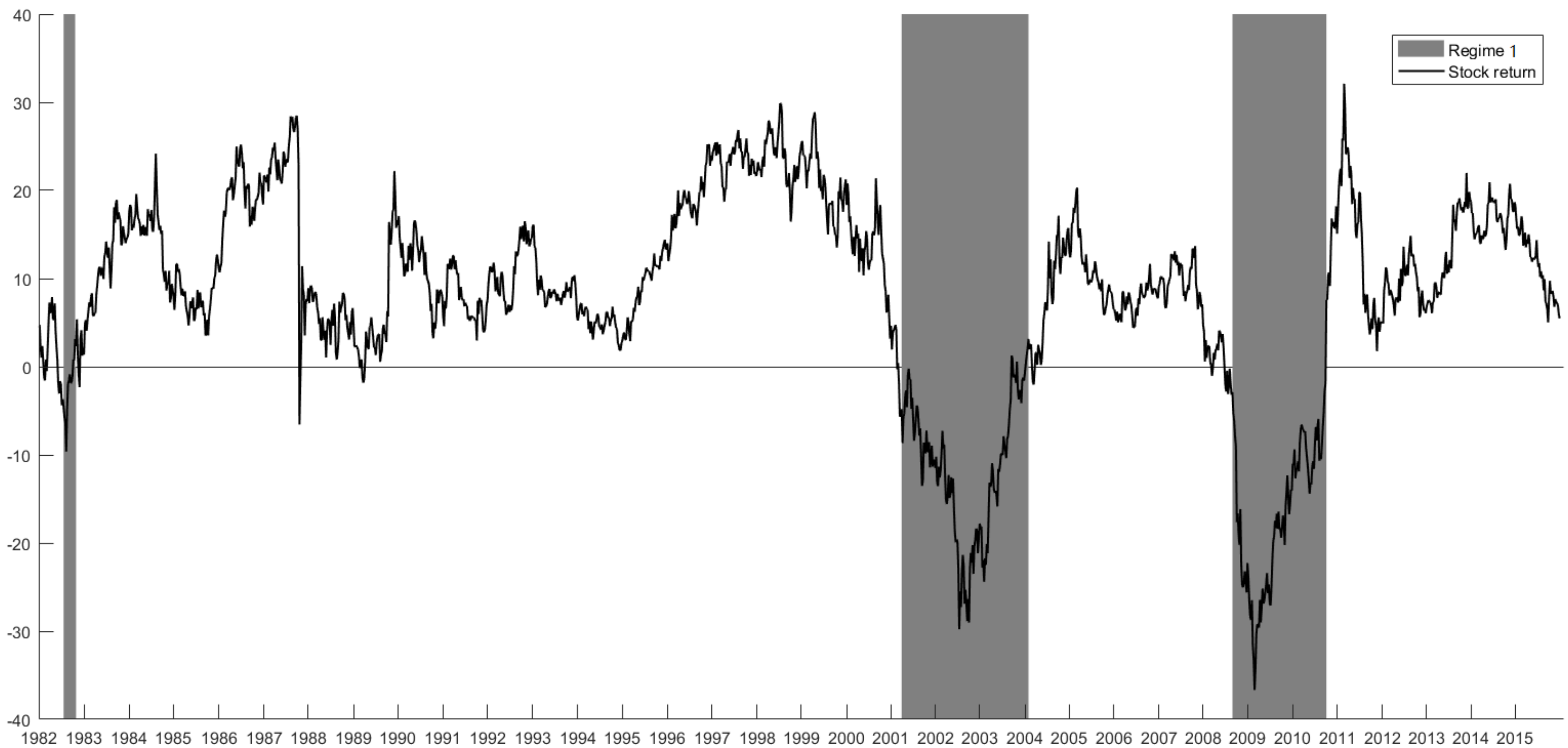
Note: the figure shows the time-series plot of the posterior total return volatilities under the MJD model during different periods. Parameters are estimated using 2-years moving window; Parameter estimation data: S&P 500 index daily log-return data 1980-2015.

Figure 3.4: S&P 500 Index Historical Volatility 1982-2015 Time-series Plot (Corrected)



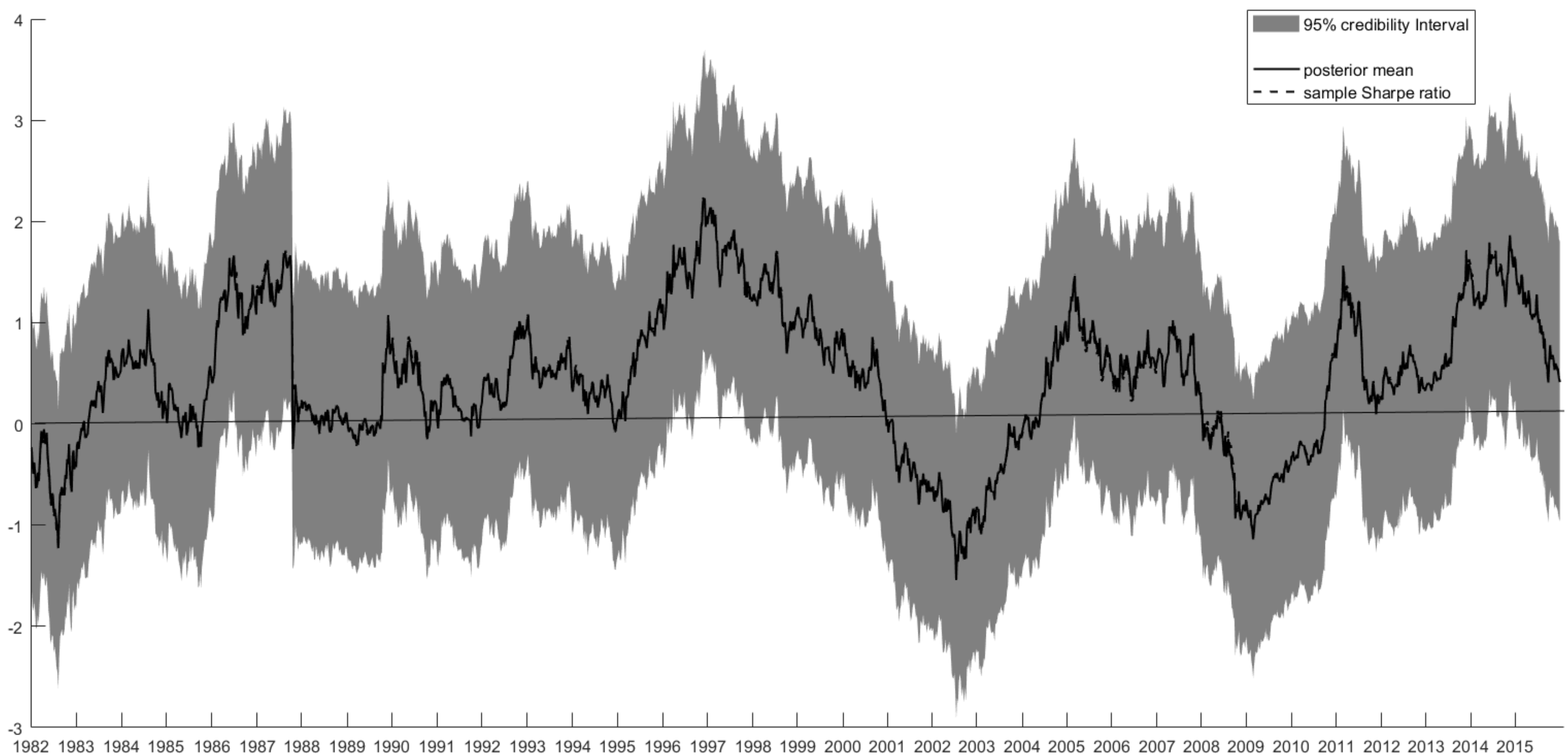
Note: the figure shows the time-series plot of the posterior total return volatilities under the MJD model during different periods (corrected for model uncertainty). Parameters are estimated using 2-years moving window; Parameter estimation data: S&P 500 index daily log-return data 1980-2015.

Figure 3.5: Regime Switch of S&P 500 Index Return



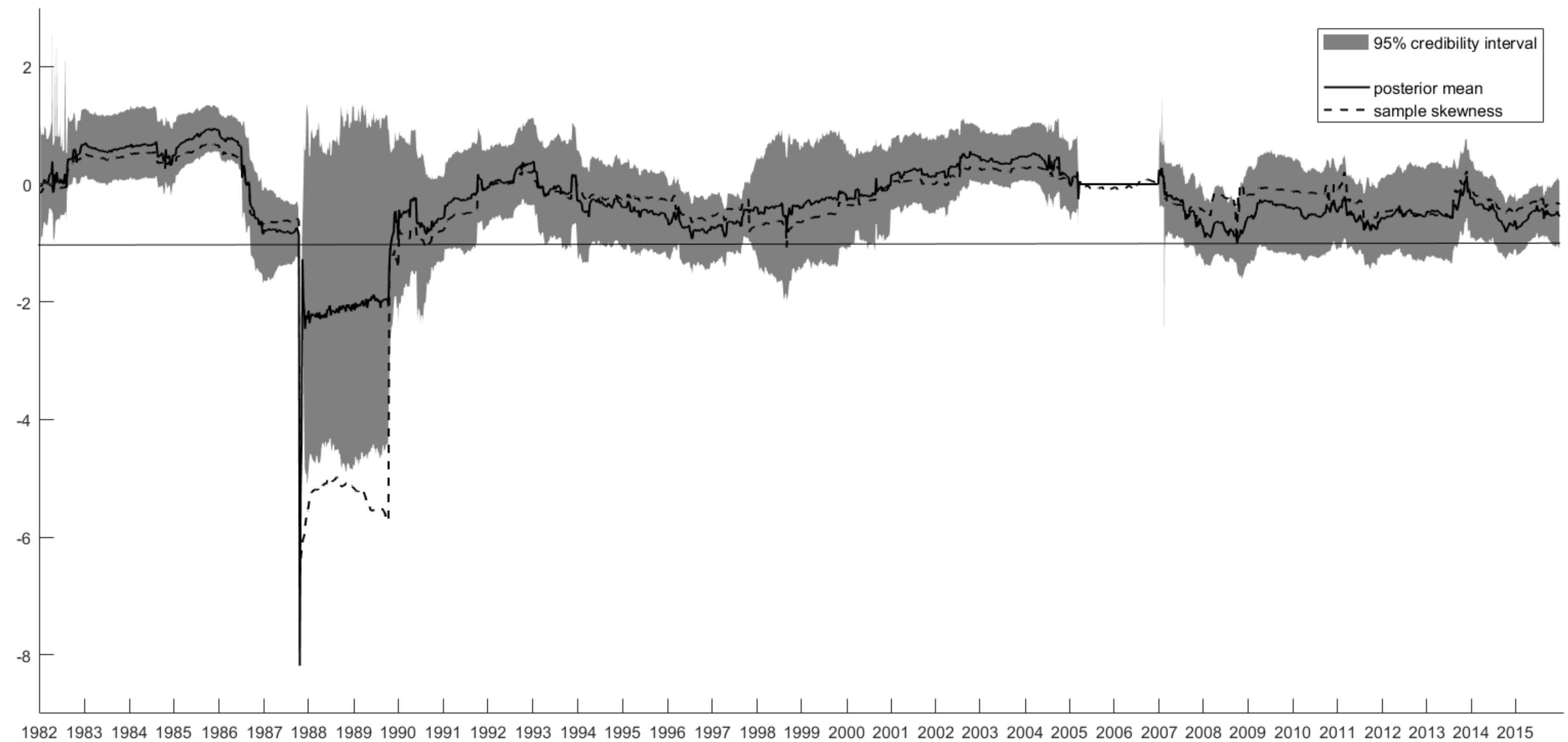
Note: The figure shows the regime switch results alone the time-series of estimated posterior return of S&P 500 index. Posterior return are estimated using 2-years moving window approach; Parameter estimation data: S&P 500 index daily log-return data 1980-2015.

Figure 3.6: S&P 500 Index Sharpe Ratio 1982-2015 Time-series Plot



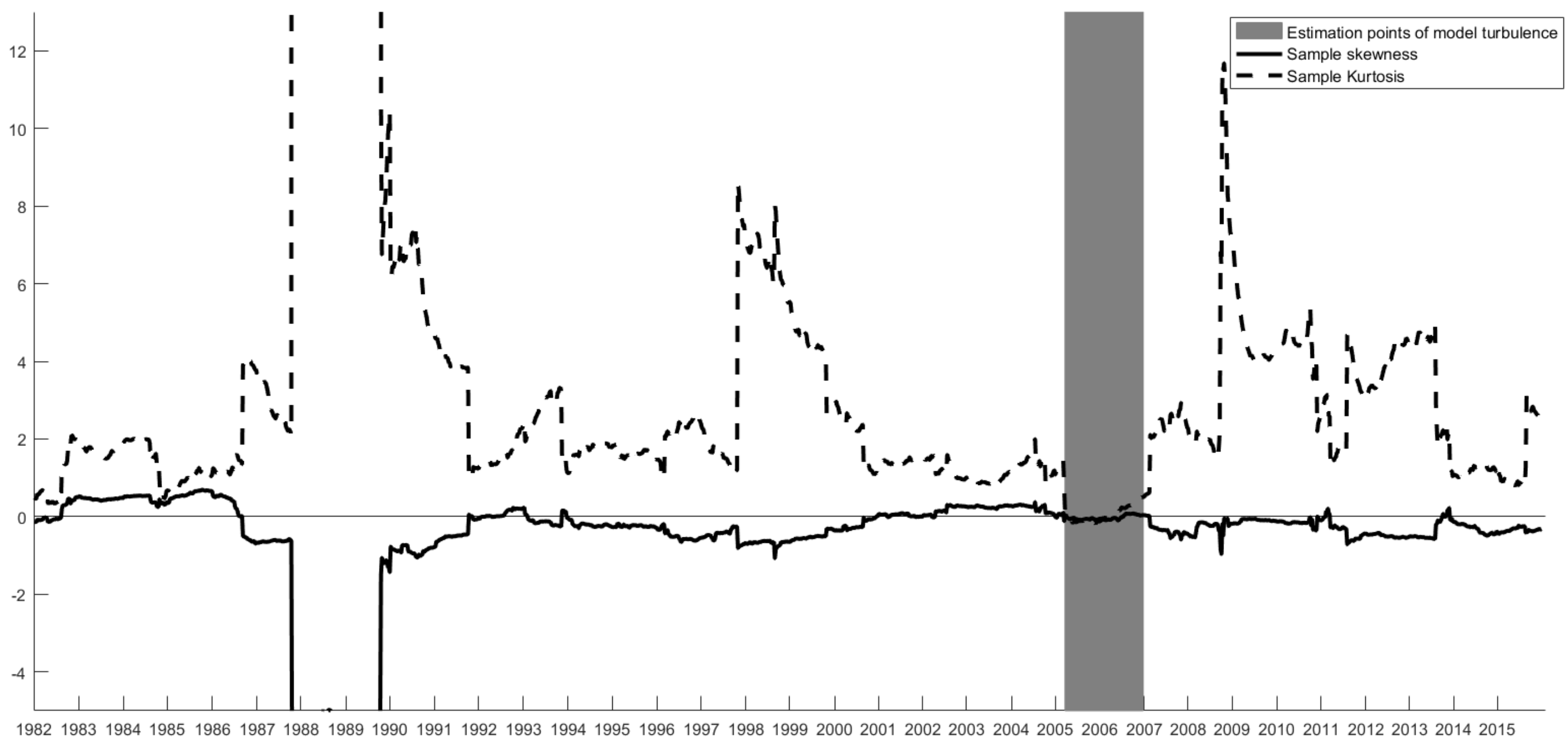
Note: The figure shows the time-series plot of the posterior Sharpe ratio of S&P 500 index calculated using estimated stock return and volatility under the MJD model. Parameters are estimated using 2-years moving window; Parameter estimation data: S&P 500 index daily log-return data 1980-2015.

Figure 3.7: S&P 500 index Historical Skewness 1982-2015 Time-series Plot (Corrected)



Note: the figure shows the time-series plot of the posterior return skewness under the MJD model during different periods (corrected for model uncertainty). Parameters are estimated using 2-years moving window; Parameter estimation data: S&P 500 index daily log-return data 1980-2015.

Figure 3.8: S&P 500 Index Historical Sample Skewness and Kurtosis 1982-2015



Note: The figure shows time-series plot of sample skewness and kurtosis calculated using 2-years moving window; Data: S&P 500 index daily log-return data 1980-2015. The grey-shaded periods are those when the MJD model indicates a calm period.

Table 3.1: Parameter Estimation Results under the Merton's Jump-Diffusion Models

Parameter	Mean	Std. Err.	Maximum	Minimum
$\mu^{'}$	9.7884	11.1698	34.2800	-31.0500
σ	13.44067	3.9921	24.0600	6.9740
λ	22.6723	14.6493	81.7740	1.4079
a	-0.2043	1.1681	1.9670	-22.2800
ζ	2.1807	1.3802	7.2120	0.8242

Note: Statistics shown above are mean, std. dev., maximum and minimum of posterior means of estimated parameters of each estimation window. Statistics of μ', σ and λ are annualised results. $\mu' = \mu - \delta$. Parameters are estimated using 2-years moving window approach; Parameter estimation data: S&P 500 index daily log-return data 1980-2015.

Table 3.2: S&P 500 Index Posterior Total Return Volatilities during 1980-2015 under the MJD model

Year	Special Events	Sample Volatility	MJD Total Volatility				
			Mean	Std. Err.	2.5%	Median	97.5%
1980-1981		15.04%	15.69%	0.81%	14.57%	15.59%	17.33%
1982-1983		16.04%	16.26%	0.72%	14.96%	16.21%	17.79%
1984-1985		11.51%	11.83%	0.56%	10.84%	11.79%	13.03%
1986-1987	"Black Monday" stock market break	26.02%	27.84%	3.79%	21.97%	27.27%	36.91%
1988-1989		15.30%	15.68%	1.04%	13.88%	15.61%	17.94%
1990-1991		15.08%	15.29%	0.62%	14.21%	15.24%	16.66%
1992-1993		9.16%	9.54%	0.41%	8.84%	9.50%	10.45%
1994-1995		8.93%	9.18%	0.42%	8.41%	9.17%	10.09%
1996-1997		15.28%	15.08%	0.70%	13.79%	15.05%	16.53%
1998-1999		19.23%	19.63%	0.90%	18.05%	19.56%	21.62%
2000-2001	"dot-com" crisis	21.79%	22.01%	0.91%	20.43%	21.94%	23.97%
2002-2003		21.99%	22.17%	0.89%	20.62%	22.12%	24.10%
2004-2005		10.68%	12.10%	2.89%	10.51%	11.17%	21.35%
2006-2007		13.33%	13.38%	0.61%	12.26%	13.36%	14.61%
2008-2009	Global financial crisis	34.87%	35.53%	2.09%	31.71%	35.41%	39.92%
2010-2011		20.85%	21.08%	1.15%	19.05%	21.02%	23.59%
2012-2013		11.98%	12.37%	0.49%	11.48%	12.34%	13.38%
2014-2015		13.57%	13.52%	0.59%	12.46%	13.48%	14.78%

Note: The table shows estimated posterior total return volatilities of S&P 500 index. Reported statistics include: posterior mean, median, s.d. and 95% credibility interval. Parameter estimation data: S&P 500 index daily log-return data 1980-2015

Table 3.3: Markov-switching Dynamic Regression Model Results of S&P 500 Index Return

Parameters	Coef.	Std. Err.	P-value	[95% Conf.	Interval]
c_1	-13.2132	0.4713	0.0000	-14.1369	-12.2896
c_2	12.0617	0.1923	0.0000	11.6847	12.4387
b	7.1779	0.1228		6.9411	7.4227
P_{11}	0.9884	0.0064		0.9662	0.9961
P_{21}	0.0020	0.0011		0.0006	0.0062

Note: the table reports parameter estimation results of a Markov-switching dynamic regression model on estimated posterior 2-years moving average return with constant variance. c_1 and c_2 are the mean of each regime, b is the standard deviation of both regimes. P_{11} is the probability of observing state 1 when the previous state is state 1, P_{21} is the probability of observing state 1 when the previous state is state 2.

Table 3.4: Normal Distributed Periods of Key Financial Market Indices Returns 2003-2007

Equity Indices	Jarque-Bera Test	Shapiro-Wilk Test
S&P 500	03/2003 - 12/2006	03/2003 - 11/2006*
Dow Jones Industrial Average	03/2003 - 12/2006	03/2003 - 11/2006
NASDAQ Composite	03/2003 - 12/2006	03/2003 - 12/2006
EURO STOXX 50	non-normal	non-normal
FTSE 100	3 periods**	5 periods***
DAX 30	non-normal	non-normal
NIKKEI 225	non-normal	non-normal
Hang Seng 50	non-normal	non-normal
Shanghai Shenzhen CSI300	non-normal	non-normal

Note: The table shows Normal distributed return periods of key market indices as indicated by the JB and SW tests at 5% critical level. Any 2-years period within the stated time interval above has normal distributed daily log returns. Tests are carried out using 2-years moving window on indices daily log-return data Mar 2003 - Dec 2006, windows move weekly.
*for SW tests at 5% critical level, daily log returns of 2-years periods ending 27/07/2005 - 03/08/2005, 06/10/2005 - 27/10/2005, 10/11/2005 - 01/02/2006, and 15/02/2006 - 09/03/2006 are non-normal. However, at 1% critical level, the null hypothesis of normal distribution cannot be rejected during these periods.
**daily log returns of 2-years periods ending 24/04/2006, 01/05/2006 and 08/05/2006.
***daily log returns of 2-years periods ending 04/05/2005, 08/06/2005, 24/04/2006, 01/05/2006 and 08/05/2006.

Table 3.5: Market Main Drivers During the Period of Normal Distributed Returns

	Full Sample		Period of Normal Returns					
	Mean	s.d.	Mean	s.d.	Maximum		Minimum	
					Value	Percentile in Full Sample	Value	Percentile in Full Sample
Return	8.18	11.66	8.54	2.29	15.71	74.55	4.43	24.81
Volatility	16.83	6.45	10.81	0.56	12.29	30.58	10.12	10.29
VIX	19.88	5.57	14.21	1.11	16.75	40.29	12.81	2.02
UOM Index	86.02	12.70	90.04	5.95	103.80	91.55	74.20	21.18
NIPO	356.24	236.79	156.07	65.84	226.00	43.04	33.00	1.97
Credit Spread	8.22	2.93	5.95	0.28	6.45	33.22	5.41	14.24

Note: Return and Volatility are sample return and realised volatilities of each of the 1715 estimation windows; VIX is the implied volatility of S&P 500 index options; UOM index is the University of Michigan index of consumer sentiment; NIPO is the sum of IPO volume in the past 12 months; Credit Spread is the series of average yield spread of Moody's Baa and AAA rated bonds. Full sample is the period from 1980-2015; Period of Normal Returns is the time interval of normal distributed returns defined by the JB test results of S&P 500 index stated in Table 3.4.

Table 3.6: Correlation of Sentiment Proxies and Future Returns

	$Return_{t+1}$	Δ UOM Index	$\Delta PERatio$	$\Delta CreditSpread$
$Return_{t+1}$	1.0000			
Δ UOM Index	-0.0932	1.0000		
Δ NIPO	-0.0717	0.1176	1.0000	
Δ Credit Spread	-0.2217	0.0662	0.2134	1.0000

Note: $Return_{t+1}$ is the 1-quarter ahead future log return of S&P 500 index adjusted for dividend; ΔUOM index is quarterly change of the University of Michigan index of consumer sentiment; ΔNIPO is the quarterly change of the sum of IPO volume in the past 12 months; Δ Credit Spread is the quarterly change of the average yield spread of Moody's Baa and AAA rated bonds. Data period: 1982-2015

Table 3.7: Regression Results of Future Returns and Normality

Forecast Horizon	1-Quarter	2-Quarters	4-Quarters	6-Quarters	8-Quarters
Δ UOM $Index_t$	-0.1499	-0.2395*	-0.0588	0.0359	-0.0014
	(0.2120)	(0.0834)	(0.7420)	(0.8652)	(0.9954)
Δ $NIPO_t$	-0.0019	-0.0021	-0.0145	-0.0234	-0.0206
	(0.8570)	(0.9141)	(0.6606)	(0.5608)	(0.6393)
Δ Credit $Spread_t$	-4.7748***	-9.1361***	-10.2179***	-11.1620***	-8.6627*
	(0.0040)	(0.0001)	(0.0023)	(0.0051)	(0.0502)
$Normality_t$	0.2714	4.9172	8.7463	8.1299	-0.5216
	(0.9170)	(0.3552)	(0.3766)	(0.5597)	(0.9766)
Industrial $Production_t$	0.1439	0.1682	0.4021	0.7750	0.9612
	(0.3910)	(0.5971)	(0.4880)	(0.3170)	(0.3018)
$Return_t$	0.0913	-0.3837	-1.0149***	-0.9948**	-0.8264*
	(0.3490)	(0.1567)	(0.0048)	(0.0191)	(0.0990)
Constant	1.7337**	5.9469	4.8165	2.4259	-2.1144
	(0.0420)	(0.1727)	(0.5596)	(0.8338)	(0.8874)
Observation	134	134	132	130	128

Note: Dependent variables are the 1-,2-,4-,6- and 8-quarters ahead future returns of S&P 500 index adjusted for dividend; ΔUOM index is quarterly change of the University of Michigan index of consumer sentiment; ΔNIPO is the quarterly change of the sum of IPO volume in the past 12 months; Δ Credit Spread is the quarterly change of the average yield spread of Moody's Baa and AAA rated bonds; Normality is a dummy variable with value 1 if the daily returns of the past 2 years are normally distributed, and 0 otherwise; Industrial production is the annual percentage change of industrial production in quarterly frequency; $Return_t$ is the the current quarter log returns of S&P 500 index. Data period: 1982-2015.

Table 3.8: Regression Results of Future Returns and Normality Interacted with Sentiment Proxies

Future Returns	1-Quarter	2-Quarters	4-Quarters	6-Quarters	8-Quarters
Δ UOM $Index_t$	-0.2144	-0.1777	0.1668	0.4377*	0.5760*
	(0.1080)	(0.2673)	(0.4483)	(0.0991)	(0.0604)
Δ $NIPO_t$	0.0006	0.0060	-0.0034	-0.0096	-0.0064
	(0.9520)	(0.7544)	(0.9197)	(0.8118)	(0.8819)
Δ Credit $Spread_t$	-4.7527***	-8.9520***	-10.2903***	-12.3409***	-10.9578**
	(0.0050)	(0.0002)	(0.0061)	(0.0079)	(0.0385)
$Normality_t$	-0.6349	1.6342	1.6318	-3.4324	-17.3651
	(0.8200)	(0.7654)	(0.8764)	(0.8177)	(0.3554)
Δ UOM $Index_t * Normality_t$	0.3039	0.0353	-0.1987	-0.3706	-1.2162**
	(0.3960)	(0.9400)	(0.6648)	(0.4884)	(0.0396)
Δ $NIPO_t * Normality_t$	-0.0892	-0.1775*	-0.3114**	-0.3334*	-0.4446**
	(0.2100)	(0.0655)	(0.0308)	(0.0729)	(0.0498)
Δ Credit $Spread_t * Normality_t$	4.9887	11.0006	19.9673	43.3005**	36.0574*
	(0.6800)	(0.5819)	(0.3646)	(0.0329)	(0.0962)
Industrial $Production_t$	0.1603	0.2720	0.5850	0.9312	1.0476
	(0.3460)	(0.4001)	(0.3315)	(0.2485)	(0.2747)
$Return_t$	0.1158	0.0660	-0.0848	-0.1018	-0.2178
	(0.2470)	(0.6387)	(0.6586)	(0.6467)	(0.3893)
Constant	1.646806*	3.7729**	8.5624**	13.2065***	19.5377***
	(0.0540)	(0.0226)	(0.0104)	(0.0074)	(0.0026)
Observation	134	134	132	130	128

Note: Dependent variables are the 1-,2-,4-,6- and 8-quarters ahead future returns of S&P 500 index adjusted for dividend; ΔUOM index is quarterly change of the University of Michigan index of consumer sentiment; ΔNIPO is the quarterly change of the sum of IPO volume in the past 12 months; Δ Credit Spread is the quarterly change of the average yield spread of Moody's Baa and AAA rated bonds; Normality is a dummy variable with value 1 if the daily returns of the past 2 years are normally distributed, and 0 otherwise; Industrial production is the annual percentage change of industrial production in quarterly frequency; $Return_t$ is the the current quarter log returns of S&P 500 index. Data period: 1982-2015.

Chapter 4

Hedge Fund Return Forecast and Portfolio Selection in the Presence of Model Risk

4.1 Introduction

Hedge funds have attracted a great deal of attention during the last 20 years. Traditionally, these instruments are private investment vehicles available to high net-worth individuals or institutional investors. Following the recent launch of investable hedge fund indices or indices-linked instruments, small- and medium-sized investors can also access this type of investments. While hedge funds have traditionally outperformed other investment strategies (partly due to their weak correlation with other financial securities), they experienced a colossal hit during the 2008 global financial crisis, which revealed the interdependencies of these funds and the rest of the financial markets (Olmo and Sanso-Navarro, 2012; Panopoulou and Vrontos, 2015). All these developments have contributed to the growing body of literature which investigates the risk-return characteristics of hedge funds. A long list of linear and non-linear risk factors have been proposed, including the

Fung and Hsieh asset based factors, the Fama-French 3 factors and Carhart's momentum factor, other general macroeconomic and financial factors (key references of this strand include Agarwal and Naik (2004), Fung and Hsieh (2004) and Bali et al. (2011)).

Similar to the general asset pricing literature, the issue of identifying the 'correct' set of factors, usually referred to as 'model specification risk' or 'model uncertainty', remains open for hedge funds with no exception. The key reason for model specification risk is that existing pricing theories do not explicitly guide us on which factors should be included in the model to explain asset returns (Vrontos and Giamouridis, 2008). Based on different data sets, different econometric methods and the set of predictors, each study defines its own best model. Furthermore, hedge funds have some unique characteristics compared to the others: the investment is flexible regarding the variety of securities and type of positions the investments take; hedge funds investment activities are not closely monitored and are not subject to public disclosure. As a result, fund managers are encouraged to construct highly dynamic, complex trading strategies. Vrontos and Giamouridis (2008) assert that the lack of transparency and the variety of trading markets and strategies combinations of hedge fund investments further renders the true set of pricing factors virtually unknown. In addition, loadings of each risk factors are time-varying, which can be referred to as 'parameter estimation risk', as fund managers rebalance portfolios dynamically. Stressed by Fung and Hsieh (2004), Fung et al. (2008), Eling and Faust (2010) and Wegener et al. (2010), both parameter estimates and the set of relevant risk factors may change over time due to the time-varying exposure of hedge funds to systematic risks.

Several recent articles have explicitly investigated model risk in hedge fund return forecast and portfolio construction. Wegener et al. (2010) test predictability of different risk factors, taking into account non-linearity, heteroscedasticity, and time-varying exposures of hedge funds to risk factors.

Specifically, they consider time-varying exposures by 1) recalibrating model using a rolling window approach; 2) using structural break models; 3) reselecting risk factors at regular intervals. They found that applying structural break models induce over-fitting and deteriorate out-of-sample hit ratio. On the other hand, results support the necessity for reselection of the risk factors in each period.

Avramov et al. (2013) consider model uncertainty by combining the forecasts of single factor models, and taking into account estimation risk by selecting funds based on t-statistics of predictive return in portfolio construction. They show that the simple strategy of combining forecasts delivers superior performance compared with individual single factor models. Additionally, Panopoulou and Vrontos (2015) combine forecasts of univariate models as well as combining the entire information set into prediction models.They confirm that the dynamic portfolio based on simple combined forecasts delivers the best performance compared with other complex combined forecasts and combined information strategies.

Vrontos and Giamouridis (2008) apply multivariate GARCH models to model time-varying covariance/correlation of hedge fund returns and consider model uncertainty by selecting the most probable model using Bayesian stochastic search algorithm. Their empirical analysis shows that introducing dynamic covariance/correlation modelling improves hedge fund portfolio construction performance out-of-sample. Nevertheless, the full factor GARCH model with Bayesian model selection underperformed the simple full factor GARCH model. Vrontos et al. (2008) further developed the method by considering model uncertainty using Bayesian model averaging (BMA). They compare the economic value of Bayesian model averaging with Bayesian model selection, stepwise regression procedure method, AIC selection and BIC selection. Results of the study show that the BMA method improves the predictive performance of model. Portfolio construction performance of the BMA method clearly outperforms the stepwise and AIC selection methods, but this does not appear to be the case for the BIC and Bayesian model selection methods.

Vrontos (2012) build further upon the earlier methodology by using student's t GARCH model with Bayesian model averaging to take into account fat-tail, non-normality, time-varying covariances and model uncertainty in forecasting hedge fund returns. By assessing the predictability of each model specification, model averaging with normal errors and GARCH demonstrates better performance in terms of mean-squared prediction error and averaging prediction error. However, model averaging with student's t errors and GARCH performs the best in terms of predictive log score. Nevertheless, economic value (i.e. portfolio construction) of the proposed methodology is not investigated in the study.

This study contributes to the literature by applying the dynamic model averaging (DMA) methodology[1] in hedge fund return forecast and portfolio construction. This methodology is introduced by Koop and Korobilis (2012) in inflation forecast. Compared with the existing literature reviewed above, it advances the research on hedge fund return forecast and portfolio construction in several ways. Firstly, in consideration of model specification risk, the DMA technique dynamically updates model probabilities at each estimation point rather than applying a constant model probability as in the conventional BMA method. With regards to parameter estimation risk, the method includes a time-varying parameter model setting, and the parameter estimation error around the expected model return can be assessed by calibrating the posterior distributions of the parameters. Heteroscedasticity of return time series is modelled by the exponentially weighted moving average (EWMA) approach. Furthermore, by considering the non-linear risk

[1]The method has been applied recently in several financial research areas, including gold or copper prices forecast (Aye et al., 2015; Baur et al., 2014; Buncic and Moretto, 2015); house prices forecast (Bork and Møller, 2015; Risse and Kern, 2016); bond portfolio strategies selection (Caldeira et al., 2016); stock return forecast and portfolio construction (Pettenuzzo and Ravazzolo, 2016). Overall, studies show a considerable statistical and economic value in incorporating model risk using the underlying methodology.

factors advocated by Fung and Hsieh (2004) and Agarwal and Naik (2004) as predictors, we also capture the non-normality in hedge fund returns. To the best of our knowledge, this is the first time that the above issues are addressed jointly. Empirical analysis are based on both prediction statistics (e.g. MSFE, log predictive likelihood) and out-of-sample portfolio performances of models. We evaluate predictive ability on hedge fund indices as well as individual funds, and investigate portfolio construction performance based on individual funds.

Our results show that considering time-varying parameters and model specification uncertainty substantially improves out-of-sample predictive ability of models. Economic value of the predictions are strongly supported by the results of certainty equivalent return evaluation. In portfolio construction, competing models deliver outstanding results in terms of risk-return trade-off when hedge funds are selected based on expected future returns. However, higher absolute returns are generated by the proposed models when selections are based on t-statistics of model predictions. In addition, we find that the time-varying feature of both regression parameters and model probabilities can be of considerable value to hedge fund portfolio construction during crisis periods.

The rest of the chapter presents as follows: Section 4.2 details the methodology; Section 4.3 summarises the data and the set of predictors; Section 4.4 reports on the empirical analysis of predictive ability; Section 4.5 evaluates out-of-sample portfolio construction performance of models; and Section 4.6 provides an overall summary and draws conclusions.

4.2 Prediction Models

4.2.1 Time-Varying Parameter Model

The Kalman Filter originally developed in Kalman et al. (1960) and Kalman and Bucy (1961) is a simple optimal estimator for normal linear state-space models. It is a commonly employed method in diverse engineering areas, such as signal processing in aerospace and underwater sonar (Meinhold and Singpurwalla, 1983). The method is also employed by applied statisticians in Bayesian forecasting (e.g. Harrison and Stevens, 1971, 1976). The Kalman Filter provides a simple way to recursively forecast the unobservable states (i.e. coefficients of prediction factors in linear regression models) given new observations under the Bayesian framework. The normal linear state-space model can be understood as a linear regression model with time-varying parameters (Koop and Korobilis, 2012; Meinhold and Singpurwalla, 1983).

The normal linear state-space model is of the following form:

$$\begin{aligned} R_t &= z_t\theta_t + \varepsilon_t \\ \theta_t &= \theta_{t-1} + \eta_t \end{aligned} \tag{4.1}$$

R_t is the hedge fund return; $z_t = \mathbf{X}_{t-1}$ is the vector of predictors with the first column being 1; θ_t is the vector of coefficients; ε_t being the regression residual of hedge fund returns, $\varepsilon_t \sim N(0, H_t)$; η_t is the residual of predictive coefficients, and $\eta_t \sim N(0, Q_t)$. The two error terms are assumed to be independent from each other.

Equivalently, the above state-space model can be interpreted as a linear regression model with time-varying parameter settings, where R_t and z_t are the dependent and independent variables, and θ_t is the vector of coefficients we seek to estimate. Different from traditional constant coefficient models, time-varying parameter (TVP) models allow coefficents to vary among each observation of data.

The ordinary Kalman filter begins with the posterior result of parameter that

$$\theta_{t-1} \mid R^{t-1} \sim N(\hat{\theta}_{t-1}, \Sigma_{t-1|t-1}) \tag{4.2}$$

where R^{t-1}, with upper subscript, stands for the information set of hedge fund return from time 0 to time $t-1$. The Kalman filter proceeds to the following predicting step:

$$\begin{aligned} \theta_t \mid R^{t-1} &\sim N(\hat{\theta}_{t-1}, \Sigma_{t|t-1}) \\ \Sigma_{t|t-1} &= \Sigma_{t-1|t-1} + Q_t \end{aligned} \tag{4.3}$$

To simplify the estimation of $\Sigma_{t|t-1}$, Raftery et al. (2010) adopted the following approximation:

$$\Sigma_{t|t-1} = \frac{1}{\upsilon}\Sigma_{t-1|t-1} \tag{4.4}$$

where $\upsilon(0 < \upsilon \leqslant 1)$ acts as a forgetting factor and is fixed to a number slightly below one, so that coefficients change gradually. Following the model specification, observations m periods in the past would be weighted by υ^m. Therefore, the impact of observations far in the past would be gradually forgotten in the estimation process. $\upsilon \to 1$ indicates that the coefficients evolved slower throughout time, and vice versa. When $\upsilon = 1$ the model returns to a constant parameter model. The important advantage of this simplification is that it eliminates the necessity to estimate or simulate Q_t.

Raftery et al. (2010) and Koop and Korobilis (2012) set the forgetting factor $\upsilon = 0.99, 0.95$ for quarterly data, so that in the case of $\upsilon = 0.99$ coefficients evolved slowly with data 5 years ago receiving a weight of 80% of the weight assigned to observation in the previous period. However, for $\upsilon = 0.95$, coefficients are more unstable, so observations 5 years ago only receive a weight of approximately 35% of the weight for the last period observation. For the monthly data of hedge fund return we used in this study, we set $\upsilon = 0.996, 0.985$ which allows similar speed of parameter evolution to monthly data.

When new data are observed, we proceed to the updating step of the Kalman filter:

$$\theta_t \mid R^t \sim N(\hat{\theta}_{t|t}, \Sigma_{t|t})$$

Where

$$\begin{aligned}\hat{\theta}_{t|t} &= \hat{\theta}_{t|t-1} + \Sigma_{t|t-1} z_t' (H_t + z_t \Sigma_{t|t-1} z_t')^{-1} (R_t - \hat{\theta}_{t|t-1} z_t) \\ \Sigma_{t|t} &= \Sigma_{t|t-1} - \Sigma_{t|t-1} z_t' (H_t + z_t \Sigma_{t|t-1} z_t')^{-1} z_t \Sigma_{t|t-1}\end{aligned} \tag{4.5}$$

The predictive distribution of recursive forecasting is:

$$R_t \mid R^{t-1} \sim N(\hat{\theta}_{t-1} z_t, H_t + z_t \Sigma_{t|t-1} z_t') \tag{4.6}$$

Therefore, results are analytical conditional on H_t, and no Markov Chain Monte Carlo (MCMC) algorithm is required in deriving the posterior distribution of parameters and the predictive distribution of hedge fund return.

Following Koop and Korobilis (2012), we adopt the EWMA estimator of H_t:

$$\hat{H}_t = \sqrt{(1-\kappa) \sum_{j=1}^{t} \kappa^{j-1} (R_j - \hat{\theta}_j z_{j-1})} \tag{4.7}$$

From the specification of EWMA formulae, forecast of H_t given information up to time $t-1$ has the analytical form:

$$\hat{H}_{t|t-1} = \kappa \hat{H}_{t-1|t-2} + (1-\kappa)(R_{t-1} - z_{t-1} \hat{\theta}_{t-1}) \tag{4.8}$$

$\kappa \in [0, 1]$ is the decay factor of the EWMA estimator, which plays a similar role to the forgetting factor υ. We adopt three different values 0.97, 0.94 and 0.92 to allow different speeds of decay.

Given all the specifications, the system can be estimated given initial conditions for θ_0 and H_0 [2].

[2] In this study, $\theta_0 \sim N(0, \frac{Var(R^{t-1})}{Var(z^{t-1})})$; $H_0 = \frac{1}{4} Var(R^{t-1})$. t is the first out of sample period.

One important drawback of the TVP model is that the set of predictors remains unchanged throughout time; thus, all of them are assumed to be relevant at all points in time. Conversely, a hedge fund manager would dynamically adjust the fund composition according to selection technique and knowledge; a constant set of relevant predictors is unlikely to hold true in hedge fund return prediction. When the number of predictors chosen is large, the problem of in-sample over-fitting is likely to be substantial which deteriorates out-of-sample forecasting performance.

4.2.2 Dynamic Model Averaging

The dynamic model averaging (DMA) technique is an extension of the conventional Bayesian model averaging. Different from the conventional Bayesian model averaging, the DMA technique allows the weights of each model to evolve dynamically and pay more attention to closer term information. Like the Bayesian model averaging, DMA enables forecast result to be based on the results of all candidate models (i.e. the full combination set of proposed independent variables); results of each individual model are weighted according to their predictive fit in the most recent periods.

For the consideration of m potential predictors, we would have a total number of $K = 2^m$ models[3] to evaluate. The DMA prediction of hedge fund return is:

$$E(R_t \mid R^{t-1}) = \sum_{k=1}^{K} \pi_{t|t-1,k} R_{t|t-1}^{(k)} \tag{4.9}$$

where $R_{t|t-1}^{(k)}$ stands for predictions by each candidate model k; $\pi_{t|t-1,k}$ is the predictive probability of model k, and also the weight of the forecast of model k in the DMA prediction result.

Unlike the ordinal estimation of $\pi_{t|t-1,k}$ which requires the estimation of

[3]Linear combination of predictors only.

a transition matrix P with $K \times K$ dimensionl. Raftery et al. (2010) replace the model probability prediction equation with:

$$\pi_{t|t-1,k} = \frac{\pi^{\alpha}_{t-1|t-1,k}}{\sum_{l=1}^{K} \pi^{\alpha}_{t-1|t-1,l}} \tag{4.10}$$

$\alpha \in [0, 1]$ is a forgetting factor similar to υ in Section 4.2.1. The approximation is used as it is proved to be suitable and not too restrictive in other academic areas (Koop and Korobilis, 2012; Raftery, 1995; Smith and Miller, 1986) . If $\alpha = 1$, we obtain the standard Bayesian model averaging (BMA). Similarly, we assigned fixed values near one to $\alpha = 0.996, 0.985$.

The updated equation of model probability is given as:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} p_k(R_t \mid R^{t-1})}{\sum_{l=1}^{K} \pi_{t|t-1,l} p_l(R_t \mid R^{t-1})} \tag{4.11}$$

where $p_k(R_t \mid R^{t-1})$ is the predictive density of (4.6) evaluated given R_t. Recall The Bayesian theorem:

$$p(M_i \mid D_{t-1}) \propto p(D_{t-1} \mid M_i) p(M_i) \tag{4.12}$$

The numerator of equation (4.11) can be more easily reconciled to equation (4.12). The denominator of equation (4.11) is to ensure that the posterior probabilities of all candidate models add up to 1. The DMA system can be estimated given an initial condition for $\pi_{0,k}$ [4].

4.2.3 Dynamic Model Selection

The dynamic model selection method predicts hedge fund return by using the forecast of the model with the highest predictive density $\pi_{t|t-1,k}$ in each period.

[4] $\pi_{0,k} = 1/\text{K}$.

4.2.4 List of Prediction Models

In the empirical analysis of this study, we employ the OLS-AR(1) model as the benchmark model:

$$R_t = \beta_1 + \beta_2 R_{t-1} \tag{4.13}$$

Competing models are classified into six categories: DMA, DMS, BMA, BMS, TVP-AR(1) and TVP-ALL models with different parametric values of α, υ and κ:

DMA: 6 DMA models. Dynamic model averaging over the full combination set of 15 predictors[5] with time-varying parameter settings (forgetting factors $\alpha = \upsilon = 0.996\, or\, 0.985$); Heteroscedasticity captured by the EWMA model with decay factor $\kappa = 0.92,\ 0.94\, or\, 0.97$.

DMS: 6 DMS models. Dynamic model selection models over the full combination set of 15 predictors with time-varying parameter and heteroscedasticity settings, assigned same parametric settings as the DMA models.

BMA: 3 BMA models. Bayesian model averaging over the full combination set of 15 predictors with constant paramter settings (i.e. with forgetting factors$\alpha = \upsilon = 1$); Heteroscedasticity captured by the EWMA model with decay factor $\kappa = 0.92,\ 0.94\, or\, 0.97$.

BMS: 3 BMS models. Bayesian model selection over the full combination set of 15 predictors with constant paramter and heteroscedasticity settings, assigned the same parametric settings as the BMA models.

TVP-AR(1): 6 TVP-AR(1) models. Linear regression models with the constant and AR(1) terms as predictors and time-varying parameter settings (forgetting factors $\upsilon = 0.996\, or\, 0.985$); Heteroscedasticity captured by the EWMA model with decay factor $\kappa = 0.92,\ 0.94\, or\, 0.97$.

TVP-ALL: 6 TVP-ALL models. Linear regression models with 15 predictors, AR(1) term and time-varying parameter settings (forgetting factors

[5]Number of candidate models K = 2^{15} = 32768

$\upsilon = 0.996\, or\, 0.985$); Heteroscedasticity captured by the EWMA model with decay factor $\kappa = 0.92,\ 0.94\, or\, 0.97$.

4.3 Data

Our hedge fund data set is from the database of BarclayHedge. The return series are between 01/1994 to 12/2014. We adopt recursive estimation and forecasting, the in-sample period ends at 12/2001, so we have 13 years (156 periods out-of-sample). Filters are applied to the database following academic and empirical practices (Avramov et al., 2011; ODoherty et al., 2015). We include funds of which returns are reported in USD and net of all fees, funds that are not closed to new investment, and funds that with asset under management (AUM) greater than $10m. We exclude those with asset under management (AUM) not uniquely listed, meaning returns of the exact same assets could be reported under a different fund ID, leading to duplication. We also exclude the first 12 months of return data of a fund to mitigate the impact of backfill/selection bias. Selection bias arises from the typical practice that hedge fund managers choose to enter a hedge fund into a database voluntarily. Naturally, only funds with good past track records would be chosen to advertise for and attract outside investors (Avramov et al., 2011). Moreover, there is a backfill period where the managers backfill its past performance. Therefore, managers are unlikely to backfill if past return record is bad (ODoherty et al., 2015).

To ensure that sufficient data are available for the estimation period, forecasting starts only when a fund accumulates at least 8 years (96 data points) of historical data. Therefore, after excluding the first 12 months of return data, each fund would have at least seven years (84 periods) of data in the sample. Each hedge fund time series are tested against unit-root using the Augmented Dickey-Fuller test, while non-stationary time series are excluded. Finally, we classify all individual funds into eleven strategies

(CTA, Emerging Markets, Event Driven, Fund of Funds, Global Macro, Long Only, Long/Short, Market Neutral, Multi-Strategy, Relative Value, Sector) following the strategy mapping method provided by Joenväärä et al. (2016). Funds that do not fall into these eleven strategies are excluded. Our final data contain 926 live funds (consisting of 265 Fund of Funds) and 1043 graveyard funds (consisting of 316 Fund of Funds). Excluding fund of funds, the total amount of funds used for portfolio construction analysis is 1388 (661 live funds and 727 graveyard funds).

[Table 4.1 about here.]

Table 4.1 Panel A reports summary statistics of the sample data. The entire sample exhibits negative skewness and excess kurtosis with an average monthly return of 0.72%. The most common trading strategies are fund of funds (581 funds) and equity long/short (444 funds). Fund of funds also accounts for the highest proportion of total asset under management ($162.50 billions), followed by Equity long/short ($97.89 billions) and CTA ($95.98 billions). Emerging market strategy shows the highest monthly return 0.91% and the highest standard deviation 5.89%. Whereas, fund of funds presents the lowest monthly return 0.47% and the lowest standard deviation 2.02%. Similarly, market neutral strategy also has a low standard deviation 2.03%. All strategies exhibit negative skewness over the sample period, except for CTA and Global Macro. Market neutral and equity long/short have close-to-zero skewness. Return distributions of all strategies are fat tailed. A summary of live funds and graveyard funds show a similar picture, except that for live funds, global macro shows the highest return and market neutral gives the lowest return volatility.

We adopt an extensive list of hedge fund return predictors that has achieved considerable prediction power according to the literature (Amenc et al., 2003; Bali et al., 2012, 2013; Fung and Hsieh, 2004; Vrontos, 2012; Agarwal and Naik, 2004; Wegener et al., 2010; Panopoulou and Vrontos, 2015). The first set of predictors includes the Fung and Hsieh's asset based factors: bond, currency, commodity, short-term interest rate and stock index lookback straddle. The Fama-French factors: SMB and HML, and the Carhart's momentum factor are also included. Other macro variables, including the S&P500 monthly log-return; change in VIX; change in 10Y T-bill yield; change in 3M T-bill yield; annual growth rate of industrial production in monthly frequency; monthly log-return of MSCI world index excluding the US.

4.4 Predictive Ability

4.4.1 Statistical Evaluation of Forecasts

To assess the predictability of each model, we provide a snap shot of the statistical evaluation of self-constructed hedge fund indices on the eleven different trading strategies CTA, emerging market (EM), event driven(ED), fund of funds(FF), global macro (GM), equity long only (LO), equity long/short(LS), market neutral(MN), multi-strategy(MS), relative value (RV) and Sector. Indices are constructed based on either an equally weighted approach or an AUM weighted approach.

To assess the forecast accuracy, we employ the measure of Theil's U:

$$Theil's\,U = \frac{MSFE_i}{MSFE_{OLS-AR(1)}} \tag{4.14}$$

where $MSFE_i$ is the mean squared forecast error of competing model i over the out-of-sample period, and $MSFE_{OLS-AR(1)}$ is the mean squared forecast error the the benchmark OLS-AR(1) model. Theil's U value of less than 1 indicates superior forecasting accuracy of the competing model.

Table 4.2 reports the Theil's U with the t-value (Clark and West, 2006) of $MSFE_i$ compared with $MSFE_{OLS-AR(1)}$. Clark and West (2007) asserts that when comparing the MSFE of a parsimonious null model with a more complex model which is nested within the null model, the more complex alternative model is expected to have a greater MSFE than the null model when the extra parameter has no explanatory power to data generation. Assuming the extra parameters have no explanatory power, the parsimonious model assigns zero value to the parameters which are their population values. On the other hand, the alternative model must induce more noises to the forecast results as it assigns some values that are deviated from the population value zero to the extra parameters. Therefore, MSFE differences between the two models do not yield good test statistics to the hypothesis test of whether the alternative provides addition explanatory power compared to the null. Clark and West (2006, 2007) present a method of how to adjust MSFEs to account for the noises and to perform tests of equal forecast accuracy. Let $MSFE_1$ be the squared forecasting error of the null model (i.e. the OLS-AR(1) model in our context), and $MSFE_2$ be the squared forecasting error of the alternative model. $\hat{y}_{1,t}$ and $\hat{y}_{2,t}$ are the forecasts of null and alternative models over the period of $t = 1, 2, ..., N$, and y_t is the observed data. The test statistics of forecast accuracy is:

$$\begin{aligned}
\bar{T} &= N^{-1} \sum_{t=1}^{N} \hat{T}_t \\
&= MSFE_1 - (MSFE_2 - adj.) \\
adj. &= N^{-1} \sum_{t=1}^{N} (\hat{y}_{1,t} - \hat{y}_{2,t})^2 \\
\hat{T}_t &= (\hat{y}_{1,t} - y_t)^2 - [(\hat{y}_{2,t} - y_t)^2 - (\hat{y}_{1,t} - \hat{y}_{2,t})^2]
\end{aligned} \tag{4.15}$$

$\bar{T}$ defines the adjusted MSFE, and the t-value of whether the alternative model excels the benchmark model in forecast accuracy is calculated by $\sqrt{N}\bar{T}$/[standard deviation of $\hat{T}$]. Clark and West (2007) prove that the null hypothesis of the alternative model has equal predictive power as the null model shall be rejected if the t-statistic is greater than +1.282 (for a one-sided 0.05 - 0.10 test), and +1.645 (for a one-sided 0.01 - 0.05 test). When the noises accounted are substantial, it is possible for an competing model to have a Theil's U > 1 and a significant positive MSFEadj. t-value at the same time. A model shall deemed to have certain level of improvement in forecasting accuracy when either of the MSFE measures gives positive indication.

Results of both equally weighted indices and AUM weighted indices are tabulated in Table 4.2 and 4.3. Overall, the competing models exhibit most advantage in forecast accuracy in the CTA, FF, MN and MS trading strategies for the equally weighted indices. The most notable improvements in MSFEs are found in the CTA strategy, TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92/0.94/0.97$) shows a Theil's U of 0.916, 0.915 and 0.912 respectively with significant t-statistics. The competing models exhibit least advantage in forecasting the EM strategy, in which none of the models generate more accurate forecast than the OLS-AR(1) model.

For AUM weighted indices (Table 4.3), competing models exhibit most advantage in forecast accuracy in the ED, FF, GM, MN and MS strategies. The most significant improvements in MSFEs are found in the MN strategy, TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92/0.94/0.97$) shows a Theil's U of 0.859, 0.868 and 0.883 respectively with very significant t-statistics. The competing models exhibit least advantage in forecasting the CTA and EM strategy, in which almost none of the models generate a more accurate forecast than the OLS-AR(1) model. The result of CTA is quite controversial compared to the equally weighted indices, indicating potential difficulty of forecasting a few high AUM funds.

Moreover, the group of TVP-AR(1) models performs best among the others, followed by BMA models, BMS model and DMA models. For the model averaging and model selection type of models, those with greater value in forgetting factors α and υ, and smaller value in κ tend to generate better forecast accuracy. This indicates that hedge fund return data favours little time variation in parameters and model predictive probability, but quicker decay in the EWMA estimation of regression residual variance H_t. On the other hand, TVP-AR(1) models do not show a consistent performance pattern in the parametric settings of υ and κ.

[Table 4.2 about here.]

[Table 4.3 about here.]

Table 4.4 and 4.5 present the sum of log predictive likelihood of each model (the OLS-AR(1) model is not Bayesian, thus we do not report its predictive likelihood results). The boldface value in Table 4.4 and 4.5 are the top three performers among the models. Quite consistently, for both equally weighted and AUM weighted indices, all strategies favour the DMS models, more specifically, DMS model with α and υ equals to 0.985. This indicates that models with highest predictive probability tend to successfully generate high log predictive likelihood. Time-varying parameters play an important role in generating higher predictive likelihood: the quicker the evolution speed, the higher the predictive likelihood. The only exceptional occasions are found in CTA for equally weighted indices and in CTA and GM for AUM weighted indices, where a TVP-AR(1) model and a BMS model also tend to show strong results.

[Table 4.4 about here.]

[Table 4.5 about here.]

4.4.2 Economic Evaluation of Forecasts

Leitch and Tanner (1991) claims that conventional forecast error measures do not fully assess return forecasts in terms of profitability. Profit- or utility-based metrics provide more direct measures of the economic value of the forecasts. Following Campbell and Thompson (2008) and Neely et al. (2014), among others, we measure the economic value of hedge fund return forecasts through an asset allocation exercise. We compute the certainty equivalent return (CER) for a mean-variance investor with a portfolio consisting of the risk-free asset and one risky asset (i.e. the hedge fund indices). The investor's problem is how to allocate wealth between the two assets at the beginning of every period. The optimal solution for the weight of wealth to be invested in the risky asset (w_t) for period t+1 is given by:

$$w_t = \left(\frac{1}{\gamma}\right)\left(\frac{\hat{R}_{t+1}}{\hat{\sigma}^2_{t+1}}\right) \tag{4.16}$$

where γ is the relative risk aversion coefficient that indicates the investor's risk appetite, we set $\gamma = 2$; $\hat{R}_{t+1}$ as a forecast of the hedge fund return in the next period; and $\hat{\sigma}^2_{t+1}$ is a forecast of its variance. Stambaugh (1999); Barberis (2000); Kandel and Stambaugh (1996) emphasise the importance of considering parameter uncertainty when determining optimal portfolio weights. Prediction of future returns associated with parameter estimation errors, and hence the historical variation of asset return can rarely capture the high uncertainty of model forecast Barberis (2000). Following Kandel and Stambaugh (1996), in particular, we adopt the variance of the model forecast given in the predictive density in equation (4.6) at each point in time to derive the optimal portfolio weights. The benchmark model OLS-AR(1) adopts model forecast with historical variance as predictive density of forecast is not available in non-Bayesian estimation. The optimal weight is subject to constraints $0 \leqslant w_t \leqslant 1.5$ to eliminate short selling and over leverage.

Portfolio returns over the out-of-sample period R_{t+1} is equal to:

$$R_{p,t+1} = w_t \cdot R_{t+1} + (1 - w_t) \cdot r_{f,t} \tag{4.17}$$

$r_{f,t}$ denotes the risk-free return. The CER of the portfolio is:

$$CER_p = \hat{\mu}_p - \frac{1}{2}\gamma\hat{\sigma}_p^2 \tag{4.18}$$

Where $\hat{\mu}_p$ and $\hat{\sigma}_p^2$ are the mean and variance of portfolio return over the out-of-sample period.

Table 4.6 and 4.7 summarise the results of CER. For the equally weighted indices, MN and CTA deliver the highest CER of 151 - 210bps. and 144 - 176bps. per month respectively, whereas LO delivers the lowest maximum CER (around 130bps. per month) among all categories. The majority of competing models are found to strongly outperform the benchmark model. The benchmark model only proves difficult to surpass in the GM strategy. Although competing models do not show significant improvement of forecast accuracy in the MSFEs of EM, LO and MN strategies, they have significantly outperformed the benchmark model in the CER measure. Such results confirm the findings of Leitch and Tanner (1991) and Rapach et al. (2013) that MSFE has a weak relationship with forecast profitability. Competing models show the greatest improvement in CER magnitude in RV (-0.32bps. compared with 63 - 204bps.), followed by FF (91.77bps. compared with 118 - 151bps.) and MN (141.87bps. compared with 151 - 210bps.). In general, TVP-AR(1) models provide the best results, followed by DMA models.

For the AUM weighted indices, MN and MS deliver the highest CER of 186 - 267bps. and 140 - 252bps. per month respectively, whereas LO delivers the lowest maximum CER (around 131bps. per month) among all categories. Different from the equally weighted indices, the competing model shows strong advantage in GM strategy, while the weakest results are found in ED strategy. The greatest improvement in CER magnitude is again found in RV, followed by MS, FF and Sector. In general, DMA models provide the best results, followed by BMS and TVP-AR(1) models.

[Table 4.6 about here.]

[Table 4.7 about here.]

4.5 Dynamic Portfolio Construction

4.5.1 Portfolio Construction Framework

A portfolio construction exercise is carried out based on the forecasts of each individual funds over the out-of-sample period Jan 2002 - Dec 2014. Fund of funds are excluded from the exercise. To mitigate the problem of selection/survival bias, graveyard funds are also included in portfolio construction. Graveyard data are sourced from the graveyard database of BarclayHedge. We apply identical data filters as stated in section 4.3. After filtering, we have 1043 graveyard funds. Selection of the fund of funds is based on two different criteria: the value of the expected future return of funds, and the t-statistics of the expected return of each model k.Avramov et al. (2013) suggest that return forecast is estimated with uncertainty, and estimation uncertainties differ across models and dataset. Selecting funds based on expected future returns or historical variance of funds overlook this type of model risk. Moreover, forecast returns of different funds using the same model cannot be compared like with like due to different estimation accuracy embedded. They developed a strategy which selects funds based on the t-statistics of the forecasts of each model at each forecast point:

$$t - stat(\hat{R}_{t+1}) = \frac{\hat{R}_{t+1}}{\hat{\sigma}_{t+1}} \tag{4.19}$$

where $\hat{R}_{t+1}$ is the expected hedge fund return in the next period; and $\hat{\sigma}_{t+1}$ is the standard deviation of the model forecast. In all competing models, $\hat{\sigma}_{t+1}$ is obtained from the predictive density of each forecast point in the equation (4.6). For the benchmark OLS-AR(1) model, $\hat{\sigma}_{t+1}$ is computed as follow:

$$\hat{\sigma}_{t+1}^{OLS-AR(1)} = \sqrt{z'_{t+1}\,\Sigma_{t+1|t}\,z_{t+1}} \tag{4.20}$$

where $z_{t+1} = X_t$ is the observation vector of predictors (i.e. return observation in the last period and the constant term in the OLS-AR(1) model) at time t, $\Sigma_{t+1|t}$ is the variance-covariance matrix of estimated coefficients.

After selecting the top 30 funds either based on the expected future returns or the forecast t-statistics, the optimal weights allocated to the 30 funds are determined by two methods: the 1/N allocation method and the mean-variance optimisation method. Constructed portfolio is reseleted and rebalanced annually.

We consider the following mean-variance optimisation problem:

$$\begin{aligned} & min\ Var(R_p) \\ s.t.\ & w_L \leqslant w_i \leqslant w_U, (i = 1, ..., n); \\ & \sum_i^n w_i = 1; \\ & E(R_p) \geqslant R_G \end{aligned} \tag{4.21}$$

where $E(R_p)$ is the expected return of the n-assets portfolio of hedge funds, $Var(R_p)$ is the variance of the portfolio return. $Var(R_p) = \mathbf{w'Vw}$, $\mathbf{w}$ is the vector which contains optimal weights $w_i\,(i = 1, ..., n)$ of each fund of the portfolio, and $\mathbf{V}$ is the $n \times n$ matrix of sample variance-covariance matrix of fund returns. Upper and lower bounds of w_i are set to be [0, 0.5] in order to eliminate short selling and facilitate diversification (Panopoulou and Vrontos, 2015; Harris and Mazibas, 2013, see for example). R_G is the target portfolio return applied to the optimisation problem, we set $R_G = 12\%$ following Panopoulou and Vrontos (2015).

4.5.2 Portfolio Performance Evaluation Criteria

The performance of the portfolios are evaluated over the out-of-sample period Jan 2002 - Dec 2014 using various performance measures.

First, we consider the average of realised portfolio return (AR) over the out-of-sample period. Given the weights $\boldsymbol{w_t} = w_{1,t}, w_{2,t}, ..., w_{n,t}$ of the n-assets portfolio, and realised returns of each included funds at t+1 $\boldsymbol{R_{t+1}} = R_{1,t+1}, R_{2,t+1}, ..., R_{n,t+1}$. Therefore, the realised portfolio return at time t+1 is calculated as:

$$R_{p,t+1} = \boldsymbol{w_t'} \boldsymbol{R_{t+1}} \tag{4.22}$$

We also consider the end of period value (EPV) which is the terminal wealth if we invest 1 unit of wealth at the beginning of the out-of-sample period.

Second, we consider risk related performance measures, including the Sharpe ratio (SR), the Sortino ratio and the Upside Potential ratio. Sharpe ratio is defined by the realised portfolio average return $E(R_p)$ and variance Var_p over the out-of-sample period.

$$SR_p = \frac{E(R_p) - E(r_f)}{\sqrt{Var(R_p)}} \tag{4.23}$$

where $E(r_f)$ is the expected risk-free return over the period. To match the monthly return data frequency, we adopt the 1-month T-bill rate as the proxy of risk-free return.

The Sortino and Satchell (2001)'s reward to lower partial moment ratio (Sortino ratio) is defined as the excessive portfolio return over a threshold value divided by the standard deviation of negative excessive returns. We use risk-free return as the threshold value:

$$Sortino(R_p) = \frac{E(R_p) - E(r_f)}{\sqrt{E[(r_f - R_p)_+^2]}} \tag{4.24}$$

Sortino et al. (1999) propose the Upside Potential ratio, which compares the positive excessive return of the managed portfolio over a threshold return value with the standard deviation of the negative excessive return of the portfolio:

$$Upside(R_p) = \frac{E[(R_p - r_f)^+]}{\sqrt{E[(r_f - R_p)_+^2]}} \tag{4.25}$$

We also consider the Omega raito by Keating and Shadwick (2002). The Omega ratio gauges the size between positive and negative excess return of the portfolio with respect to a threshold return value. Similarly, we adopt the risk-free return as the threshold value.

$$Omega(R_p) = \frac{E[(R_p - r_f)^+]}{E[(r_f - R_p)_+]} \tag{4.26}$$

Finally, tail risk of portfolio returns are also assessed using value-at-risk $VaR_{1\%}$, $VaR_{5\%}$ and $VaR_{10\%}$ based on the realised portfolio average return $E(R_p)$ and variance $Var(R_p)$.

4.5.3 Out-of-sample Portfolio Performance Results

Table 4.8 reports the results of portfolios constructed based on expected returns with 1/N allocation method. Overall, the result suggest that competing models outperform the benchmark model absolutely in risk-return trade-off and risk related performance measures, whereas some models do not beat the benchmark model in terms of absolute return measures (i.e. average portfolio return and end of period value). These results indicate that portfolios constructed based on forecasts of competing models generate less volatility in return time series compared to the benchmark portfolio. Hence, these portfolios deliver a higher return per unit risk ratio and smaller loss value at tail area. The Omega ratio of all portfolios are greater than 1, indicating that all portfolios generate higher positive gain than losses. Better performance in the Omega ratio demonstrates that the portfolio generates higher value in positive return compared with the magnitude of portfolio negative returns over the test period. Better performance in Sortino and Upside Potential ratio shows higher return generated per unit downside risk taken.

The TVP-AR(1) ($\upsilon = 0.996, \kappa = 0.92$) model presents the highest absolute return (1.131% monthly return and 4.922 EPV), followed by TVP-AR(1) ($\upsilon = 0.996, \kappa = 0.94$) model (AR 1.109%, EPV 4.725). Their Shapre, Omega, Sortino and Upside Potential ratios take the top rankings among the others. In general, the group of TVP-AR(1) models with $\upsilon = 0.996$ shows the strongest performance, followed by TVP-AR(1) models with $\upsilon = 0.985$, DMA and DMS models with $\alpha = \upsilon = 0.996$, then BMA models.

[Table 4.8 about here.]

Moving to the mean-variance optimised expected return based portfolios, competing models remain very strong positions compared to the benchmark model. However, the TVP-AR(1) models have loss some of their superiority compared to the benchmark. Instead, TVP-ALL models show the greatest improvement compared to their results in the 1/N portfolios. BMA models deliver the best performance, followed by TVP-ALL models, DMA models and BMS models.

Mean-variance optimisation takes the variance of portfolio return into account in order to minimise the risk while achieving the target portfolio return. While the majority of competing models showed improved performance compared to the 1/N allocation, results of both the benchmark model and the TVP-AR(1) models deteriorated, showing that the mean-variance optimisation approach adds more value to complex forecast models.

[Table 4.9 about here.]

Table 4.10 and 4.11 present the results of portfolios constructed based on t-statistics of model forecasts. Focused on the 1/N allocated portfolio, overall, the AR and EPV statistics of all models decreased by 10%-30% compared to the expected return portfolios. However, the performance in the Sharpe, Omega, Sortino and Upside Potential raios generally improved compared with the expected return portfolios. Moreover, the performance in tail risk presented by VaRs are 2.5-5 times less than the expected return based portfolios. These results suggest that portfolio risk is reduced considerably when constructed based on t-statistics of model forecast. As a result, absolute returns of portfolios show a decrease due to risk-return trade-off. The improvement in the benchmark OLS-AR(1) model is most noteworthy among the rest, the Sharpe, Omega, Sortino and Upside Potential ratios are 2 or more than 2 times better than before, whereas the VaR performances are more than 5 times better than before. Consequently, we do not observe any competing model which can outperform the benchmark models in all risk-related measures. However, all competing models still retain a strong advantage in absolute return measures. Among all competing models, the TVP-AR(1) group of models is the one which achieved greatest improvements in risk related measures. As a result, the TVP-AR(1) models with certain parametric settings are still capable of outperforming the benchmark model in all measures except portfolio VaR, models with $\upsilon = 0.996$ which perform better than models with $\upsilon = 0.985$. The highest returns are found in the DMA ($\alpha = \upsilon = 0.985, \kappa = 0.97$) model with monthly AR 0.760% and EPV 3.133, followed by the TVP-ALL models with $\upsilon = 0.996$.

Moving from 1/N allocation to mean-variance allocation, the overall picture is similar to the mean based portfolios when moving to mean-variance portfolio allocation. Statistics show that for all models, both absolute portfolio returns and risk are reduced, but the risk-return trade-off performance of the mean-variance portfolio seems to be improved greatly in the complex models compared with the statistics in the 1/N portfolio. Despite the portfolio VaR, we find the majority of competing models outperform the benchmark in all aspects. Furthermore, TVP-AR(1) models and BMA models with certain parametric settings are able to outstrip the benchmark in all performance measures. TVP-AR(1) models with $\upsilon = 0.985$ show the strongest performance, followed by BMA models. Results show the same evidence as in Table 4.9 that mean-variance allocation works more efficiently for complex models.

[Table 4.10 about here.]

[Table 4.11 about here.]

To gauge the model performances during volatile market conditions, we evaluate the portfolio performance during the global financial crisis periods from Jan 2007 - Dec 2009 in particular. Portfolios held during this period are first constructed at the end of 2006 and rebalanced at the end of each year. Results of the four types of portfolios construction approaches utilised are reported in Table 4.12 - 4.15.

Results show that expected return based portfolios with 1/N allocation still remain the one which has the most competing models outperforming the benchmark model, and the advantages are shown in all types of performance measures. In general, the group of TVP-AR(1) models with $\upsilon = 0.985$ shows the strongest performance, followed by TVP-AR(1) models with $\upsilon = 0.996$, BMA models and DMS models with $\alpha = \upsilon = 0.996$. These are a similar list of top models as evidenced in the full out-of-sample periods. Switching to mean-variance allocation, similarly BMA models ranked the top performance group, followed by DMA models with $\alpha = \upsilon = 0.985$ and TVP-ALL models.

Moving to t-statistics based portfolios with 1/N allocation (Table 4.14), while portfolio absolute returns and VaR are decreased which is consistent with the full out-of-sample periods results, it is noteworthy that during the crisis period the risk-return trade-off type of measures do not improve when portfolios are selected based on t-statistics. The results indicate the increased difficulty in achieving low hedge fund return volatility during the period of financial crisis. It is hard to identify any group of competing models which is particularly salient. Instead, models with $\kappa = 0.94\, or\, 0.97$ tend to show stronger performance indicating the clustering of volatility during the tested periods. With mean-variance allocation, results show the same picture, but with further deterioration in portfolio returns, many models generate negative average returns or negative Sharpe ratio including the benchmark OLS-AR(1) model. However, despite models with $\kappa = 0.94\, or\, 0.97$ continue to show better results in general, BMA models and TVP-AR(1) models with $\upsilon = 0.985$ exhibit very strong performance in all aspects.

These results demonstrate that the time varying feature and faster decay of parameters are essential in gauging valuable investments during volatile market periods. On the other hand, slower decay factor of the EWMA estimator is preferred indicating volatility clustering during periods of financial stress in the markets.

[Table 4.12 about here.]

[Table 4.13 about here.]

[Table 4.14 about here.]

[Table 4.15 about here.]

4.5.4 Portfolio Composition

Figure 4.1 and 4.2 show the portfolio compositions of selected models[6]at four rebalance points Jan 2002, Jan 2006, Jan 2009 and Jan 2013 respectively. In expected value based portfolios, equity long/short and CTA strategies account for the highest proportion in selected funds, followed by sector and emerging market strategies. There is an increasing trend of selecting the CTA funds, whereas a decreasing trend of selecting LS and Sector funds is observed throughout the periods. For t-statistics based portfolios, funds with MS,EM and RV strategies are selected the most. Portfolios selected based on t-statistics are more diversified compared to the expected value portfolios. Moreover, the ED strategy received zero weight in all four models, whereas the Sector strategy received zero weight in the OLS-AR(1), but some weights in the other 3 models. Portfolio composition of the BMA and DMA models seems to show a high degree of similarity when selected based on t-statistics. Equity long/short strategy attracted a tremendous amount of investment at

[6]Selected models: OLS-AR(1), BMA ($\alpha = \upsilon = 1, \kappa = 0.97$), DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$), TVP-AR(1) ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)

the crisis period Jan 2009. Overall, taking into account model uncertainty in hedge fund return forecast and portfolio construction tends to diversify the selection among different trading strategies when benchmarking the OLS-AR(1) model.

[Figure 4.1 about here.]

[Figure 4.2 about here.]

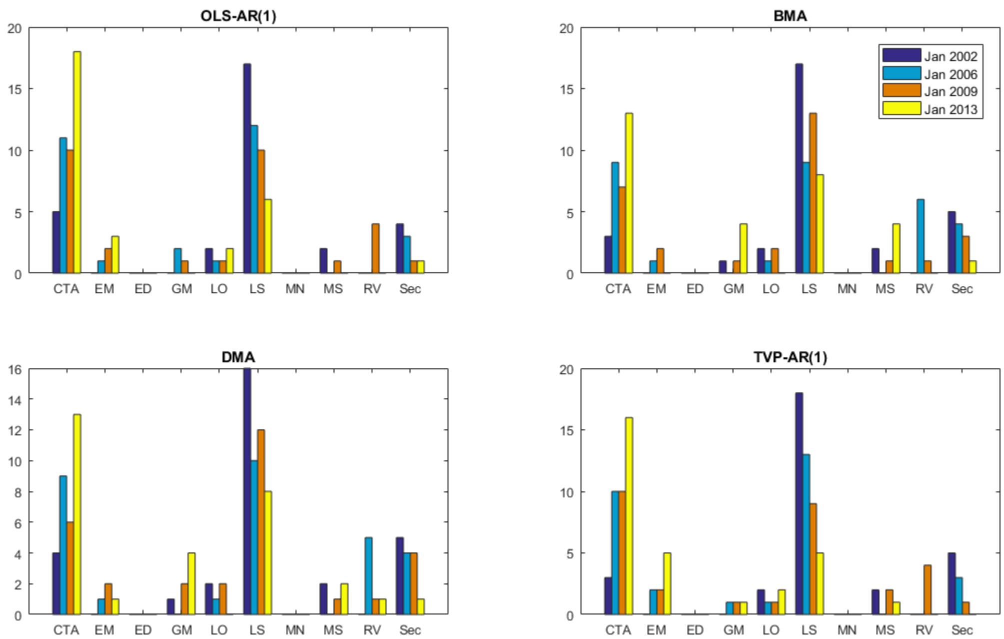
4.6 Conclusion

In this study, we jointly investigate the statistical and economic value of incorporating heteroscedasticity, non-normality, time-varying parameter of predictive regression models, model specification uncertainty and parameter uncertainty in hedge fund return forecast and portfolio construction. We employ the methods introduced by Koop and Korobilis (2012). Parameter uncertainty is dealt with by the time-varing parameter structure, and model specification uncertainty is mitigated by dynamic model averaging or model selection. Empirical results show that addressing model risk by the proposed method significantly improved forecast accuracy and portfolio performance of hedge funds.

With respect to the benchmark OLS-AR(1) model, we find that the proposed methods have good statistical value in terms of forecast accuracy as measured by MSFE and log predictive likelihood. In the analysis of certainty equivalent return (CER), competing models deliver superior CER results compared to the benchmark when estimation risk is further mitigated by adopting estimation variance of the model forecasts as the forecast variance of hedge fund returns, indicating very strong economic value of the model forecasts.

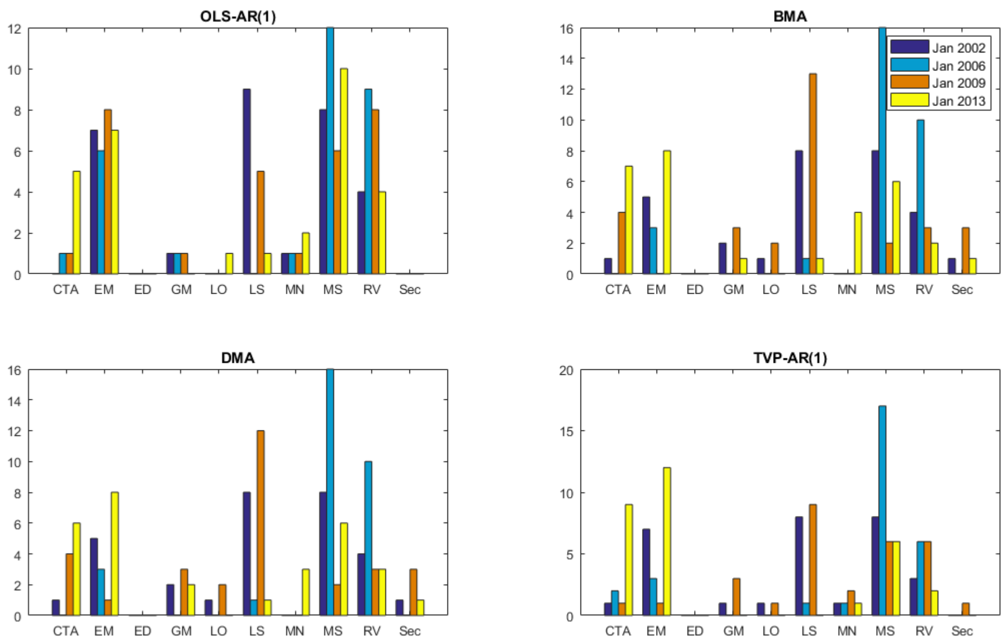
Regarding portfolio construction exercises, in the full out-of-sample period (Jan 2002 - Dec 2014), we find the majority of competing models outperform the benchmark model, indicating that consideration of parameter or model selection risk adds significant economic value to portfolio construction. Among the four different portfolio construction approaches, BMA and TVP-AR(1) models with gradual evolution speed of parameters show the best and most stable performance among others. On the other hand, the results suggest that the DMA models with the time-varying parameter settings does not contribute significant extra value compared to the conventional Bayesian model averaging approach in terms of constructing fund of funds. Comparison between the portfolios selected based on expected returns or t-statistics shows that t-statistics portfolios have a lower level of risk, which is evidenced by reduced portfolio return, yet increased Sharpe Ratio and reduced VaR. The outstanding performance of competing models persists during the crisis period 2007 - 2009, but with lower absolute returns and much higher return volatilities and tail risk. Models with decay factor closer to 1 in the EWMA forecast of volatility generate better results overall, which supports volatility clustering in market stress periods.

Figure 4.1: Portfolio Composition of Top Expected Value Portfolios



Note: The figure shows composition of portfolios selected based on forecast expected value of fund returns at time Jan 2002, Jan 2006, Jan 2009 and Jan 2013 respectively. Selected models are OLS-AR(1), BMA ($\alpha = \upsilon = 1, \kappa = 0.97$), DMA ($\alpha = \upsilon = 0.996, \kappa = 0.97$), TVP-AR(1) ($\alpha = \upsilon = 0.996, \kappa = 0.97$).

Figure 4.2: Portfolio Composition of Top t-statistics Portfolios



Note: The figure shows composition of portfolios selected based on t-statistics of expected return at time Jan 2002, Jan 2006, Jan 2009 and Jan 2013 respectively. Selected models are OLS-AR(1), BMA ($\alpha = \upsilon = 1, \kappa = 0.97$), DMA ($\alpha = \upsilon = 0.996, \kappa = 0.97$), TVP-AR(1) ($\alpha = \upsilon = 0.996, \kappa = 0.97$).

Table 4.1: Summary Statistic of Monthly Hedge Fund Data

Category	N	AUM ($bn.)	Mean	Std. Dev.	Skewness	Excess Kurtosis
Panel A: All Funds						
All	1969	731.02	0.72	3.50	-0.55	5.54
CTA	55	95.98	0.77	4.22	0.56	4.39
Emerging Market	164	54.04	0.91	5.89	-0.48	6.18
Event Driven	152	86.71	0.80	3.05	-0.59	5.25
Fund of Funds	581	162.50	0.47	2.02	-1.07	5.80
Global Macro	92	29.01	0.82	3.78	0.34	4.40
Equity Long Only	65	26.16	0.81	4.83	-0.14	2.43
Equity Long/Short	444	97.89	0.86	4.37	-0.03	3.11
Market Neutral	36	4.76	0.53	2.03	0.10	3.26
Multi-Strategy	93	95.29	0.75	2.76	-0.70	6.41
Relative Value	172	59.97	0.74	3.08	-1.45	13.90
Sector	115	18.71	0.86	5.09	-0.14	3.76
Panel B: Live Funds						
All	926	429.54	0.73	3.55	-0.51	5.38
CTA	27	91.79	0.81	4.16	0.54	5.16
Emerging Market	89	27.71	0.86	5.82	-0.35	4.78
Event Driven	66	23.93	0.75	2.96	-0.47	5.48
Fund of Funds	265	93.70	0.48	2.02	-1.05	6.14
Global Macro	42	18.81	0.92	3.99	0.35	4.29
Equity Long Only	34	10.12	0.83	5.03	-0.23	2.61
Equity Long/Short	211	54.12	0.87	4.39	-0.10	2.89
Market Neutral	18	3.32	0.62	1.82	0.04	3.21
Multi-Strategy	41	54.79	0.82	2.97	-0.49	5.63
Relative Value	71	40.04	0.81	3.01	-1.36	14.87
Sector	62	11.22	0.77	4.77	-0.26	3.30
Panel C: Graveyard Funds						
All	1043	301.48	0.71	3.45	-0.58	5.68
CTA	28	4.20	0.72	4.28	0.59	3.66
Emerging Market	75	26.33	0.98	5.98	-0.63	7.84
Event Driven	86	62.78	0.83	3.12	-0.68	5.07
Fund of Funds	316	68.80	0.47	2.02	-1.08	5.52
Global Macro	50	10.20	0.73	3.61	0.33	4.49
Equity Long Only	31	16.04	0.79	4.61	-0.04	2.22
Equity Long/Short	233	43.77	0.86	4.36	0.04	3.31
Market Neutral	18	1.44	0.45	2.25	0.17	3.32
Multi-Strategy	52	40.50	0.70	2.60	-0.87	7.02
Relative Value	101	19.93	0.70	3.14	-1.51	13.21
Sector	53	7.49	0.96	5.46	0.01	4.29

Note: The table reports the total number and the total asset under management of funds under each category. The summary statistics are the average monthly return, standard deviation, skewness and excess kurtosis. Sample period: Jan 1994 - Dec 2014.

Table 4.2: Statistical evaluation - Theil's U of equally weighted indices

	CTA	EM	ED	FF	GM	LO	LS	MN	MS	RV	Sector
OLS-AR(1)	1.676	11.156	3.140	1.923	1.829	12.358	6.526	0.390	1.889	1.847	7.030
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.960***	1.030	**0.990**	**0.987***	1.003*	1.011	1.003	1.015*	**0.976***	**0.959***	1.021
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.963***	1.036	**0.994**	**0.988***	1.003*	1.018	1.009	1.024*	**0.976***	**0.985**	1.030
BMA ($\alpha = \upsilon = 1$,$\kappa = 0.97$)	**0.974***	1.056	1.011	**0.990***	1.002*	1.036	1.021	1.052*	**0.976***	1.045	1.044
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.960***	1.021	1.001	**0.989***	1.044	1.012	1.004	1.051	**0.997***	**0.972***	1.042
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.967***	1.021	1.007	**0.991***	1.026	1.007	1.004	1.058	1.007	**0.971***	1.046
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**0.975***	1.050	1.013	1.003*	1.003*	1.045	1.037	1.134	1.013	1.044	1.062
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**0.942***	1.048	**0.997**	**0.986***	1.008	1.030	1.013	1.011*	**0.971***	**0.990**	1.027
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.942***	1.057	1.002	**0.989***	1.008	1.039	1.020	1.017*	**0.972***	1.019	1.032
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.948***	1.077	1.017	**0.996***	1.007	1.056	1.032	1.033*	**0.975***	1.062	1.043
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	**0.950***	1.136	1.036	1.016*	1.047	1.085	1.046*	1.017*	**0.990***	1.132	1.059*
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.945***	1.150	1.043	1.030*	1.043	1.094	1.051	1.017*	**0.994***	1.173	1.062*
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.933***	1.158	1.056	1.059*	1.036	1.107	1.057	1.011*	1.007*	1.240	1.057*
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**0.947***	1.024	1.038	1.019	1.024	1.065	1.034	1.074	1.018	1.064	1.051
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.945***	1.021	1.038	1.035	1.028	1.043	1.037	1.084	**0.995**	1.040	1.092
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.948***	1.085	1.018	1.090	1.006*	1.041	1.051	1.138	1.013	1.086	1.117
DMS ($\alpha = \upsilon = 0.985$,$\kappa = 0.92$)	**0.945***	1.157	1.094	1.041*	1.088	1.100	1.044	1.110*	1.084	1.130	1.158
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.946***	1.245	1.104	1.128	1.065	1.129	1.073	1.119	1.084	1.190	1.150
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.944***	1.241	1.085	1.172*	1.076	1.135	1.084	1.049*	1.061*	1.237	1.112*
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	1.143*	1.349	1.206	1.158	1.133	1.166	1.153	1.293	1.061*	1.324	1.185
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	1.138*	1.374	1.206	1.166	1.127	1.170	1.156	1.307	1.061*	1.365	1.195
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	1.128*	1.381	1.190	1.178	1.110	1.168	1.149	1.347	1.059*	1.332	1.200
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	1.119*	1.556	1.293	1.328*	1.225	1.278	1.270	1.376*	1.146*	1.532	1.285*
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	1.107*	1.586	1.299	1.350*	1.219	1.283	1.274	1.390	1.151*	1.579	1.296*
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	1.073*	1.581	1.285	1.387*	1.198	1.269	1.253	1.381	1.156*	1.575	1.297
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	**0.942***	1.014	**0.987***	**0.994***	**0.994**	**0.986***	**0.988***	**0.957***	**0.990***	**0.956***	**0.977***
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	**0.943***	1.012	**0.988***	**0.996**	**0.994**	**0.987***	**0.990***	**0.958***	**0.990***	**0.960***	**0.979***
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**0.948***	1.005	**0.991***	**0.996**	**0.993***	**0.991***	**0.995**	**0.966***	**0.993***	**0.970**	**0.986**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	**0.916***	1.030	**0.982**	1.007	1.002	**0.998**	**0.997**	**0.926***	**0.997**	**0.961**	**0.984**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.915***	1.028	**0.984**	1.010	1.000	**0.999**	**0.998**	**0.922***	**0.997**	**0.969**	**0.986**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.912***	1.024	**0.988**	1.011	**0.995**	1.003	1.002	**0.915***	**0.999**	1.000	**0.991**

Note: The table reports Theil's U statistics of each model calculating based on the out-of-sample MSFEs of equally weighted hedge fund strategy indices . Theil's U less than 1 (in boldface) indicates better performance of the underlying model compared to the benchmark model OLS-AR(1). * indicates the underlying Clark and West (2007) MSFE-adj. t-statistics are greater than +1.282, which means the model shows statistical improvement in forecast accuracy at a significant level between 5% to 10%. Out-of-sample forecasting period: Jan 2002 - Dec 2014.

Table 4.3: Statistical evaluation - Theil's U of AUM weighted indices

	CTA	EM	ED	FF	GM	LO	LS	MN	MS	RV	Sector
OLS-AR(1)	6.448	7.181	2.671	1.557	1.872	18.679	5.429	0.557	0.982	1.293	6.938
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	1.071	1.028	**0.970***	1.007	**0.972***	1.036	**0.996***	**0.941***	**0.966***	**0.930***	1.017
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	1.104	1.037	**0.970***	1.005	**0.968***	1.042	**0.999**	**0.964***	**0.970***	**0.949**	1.024
BMA ($\alpha = \upsilon = 1$,$\kappa = 0.97$)	1.139	1.074	**0.971***	**0.998***	**0.971***	1.043	1.008	1.006*	**0.991***	1.014	1.029
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	1.114	1.010	**0.961***	1.010	**0.951***	1.063	1.012	**0.981***	**0.963***	**0.927***	1.025*
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	1.209	1.014	**0.974***	**0.993***	**0.946***	1.050	1.011	1.012*	**0.964***	**0.952***	1.025*
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	1.176	1.050	**0.963***	**0.988***	**0.981***	1.025	1.018	1.052*	1.022	1.077	1.043
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	1.116	1.045	**0.978***	**0.996***	**0.983***	1.052	1.007	**0.927***	**0.964***	**0.945**	1.023
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	1.135	1.057	**0.978***	1.000	**0.978***	1.058	1.011	**0.947***	**0.972***	**0.977**	1.026
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	1.151	1.086	**0.979***	1.005*	**0.977***	1.062	1.020	**0.975***	1.002*	1.012	1.027
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	1.260	1.131	1.042*	1.017*	1.043*	1.112	1.046	**0.932***	1.017*	1.072	1.052*
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	1.271	1.150	1.042*	1.034*	1.033*	1.116	1.051	**0.946***	1.035*	1.105	1.052*
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	1.253	1.183	1.030*	1.066*	1.007*	1.117	1.059	**0.946***	1.084*	1.114	1.036*
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	1.168	1.009	1.002*	1.021	**0.953***	1.099	1.014	**0.976***	1.080	**0.945***	1.086
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	1.165	1.028	1.001*	1.039	**0.947***	1.087	1.019	**0.993***	1.068	**0.995**	1.062*
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	1.208	1.191	1.008*	1.014	**0.973***	1.091	1.061	1.024*	1.105	**0.999**	1.093
DMS ($\alpha = \upsilon = 0.985$,$\kappa = 0.92$)	1.403	1.153	1.080*	1.083*	1.120*	1.137	1.069	1.007*	1.129	1.106	1.134*
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	1.396	1.175	1.072*	1.110	1.048*	1.151	1.087	**0.991***	1.136	1.168	1.124*
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	1.344	1.279	1.080*	1.143*	1.015*	1.215	1.101	**0.990***	1.242	1.262	1.136
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	1.219	1.360	1.218	1.273	1.494*	1.226	1.134	1.226*	1.130*	1.378	1.258
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	1.220	1.405	1.200	1.286	1.508*	1.225	1.137	1.256*	1.142*	1.416	1.265
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	1.209	1.477	1.151*	1.303	1.543	1.215	1.135	1.315*	1.156*	1.396	1.253
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	1.354	1.571	1.393	1.420*	1.514*	1.324	1.260	1.312*	1.294*	1.499	1.376
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	1.356	1.629	1.367*	1.443*	1.515*	1.320	1.262	1.337*	1.320*	1.545	1.383
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	1.340	1.720	1.293*	1.483*	1.496*	1.294	1.250	1.324*	1.367*	1.521	1.364
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	**0.998**	1.015	**0.996**	**0.998**	**0.953***	**0.995**	**0.994**	**0.889***	**0.962***	**0.902***	**0.988**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	1.001	1.016	**0.995**	**0.999**	**0.948***	**0.996**	**0.994**	**0.901***	**0.963***	**0.916***	**0.990**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	1.005	1.012	**0.995**	**0.996**	**0.937***	**0.997**	**0.995**	**0.935***	**0.970***	**0.950***	**0.996**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	1.028	1.038	1.006	1.012	**0.965***	1.003	1.006	**0.859***	**0.964***	**0.902***	**0.996**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	1.039	1.038	1.005	1.015	**0.958***	1.005	1.006	**0.868***	**0.965***	**0.914***	**0.999**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	1.059	1.036	1.004	1.016	**0.946***	1.008	1.006	**0.883***	**0.971***	**0.951***	1.004

Note: The table reports Theil's U statistics of each model calculating based on the out-of-sample MSFEs of AUM weighted hedge fund strategy indices . Theil's U less than 1 (in boldface) indicates better performance of the underlying model compared to the benchmark model OLS-AR(1). * indicates the underlying Clark and West (2007) MSFE-adj. t-statistics are greater than +1.282, which means the model shows statistical improvement in forecast accuracy at a significant level between 5% to 10%. Out-of-sample forecasting period: Jan 2002 - Dec 2014.

Table 4.4: Statistical evaluation - Sum of logPL equally weighted indices

	CTA	EM	ED	FF	GM	LO	LS	MN	MS	RV	Sector
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	-253.3	-408.1	-309.2	-262.5	-272.7	-410.5	-364.0	-149.1	-264.7	-253.1	-370.2
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	-253.5	-410.8	-309.8	-265.1	-271.6	-412.9	-365.5	-150.5	-265.5	-261.4	-373.4
BMA ($\alpha = \upsilon = 1$,$\kappa = 0.97$)	-256.8	-418.2	-312.7	-271.7	-270.1	-418.3	-368.5	-156.0	-268.0	-274.8	-380.2
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	-252.9	-405.4	-305.4	-261.0	-266.7	-407.1	-360.0	-141.7	-262.7	-235.9	-364.5
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	-253.1	-407.6	-305.2	-263.5	-265.9	-407.5	-361.2	-145.2	-263.3	-249.8	-367.3
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	-256.0	-413.3	-305.7	-268.6	-267.8	-412.8	-365.2	-147.7	-262.8	-264.1	-373.2
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	-251.1	-409.0	-310.0	-263.4	-272.6	-411.8	-365.1	-148.6	-264.8	-253.8	-371.2
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	-251.2	-411.9	-310.5	-266.2	-271.7	-414.1	-366.4	-149.8	-265.5	-261.9	-374.2
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	-254.5	-419.4	-312.7	-272.7	-270.5	-419.1	-369.2	-154.8	-267.9	-274.0	-380.4
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	-253.1	-412.0	-312.2	-267.1	-274.7	-415.1	-367.4	-147.9	-266.5	-254.2	-374.5
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	-253.0	-415.3	-312.5	-270.1	-274.1	-417.1	-368.5	-149.5	-267.1	-262.5	-377.2
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	-256.5	-423.2	-313.8	-276.5	-273.4	-421.0	-370.7	-154.6	-269.1	-275.8	-382.4
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	-249.1	**-404.6**	-303.8	**-259.6**	-262.6	-406.3	-359.2	**-139.3**	-258.5	**-232.9**	**-363.4**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	-248.8	-406.7	-303.3	-262.3	-264.3	-407.2	-360.5	-139.4	-259.4	-237.1	-365.4
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	-252.9	-412.6	-303.0	-266.3	-264.2	-410.0	-361.5	-144.1	-257.0	-255.1	-370.5
DMS ($\alpha = \upsilon = 0.985$,$\kappa = 0.92$)	**-243.3**	**-391.6**	**-292.7**	**-254.6**	**-256.9**	**-396.5**	**-352.8**	**-130.4**	**-250.5**	**-224.9**	**-353.5**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**-243.5**	**-398.3**	**-294.7**	**-257.0**	**-258.3**	**-399.1**	**-353.2**	**-132.1**	**-250.3**	**-229.1**	**-358.1**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	-247.7	-408.7	**-294.0**	-260.4	**-259.0**	**-403.5**	**-355.2**	-139.9	**-252.3**	-246.1	-365.7
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	-271.7	-427.3	-321.4	-279.3	-278.0	-420.7	-373.0	-164.6	-269.4	-262.3	-381.8
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	-273.6	-432.0	-322.0	-281.9	-278.0	-422.6	-374.3	-166.6	-270.2	-270.7	-385.1
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	-284.2	-441.1	-322.2	-287.4	-278.4	-425.8	-376.6	-173.1	-271.8	-280.0	-392.6
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	-275.8	-434.8	-323.8	-287.7	-285.0	-425.7	-378.3	-167.4	-274.8	-267.3	-389.1
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	-277.8	-440.0	-324.6	-290.8	-285.2	-427.5	-379.6	-170.2	-275.6	-275.3	-392.1
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	-288.1	-449.6	-325.1	-296.8	-285.9	-429.8	-381.6	-176.3	-276.6	-285.6	-399.3
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	-250.9	-405.5	-306.5	-261.2	-271.0	-407.8	-361.7	-142.9	-263.5	-243.9	-366.0
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	-250.9	-407.7	-307.0	-263.9	-270.0	-410.0	-363.0	-144.0	-264.5	-252.4	-368.9
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	-253.1	-413.5	-309.5	-270.2	-268.5	-414.6	-365.7	-148.3	-267.6	-268.3	-375.1
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	-246.9	-406.1	-306.4	-262.0	-271.1	-408.6	-361.8	-140.3	-263.7	-243.7	-366.2
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**-246.7**	-408.4	-306.9	-264.8	-270.0	-410.8	-363.2	-141.1	-264.7	-251.9	-369.2
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	-249.0	-414.4	-309.1	-271.1	-268.5	-415.1	-365.9	-144.6	-267.8	-267.4	-375.2

Note: The table reports the sum of log predictive likelihood of each model. The top three results of each hedge fund strategy index are in boldface. Out-of-sample forecasting period: Jan 2002 - Dec 2014.

Table 4.5: Statistical evaluation - Sum of logPL AUM weighted indices

	CTA	EM	ED	FF	GM	LO	LS	MN	MS	RV	Sector
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	-372.3	-367.1	-302.0	-249.2	-269.9	-447.4	-350.0	-168.9	-208.9	-217.3	-374.0
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	-374.7	-371.9	-301.1	-250.8	-273.0	-449.7	-350.7	-171.0	-211.0	-225.7	-378.0
BMA ($\alpha = \upsilon = 1$,$\kappa = 0.97$)	-379.8	-385.3	-299.5	-256.3	-288.3	-452.9	-353.0	-179.2	-217.1	-243.3	-385.7
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**-360.3**	-364.8	-297.8	-246.7	**-266.9**	-441.6	-347.6	-159.1	-207.2	-215.7	-368.0
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	-361.2	-367.9	-296.1	-248.4	-270.1	-442.1	-347.6	-161.2	-209.4	-222.7	-371.9
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	-367.2	-378.9	-295.6	-254.0	-285.7	-447.5	-348.7	-173.7	-211.9	-236.8	-379.1
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	-371.6	-367.7	-302.7	-248.8	-271.5	-447.8	-350.8	-166.6	-209.2	-217.1	-375.6
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	-373.4	-372.8	-301.9	-251.0	-274.9	-450.2	-351.5	-168.6	-211.4	-225.9	-379.4
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	-378.1	-385.8	-300.2	-257.4	-290.2	-453.7	-353.5	-177.0	-217.5	-241.5	-386.5
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	-374.6	-371.0	-304.9	-250.9	-277.2	-450.7	-353.1	-165.6	-211.6	-221.2	-379.0
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	-376.3	-376.4	-304.1	-253.7	-281.1	-452.9	-353.7	-168.0	-214.1	-229.6	-382.5
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	-380.1	-389.9	-302.0	-260.7	-296.9	-456.3	-355.1	-176.8	-220.2	-243.7	-389.6
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	-360.4	**-363.5**	-294.1	**-243.6**	-267.0	-439.6	-344.2	**-153.9**	-201.8	**-212.2**	**-365.3**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	-360.7	-366.7	-290.7	-246.3	-270.2	-440.6	-344.7	-157.1	-203.2	-215.3	-369.9
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	-366.6	-375.9	-289.2	-251.4	-285.7	-446.5	-345.3	-168.2	-205.5	-230.1	-377.7
DMS ($\alpha = \upsilon = 0.985$,$\kappa = 0.92$)	**-355.3**	**-356.9**	**-282.5**	**-238.6**	**-256.4**	**-434.2**	**-338.6**	**-146.2**	**-190.4**	**-199.7**	**-358.8**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**-358.9**	**-359.9**	**-282.4**	**-238.3**	**-265.0**	**-437.8**	**-339.3**	**-147.7**	**-193.2**	**-203.6**	**-362.8**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	-363.9	-373.2	**-283.0**	-244.2	-286.5	**-439.0**	**-339.3**	-160.1	**-197.5**	-224.8	-371.9
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	-375.5	-387.1	-311.5	-271.0	-302.0	-456.3	-357.7	-182.2	-217.4	-244.7	-388.3
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	-377.2	-393.7	-310.5	-273.7	-308.8	-458.2	-358.3	-186.5	-220.4	-253.7	-392.4
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	-382.2	-408.6	-308.2	-280.3	-331.0	-461.4	-359.0	-199.6	-226.6	-266.6	-402.6
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	-382.2	-396.4	-315.8	-277.2	-310.2	-462.0	-363.0	-187.9	-224.7	-248.8	-397.6
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	-384.6	-403.6	-315.0	-280.3	-317.1	-463.7	-363.7	-192.5	-228.4	-258.0	-401.6
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	-390.2	-419.1	-313.1	-288.0	-339.1	-465.8	-363.7	-203.9	-235.8	-271.7	-411.9
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	-371.3	-364.2	-299.8	-245.3	-267.0	-442.5	-348.6	-161.2	-206.8	-213.3	-370.4
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	-372.2	-368.2	-299.1	-247.8	-270.2	-444.8	-349.4	-163.1	-209.1	-221.9	-374.3
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	-373.5	-378.8	-298.5	-254.5	-285.0	-448.5	-351.3	-171.7	-215.1	-238.7	-381.9
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	-371.6	-365.1	-300.4	-246.2	-267.6	-443.3	-349.0	-158.2	-206.1	-212.4	-371.0
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	-372.4	-369.3	-299.7	-248.8	-271.0	-445.5	-349.8	-160.0	-208.6	-220.7	-375.0
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	-373.8	-380.3	-299.1	-255.6	-286.0	-449.2	-351.8	-167.9	-215.0	-236.5	-382.7

Note: The table reports the sum of log predictive likelihood of each model. The top three results of each hedge fund strategy index are in boldface. Out-of-sample forecasting period: Jan 2002 - Dec 2014.

Table 4.6: Economic evaluation - equally weighted indices CER

	CTA	EM	ED	FF	GM	LO	LS	MN	MS	RV	Sector
OLS-AR(1)	142.06	113.59	139.53	91.77	171.01	115.08	112.60	141.87	129.83	-0.32	88.76
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**161.42**	**130.68**	**147.99**	**150.41**	149.69	**121.13**	**123.71**	**169.49**	**163.58**	**154.48**	**119.67**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**166.98**	**131.37**	**150.10**	**149.31**	157.01	**122.24**	**127.48**	**184.22**	**164.10**	**140.41**	**122.07**
BMA ($\alpha = \upsilon = 1$,$\kappa = 0.97$)	**175.94**	**135.04**	**150.92**	**145.60**	168.31	**125.18**	**134.43**	**200.36**	**157.82**	**143.14**	**130.26**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**159.97**	**129.42**	**146.16**	**148.41**	147.60	**124.48**	**125.52**	**156.36**	**158.80**	**142.69**	**118.18**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**164.95**	**132.15**	**145.10**	**147.85**	163.18	**125.92**	**129.36**	**174.88**	**155.20**	**129.68**	**120.83**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**175.66**	**132.30**	**144.92**	**145.73**	162.45	**127.87**	**132.65**	**185.19**	**136.96**	**115.39**	**131.18**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**162.88**	**131.98**	**146.27**	**151.37**	151.68	**120.79**	**124.07**	**175.79**	**165.33**	**155.01**	**125.12**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**168.30**	**131.70**	**148.19**	**149.70**	157.26	**122.24**	**127.77**	**187.62**	**165.44**	**138.85**	**127.14**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**176.70**	**134.72**	**152.39**	**144.60**	167.16	**125.32**	**134.43**	**200.50**	**158.82**	**149.14**	**133.77**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	**163.76**	**138.05**	**151.43**	**150.09**	155.43	**124.91**	**132.27**	**191.23**	**162.84**	**163.34**	**133.97**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**167.81**	**135.84**	**154.30**	**149.49**	158.35	**126.23**	**135.82**	**195.43**	**162.38**	**145.92**	**135.43**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**173.17**	**134.89**	**157.87**	**145.85**	163.50	**129.93**	**140.41**	**198.07**	**156.16**	**156.69**	**139.60**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**161.09**	**130.04**	138.20	**146.34**	115.55	**121.21**	**118.05**	**151.27**	**148.18**	**146.33**	**122.37**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**165.63**	**129.96**	**141.59**	**146.41**	133.70	**122.27**	**121.58**	**175.77**	**152.38**	**135.04**	**118.10**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**176.37**	**134.03**	**142.66**	**139.01**	162.77	**129.74**	**130.20**	**182.05**	**131.14**	**123.04**	**130.95**
DMS ($\alpha = \upsilon = 0.985$,$\kappa = 0.92$)	**162.84**	**125.83**	121.68	**137.98**	100.24	**121.39**	**123.50**	**159.38**	122.92	**63.78**	**94.77**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**167.37**	**120.76**	120.30	**131.06**	117.95	**119.01**	**128.54**	**169.66**	120.76	**80.02**	**96.88**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**173.39**	**119.55**	**144.97**	**127.05**	134.88	**126.93**	**131.20**	**192.93**	126.81	**89.42**	**123.17**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	**154.69**	**128.51**	118.17	**136.44**	143.04	112.42	**122.82**	**176.42**	**173.95**	**161.28**	**129.39**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	**155.48**	**125.51**	120.55	**133.83**	146.31	114.30	**127.02**	**174.10**	**174.73**	**156.45**	**130.06**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	**155.05**	**123.85**	136.19	**130.46**	152.82	**118.16**	**133.84**	**168.62**	**177.17**	**179.56**	**132.78**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	**144.00**	**136.73**	**155.30**	**144.98**	150.11	**122.85**	**133.55**	**187.10**	**167.29**	**164.40**	**136.33**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	**146.41**	**133.84**	**156.81**	**143.53**	151.83	**124.08**	**135.60**	**183.95**	**167.78**	**160.82**	**136.75**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	**148.53**	**132.76**	**161.85**	**142.03**	154.43	**127.38**	**139.30**	**177.33**	**169.89**	**175.28**	**137.99**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	**160.07**	**130.81**	**148.54**	**146.34**	162.39	**124.68**	**126.12**	**173.20**	**162.12**	**198.83**	**124.04**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	**166.16**	**133.50**	**153.22**	**146.84**	168.16	**126.15**	**129.04**	**189.98**	**161.80**	**184.55**	**125.33**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**176.62**	**139.77**	**159.55**	**144.06**	**176.45**	**130.21**	**134.95**	**210.38**	**152.99**	**171.22**	**133.60**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	**164.15**	**127.96**	**147.68**	**143.20**	164.44	**124.56**	**129.18**	**180.95**	**162.84**	**204.10**	**127.49**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**169.18**	**130.42**	**152.58**	**143.29**	168.92	**126.25**	**131.75**	**193.52**	**162.25**	**191.01**	**127.96**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	**177.47**	**137.37**	**159.69**	**139.45**	**175.22**	**130.76**	**137.15**	**208.34**	**153.82**	**179.57**	**133.91**

Note: The table reports the CER of a portfolio consisting of a risk-free asset and one risky asset (i.e. the underlying hedge fund strategy index). CERs of competing models are computed based on the model forecast expected returns and variance. Results which outperform the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2002 - Dec 2014.

Table 4.7: Economic evaluation - AUM weighted indices CER

	CTA	EM	ED	FF	GM	LO	LS	MN	MS	RV	Sector
OLS-AR(1)	35.63	92.90	175.59	94.15	161.44	121.74	144.55	204.98	77.47	-2.66	90.70
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**130.08**	**164.01**	169.47	**169.24**	**171.22**	114.95	**156.06**	**224.44**	**193.74**	**209.68**	**149.30**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**133.91**	**155.03**	172.73	**166.11**	**184.93**	115.34	**157.04**	**242.22**	**202.90**	**199.03**	**150.99**
BMA ($\alpha = \upsilon = 1$,$\kappa = 0.97$)	**135.14**	**148.76**	**178.26**	**163.02**	**193.42**	116.68	**156.88**	**254.00**	**235.75**	**200.93**	**159.84**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**124.87**	**168.21**	165.67	**169.23**	**172.96**	115.77	**157.79**	**232.76**	**191.10**	**223.52**	**150.50**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**127.58**	**152.73**	168.97	**167.04**	**186.64**	115.42	**159.88**	**253.87**	**200.87**	**207.64**	**155.09**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**136.19**	**140.23**	**198.30**	**165.09**	**197.29**	116.36	**154.69**	**251.74**	**240.99**	**199.15**	**163.40**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**139.38**	**166.94**	165.33	**172.19**	**176.20**	118.78	**153.15**	**230.31**	**212.59**	**216.25**	**155.97**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**140.80**	**157.59**	167.58	**168.68**	**186.32**	119.28	**154.13**	**242.16**	**220.47**	**200.91**	**156.61**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**141.94**	**149.26**	171.40	**164.30**	**191.23**	119.85	**154.85**	**251.60**	**244.27**	**204.88**	**162.25**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	**136.87**	**172.15**	170.02	**171.25**	**186.52**	**124.91**	**153.09**	**241.21**	**238.53**	**204.91**	**167.81**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**137.67**	**162.74**	173.25	**170.09**	**190.04**	**126.27**	**154.35**	**243.64**	**238.16**	**184.80**	**167.50**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**138.66**	**150.21**	**178.34**	**165.61**	**184.47**	**128.49**	**156.28**	**240.78**	**244.55**	**204.26**	**169.53**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**132.18**	**166.19**	158.07	**161.91**	**172.78**	113.56	**155.27**	**225.95**	**162.45**	**224.74**	**130.78**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**139.85**	**157.18**	142.49	**164.22**	**186.69**	117.42	**154.59**	**248.91**	**184.88**	**209.07**	**159.62**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**144.75**	**137.05**	165.03	**161.51**	**197.10**	113.16	**150.98**	**267.78**	**209.99**	**191.75**	**165.04**
DMS ($\alpha = \upsilon = 0.985$,$\kappa = 0.92$)	**136.63**	**164.16**	167.83	**152.97**	153.70	**122.08**	133.99	**208.17**	**141.62**	**178.64**	**140.96**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**136.55**	**151.50**	174.15	**151.94**	**166.49**	**122.62**	142.18	**216.15**	**140.47**	**42.65**	**142.14**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**139.34**	**123.86**	117.02	**145.11**	**196.53**	120.89	137.68	**251.04**	**165.83**	**172.65**	**163.19**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	**137.17**	**152.34**	149.86	**138.85**	**164.74**	**121.98**	**147.60**	**225.35**	**234.63**	**196.83**	**156.54**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	**137.65**	**143.95**	155.32	**135.56**	**162.70**	**121.99**	**148.05**	**216.06**	**236.63**	**190.32**	**155.62**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	**139.48**	**133.00**	169.49	**130.89**	154.55	120.93	**150.87**	196.46	**246.79**	**209.31**	**155.73**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	**136.62**	**155.74**	**182.06**	**155.37**	160.87	**128.40**	**152.74**	**221.83**	**232.49**	**172.37**	**157.70**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	**136.55**	**148.59**	**185.60**	**153.23**	158.42	**129.33**	**153.20**	**213.64**	**228.42**	**167.79**	**156.99**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	**137.79**	**139.42**	**192.09**	**150.12**	151.30	**131.28**	**154.77**	197.60	**226.12**	**189.57**	**155.97**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	**101.57**	**163.61**	**177.91**	**165.21**	**172.78**	**125.17**	**158.58**	**229.15**	**197.35**	**226.64**	**151.24**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	**96.16**	**159.84**	**183.63**	**164.98**	**186.66**	**126.65**	**158.47**	**241.39**	**206.76**	**208.61**	**152.38**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**86.67**	**158.50**	**193.27**	**162.63**	**196.66**	**129.78**	**157.22**	**255.34**	**233.66**	**203.11**	**163.17**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	**104.29**	**160.33**	**176.14**	**160.02**	**174.65**	**124.28**	**155.69**	**228.90**	**215.76**	**236.38**	**154.48**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**103.18**	**157.30**	**182.33**	**159.96**	**187.36**	**126.12**	**155.21**	**237.71**	**223.87**	**218.13**	**154.38**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	**102.22**	**155.32**	**193.11**	**157.19**	**194.45**	**129.46**	**153.73**	**245.28**	**238.07**	**208.15**	**160.94**

Note: The table reports the CER of a portfolio consisting of a risk-free asset and one risky asset (i.e. the underlying hedge fund strategy index). CERs of competing models are computed based on the model forecast expected returns and variance. Results which outperform the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2002 - Dec 2014.

Table 4.8: Out-of-sample performance of top expected return portfolios with 1/N allocation

	AR	EPV	Sharpe Ratio	Omega	Sortino	Upside	VaR 1%	VaR 5%	VaR 10%
OLS-AR(1)	0.999	3.998	0.640	1.662	0.281	0.706	10.087	6.839	5.108
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.999**	**4.126**	**0.744**	**1.802**	**0.335**	**0.753**	**8.543**	**5.748**	**4.257**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**1.037**	**4.353**	**0.787**	**1.841**	**0.354**	**0.776**	**8.366**	**5.611**	**4.143**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**1.058**	**4.455**	**0.783**	**1.838**	**0.360**	**0.789**	**8.610**	**5.778**	**4.268**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	0.972	3.956	**0.702**	**1.725**	**0.311**	**0.739**	**8.821**	**5.952**	**4.423**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	0.992	**4.105**	**0.728**	**1.752**	**0.331**	**0.771**	**8.681**	**5.847**	**4.337**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**1.009**	**4.124**	**0.754**	**1.805**	**0.332**	**0.744**	**8.508**	**5.720**	**4.234**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**1.023**	**4.279**	**0.770**	**1.815**	**0.347**	**0.772**	**8.440**	**5.668**	**4.190**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**1.058**	**4.464**	**0.793**	**1.834**	**0.358**	**0.787**	**8.494**	**5.696**	**4.204**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**1.071**	**4.557**	**0.817**	**1.871**	**0.376**	**0.807**	**8.328**	**5.574**	**4.107**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	0.954	3.863	**0.727**	**1.759**	**0.319**	**0.740**	**8.303**	**5.591**	**4.146**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	0.977	**4.014**	**0.761**	**1.793**	**0.338**	**0.763**	**8.110**	**5.448**	**4.029**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	0.984	**4.045**	**0.747**	**1.760**	**0.326**	**0.756**	**8.356**	**5.620**	**4.161**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**1.020**	**4.242**	**0.743**	**1.794**	**0.340**	**0.768**	**8.762**	**5.897**	**4.369**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**1.060**	**4.514**	**0.774**	**1.830**	**0.350**	**0.772**	**8.736**	**5.866**	**4.337**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**1.015**	**4.183**	**0.766**	**1.810**	**0.334**	**0.746**	**8.414**	**5.652**	**4.179**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	0.930	3.805	**0.747**	**1.746**	**0.339**	**0.793**	**7.824**	**5.260**	**3.893**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	0.971	3.992	**0.729**	**1.734**	**0.326**	**0.770**	**8.452**	**5.691**	**4.220**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**1.057**	**4.498**	**0.774**	**1.795**	**0.353**	**0.797**	**8.709**	**5.848**	**4.323**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	0.963	3.993	**0.766**	**1.823**	**0.352**	**0.779**	**7.927**	**5.323**	**3.934**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	0.982	**4.128**	**0.785**	**1.843**	**0.367**	**0.803**	**7.875**	**5.280**	**3.897**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	0.987	**4.188**	**0.828**	**1.897**	**0.383**	**0.810**	**7.467**	**4.991**	**3.670**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	0.867	3.465	**0.711**	**1.731**	**0.316**	**0.748**	**7.609**	**5.126**	**3.803**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	0.925	3.772	**0.763**	**1.799**	**0.347**	**0.781**	**7.596**	**5.100**	**3.769**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	0.915	3.707	**0.720**	**1.738**	**0.324**	**0.762**	**8.001**	**5.389**	**3.997**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	**1.131**	**4.922**	**0.813**	**1.911**	**0.381**	**0.798**	**8.904**	**5.964**	**4.397**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	**1.109**	**4.725**	**0.790**	**1.864**	**0.365**	**0.788**	**8.994**	**6.034**	**4.457**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**1.073**	**4.472**	**0.766**	**1.828**	**0.359**	**0.792**	**8.968**	**6.027**	**4.459**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	**1.101**	**4.750**	**0.807**	**1.918**	**0.387**	**0.808**	**8.711**	**5.836**	**4.304**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**1.085**	**4.626**	**0.779**	**1.860**	**0.373**	**0.806**	**8.910**	**5.982**	**4.421**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	**1.032**	**4.231**	**0.719**	**1.767**	**0.327**	**0.753**	**9.196**	**6.200**	**4.603**

Note: The table reports out-of-sample performances of portfolio constructed based on forecast expected future return, asset weights are allocated by 1/N method. Performance measures include: average monthly return (AR), end of period value (EPV), annualised Sharpe ratio, Omega ratio, Sortino ratio, Upside Potential ratio, and VaR 1%, 5% and 10%. Results which outperforms the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2009 - Dec 2014.

Table 4.9: Out-of-sample performance of top expected return portfolios with mean-variance optimisation allocation

	AR	EPV	Sharpe Ratio	Omega	Sortino	Upside	VaR 1%	VaR 5%	VaR 10%
OLS-AR(1)	0.858	3.170	0.494	1.666	0.264	0.661	11.197	7.666	5.783
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**1.073**	**4.616**	**0.807**	**2.450**	**0.537**	**0.907**	**8.462**	**5.669**	**4.180**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**1.127**	**4.810**	**0.763**	**2.340**	**0.500**	**0.873**	**9.516**	**6.398**	**4.736**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**1.121**	**4.584**	**0.705**	**2.180**	**0.424**	**0.784**	**10.333**	**6.977**	**5.189**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.952**	**3.805**	**0.691**	**2.057**	**0.387**	**0.754**	**8.773**	**5.924**	**4.405**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.905**	**3.651**	**0.742**	**2.272**	**0.445**	**0.796**	**7.639**	**5.136**	**3.802**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**1.044**	**4.297**	**0.774**	**2.099**	**0.450**	**0.860**	**8.589**	**5.767**	**4.262**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	0.673	2.652	**0.770**	**1.878**	**0.375**	**0.801**	**5.128**	**3.429**	**2.523**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.904**	**3.660**	**0.770**	**2.124**	**0.446**	**0.842**	**7.312**	**4.905**	**3.622**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**1.005**	**3.916**	**0.637**	**1.950**	**0.387**	**0.794**	**10.214**	**6.927**	**5.175**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	**1.058**	**4.483**	**0.698**	**2.001**	**0.425**	**0.850**	**9.789**	**6.612**	**4.917**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**1.103**	**4.610**	**0.693**	**2.033**	**0.442**	**0.869**	**10.331**	**6.982**	**5.196**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**1.107**	**4.487**	**0.621**	**1.925**	**0.382**	**0.795**	11.717	7.960	5.958
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**1.038**	**4.214**	**0.685**	**1.965**	**0.388**	**0.789**	**9.780**	**6.611**	**4.922**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**1.007**	**3.975**	**0.622**	**1.887**	**0.342**	**0.729**	**10.510**	**7.136**	**5.337**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	0.759	2.842	**0.633**	**1.727**	**0.315**	**0.748**	**7.399**	**5.009**	**3.735**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	0.689	2.652	**0.641**	1.657	**0.308**	**0.777**	**6.489**	**4.386**	**3.265**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	0.573	2.174	0.445	1.436	0.197	0.648	**7.662**	**5.250**	**3.963**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	0.855	3.143	**0.504**	**1.672**	**0.272**	**0.676**	**10.915**	**7.467**	**5.629**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	**1.279**	**5.834**	**0.776**	**2.706**	**0.559**	**0.887**	**10.777**	**7.245**	**5.363**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	**1.244**	**5.440**	**0.736**	**2.380**	**0.478**	**0.824**	**11.079**	**7.469**	**5.545**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	**1.025**	**4.266**	**0.761**	**2.533**	**0.464**	**0.766**	**8.568**	**5.758**	**4.260**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	**1.217**	**5.400**	**0.695**	**2.234**	**0.489**	**0.886**	11.524	7.792	5.802
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	**1.008**	**4.121**	**0.745**	**2.110**	**0.459**	**0.872**	**8.616**	**5.797**	**4.294**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	**1.109**	**4.707**	**0.727**	**2.150**	**0.474**	**0.886**	**9.875**	**6.657**	**4.942**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	0.851	3.007	**0.505**	**1.714**	0.245	0.587	**10.845**	**7.419**	**5.592**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	0.836	2.899	0.477	1.619	0.228	0.596	11.278	7.729	5.837
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**1.019**	**3.958**	**0.638**	**1.901**	**0.336**	**0.708**	**10.349**	**7.019**	**5.243**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	0.815	3.142	**0.680**	**1.868**	**0.320**	**0.688**	**7.439**	**5.021**	**3.732**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.995**	**3.862**	**0.633**	**1.937**	**0.374**	**0.774**	**10.154**	**6.888**	**5.147**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	0.844	**3.243**	**0.677**	**1.749**	**0.306**	**0.715**	**7.795**	**5.264**	**3.915**

Note: The table reports out-of-sample performances of portfolio constructed based on forecast expected future return. Asset weights are allocated by the mean-variance optimisation method with target annual return 12%. Performance measures include: average monthly return (AR), end of period value (EPV), annualised Sharpe ratio, Omega ratio, Sortino ratio, Upside Potential ratio, and VaR 1%, 5% and 10%. Results which outperform the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2002 - Dec 2014.

Table 4.10: Out-of-sample performance of top t-statistics portfolios with 1/N allocation

	AR	EPV	Sharpe Ratio	Omega	Sortino	Upside	VaR 1%	VaR 5%	VaR 10%
OLS-AR(1)	0.570	2.392	1.258	2.763	0.583	0.913	2.319	1.473	1.022
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.575**	**2.392**	1.164	2.652	0.507	0.813	2.584	1.659	1.165
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.649**	**2.665**	1.182	**2.826**	**0.584**	0.904	2.965	1.906	1.342
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**0.650**	**2.661**	1.106	2.659	0.547	0.876	3.222	2.087	1.483
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.626**	**2.583**	1.177	2.718	0.567	0.898	2.846	1.829	1.287
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.628**	**2.591**	**1.260**	**2.871**	**0.589**	0.904	2.634	1.678	1.169
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**0.637**	**2.614**	1.061	2.553	0.522	0.857	3.297	2.145	1.531
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**0.589**	**2.447**	1.252	**2.795**	**0.586**	**0.913**	2.437	1.550	1.078
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.679**	**2.787**	1.252	**2.974**	**0.662**	**0.997**	2.928	1.871	1.308
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.662**	**2.706**	1.156	2.757	0.561	0.880	3.127	2.017	1.426
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	**0.624**	**2.564**	1.170	2.719	0.568	0.899	2.858	1.838	1.294
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.670**	**2.747**	1.178	**2.873**	**0.622**	**0.953**	3.102	1.997	1.408
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.760**	**3.133**	1.138	**3.108**	**0.711**	**1.048**	3.784	2.453	1.743
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**0.607**	**2.509**	1.138	2.593	0.532	0.866	2.849	1.836	1.297
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.619**	**2.542**	1.131	2.594	0.512	0.833	2.945	1.901	1.344
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.625**	**2.559**	1.045	2.503	0.498	0.830	3.282	2.137	1.527
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	**0.647**	**2.649**	1.093	2.556	0.500	0.821	3.248	2.107	1.498
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.692**	**2.830**	1.130	2.648	0.562	0.903	3.394	2.197	1.559
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.694**	**2.843**	1.198	2.630	0.569	**0.919**	3.175	2.042	1.438
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	**0.743**	**3.065**	1.221	**2.920**	**0.704**	**1.071**	3.375	2.169	1.526
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	**0.736**	**3.023**	1.122	2.730	**0.640**	**1.010**	3.701	2.401	1.708
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	**0.744**	**3.079**	**1.302**	**2.916**	**0.714**	**1.087**	3.126	1.992	1.388
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	**0.656**	**2.677**	1.063	2.367	0.504	0.873	3.416	2.223	1.587
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.670**	**2.723**	1.022	2.336	0.504	0.882	3.681	2.407	1.727
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.756**	**3.113**	1.121	2.609	**0.584**	**0.946**	3.825	2.483	1.768
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	**0.595**	**2.464**	**1.265**	**2.770**	**0.608**	**0.951**	2.439	1.550	1.076
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	**0.623**	**2.567**	1.241	**2.777**	**0.598**	**0.934**	2.651	1.692	1.181
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**0.692**	**2.846**	**1.289**	**3.010**	**0.712**	**1.067**	2.896	1.845	1.284
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	**0.616**	**2.546**	1.255	2.721	0.579	**0.915**	2.580	1.644	1.145
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.647**	**2.659**	1.258	**2.777**	**0.606**	**0.946**	2.735	1.745	1.216
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.715**	**2.942**	**1.320**	**3.032**	**0.694**	**1.035**	2.925	1.859	1.290

Note: The table reports out-of-sample performances of portfolio constructed based on the t-statistics of forecast expected future returns, asset weights are allocated by 1/N method. Performance measures include: average monthly return (AR), end of period value (EPV), annualised Sharpe ratio, Omega ratio, Sortino ratio, Upside Potential ratio, and VaR 1%, 5% and 10%. Results which outperform the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2002 - Dec 2014.

Table 4.11: Out-of-sample performance of top t-statistics portfolios with mean-variance optimisation allocation

	AR	EPV	Sharpe Ratio	Omega	Sortino	Upside	VaR 1%	VaR 5%	VaR 10%
OLS-AR(1)	0.350	1.709	0.726	1.825	0.297	0.657	2.218	1.466	1.065
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.426**	**1.913**	**1.353**	**3.681**	**0.810**	**1.111**	**1.405**	**0.868**	**0.582**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.471**	**2.029**	**0.780**	**2.261**	**0.423**	**0.759**	3.166	2.101	1.533
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**0.632**	**2.605**	**1.414**	**4.445**	**1.055**	**1.362**	2.295	**1.437**	**0.980**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.401**	**1.829**	**0.774**	**2.046**	**0.334**	0.654	2.539	1.678	1.219
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.368**	**1.737**	0.601	**1.868**	0.233	0.501	2.976	1.997	1.474
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**0.802**	**3.314**	**1.040**	**4.178**	**0.961**	**1.263**	4.496	2.944	2.117
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	0.213	1.366	0.228	1.302	0.078	0.335	3.139	2.157	1.633
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	0.350	1.653	0.381	1.528	0.143	0.413	4.531	3.101	2.339
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.507**	**2.096**	0.620	**1.890**	**0.329**	**0.698**	4.540	3.061	2.273
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	0.252	1.430	0.244	1.318	0.081	0.334	4.177	2.879	2.188
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	0.349	1.671	0.460	1.593	0.162	0.436	3.687	2.505	1.874
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.713**	**2.875**	**0.861**	**2.514**	**0.501**	**0.833**	4.857	3.225	2.355
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	0.283	1.522	0.387	1.472	0.142	0.443	3.148	2.143	1.607
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.386**	**1.773**	0.543	1.758	0.211	0.490	3.590	2.425	1.804
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.761**	**3.131**	**1.152**	**4.026**	**1.004**	**1.336**	3.731	2.415	1.714
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	0.298	1.552	0.430	1.565	0.152	0.420	3.070	2.083	1.557
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.455**	**1.975**	0.644	**1.871**	0.277	0.595	3.756	2.522	1.864
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.444**	**1.947**	**0.862**	**2.094**	**0.357**	**0.683**	2.603	1.710	1.234
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	**0.627**	**2.543**	**0.953**	**2.680**	**0.643**	**1.026**	3.671	2.412	1.741
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	**0.741**	**2.977**	**0.880**	**2.896**	**0.706**	**1.079**	4.955	3.287	2.397
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	**0.499**	**2.120**	**1.084**	**2.458**	**0.531**	**0.895**	2.329	1.501	**1.059**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	**0.405**	**1.832**	**0.786**	**2.030**	**0.323**	0.637	2.536	1.674	1.215
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.504**	**2.127**	**0.975**	**2.311**	**0.439**	**0.773**	2.683	1.749	1.252
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.508**	**2.140**	**0.966**	**2.277**	**0.450**	**0.803**	2.743	1.790	1.283
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	**0.469**	**2.057**	**1.473**	**3.708**	**0.859**	**1.176**	**1.448**	**0.887**	**0.587**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	0.193	1.301	0.130	1.183	0.042	0.270	4.390	3.048	2.332
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**0.613**	**2.526**	**1.204**	**2.745**	**0.655**	**1.030**	2.698	1.728	1.211
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	**0.584**	**2.450**	**1.596**	**4.752**	**1.130**	**1.431**	**1.766**	**1.078**	**0.711**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.560**	**2.370**	**1.532**	**4.350**	**1.061**	**1.378**	**1.762**	**1.082**	**0.719**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.616**	**2.526**	**1.133**	**2.680**	**0.569**	**0.908**	2.923	1.887	1.334

Note: The table reports out-of-sample performances of portfolio constructed based on the t-statistics of forecast expected future returns, asset weights are allocated by the mean-variance optimisation method with target annual return 12%. Performance measures include: average monthly return (AR), end of period value (EPV), annualised Sharpe ratio, Omega ratio, Sortino ratio, Upside Potential ratio, and VaR 1%, 5% and 10%. Results which outperform the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2002 - Dec 2014.

Table 4.12: Crisis periods - Out-of-sample performance of top expected return portfolios with 1/N allocation

	AR	EPV	Sharpe Ratio	Omega	Sortino	Upside	VaR 1%	VaR 5%	VaR 10%
OLS-AR(1)	0.667	1.100	0.133	1.112	0.054	0.536	16.231	11.281	8.642
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.908**	**1.221**	**0.285**	**1.249**	**0.122**	**0.614**	**13.727**	**9.440**	**7.154**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.742**	**1.167**	**0.209**	**1.179**	**0.087**	**0.573**	**12.849**	**8.867**	**6.745**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**0.814**	**1.190**	**0.242**	**1.213**	**0.104**	**0.593**	**13.298**	**9.164**	**6.960**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.695**	**1.130**	**0.162**	**1.133**	**0.066**	**0.563**	**14.479**	**10.034**	**7.664**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.848**	**1.198**	**0.254**	**1.220**	**0.109**	**0.602**	**13.676**	**9.421**	**7.153**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**0.677**	**1.144**	**0.166**	**1.143**	**0.066**	0.529	**13.272**	**9.186**	**7.007**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**0.912**	**1.237**	**0.307**	**1.266**	**0.129**	**0.613**	**12.825**	**8.800**	**6.655**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.823**	**1.199**	**0.252**	**1.215**	**0.105**	**0.596**	**13.020**	**8.965**	**6.803**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.809**	**1.193**	**0.251**	**1.220**	**0.107**	**0.596**	**12.690**	**8.736**	**6.628**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	**0.704**	**1.168**	**0.188**	**1.156**	**0.074**	**0.551**	**12.810**	**8.851**	**6.741**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.766**	**1.199**	**0.234**	**1.194**	**0.094**	**0.576**	**12.225**	**8.419**	**6.390**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.764**	**1.182**	**0.229**	**1.191**	**0.090**	**0.560**	**12.437**	**8.570**	**6.508**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	**0.849**	**1.193**	**0.246**	**1.209**	**0.104**	**0.601**	**14.224**	**9.808**	**7.455**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.935**	**1.232**	**0.295**	**1.258**	**0.124**	**0.608**	**13.960**	**9.596**	**7.270**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.703**	**1.163**	**0.191**	**1.163**	**0.074**	0.529	**12.583**	**8.691**	**6.616**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	**0.693**	**1.183**	**0.204**	**1.164**	**0.085**	**0.599**	**11.302**	**7.788**	**5.915**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.738**	**1.181**	**0.211**	**1.174**	**0.085**	**0.577**	**12.576**	**8.675**	**6.596**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.807**	**1.199**	**0.248**	**1.206**	**0.103**	**0.602**	**12.790**	**8.807**	**6.684**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	**0.879**	**1.244**	**0.318**	**1.278**	**0.142**	**0.653**	**11.511**	**7.882**	**5.947**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	**0.882**	**1.245**	**0.320**	**1.281**	**0.142**	**0.647**	**11.547**	**7.906**	**5.965**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	**0.792**	**1.211**	**0.280**	**1.237**	**0.118**	**0.618**	**10.791**	**7.398**	**5.589**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	0.593	**1.143**	**0.140**	1.111	**0.058**	**0.580**	**11.097**	**7.672**	**5.847**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	0.641	**1.164**	**0.174**	**1.138**	**0.073**	**0.602**	**11.019**	**7.603**	**5.782**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	0.609	**1.139**	**0.149**	**1.119**	**0.061**	**0.575**	**11.264**	**7.786**	**5.932**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	**1.020**	**1.255**	**0.324**	**1.289**	**0.143**	**0.636**	**14.690**	**10.087**	**7.634**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	**0.916**	**1.217**	**0.274**	**1.239**	**0.119**	**0.618**	**14.551**	**10.020**	**7.604**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**1.028**	**1.270**	**0.339**	**1.303**	**0.152**	**0.652**	**14.153**	**9.705**	**7.335**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	**1.181**	**1.339**	**0.412**	**1.378**	**0.189**	**0.688**	**14.311**	**9.772**	**7.353**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**1.122**	**1.311**	**0.381**	**1.346**	**0.175**	**0.680**	**14.370**	**9.832**	**7.412**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.878**	**1.213**	**0.257**	**1.223**	**0.110**	**0.602**	**14.440**	**9.952**	**7.560**

Note: The table reports out-of-sample performances of portfolio constructed based on forecast expected future returns, asset weights are allocated by the 1/N method. Performance measures include: average monthly return (AR), end of period value (EPV), annualised Sharpe ratio, Omega ratio, Sortino ratio, Upside Potential ratio, and VaR 1%, 5% and 10%. Results which outperform the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2009 - Dec 2010.

Table 4.13: Crisis periods - Out-of-sample performance of top expected return portfolios with mean-variance optimisation allocation

	AR	EPV	Sharpe Ratio	Omega	Sortino	Upside	VaR 1%	VaR 5%	VaR 10%
OLS-AR(1)	2.023	1.598	0.606	1.799	0.353	0.796	19.719	13.350	9.954
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**2.574**	**2.072**	**1.005**	**2.655**	**0.744**	**1.193**	**14.952**	**9.818**	**7.081**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**2.604**	**2.036**	**0.927**	**2.536**	**0.682**	**1.126**	**16.638**	**11.001**	**7.996**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**2.599**	**1.968**	**0.836**	**2.135**	**0.532**	**1.000**	**18.704**	**12.464**	**9.137**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	1.738	1.558	**0.630**	1.784	**0.381**	**0.867**	**15.522**	**10.466**	**7.770**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	1.825	**1.661**	**0.776**	**2.158**	**0.515**	**0.960**	**13.093**	**8.723**	**6.393**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	1.985	**1.709**	**0.765**	**1.958**	**0.470**	0.960	**14.828**	**9.903**	**7.277**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	1.007	1.394	**0.767**	**1.815**	**0.390**	**0.868**	**5.486**	**3.583**	**2.569**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	1.977	**1.767**	**0.882**	**2.151**	**0.557**	**1.041**	**12.535**	**8.283**	**6.017**
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**2.492**	**1.923**	**0.830**	**2.171**	**0.554**	**1.027**	**17.930**	**11.947**	**8.758**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	**2.145**	**1.709**	**0.694**	1.745	**0.414**	**0.969**	**18.257**	**12.280**	**9.094**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**2.468**	**1.884**	**0.793**	**2.174**	**0.551**	**1.021**	**18.648**	**12.462**	**9.165**
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**2.819**	**2.032**	**0.824**	**2.267**	**0.553**	**0.989**	20.949	13.986	10.275
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	1.683	1.468	0.526	1.633	0.306	0.789	**18.128**	**12.324**	**9.230**
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	1.740	1.454	0.506	1.596	0.283	0.758	19.787	13.481	10.119
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	0.107	0.966	-0.178	0.843	-0.085	0.458	**12.664**	**8.923**	**6.928**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	0.653	1.203	0.276	1.259	0.121	0.590	**7.032**	**4.781**	**3.581**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	0.518	1.076	0.086	1.076	0.036	0.516	**11.503**	**7.982**	**6.104**
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	1.709	1.417	0.477	1.547	0.265	0.748	20.578	14.049	10.568
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	**3.314**	**2.463**	**1.050**	**3.435**	**0.892**	**1.258**	**19.129**	**12.555**	**9.050**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	**2.895**	**2.123**	**0.890**	**2.736**	**0.659**	**1.039**	19.787	**13.142**	**9.600**
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	**2.068**	**1.736**	**0.784**	**2.337**	**0.520**	**0.908**	**15.187**	**10.132**	**7.438**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	**3.432**	**2.526**	**1.048**	**3.474**	**0.897**	**1.260**	19.978	**13.120**	**9.464**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	**2.366**	**1.960**	**0.972**	**2.910**	**0.792**	**1.206**	**14.025**	**9.223**	**6.664**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	**2.878**	**2.220**	**1.015**	**2.661**	**0.760**	**1.218**	**16.888**	**11.098**	**8.011**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	1.139	1.164	0.273	1.288	0.128	0.574	21.022	14.530	11.069
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	1.214	1.190	0.297	1.318	0.143	0.592	21.157	14.604	11.110
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	1.549	1.388	0.461	1.544	0.247	0.702	**18.743**	**12.799**	**9.630**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	0.805	1.195	0.247	1.248	0.105	0.527	**12.740**	**8.772**	**6.657**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	1.767	1.496	0.547	1.692	0.326	**0.797**	**18.530**	**12.584**	**9.414**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	1.051	1.327	0.433	1.426	0.188	0.630	**11.255**	**7.650**	**5.728**

Note: The table reports out-of-sample performances of portfolio constructed based on forecast expected future returns, asset weights are allocated by the mean-variance optimisation method with target annual return 12%. Performance measures include: average monthly return (AR), end of period value (EPV), annualised Sharpe ratio, Omega ratio, Sortino ratio, Upside Potential ratio, and VaR 1%, 5% and 10%. Results which outperform the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2009 - Dec 2010.

Table 4.14: Crisis periods - Out-of-sample performance of top t-statistics portfolios with 1/N allocation

	AR	EPV	Sharpe Ratio	Omega	Sortino	Upside	VaR 1%	VaR 5%	VaR 10%
OLS-AR(1)	0.542	1.189	0.258	1.228	0.105	0.565	4.227	2.830	2.085
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	0.378	1.113	-0.017	0.986	-0.006	0.430	4.840	3.311	2.496
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.722**	**1.239**	**0.424**	**1.426**	**0.189**	**0.635**	5.604	3.751	2.763
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**0.758**	**1.248**	**0.438**	**1.450**	**0.201**	**0.647**	6.016	4.032	2.974
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.567**	1.183	0.242	1.217	0.103	**0.580**	5.343	3.612	2.689
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	0.486	1.156	0.148	1.134	0.059	0.498	4.787	3.242	2.419
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**0.732**	**1.238**	**0.404**	**1.412**	**0.187**	**0.642**	6.110	4.106	3.037
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	0.424	1.133	0.059	1.050	0.023	0.485	4.375	2.969	2.220
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.762**	**1.256**	**0.481**	**1.494**	**0.233**	**0.704**	5.476	3.649	2.675
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.763**	**1.254**	**0.463**	**1.496**	**0.206**	**0.622**	5.755	3.845	2.828
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	0.530	1.163	0.195	1.192	0.085	0.528	5.305	3.596	2.684
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.780**	**1.260**	**0.482**	**1.545**	**0.242**	**0.685**	5.746	3.834	2.815
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**1.246**	**1.467**	**0.836**	**2.222**	**0.527**	**0.958**	7.018	4.597	3.306
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	0.474	1.149	0.120	1.104	0.049	0.521	5.229	3.559	2.668
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	0.450	1.137	0.082	1.072	0.032	0.482	5.488	3.749	2.821
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**0.691**	**1.218**	**0.358**	**1.362**	**0.159**	**0.600**	6.098	4.110	3.049
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	0.468	1.139	0.094	1.084	0.038	0.494	6.293	4.313	3.257
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.736**	**1.242**	**0.396**	**1.393**	**0.182**	**0.645**	6.317	4.251	3.149
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.667**	**1.229**	**0.387**	**1.388**	**0.166**	**0.595**	5.101	3.412	2.511
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	**0.974**	**1.341**	**0.669**	**1.786**	**0.368**	**0.837**	6.070	4.006	2.906
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	**0.981**	**1.340**	**0.624**	**1.727**	**0.348**	**0.828**	6.657	4.420	3.227
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	**1.021**	**1.381**	**0.901**	**2.137**	**0.530**	**0.996**	4.626	2.972	2.090
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	0.526	1.156	0.168	1.149	0.071	0.548	6.040	4.117	3.091
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.678**	**1.215**	**0.335**	**1.319**	**0.155**	**0.641**	6.282	4.243	3.156
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.985**	**1.347**	**0.649**	**1.750**	**0.337**	**0.787**	6.414	4.246	3.091
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	0.528	1.185	0.233	1.217	0.096	0.537	4.256	2.855	2.108
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	**0.558**	**1.194**	**0.267**	**1.256**	**0.111**	0.544	4.533	3.042	2.247
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**0.931**	**1.356**	**0.777**	**1.963**	**0.425**	**0.866**	4.692	3.044	2.166
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	0.484	1.168	0.154	1.136	0.059	0.495	4.456	3.009	2.238
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.563**	**1.201**	**0.268**	**1.249**	**0.110**	0.555	4.666	3.134	2.318
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.855**	**1.323**	**0.675**	**1.796**	**0.338**	**0.762**	4.705	3.076	2.208

Note: The table reports out-of-sample performances of portfolio constructed based on the t-statistics of forecast expected future returns, asset weights are allocated by the 1/N method. Performance measures include: average monthly return (AR), end of period value (EPV), annualised Sharpe ratio, Omega ratio, Sortino ratio, Upside Potential ratio, and VaR 1%, 5% and 10%. Results which outperform the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2009 - Dec 2010.

Table 4.15: Crisis periods - Out-of-sample performance of top t-statistics portfolios with mean-variance optimisation allocation

	AR	EPV	Sharpe Ratio	Omega	Sortino	Upside	VaR 1%	VaR 5%	VaR 10%
OLS-AR(1)	-0.081	0.967	-1.209	0.206	-0.340	0.088	3.212	2.295	1.806
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.570**	**1.226**	**0.959**	**2.582**	**0.778**	**1.269**	**0.947**	**0.503**	**0.266**
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.700**	**1.226**	**0.386**	**1.423**	**0.197**	**0.662**	5.776	3.879	2.868
BMA ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**1.160**	**1.466**	**1.393**	**5.754**	**1.326**	**1.605**	3.298	**1.992**	**1.296**
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.92$)	**0.287**	**1.088**	**-0.187**	**0.854**	**-0.067**	**0.392**	4.116	2.826	2.139
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.94$)	**0.223**	**1.057**	**-0.226**	**0.803**	**-0.077**	**0.314**	5.707	3.970	3.044
BMS ($\alpha = \upsilon = 1$, $\kappa = 0.97$)	**1.984**	**1.844**	**1.303**	**5.629**	**1.317**	**1.601**	7.881	4.991	3.451
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	-0.371	0.864	**-1.039**	**0.291**	**-0.302**	**0.124**	6.265	4.539	3.618
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.066**	0.964	**-0.279**	**0.768**	**-0.097**	**0.320**	9.284	6.545	5.085
DMA ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**1.155**	**1.409**	**0.710**	**2.019**	**0.432**	**0.855**	7.537	4.990	3.633
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	-0.505	0.808	**-0.904**	**0.324**	**-0.262**	**0.125**	8.471	6.137	4.893
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	-0.335	0.861	**-0.877**	**0.360**	**-0.256**	**0.144**	6.994	5.043	4.003
DMA ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**1.287**	**1.433**	**0.690**	**1.885**	**0.386**	**0.822**	9.208	6.134	4.495
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.92$)	-0.295	0.890	**-1.007**	**0.350**	**-0.299**	**0.161**	5.766	4.163	3.309
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.94$)	**0.252**	**1.058**	**-0.155**	**0.871**	**-0.054**	**0.366**	6.878	4.790	3.676
DMS ($\alpha = \upsilon = 0.996$, $\kappa = 0.97$)	**1.760**	**1.737**	**1.371**	**5.912**	**1.425**	**1.715**	6.297	3.937	2.679
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.92$)	-0.208	0.910	**-0.815**	**0.421**	**-0.243**	**0.177**	6.109	4.380	3.459
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.94$)	**0.609**	**1.194**	**0.233**	**1.212**	**0.091**	**0.522**	7.009	4.777	3.587
DMS ($\alpha = \upsilon = 0.985$, $\kappa = 0.97$)	**0.197**	**1.038**	**-0.348**	**0.720**	**-0.113**	**0.293**	4.242	2.941	2.248
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.92$)	**1.174**	**1.434**	**0.824**	**2.239**	**0.570**	**1.031**	6.503	4.254	3.055
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.94$)	**1.676**	**1.650**	**0.967**	**2.926**	**0.814**	**1.237**	9.049	5.907	4.232
TVP-ALL ($\upsilon = 0.996$, $\kappa = 0.97$)	**0.559**	**1.197**	**0.345**	**1.358**	**0.147**	**0.558**	3.412	**2.249**	**1.629**
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.92$)	**0.175**	**1.053**	**-0.396**	**0.660**	**-0.128**	**0.249**	4.198	2.917	2.234
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.474**	**1.159**	**0.131**	**1.134**	**0.050**	**0.426**	4.710	3.191	2.382
TVP-ALL ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.600**	**1.208**	**0.324**	**1.354**	**0.133**	**0.507**	4.640	3.105	2.287
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.92$)	**0.731**	**1.292**	**1.567**	**5.399**	**1.362**	**1.672**	**1.025**	**0.511**	**0.236**
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.94$)	-0.684	0.759	**-1.011**	**0.248**	**-0.290**	**0.095**	9.236	6.730	5.395
TVP-AR(1) ($\upsilon = 0.996$, $\kappa = 0.97$)	**0.988**	**1.382**	**0.937**	**2.068**	**0.465**	**0.899**	4.160	2.652	1.848
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.92$)	**0.672**	**1.269**	**1.328**	**3.976**	**1.209**	**1.616**	**1.042**	**0.540**	**0.272**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.94$)	**0.659**	**1.263**	**1.223**	**3.378**	**0.982**	**1.394**	**1.116**	**0.596**	**0.319**
TVP-AR(1) ($\upsilon = 0.985$, $\kappa = 0.97$)	**0.823**	**1.304**	**0.634**	**1.686**	**0.273**	**0.671**	4.693	3.077	2.215

Note: The table reports out-of-sample performances of portfolio constructed based on the t-statistics of forecast expected future returns, asset weights are allocated by the mean-variance optimisation method with target annual return 12%. Performance measures include: average monthly return (AR), end of period value (EPV), annualised Sharpe ratio, Omega ratio, Sortino ratio, Upside Potential ratio, and VaR 1%, 5% and 10%. Results which outperform the benchmark OLS-AR(1) model are in boldface. Out-of-sample forecasting period: Jan 2009 - Dec 2010.

Chapter 5

Conclusion

Motivated by current post-crisis discussions and the corresponding shift in regulatory requirements, this thesis is dedicated to the study of model risk in financial modelling. It is well-known that the majority of finance quantities that are involved in asset pricing, trading, and risk management activities are dependent on the chosen financial models. This gives rise to model risk in all financial activities. Even when the chosen model form is appropriate, model outputs are still subject to parameter estimation uncertainty. Therefore, among different sources of model risk, we mainly focused on investigating the impact of parameter estimation risk and model selection risk in different financial models. Models investigated in this thesis are key models in option pricing, credit risk management, stochastic process of security returns and hedge fund return forecasting.

We provoke a solution which naturally stems from the Bayesian framework. Regarding parameter estimation risk, instead of focusing on point estimation value, it is possible to gauge the rich information about parameter uncertainty from the posterior distribution of parameters. Subsequent impact to model final outputs can be easily accessed by inserting the posterior distribution of parameters into the model. Depending on the related financial activities, model users may find it useful to adopt the estimated value at a certain percentile (e.g. 97.5%) of the posterior distribution as an overlay to the estimated mean value. While more than one candidate model is considered, posterior or predictive probability of a candidate model derived from the likelihood of the model output in fitting the data is applied for a model averaging exercise to account for model selection risk.

Summaries of the key findings in each area are presented in Section 5.1 to 5.3. Further research directions are discussed in Section 5.4.

5.1 Option Pricing and Credit Risk Management

In Chapter 2, we carry out an investigation of the performance of the Black-Scholes (BS) and Merton's Jump-Diffusion (MJD) models in option-pricing activities, incorporating parameter estimation risk. Bayesian posterior distributions are simulated by the Markov Chain Monte Carlo (MCMC) simulation techniques. The MJD model was developed by Merton (1976) as an alternative model to the BS model, capable of generating kurtosis and skewness in line with empirical literature on stock returns; see Bakshi et al. (1997) and Dahlbokum (2010). To this end, we show how to construct the entire distributions of option prices underpinned by distributions of parameters for the two models investigated. Our results show that, among the three in-sample fitting tests we adopted, the MJD model significantly outperforms the BS model, except when the Bayesian p-value test is utilised. On the other hand, the MJD model outperforms the BS model in all out-of-sample pricing performance measures when parameter values are calibrated from market option prices rather than estimated from historical returns. Furthermore, while results of the BS model shows less parameter uncertainty as indicated by the narrower 95% credibility intervals of posterior model price distributions, the posterior MJD model price distribution with wider range better captures the realised market option value. Therefore, simpler models may show less parameter uncertainty but this could in fact give a false security to model users.

In addition, we also apply the Bayesian MCMC technology to look at the Greeks parameters, and subsequently derive a VaR-type measure for parameter estimation risk to option pricing. Taking parameter estimation risk into account, movement of Greek parameters can show a different picture (e.g. more volatile) compared to the movement when only point estimation values are considered. Our VaR-type parameter estimation risk measure reveals that the risk exposure is of a different magnitude to the two parties of European option holders and accounts for a substantial amount of the option trading price.

It is very important for model users to be aware of the existence and magnitude of any model they adopt for option pricing or any type of asset pricing. Traditional model performance metrics, such as mean pricing error, mean absolute pricing error or mean square pricing error might not show sufficient merit to fully assess a model's performance in a risk management aspect. It is possible that a complex model might not outperform the simpler model due to the difficulty in parameter estimation, but after incorporating parameter uncertainty the model could perform considerably better than the simpler model in capturing the true asset value.

Furthermore, when assessing potential risk of the option trading portfolio, financial institutions might want to consider parameter estimation risk in parallel to their calibrated market risk metrics, such as portfolio VaR. Either the parameter estimation risk measured by the VaR-type method proposed can be incorporated in extra to the calibrated portfolio VaR, or the maximum of the two values can be adopted to achieve a more prudent estimation of the portfolio risk. Further research can be carried out on the topic of how to incorporate parameter estimation risk VaR into the VaR estimation of traded portfolio; this is not constraint to options but all types of securities. In fact, the Bayesian estimation framework and the VaR-type parameter estimation risk measure advocated in Chapter 2 can be extended to all types of asset pricing whenever a parametric asset pricing model is used.

Also in Chapter 2, we describe how to apply the Bayesian approach to the Merton's Credit Risk model with a focus on capturing parameter estimation risk in computing the probability of default. Results show that the impact of parameter uncertainty becomes more severe with increase in gearing level. Taking into account parameter estimation risk can result in different credit rating, regulatory capital requirement and internal investment decision of a company. This result indicates that parameter estimation risk in computing default probabilities skews towards more stress funding scenarios. In other words, the impact of parameter estimation risk is more material when a firm's funding stress increases. This finding shows that neglecting estimation risk may significantly deteriorate the prudence of credit risk management metrics in a situation where the default risk is higher.

5.2 The Volatility and Skewness Crystal Ball

It is a common knowledge that during market stress periods, models tend to perform poorly in both in-sample fitting and out-of-sample prediction. Increases in market uncertainty (indicated by increased volatility and noisiness of market data) and structural change in key market statistics offer an important contribution to the poor model performance. While increased in the noisiness of market data is very likely to result in higher level of parameter estimation risk, structural change is a matter of model selection issue. In Chapter 3, we employ Merton's jump-diffusion model to analyse the evolution of the S&P 500 index returns volatility and skewness between 1980 and 2015, accounting for parameter estimation risk. We compare and contrast the market dynamics among the significant financial crises during our study period: the Black Monday Crash in 1987, the "dot-com" crisis in early 2000s and the global financial crisis in 2008. The underlying evolution of key components of the market index and the associate model risk provide insights into the different periods of market distress. Through in-depth analysis, we find empirical results in line with the Minsky theory(Minsky, 1982; Minsky and Kaufman, 2008; Minsky, 1992), and also confirms the theory of endogenous risk (Danielsson and Shin, 2003).

In particular, the 1987 market crash is found to have distinct market dynamics compared to the "dot-com" crisis and the 2008 global financial crisis. Evidenced by the evolution of estimated market volatility and associated estimation risk, endogenous risk can result in a tremendous crash through feedback effect of market participants, when the feedback effect does not root on or reinforces the need of market prices adjustment, the crash will be rather short-lived, and may not even bring the market return into negative. In contrast, when the initial market collapse is reinforced, volatility continues to surge and market return continues to fall after the initial shock. The former explains the market behaviour as observed in the 1987 crash, and the latter explains the other two stress periods in the sample data. Our findings reveal the deficiency of the endogenous risk theory. While the shifts in market participants' beliefs and their subsequent reactions can result in market crashes, whether these beliefs are rooted in a substantial over-valuation problem and reinforces that problem or not would make a big difference to the final outcome. In short, the "endogenous risk" defined by Danielsson and Shin (2003) can result in dramatic market crash, yet alone is insufficient to generate a real financial crisis.

Another notable contribution of this research is that, we are the first to identify and publish the discovery of a long consecutive period prior to the 2008 crisis when the market return distribution returned to normal in the US market. During this period, estimation risk of the MJD model in both volatility and skewness estimation deviated from their normal range. We found estimation failure of the MJD model during these periods as the underlying market distribution returned to Gaussian. Further analysis has been carried out regarding this structural change as identified by the exhibited model selection uncertainty. The importance of this finding is threefold. Firstly, it is in contradiction to common empirical findings that stock returns deviate from normal distribution and exhibit negative skewness and excess kurtosis. Our test results show that stock returns did return to gaussian, for a long period. Secondly, it indicates the necessity of considering model uncertainty in stock return parameter estimation. Throughout the changes of market performance, return distributions can deviate significantly from one model and switching onto another. Finally, the gaussian period happened in parallel with the period of very low market volatility before the 2008 crisis.

We carried out further analysis and found that when a calm period is observed, a high level of sentiment leads to an extra negative impact to the subsequent market returns. This is aligned with the Minsky's theory of "stability is destabilsing". The explanation is that when normality interacted with high levels of sentiment, it encourages extra risk taking and an overestimation of future growth, resulting in overvaluation or bubbles of assets prices which requires correction in a later period. When the magnitude of bubble is remarkable, the consequential correction would be a market crash or crisis similar to what we observed in the 2008 crisis episode.

These results shed light on the policy makers and regulators in two ways: firstly, parameter estimation risk may effectively act as an indicator to potential market distress and to distinguish different features of market crash in the after maths analysis. Secondly, when market return distribution exhibits a shift to Gaussian in parallel to a high level of market investor sentiment, it is worth considering it as an alert of potential forthcoming crisis. Policy makers and regulators may consider taking actions accordingly to shape market behaviour and maintain market stabilities.

5.3 Hedge Fund Return Forecasting and Portfolio Construction

In Chapter 4, we show both statistical and economic value of considering both parameter estimation risk and model selection risk in portfolio construction. We apply the Bayesian estimation paradigm particularly to hedge fund return forecasting and portfolio construction. Different from Chapter 2 and 3, instead of using the MCMC simulation techniques, we calibrate posterior and predictive distributions of parameters and model output by adopting the Kalman filter algorithm. We employ the methods introduced by Koop and Korobilis (2012). Parameter uncertainty is dealt with by the time-varing parameter structure, and model selection uncertainty is mitigated by model averaging or model selection. Six categories of alternative models are considered dealing with estimation risk or both types of model risk: time-varying parameter AR(1) model (TVP-AR(1)); time-varying parameter model with all risk factors (TVP-ALL); Bayesian model averaging (BMA); Bayesian model selection (BMS); dynamic model averaging (DMA); and finally dynamic model selection (DMS). In total, 30 alternative models are investigated comparing to the benchmark model OLS-AR(1).

We find that the proposed methods have good statistical value in terms of forecast accuracy as measured by MSFE and log predictive likelihood. In the analysis of certainty equivalent return (CER), competing models deliver superior CER results, compared to the benchmark when estimation risk are further mitigated by adopting estimation variance of the model forecasts as the forecast variance of hedge fund returns, indicating the very strong economic value of the model forecasts.

Regarding portfolio construction exercises, the majority of competing models outperform the benchmark model. Among all, BMA and TVP-AR(1) models with gradual evolution speed of parameters show the best and most stable performance. On the other hand, the results suggest that, the DMA models with the time-varying parameter settings do not contribute significant extra value compared to the conventional Bayesian model averaging approach in terms of constructing hedge fund portfolios. Comparison between the portfolios selected based on expected returns or t-statistics of expected returns shows that t-statistics portfolios have lower level of risk, this is evidenced by reduced portfolio return, yet increased Sharpe Ratio and reduced VaR. The outstanding performance of competing models persisted during the crisis period 2007 - 2009, but with lower absolute returns and much higher return volatilities and tail risk. Models with a decay factor closer to 1 in the EWMA forecast of regression residuals variance generate better results overall, which supports volatility clustering in market stress periods.

5.4 Limitations and Further Research

In this thesis, we advocate the Bayesian estimation framework as a natural way to incorporate and account for model risk in various financial models, as it delivers the joint posterior distribution of parameters, which contains all possible value of the parameters given the model and data. Therefore, under the Bayesian framework, uncertainty in parameter estimation is accounted for because all parameters are treated as stochastic. While the posterior density of parameter is determined by the selected likelihood of data conditioned on parameters and your prior beliefs of parameters. We acknowledge that influence of prior distribution selection towards the conclusion of model outcome remains, particularly in the case when non-informative priors cannot be adopted. Prior distribution as another imposed model assumption subject to researcher's own judgement becomes another source of model risk. In the Bayesian parameter inference exercises carried out in the research of this thesis, the impact of prior selections to each of our model outcome has not been evaluated. Instead, we adopt the priors suggested by existing literature. It would be interesting to look further into the potential impact of different prior selections. Brief discussion of how to select prior distribution and potential impact is presented in Lancaster (2004) and Gelman et al. (2014). Nevertheless, very few of the related financial literature has carried out thorough investigation regarding this issue, thus exposing a gap in the financial literature. Even in the case of the MJD model, where informative priors are unavoidable due to the unbounded likelihood, it is still interesting to see how different forms of informative priors may affect the final model outcome. For example, instead of adopting gamma distribution, beta, uniform or log-normal distribution can also be possible choices for the precision of diffuse returns and jump component.

In the topic of model risk in option pricing, we have constructed the posterior distribution of the Greeks parameters, and show that even the movement of Greeks with or without consideration of parameter estimation risk can be quite different. Further empirical study can follow to investigate how model users can obtain economic value through the rich information contained in the posterior distributions and provide practical strategies. Bayesian model averaging exercise to further incorporate model selection risk and related empirical analysis is another interesting direction where gaps remain in the Bayesian option-pricing literature. Other important option-pricing models include stochastic volatility models (Hull and White, 1987; Scott, 1987; Heston, 1993), jump-diffusion models with exponential jumps (Kou, 2002) and stochastic volatility with jump-diffusion models (Bates, 1996; Eraker et al., 2003; Kaeck and Alexander, 2013) and etc. The proposed VaR-type measure of parameter estimation risk can be used as a supplementary element to the conventional VaR metrics in assessing portfolio market risk. Further research can be carried out on the topic of how to incorporate this parameter estimation risk VaR into the VaR estimation of trading portfolio. This is not constrained to options but can be extended to all types of securities whenever a parametric asset pricing model is used.

In Chapter 3, we have observed the model convergence failure due to the fact that market returns in a two-year rolling window from March 2003 - December 2006 are asymptotically normal. It would be very interest to investigate how the parameters of stochastic volatility models (e.g. the Heston model) will be calibrated throughout the same periods. Whether the calibrated parameters would suggest constant volatility instead of stochastic volatility may shed more lights to the empirical process of market returns, or may further support our conclusion of market returns returned to Gaussian.

In Chapter 4, we have been focused on constructing portfolios using better forecast of hedge funds' expected returns. We either select the hedge funds directly based on forecast expected returns, or we work on the t-statistics of the forecast value (i.e. the expected returns are penalised by the embedded estimation error). In terms of portfolio variance, we assumed that variance of hedge fund returns is constant, and hence adopted historical variance of the series as a good proxy. However, this assumption can be relaxed, and existing literature has tried to gauge a better forecast of portfolio variance. For example, see Roumpis and Syriopoulos (2014); Giamouridis and Vrontos (2007). The current research of Chapter 4 can be extended to incorporate the forecast of variance and covariance metrics of hedge funds under the Bayesian framework, and construct portfolio using the estimated quantities with minimum variance or mean-variance optimisation method.

References

Acharya, V. V. and Viswanathan, S. (2011), 'Leverage, moral hazard, and liquidity', *Journal of Finance* **66**(1), 99–138.

Adrian, T. and Brunnermeier, M. K. (2011), CoVaR, Technical report, National Bureau of Economic Research.

Agarwal, V. and Naik, N. Y. (2004), 'Risks and portfolio decisions involving hedge funds', *Review of Financial studies* **17**(1), 63–98.

Aït-Sahalia, Y. and Brandt, M. W. (2001), 'Variable selection for portfolio choice', *The Journal of Finance* **56**(4), 1297–1351.

Altunbas, Y., Gambacorta, L. and Marques-Ibanez, D. (2010), 'Does monetary policy affect bank risk-taking?'. ECB Working Paper.

Amaya, D., Christoffersen, P., Jacobs, K. and Vasquez, A. (2015), 'Does realized skewness predict the cross-section of equity returns?', *Journal of Financial Economics* **118**(1), 135–167.

Amenc, N., El Bied, S. and Martellini, L. (2003), 'Predictability in hedge fund returns', *Financial Analysts Journal* pp. 32–46.

Andersen, T. G., Fusari, N. and Todorov, V. (2015), 'The risk premia embedded in index options', *Journal of Financial Economics* **117**, 558–584.

Arditti, F. D. (1967), 'Risk and the required return on equity', *Journal of Finance* **22**(1), 19–36.

Avellaneda, M., Levy, A. and Parás, A. (1995), 'Pricing and hedging derivative securities in markets with uncertain volatilities', *Applied Mathematical Finance* **2**(2), 73–88.

Avramov, D. (2002), 'Stock return predictability and model uncertainty', *Journal of Financial Economics* **64**(3), 423–458.

Avramov, D., Barras, L. and Kosowski, R. (2013), 'Hedge fund return predictability under the magnifying glass', *Journal of Financial and Quantitative Analysis* **48**(04), 1057–1083.

Avramov, D., Kosowski, R., Naik, N. Y. and Teo, M. (2011), 'Hedge funds, managerial skill, and macroeconomic variables', *Journal of Financial Economics* **99**(3), 672–692.

Aye, G., Gupta, R., Hammoudeh, S. and Kim, W. J. (2015), 'Forecasting the price of gold using dynamic model averaging', *International Review of Financial Analysis* **41**, 257–266.

Backus, D. K., Foresi, S. and Wu, L. (2004), 'Accounting for biases in black-scholes', *SSRN* .

Baig, T. and Goldfajn, I. (1999), 'Financial market contagion in the Asian crisis', *IMF Staff Papers* **46**(2), 167–195.

Baker, M. and Wurgler, J. (2007), 'Investor sentiment in the stock market', *Journal of Economic Perspectives* **21**(2), 129–151.

Bakshi, G., Cao, C. and Chen, Z. (1997), 'Empirical performance of alternative option pricing models', *The Journal of Finance* **52**(5), 2003–2049.

Bakshi, G., Kapadia, N. and Madan, D. (2003), 'Stock return characteristics, skew laws, and the differential pricing of individual equity options', *Review of Financial Studies* **16**(1), 101–143.

Bali, T., Atilgan, Y. and Demirtas, O. (2013), *Investing in hedge funds: a guide to measuring risk and return characteristics*, Academic Press.

Bali, T. G., Brown, S. J. and Caglayan, M. O. (2011), 'Do hedge funds' exposures to risk factors predict their future returns?', *Journal of financial economics* **101**(1), 36–68.

Bali, T. G., Brown, S. J. and Caglayan, M. O. (2012), 'Systematic risk and the cross section of hedge fund returns', *Journal of Financial Economics* **106**(1), 114–131.

Ball, C. A. and Torous, W. N. (1985), 'On jumps in common stock prices and their impact on call option pricing', *The Journal of Finance* **40**(1), 155–173.

Bannör, K. F. and Scherer, M. (2013), 'Capturing parameter risk with convex risk measures', *European Actuarial Journal* **3**(1), 97–132.

Barberis, N. (2000), 'Investing for the long run when returns are predictable', *The Journal of Finance* **55**(1), 225–264.

Basel Committee on Banking Supervision (2006), *International Convergence of Capital Measurement and Capital Standards*, Bank for International Settlements, Basel.

Basel Committee on Banking Supervision (2010), *Sound practices for backtesting counterparty credit risk models*, Bank for International Settlements, Basel.

Basel Committee on Banking Supervision (2011), *Revisions to the Basel II Market Risk Framework*, Bank for International Settlements, Basel.

Bates, D. S. (1996), 'Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options', *Review of Financial Studies* **9**(1), 69–107.

Batten, J. A. and Ellis, C. A. (2005), 'Paramater estimation bias and volatility scaling in Black–Scholes option prices', *International Review of Financial Analysis* **14**(2), 165–176.

Baur, D. G., Beckmann, J. and Czudaj, R. (2014), 'Gold price forecasts in a dynamic model averaging framework–have the determinants changed over time?', *SSRN* .

Bauwens, L. and Lubrano, M. (2002), 'Bayesian option pricing using asymmetric GARCH models', *Journal of Empirical Finance* **9**(3), 321–342.

Bekaert, G. and Wu, G. (2000), 'Asymmetric volatility and risk in equity markets', *Review of Financial Studies* **13**, 1–42.

Bernhardt, D. and Eckblad, M. (1987), 'Black Monday: The stock market crash of 1987'. Federal Reserve History.

Bernstein, W. J. and Fridson, M. S. (2016), 'Why Minsky matters: An introduction to the work of a maverick economist (a review)', *Book Reviews* **11**(1), 1–1.

Bork, L. and Møller, S. V. (2015), 'Forecasting house prices in the 50 states using dynamic model averaging and dynamic model selection', *International Journal of Forecasting* **31**(1), 63–78.

Boucher, C. M., Daníelsson, J., Kouontchou, P. S. and Maillet, B. B. (2014), 'Risk models-at-risk', *Journal of Banking & Finance* **44**, 72–92.

Brandt, M. and Kang, Q. (2004), 'On the relationship between the conditional mean and volatility of stock returns: a latent VAR approach', *Journal of Financial Economics* **72**, 217–257.

Branger, N., Krautheim, E., Schlag, C. and Seeger, N. (2012), 'Hedging under model misspecification: All risk factors are equal, but some are more equal than others', *Journal of Futures Markets* **32**(5), 397–430.

Britten-Jones, M., Neuberger, A. and Nolte, I. (2011), 'Improved inference in regression with overlapping observations', *Journal of Business Finance & Accounting* **38**(5-6), 657–683.

Broadie, M., Chernov, M. and Johannes, M. (2007), 'Model specification and risk premia: Evidence from futures options', *Journal of Finance* **62**, 1453–1490.

Brooks, S. and Gelman, A. (1998), 'Some issues for monitoring convergence of iterative simulations', *Computing Science and Statistics* pp. 30–36.

Brown, G. W. and Cliff, M. T. (2005), 'Investor sentiment and asset valuation', *Journal of Business* **78**(2), 405–440.

Buncic, D. and Moretto, C. (2015), 'Forecasting copper prices with dynamic averaging and selection models', *The North American Journal of Economics and Finance* **33**, 1–38.

Bunnin, F. O., Guo, Y. and Ren, Y. (2002), 'Option pricing under model and parameter uncertainty using predictive densities', *Statistics and Computing* **12**(1), 37–44.

Burger, P. and Kliaris, M. (2013), 'Jump diffusion models for option pricing vs. the black scholes model', *Working Paper, University of Applied Sciences bfi Vienna* .

Butler, J. and Schachter, B. (1997), 'Estimating value-at-risk with a precision measure by combining kernel estimation with historical simulation', *Review of Derivatives Research* **1**, 371–390.

Caldeira, J. F., Moura, G. V. and Santos, A. A. (2016), 'Bond portfolio optimization using dynamic factor models', *Journal of Empirical Finance* **37**, 128–158.

Campbell, J. Y. and Thompson, S. B. (2008), 'Predicting excess stock returns out of sample: Can anything beat the historical average?', *Review of Financial Studies* **21**(4), 1509–1531.

Chang, B. Y., Christoffersen, P. and Jacobs, K. (2013), 'Market skewness risk and the cross section of stock returns', *Journal of Financial Economics* **107**(1), 46–68.

Chatfield, C. (1995), 'Model uncertainty, data mining and statistical inference', *Journal of the Royal Statistical Society. Series A (Statistics in Society)* **158**(3), pp. 419–466.

Chen, Z. et al. (2003), 'Bayesian filtering: From Kalman filters to particle filters, and beyond', *Statistics* **182**(1), 1–69.

Christoffersen, P. and Gonçalves, S. (2005), 'Estimation risk in financial risk management', *Journal of Risk* **7**(3), 1–28.

Chung, T.-K., Hui, C.-H. and Li, K.-F. (2013), 'Explaining share price disparity with parameter uncertainty: Evidence from Chinese A-and H-shares', *Journal of Banking & Finance* **37**(3), 1073–1083.

Chung, Y. P., Johnson, H. and Schill, M. J. (2006), 'Asset pricing when returns are nonnormal: Fama-French factors versus higher-order systematic comoments', *Journal of Business* **79**(2), 923–940.

Chunhachinda, P., Dandapani, K., Hamid, S. and Prakash, A. J. (1997), 'Portfolio selection and skewness: Evidence from international stock markets', *Journal of Banking & Finance* **21**(2), 143–167.

Clark, T. E. and West, K. D. (2006), 'Using out-of-sample mean squared prediction errors to test the martingale difference hypothesis', *Journal of Econometrics* **135**(1), 155–186.

Clark, T. E. and West, K. D. (2007), 'Approximately normal tests for equal predictive accuracy in nested models', *Journal of econometrics* **138**(1), 291–311.

Congdon, P. (2014), *Applied bayesian modelling*, Vol. 595, John Wiley & Sons.

Cont, R. (2001), 'Empirical properties of asset returns: stylized facts and statistical issues', *Quantitative Finance* **1**(2), 223–236.

Cont, R. (2006), 'Model uncertainty and its impact on the pricing of derivative instruments', *Mathematical Finance* **16**(3), 519–547.

Corsi, F., Mittnik, S., Pigorsch, C. and Pigorsch, U. (2008), 'The volatility of realized volatility', *Econometric Reviews* **27**(1-3), 46–78.

Dahlbokum, A. (2010), 'Empirical performance of option pricing models based on time-changed lévy processes', *SSRN* .

Danielsson, J. and Shin, H. S. (2003), *Endogenous risk*, Risk Books.

Danielsson, J., Shin, H. S. and Zigrand, J.-P. (2010), *Risk appetite and endogenous risk*, Financial Markets Group.

Das, S. R. and Sundaram, R. K. (1999), 'Of smiles and smirks: A term structure perspective', *Journal of Financial and Quantitative Analysis* **34**(02), 211–239.

Detering, N. and Packham, N. (2016), 'Model risk of contingent claims', *Quantitative Finance* **16**(9), 1357–1374.

Diamond, P. A. (1967), 'The role of a stock market in a general equilibrium model with technological uncertainty', *The American Economic Review* pp. 759–776.

Dowd, K. (2003), *An introduction to market risk measurement*, John Wiley & Sons.

Draper, D. (1995), 'Assessment and propagation of model uncertainty', *Journal of the Royal Statistical Society. Series B (Methodological)* pp. 45–97.

Draper, D., Hodges, J. S., Leamer, E. E., Morris, C. N. and Rubin, D. B. (1987), *A research agenda for assessment and propagation of model uncertainty*, Vol. 2683, Rand.

Drost, F. C., Nijman, T. E. and Werker, B. J. (1998), 'Estimation and testing in models containing both jumps and conditional heteroscedasticity', *Journal of Business & Economic Statistics* **16**(2), 237–243.

Dumas, B., Fleming, J. and Whaley, R. E. (1998), 'Implied volatility functions: Empirical tests', *The Journal of Finance* **53**(6), 2059–2106.

Elices, A. and Giménez, E. (2013), 'Applying hedging strategies to estimate model risk and provision calculation', *Quantitative Finance* **13**(7), 1015–1028.

Eling, M. and Faust, R. (2010), 'The performance of hedge funds and mutual funds in emerging markets', *Journal of Banking & Finance* **34**(8), 1993–2009.

Ellsberg, D. (1961), 'Risk, ambiguity, and the savage axioms', *The Quarterly Journal of Economics* **75**(4), 643–669.

Embrechts, P., Puccetti, G. and Rüschendorf, L. (2013), 'Model uncertainty and VaR aggregation', *Journal of Banking & Finance* **37**(8), 2750–2764.

Eraker, B. (2001), 'MCMC analysis of diffusion models with application to finance', *Journal of Business & Economic Statistics* **19**(2), 177–191.

Eraker, B., Johannes, M. and Polson, N. (2003), 'The impact of jumps in volatility and returns', *The Journal of Finance* **58**(3), 1269–1300.

Escanciano, J. C. and Olmo, J. (2010), ‘Backtesting parametric value-at-risk with estimation risk’, *Journal of Business & Economic Statistics* **28**(1).

Ferreira, M. a. and Gama, P. M. (2005), ‘Have world, country, and industry risks changed over time? An investigation of the volatility of developed stock markets’, *Journal of Financial and Quantitative Analysis* **40**(1), 195–222.

Figlewski, S. (2016), ‘What goes into risk-neutral volatility? Empirical estimates of risk and subjective risk preferences’, *Journal of Portfolio Management* **43**(1), 29–42.

Frey, R. (2013), Essays On Jump-Diusion Models In Asset Pricing And On The Prediction Of Aggregate Stock Returns, PhD thesis, University of St. Gallen.

Frühwirth-Schnatter, S. (2006), *Finite Mixture and Markov Switching Models*, Springer Science & Business Media.

Fung, W. and Hsieh, D. A. (2004), ‘Hedge fund benchmarks: A risk-based approach’, *Financial Analysts Journal* **60**(5), 65–80.

Fung, W., Hsieh, D. A., Naik, N. Y. and Ramadorai, T. (2008), ‘Hedge funds: Performance, risk, and capital formation’, *The Journal of Finance* **63**(4), 1777–1803.

Gardon, A. (2011), ‘The normality of financial data after an extraction of jumps in the jump-diffusion model’, *Mathematical Economics* **7**(14), 93–106.

Gelfand, A. E. and Sahu, S. K. (1999), ‘Identifiability, improper priors, and Gibbs sampling for generalized linear models’, *Journal of the American Statistical Association* **94**(445), 247–253.

Gelman, A., Carlin, J. B., Stern, H. S. and Rubin, D. B. (2014), *Bayesian Data Analysis*, Vol. 2, Taylor & Francis.

Geyer, C. J. and Thompson, E. A. (1995), 'Annealing Markov Chain Monte Carlo with Application to Ancestral Inference', *Journal of the American Statistical Association* **90**(431), 909–920.

Giamouridis, D. and Vrontos, I. D. (2007), 'Hedge fund portfolio construction: A comparison of static and dynamic approaches', *Journal of Banking & Finance* **31**(1), 199–217.

Glasserman, P. and Xu, X. (2014), 'Robust risk measurement and model risk', *Quantitative Finance* **14**(1), 29–58.

Goldfarb, B., Kirsch, D. and Miller, D. A. (2007), 'Was there too little entry during the Dot Com era?', *Journal of Financial Economics* **86**(1), 100–144.

Gupta, A. and Reisinger, C. (2014), 'Robust calibration of financial models using Bayesian estimators', *Journal of Computational Finance* **17**(4), 3–36.

Hansen, B. (1994), 'Autoregressive conditional density estimation', *International Economic Review* **35**, 705–730.

Hanson, F. B. and Westman, J. J. (2002), Stochastic analysis of jump-diffusions for financial log-return processes, *in* 'Stochastic Theory and Control', Springer, pp. 169–183.

Harris, R. D. and Mazibas, M. (2013), 'Dynamic hedge fund portfolio construction: A semi-parametric approach', *Journal of Banking & Finance* **37**(1), 139–149.

Harrison, P. J. and Stevens, C. F. (1976), 'Bayesian forecasting', *Journal of the Royal Statistical Society. Series B (Methodological)* pp. 205–247.

Harrison, P. and Stevens, C. F. (1971), 'A Bayesian approach to short-term forecasting', *Operational Research Quarterly* pp. 341–362.

Harvey, C. and Siddique, A. (2000), ‘Conditional skewness in asset pricing tests’, *Journal of Finance* **55**, 1263–1295.

Hénaff, P. and Martini, C. (2011), ‘Model validation: theory, practice and perspectives’, *The Journal of Risk Model Validation* **5**(3), 15.

Hengelbrock, J., Theissen, E. and Westheide, C. (2013), ‘Market response to investor sentiment’, *Journal of Business Finance & Accounting* **40**(7-8), 901–917.

Heston, S. L. (1993), ‘A closed-form solution for options with stochastic volatility with applications to bond and currency options’, *Review of Financial Studies* **6**(2), 327–343.

Hodges, J. S. (1987), ‘Uncertainty, policy analysis and statistics’, *Statistical Science* pp. 259–275.

Hoeting, J. A., Madigan, D., Raftery, A. E. and Volinsky, C. T. (1999), ‘Bayesian model averaging: a tutorial’, *Statistical Science* pp. 382–401.

Honore, P. (1998), ‘Pitfalls in estimating jump-diffusion models’, *SSRN* .

Hsieh, D. A. (1989), ‘Modeling heteroscedasticity in daily foreign-exchange rates’, *Journal of Business & Economic Statistics* **7**(3), 307–317.

Hull, J., Nelken, I. and White, A. (2004), ‘Mertons model, credit risk, and volatility skews’, *Journal of Credit Risk Volume* **1**(1), 8–23.

Hull, J. and White, A. (1987), ‘The pricing of options on assets with stochastic volatilities’, *The journal of finance* **42**(2), 281–300.

In, F., Kim, S., Yoon, J. H. and Viney, C. (2001), ‘Dynamic interdependence and volatility transmission of Asian stock markets: Evidence from the Asian crisis’, *International Review of Financial Analysis* **10**(1), 87–96.

Jacque, L. L. (2015), *Global derivative debacles: from theory to malpractice*, World Scientific.

Jacquier, E. and Jarrow, R. (2000), 'Bayesian analysis of contingent claim model error', *Journal of Econometrics* **94**(1-2), 145–180.

Jacquier, E. and Polson, N. (2010), Bayesian methods in finance, *in* 'Oxford Handbook of Bayesian Econometrics', Oxford University Press, pp. 439–512.

Jacquier, E., Polson, N. G. and Rossi, P. E. (1994), 'Bayesian analysis of stochastic volatility models', *Journal of Business & Economic Statistics* **12**(4), 69–87.

Jarrow, R. A. (2012), 'Hedging derivatives with model error', *Quantitative Finance* **12**(6), 855–863.

Jarrow, R. A. and Rosenfeld, E. R. (1984), 'Jump risks and the intertemporal capital asset pricing model', *Journal of Business* **57**(3), 337–351.

Joenväärä, J., Kosowski, R. and Tolonen, P. (2016), 'Hedge fund performance: What do we know?', *SSRN* .

Johannes, M., Korteweg, A. and Polson, N. (2014), 'Sequential learning, predictability, and optimal portfolio returns', *The Journal of Finance* **69**(2), 611–644.

Johannes, M. and Polson, N. (2010), MCMC methods for continuous-time financial econometrics, *in* 'Handbook of Financial Econometrics, Vol 2', Elsevier Inc., pp. 1–72.

Johannes, M. S., Polson, N. G. and Stroud, J. R. (2009), 'Optimal filtering of jump diffusions: Extracting latent states from asset prices', *Review of Financial Studies* **22**, 2559–2599.

Jondeau, E. and Rockinger, M. (2003), 'Testing for differences in the tails of stock-market returns', *Journal of Empirical Finance* **10**(5), 559–581.

Jones, C. S. (2003), 'The dynamics of stochastic volatility: evidence from underlying and options markets', *Journal of Econometrics* **116**(1), 181–224.

Jones, G. L. et al. (2004), 'On the markov chain central limit theorem', *Probability Surveys* **1**(299-320), 5–1.

Jorion, P. (1988), 'On jump processes in the foreign exchange and stock markets', *Review of Financial Studies* **1**(4), 427–445.

Julier, S. J. and Uhlmann, J. K. (1997), 'A new extension of the Kalman filter to nonlinear systems', *Int. symp. aerospace/defense sensing, simul. and controls* **3**(26), 182–193.

Kaeck, A. (2013), 'Hedging surprises, jumps, and model misspecification: A risk management perspective on hedging S&P 500 options', *Review of Finance* **17**, 1535–1569.

Kaeck, A. and Alexander, C. (2013), 'Stochastic volatility jump-diffusions for European equity index dynamics', *European Financial Management* **19**(3), 470–496.

Kalman, R. E. and Bucy, R. S. (1961), 'New results in linear filtering and prediction theory', *Journal of basic engineering* **83**(1), 95–108.

Kalman, R. E. et al. (1960), 'A new approach to linear filtering and prediction problems', *Journal of basic Engineering* **82**(1), 35–45.

Kandel, S. and Stambaugh, R. F. (1996), 'On the predictability of stock returns: an asset-allocation perspective', *The Journal of Finance* **51**(2), 385–424.

Keating, C. and Shadwick, W. F. (2002), ‘A universal performance measure’, *Journal of performance measurement* **6**(3), 59–84.

Keen, S. (2013), ‘A monetary Minsky model of the Great Moderation and the Great Recession’, *Journal of Economic Behavior & Organization* **86**, 221–235.

Kerkhof, J., Melenberg, B. and Schumacher, H. (2010), ‘Model risk and capital reserves’, *Journal of Banking & Finance* **34**(1), 267–279.

Kim, M.-J., Oh, Y.-H. and Brooks, R. (1994), ‘Are jumps in stock returns diversifiable? Evidence and implications for option pricing’, *Journal of Financial and Quantitative Analysis* **29**(4), 609–631.

Kim, T. Y., Hwang, C. and Lee, J. (2004), ‘Korean economic condition indicator using a neural network trained on the 1997 crisis’, *Journal of Data Science* **2**(4), 371–381.

Knight, F. H. (1921), ‘Risk, uncertainty and profit’, *New York: Hart, Schaffner and Marx* .

Kogan, L. and Wang, T. (2002), ‘A simple theory of asset pricing under model uncertainty’, *Working Paper, MIT and The University of British Columbia* .

Koop, G. and Korobilis, D. (2012), ‘Forecasting inflation using dynamic model averaging’, *International Economic Review* **53**(3), 867–886.

Kostrzewski, M. (2014), ‘Bayesian inference for the jump-diffusion model with M jumps’, *Communications in Statistics - Theory and Methods* **43**(18), 3955–3985.

Kou, S. G. (2002), ‘A jump-diffusion model for option pricing’, *Management science* **48**(8), 1086–1101.

Kou, S., Yu, C. and Zhong, H. (2016), 'Jumps in equity index returns before and during the recent financial crisis: A Bayesian analysis', *Management Science* . Forthcoming.

Kuha, J. (2004), 'AIC and BIC comparisons of assumptions and performance', *Sociological Methods & Research* **33**(2), 188–229.

Lancaster, T. (2004), *An introduction to modern Bayesian econometrics*, Blackwell Oxford.

Laurini, M. P. and Hotta, L. K. (2010), 'Bayesian extensions to Diebold-Li term structure model', *International Review of Financial Analysis* **19**(5), 342–350.

Leitch, G. and Tanner, J. E. (1991), 'Economic forecast evaluation: profits versus the conventional error measures', *The American Economic Review* pp. 580–590.

Lemmon, M. and Portniaguina, E. (2006), 'Consumer confidence and asset prices: Some empirical evidence', *Review of Financial Studies* **19**(4), 1499–1529.

Liechty, J. (2013), Regime switching models and risk measurement tools, *in* 'Handbook on Systemic Risk', Cambridge University Press, pp. 180–190.

Lönnbark, C. (2013), 'On the role of the estimation error in prediction of expected shortfall', *Journal of Banking & Finance* **37**(3), 847–853.

Lunn, D., Jackson, C., Best, N., Thomas, A. and Spiegelhalter, D. (2012), *The BUGS Book: A Practical Introduction to Bayesian Analysis*, CRC press.

Lybeck, J. A. (2011), *A Global History of the Financial Crash of 2007–10*, Cambridge University Press.

Maekawa, K., Lee, S., Morimoto, T. and Kawai, K. (2008), 'Jump Diffusion Model : An Application to the Japanese Stock Market', *Mathematics and Computers in Simulation* **78**(2), 223–236.

Matsuda, K. (2004), 'Introduction to Merton jump diffusion model', *Department of Economics. The Graduate Center, The City University of New York* .

McDonald, R. L., Cassano, M. and Fahlenbrach, R. (2006), *Derivatives markets*, Vol. 2, Addison-Wesley Boston.

Meier, C. (2015), 'Aggregate investor confidence in the stock market'. Working Paper. SSRN.

Meinhold, R. J. and Singpurwalla, N. D. (1983), 'Understanding the Kalman filter', *The American Statistician* **37**(2), 123–127.

Merton, R. C. (1974), 'On the pricing of corporate debt: The risk structure of interest rates', *Journal of Finance* **29**(2), 449–470.

Merton, R. C. (1976), 'Option pricing when underlying stock returns are discontinuous', *Journal of Financial Economics* **3**(1), 125–144.

Miller, E. M. (1977), 'Risk, uncertainty, and divergence of opinion', *The Journal of Finance* **32**(4), 1151–1168.

Minsky, H. P. (1977), 'The financial instability hypothesis: an interpretation of Keynes and an alternative to "standard" theory', *Challenge* **20**(1), 20–27.

Minsky, H. P. (1982), 'Can "it" happen again? A reprise', *Challenge* **25**(3), 5–13.

Minsky, H. P. (1992), 'The financial instability hypothesis', (74). Working Paper. The Jerome Levy Economics Institute.

Minsky, H. P. and Kaufman, H. (2008), *Stabilizing an Unstable Economy*, Vol. 1, McGraw-Hill New York.

Mun, M. and Brooks, R. (2012), 'The roles of news and volatility in stock market correlations during the global financial crisis', *Emerging Markets Review* **13**(1), 1–7.

Naser, H. and Alaali, F. (2017), 'Can oil prices help predict US stock market returns? Evidence using a dynamic model averaging (DMA) approach', *Empirical Economics* pp. 1–21.

Navas, J. F. (2003), 'Correct calculation of volatility in a Jump-Diffusion model', *Journal of Derivatives* **11**(2), 66–72.

Neely, C. J., Rapach, D. E., Tu, J. and Zhou, G. (2014), 'Forecasting the equity risk premium: the role of technical indicators', *Management Science* **60**(7), 1772–1791.

ODoherty, M. S., Savin, N. E. and Tiwari, A. (2015), 'Evaluating hedge funds with pooled benchmarks', *Management Science* **62**(1), 69–89.

Oh, K. J., Kim, T. Y. and Kim, C. (2006), 'An early warning system for detection of financial crisis using financial market volatility', *Expert Systems* **23**(2), 83–98.

Olmo, J. and Sanso-Navarro, M. (2012), 'Forecasting the performance of hedge fund styles', *Journal of Banking & Finance* **36**(8), 2351–2365.

Panopoulou, E. and Vrontos, S. (2015), 'Hedge fund return predictability; to combine forecasts or combine information?', *Journal of Banking & Finance* **56**, 103–122.

Pettenuzzo, D. and Ravazzolo, F. (2016), 'Optimal portfolio choice under decision-based model combinations', *Journal of Applied Econometrics* **31**(7), 1312–1332.

Porras, E. (2016), *Bubbles and Contagion in Financial Markets, Volume 1: An Integrative View*, Springer.

Raftery, A. E. (1995), 'Bayesian model selection in social research', *Sociological Methodology* **25**, 111–164.

Raftery, A. E., Kárnỳ, M. and Ettler, P. (2010), 'Online prediction under model uncertainty via dynamic model averaging: Application to a cold rolling mill', *Technometrics* **52**(1), 52–66.

Ramezani, C. A. and Zeng, Y. (1998), 'Maximum likelihood estimation of asymmetric jump-diffusion processes: Application to security prices', *SSRN* .

Rapach, D. E., Zhou, G. et al. (2013), 'Forecasting stock returns', *Handbook of economic forecasting* **2**(Part A), 328–383.

Razali, N. M., Wah, Y. B. et al. (2011), 'Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests', *Journal of Statistical Modeling and Analytics* **2**(1), 21–33.

Riedle, T. (2016), 'The relationship between bubbles and stock market crises in the US'. Working Paper. University of Kent.

Risse, M. and Kern, M. (2016), 'Forecasting house-price growth in the Euro area with dynamic model averaging', *The North American Journal of Economics and Finance* **38**, 70–85.

Rodríguez, A., Horst, E. T. and Malone, S. (2015), 'Bayesian inference for a structural credit risk model with stochastic volatility and stochastic interest rates', *Journal of Financial Econometrics* **13**(4), 839–867.

Rose, A. K. and Spiegel, M. M. (2012), 'Cross-country causes and consequences of the 2008 crisis: early warning', *Japan and the World Economy* **24**(1), 1–16.

Roumpis, E. and Syriopoulos, T. (2014), 'Dynamics and risk factors in hedge funds returns: implications for portfolio construction and performance evaluation', *The Journal of Economic Asymmetries* **11**, 58–77.

Scheinkman, J. A. and Xiong, W. (2003), 'Overconfidence and speculative bubbles', *Journal of Political Economy* **111**(6), 1183–1220.

Schmeling, M. (2009), 'Investor sentiment and stock returns: Some international evidence', *Journal of Empirical Finance* **16**(3), 394–408.

Schrimpf, A. (2010), 'International stock return predictability under model uncertainty', *Journal of International Money and Finance* **29**(7), 1256–1282.

Schwert, G. W. (1990), 'Stock volatility and the crash of '87', *Review of Financial Studies* **3**(1), 77–102.

Schwert, G. W. (2011), 'Stock volatility during the recent financial crisis', *European Financial Management* **17**(5), 789–805.

Scott, L. O. (1987), 'Option pricing when the variance changes randomly: Theory, estimation, and an application', *Journal of Financial and Quantitative Analysis* **22**(04), 419–438.

Shapiro, S. S., Wilk, M. B. and Chen, H. J. (1968), 'A comparative study of various tests for normality', *Journal of the American Statistical Association* **63**(324), 1343–1372.

Smith, R. and Miller, J. (1986), 'A non-gaussian state space model and application to prediction of records', *Journal of the Royal Statistical Society. Series B (Methodological)* pp. 79–88.

Sortino, F. A., Meer, R. V. D. and Plantinga, A. (1999), 'The Dutch triangle', *The Journal of Portfolio Management* **26**(1), 50–57.

Sortino, F. A. and Satchell, S. (2001), *Managing downside risk in financial markets*, Butterworth-Heinemann.

Spiegelhalter, D. (2006), 'Some DIC slides', *MRC Biostatistics Unit, Cambridge. IceBUGS: Finland* .

Spiegelhalter, D. J., Best, N. G., Carlin, B. P. and Linde, A. V. D. (2002), 'Bayesian measures of model complexity and fit', *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* **64**(4), 583–639.

Spiegelhalter, D. J., Best, N. G., Gilks, W. R. and Inskip, H. (1996), 'Hepatitis B: a case study in MCMC methods', *Markov chain Monte Carlo in practice* pp. 45–58.

Stambaugh, R. F. (1999), 'Predictive regressions', *Journal of Financial Economics* **54**(3), 375–421.

Stanescu, S., Tunaru, R. and Candradewi, M. R. (2014), 'Forward–futures price differences in the uk commercial property market: Arbitrage and marking-to-model explanations', *International Review of Financial Analysis* **34**, 177–188.

Tarashev, N. (2010), 'Measuring portfolio credit risk correctly: Why parameter uncertainty matters', *Journal of Banking & Finance* **34**(9), 2065–2076.

Tarashev, N. and Zhu, H. (2008), 'Specification and calibration errors in measures of portfolio credit risk: The case of the ASRF model', *International Journal of Central Banking* **4**(2), 129–173.

Thadewald, T. and Büning, H. (2007), 'Jarque-Bera test and its competitors for testing normality–a power comparison', *Journal of Applied Statistics* **34**(1), 87–105.

Tunaru, R. (2015), *Model Risk in Financial Markets: From Financial Engineering to Risk Management*, World Scientific.

Tunaru, R. and Zheng, T. (2017), 'Parameter estimation risk in asset pricing and risk management: A Bayesian approach', *International Review of Financial Analysis* **53**, 80–93.

Uppal, R. and Wang, T. (2003), 'Model misspecification and underdiversification', *The Journal of Finance* **58**(6), 2465–2486.

Vrontos, I. (2012), 'Evidence for hedge fund predictability from a multivariate student's t full-factor garch model', *Journal of Applied Statistics* **39**(6), 1295–1321.

Vrontos, I. D. and Giamouridis, D. (2008), 'Hedge fund return predictability in the presence of model uncertainty and implications for wealth allocation', *SSRN* .

Vrontos, S. D., Vrontos, I. D. and Giamouridis, D. (2008), 'Hedge fund pricing and model uncertainty', *Journal of Banking & Finance* **32**(5), 741–753.

Wegener, C., von Nitzsch, R. and Cengiz, C. (2010), 'An advanced perspective on the predictability in hedge fund returns', *Journal of Banking & Finance* **34**(11), 2694–2708.

Wen, F. and Yang, X. (2009), 'Skewness of return distribution and coefficient of risk premium', *Journal of Systems Science and Complexity* **22**(3), 360–371.

Wolfson, M. H. (2002), 'Minsky's theory of financial crises in a global context', *Journal of Economic Issues* **36**(2), 393–400.

Wooldridge, J. (2012), *Introductory econometrics: A modern approach*, Cengage Learning.

Wray, L. R. (2015), *Why Minsky Matters: An Introduction to the Work of a Maverick Economist*, Princeton University Press.

Wu, Y. (2010), Uncertainty in Portfolio Credit Risk Modelling, PhD thesis, Warwick Business School.

Xia, Y. (2001), 'Learning about predictability: The effects of parameter uncertainty on dynamic asset allocation', *The Journal of Finance* **56**(1), 205–246.

Yu, C., Li, H. and Wells, M. (2011), 'MCMC estimation of Levy jump models using stock and option prices', *Mathematical Finance* **21**(3), 383–422.

Yun, J. (2014), 'Out-of-sample density forecasts with affine jump diffusion models', *Journal of Banking & Finance* **47**, 74–87.

Research Contributions

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