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1 MAINTENANCE POLICIES CONSIDERING DEGRADATION AND
2 COST PROCESSES FOR A MULTICOMPONENT SYSTEM

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5 **ABSTRACT.** Condition-based maintenance (CbM) is a method for reducing the probability
6 of system failures as well as the operating cost. Nowadays, a system is composed of multiple
7 components. If the deteriorating process of each component can be monitored and then modelled
8 by a stochastic process, the deteriorating process of the system is a stochastic process. The cost
9 of repairing failures of the components in the system forms a stochastic process as well, and is
10 known as a cost process.

11 This paper models the deterioration process of a multi-component system. Each dete-
12 rioration process is modelled by the Wiener process. When a linear combination of the
13 processes, which can be the deterioration processes and the cost processes, exceeds a
14 pre-specified threshold, a replacement policy will be carried out to preventively maintain
15 the system. Under this setting, this paper investigates maintenance policies based on the
16 deterioration process and the cost process. Numerical examples are given to illustrate
17 the optimisation process.

18 **Keywords:** Condition-based maintenance; age replacement policy; block replacement policy;
19 cost process; Wiener process

20 **1 Introduction**

21 Condition-based maintenance (CbM) is a class of methods for scheduling maintenance
22 policies that aims to reduce the probability of failures, help reduce the operation cost,
23 and ensure the stable quality of the products. In the CbM related literature, stochastic
24 processes such as the gamma process (Lawless and Crowder, 2004; Cholette et al., 2019),
25 the inverse Gaussian process (Li et al., 2017; Hao et al., 2019), and the Wiener process
26 (WP) (Wen et al., 2018; Xie et al., 2019; Wang and Kang, 2020) are widely used for
27 modelling the deterioration processes under different applications.

28 Basically, CbM is performed on a piece of equipment once a parameter(s) related to the
29 condition of the monitored system reaches a pre-specified value. Its purpose is to prevent
30 the efficiency of the system from deteriorating to an unacceptable condition or the system
31 stop working completely, due to the ageing or deterioration of the system. It is therefore
32 important to assess the status or remaining useful life of a system, which can further be
33 used in deciding the future operation in order to maintain the system at a certain level
34 of availability.

35 In the real world, an engineering system is normally composed of multiple components. If
36 the deterioration process of each component can be observed and modelled by a stochas-
37 tic process, the deterioration process of the system forms a stochastic process. Sun et al.

38 (2017) optimised maintenance policies when the combination of multi-deterioration pro-
39 cesses is assumed nonlinear using a Markov decision process for a k -out-of- n system, in
40 which the deterioration process of each component follows the Wiener process. Other
41 research considers multi-component systems under linear combinations. A real example
42 is pavement defects, as discussed in Wu and Castro (2020), where the deterioration of
43 a pavement was due to different defects such as cracking and potholes. Similar research
44 could be seen in Coraddu et al. (2016) and Cheng et al. (2019). Zhang et al. (2018)
45 discussed the application of both non-linear and linear Wiener processes degradation
46 processes. Wu and Castro (2020) proposed the concept of the cost process for the sce-
47 nario in which maintenance policies are considered for a linear combination of multiple
48 gamma processes. Nevertheless, the concept of the cost process, has not been studied
49 in other scenarios, including a linear combination of multiple Wiener processes. This
50 knowledge gap is the main motivation for this research.

51 1.1 Related work

52 In the literature, publications relating to CbM are enormous. For example, Li and Nilk-
53 itsaranont (2009) proposed the combined regression techniques for CbM to assess the
54 remaining useful life of gas turbine engines, which improves engine reliability and avail-
55 ability and reduces life cycle costs. Coraddu et al. (2016) used some machines approaches
56 to effectively predicting potential failures of naval propulsion plants. Other research such
57 as Zhu et al. (2015) presented a deterioration model that included two system deteri-
58 oration processes: wear and shock and presents an optimal maintenance policy for the
59 minimal cost criterion. These studies pointed a direction for future research: how to
60 build a model which is more suitable for a complex system with multiple components or
61 failure types.

62 As aforementioned, in existing literature, stochastic processes such as the gamma process
63 citeplawless2004covariates,wu2020maintenance,wang2022condition, the inverse Gaussian
64 process (Li et al., 2017; Chen et al., 2015), and the WP (Ebrahimi, 2005; Wen et al.,
65 2018; Pedersen and Vatn, 2022) are widely used for different applications in CbM. Plenty
66 of research is concentrated on the combination approach to dealing with the increasingly
67 complex system (see Galar et al. (2013); Feng et al. (2017); Chang et al. (2019), for
68 example). Caballé et al. (2015) proposed a condition-based maintenance policy by com-
69 bining the non-homogeneous Poisson process(NHPP) and the gamma process(GP). They
70 modelled a multiple deterioration processes with dependent deterioration-threshold-shock
71 models. This was a typical example for multi-failure modes. It carried out two incre-
72 mental processes in two different methodologies, so as to achieve the situation where the
73 decline mode of a single system changes. They also pointed out that the dependence
74 analysis between the causes of failures was a potential development and the variability of
75 the threshold should be considered in future. Zhu et al. (2015) simulated the wear dam-
76 age by a non-stationary gamma process and the random shock damage with a generalized
77 Pareto distribution following Poisson arrivals. They derived the mathematical expression
78 of the stationary behaviour of the system and calculated the long-term average total cost
79 by using the semi-regenerative properties. It is worthwhile to notice that this study did
80 not consider the impact of shocks or inspection costs which may influence the result of a
81 long-term optimised maintenance policy. Liu et al. (2017a) proposed a new CbM model
82 based on three-state deterioration and the influence of external environmental shocks.
83 The deterioration process of the system was modelled by a two-state WP with a dou-

84 ble stochastic Poisson process (DSPP). It considered two different thresholds, namely a
85 normal threshold and a defective threshold, both of which depends on the system state.
86 Other common methods such as the geometric process, regression analysis, artificial neu-
87 ral networks and support vector regression can be seen in these examples: Dong et al.
88 (2014); Liu et al. (2017b); Lo et al. (2019). Zhang et al. (2018) reviewed some develop-
89 ments and applications of the WP. They also summarized some challenges and problems
90 which mainly include: the WP with multiple time-scales, the WP integrating various
91 types of data, the WP with state recoveries and the WP with non-Markovian feature.
92 Change points on deterioration modelling and prognostics were largely occur randomly.
93 Yang et al. (2019) proposed a two-phase preventive maintenance policy for a single-
94 component system. The first stage was the imperfect maintenance phase which aims to
95 keep the system working. The second stage was the postponed replacement phase which
96 considers a preventive replacement. This meant that this maintenance policy would be
97 sufficient and flexible for resource allocation due to its phase variability. Zhao et al.
98 (2021) proposed a multi-criteria mission abort policy that considered the normal and
99 defective stages based on the time threshold. They also indicated that performance of
100 the optimal policy was compared in detail against several heuristic policies. Besides,
101 the dynamic risk for controlling policy was also a possible extension for phased mission
102 systems. Liu et al. (2021a) proposed a condition-based maintenance model in a finite-
103 time horizon that consider a system with two heterogeneous dependent components with
104 economic dependence. Moreover, this research pointed that the two-unit system in this
105 paper could be extended to multi-unit systems by generalizing the deterioration process
106 and Bellman equation, and the maintenance level could be extended to imperfect repair
107 in future. For a multi-component system, in which each component had an observable
108 deteriorating process, Wu and Castro (2020) developed a weighted linear combination
109 of deterioration processes to optimise the time interval of maintenance for a pavement
110 network.

111 Most existing maintenance policy optimisation approaches, such as Zhang et al. (2022a),
112 Shi et al. (2020), and Liu et al. (2021b), aimed to minimise the relevant cost.

113 For a component in a system, it may have different failure modes. The deterioration
114 process of a system with different failure modes can be modelled by multiple deterioration
115 processes. Maintenance policies on such systems have been discussed in several papers.
116 Zhu et al. (2016) studied the maintenance policies of a multi-component system with two
117 independent failure modes. Qiu et al. (2017) considered an optimal maintenance policy by
118 both maximizing steady-state availability and minimizing long-term average cost for a
119 system with multiple failure modes. They assumed that failure modes are independent.
120 Zheng and Makis (2020) considered the failure state of a system changed from a soft
121 failure to a hard failure and assumes that under different state, different maintenance
122 activities can be taken (such as corrective replacement for soft failure and minimal repair
123 for hard failure). Pedersen and Vatn (2022) considered a risk-averse decision maker of
124 the CbM based on the Wiener process. They pointed out that a policy for reducing the
125 cost of renewals or replacements may increase the risk of long downtime, and associated
126 losses cannot be ignored. Zhang et al. (2022b) used the Wiener process to predict the
127 remaining useful life of a system. The random effect of the operating environments and
128 loading conditions were estimated by a continuous-time random walk.

129 In what follows, for convenience of expression, we regard the term *components* and *fail-*
130 *ure modes* interchangeable. That is, a system is composed of n components, or the
131 deterioration process of a system is composed of n failure modes.

132 1.2 Novelty and contributions

133 From the above review, there is a need to explore the problem of the deterioration process
134 of multi-component systems with. Consequently, this paper investigates the cost process
135 relating to the linear combination, based on which maintenance policies are developed.

136 Hence, the contributions of this paper includes

- 137 • development of a cost process related to the linear combination of the deterioration
138 processes.;
- 139 • development of maintenance policies for a system whose cost process can be mod-
140 elled by a linear combination of Wiener processes.

141 1.3 Overview

142 The remainder of this paper is structured as follows. Section 2 describes notation and
143 assumptions used in this paper. Section 3 develops deterioration processes and cost
144 processes. Section 4 describes our maintenance policies under four situations. Section 5
145 shows some numerical examples. Section 6 concludes the paper.

146 2 Notation and Assumptions

147 2.1 Notation

148 Table 1 shows the notations used in this paper.

149 2.2 Assumption

- 150 • The system is new at time $t = 0$.
- 151 • Replacement is carried out every T_a units of time for age replacement policy or T_b
152 for block replacement policy.
- 153 • Degradation processes of different failure modes are modelled by Wiener processes
154 with different parameters.
- 155 • The deterioration process of each component develops from time $t = 0$. When a
156 linear combination of the magnitudes of the deterioration exceeds a pre-specified
157 value, the system needs replacement.
- 158 • The deterioration processes are independent from each other.

159 Although we assume that the deterioration processes are independent, other existing
160 studies have discussed the different dependences between components of a multi-components
161 system.

162 Tian and Liao (2011) proposed a proportional hazards model based CbM policy with the
163 economic dependency among different components. In their work, the components are

k	Index of the k failure mode.
n	Number of components, or failure modes, in the system under consideration, $k = 1, 2, \dots, n$
$X_k(t)$	Degradation state of k th failure modes at time t .
$Y(t)$	Overall deterioration of one system at time t .
μ_k	Drift of k th failure modes.
σ_k	Infinitesimal variance of k th failure modes.
a_k	Weight of failure mode k .
μ_Y	Drift of the overall deterioration of one system.
σ_Y	Infinitesimal variance of the overall deterioration of one system.
c_k	PM cost for every unit of the k th failure modes.
$U(t)$	Overall cost of a system at time t .
L	Threshold of the deterioration level for a system.
L_c	Threshold of the cost for a system.
$C_k(t)$	Total repair cost of the k th failure modes at t .
$C_{A,i}(T_a)$	Expected cost per time unit under the age replacement policy.
$C_{B,i}(T_b)$	Expected cost per time unit under the age replacement policy.
c_m	Expected repair cost incurred due to failures
c_r	Expected replacement cost
T_a	Interval time for the age replacement policy.
T_b	Interval time for the block replacement policy.

Table 1: Notation table

164 independent in their degradation and failure processes. They assumed that different com-
165 ponents had different thresholds for determining which component should be preventively
166 maintained. Song et al. (2014) studied the deterioration process of multi-components sys-
167 tem under shocks. The number of shocks, which is caused by one component, has an
168 effect on other components. The larger sum of the shocks leads to larger probabilities
169 of failures. Li et al. (2016) considered both of stochastic dependence and economic de-
170 pendences. The former is modelled by Levy copulas, and it will influenced by different
171 dependence degrees. The latter will influence the performance of several maintenance
172 policies, and the policy with the smallest long-term cost would be chosen by its decision
173 rule. Liu et al. (2020) considered a life cycle cost model with multiple dependent degra-
174 dation processes with random effect, which is due to environment. The dependence of
175 the degradation process is evaluated by a copula in their work.

176 3 Model development

177 3.1 Deterioration process

178 We assume that the system has k deterioration processes, each of which follows a WP.
179 Let $X_k(t)$ be the deterioration level of the k th deterioration process at time t , where
180 $k = 1, 2, \dots, n$. Then, $X_k(t)$ have the following assumptions:

- 181 • $X_k(0) = 0$, which also means that $W_k(0) = 0$;
- 182 • $W_k(t)$ has independent increments that follows the normal distribution. That is,

183 for $0 < s < t$, $W_k(t - s) - W_k(s)$ follows $N(0, (t - s))$.

184 • $W_k(t)$ is continuous in t .

185 $X_k(t)$ is said having drift coefficient μ_k and variance parameter σ_k^2 , the associated stochastic process is:

$$187 \quad X_k(t) = \mu_k t + \sigma_k W_k(t), \quad (1)$$

188 where μ_k and σ_k are the parameters of failure mode k , respectively, $W_k(\cdot)$ is the standard WP, which also can be called as the Brownian motion. The estimation method of parameters can be seen in Shah et al. (2013).

191 3.1.1 Basic Properties

192 The unconditional probability density function, which follows the normal distribution with mean = 0 and variance = t , at a fixed time t :

$$194 \quad f_{W_t}(x) = \frac{1}{\sqrt{2\pi t}} e^{-x^2/(2t)}.$$

195 We have $E[W_k(t)] = 0$ and $\text{Var}[W_k(t)] = t$.

196 These results follow immediately from the definition that increments have a normal distribution, centred at zero.

198 Thus, the expected value and the variance of $X_k(t)$ are given by: $E(X_k(t)) = \mu_k t$ and $V(X_k(t)) = \sigma_k^2 t$.

200 3.1.2 A linear combination of WPs

201 Now let us assume $Y(t)$ is a linear combination of n WPs. The overall deterioration $Y(t)$ of the system is represented by

$$203 \quad Y(t) = \sum_{k=1}^n a_k X_k(t), t \geq 0, a_k \geq 0, \quad (2)$$

204 where a_k is the weight of failure mode k . Fig. 1 shows the realisation of a linear combination of two WPs.

206 Furthermore, the overall deterioration process $\{Y(t), t > 0\}$ is a stochastic process with the following properties (without the skew-normal random effects):

- 208 • $Y(0) = \sum_{k=1}^n a_k X_k(0) = 0$,
- 209 • $\Delta Y(t) = \sum_{k=1}^n a_k \Delta X_k(t)$ is an independent increment as well.

210 Thus, $Y(t)$ is given by

$$211 \quad Y(t) = t \sum_{k=1}^n a_k \mu_k + \sum_{k=1}^n a_k \sigma_k W_k(t). \quad (3)$$

212 Let $\mu_Y = \sum_{k=1}^n a_k \mu_k$ and $\sigma_Y^2 = \sum_{k=1}^n a_k^2 \sigma_k^2$. Then $Y(t)$ follows the normal distribution $N(\mu_Y t, \sigma_Y^2 t)$.

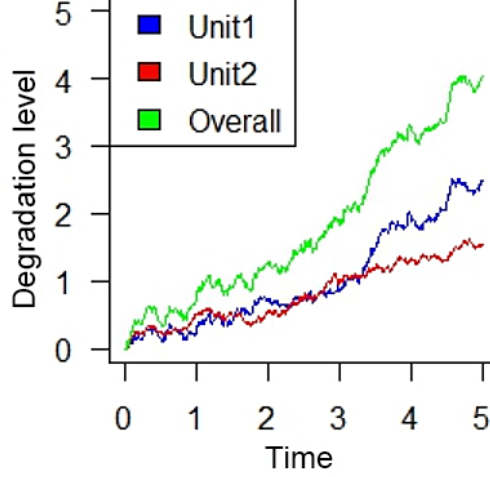


Figure 1: Example of two deterioration processes and a linear combination

214 3.1.3 First time to exceed the pre-specified threshold L

215 The distribution of the first hitting time of the process $\{Y(t), t \geq 0\}$, which starts from
 216 $Y(0) = 0$ should be obtained. The first hitting time $\omega_{Y(t)}$ is defined when $Y(t)$ reaches
 217 the deterioration level L , according to the statistical characteristic of a WP, the first-
 218 passage-time, which is $\omega_{Y(t)}$, follows an inverse Gaussian distribution (Ross et al., 1996;
 219 Pan et al., 2017; Ye and Chen, 2014), then

$$220 \quad \omega_L = \inf\{t > 0: Y(t) \geq L\}, \quad (4)$$

221 then, the pdf of ω_L can be obtained by

$$222 \quad f_{\omega_L}(t) = \frac{L}{\sigma_Y \sqrt{2\pi t^3}} \exp\left(\frac{-(L - \mu_Y t)^2}{2\sigma_Y^2 t}\right)$$

$$223 \quad = \frac{L}{\sigma_Y \sqrt{\pi t^3}} \phi\left(\frac{-(L - \mu_Y t)}{\sigma_Y \sqrt{t}}\right), \quad (5)$$

225 where $\phi(\cdot)$ denotes the standard normal pdf. Then, the cdf of ω_L is obtained by

$$226 \quad F_{\omega_L}(t) = P(Y(t) \geq L)$$

$$227 \quad = \Phi\left(\frac{-(L - \mu_Y t)}{\sigma_Y \sqrt{t}}\right) - \exp\left(\frac{2\mu_Y L}{\sigma_Y^2}\right), \quad (6)$$

229 where $\Phi(\cdot)$ denotes the standard normal cdf.

230 3.2 Repair cost process

231 The repair costs of different failure modes are normally different. We consider that the
 232 actual cost is dependent on the deterioration level of the failure model. For example,
 233 the repair cost for a system with longer usage time is normally higher than a system

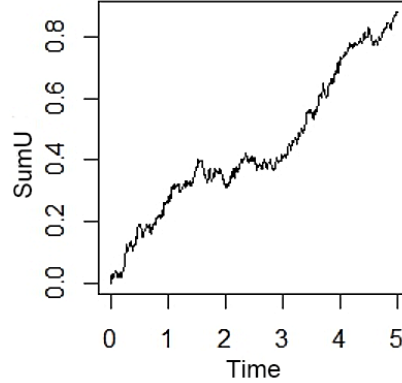


Figure 2: Example of the cost process of $C(t)$

234 with shorter usage time. Several references have considered this situation, see Liu et al.
 235 (2017a); Wu and Castro (2020), for example. It is worth noticing that, according to Wu
 236 and Castro (2020), the total cost $U(t)$, which is associated to $Y(t)$, is also a stochastic
 237 process and does not have a linear relationship with $Y(t)$. As $Y(t)$ is a WP, $U(t)$ is a
 238 WP which is a sum of $Y(t)$ with a drift.

239 Thus, the maintenance cost for the k th failure mode which is related to the deterioration
 240 level is given by,

$$241 \quad C_k(t) = a_k c_k X_k(t), \quad (7)$$

242 where c_k is the maintenance cost for the k th failure mode. Then, the total cost of the
 243 whole system with multiple components or failure modes is given by

$$244 \quad U(t) = \sum_{k=1}^n C_k(t) = \sum_{k=1}^n a_k c_k X_k(t), \quad (8)$$

245 where $U(t)$ is a WP with a linear drift related to its deterioration level.

247 3.2.1 Basic Properties

248 As $X_k(t)$ follows the normal distribution with mean = $\mu_k t$ and variance = $\sigma_k^2 t$, the ex-
 249 pected value and the variance of $C_k(t)$ are given by: $E(C_k(t)) = a_k c_k \mu_k t$ and $V(C_k(t)) =$
 250 $a_k^2 c_k^2 \sigma_k^2 t$.

251 Then $U(t)$ has expected value and variance,

$$252 \quad E(U(t)) = \sum_{k=1}^n a_k c_k \mu_k t = \mu_U, \quad (9)$$

253 and

$$254 \quad V(U(t)) = \sum_{k=1}^n a_k^2 c_k^2 \sigma_k^2 t = \sigma_U^2, \quad (10)$$

255 respectively.

258 Obviously, both of $Y(t)$ and $U(t)$ have the same values μ_k and σ_k , respectively, so the
 259 covariance between $Y(t)$ and $U(t)$ is given by

$$\begin{aligned}
 260 \quad \text{Cov}(Y(t), U(t)) &= \text{Cov}\left(\sum_{k=1}^n a_k X_k(t), \sum_{j=1}^n c_k X_j(t)\right) \\
 261 &= \sum_{k=1}^n \sum_{j=1}^n a_k c_k \text{Cov}(X_k(t), X_j(t)) \\
 262 &= \sum_{k=1}^n a_k c_k \mu_k^2 t. \tag{11} \\
 263
 \end{aligned}$$

264 The characteristic function of the bivariate normal distribution is given by

$$\begin{aligned}
 265 \quad \phi_{(Y(t), U(t))}(t_1, t_2) &= \mathbb{E}[\exp(it_1 Y(t) + it_2 U(t))] \\
 266 &= \mathbb{E}[\exp(it_1 \sum_{k=1}^n a_k X_k(t) + it_2 \sum_{k=1}^n a_k c_k X_k(t))] \\
 267 &= \mathbb{E}[\exp(i \sum_{k=1}^n (a_k t_1 + a_k c_k t_2) X_k(t))] \\
 268 &= \mathbb{E}[\exp(i \sum_{k=1}^n (a_k t_1 + a_k c_k t_2) X_k(t))] \\
 269 &= \prod_{k=1}^n \mathbb{E}[\exp(i(a_k t_1 + a_k c_k t_2) X_k(t))] \\
 270 &= \prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + a_k c_k t_2), \tag{12} \\
 271
 \end{aligned}$$

272 then we can obtain

$$273 \quad f_{Y(t), U(t)}(y, u) \tag{13}$$

$$\begin{aligned}
 274 &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{(Y(t), U(t))}(t_1, t_2) e^{-it_1 y - it_2 u} dt_1 dt_2 \\
 275 &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + c_k t_2)\right)^{-it_1 y - it_2 u} dt_1 dt_2, \\
 276 & \tag{14}
 \end{aligned}$$

278 the conditional probability $f_{U(t)|Y(t)}(y, u)$ is hence obtained by

$$\begin{aligned}
 279 \quad f_{U(t)|Y(t)}(y, u) &= \frac{f_{U(t), Y(t)}(y, u)}{f_{Y(t)}(y)} \\
 280 &= \frac{1}{4\pi^2 f_{Y(t)}(y)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + c_k t_2)\right)^{-it_1 y - it_2 u} dt_1 dt_2. \\
 281 & \tag{15}
 \end{aligned}$$

283 where

$$284 \quad \phi_{X_k(t)}(a_k t_1 + c_k t_2) = \exp\left\{\frac{\sigma_k [1 - (1 - 2i\mu_k^2(a_k t_1 + a_k c_k t_2)\sigma_k^{-1})^{1/2}]}{\mu_k}\right\}. \quad (16)$$

286 3.2.2 First time to exceed the pre-specified threshold L_U

287 However, if we consider a real situation: after a period of time, $U(t)$ becomes so high
 288 that using a new piece of equipment to replace the old one may be a better choice. Also,
 289 the owner of the equipment may have an expectation overall cost: when $U(t)$ is larger
 290 than this expectation, they will buy a new piece of equipment. For example, we assume
 291 this expectation cost is L_U , which will be described in the next section. Similarly, we
 292 define

$$293 \quad \omega_U = \inf\{t > 0 : U(t) \geq L_U\}, \quad (17)$$

294 Then, the pdf of ω_U can be obtained as

$$295 \quad f_{\omega_U}(t) = \frac{L_U}{\sigma_U \sqrt{2\pi t^3}} \exp\left(\frac{-(L_U - \mu_U t)^2}{2\sigma_U^2 t}\right)$$

$$296 \quad = \frac{L_U}{\sigma_U \sqrt{\pi t^3}} \phi\left(\frac{-(L_U - \mu_U t)}{\sigma_U \sqrt{t}}\right). \quad (18)$$

298 Then, the cdf of ω_U is obtained by

$$299 \quad F_{\omega_{L_U}}(t) = P(U(t) \geq L_U) = \Phi\left(\frac{-(L_U - \mu_U t)}{\sigma_U \sqrt{t}}\right) - \exp\left(\frac{2\mu_U L_U}{\sigma_U^2}\right). \quad (19)$$

301 4 Maintenance policies

302 In this section, we will consider the maintenance policy under age replacement and block
 303 replacement policies.

304 We consider the following four maintenance policies:

- 305 • *Maintenance Policy A*: Under the deterioration process, when the deterioration
 306 level exceeds the pre-specified threshold L , then maintenance activities will be
 307 taken. We denote this event as A_1 .
- 308 • *Maintenance Policy B*: Under the cost process, when the cost level exceeds the
 309 pre-specified threshold L_U , then maintenance activities will be taken. We denote
 310 this event as A_2 .
- 311 • *Maintenance Policy C*: Only if both A_1 and A_2 have occurred, the age replacement
 312 will be conducted. Denote this event as $A_3 = A_1 \cap A_2$.
- 313 • *Maintenance Policy D*: If one of the two events, A_1 and A_2 , occurs, the age re-
 314 placement will be conducted. Denote this event as $A_4 = A_1 \cup A_2$.

315 Therefore, $G_1(t) := P(A_1) = F_{\omega_L}(t)$ and $G_2(t) := P(A_2) = F_{\omega_{LU}}(t)$ and these functions
 316 can be obtained

$$\begin{aligned}
 317 \quad G_3(t) &:= P(A_3) \\
 318 \quad &= P(A_1 \cap A_2) \\
 319 \quad &= P(A_1)P(A_2|A_1) \\
 320 \quad &= F_{\omega_L}(t)F_{\omega_{LU}}(t|\omega_L), \tag{20} \\
 321
 \end{aligned}$$

322 and

$$\begin{aligned}
 323 \quad G_4(t) &:= P(A_4) \\
 324 \quad &= P(A_1 \cup A_2) \\
 325 \quad &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\
 326 \quad &= P(A_1) + P(A_2) - P(A_3), \tag{21} \\
 327
 \end{aligned}$$

328 where symbol $:=$ is used to denote a definition.

329 We have already obtained the conditional probability $f_{U(t)|Y(t)}(y,u)$, using $f_{\omega_L}(t)$ and
 330 $f_{\omega_{LU}}(t)$ to replace $f_{Y(t)}$ and $f_{U(t)}$, respectively, then

$$\begin{aligned}
 331 \quad f_{\omega_{LU}|\omega_L}(y,u) &= \frac{f_{\omega_{LU},\omega_L}(y,u)}{f_{\omega_L}(y)} \\
 332 \quad &= \frac{1}{4\pi^2 f_{\omega_L}(y)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + c_k t_2) \right)^{-it_1 y - it_2 u} dt_1 dt_2, \tag{22} \\
 333
 \end{aligned}$$

334 where

$$\begin{aligned}
 335 \quad \phi_{X_k(t)}(a_k t_1 + c_k t_2) &= \exp\left\{ \frac{\sigma_k [1 - (1 - 2i\mu_k^2 (a_k t_1 + a_k c_k t_2) \sigma_k^{-1})^{1/2}]}{\mu_k} \right\}, \tag{23} \\
 336
 \end{aligned}$$

337 and

$$\begin{aligned}
 338 \quad F_{\omega_{LU}}(t|\omega_L) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{\omega_L}^{-1}(t) \left(\prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + c_k t_2) \right)^{-it_1 t - it_2 u} dt_1 dt_2 dt \\
 339 \quad &= \frac{\ln f_{\omega_L}(t)}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + c_k t_2) \right)^{-it_1 t - it_2 u} dt_1 dt_2. \tag{24} \\
 340
 \end{aligned}$$

341 Therefore, the distribution of both $G_3(t)$ and $G_4(t)$ can be obtained.

342 The distribution of $G_3(t)$ is given by

$$\begin{aligned}
 343 \quad G_3(t) &:= \frac{F_{\omega_L}(t) \ln f_{\omega_L}(t)}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + c_k t_2) \right)^{-it_1 t - it_2 u} dt_1 dt_2, \tag{25} \\
 344
 \end{aligned}$$

345 and $G_4(t)$ now can be presented by

$$\begin{aligned}
 346 \quad G_4(t) &:= F_{\omega_L}(t) + F_{\omega_{LU}}(t) - \frac{F_{\omega_L}(t) \ln f_{\omega_L}(t)}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + c_k t_2) \right)^{-it_1 t - it_2 u} dt_1 dt_2. \\
 347 \quad &\tag{26}
 \end{aligned}$$

348 4.1 Age replacement policy

349 For the age replacement policy, a preventive replacement is conducted after a continuous
350 working time T_a when there is no failure occurs (Barlow and Hunter, 1960).

- 351 • The replacement time interval is T_a .
- 352 • Immediately after a preventive or corrective maintenance, the system rests its age
353 to 0.
- 354 • Both c_r and c_m are constants.

355 Then the mean time between replacements $M(T_a)$ will be

$$\begin{aligned}
 356 \quad M(T_a) &= \int_0^{T_a} t f(t) dt + t_0 P(X > T_a) \\
 357 \quad &= \int_0^{T_a} t f(t) dt + t_0 (t - F(T_a)) \\
 358 \quad &= \int_0^{T_a} (1 - F(t)) dt. \tag{27} \\
 359
 \end{aligned}$$

360 Then, the expected cost per time unit is given by

$$361 \quad C_{A,i}(T_a) = \frac{c_r + c_m G_i(T_a)}{\int_0^{T_a} (1 - G_i(t)) dt}, \tag{28} \\
 362$$

363 where $i = 1, 2, 3, 4$, corresponding to maintenance policies A, B, C, and D, respectively
364 and T_a is the decision variable, c_r is the expected replacements cost and c_m is the expected
365 repair cost incurred due to failures.

366 **Property 1.** For given t , if $G_1(t) \geq G_3(t)$, $G_2(t) \geq G_3(t)$, $G_1(t) \leq G_4(t)$ and $G_2(t) \leq$
367 $G_4(t)$, then $C_{A,1}(T_a) \geq C_{A,3}(T_a)$, $C_{A,2}(T_a) \geq C_{A,3}(T_a)$, $C_{A,1}(T_a) \leq C_{A,4}(T_a)$, and
368 $C_{A,2}(T_a) \leq C_{A,4}(T_a)$.

369 **Proof.** Since $G_1(t) \geq G_3(t)$, $c_r + c_m G_1(T_a) \geq c_r + c_m G_3(T_a)$ and $\int_0^{T_a} (1 - G_1(t)) dt \leq$
370 $\int_0^{T_a} (1 - G_3(t)) dt$. Hence, $C_{A,1}(T_a) = \frac{c_r + c_m G_1(T_a)}{\int_0^{T_a} (1 - G_1(t)) dt} \geq \frac{c_r + c_m G_3(T_a)}{\int_0^{T_a} (1 - G_3(t)) dt} = C_{A,3}(T_a)$.

371 Similar proofs can be established on the other inequality. ■

372 By minimising $C_{A,i}(T_a)$, we can obtain the optimum T_a^* for the age replacement policy
373 based on maintenance policies A, B, C, and D, respectively.

374 4.2 Block replacement policy

375 For the block replacement policy, which is introduced by Barlow and Hunter (1960), a
376 unit is replaced at a scheduled time regardless of time since its last repair. Any failure
377 between replacements will be repaired with the minimal repair, which restores the failed
378 system to the status just before the failure occurred.

379 We have following assumptions.

- 380 • The inspection will be taken every T_b .
- 381 • Immediately after a preventive or corrective maintenance, the system rests its age
382 to 0.
- 383 • Both c_r and c_m are constants.

384 Then, the expected cost per time unit for the block replacement policy is given by

$$385 \quad C(T) = \frac{c_r + c_m M(T)}{T}, \quad (29)$$

387 where $M(t)$ is a renewal functions. To approximate this renewal function, given a

$$388 \quad C_{B,i}(T_b) = \frac{c_r + c_m M_{\omega_L}(T_b)}{T_b}, \quad (30)$$

390 where $M_{\omega_L}(T_b)$ is the expected number of failed units with the CDF (cumulative dis-
391 tribution function) $F_{\omega_L}(t)$, during the interval $(0, T_b]$, c_r is the replacement cost and
392 c_m is the maintenance cost. Assume that the replacement interval is so short that the
393 probability of two or more failures occurring within $(0, T_b)$ is zero. Denote that $N(T_b)$ is
394 the number of failures within an interval of length T_b , then

$$395 \quad B(T_b) = E[M_{\omega_L}(T_b)], \quad (31)$$

397 then the expected cost per time unit is given by

$$398 \quad C_{B,i}(T_b) = \frac{c_r + c_m B(T_b)}{T_b}, \quad (32)$$

400 According to our four maintenance policies, then

$$401 \quad C_{B,i}(T_b) = \frac{c_r + c_m B_i(T_b)}{T_b}, \quad (33)$$

403 where $i = 1, 2, 3, 4$, corresponding to maintenance policies A, B, C, and D, the optimal
404 scheduled replacement time T_b could be obtained by minimizing the $C_{B,i}(T_b)$. Similarly,
405 we can obtain this property.

406 **Property 2.** For given t , $G_1(t) \geq G_3(t)$, $G_2(t) \geq G_3(t)$, $G_1(t) \leq G_4(t)$, and $G_2(t) \leq$
407 $G_4(t)$, then $C_{B,1}(T_b) \geq C_{B,3}(T_b)$, $C_{B,2}(T_b) \geq C_{B,3}(T_b)$, $C_{B,1}(T_b) \leq C_{B,4}(T_b)$, and
408 $C_{B,2}(T_b) \leq C_{B,4}(T_b)$.

409 5 Numerical examples

410 We consider a system with two different failure modes. The deterioration process of the
411 two failure modes is modelled with two WPs, respectively, each of which has different
412 parameters α , β and σ . We assume that two modes have weights as following $a_1 = 0.3$
413 and $a_2 = 0.7$. α_1 , β_1 and σ_1 are 0.8, 0.5 and 0.2 for the first failure mode, respectively.
414 α_2 , β_2 and σ_2 are 0.7, 1 and 0.5, respectively. We also assume that $c_r = 100$ and $c_m = 50$,
415 then we can obtain the following result.

416 Thus, the linear combination of the two processes is given by

$$417 \quad Y(t) = 0.3X_1 + 0.7X_2.$$

418
 419 We assume that the system needs to be repaired when the deterioration levels exceed the
 420 threshold L_{w_L} and the threshold $L_{w_{L_c}}$, respectively. Replacement activities will be taken
 421 and the deterioration level will be restored to zero when the component is completely
 422 replaced. We obtain the result under $L_{w_L} = \{3, 3.5, 2\}$ and $L_{w_{L_c}} = \{1.5, 1, 2.5\}$ under
 423 policies A, B, C and D, respectively. It is worth noticing that all parameters can be
 424 estimated based on historical data or expert elicitation (Shah et al., 2013).

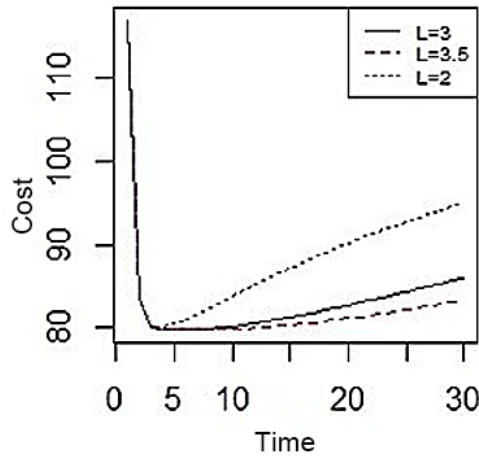


Figure 3: Maintenance Policy A

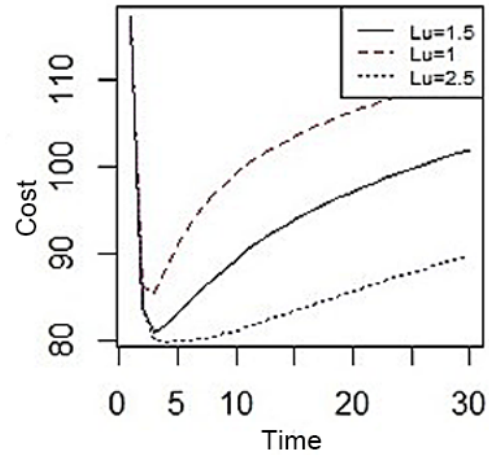


Figure 4: Maintenance Policy B

425 Figure 3 shows the expected cost per unit time under the maintenance policy A.

- 426 • When the threshold L_{w_L} is 3, the optimised time interval is ($T_{opt} = 4.318$) and the
 427 expected unit cost per time is 79.793.
- 428 • When the threshold L_{w_L} is 3.5, the optimised time interval is ($T_{opt} = 4.745$) and
 429 the expected unit cost per time is 79.789.
- 430 • When the threshold L_{w_L} is 2, the optimized time interval is ($T_{opt} = 3.410$) and the
 431 expected unit cost per time is 79.966

432 Figure 4 shows the expected cost per unit time under the maintenance policy B.

- 433 • When the threshold $L_{w_{L_c}}$ is 1.5, the optimized time interval is ($T_{opt} = 2.934$) and
 434 the expected unit cost per time is 80.787.
- 435 • When the threshold $L_{w_{L_c}}$ is 1, the optimized time interval is ($T_{opt} = 2.483$) and
 436 the expected unit cost per time is 84.731.
- 437 • When the threshold $L_{w_{L_c}}$ is 2.5, the optimized time interval is ($T_{opt} = 3.874$) and
 438 the expected unit cost per time is 79.817.

439 Figure 5 shows the expected cost per unit time under the maintenance policy C.

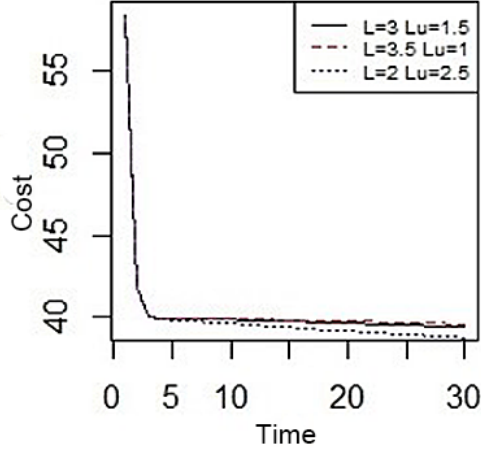


Figure 5: Maintenance Policy C

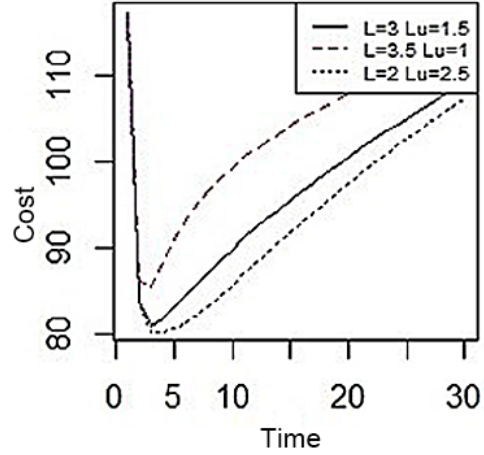


Figure 6: Maintenance Policy D

- 440 • When the thresholds for L_{wL} and L_{wL_c} are 3 and 1.5, respectively, the expected
441 unit cost per time is 39.85863.
- 442 • When the thresholds for L_{wL} and L_{wL_c} are 3.5 and 1, respectively, the expected
443 unit cost per time is 39.88409.
- 444 • When the thresholds for L_{wL} and L_{wL_c} are 2 and 2.5, respectively, the expected
445 unit cost per time is 39.62653.

446 Figure 6 shows the expected cost per unit time under the maintenance policy D.

- 447 • When the thresholds for L_{wL} and L_{wL_c} are 3 and 1.5, respectively, the optimized
448 time interval is ($T_{opt} = 2.934$) and the expected unit cost per time is 80.787.
- 449 • When the thresholds for L_{wL} and L_{wL_c} are 3.5 and 1, respectively, the optimized
450 time interval is ($T_{opt} = 2.483$) and the expected unit cost per time is 84.731.
- 451 • When the thresholds for L_{wL} and L_{wL_c} are 2 and 2.5, respectively, the optimized
452 time interval is ($T_{opt} = 3.360$) and the expected unit cost per time is 79.994.

453 Figure 7 shows the comparison among policy A, B, C and D. Table 2 is the optimized
454 result which is related to Figure 7.

Optimized result	A1	A2	A3	A4
Optimized expected unit cost per time	79.803	81.518	39.819	81.518
Time interval	4.024	2.777	-	2.777

Table 2: Comparison result among policy A, B, C and D

455 Then, we set 10 scenarios. The following table shows parameters we used for these 10
456 scenarios. Table 3 shows parameters we used for 10 scenarios.

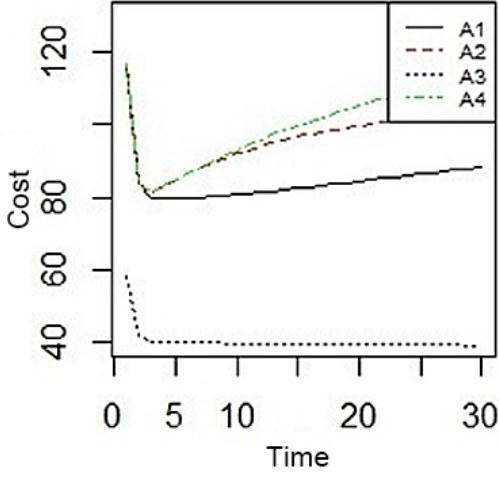


Figure 7: Comparison among policy A, B, C and D

Parameters	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
c_r	100	100	100	100	80	85	90	100	120	100
c_m	50	50	50	50	40	80	70	80	90	120
L	3.00	2.00	2.50	3.50	3.00	3.00	2.00	2.50	3.50	3.00
L_u	1.50	1.80	2.00	2.50	1.50	1.50	1.80	2.00	2.50	1.50
a_1	0.30	0.30	0.40	0.50	0.30	0.30	0.30	0.40	0.50	0.30
a_2	0.70	0.70	0.60	0.50	0.70	0.70	0.70	0.60	0.50	0.70
α_1	0.80	0.60	0.70	0.65	0.80	0.80	0.60	0.70	0.65	0.80
α_2	0.70	0.50	0.80	0.55	0.70	0.70	0.50	0.80	0.55	0.70
β_1	0.50	0.60	0.80	1.20	0.50	0.50	0.60	0.80	1.20	0.50
β_2	1.00	0.90	0.80	0.70	1.00	1.00	0.90	0.80	0.70	1.00
σ_1	0.20	0.40	0.60	0.80	0.20	0.20	0.40	0.60	0.80	0.20
σ_2	0.50	0.55	0.65	0.45	0.50	0.50	0.55	0.65	0.45	0.50

Table 3: Parameters for 10 scenario

- 457 • S1, S5, S6 and S10 have same parameters exclude the replacement cost and repair
458 cost.
- 459 • S2 and S7 have same parameters exclude the replacement cost and repair cost.
- 460 • S3 and S8 have same parameters exclude the replacement cost and repair cost.
- 461 • S4 and S9 have same parameters exclude the replacement cost and repair cost.
- 462 • S1, S2, S3, S4 have same replacement cost and repair cost. However, other param-
463 eters are different.

464 The following table shows the expected cost per time unit with its time interval based
465 on our 10 scenarios. The value outside the brackets is the optimized expected cost per
466 time unit and the value inside the brackets is the time interval.

Scenario	A1	A2	A3	A4
S1	80.787(2.934)	80.787(2.934)	39.859	80.787(2.934)
S2	80.197(3.186)	80.534(3.018)	39.498	80.776(2.894)
S3	80.006(3.358)	80.621(2.988)	39.600	80.715(2.929)
S4	79.801(4.067)	80.019(3.354)	39.827	80.012(3.348)
S5	63.835(4.280)	64.769(2.870)	31.880	64.770(2.870)
S6	67.826(4.210)	69.154(2.752)	33.853	69.154(2.752)
S7	72.312(3.074)	72.731(2.897)	35.351	73.009(2.775)
S8	80.091(3.247)	80.956(2.856)	39.424	81.073(2.801)
S9	95.766(3.993)	96.099(3.258)	47.752	96.102(3.253)
S10	79.796(4.169)	81.644(2.682)	39.809	81.644(2.682)

Table 4: Numerical examples for 10 scenario

467 According to Table 4, we can find that the result is satisfied with property 1 in section
468 4, $C_{A,1}(T_a) \geq C_{A,3}(T_a)$, $C_{A,2}(T_a) \geq C_{A,3}(T_a)$, $C_{A,1}(T_a) \leq C_{A,4}(T_a)$, and $C_{A,2}(T_a) \leq$
469 $C_{A,4}(T_a)$.

470 We compare these results from two aspects: the influence of cost and the influence of
471 other parameter exclude cost. According to Table 4, we use results of S1, S5, S6, S10 for
472 the first aspect and S1, S2, S3, S4 for the second aspect.

473 5.1 Comparison among S1, S5, S6, S10

474 We focus on the influence of cost in this part.

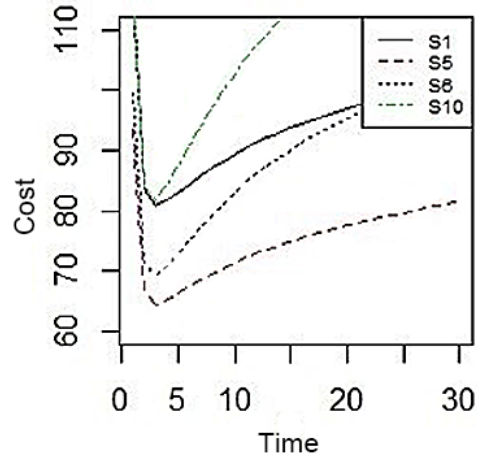
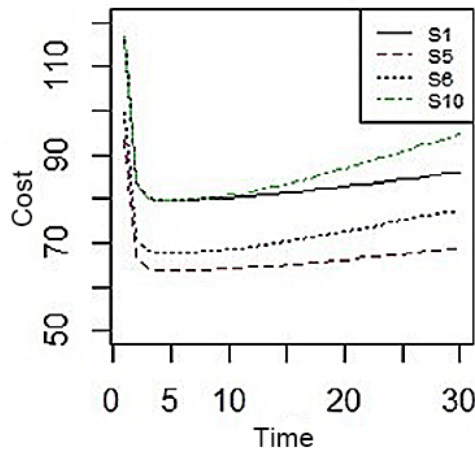


Figure 8: Policy A for S1, S5, S6 and S10

Figure 9: Policy B for S1, S5, S6 and S10

- 475 • According to Figures 8, 9 and 10, with the increase of cost, all of policy A, B and
476 D have increasing expected cost.
- 477 • Among them, maintenance policy D is the most sensitive to price changes. The
478 expected cost of S6 is gradually higher than that of S5.

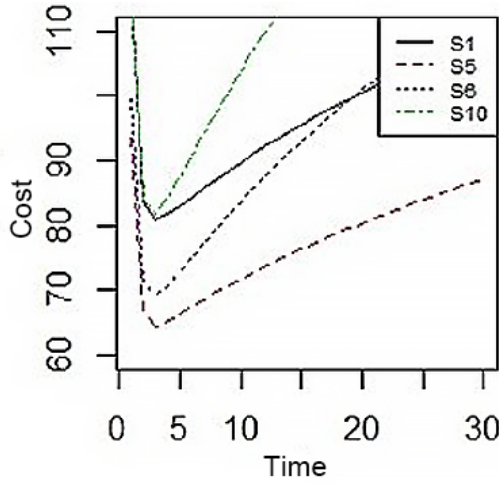


Figure 10: Policy D for S1, S5, S6 and S10

479 **5.2 Comparison among S1, S2, S3, S4**

480 We focus on the influence of other parameters exclude cost in this part.

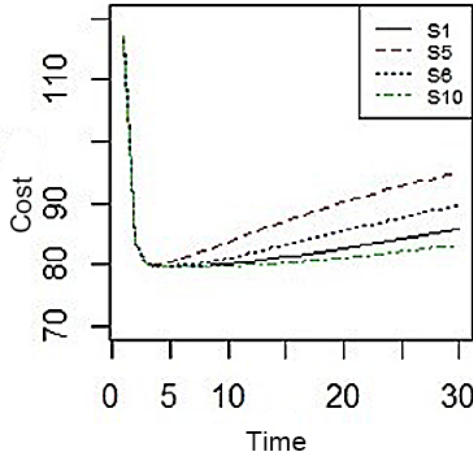


Figure 11: Policy A for S1, S2, S3 and S4

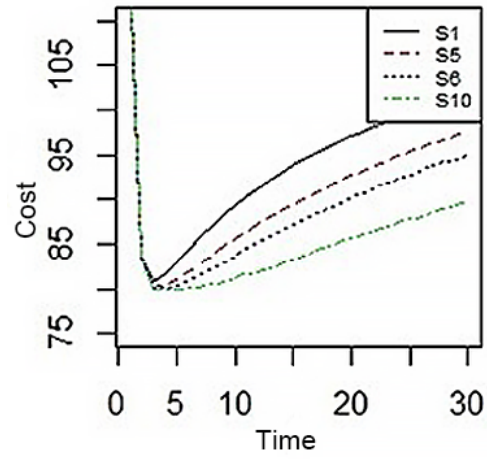


Figure 12: Policy B for S1, S2, S3 and S4

- 481 • According to Figure 11, the ratios of cost changing from the highest to the lowest
- 482 are: $S5 > S6 > S1 > S10$.
- 483 • According to Figure 12, the ratios of cost changing from the highest to the lowest
- 484 are: $S1 > S5 > S6 > S10$.
- 485 • According to Figure 13, the ratios of cost changing from the highest to the lowest
- 486 are: $S1 > S5 > S6 > S10$ before the turning point $t = 10$ and $S5 > S1 > S6 > S10$
- 487 after the turning point.

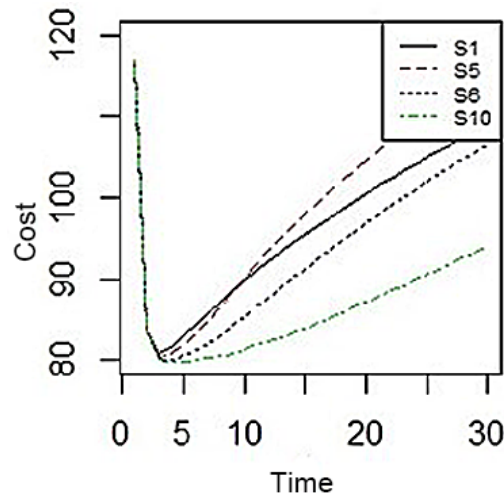


Figure 13: Policy D for S1, S2, S3 and S4

488 6 Conclusions

489 This paper investigated maintenance policies for a system whose deterioration process
 490 is a linear combination of Wiener processes. It proposed four maintenance policies with
 491 both degradation and cost thresholds for a multi-component system and then compared
 492 them. This paper also discussed two properties based on these four maintenance policies.
 493 Numerical examples were given to illustrate the optimisation process.

494 However, there are several limitations in our research.

- 495 1. The deterioration process of a system may be a non-linear combination of deteriora-
 496 tion processes. A non-linear combination of deterioration processes based on other
 497 models, such as the gamma process and the geometric process, can be considered
 498 in future.
- 499 2. The dependence among failure modes or failure components has not been considered
 500 in this paper. Besides, the economic dependence is another possible problem for
 501 designing the maintenance policy. Such problems will be investigated in our future
 502 work.

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