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ORIGINAL RESEARCH

Two-timescale design for RIS-aided full-duplex MIMO systems with transceiver hardware impairments

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Abstract

This paper focuses on a reconfigurable intelligent surface (RIS)-aided full-duplex multi-user massive multiple-input multiple-output system with transceiver hardware impairments (THWIs). Different from the existing works, the two-timescale design scheme is considered. The phase shift at the RIS is designed only based on the statistical channel state information. Firstly, the closed-form expression of the uplink and downlink achievable rate is derived. Then, the power scaling laws are revealed. Finally, the impact of THWIs on the system performance is analysed and genetic algorithm to maximize the achievable rate is used.

1 | INTRODUCTION

In recent years, reconfigurable intelligent surface (RIS) has been widely studied because of its advantages in improving spectral efficiency and energy efficiency [1, 2]. RIS-aided communication systems have been widely studied in various applications [3–9]. In [3], the authors investigated an RIS-aided downlink communication system in a single-cell network, and minimized the total transmit power at the access point. The authors in [4] focused on an RIS-aided downlink multi-user system, and designed both the RIS's phase shifts and the transmit power allocation. In [5], an RIS-aided full-duplex (FD) communication system was investigated, where the authors considered the passive beamforming design and minimized the sum transmit power. The authors in [6] investigated a multi-user FD two-way communications system, and maximized the weighted minimum data rate of users by jointly optimizing the precoding matrix of the base station (BS) and the reflection matrix of the RIS. In [7], the authors considered the beamforming optimization for an RIS-aided full-duplex (FD) communication system, and maximized the sum data rate of bi-directional transmissions. The authors in [8] and [9] considered the RIS-aided FD communication systems with hardware impairments (HWIs). In [8], the authors studied a RIS-aided multiuser FD secure communication system with HWIs at RIS and transceivers and explored a deep reinforcement learning-based algorithm to maximize the sum secrecy rate. In [9], a multi-RIS-aided FD system with HWIs was investigated, where the authors proposed an efficient iterative alternating approach to solve the system weighted sum-rate optimization problem.

However, the above works are based on instantaneous channel state information (CSI), which requires high channel training overhead. To address this problem, the two-timescale design which is more suitable for RIS-aided systems was firstly proposed in [10]. Specifically, the beamforming at the BS is designed based on the instantaneous effective CSI, while the phase shift at the RIS is designed based on the statistical CSI.

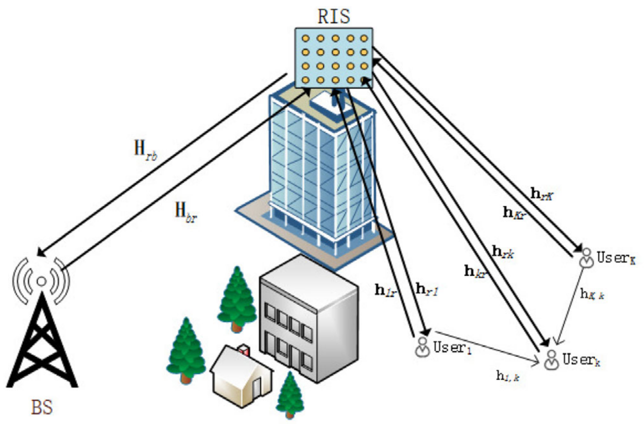

FIGURE 1 A typical RIS-aided FD multi-user mMIMO system

Against this background, we consider the two-timescale design for RIS-aided FD massive multiple-input multiple-output (mMIMO) system with THWIs. The downlink (DL) and uplink (UL) transmission can occur simultaneously at the same frequency. Since statistical CSI varies more slowly than instantaneous CSI, the two-timescale design can significantly reduce the frequency of reconfiguring the RISs and feedback overhead [11, 12]. We assume that the channel can be perfectly estimated as in [10], and the scenario of imperfect CSI is left for our further work. In addition, the spatial correlation can be avoided through increasing the element spacing of RIS. First, the closed-form expressions of the UL and DL rate are derived. Then, we adopt genetic algorithm (GA) to optimize the RIS's phase shifts only based on statistical CSI. Finally, we analyse the system performance of RIS-aided FD mMIMO system with THWIs.

Notations: $\circ$ represents the Hadamard product. $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{A})$ denotes that $\mathbf{x}$ follows the complex Gaussian distribution with mean $\mathbf{0}$ and covariance matrix $\mathbf{A}$.

2 | SYSTEM MODEL

Consider an RIS-aided FD multi-user mMIMO system with THWIs, where the direct link is blocked. The RIS is composed of N reflecting elements and the BS is equipped with Q transmit antennas and M receive antennas. K users communicate with the BS through the RIS and each user equipped with a pair of transmit and receive antennas. The reflection matrix of the RIS can be expressed as $\mathbf{\Theta} = \mathrm{diag}\left\{e^{j\theta_1}, \dots, e^{j\theta_n}, \dots, e^{j\theta_N}\right\}$, where $\theta_n \in [0, 2\pi)$ represents the phase shift of the reflecting element n (Figure 1).

For the channel between the BS and users, we consider the Rician fading model. Therefore, we can respectively model the UL channel $\mathbf{H}_{ur}$ and $\mathbf{H}_{rb}$ as

$$\mathbf{H}_{ur} = [\mathbf{h}_{1r}, \dots, \mathbf{h}_{kr}, \dots, \mathbf{h}_{Kr}], \tag{1}$$

$$\mathbf{h}_{kr} = \sqrt{\frac{\mu_{U,k}\epsilon_{U,k}}{\epsilon_{U,k}+1}}\overline{\mathbf{h}}_{kr} + \sqrt{\frac{\mu_{U,k}}{\epsilon_{U,k}+1}}\widetilde{\mathbf{h}}_{kr}, \tag{2}$$

$$\mathbf{H}_{rb} = \sqrt{\frac{\nu_U\rho_U}{\rho_U+1}}\overline{\mathbf{H}}_{rb} + \sqrt{\frac{\nu_U}{\rho_U+1}}\widetilde{\mathbf{H}}_{rb}, \tag{3}$$

where $\mathbf{H}_{ur} \in \mathbb{C}^{N\times K}$ is the channel from users to RIS, $\mathbf{h}_{kr}$ is the channel from k-th user to RIS, and $\mathbf{H}_{rb} \in \mathbb{C}^{M\times N}$ is the channel from RIS to BS. $\mu_{U,k}$ and $\epsilon_{U,k}$ are respectively the path-loss coefficients and Rician factors from k-th user to RIS. ν_U and ρ_U are respectively the path-loss coefficients and Rician factors from RIS to BS. $\widetilde{\mathbf{H}}_{rb}$ and $\overline{\mathbf{H}}_{rb}$ are respectively the non-line-of-sight (NLoS) and Los components of channel $\mathbf{H}_{rb}$. $\widetilde{\mathbf{h}}_{kr}$ and $\overline{\mathbf{h}}_{k}$ are respectively the NLoS and Los components of channel $\mathbf{h}_{kr}$. The elements of $\widetilde{\mathbf{H}}_{rb}$ and $\widetilde{\mathbf{h}}_{kr}$ are independent and follow the distribution of $\mathcal{CN}(0, 1)$. In addition, we consider the uniform square planar array model. Thus, the expressions of $\overline{\mathbf{h}}_{k}$ and $\overline{\mathbf{H}}_{rb}$ are

$$\overline{\mathbf{h}}_{kr} = \mathbf{a}_N\left(\psi^a_{kr}, \psi^e_{kr}\right), \tag{4}$$

$$\overline{\mathbf{H}}_{rb} = \mathbf{a}_M\left(\phi^a_{rb}, \phi^e_{rb}\right)\mathbf{a}^H_N\left(\psi^a_{rb}, \psi^e_{rb}\right), \tag{5}$$

where the superscript a and e respectively represent the azimuth and elevation angles. The ψ and ϕ respectively denote the angles of arrival (AoA) and the angles of departure (AoD). The subscripts kr and rb, respectively denote k-th user-RIS and RIS-BS. We can express form of $\mathbf{a}_X(v^a_{rb}, v^e_{rb})$ as

$$\begin{aligned}\mathbf{a}_X(v^a_{rb}, v^e_{rb}) = \Big[&1, \dots, e^{j\frac{2\pi d}{\lambda}\left(p\sin v^a_{rb}\sin v^e_{rb} + q\cos v^e_{rb}\right)}, \\ &\dots, e^{j\frac{2\pi d}{\lambda}\left(\left(\sqrt{M}-1\right)\sin v^a_{rb}\sin v^e_{rb} + \left(\sqrt{M}-1\right)\cos v^e_{rb}\right)}\Big]^T,\end{aligned} \tag{6}$$

where $0 \le p, q \le \sqrt{X} - 1$, λ is the carrier wavelength and d is the element spacing.

Based on the above definitions, the whole UL cascaded channel can be expressed as $\mathbf{G} = \mathbf{H}_{rb}\mathbf{\Theta}\mathbf{H}_{ur}$, and the k-th column of $\mathbf{G}$ can be denoted by $\mathbf{g}_k = \mathbf{H}_{rb}\mathbf{\Theta}\mathbf{h}_{kr}$.

Similar to the UL channel, the DL channel $\mathbf{H}_{ru}$ and $\mathbf{H}_{br}$ can be modelled as

$$\mathbf{H}_{ru} = [\mathbf{h}_{r1}, \dots \mathbf{h}_{rk}, \dots, \mathbf{h}_{rK}], \tag{7}$$

$$\mathbf{h}_{rk} = \sqrt{\frac{\mu_{D,k}\epsilon_{D,k}}{\epsilon_{D,k}+1}}\overline{\mathbf{h}}_{rk} + \sqrt{\frac{\mu_{D,k}}{\epsilon_{D,k}+1}}\widetilde{\mathbf{h}}_{rk}, \tag{8}$$

$$\mathbf{H}_{br} = \sqrt{\frac{\nu_D\rho_D}{\rho_D+1}}\overline{\mathbf{H}}_{br} + \sqrt{\frac{\nu_D}{\rho_D+1}}\widetilde{\mathbf{H}}_{br}, \tag{9}$$

where $\mathbf{H}_{ru} \in \mathbb{C}^{N\times K}$, $\mathbf{H}_{br} \in \mathbb{C}^{N\times Q}$, $\overline{\mathbf{h}}_{rk} = \mathbf{a}_N(\phi^a_{rk}, \phi^e_{rk})$, $\overline{\mathbf{H}}_{br} = \mathbf{a}_N(\psi^a_{br}, \psi^e_{br})\mathbf{a}^H_Q(\phi^a_{br}, \phi^e_{br})$. The elements of $\widetilde{\mathbf{H}}_{br}$, $\widetilde{\mathbf{h}}_{rk}$ are independent and follow the distribution of $\mathcal{CN}(0, 1)$. The detailed description of the DL channel is similar to that of the UL channel. The whole DL cascaded channel can be expressed as $\mathbf{F} = \mathbf{H}_{ru}^{H}\mathbf{\Theta}\mathbf{H}_{br}$, and the k-th row of $\mathbf{F}$ can be denoted by $\mathbf{f}_k^H = \mathbf{h}_{rk}^H\mathbf{\Theta}\mathbf{H}_{br}$.

We consider the existence of the THWIs. There are transmit distortion and receive distortion at BS and users. Specifically, the transmit distortion vector of users is denoted by $\mathbf{z}_{t,U} = [z_{t,1}; \dots z_{t,k}; \dots ; z_{t,K}]$ and $z_{t,k}$ represents the transmit distortion of user k. The receive distortion of user k is denoted by $z_{r,k}$. The transmit distortion vector of the BS is denoted by $\mathbf{z}_{t,B}$. The receive distortion vector of the BS is denoted by $\mathbf{z}_{r,B}$.

In addition, the direct link from user i to user k is denoted by $h_{i,k}, i = 1, 2 \dots , K$ and $h_{i,k} \sim \mathcal{CN}(0, \alpha_i)$. The loop channel of the BS is denoted by $\mathbf{H}_B \in \mathbb{C}^{M\times Q}$ whose elements are independent and follow the distribution of $\mathcal{CN}(0, \alpha_B)$. $\mathbf{s} = [s_1, s_2, \dots , s_K]^T$ and $\mathbf{x} = [x_1, x_2, \dots , x_K]^T$ are respectively the transmit signal vector of BS and users. We use maximum ratio transmission and low-complexity maximal ratio combining (MRC) technique at the BS.

Based on the above definition, we can express the signal received by user k as

$$y_{U,k} = \mathbf{f}_k^H(\sqrt{p_B}\mathbf{W}\mathbf{s} + \mathbf{z}_{t,B}) + (h_{k,k} + \mathbf{h}_{rk}^H\mathbf{\Theta}\mathbf{h}_{kr})(\sqrt{p_k}x_k + z_{t,k}) + \sum_{i=1,i\neq k}^{K}(h_{i,k} + \mathbf{h}_{ri}^H\mathbf{\Theta}\mathbf{h}_{ir})(\sqrt{p_k}x_i + z_{t,i}) + z_{r,k} + n_k \tag{10}$$

The path loss of $\mathbf{h}_{ri}^H\mathbf{\Theta}\mathbf{h}_{ir}$ is much larger than that of $h_{i,k}$, , so we can ignore $\mathbf{h}_{ri}^H\mathbf{\Theta}\mathbf{h}_{ir}$ as in [13] and simplify Equation (10) to

$$\begin{aligned} y_{U,k} &= \mathbf{f}_k^H(\sqrt{p_B}\mathbf{W}\mathbf{s} + \mathbf{z}_{t,B}) + h_{k,k}(\sqrt{p_k}x_k + z_{t,k}) + \sum_{i=1,i\neq k}^{K} h_{i,k}(\sqrt{p_i}x_i + z_{t,i}) + z_{r,k} + n_k \\ &= \underbrace{\sqrt{p_B}\mathbf{f}_k^H\mathbf{w}_k s_k}_{\text{Signal}} + \underbrace{\sum_{i=1,i\neq k}^{K}(\sqrt{p_B}\mathbf{f}_k^H\mathbf{w}_i s_i + \sqrt{p_i}h_{i,k}x_i)}_{\text{Multi-user interference (MUI)}} \\ &+ \underbrace{\sqrt{p_k}h_{k,k}x_k}_{\text{Loop interference (LI)}} + \underbrace{\mathbf{f}_k^H\mathbf{z}_{t,B} + \sum_{i=1}^{K} h_{i,k}z_{t,i} + z_{r,k}}_{\text{THWIs}} + \underbrace{n_k}_{\text{Noise}}. \\ &\triangleq \widetilde{y}_{U,k} + z_{r,k} \end{aligned} \tag{11}$$

where $n_k \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise (AWGN), $\mathbf{W}$ is the precoding matrix of the BS and can be denoted by $\mathbf{W} = \frac{F^H}{\sqrt{\mathrm{Tr}(FF^H)}} = [\mathbf{w}_1, \mathbf{w}_2, \dots , \mathbf{w}_K] \in \mathbb{C}^{Q\times K}$. We consider the conditional distributions $\mathbf{z}_{t,B} \sim \mathcal{CN}(0, k_b\mathbf{I}_M \circ \mathbf{W}\mathbf{W}^H)$, $z_{r,k} \sim \mathcal{CN}(0, k_u|\widetilde{y}_{U,k}|^2)$ for given channel realizations. k_b and k_u express the severity of the residual impairments of transceiver at the BS and users, respectively.

Similarly, for the signal received by the BS

$$\mathbf{y} = \mathbf{G}\left(\sqrt{p_k}\mathbf{x} + \mathbf{z}_{t,U}\right) + (\mathbf{H}_B + \mathbf{H}_{rb}\mathbf{\Theta}\mathbf{H}_{br})\left(\sqrt{p_B}\mathbf{W}\mathbf{s} + \mathbf{z}_{t,B}\right) + \mathbf{z}_{r,B} + \mathbf{n}, \tag{12}$$

We ignore the effect of $\mathbf{H}_{rb}\mathbf{\Theta}\mathbf{H}_{br}$ and apply MRC technique. Therefore, the received signal of the k-th user is

$$\begin{aligned} y_k &= \mathbf{g}_k^H\left(\mathbf{G}(\sqrt{p_k}\mathbf{x} + \mathbf{z}_{t,U}) + \mathbf{H}_B(\sqrt{p_B}\mathbf{W}\mathbf{s} + \mathbf{z}_{t,B}) + \mathbf{z}_{r,B} + \mathbf{n}\right) \\ &= \underbrace{\mathbf{g}_k^H\mathbf{g}_k\sqrt{p_k}x_k}_{\text{Signal}} + \underbrace{\sum_{i=1,i\neq k}^{K}\mathbf{g}_k^H\mathbf{g}_i\sqrt{p_i}x_i}_{\text{MUI}} + \underbrace{\mathbf{g}_k^H\mathbf{H}_B\sqrt{p_B}\mathbf{W}\mathbf{s}}_{\text{LI}} \\ &+ \underbrace{\mathbf{g}_k^H(\mathbf{G}\mathbf{z}_{t,U} + \mathbf{H}_B\mathbf{z}_{t,B} + \mathbf{z}_{r,B})}_{\text{THWIs}} + \underbrace{\mathbf{g}_k^H\mathbf{n}}_{\text{noise}} \end{aligned} \tag{13}$$

where $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_M)$ is the AWGN vector and we consider the conditional distributions $z_{t,i} \sim \mathcal{CN}(0, k_u p_i)$, $z_{r,B} \sim \mathcal{CN}(0, k_b p_k \sum_{i=1}^{K}\mathbf{I}_M \circ \mathbf{g}_i\mathbf{g}_i^H)$ for given channel realizations.

3 | ANALYSIS OF UPLINK AND DOWNLINK ACHIEVABLE RATE

We can express the UL achievable rate of user k as $R_k^{UL} = \mathbb{E}\{\log_2(1 + \gamma_k^{\mathrm{UL}})\}$. γ_k^{UL} is the UL signal-to-interference-plus-noise ratio (SINR) of user k and can be expressed as Equation (14), where $\Gamma \triangleq \mathrm{Tr}\{\mathbf{F}\mathbf{F}^H\}$.

$$\begin{aligned} \gamma_k^{\mathrm{UL}} &= \frac{p_k|\mathbf{g}_k^H\mathbf{g}_k|^2}{\sum_{i=1,i\neq k}^{K} p_i|\mathbf{g}_k^H\mathbf{g}_i|^2 + \sum_{i=1}^{K} p_B|\mathbf{g}_k^H\mathbf{H}_B\mathbf{w}_i|^2 + |\mathbf{g}_k^H(\mathbf{G}\mathbf{z}_{t,U} + \mathbf{H}_B\mathbf{z}_{t,B} + \mathbf{z}_{r,B})|^2 + |\mathbf{g}_k^H\mathbf{n}|^2} \\ &= \frac{p_k\Gamma|\mathbf{g}_k^H\mathbf{g}_k|^2}{\sum_{i=1,i\neq k}^{K} p_i\Gamma|\mathbf{g}_k^H\mathbf{g}_i|^2 + \sum_{i=1}^{K} p_B|\mathbf{g}_k^H\mathbf{H}_B\mathbf{f}_i|^2 + \Gamma|\mathbf{g}_k^H(\mathbf{G}\mathbf{z}_{t,U} + \mathbf{H}_B\mathbf{z}_{t,B} + \mathbf{z}_{r,B})|^2 + \Gamma|\mathbf{g}_k^H\mathbf{n}|^2}, \end{aligned} \tag{14}$$

Based on [14, Lemma1], we can approximate the UL achievable rate R_k^{UL} as

$$R_k^{UL} \approx \log_2\left(1 + \frac{p_k \mathbb{E}_{U,k}^{\text{signal}}}{\sum_{i\neq k}^{K} p_i \mathbb{E}_{U,k}^{\text{MUI}} + \sum_{i=1}^{K} p_B \mathbb{E}_{U,k}^{\text{LI}} + \mathbb{E}_{U,k}^{\text{THWIs}} + \sigma^2 \mathbb{E}_{U,k}^{\text{noise}}}\right), \tag{15}$$

where $\mathbb{E}_{U,k}^{\text{signal}} = \mathbb{E}\{\Gamma\}\mathbb{E}\{|\mathbf{g}_k^H \mathbf{g}_k|^2\}$, $\mathbb{E}_{U,k}^{\text{MUI}} = \mathbb{E}\{\Gamma\}\mathbb{E}\{|\mathbf{g}_k^H \mathbf{g}_i|^2\}$, $\mathbb{E}_{U,k}^{\text{THWIs}} = \mathbb{E}\{\Gamma\}\mathbb{E}\{|\mathbf{g}_k^H(\mathbf{G}\mathbf{z}_{t,U} + \mathbf{H}_B\mathbf{z}_{t,B} + \mathbf{z}_{r,B})|^2\}$, $\mathbb{E}_{U,k}^{\text{LI}} = \mathbb{E}\{|\mathbf{g}_k^H \mathbf{H}_B \mathbf{f}_i|^2\}$, $\mathbb{E}_{U,k}^{\text{noise}} = \mathbb{E}\{\Gamma\}\mathbb{E}\{\mathbf{g}_k^H \mathbf{g}_k\}$ are respectively given by Equations (16), (17), (18), (19) and (22). $\mathbb{E}\{|\mathbf{g}_k^H \mathbf{g}_k|^2\}$ and $\mathbb{E}\{|\mathbf{g}_k^H \mathbf{g}_i|^2\}$ can be found in [15]. $\mathbb{E}\{\Gamma\}$, $\mathbb{E}_{U,k}^{\text{LI}}$, $\mathbb{E}_{U,k}^{\text{THWIs}}$, and $\mathbb{E}_{U,k}^{\text{noise}}$ are proved in Appendix. In addition, $a_{U,k} \overset{\Delta}{=} \frac{\nu_U \mu_{U,k}}{(\rho_U+1)(\epsilon_{U,k}+1)}$, $\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta}) \overset{\Delta}{=} \mathbf{a}_N^H(\psi_{rb}^a, \psi_{rb}^e)\boldsymbol{\Theta}\overline{\mathbf{h}}_{kr}$, g_{k_m} is the mth element of $\mathbf{g}_k$.

$$\begin{aligned}
\mathbb{E}_{U,k}^{\text{signal}} = {} & MQa_{U,k}^2 \sum_{i=1}^{K} a_{D,i}(\epsilon_{D,i}\rho_D|\boldsymbol{\Phi}_i^D(\boldsymbol{\Theta})|^2 + N\epsilon_{D,i} + N\rho_D + N) \\
& \times \Big\{ M(\rho_U\epsilon_{U,k})^2|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})|^4 \\
& + MN^2(2\rho_U^2 + \epsilon_{U,k}^2 + 2\rho_U\epsilon_{U,k} + 2\rho_U + 2\epsilon_{U,k} + 1) \\
& + 2\rho_U\epsilon_{U,k}|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})|^2(2MN\rho_U + MN\epsilon_{U,k} \\
& + MN + 2M + N\epsilon_{U,k} + N + 2) + N^2(\epsilon_{U,k}^2 \\
& + 2\rho_U\epsilon_{U,k} + 2\rho_U + 2\epsilon_{U,k} + 1) + (M+1)N(2\rho_U \\
& + 2\epsilon_{U,k} + 1)\Big\}
\end{aligned} \tag{16}$$

$$\begin{aligned}
\mathbb{E}_{U,k}^{\text{MUI}} = {} & MQa_{U,k}a_{U,i} \sum_{j=1}^{K} a_{D,j}(\epsilon_{D,j}\rho_D|\boldsymbol{\Phi}_j^D(\boldsymbol{\Theta})|^2 + N\epsilon_{D,j} + N\rho_D + N) \\
& \times \Big\{ M\rho_U^2\epsilon_{U,k}\epsilon_{U,i}|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})|^2|\boldsymbol{\Phi}_i^U(\boldsymbol{\Theta})|^2 \\
& + \rho_U\epsilon_{U,k}|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})|^2(\rho_U MN + N\epsilon_{U,i} + N + 2M) \\
& + \rho_U\epsilon_{U,i}|\boldsymbol{\Phi}_i^U(\boldsymbol{\Theta})|^2(N(\rho_U M + \epsilon_{U,k} + 1) + 2M) \\
& + N^2(M\rho_U^2 + \rho_U(\epsilon_{U,i} + \epsilon_{U,k} + 2) \\
& + (\epsilon_{U,k} + 1)(\epsilon_{U,i} + 1)) + MN(2\rho_U + \epsilon_{U,i} + \epsilon_{U,k} + 1) \\
& + M\epsilon_{U,k}\epsilon_{U,i}|\overline{\mathbf{h}}_{kr}^H\overline{\mathbf{h}}_{ir}|^2 + 2M\rho_U\epsilon_{U,k}\epsilon_{U,i} \\
& \times \mathrm{Re}\Big\{\big(\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})\big)^H \boldsymbol{\Phi}_i^U(\boldsymbol{\Theta})\overline{\mathbf{h}}_{ir}^H\overline{\mathbf{h}}_{kr}\Big\}\Big\}
\end{aligned} \tag{17}$$

$$\begin{aligned}
\mathbb{E}_{U,k}^{\text{LI}} = {} & MQ\alpha_B a_{U,k}a_{D,i}\Big\{\epsilon_{U,k}\rho_U\epsilon_{D,i}\rho_D|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})|^2|\boldsymbol{\Phi}_i^D(\boldsymbol{\Theta})|^2 \\
& + N\epsilon_{U,k}\rho_U|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})|^2(\epsilon_{D,i} + \rho_D + 1) \\
& + N\epsilon_{D,i}\rho_D|\boldsymbol{\Phi}_i^D(\boldsymbol{\Theta})|^2(\epsilon_{U,k} + \rho_U + 1) + N^2(\epsilon_{U,k} \\
& + \rho_U + 1)(\epsilon_{D,i} + \rho_D + 1)\Big\}
\end{aligned} \tag{18}$$

$$\begin{aligned}
\mathbb{E}_{U,k}^{\text{THWIs}} = {} & k_u(p_k\mathbb{E}_{U,k}^{\text{signal}} + \sum_{i=1,i\neq k}^{K} p_i\mathbb{E}_{U,k}^{\text{MUI}}) + k_b p_B \mathbb{E}_{U,k}^{\text{LI}} \\
& + Q\sum_{i=1}^{K} a_{D,i}(\epsilon_{D,i}\rho_D|\boldsymbol{\Phi}_i^D(\boldsymbol{\Theta})|^2 + N\epsilon_{D,i} + N\rho_D + N) \\
& \times k_b M\left(p_k\mathbb{E}\{|g_{k_m}|^4\} + \sum_{i=1,i\neq k}^{K} p_i\mathbb{E}\{|g_{i_m}|^2|g_{k_m}|^2\}\right)
\end{aligned} \tag{19}$$

where $\mathbb{E}\{|g_{k_m}|^4\}$ and $\mathbb{E}\{|g_{i_m}|^2|g_{k_m}|^2\}$ are expressed as follows:

$$\begin{aligned}
\mathbb{E}\{|g_{k_m}|^4\} = {} & a_{U,k}^2\rho_U^2\epsilon_{U,k}^2|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})|^4 \\
& + 4\rho_U a_{U,k}^2\epsilon_{U,k}N(\rho_U + \epsilon_{U,k} + 1)\left|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})\right|^2 + 4\rho_U a_{U,k}^2\epsilon_{U,k}N^2 \\
& + 2a_{U,k}^2 N(\rho_U^2 N + \epsilon_{U,k}^2 N + N + 1) \\
& + a_{U,k}^2(\rho_U + \epsilon_{U,k})(2N(N+1)) + 2a_{U,k}^2(\epsilon_{U,k} + \rho_U)(N^2 + N).
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mathbb{E}\{|g_{i_m}|^2|g_{k_m}|^2\} = {} & a_{U,k}a_{U,i}N + a_{U,k}a_{U,i}\rho_U N^2 \\
& + a_{U,k}a_{U,i}(\epsilon_{U,k} + 1)N\Big((\rho_U + 1)N + \epsilon_{U,i}(N+1)\Big) \\
& + a_{U,k}a_{U,i}\rho_U\left(N + \epsilon_{U,k}\left|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})\right|^2\right) \\
& \times\left(\rho_U\epsilon_{U,i}\left|\boldsymbol{\Phi}_i^U(\boldsymbol{\Theta})\right|^2 + \rho_U N + \epsilon_{U,i}N\right) \\
& + a_{U,k}a_{U,i}\rho_U\epsilon_{U,k}\left|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})\right|^2 \\
& + a_{U,k}a_{U,i}\Big(2\rho_U\epsilon_{U,i}\epsilon_{U,k}\mathbf{Re}\Big\{\big(\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})\big)^H\boldsymbol{\Phi}_i^U(\boldsymbol{\Theta})\overline{\mathbf{h}}_{rk}^H\overline{\mathbf{h}}_{ri}\Big\} \\
& + 4\rho_U\epsilon_{U,k}|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})|^2\Big) \\
& + a_{U,k}a_{U,i}\rho_U N\Big(\big(\epsilon_{U,k}^2 + \epsilon_{U,i}\big)\left|\boldsymbol{\Phi}_i^U(\boldsymbol{\Theta})\right|^2 + 2\Big).
\end{aligned} \tag{21}$$

$$\begin{aligned}
\mathbb{E}_{U,k}^{\text{noise}} = {} & MQ\sum_{i=1}^{K} a_{U,k}a_{D,i}(\epsilon_{D,i}\rho_D|\boldsymbol{\Phi}_i^D(\boldsymbol{\Theta})|^2 + N\epsilon_{D,i} \\
& + N\rho_D + N)(\epsilon_{U,k}\rho_U|\boldsymbol{\Phi}_k^U(\boldsymbol{\Theta})|^2 + N\epsilon_{U,k} + N\rho_U + N)
\end{aligned} \tag{22}$$

The DL achievable rate of user k can be expressed as $R_k^{DL} = \mathbb{E}\{\log_2(1 + \gamma_k^{\text{DL}})\}$, where γ_k^{DL} can be expressed as Equation (23).

$$\gamma_k^{\mathrm{DL}} = \frac{p_B\left|\mathbf{f}_k^H\mathbf{w}_k\right|^2}{\sum_{i=1,i\neq k}^{K} p_B\left|\mathbf{f}_k^H\mathbf{w}_i\right|^2 + \sum_{i=1}^{K} p_i\left|h_{i,k}\right|^2 + \left|\mathbf{f}_k^H\mathbf{z}_{t,B} + z_{r,k} + \sum_{i=1}^{K} h_{i,k}z_{t,i}\right|^2 + \left|n_k\right|^2}$$
$$= \frac{p_B\left|\mathbf{f}_k^H\mathbf{f}_k\right|^2}{\sum_{i=1,i\neq k}^{K} p_B\left|\mathbf{f}_k^H\mathbf{f}_i\right|^2 + \sum_{i=1}^{K} p_i\Gamma\left|h_{i,k}\right|^2 + \Gamma\left|\mathbf{f}_k^H\mathbf{z}_{t,B} + z_{r,k} + \sum_{i=1}^{K} h_{i,k}z_{t,i}\right|^2 + \Gamma\left|n_k\right|^2} \tag{23}$$

Similarly, R_k^{DL} can be approximated as

$$R_k^{DL} \approx \log_2\left(1 + \frac{p_B\mathbb{E}_{D,k}^{\mathrm{signal}}}{\sum_{i\neq k}^{K} p_B\mathbb{E}_{D,k}^{\mathrm{MUI}} + p_i\mathbb{E}_{D,k}^{\mathrm{LI}} + \mathbb{E}_{D,k}^{\mathrm{THWIs}} + \sigma^2\mathbb{E}_{D,k}^{\mathrm{noise}}}\right) \tag{24}$$

where $\mathbb{E}_{D,k}^{\mathrm{signal}} = \mathbb{E}\{|\mathbf{f}_k^H\mathbf{f}_k|^2\}$, $\mathbb{E}_{D,k}^{\mathrm{LI}} = \mathbb{E}\{\Gamma\sum_{i=1}^{K}|h_{i,k}|^2\}$, $\mathbb{E}_{D,k}^{\mathrm{MUI}} = p_B\mathbb{E}\{|\mathbf{f}_k^H\mathbf{f}_i|^2\} + p_i\mathbb{E}\{\Gamma|h_{i,k}|^2\}$, $\mathbb{E}_{D,k}^{\mathrm{THWIs}} = \mathbb{E}\{\Gamma|\mathbf{f}_k^H\mathbf{z}_{t,B} + z_{r,k} + \sum_{i=1}^{K} h_{i,k}z_{t,i}|^2\}$, $\mathbb{E}_{U,k}^{\mathrm{noise}} = \mathbb{E}\{\Gamma\}$ are respectively given by Equations (25), (26), (27), (28) and (29). The derivation of $\mathbb{E}\{|\mathbf{f}_k^H\mathbf{f}_k|^2\}$ and $\mathbb{E}\{|\mathbf{f}_k^H\mathbf{f}_i|^2\}$ can refer to $\mathbb{E}\{|\mathbf{g}_k^H\mathbf{g}_k|^2\}$ and $\mathbb{E}\{|\mathbf{g}_k^H\mathbf{g}_i|^2\}$, respectively. $\mathbb{E}_{D,k}^{\mathrm{LI}}$, $\mathbb{E}_{D,k}^{\mathrm{THWIs}}$ and $\mathbb{E}_{D,k}^{\mathrm{noise}}$ are proved in Appendix. Besides, $a_{D,k} \triangleq \frac{\nu_D\mu_{D,k}}{(\rho_D+1)(\epsilon_{D,k}+1)}$, $\Phi_k^D(\mathbf{\Theta}) \triangleq \bar{\mathbf{h}}_{rk}^H\mathbf{\Theta}\mathbf{a}_N(\psi_{br}^a,\psi_{br}^e)$, f_{k_q} is the q-th element of $\mathbf{f}_k$.

$$\begin{aligned}\mathbb{E}_{D,k}^{\mathrm{signal}} = {} & Qa_{D,k}^2\Big\{Q(\rho_D\epsilon_{D,k})^2|\Phi_k^D(\mathbf{\Theta})|^4 \\ & + QN^2(2\rho_D^2 + \epsilon_{D,k}^2 + 2\rho_D\epsilon_{D,k} + 2\rho_D + 2\epsilon_{D,k} + 1) \\ & + 2\rho_D\epsilon_{D,k}|\Phi_k^D(\mathbf{\Theta})|^2(2QN\rho_D \\ & + QN\epsilon_{D,k} + QN + 2Q + N\epsilon_{D,k} + N + 2) + N^2(\epsilon_{D,k}^2 \\ & + 2\rho_D\epsilon_{D,k} + 2\rho_D + 2\epsilon_{D,k} + 1) \\ & + (Q+1)N(2\rho_D + 2\epsilon_{D,k} + 1)\Big\}\end{aligned} \tag{25}$$

$$\begin{aligned}\mathbb{E}_{D,k}^{\mathrm{MUI}} = {} & Qa_{D,k}a_{D,i}\times\Big\{Q\rho_D^2\epsilon_{D,k}\epsilon_{D,i}|\Phi_k^D(\mathbf{\Theta})|^2|\Phi_i^D(\mathbf{\Theta})|^2 \\ & + \rho_D\epsilon_{D,k}|\Phi_k^D(\mathbf{\Theta})|^2(N(\rho_DQ + \epsilon_{D,i} + 1) + 2Q) \\ & + \rho_D\epsilon_{D,i}|\Phi_i^D(\mathbf{\Theta})|^2(N(\rho_DQ + \epsilon_{D,k} + 1) + 2Q) \\ & + N^2\left(Q\rho_D^2 + \rho_D(\epsilon_{D,i} + \epsilon_{D,k} + 2) + (\epsilon_{D,k} + 1)(\epsilon_{D,i} + 1)\right) \\ & + QN(2\rho_D + \epsilon_{D,i} + \epsilon_{D,k} + 1) + Q\epsilon_{D,k}\epsilon_{D,i}|\bar{\mathbf{h}}_{rk}^H\bar{\mathbf{h}}_{ri}|^2 \\ & + 2Q\rho_D\epsilon_{D,k}\epsilon_{D,i}\mathrm{Re}\left\{(\Phi_k^D(\mathbf{\Theta}))^H\Phi_i^D(\mathbf{\Theta})\bar{\mathbf{h}}_{ri}^H\bar{\mathbf{h}}_{rk}\right\}\Big\}\end{aligned} \tag{26}$$

$$\mathbb{E}_{D,k}^{\mathrm{LI}} = \sum_{i=1}^{K}\alpha_i\times Q\sum_{j=1}^{K}a_{D,j}(\epsilon_{D,j}\rho_D|\Phi_j^D(\mathbf{\Theta})|^2 + N\epsilon_{D,j} + N\rho_D + N) \tag{27}$$

$$\mathbb{E}_{D,k}^{\mathrm{noise}} = \mathbb{E}\{\Gamma\} = Q\sum_{i=1}^{K}a_{D,i}(\epsilon_{D,i}\rho_D|\Phi_i^D(\mathbf{\Theta})|^2 + N\epsilon_{D,i} + N\rho_D + N) \tag{28}$$

$$\begin{aligned}\mathbb{E}_{D,k}^{\mathrm{THWIs}} = {} & k_u\left(p_B\mathbb{E}_{D,k}^{\mathrm{signal}} + \sum_{i=1}^{K}p_B\mathbb{E}_{D,k}^{\mathrm{MUI}} + \sum_{i=1}^{K}p_i\mathbb{E}_{D,k}^{\mathrm{LI}} + \sigma^2\mathbb{E}_{D,k}^{\mathrm{noise}}\right) \\ & + (1+k_u)\Bigg(k_u\sum_{i=1}^{K}p_i\alpha_i\mathbb{E}\{\Gamma\} + k_bQ \\ & \times\Bigg(\mathbb{E}\{|\mathrm{f}_{k_q}|^4\} + \sum_{i=1,i\neq k}^{K}\mathbb{E}\left\{|\mathrm{f}_{i_q}|^2|\mathrm{f}_{k_q}|^2\right\}\Bigg)\Bigg)\end{aligned} \tag{29}$$

where $\mathbb{E}\{|\mathrm{f}_{k_q}|^4\}$ and $\mathbb{E}\{|\mathrm{f}_{i_q}|^2|\mathrm{f}_{k_q}|^2\}$ are expressed as follows:

$$\begin{aligned}\mathbb{E}\{|\mathrm{f}_{k_q}|^4\} = {} & a_{D,k}^2\rho_D^2\epsilon_{D,k}^2|\Phi_k^D(\mathbf{\Theta})|^4 \\ & + 4\rho_Da_{D,k}^2\epsilon_{D,k}N(\rho_D + \epsilon_{D,k} + 1)|\Phi_k^D(\mathbf{\Theta})|^2 \\ & + 4\rho_Da_{D,k}^2\epsilon_{D,k}N^2 \\ & + 2a_{D,k}^2N(\rho_D^2N + \epsilon_{D,k}^2N + N + 1) \\ & + a_{D,k}^2(\rho_D + \epsilon_{D,k})(2N(N+1)) \\ & + 2a_{D,k}^2(\epsilon_{D,k} + \rho_D)(N^2 + N).\end{aligned} \tag{30}$$

$$\begin{aligned}\mathbb{E}\left\{|\mathrm{f}_{i_q}|^2|\mathrm{f}_{k_q}|^2\right\} = {} & a_{D,k}a_{D,i}N + a_{D,k}a_{D,i}\rho_DN^2 + a_{D,k}a_{D,i}(\epsilon_{D,k} + 1) \\ & \times N\Big((\rho_D + 1)N + \epsilon_{D,i}(N+1)\Big) \\ & + a_{D,k}a_{D,i}\rho_D\Big(N + \epsilon_{D,k}|\Phi_k^D(\mathbf{\Theta})|^2\Big) \\ & \times\Big(\rho_D\epsilon_{D,i}|\Phi_i^D(\mathbf{\Theta})|^2 + \rho_DN + \epsilon_{D,i}N\Big) \\ & + a_{D,k}a_{D,i}\rho_D\epsilon_{D,k}|\Phi_k^D(\mathbf{\Theta})|^2 \\ & + a_{D,k}a_{D,i}\Bigg(2\rho_D\epsilon_{D,i}\epsilon_{D,k}\mathbf{Re}\left\{\left(\Phi_k^D(\mathbf{\Theta})\right)^H \right. \\ & \left. \times\Phi_i^D(\mathbf{\Theta})\bar{\mathbf{h}}_{rk}^H\bar{\mathbf{h}}_{ri}\right\} + 4\rho_D\epsilon_{D,k}|\Phi_k^D(\mathbf{\Theta})|^2\Bigg)\end{aligned}$$

$$+a_{D,k}a_{D,i}\rho_D N\Big((\epsilon_{D,k}^2+\epsilon_{D,i})|\Phi_i^D(\boldsymbol{\Theta})|^2+2\Big). \tag{31}$$

We assume that the power is scaled as $p_B = E_B/M, p_i = E_i/M, \forall i$, where E_B, E_k is a fixed value. When $M \to \infty$, we have

$$R_k^{UL} \to \log_2\left(1+\frac{E_k A_1}{\sum_{i=1,i\neq k}^K E_i A_2 + A_3 + \sigma^2 A_4}\right), \tag{32}$$

where

$$\begin{aligned}A_1 = a_{U,k}\Big(&\rho_U^2\epsilon_{U,k}^2|\Phi_k^U(\boldsymbol{\Theta})|^4+2\rho_U\epsilon_{U,k}|\Phi_k^U(\boldsymbol{\Theta})|^2(2N\rho_U\\&+N\epsilon_{U,k}+N+2)+N^2(2\rho_U^2+\epsilon_{U,k}^2+2\rho_U\epsilon_{U,k}\\&+2\rho_U+2\epsilon_{U,k}+1)+N(2\rho_U+2\epsilon_{U,k}+1)\Big)\end{aligned} \tag{33}$$

$$\begin{aligned}A_2 = a_{U,i}\Big(&\rho_U^2\epsilon_{U,k}\epsilon_{U,i}|\Phi_k^U(\boldsymbol{\Theta})|^2|\Phi_i^U(\boldsymbol{\Theta})|^2+N^2\rho_U^2\\&+(\rho_U^2N+2\rho_U)(\epsilon_{U,k}|\Phi_k^U(\boldsymbol{\Theta})|^2+\epsilon_{U,i}|\Phi_i^U(\boldsymbol{\Theta})|^2)\\&+N(2\rho_U+\epsilon_{U,i}+\epsilon_{U,k}+1)+\epsilon_{U,k}\epsilon_{U,i}|\overline{\mathbf{h}}_{kr}^H\overline{\mathbf{h}}_{ir}|^2\\&+2\rho_U\epsilon_{U,k}\epsilon_{U,i}\mathrm{Re}\Big\{\Big(\Phi_k^U(\boldsymbol{\Theta})\Big)^H\Phi_i^U(\boldsymbol{\Theta})\overline{\mathbf{h}}_{ir}^H\overline{\mathbf{h}}_{kr}\Big\}\Big)\end{aligned} \tag{34}$$

$$A_3 = k_u(E_k A_1 + \sum_{i=1,i\neq k}^K E_i A_2) \tag{35}$$

$$A_4 = \epsilon_{U,k}\rho_U|\Phi_k^U(\boldsymbol{\Theta})|^2 + N\epsilon_{U,k} + N\rho_U + N \tag{36}$$

Similarly, we assume that the power is scaled as $p_B = E_B/Q, p_i = E_i/Q, \forall i$, where E_B, E_k is a fixed value. When $Q \to \infty$, we have

$$R_k^{DL} \to \log_2\left(1+\frac{E_B B_1}{\sum_{i=1,i\neq k}^K E_B B_2 + B_3 + \sigma^2 B_4}\right), \tag{37}$$

where

$$\begin{aligned}B_1 = a_{D,k}^2\Big(&\rho_D^2\epsilon_{D,k}^2|\Phi_k^D(\boldsymbol{\Theta})|^4+2\rho_D\epsilon_{D,k}|\Phi_k^D(\boldsymbol{\Theta})|^2(2N\rho_D\\&+N\epsilon_{D,k}+N+2)+N^2(2\rho_D^2+\epsilon_{D,k}^2+2\rho_D\epsilon_{D,k}\\&+2\rho_D+2\epsilon_{D,k}+1)+N(2\rho_D+2\epsilon_{D,k}+1)\Big)\end{aligned} \tag{38}$$

$$\begin{aligned}B_2 = a_{D,i}a_{D,k}\Big(&\rho_D^2\epsilon_{D,k}\epsilon_{D,i}|\Phi_k^D(\boldsymbol{\Theta})|^2|\Phi_i^D(\boldsymbol{\Theta})|^2\\&+(\rho_D^2N+2\rho_D)(\epsilon_{D,k}|\Phi_k^D(\boldsymbol{\Theta})|^2+\epsilon_{D,i}|\Phi_i^D(\boldsymbol{\Theta})|^2)\\&+N^2\rho_D^2+N(2\rho_D+\epsilon_{D,i}+\epsilon_{D,k}+1)+\epsilon_{D,k}\epsilon_{D,i}|\overline{\mathbf{h}}_{rk}^H\overline{\mathbf{h}}_{ri}|^2\\&+2\rho_D\epsilon_{D,k}\epsilon_{D,i}\mathrm{Re}\Big\{\Big(\Phi_k^D(\boldsymbol{\Theta})\Big)^H\Phi_i^D(\boldsymbol{\Theta})\overline{\mathbf{h}}_{ri}^H\overline{\mathbf{h}}_{rk}\Big\}\Big)\end{aligned} \tag{39}$$

$$\begin{aligned}B_3 = {}& k_u(E_B B_1 + \sum_{i=1,i\neq k}^K E_B B_2 + \sigma^2 B_4)\\&+k_b(1+k_u)\Big(\mathbb{E}\{|\mathrm{f}_{k_q}|^4\}+\sum_{i=1,i\neq k}^K\mathbb{E}\{|\mathrm{f}_{i_q}|^2|\mathrm{f}_{k_q}|^2\}\Big)\end{aligned} \tag{40}$$

$$B_4 = \sum_{i=1}^K a_{D,i}(\epsilon_{D,k}\rho_D|\Phi_k^D(\boldsymbol{\Theta})|^2 + N\epsilon_{D,k} + N\rho_D + N) \tag{41}$$

From Equations (32) and (37), we can find that BS and users in RIS-aided FD systems with THWIs can scale down their transmit power by a factor of $1/M, M \to \infty$ or $1/Q, Q \to \infty$ while the UL or DL sum rate will converge to a non-zero value. In addition, by scaling down the transmit power, the effect of LI can be eliminated, but the effect of THWIs cannot be eliminated.

4 | PHASE SHIFT OPTIMIZATION

By observing Equations (15) and (24), we can find that the achievable rate only depend on the Rician factors, the NLoS components of channel and the path-loss coefficients. Thus, we can optimize the phase shifts only based on the statistical CSI. Specifically, we formulate the optimization problems as follows:

$$\max_{\boldsymbol{\Theta}} \sum_{k=1}^K R_k^{UL} \tag{42}$$

$$s.t. \quad \theta_n \in [0, 2\pi), \forall n. \tag{43}$$

$$\max_{\boldsymbol{\Theta}} \sum_{k=1}^K R_k^{DL} \tag{44}$$

$$s.t. \quad \theta_n \in [0, 2\pi), \forall n. \tag{45}$$

and

$$\max_{\boldsymbol{\Theta}} \sum_{k=1}^K \Big(R_k^{DL} + R_k^{UL}\Big) \tag{46}$$

$$s.t. \quad \theta_n \in [0, 2\pi), \forall n. \tag{47}$$

where the expressions of R_k^{UL} and R_k^{DL} are respectively given in Equations (15) and (24).

The rate expression is so complex that we cannot obtain a optimal solution by using the traditional optimization methods. Therefore, we adopt GA to solve the above optimization problems. The complexity of GA is proportional to tSN, where S represents the population size, t represents the number of iterations. The detailed steps of GA are as follows:

(1) Initialize population: Generate S individuals, and each individual has N randomly generated chromosomes corresponding to the phase shifts of RIS.

ALGORITHM 1 Mutation Algorithm

for $s = 1 : S_3$ **do**
  for $n = 1 : N$ **do**
    $r = \text{rand}(1)$
    if $r < 0.1$ **then**
      the n-th chromosome of the s-th parent mutates to $2\pi \times r$;
    end if
  end for
end for

ALGORITHM 2 Two Points Crossover Algorithm

Initialize $t = 1$
for $s = 1 : S_2$ **do**
  Select the t-th and the $(t+1)$-th parent from the $2S_2$ parents;
  Randomly create integers a, b from $[1, N-1]$, and $a < b$;
  Generate the s-th offspring and the s-th offspring = [t-th parent$(1 : a)$, $(t+1)$-th parent$(a+1 : b)$, t-th parent$(b+1 : N)$] ;
  $t = t + 2$
end for

(2) Evaluate fitness: We first calculate the fitness of each individual through objective function in Equation (42) or (44) in the current population. Then, we sort them in a descending order.
(3) Select elite: In the descending order, we select the top S_1 individuals as elites and pass them to the next generation.
(4) Mutate parents: In the descending order, we select the last S_3 individuals as parents and mutate them with probability 0.1 to generate S_3 offspring. The mutation method is shown in Algorithm 1.
(5) Cross parents: Firstly, we use stochastic universal sampling to create $2S_2$ parents from the remaining $S_2 = S - S_1 - S_3$ individuals. Then, we use two points crossover method to generate S_2 offspring from $2S_2$ parents. If $N \leq 2$, we adopt single point crossover method. The two points crossover method is shown in Algorithm 2.
Finally, we combine S_1 elites and $(S_2 + S_3)$ offspring to produce the next generation. GA will stop when the number of generations $> 100 \times N$. The chromosome of the individual with the highest fitness corresponds to the optimal phase shifts of RIS.

5 | SIMULATIONS RESULTS

Numerical simulations are provided for validating analytical results in this section. The simulation parameters are set as follows [16]: $K = 4, M = Q = 49,\ N = 49,\ k_b = k_u = 0.01$, $\sigma^2 = -104$ dBm, $p_k = 20$ dBm, $\forall k$, $p_B = 30$ dBm, $\alpha_i = \alpha_B = -114$ dBm. The AoA and the AoD are randomly generated from $[0, 2\pi]$. The Rician factors $\epsilon_{U,k} = \epsilon_{D,k} = 1, \forall k,\ \rho_U = \rho_D = 10$. We assume that the distance of RIS-BS is l_{rb} m and users are located on a semicircle centered at the RIS with a radius of l_{ur} m, where $l_{rb} = 1000$, $l_{ur} = 30$. Therefore, the path-loss coefficients are set as $\nu_U = \nu_D = 10^{-3} l_{rb}^{-2.5} \forall k$, $\mu_{U,k} = \mu_{D,k} = 10^{-3} l_{ur}^{-2}$.

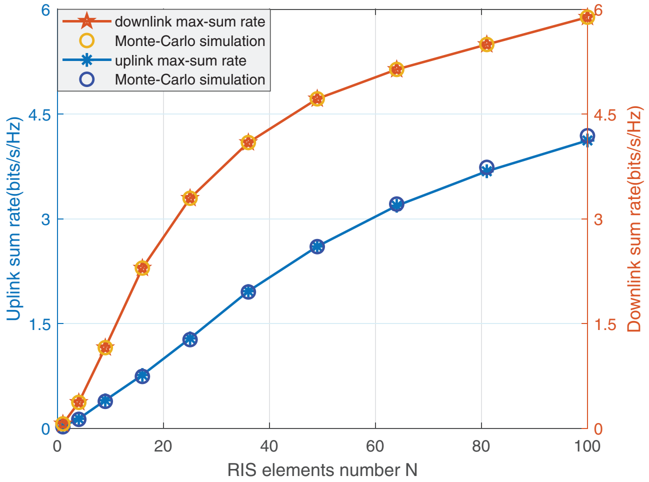

FIGURE 2 Max-sum rate versus N

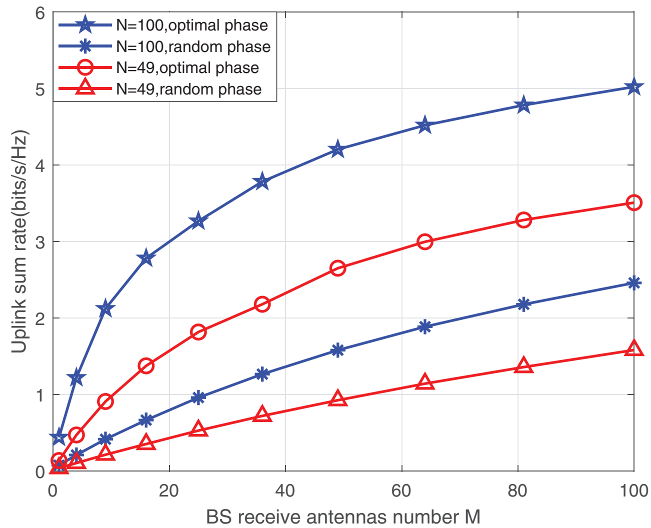

FIGURE 3 Uplink sum rate versus M

Figure 2 plots the UL and DL sum rate respectively achieved by the optimization problems in Equation (42) and (44). We obtain the Monte-Carlo (MC) simulation results through averaging over 10^4 random realizations. The MC simulation results agree with our derived results, which verify the accuracy of the derived expression. Figures 3 and 4 plot the achievable rate of different schemes. As the number of antennas at the BS increases, the performance of the optimal phase will be much better than that of the random phase. Especially, the system performance of "$N = 49$, optimal phase" is better than that of "$N = 100$, random phase". It shows the necessity of optimizing the phase shifts in RIS-aided FD mMIMO system with THWIs.

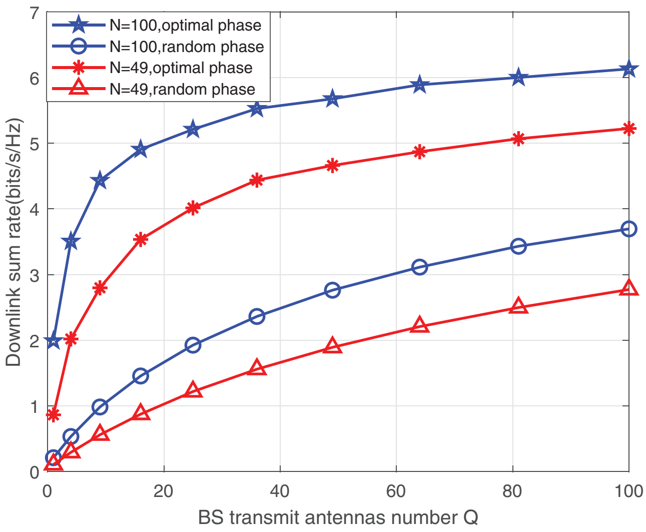

FIGURE 4 Downlink sum rate versus Q

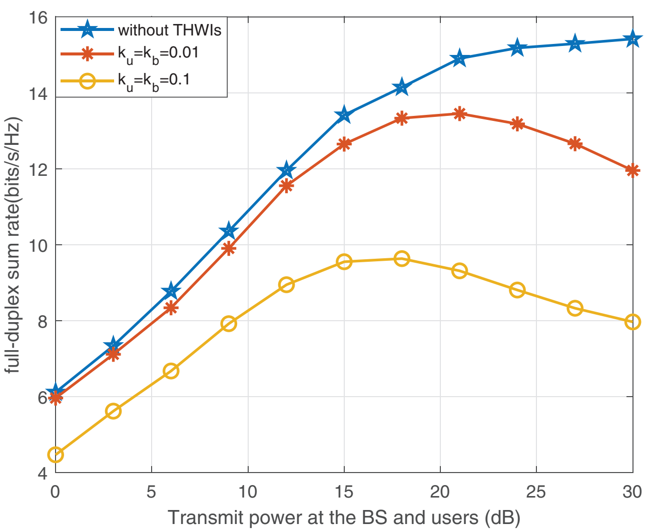

FIGURE 5 Full-duplex sum rate versus transmit power

Figure 5 shows the impact of THWIs on the system performance. When the transmit power increases to a certain extent, the system performance will degrade with the increase of transmit power. This is because the THWIs are closely coupled with the transmit signal, and this coupling will become stronger with the increase of THWIs coefficients k_u, k_b. Figure 6 plots FD sum rate. The horizontal comparison shows that we should increase the number of RIS's elements instead of increasing the number of transceiver antennas in the RIS-aided FD mMIMO system with THWIs. For example, the system performance of '$M = Q = 16, N = 100$' is similar to that of '$M = Q = 64, N = 49$' and even better than that of '$M = Q = 256, N = 16$'. This means that we can significantly reduce the number of transceiver antennas at the BS through appropriately increasing the elements of RIS.

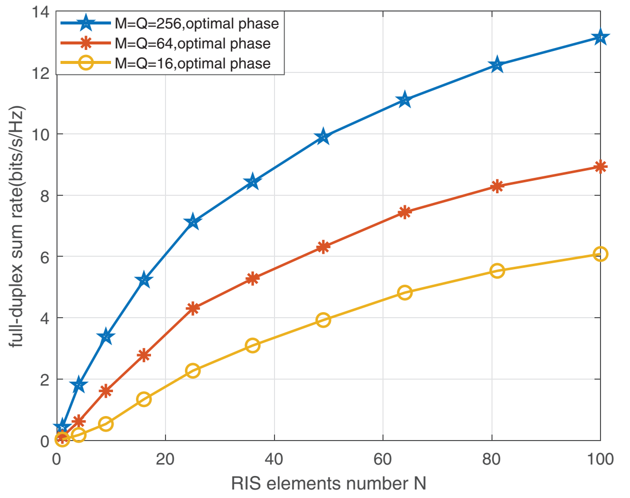

FIGURE 6 Full-duplex sum rate versus N

6 | CONCLUSION

In this paper, we investigated an RIS-aided FD mMIMO system with THWIs based on the two-timescale scheme. The closed-form expressions of UL and DL rate are derived. We verified the correctness of our derived expressions with MC simulation. Then, we optimized the phase shifts of RIS only based on statistical CSI. Finally, we analysed the system performance and found that it is more worthwhile to increase N than increase M in RIS-aided FD mMIMO system with THWIs. In addition, the scenario where each user has a large number of antennas [17] will be considered in our future work.

AUTHOR CONTRIBUTIONS

Jianxin Dai: Conceptualization, Resources, Supervision, Funding acquisition, writing - review and editing. Feng Zhu: Investigation, methodology, software, validation, writing - original draft. Cunhua Pan: Conceptualization, investigation, methodology, resources, writing - review and editing. Jiangzhou Wang: Resources, writing - review and editing.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as the data of this paper is the result of simulation and all the data are presented in the form of graphs inside the paper.

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APPENDIX A

A.1 | Derivation of $\mathbb{E}\{\boldsymbol{\Gamma}\}$

$$
\begin{aligned}
&\mathbb{E}\left\{\Gamma\right\}\\
&\quad=\sum_{i=1}^{K}\mathbb{E}\left\{\mathbf{h}_{ri}^{H}\boldsymbol{\Theta}\mathbf{H}_{br}\mathbf{H}_{br}^{H}\boldsymbol{\Theta}^{H}\mathbf{h}_{ri}\right\}\\
&\quad\overset{(o_1)}{=}\sum_{i=1}^{K}a_{D,i}\Big(\mathbb{E}\left\{\epsilon_{D,i}\rho_{D}\overline{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\overline{\mathbf{H}}_{br}\overline{\mathbf{H}}_{br}^{H}\boldsymbol{\Theta}^{H}\overline{\mathbf{h}}_{ri}\right\}\\
&\qquad+\mathbb{E}\left\{\epsilon_{D,i}\overline{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\widetilde{\mathbf{H}}_{br}\widetilde{\mathbf{H}}_{br}^{H}\boldsymbol{\Theta}^{H}\overline{\mathbf{h}}_{ri}\right\}\\
&\qquad+\mathbb{E}\left\{\rho_{D}\widetilde{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\overline{\mathbf{H}}_{br}\overline{\mathbf{H}}_{br}^{H}\boldsymbol{\Theta}^{H}\widetilde{\mathbf{h}}_{ri}\right\}\\
&\qquad+\mathbb{E}\left\{\widetilde{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\widetilde{\mathbf{H}}_{br}\widetilde{\mathbf{H}}_{br}^{H}\boldsymbol{\Theta}^{H}\widetilde{\mathbf{h}}_{ri}\right\}\Big)
\end{aligned}
\tag{A.1}
$$

where (o_1) is obtained by removing the zero terms and

$$
\begin{aligned}
&\mathbb{E}\left\{\epsilon_{D,i}\rho_{D}\overline{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\overline{\mathbf{H}}_{br}\overline{\mathbf{H}}_{br}^{H}\boldsymbol{\Theta}^{H}\overline{\mathbf{h}}_{ri}\right\}\\
&\quad=\epsilon_{D,i}\rho_{D}\mathbb{E}\Big\{\overline{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\mathbf{a}_{N}(\psi_{br}^{a},\psi_{br}^{e})\mathbf{a}_{Q}^{H}(\phi_{br}^{a},\phi_{br}^{e})\\
&\qquad\times\mathbf{a}_{Q}(\phi_{br}^{a},\phi_{br}^{e})\mathbf{a}_{N}^{H}(\psi_{br}^{a},\psi_{br}^{e})\boldsymbol{\Theta}^{H}\overline{\mathbf{h}}_{ri}\Big\}\\
&\quad=Q\epsilon_{D,i}\rho_{D}|\Phi_{i}^{D}(\boldsymbol{\Theta})|^{2}
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
&\mathbb{E}\left\{\epsilon_{D,i}\overline{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\widetilde{\mathbf{H}}_{br}\widetilde{\mathbf{H}}_{br}^{H}\boldsymbol{\Theta}^{H}\overline{\mathbf{h}}_{ri}\right\}\\
&\quad=Q\epsilon_{D,i}\mathbb{E}\left\{\mathbf{a}_{N}^{H}(\phi_{ri}^{a},\phi_{ri}^{e})\boldsymbol{\Theta}\boldsymbol{\Theta}^{H}\mathbf{a}_{N}(\phi_{ri}^{a},\phi_{ri}^{e})\right\}\\
&\quad=QN\epsilon_{D,i}
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
&\mathbb{E}\left\{\rho_{D}\widetilde{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\overline{\mathbf{H}}_{br}\overline{\mathbf{H}}_{br}^{H}\boldsymbol{\Theta}^{H}\widetilde{\mathbf{h}}_{ri}\right\}\\
&\quad=\rho_{D}\mathbb{E}\Big\{\widetilde{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\mathbf{a}_{N}(\psi_{br}^{a},\psi_{br}^{e})\mathbf{a}_{Q}^{H}(\phi_{br}^{a},\phi_{br}^{e})\\
&\qquad\times\mathbf{a}_{Q}(\phi_{br}^{a},\phi_{br}^{e})\mathbf{a}_{N}^{H}(\psi_{br}^{a},\psi_{br}^{e})\boldsymbol{\Theta}^{H}\widetilde{\mathbf{h}}_{ri}\Big\}\\
&\quad=Q\rho_{D}\mathbb{E}\left\{\mathbf{a}_{N}^{H}(\psi_{br}^{a},\psi_{br}^{e})\boldsymbol{\Theta}^{H}\widetilde{\mathbf{h}}_{ri}\widetilde{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\mathbf{a}_{N}(\psi_{br}^{a},\psi_{br}^{e})\right\}\\
&\quad=QN\rho_{D}
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
&\mathbb{E}\left\{\widetilde{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\widetilde{\mathbf{H}}_{br}\widetilde{\mathbf{H}}_{br}^{H}\boldsymbol{\Theta}^{H}\widetilde{\mathbf{h}}_{ri}\right\}\\
&\quad=Q\mathbb{E}\left\{\widetilde{\mathbf{h}}_{ri}^{H}\boldsymbol{\Theta}\boldsymbol{\Theta}^{H}\widetilde{\mathbf{h}}_{ri}\right\}\\
&\quad=QN
\end{aligned}
\tag{A.5}
$$

Substituting Equations (A.2)−(A.5) into Equation (A.1), we complete the derivation of $\mathbb{E}\{\Gamma\}$.

A.2 | Derivation of $\mathbb{E}^{\mathbf{LI}}_{D,k}$, $\mathbb{E}^{\mathbf{LI}}_{U,k}$, $\mathbb{E}^{\mathbf{noise}}_{D,k}$ and $\mathbb{E}^{\mathbf{noise}}_{U,k}$

$$\begin{aligned}\mathbb{E}^{\mathrm{LI}}_{D,k} &= \mathbb{E}\left\{\Gamma\sum_{i=1}^{K}|h_{i,k}|^2\right\}\\ &= \mathbb{E}\{\Gamma\}\mathbb{E}\left\{\sum_{i=1}^{K}|h_{i,k}|^2\right\}\\ &= \alpha_i\mathbb{E}\{\Gamma\}\end{aligned} \tag{A.6}$$

where $\mathbb{E}\{\Gamma\}$ has been derived. Substituting $\mathbb{E}\{\Gamma\}$ into Equation (A.6), we complete the derivation of $\mathbb{E}^{\mathrm{LI}}_{D,k}$.

$$\begin{aligned}\mathbb{E}^{\mathrm{LI}}_{U,k} &= \mathbb{E}\left\{|\mathbf{g}_k^H\mathbf{H}_B\mathbf{f}_i|^2\right\}\\ &= \mathbb{E}\left\{\mathbf{g}_k^H\mathbf{H}_B\mathbf{f}_i\mathbf{f}_i^H\mathbf{H}_B^H\mathbf{g}_k\right\}\\ &= \mathbb{E}\left\{\mathbf{g}_k^H\mathrm{Tr}\left\{\mathbf{f}_i\mathbf{f}_i^H\right\}\mathbf{I}_M\mathbf{g}_k\right\}\\ &= \mathbb{E}\left\{\mathbf{g}_k^H\mathbf{g}_k\right\}\mathbb{E}\left\{\mathbf{f}_i^H\mathbf{f}_i\right\}\end{aligned} \tag{A.7}$$

where $\mathbb{E}\{\mathbf{f}_i^H\mathbf{f}_i\}$ has been derived in $\mathbb{E}\{\Gamma\}$, and $\mathbb{E}\{\mathbf{g}_k^H\mathbf{g}_k\}$ is derived as follows:

$$\begin{aligned}&\mathbb{E}\left\{\mathbf{g}_k^H\mathbf{g}_k\right\}\\ &= \mathbb{E}\left\{\mathbf{h}_{kr}^H\mathbf{\Theta}^H\mathbf{H}_{rb}^H\mathbf{H}_{rb}\mathbf{\Theta}\mathbf{h}_{kr}\right\}\\ &= a_{U,k}\Big(\mathbb{E}\left\{\epsilon_{U,k}\rho_U\bar{\mathbf{h}}_{kr}^H\mathbf{\Theta}^H\bar{\mathbf{H}}_{rb}^H\bar{\mathbf{H}}_{rb}\mathbf{\Theta}\bar{\mathbf{h}}_{kr}\right\}\\ &\quad+\mathbb{E}\left\{\epsilon_{U,k}\bar{\mathbf{h}}_{kr}^H\mathbf{\Theta}^H\tilde{\mathbf{H}}_{rb}^H\tilde{\mathbf{H}}_{rb}\mathbf{\Theta}\bar{\mathbf{h}}_{kr}\right\}\\ &\quad+\mathbb{E}\left\{\rho_U\tilde{\mathbf{h}}_{rk}^H\mathbf{\Theta}^H\bar{\mathbf{H}}_{rb}^H\bar{\mathbf{H}}_{br}\mathbf{\Theta}\tilde{\mathbf{h}}_{kr}\right\}\\ &\quad+\mathbb{E}\left\{\tilde{\mathbf{h}}_{kr}^H\mathbf{\Theta}^H\tilde{\mathbf{H}}_{rb}^H\tilde{\mathbf{H}}_{rb}\mathbf{\Theta}\tilde{\mathbf{h}}_{kr}\right\}\Big)\\ &= a_{U,k}\left(\epsilon_{U,k}\rho_U|\Phi_k^U(\mathbf{\Theta})|^2+MN\epsilon_{U,k}+MN\rho_U+MN\right)\end{aligned} \tag{A.8}$$

Substituting $\mathbb{E}\{\mathbf{f}_i^H\mathbf{f}_i\}$ and Equation (A.8) into Equation (A.7), we complete the derivation of $\mathbb{E}^{\mathrm{LI}}_{U,k}$.

Based on Equations (A.1) and (A.8), we can easily complete the derivation of $\mathbb{E}^{\mathrm{noise}}_{D,k}$ and $\mathbb{E}^{\mathrm{noise}}_{U,k}$.

A.3 | Derivation of $\mathbb{E}^{\mathbf{THWIs}}_{U,k}$ and $\mathbb{E}^{\mathbf{THWIs}}_{D,k}$

$$\begin{aligned}\mathbb{E}^{\mathrm{THWIs}}_{U,k} &= \mathbb{E}\left\{\Gamma|\mathbf{g}_k^H(\mathbf{G}\mathbf{z}_{t,U}+\mathbf{H}_B\mathbf{z}_{t,B}+\mathbf{z}_{r,B})|^2\right\}\\ &\overset{(o_2)}{=} \mathbb{E}\left\{\Gamma|\mathbf{g}_k^H\mathbf{G}\mathbf{z}_{t,U}|^2\right\}+\mathbb{E}\left\{\Gamma|\mathbf{g}_k^H\mathbf{H}_B\mathbf{z}_{t,B}|^2\right\}\\ &\quad+\mathbb{E}\left\{\Gamma|\mathbf{g}_k^H\mathbf{z}_{r,B}|^2\right\}\\ &= k_u\Big(p_k\mathbb{E}^{\mathrm{signal}}_{U,k}+\sum_{i=1,i\neq k}^{K}p_i\mathbb{E}^{\mathrm{MUI}}_{U,k}\Big)+k_bp_B\mathbb{E}^{\mathrm{LI}}_{U,k}+\mathbb{E}\{\Gamma\}\\ &\quad\times k_bM\left(p_k\mathbb{E}\{|\mathrm{g}_{k_m}|^4\}+\sum_{i=1,i\neq k}^{K}p_i\mathbb{E}\left\{|\mathrm{g}_{i_m}|^2|\mathrm{g}_{k_m}|^2\right\}\right)\end{aligned} \tag{A.9}$$

where (o_2) is obtained by removing the zero terms and $\mathbb{E}\{|\mathrm{g}_{k_m}|^4\}$ has been derived in [15]. Therefore, we only need to derive $\mathbb{E}\{|\mathrm{g}_{i_m}|^2|\mathrm{g}_{k_m}|^2\}$.

We can express g_{k_m} in the form of Equation (A.10), where a_{Mm} and a_{Nn} are respectively the mth element of $\mathbf{a}_M$ and nth element of $\mathbf{a}_N$. Based on Equation (A.10), we can express $\mathbb{E}\{|\mathrm{g}_{i_m}|^2|\mathrm{g}_{k_m}|^2\}$ as Equation (A.11) and we will calculate the terms in Equation (A.11) one by one.

$$\begin{aligned}\mathrm{g}_{k_m} = \sqrt{\frac{\nu_U\mu_{U,k}}{(\rho_U+1)(\epsilon_{U,k}+1)}}\times\Bigg(&\underbrace{\sqrt{\rho_U\epsilon_{U,k}}\mathrm{a}_{Mm}\left(\phi^a_{rb},\phi^e_{rb}\right)\Phi_k^U(\mathbf{\Theta})}_{\mathrm{g}^1_{k_m}}\\ &+\underbrace{\sqrt{\rho_U}\mathrm{a}_{Mm}\left(\phi^a_{rb},\phi^e_{rb}\right)\sum_{n=1}^{N}\mathrm{a}^*_{Nn}\left(\psi^a_{rb},\psi^e_{rb}\right)e^{j\theta_n}\tilde{h}_{kr_n}}_{\mathrm{g}^2_{k_m}}\\ &+\underbrace{\sqrt{\epsilon_{U,k}}\sum_{n=1}^{N}[\tilde{\mathbf{H}}_{rb}]_{mn}e^{j\theta_n}\mathrm{a}_{Nn}\left(\psi^a_{kr},\psi^e_{kr}\right)}_{\mathrm{g}^3_{k_m}}\\ &+\underbrace{\sum_{n=1}^{N}[\tilde{\mathbf{H}}_{rb}]_{mn}e^{j\theta_n}\tilde{h}_{kr_n}}_{\mathrm{g}^4_{k_m}}\Bigg),\end{aligned} \tag{A.10}$$

$$\begin{aligned}&\mathbb{E}\left\{|\mathrm{g}_{i_m}|^2|\mathrm{g}_{k_m}|^2\right\}\\ &= a_{U,i}a_{U,k}\mathbb{E}\Bigg\{\left(\sum_{\omega=1}^{4}|\mathrm{g}^\omega_{k_m}|^2+2\sum_{\omega=1}^{3}\sum_{\psi=\omega+1}^{4}Re\left\{\mathrm{g}^\omega_{k_m}\left(\mathrm{g}^\psi_{k_m}\right)^*\right\}\right)\\ &\quad\times\left(\sum_{\omega=1}^{4}|\mathrm{g}^\omega_{i_m}|^2+2\sum_{\omega=1}^{3}\sum_{\psi=\omega+1}^{4}Re\left\{\mathrm{g}^\omega_{i_m}\left(\mathrm{g}^\psi_{i_m}\right)^*\right\}\right)\Bigg\}\\ &= a_{U,i}a_{U,k}\mathbb{E}\left\{\sum_{\omega=1}^{4}|\mathrm{g}^\omega_{k_m}|^2\sum_{\omega=1}^{4}|\mathrm{g}^\omega_{i_m}|^2\right\}\\ &\quad+4a_{U,i}a_{U,k}\Bigg(\mathbb{E}\left\{Re\left\{\mathrm{g}^1_{k_m}\left(\mathrm{g}^3_{k_m}\right)^*\right\}Re\left\{\mathrm{g}^1_{i_m}\left(\mathrm{g}^3_{i_m}\right)^*\right\}\right\}\\ &\quad+\mathbb{E}\left\{Re\left\{\mathrm{g}^1_{k_m}\left(\mathrm{g}^3_{k_m}\right)^*\right\}Re\left\{\mathrm{g}^2_{i_m}\left(\mathrm{g}^4_{i_m}\right)^*\right\}\right\}\\ &\quad+\mathbb{E}\left\{Re\left\{\mathrm{g}^2_{k_m}\left(\mathrm{g}^4_{k_m}\right)^*\right\}Re\left\{\mathrm{g}^1_{i_m}\left(\mathrm{g}^3_{i_m}\right)^*\right\}\right\}\\ &\quad+\mathbb{E}\left\{Re\left\{\mathrm{g}^2_{k_m}\left(\mathrm{g}^4_{k_m}\right)^*\right\}Re\left\{\mathrm{g}^2_{i_m}\left(\mathrm{g}^4_{i_m}\right)^*\right\}\right\}\Bigg).\end{aligned} \tag{A.11}$$

The first one is

$$\mathbb{E}\left\{\sum_{\omega=1}^{4}\left|g_{k_m}^{\omega}\right|^2\sum_{\omega=1}^{4}\left|g_{i_m}^{\omega}\right|^2\right\}$$
$$=\rho_U\epsilon_{U,k}\left(\rho_U\epsilon_{U,i}+\rho_U N+\epsilon_{U,i}N+1\right)$$
$$+\rho_U N\left(\rho_U\epsilon_{U,i}+N+\rho_U N+\epsilon_{U,i}N\right)$$
$$+\epsilon_{U,k}N\left(\rho_U\epsilon_{U,k}+\rho_U N+\epsilon_{U,i}(N+1)+N\right)$$
$$+N\left(\rho_U\epsilon_{U,i}+\rho_U N+\epsilon_{U,i}(N+1)+N+1\right). \tag{A.12}$$

The second one is

$$\mathbb{E}\left\{Re\left\{g_{k_m}^1\left(g_{k_m}^3\right)^*\right\}Re\left\{g_{i_m}^1\left(g_{i_m}^3\right)^*\right\}\right\}$$
$$=\mathbb{E}\left\{\rho_U\epsilon_{U,k}\epsilon_{U,i}\mathrm{Re}\left\{a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\Phi_k^U(\mathbf{\Theta})\right.\right.$$
$$\left.\times\sum_{n=1}^{N}\left[\widetilde{\mathbf{H}}_{rb}\right]_{mn}^{*}e^{-j\theta_n}a_{Nn}^{*}(\psi_{kr}^a,\psi_{kr}^e)\right\} \tag{A.13}$$
$$\times\mathrm{Re}\left\{a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\Phi_i^U(\mathbf{\Theta})\right.$$
$$\left.\left.\times\sum_{n=1}^{N}\left[\widetilde{\mathbf{H}}_{rb}\right]_{mn}^{*}e^{-j\theta_n}a_{Nn}^{*}(\psi_{ir}^a,\psi_{ir}^e)\right\}\right\}$$

Assume that

$$a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\Phi_k^U(\mathbf{\Theta})e^{-j\theta_n}a_{Nn}^{*}(\psi_{kr}^a,\psi_{kr}^e)=\sigma_c^{kn}+j\sigma_s^{kn},$$
$$a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\Phi_i^U(\mathbf{\Theta})e^{-j\theta_n}a_{Nn}^{*}(\psi_{ir}^a,\psi_{ir}^e)=\sigma_c^{in}+j\sigma_s^{in}, \tag{A.14}$$
$$\left[\widetilde{\mathbf{H}}_{rb}\right]_{mn}=s_{mn}+jt_{mn},$$

and after some algebraic simplifications, we can obtain

$$\mathbb{E}\left\{\mathrm{Re}\left\{g_{k_m}^1(g_{k_m}^3)^*\right\}\mathrm{Re}\left\{g_{i_m}^1(g_{i_m}^3)^*\right\}\right\}$$
$$=\rho_U\epsilon_{U,k}\epsilon_{U,i}\mathbb{E}\left\{\sum_{n=1}^{N}\sigma_c^{kn}\sigma_c^{in}s_{mn}^2+\sigma_s^{kn}\sigma_s^{in}t_{mn}^2\right\}$$
$$=\frac{\rho_U\epsilon_{U,k}\epsilon_{U,i}}{2}\mathrm{Re}\left\{\sum_{n=1}^{N}a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\Phi_k^U(\mathbf{\Theta})e^{-j\theta_n}a_{Nn}^{*}(\psi_{kr}^a,\psi_{kr}^e)\right.$$
$$\left.\times a_{Nn}(\psi_{ir}^a,\psi_{ir}^e)e^{j\theta_n}\left(\Phi_i^U(\mathbf{\Theta})\right)^H a_{Mm}^{*}(\phi_{rb}^a,\phi_{rb}^e)\right\}$$
$$=\frac{\rho_U\epsilon_{U,k}\epsilon_{U,i}}{2}\mathrm{Re}\left\{\left(\Phi_i^U(\mathbf{\Theta})\right)^H\Phi_k^U(\mathbf{\Theta})\overline{\mathbf{h}}_{kr}^H\overline{\mathbf{h}}_{ir}\right\} \tag{A.15}$$

Likewise, the third one is

$$\mathbb{E}\left\{\mathrm{Re}\left\{g_{k_m}^1(g_{k_m}^3)^*\right\}\mathrm{Re}\left\{g_{i_m}^2(g_{i_m}^4)^*\right\}\right\}$$
$$=\rho_U\epsilon_{U,k}\mathbb{E}\left\{\mathrm{Re}\left\{a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\Phi_k^U(\mathbf{\Theta})\right.\right.$$
$$\left.\times\sum_{n=1}^{N}\left[\widetilde{\mathbf{H}}_{rb}\right]_{mn}^{*}e^{-j\theta_n}a_{Nn}^{*}(\psi_{kr}^a,\psi_{kr}^e)\right\}$$
$$\times\mathrm{Re}\left\{a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\sum_{n=1}^{N}a_{Nn}^{*}(\psi_{rb}^a,\psi_{rb}^e)e^{j\theta_n}\widetilde{h}_{ir_n}\right. \tag{A.16}$$
$$\left.\left.\times\sum_{n=1}^{N}\left[\widetilde{\mathbf{H}}_{rb}\right]_{mn}^{*}e^{-j\theta_n}\widetilde{h}_{ir_n}^{*}\right\}\right\}$$
$$=\frac{\rho_U\epsilon_{U,k}}{2}\mathrm{Re}\left\{\sum_{n=1}^{N}a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\Phi_k^U(\mathbf{\Theta})e^{-j\theta_n}a_{Nn}^{*}(\psi_{kr}^a,\psi_{kr}^e)\right.$$
$$\left.\times a_{Nn}(\psi_{rb}^a,\psi_{rb}^e)a_{Mm}^{*}(\phi_{rb}^a,\phi_{rb}^e)\right\}$$
$$=\frac{\rho_U\epsilon_{U,k}}{2}|\Phi_k^U(\mathbf{\Theta})|^2$$

The fourth one can be easily obtained

$$\mathbb{E}\left\{\mathrm{Re}\left\{g_{k_m}^2(g_{k_m}^4)^*\right\}\mathrm{Re}\left\{g_{i_m}^1(g_{i_m}^3)^*\right\}\right\}=\frac{\rho_U\epsilon_{U,i}}{2}|\Phi_i^U(\mathbf{\Theta})|^2 \tag{A.17}$$

The last one can be derived as follows

$$\mathbb{E}\left\{\mathrm{Re}\left\{g_{k_m}^2(g_{k_m}^4)^*\right\}\mathrm{Re}\left\{g_{i_m}^2(g_{i_m}^4)^*\right\}\right\}$$
$$=\rho_U\mathbb{E}\left\{\mathrm{Re}\left\{a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\sum_{n=1}^{N}a_{Nn}^{*}(\psi_{rb}^a,\psi_{rb}^e)e^{j\theta_n}\widetilde{h}_{kr_n}\right.\right.$$
$$\left.\times\sum_{n=1}^{N}\left[\widetilde{\mathbf{H}}_{rb}\right]_{mn}^{*}e^{-j\theta_n}\widetilde{h}_{kr_n}^{*}\right\}$$
$$\times\mathrm{Re}\left\{a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)\sum_{n=1}^{N}a_{Nn}^{*}(\psi_{rb}^a,\psi_{rb}^e)e^{j\theta_n}\widetilde{h}_{ir_n}\right.$$
$$\left.\left.\times\sum_{n=1}^{N}\left[\widetilde{\mathbf{H}}_{rb}\right]_{mn}^{*}e^{-j\theta_n}\widetilde{h}_{ir_n}^{*}\right\}\right\}$$
$$=\frac{\rho_U}{2}\mathrm{Re}\left\{\sum_{n=1}^{N}a_{Mm}(\phi_{rb}^a,\phi_{rb}^e)a_{Nn}^{*}(\phi_{rb}^a,\phi_{rb}^e)\right.$$
$$\left.\times a_{Nn}(\phi_{rb}^a,\phi_{rb}^e)a_{Mm}^{*}(\phi_{rb}^a,\phi_{rb}^e)\right\}$$
$$=\frac{\rho_U N}{2} \tag{A.18}$$

Substituting Equations (A.12)–(A.18) into Equation (A.11), we complete the derivation of $\mathbb{E}\{|g_{i_m}|^2|g_{k_m}|^2\}$.

$$\mathbb{E}_{D,k}^{\mathrm{THWIs}} = \mathbb{E}\left\{\Gamma\left|\mathbf{f}_k^H\mathbf{z}_{t,B} + z_{r,k} + \sum_{i=1}^{K} h_{i,k}z_{t,i}\right|^2\right\}$$

$$\overset{(o_3)}{=} \mathbb{E}\left\{\Gamma\left|\mathbf{f}_k^H\mathbf{z}_{t,B}\right|^2\right\} + \mathbb{E}\left\{\Gamma\sum_{i=1}^{K}\left|h_{i,k}z_{t,i}\right|^2\right\} + \mathbb{E}\left\{\Gamma\left|z_{r,k}\right|^2\right\}$$

$$= k_u\left(p_B\mathbb{E}_{D,k}^{\mathrm{signal}} + \sum_{i=1}^{K} p_B\mathbb{E}_{D,k}^{\mathrm{MUI}} + \sum_{i=1}^{K} p_i\mathbb{E}_{D,k}^{\mathrm{LI}} + \sigma^2\mathbb{E}_{D,k}^{\mathrm{noise}}\right)$$

$$+ (1+k_u)\left(k_u\sum_{i=1}^{K} p_i\alpha_i\mathbb{E}\{\Gamma\} + k_b Q\left(\mathbb{E}\{|f_{k_q}|^4\}\right.\right.$$

$$\left.\left.+ \sum_{i=1, i\neq k}^{K}\mathbb{E}\left\{|f_{i_q}|^2|f_{k_q}|^2\right\}\right)\right) \quad \text{(A.19)}$$

where (o_3) is obtained by removing the zero terms. The derivation of $\mathbb{E}\{|f_{k_q}|^4\}$ and $\mathbb{E}\{|f_{i_q}|^2|f_{i_q}|^2\}$ can refer to $\mathbb{E}\{|g_{k_m}|^4\}$ and $\mathbb{E}\{|g_{i_m}|^2|g_{k_m}|^2\}$, respectively.