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Innovative Applications of O.R.

Bidding for an optimal portfolio of keywords in sponsored search advertising: From generic to branded keywords

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ABSTRACT

With the increasing prominence of digital media, retailers attempt to attract consumers to their websites by investing in sponsored search advertising. However, due to stiff competition among retailers, sponsored search advertising can be expensive. This paper develops a multi-period, dynamic programming model that provides a retailer with an optimal portfolio of generic and branded bids. We model two critical aspects of consumer search behavior: (i) the spillover effect of generic searches leading to branded search arrivals in subsequent periods and (ii) the memory effect that leads to a decline of consumer awareness of a brand over time. We find that the retailer can effectively shuffle his investments on generic and branded keywords depending on several consumer parameters, e.g., awareness level, brand retention and reservation price variances. We develop a bidding policy framework to highlight the shift in bid shares from generic to branded at different levels of consumer awareness. We find that harnessing the benefits of spillover from generic to branded keywords allows the retailer to save on generic bids at higher awareness and retention levels and lower variance in consumers' reservation prices. Further, we extend our model to different consumer purchase situations/ product classifications, viz., Convenience, Shopping and Specialty purchasing. Our analysis suggests prevalence of generic bids for certain purchase/product situations, whereas branded bids remain salient in other situations.

1. Introduction

We have witnessed a phenomenal growth in digital advertising largely aided by the increasing access to cheaper internet among users worldwide. Contrary to earlier expectations, the market for digital advertising continued to grow in 2020 and in 2021 in spite of the pandemic, by 15% to USD378 billion in 2020 and then by 25% to USD491 billion in 2021, and is expected to reach approximately USD565 billion in 2022 (Statista, 2021). While display advertising still an important component of entire digital advertising expenditure with 40% share, search advertising is experiencing a strong growth, at almost 36% (eMarketer, 2021; Statista, 2021). In spite of early skepticism around the efficacy of digital advertising (Lee & Cho, 2020), digital advertising has grown over the past decade mainly owing to the ability to target individual consumers and clear metrics of measurement of the advertising dollars spent (Abhishek & Hosanagar, 2013). Since consumers actively search for product and brand related information through search engines, retailers try to ensure that their brands appear as search results in these product/brand related searches. Retailers bid for keywords, aiming to occupy top positions in search results and opt between various advertising models, depending on their specific preferences and marketing goals.

Search advertising essentially provides a two-sided platform, where consumers search for product and brand related information, while retailers and manufacturers are trying to showcase their offerings to the consumers (Varian, 2007). Retailers bid for keywords, which the consumers might use to search- and the highest bidders' (retailers) sites would appear as the top search results (Ghose & Yang, 2009). Bidding for keywords is one of the most critical decisions for the retailer as far as promotion is concerned. Search Engines (SE) offer a range of bidding options for retailers; Cost-Per-(*Mille*) Impressions (i.e., *CPM*), Cost-Per-Click (*CPC*), or, Cost-Per-Acquisition (*CPA*) depending on the business

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model and promotional objectives of the retailer and consumer's decision making stages (Jerath, Ma, & Park, 2014).

As mentioned earlier, the choice of keywords will also depend on the stage of decision-making process for their target consumers; e.g., consumers at their early stage of decision making process are expected to initiate a search with product/category-specific keywords, or, 'generic' keywords. In case of consumers, who have already evaluated multiple brand offerings under a product category, and at an advanced stage of decision making, would be interested in brand related searches ('branded' keywords). For example, consumers considering the purchase of 'jackets' for the winter would ideally initiate their search with the generic keywords,"*jacket/jackets*". Consumers click on the links and browse some of the options for information on the range of features, products, and brands. However, this search could merely be intended towards information gathering, and may not necessarily lead to a purchase. However, consumers, when they have converged on the choice of a particular product quality, or a brand, could return with a more specific search, say, 'woolen jackets', or 'Nike jackets', rather than merely using 'jackets' as their keyword to search for. These specific, brand-directed search are considered as 'branded keyword search'.

Since the investment in search engine advertising can be quite substantial for retailers, they need to be judicious regarding the choice, and nature of keywords. The sequential nature of consumer search shifting from generic to branded keyword search depends on an optimal investment in generic keywords to ensure a lasting impression on the consumers' minds, which can be followed up with investments in branded keywords. Studies (e.g., Rutz & Bucklin, 2011) suggests that branded keywords are usually less expensive compared to generic keywords, which are more competitive.

1.1. Research context and contributions

Consumers' using branded keywords to choose their preferred brands essentially suggests that they retain information from their previous exposures to the brands through generic keyword searches. Consumers' awareness of brands offered in a particular category of products results from a generic search and this awareness and preference carries over to later periods. Several researchers (Aravindakshan & Naik, 2015; Nerlove & Arrow, 1962) have pointed out that awareness declines over time; and as Wyer & Srull (1986) suggests, consumers would tend to remember recent experiences, which translates into last visited sites, according to our research context. The retailer can potentially capitalize on this retention by investing on a mix of generic and branded keywords, rather than focusing on simply one set of keywords. It is well understood that generic keywords would invite more aggressive bidding, and therefore, might be a more expensive proposition as compared to branded keywords (Du, Su, Zhang, & Zheng, 2017), which would have significantly less number of interested parties. Therefore, a portfolio arising out of an optimal mix of generic and branded keywords would potentially lead to significant savings and more efficient allocation of the promotional resources for the retailer.

In this paper, we develop a multi-period dynamic programming model for the retailer to create a portfolio of generic and branded keywords depending on consumers' capability to retain product and brand related awareness over time. The spillover of the awareness to the subsequent periods results in significant savings for the retailer. We find that the retailer can effectively shuffle his investments on generic and branded keywords depending on several consumer parameters, e.g. ability to retain awareness and consumers' reservation price variances. The numerical analysis suggests that there could be significant savings as far as advertising expenditures are concerned by relying on branded keyword search for certain groups of consumers, rather than investing on the more expensive generic keywords. We also develop a bidding policy framework to highlight the shift in bid shares for different levels of awareness taking into account different consumer characteristics. Harnessing the benefits of spillover from generic to branded keywords allows retailers to save on generic bids at higher awareness levels without losing out on brand visibility. The results from the numerical analyses show significant lowering of investment in generic bids for consumers with high levels of retention. Interestingly, we find that retailers can significantly improve their advertisement *ROI* by consciously avoiding consumers with higher reservation price variances since these consumers would always vacillate between a larger set of brands, therefore, spending more on such consumers may not essentially lead them to purchases. We extend these findings to select product categories depending on purchase situation and arrive at optimal portfolio possibilities. The model suggests that investment in branded keywords needs to be consistently maintained across the awareness levels, to ensure conversions happen.

Our model formulation is guided by the seminal work of Rutz & Bucklin (2011) who established empirically that generic bids positively impact branded search activity, and later research by Du et al. (2017), who found that using multiple keywords like generic and branded in a sponsored search ad campaign lead to higher returns. Our paper contributes to this literature through the following key aspects:

- Provides a broad, analytical framework for the optimal portfolio of generic and branded bids, which yields maximum profits for the retailer. Although few researchers have highlighted the benefits by testing the phenomenon empirically, according to our understanding, this is the first attempt in developing an analytical rationale in a multi-period stochastic setting.
- The study identifies and incorporates memory effect and consumers' choice variances as key determinants in retailers' bidding strategy; and derives the optimal portfolio of generic and branded bids.
- Also explores the generic and branded bids for select product categories according to consumers' decision parameters in terms low/high willingness-to-pay and variance in prices. We find clear evidence that consumers' decision parameters have significant effect on retailers' optimal bidding choices. Our results show that the relative importance of branded bids is more evident for customers, who are high-end shoppers as well as *Specialty* shoppers. Whereas for *Convenience* shoppers, retailers would need to focus more on generic bids even at moderate to high awareness levels.

The manuscript is organized in this manner; the following section contains a brief coverage of the extant literature in the areas of bidding policies for sponsored search, advertising spillovers and memory effects. Then we present the models for generic and branded search and the spillover effects followed by the numerical analyses. We conclude the paper by presenting the managerial insights derived from our model.

2. Literature review

The relevant literature for this study could be segregated under the following broad headings: (i) Bidding Policies in Search advertising, (ii) Advertising spillovers and (iii) Memory Effects in Sponsored search advertising. While the literature exploring the dynamics of online advertising space, especially search advertising and spillover from other online media, is quite extensive, the same is not for memory effects across various types of search. We present a brief overview of each of these streams in this section.

2.1. Bidding policies in search advertising

The sponsored search advertising space is greatly indebted to the seminal work by several prominent (notably, Varian, 2007) researchers in auction theory. Empirical studies in the area of Search Engine Marketing (SEM) (e.g., Ghose & Yang, 2009; Narayanan & Kalyanam, 2015) suggest that consumers have a greater preference to click results, which appear higher on the search rank and therefore, advertisers would ideally want to occupy higher ranks to ensure visibility (and, subsequent visits by prospective consumers). While both works focus on advertisers' optimal Cost-Per-Click (*CPC*), Narayanan & Kalyanam (2015) extend the effect of position on Click-Through-Rates (*CTR*s) and resultant sales of the online retailer. Shin (2015) extends this stream of work by presenting an interesting work for a retailer on a budget could find it optimal to bid lower and ensure an extended visibility over a longer period to ensure recall. While advertising ensures visibility to an extent, it does not necessarily guarantee a sale, which can depend on the price posted on the website, among other variables. Ye, Aydin, & Hu (2015) explore an on-off bidding policy where the retailer uses both price and bid as levers to increase sales. Assuming the click/conversion rate probabilities to be a function of bid, an algorithm is developed to yield the optimal bids and prices. The present study follows similar logic as Ye et al. (2015), where consumer's reservation price is considered for the conversion of a sale.

Most studies in the sponsored search advertising space consider advertising investments to be 'episodic'; the investment loses its value if no transaction takes place, which is also highlighted by Katona & Sarvary (2010). Their studies tend to overlook the possibility of a spillover effect, or a memory effect, where current advertising exposure may not immediately yield results, but could lead consumers to a future purchase. In this paper, we incorporate the spillover effect of exposure and also account for consumers' decay in recall to plan for the retailer's optimal bidding portfolio of keywords.

2.2. Generic and branded keywords

The work by Kireyev, Pauwels, & Gupta (2016) suggest that consumers who engage in search on a publisher's site are often at different stages of the decision-funnel. While some of the consumers could be merely gathering information on a product category, others could be comparing brands, and while others could be about to close a purchase. Other researchers (Moe, 2003; Moe & Fader, 2004; Montgomery, Hosanagar, Krishnan, & Clay, 2004) classify consumers into two types: buyers and information seekers. A study by Im, Jun, Oh, & Jeong (2016) consider another group of buyers: 'Deal Seekers'. Consumers cannot be expected to instantaneously convert to a purchase after being exposed to a site. According to a study by Agarwal, Hosanagar, & Smith (2011), users' purchase intent maybe gauged by the choice of keywords in the search; use of more specific keywords, or directed search result in higher probability of purchase (Montgomery et al., 2004). Yoo (2014) examined the effects of ranks in search lists across well-known and relatively unknown brands. It suggests that top ranked keyword search listings generated greater recognition and more favorable brand evaluations than the ones ranked lower than well-known brands. Moe & Fader (2004) attempted to predict the probability of a purchase based on an observed history of related purchases and site visits. The model attempted to account for both categories of consumers; i.e. buyers and information seekers. The work by Rutz & Bucklin (2011) highlights how consumers would initially engage in generic (possibly, product category related) searches, and then, move towards branded keyword searches in subsequent periods as they progress through the decision-making funnel.

Given a competitive market scenario, branded keywords are not only critical for the focal brands, but also for the competitive brands, which can ensure that by bidding on some specific competitor branded keywords, they can ensure their visibility in the search results (Desai, Shin, & Staelin, 2014). Simonov, Nosko, & Rao (2018) also highlight the threat to reputed brands when faced with the prospect of 'poaching' of keywords by competing brands. The study suggests 'defensive advertising' to protect branded keywords provides strong justification. Our paper essentially suggests that an optimal mix consisting both branded, as well as generic keywords is critical from a strategic advertising investment viewpoint.

2.3. Measuring online advertising effectiveness- and spillovers

One of the greatest benefits of online advertising is its measurability; however, retailers can assign revenues from online advertising expenditures, often it might be challenging to allocate budgets for each of the advertising elements. A stream of research studies have since studied the complimentary effects of television (offline) as well as banner advertisements (online) on search advertising- whether consumers' choice of keywords have been influenced by their exposure to these offline and online promotional elements (Joo, Wilbur, Cowgill, & Zhu, 2014; Joo, Wilbur, & Zhu, 2016; Lewis & Nguyen, 2015; Lobschat, Osinga, & Reinartz, 2017). Li & Kannan (2014) underline the importance of attributing the sales to the specific channel through which the sale has been made. They use the estimated carryover and spill-over effects to analyze consumer's consideration of online channels at different stages in the purchase process. Yang & Ghose (2010) analyze the interdependence and the extent of spillover between sponsored listings and organic listings and demonstrated that a combination of organic and paid listings yielded higher clicks/conversions than with solely organic listings. Lewis & Nguyen (2015) observe the complementarity between display (online) and search advertisements; their study indicated that display advertisements can often prime potential consumers about brands, which increase instances of branded keyword searches. Again, depending on consumer readiness (and status in decision-funnel) offline purchase incidences are often positively impacted due to cross-campaign effects. An earlier work by Joo et al. (2016) also suggests that banner advertising tends to increase instances of branded keyword search.

As advertisers plan to optimize their promotional budgets allocated for search advertising, the retailer's decision making would involve an optimal choice of generic and branded keywords. Rutz & Bucklin (2011) explore the nature of shift in search from generic to branded keywords during the consumers' choice process. The present study adopts the study by Rutz & Bucklin (2011) to explain the consumers' search logic. All these models discuss the possibility of a spill-over effect in various contexts. In our paper, we consider the context of a consumer visiting the website in one time period, remembering and returning to the website in a subsequent time period to make the purchase. Lambrecht & Tucker (2013) highlight that advertisers need to focus on a mix of generic and detailed advertisements for the consumer to improve the click rate on search results. Users who visit several review websites tend to possess narrowly construed preferences and detailed advertisements would lead to visits but for those with broadly construed preferences, generic advertisements are found to have a better impact. These advertisements entice the users to visit their respective websites by displaying customized advertisements alongside their browsing activity. The current model suggests that consumers resort to branded keywords search by relying on their memory of past exposures (display advertisements, or more importantly, previous generic keyword searches).

2.4. Memory effects of advertising

One of the earliest and most impactful models illustrating the memory effect was developed by Nerlove & Arrow (1962); widely known as the 'N-A Model'. This model uses the concept of a 'leaky-bucket' to explain the decay effect of advertising, which sets in instantaneously, post exposure. The model employs a linear decay factor and devises the optimal advertising policies. Several models have been developed considering the decay to commence instantly (Naik & Raman, 2003; Srinivasan, Vanhuele, & Pauwels, 2010). Aravindakshan & Naik (2015) explore the possibility of further developing the memory models by delayed differential equations to formulate an optimal advertising policy where memory decay is 'delayed'. The concept of memory decay, is however, not unique; Wyer & Srull (1986) explore the phenomenon of decay, in case consumers are exposed to large quantity of advertising stimuli. Their findings were intuitive; as the lag between exposure and purchase increases, the retention of the advertisement by user also decreases due to recency effect, which is corroborated by the works of Keller (1987). Keller (1987) establish that consumer memory for advertising is affected by the number of competing advertisements owing to the recency effects. Further, Kent & Allen (1994) conducted a similar experiment by varying the brand familiarity and capturing its impact on the memory of the consumers. The findings suggest that established brands enjoy greater memory-based recall even in the face of competing advertisements. Mahajan & Muller (1986) model the evolution of awareness as an extension to the N-A model as a function of current advertising level and the accumulated awareness till date. In the online context, Katona & Sarvary (2010) highlight that advertisers cannot treat sponsored search efforts as a 'one-off' investment, rather, suggest there would be a lagged component; and therefore, suggests that bidding strategies should be dynamic rather than optimized for a single-period. We have already mentioned the important contribution from Rutz & Bucklin (2011), where they investigate the spillover from generic to branded keywords. We attempt to build on Rutz & Bucklin (2011) model to capture this effect in our model. Based on our understanding, our work attempts to cover the following areas: (*i*) implications of potential spillover of advertising efforts for retailers in future periods in the area of sponsored search advertising, (*ii*) impact of consumer characteristics on bidding strategies, and finally, (*iii*) developing a 'demand-side' driven mechanism for optimal bid allocation across generic and branded keywords under spillover conditions.

3. Model

In our model, a retailer uses generic and branded keywords to generate traffic to his website. The consumer search journey is based on the work of Rutz & Bucklin (2011), where they suggest that generic searches for a particular category impacts branded search arrivals in later periods. Considering the consumer decision-making funnel, a generic search follows a consumers' awareness of a product category. This is followed by the preparation of the consideration set, which narrows the preferences to a few brands following a visit to each of the brands appearing on the search. It is expected that subsequently, consumers will focus on branded search to close their purchases.

We assume that consumers have a residual level of product and brand level awareness given by N_t, where t is number of periods left in the selling horizon for the retailer. At this point, only generic exposures are responsible for the increase in level of awareness of consumers. N_{gt} represents the awareness created through generic search during that particular time period. Branded exposures apply to those consumers who are already aware of the product category (through previous generic search). Awareness (akin to memory) decays with time, with a periodic discount rate, δ[1], then, the evolution of awareness with time is shown below in Fig. 1. For our paper, we have used the terms,'recall' and 'retention' interchangeably throughout.

A profit-maximizing retailer needs to choose an optimal bid portfolio of both generic and branded keywords based on the prevailing levels of consumer awareness. In this paper, we develop a stochastic dynamic programming model for a retailer employing the Cost-per-Click (*CPC*) bidding option,which provides the optimal bid for generic keyword(s) and branded keyword(s) in a given time period over a finite selling horizon. Price is considered as an exogenous parameter to our model, which is determined based on market forces.

There are three possible scenarios that arise with the usage of generic and branded keywords:

- Consumer arrives through a generic keyword search, e.g. "jackets".
- Consumer arrives through a branded keyword search, i.e. with prior brand knowledge, e.g. "Nike+jackets"
- Consumer does not proceed to purchase through generic search, however, decides to return through branded search, e.g. generic keyword search "Jackets", followed by a return search using branded keyword search e.g., "Nike+jackets".

We evaluate the expected payoffs in each of these scenarios to derive the optimal bid strategy. For notional simplicity, we drop the subscript 't'. The following table provides the key to the symbols used in the model (please refer Table 1).

3.1. Arrivals through generic search

Most online retailers extensively utilize the services of search-engines. Retailers stake their claim by bidding on a keyword that is expected to be used by consumers while searching for products. When consumers type in a keyword for searching, the search engines showcase the 'search results'. Qualitative aspects of their sites being similar, it is expected that retailers who bid higher amounts for these keywords would appear higher in the search results- providing them greater visibility. The appearance of a retailer on this search result is an *Impression*. For our model, the retailer has choices in terms of bidding for a generic (product-category) keyword (bid b_g), a branded (focal brand) keyword b_b, and a combination of both. Bids for generic keywords are usually higher than branded keywords as several retailers would be competing against each other (Abhishek & Hosanagar, 2013; Ghose & Yang, 2009; Rutz & Bucklin, 2011), where specific brand level keywords would have much less competition.

When consumers view the search results on the page (i.e. *impression*), they are expected to 'click' on the retailer's link that grabs their attention. Extant literature (Johansson, 1979; Little, 1979; Villas-Boas, 1993) models the 'click' and subsequent arrival at the retailer's site as a *Poisson* arrival process that emulates an *S-Shaped Curve* that increases in the value of the bid. An *S-Shaped* curve is defined by the parametric considerations given by $[\alpha, \beta]$. Assuming that the consumer enters a generic keyword search, then the probability $\lambda_g(b_g)$ of clicking on the retailer's link would be given by:

$$\lambda_g(b_g) = \frac{\lambda_{g\infty} + \lambda_{g0}.e^{\beta_g - \alpha_g b_g}}{1 + e^{\beta_g - \alpha_g b_g}} \quad (1)$$

[1] Although in our analysis, we predominately adopt a linear decay coefficient, we have explored the possibilities of using a non-linear decay coefficient, the impact is not significant. For the benefit of readers, we have presented the analysis in the Appendix

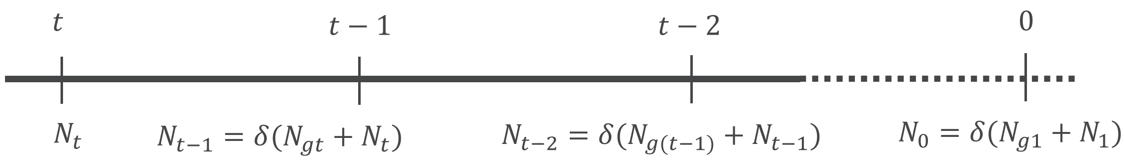


Fig. 1. Decay of Consumer Awareness over time.

Table 1
Summary of Symbols used in the model.

Symbol	Description
t	Number of Periods remaining in the sales horizon
N	Awareness at the beginning of the period
p	Price of the product
δ	Decay of awareness across a period
N_g	Number of generic impressions
N_b	Number of branded impressions
b_g	Bid for generic keyword (Decision Variable)
b_b	Bid for branded keyword (Decision Variable)
$\lambda_g(b_g)$	Click-rate function for generic keyword search
$\lambda_b(b_b)$	Click-rate function for branded keyword search
$\bar{F}(p\|b_g)$	Conversion Rate- Generic Search
$\bar{F}(p\|b_b)$	Conversion Rate- Branded Search
$\vartheta_g(b_g)$	Distribution for Reservation prices at bid b_g - generic search
$\vartheta_b(b_b)$	Distribution for Reservation prices at bid b_b - branded search
R_g	Distribution for Reservation prices at bid $b_g = 0$ - generic search
R_b	Distribution for Reservation prices at bid $b_b = 0$ - branded search
$G_g(.)$	Cumulative Distribution function for R_g
$G_b(.)$	Cumulative Distribution function for R_b
$L_g(i_g, j_g)$	Probability of having j_g conversions, given i_g clicks
$L_b(i_b, j_b)$	Probability of having j_b conversions, given i_b clicks
$L_{gb}(i_{gb}, j_{gb})$	Probability of having j_{gb} conversions, given i_{gb} clicks
$L_{g'}(i_{g'}, j_{g'})$	Probability of having $j_{g'}$ non-conversions, given 'cumulative' $i_{g'}$ clicks
$\tau_g(t, N_g)$	The immediate payoff (scenario 1) through generic search
$\tau_b(t, N_b)$	The immediate payoff (scenario 2) through branded search
$\tau_{gb}(t, N)$	The payoff (scenario 3) through first a generic search, followed by branded search
$\pi(t, N)$	Overall Expected Profit

Where:	
α_g=	Sharpness parameter of the *S-Shaped* curve at b_g
$\lambda_{g\infty}$=	*Maximum* Click-rate at $b_g = \infty$
λ_{g0}=	*Minimum* Click-rate at $b_g = 0$
β_g=	Steepness parameter of the *S-Shaped* curve at b_g.

Once a consumer arrives at the retailer's site, her decision to purchase is a function of her *reservation price*; in case the reservation price is higher than the price posted at the site, the purchase will occur. The higher willingness to pay is driven by her perception of a 'reliable' product (Agarwal et al., 2011; Ghose & Yang, 2009; Ye et al., 2015); the same perception leads her to click among the top-ranked results from the search engine, which in turn, is governed by a higher bid placed by the retailer. Consequently, if μ and σ denote the mean and standard deviation of the reservation price of the consumers, both $\mu_g(b_g)$ and $\sigma_g(b_g)$ increase monotonically with the increase in bid (Ye et al., 2015).

Assuming R_g denotes the distribution of reservation price of consumers when there is no bid for generic keyword ($b_g = 0$), then the reservation price of consumers coming through generic search can be represented as:

$$\vartheta_g(b_g) = \mu_g(b_g) + \sigma_g(b_g)R_g \tag{2}$$

Where: $\mu_g(0) = 0, \sigma_g(0) = 1$.

We assume that R_g follows a Gamma distribution whose CDF is given by $G_g(.)$ (Ye et al., 2015). Let the *cdf* of $\vartheta_g(b_g)$ be $F_g(.|b_g)$, then,

$$F_g(p|b_g) = G_g(\frac{p - \mu_g(b_g)}{\sigma_g(b_g)}) \tag{3}$$

Which implies that the probability of consumer completing the purchase, i.e. her reservation price being higher than the retailer's posted price is given by:

$$\bar{F}_g(p|b_g) = 1 - F_g(p|b_g) \tag{4}$$

Therefore, to sum up, the demand expected to be generated at a given time period can be expressed as a function of the price of the product as well as the Cost-per-Click (CPC), (or, bid amount) for the generic keyword. We adopt the Ghose & Yang (2009) model to arrive at the aggregate demand $L_g(i_g, j_g)$ for a generic search to be:

$$L_g(i_g, j_g) = (\frac{N_g!}{j_g!\,(i_g - j_g)!\,(N_g - i_g)!}) * (\lambda_g(b_g)\bar{F}_g(p|b_g))^{j_g} * \$\lambda_g(b_g)F_g(p|b_g))^{(i_g - j_g)} * (1 - \lambda_g(b_g))^{(N_g - i_g)} \tag{5}$$

Where:

N_g= Number of generic impressions; i_g= Number of clicks and j_g= Number of consumers who buy after clicking the link

The schematic flow of generic keyword search is presented in Fig. 2 for better understanding. The immediate pay-off through generic keyword search is $\tau_g(t, N_g)$:

$$\tau_g(t, N_g) = \sum_{i_g=0}^{N_g} \sum_{j_g=0}^{i_g} L_g(i_g, j_g)(p.j_g - b_g.i_g) \tag{6}$$

3.2. Arrivals through branded keyword search

While the branded keyword search follows similar logic as the generic keyword search, there is an intuitive difference; since

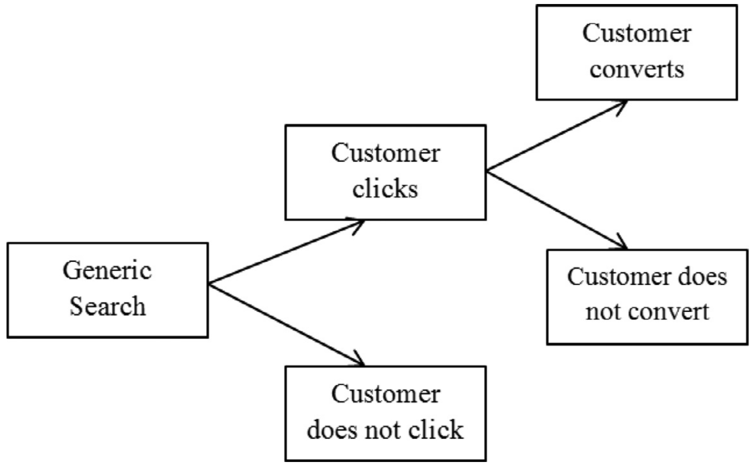


Fig. 2. Consumer's decision flow model for 'Generic' Search.

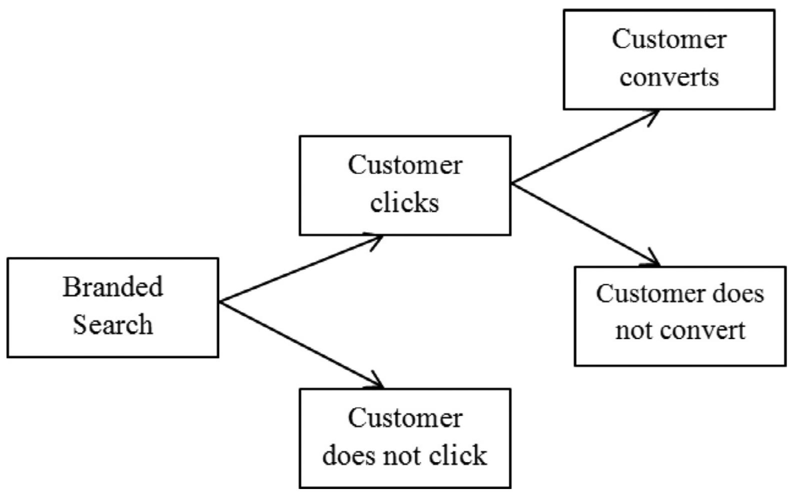


Fig. 3. Consumer's decision flow model for 'Branded' Search.

branded keywords would yield more specific results, the number of consumers who would arrive at the site through such a search would be much less in number, compared to a generic keyword search. Due to less competition for such specific keywords, the bid (b_b) is also expected to be less compared to one for a generic keyword (for example, Abhishek & Hosanagar, 2013; Rutz & Bucklin, 2011).

It is obvious that whenever a consumer enters a branded keyword for search, she is already aware of the brand and is specifically looking for it, and therefore has a higher proclivity for clicking the link corresponding to the brand. While the inherent search mechanisms remain the same for both generic and branded keyword searches, branded search click-rates have a narrow range; i.e. probability of a link being clicked is significantly higher for a corresponding branded search, as compared to a generic keyword search (Abhishek & Hosanagar, 2013; Rutz & Bucklin, 2011). For our model, the click rate for branded search is given by:

$$\lambda_b(b_b) = \frac{\lambda_{b_\infty} + \lambda_{b_0} e^{\beta_b - \alpha_b b_b}}{1 + e^{\beta_b - \alpha_b b_b}} \quad (7)$$

The distribution of reservations prices for consumers arriving through branded search ($\vartheta_b(b_b)$) is similar in nature to the distribution characteristics for generic keyword search (i.e. $\vartheta_g(b_g)$). However, there is a critical difference; consumers who are arriving through branded search obviously have both greater awareness as well as preference for the brand, and therefore, the variance of reservation prices would be less compared to consumers arriving through generic search.

$$\vartheta_b(b_b) = \mu_b(b_b) + \sigma_b(b_b)R_b, \qquad \text{where } \mu_b = 0, \text{ and } \sigma_b = 1 \quad (8)$$

As stated earlier, μ_b and σ_b represent the mean and standard deviation of the reservation price of consumers through branded search. The behavior of μ_b and σ_b with corresponding bid of b_b is similar to that of μ_g and σ_g with bid b_g, i.e. increasing with the bid. Similarly, R_b represents the distribution of reservation price of consumers when $b_b = 0$. The CDF of R_b is given by $G_b(.)$. If the CDF of $\vartheta_b(b_b)$ is $F_b(.|b_b)$, then:

$$F_b(p|b_b) = G_b\left(\frac{p - \mu_b(b_b)}{\sigma_b(b_b)}\right) \quad (9)$$

Similar to the context of generic search arrivals, a purchase will only happen when the reservation price exceeds the ask price of the retailer. Therefore, the conversion rate during a branded search can be presented as:

$$\bar{F_b}(p|b_b) = 1 - F_b(p|b_b) \quad (10)$$

Since the underlying logic for demand generation remains same even for a branded keyword search, the demand expression $L_b(i_b, j_b)$ is given by:

$$L_b(i_b, j_b) = \left(\frac{N_b!}{j_b!\,(i_b - j_b)!\,(N_b - i_b)!}\right) * (\lambda_b(b_b)\bar{F_b}(p|b_b))^{j_b} * \$\lambda_b(b_b)F_b(p|b_b))^{(i_b - j_b)} * (1 - \lambda_b(b_b))^{(N_b - i_b)} \quad (11)$$

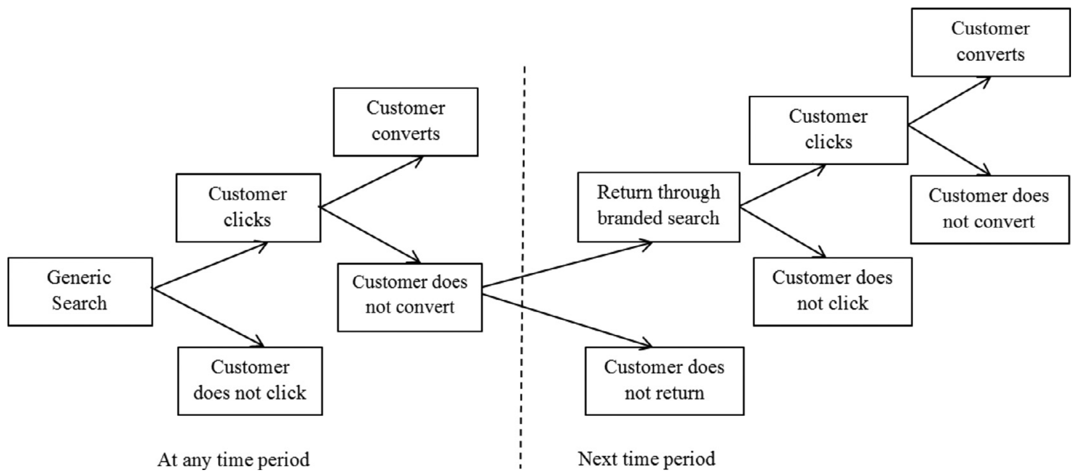


Fig. 4. Consumer's decision flow model for 'Branded' Search- following a generic keyword search

Similarly, the immediate payoff through a branded search $\tau_b(t, N_b)$ would be given by:

$$\tau_b(t, N_b) = \sum_{i_b=0}^{N_b} \sum_{j_b=0}^{i_b} L_b(i_b, j_b)(p.j_b - b_b.i_b) \tag{12}$$

3.3. Arrivals returning through Branded Search (following a Generic search)

In this section, we include consumers who do not complete the purchase in their initial visit; they could initiate a generic search, click on a particular link, visit the site, gather information, and leave. Post evaluation phase, they could follow up on their retained brand recall (of the visited site) and initiate a branded keyword search in the next stage and arrive through the branded search. For our model, we assume that generic keyword search and mere 'impressions' do not impact their brand recall (Abhishek & Hosanagar, 2013; Rutz & Bucklin, 2011). Consumers can only retain recall when they have visited the brand website in the previous stage. The consumer decision flow is mapped in the Fig. 4.

Empirical studies (Rutz & Bucklin, 2011) suggest that the click-rates for such a 'return' branded keyword search are higher; due to their prior knowledge of the brand- aided by the prior visit through generic keyword search. The probability of such a consumer arriving through a branded search, and clicking (following a generic one) is given by:

$$\lambda_{gb}(b_b) = \lambda_b(b_b(1+\Delta)) \tag{13}$$

Where Δ captures the contribution from a generic click on branded click. While for our model we assume Δ to be exogenous, empirical studies could estimate the value of Δ.

Although for this case, the nature of consumer arrivals is different, we assume the conversion would only depend on the consumers' willingness to pay the price posted at the website. Therefore, it would be similar to the expression presented in Eq. 10, i.e. $\bar{F}_b(p|b_b)$. For this analysis, all the consumers during the previous periods, who had clicked the link, which appeared as a generic search result page ($i_{g'}$), visited the website of the retailer, however, they did not convert ($j_{g'}$). Since then, they have returned to the page through the branded keyword search. Therefore, the probability of arrival (for consumers, who return after a generic search, not having completed their purchases in the previous search) would be given by:

$$L_{g'}(i_{g'}, j_{g'}) = \binom{N}{i_{g'}}[\lambda_g(\tilde{b}_N)]^{i_{g'}}(1-[\lambda_g(\tilde{b}_N)]^{N-i_{g'}} * \binom{i_{g'}}{j_{g'}}\bar{F}_g(p|\tilde{b}_N)^{j_{g'}}(1-\bar{F}_g(p|\tilde{b}_N))^{i_{g'}-j_{g'}} \tag{14}$$

Where: $\tilde{b}_N$ is the 'equivalent bid' corresponding to the total amount of awareness (N) at beginning of time period t. We understand that consumer awareness of products/brands is driven by the appearances in searches; generic, or branded. Given that appearances depend on retailer's bids, we introduce an 'equivalent bid, $\tilde{b}_N$ to substitute for the awareness level at the beginning of a given time period.

Again, the probability of having i_{gb} clicks and then j_{gb} conversions from those 'visitors' would be given by:

$$L_{gb}(i_{gb}, j_{gb}) = \binom{j_{g'}}{i_{gb}}[\lambda_{gb}(b_b)]^{i_{gb}}(1-[\lambda_{gb}.(b_b)]^{j_{g'}-i_{gb}} * \binom{i_{gb}}{j_{gb}}\bar{F}_b(p|b_b)^{j_{gb}}(1-\bar{F}_b(p|b_b))^{i_{gb}-j_{gb}} \tag{15}$$

Combining the relevant Eqs. 13–15, the immediate payoff from these visitors can be calculated by:

$$\tau_{gb}(t, N) = \sum_{i_{g'}=0}^{N} \sum_{j_{g'}=0}^{i_{g'}} \sum_{i_{gb}=0}^{j_{g'}} \sum_{j_{gb}=0}^{i_{gb}} L_{g'}(i_{g'}, j_{g'}) * L_{gb}(i_{gb}, j_{gb}) * (p * j_{gb} - b_b * i_{gb}) \tag{16}$$

3.4. Profit maximization model

It is obvious from the discussions in the previous sections that the retailer's profit maximization model is critically dependent on the decision surrounding the bids; for generic, or branded keywords. Consumers in the initial stages of their purchase journey would ideally rely on generic keyword based search, while consumers who have narrowed down to a brand level preference, would seek more compelling, brand-specific information by relying on branded keywords. We have also modeled for consumers who are exposed to brands through search results of their generic search. Following the 'impression', i.e. the brand appearing on their search page, they visited focal brand website, however, did not complete the purchase and came back at a later period through a branded keyword search. We have also assumed an equivalent bid value ($\tilde{b}_N$) for the sum of awareness carried over from previous periods (N). Since within the modeling framework, all awareness may be attributed to retailer's bidding activities, the adoption of an equivalent bid level for the current level of awareness can be justified.

Consumers who arrive through a generic keyword search are exposed a set of search results; the page impressions as well as when they click on the link provided on the search page increases their brand awareness, however, similar effect is not there for branded keyword search, since consumers were already aware of the brand as they initiated the search, and no incremental awareness occurs. However, in spite of this, retailers still need to bid for a combination of generic and branded keywords to ensure consumer arrivals are optimally captured. There are two parameters in our model, which impact awareness, viz., the decay of awareness $\delta \in [0, 1]$, (refer Basu & Nair, 2015; Chintagunta & Vilcassim, 1992) across periods, while we consider an 'enhancing' impact (Δ) of generic clicks on subsequent branded click possibilities.

The state variable in the dynamic programming model is N, which is the residual awareness at the beginning of each period. The awareness at the beginning of the next period is given by $\delta(N + N_g)$[2], where: N_g is the impression generated in the current period based on generic bids. The maximum awareness that can be generated is given by N_M.

The retailer's total payoff, therefore, would cover all three bidding and arrival scenarios (explained in Sections 3.1-3.3) over the entire selling horizon. We adopt dynamic programming to solve the model to arrive at the retailer's optimal portfolio of bids covering generic and branded keywords.

$$\pi(t, N) = \tau_g(t, N_g) + \tau_b(t, N_b) + \tau_{gb}(t, N) + \pi(t-1, \delta(N + N_g)),$$
The boundary condition is given by: $\pi(0, x) = 0$ for all $x \in N$. (17)

4. Analytical results

Analytically, we attempt to show the behavior of the optimal branded and generic bids for a higher level of awareness in a given time period.

We use the monotone likelihood ratio property to analyze the results (Ferguson, 1967). We slightly modify the existing proof of monotone likelihood ratio property taking into account the subtleties of our model.

Let $\mathbb{Z}^+$ be the set of non-negative integers, and X be a discrete random variable with probability mass function, $F(x, \theta), x \in \mathbb{Z}^+$, which involves a parameter θ.

$$F(x, \theta) = P_\theta(X \geq x) = \sum_{K=x}^{\infty} f(K, \theta) \tag{18}$$

Assume that for any $x \in \mathbb{Z}^+$, $f(x, \theta) = 0 \Rightarrow f(K, \theta) = 0$ for every $K(\geq x) \in \mathbb{Z}^+$,

$$\bar{F}(x, \theta) = 0 \tag{19}$$

Let $g(.)$ be a non-decreasing function over $\mathbb{Z}^+$ such that,

$$E_\theta[g(x)] = \sum_{x=0}^{\infty} g(x) f(x, \theta) \tag{20}$$

exists finitely for every θ. In the application considered later, the effective range of X is finite and hence the existence of $E[g(x)]$ is always guaranteed.

Lemma 1. *The distribution of X has monotone likelihood ratio in the sense that:*
$f(x_2, \theta_2).f(x_1, \theta_1) \geq f(x_2, \theta_1) f(x_1, \theta_2)$
for every $x_1, x_2 \in \mathbb{Z}^+, x_1 < x_2$. and every θ_1, θ_2 and , $\theta_1 < \theta_2$. Then,
(a) For every $x \in \mathbb{Z}^+, \bar{F}(x, \theta)$ is non-decreasing in θ.
(b) $E_\theta[g(x)]$ is non-decreasing in θ

Proof. See Appendix. □

Lemma 2. *If X follows a binomial distribution with parameters n and ϕ, then for every non-decreasing function $g(.)$ over $\mathbb{Z}^+$, $E[g(x)]$ is non-decreasing in n as well as in ϕ.*

Proof. See Appendix. □

Proposition 1. *Let $0 < \phi_1, \phi_2 < 1$ and N be a positive integer. Then, $L = \sum_{i=0}^{N} \sum_{j=0}^{i} \binom{N}{i} \phi_1^i (1-\phi_1)^{N-i} \binom{i}{j} \phi_2^j (1-\phi_2)^{i-j}$ is non-decreasing in N, and ϕ_1.*

Proof. See Appendix. □

For computational purposes and managerial insights, we now examine the trend of the optimal branded and generic bids separately by holding one of them constant as awareness in a time period changes.

Proposition 2. *In order for the retailer to continue maximizing his profits, the optimal branded bid must be non-increasing as awareness in the same time period increases at a fixed level of generic bid when $\frac{b_b}{p} > \frac{j_b}{N_b}$.*

Proof. See Appendix. □

The above Proposition provides a monotonic structural property of the optimal branded bid when $\frac{b_b}{p} > \frac{j_b}{N_b}$. In practical scenarios, the ratio of the bid price to the product price is much higher than the ratio of conversions to the number of impressions per bid (Ghose & Yang, 2009; Ye et al., 2015). Hence, the above mathematical condition will hold in most business scenarios.

Proposition 3. *In order for the retailer to continue maximizing his profits, the optimal generic bid must be non-increasing as awareness in the same time period increases at a fixed level of branded bid when $\frac{b_g}{p} > \frac{j_g}{N_g}$.*

Proof. See Appendix. □

The above theoretical results provide useful structural properties for the optimal branded and generic bids at changing levels of customer awareness. We use these results to derive critical managerial insights in the next section along with extensive numerical analyses.

5. Numerical analysis and managerial insights

We present a set of numerical analyses to highlight the nature of decision-making framework for a retailer, focused on optimal allocations for generic and branded keyword bids. As mentioned earlier, for our model, we consider the following, i.e., price of product (p), generic-to-branded search spillover factor (Δ), consumers' memory decay coefficient (δ), total potential awareness level (N_M) to be exogenous. We consider the *gamma*-distribution for the distribution of the reservation prices (Hong & Shum, 2006) as it is one of the two-parameter distributions, which can assume various shapes based on the parameter values. We try to solve the retailer's problem at the beginning of each period; that of allocating his resources to a combination of generic and branded keyword bids. We have already mentioned earlier, that due to the inherent nature of the appearance of search results, we can assume that consumers' reservation prices to be a function of the

[2] The state transition equation is given by: $N_{t-1} = \delta(N_t + N_{gt})$, which is depicted in Fig. 1.

Table 2
Fixed parameter values for the Simulated Results

Price of the Product (p)	$= 20$
Max. Awareness (N_M)	$= 80$
Generic Bid (b_g)	$\in [0, 40]$
Branded Bid (b_b)	$\in [0, 20]$

retailer's bid for the keyword. As for the S-shaped click-rate functions, we assume the following 'sharpness' and 'steepness' parameters for generic (α_g, β_g) to be (0.3,8), and for branded (α_b, β_b), the parameter values to be (0.5,5), since the two keyword searches demonstrate distinctive characteristics. We are implicitly assuming through the choice of these parameters that generic bids usually will be 'slower' to gain attention and will gradually reach a saturation, when higher bids would not generate incremental awareness; on the other hand, branded bids will immediately pickup, and reach saturation much earlier, compared to generic bids.

The following parametric expressions for the means $[\mu_g(b_g), \mu_b(b_b)]$ and standard deviations $[\sigma_g(b_g), \sigma_b(b_b)]$ for consumers' reservation prices are considered:

$\mu_g(b_g)$	$= \gamma_{g\mu} * b_g^{k_{g1}}$
$\sigma_g(b_g)$	$= 1 + \gamma_{g\sigma} * b_g^{k_{g2}}$
$\mu_b(b_b)$	$= \gamma_{b\mu} * b_b^{k_{b1}}$
$\sigma_b(b_b)$	$= 1 + \gamma_{b\sigma} * b_b^{k_{b2}}$
Where:	$\gamma_{g\mu}, \gamma_{g\sigma}, \gamma_{b\mu}, \gamma_{b\sigma}, k_{g1}, k_{g2}, k_{b1}, k_{b2}$ are constants.

Although for our model we have assigned some 'sensitivity' coefficients (e.g., $\gamma_{g\mu}, \gamma_{g\sigma}, \gamma_{b\mu}, \gamma_{b\sigma}$), they can be tested empirically with consumer data. We use these coefficients to adjust the consumer willingness-to-pay to the prices posted by the retailer. The other constants, i.e. $k_{g1}, k_{g2}, k_{b1}, k_{b2}$ are essentially to simulate the retailer's tweaking of the bid amounts corresponding to the category of bids (i.e., generic, or branded). As stated earlier, consumers who arrive through branded keyword searches would have lower variances in reservation prices compared to 'generic' consumers, since they already have prior knowledge of the brand (and its prices). The parameters used in the numerical analysis are provided in Table 2. The relative values of the price and the bids are chosen in line with Ye et al. (2015) and satisfy the structural condition mentioned in Section 4. For robustness, we ran the models for other parameter values and obtained similar results. In subsequent sections, we present the insights from simulations using other sets of parametric values.

5.1. Varying bid portfolio with consumer awareness

The retailer's foremost decision problem is to find the optimal mix of generic (b_g) and branded (b_b) keyword bids at the beginning of each period, which is dependent on the extant awareness of the brand in question in consumers' minds. Given that the retailer has information on the awareness, he can then mix the bids on generic and branded keywords optimally. Intuitively, branded keyword bids would be lower compared to generic bids, given that there is more competition for those keywords. In the first set of analyses, we manipulate across the various levels of consumer awareness (maximum being $N_M = 80$) to find the optimal bids for each category of keywords. Fig. 5 shows the retailer's optimal mix of generic and branded keyword bids to drive consumer awareness. The different awareness levels are reflective of actual impressions and their cumulative effect on awareness (due to memory effects) and to bring parity to real-life situations, could be considered as being in units of thousand ('000), or *CPM*.

Results as shown in Fig. 5 corroborate with the intuitive assessment of the scenario. At very low levels of extant consumer awareness, the retailer's optimal investment should be in generic keyword bids, with somewhat less focus on branded bids. However, the retailer gradually reduces the focus on generic keywords as the awareness among the target audience increases. In case the awareness grows higher, the retailer's investment in generic keywords will reduce further and the optimal strategy would be to focus on maintaining the a minimum level of bids for generic keywords, and a relatively higher level of spend on branded keywords. The results also indicate that the retailer's key aim should be develop a category-level connect with the brand at lower levels of awareness, when the brand awareness is relatively higher, and consumers have strongly identified the brand with the particular product category, he should focus on pursuing a predominantly brand-focused (i.e., branded keyword) promotional strategy.

5.2. Effect of consumers' recall on optimal mix of bids

In the next set of simulations, we try to assess the impact of consumer's recall (δ) on retailer's optimal mix of bids (b_g, b_b). Since the retailer's decision to invest on bids at the beginning of each period is dependent on the level of awareness at that point, it is obvious that 'recall' due to previous period's bid investments would play a significant part. We assume the presence of two types of consumers, one with high recall, and the other with lower recall capabilities. We observe the change in optimal mix of bids for various levels of awareness, for each of the two types of consumers (high and low recall), keeping other parameters constant (please refer: Fig. 6).

In Fig. 6, we find that the nature of the optimal mix is similar to Fig. 5, as would be expected, as recall directly impacts the awareness at the beginning of the next period. They both indicate that the retailer's optimal strategy would be to initially have a larger investment for generic bids giving way to branded bids as the awareness levels increase. However, this set of results critically shows that for consumers with higher recall values, the retailer can drop his investment in generic bids at the earlier stages, rather than having to maintain them for a longer duration, as in the case of lower recall consumers. So, given that consumers possess higher recall capacities (exogenous to the model), the retailer could benefit from obvious savings in investment on generic bids. Both for Figs. 5 and 6, we observe that the retailer has to maintain a steady level of investment in branded keywords to ensure recall/awareness among consumers.

Although the high-recall consumers lead to retailer's lower investments, Fig. 7 highlights a seemingly counterintuitive result. For medium levels of awareness (i.e., 20–25), we find that the retailer would prefer to have a higher level of generic bids (i.e. $b_g > b_b$), compared to the case of lower-recall consumers (where it is $b_g < b_b$). This behavior can be explained by the logic of overall profitable mix of bids by the retailer; the retailer would maintain a higher level of investment on generic bids for high-recall consumers during medium levels of awareness so as to ensure high recall over a substantial period, which leads to much lower investments (compared to low-recall consumers) for higher awareness levels. By holding the generic bid investment at a higher level (only briefly) the retailer ensures higher profitability as his investments in generic bids fall sharply at higher levels of awareness.

5.3. Effect of variance in reservation prices for arrivals through generic search

Next, we explore how the consumers' reservation price variance impacts the optimal choice of bids based on the awareness generated. It is generally expected that consumers who are arriving through generic search would have higher variance in their reservation prices since they have carried out a more product-focused search, and not for a specific brand. Since the particular product

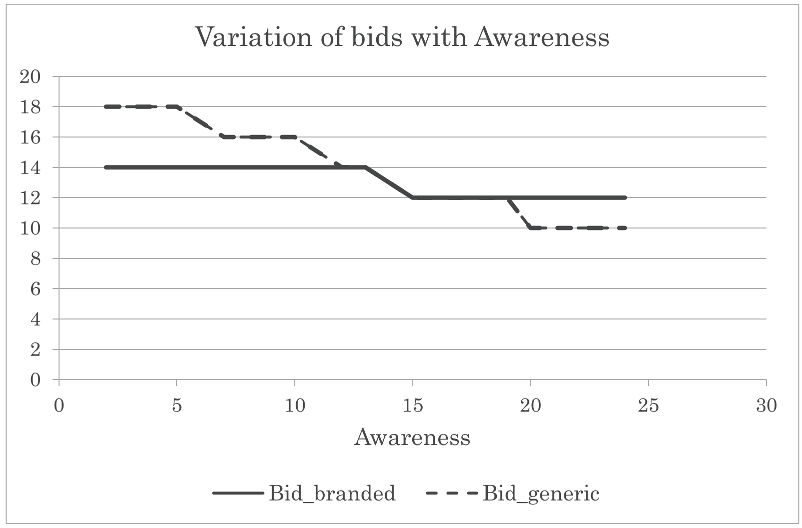
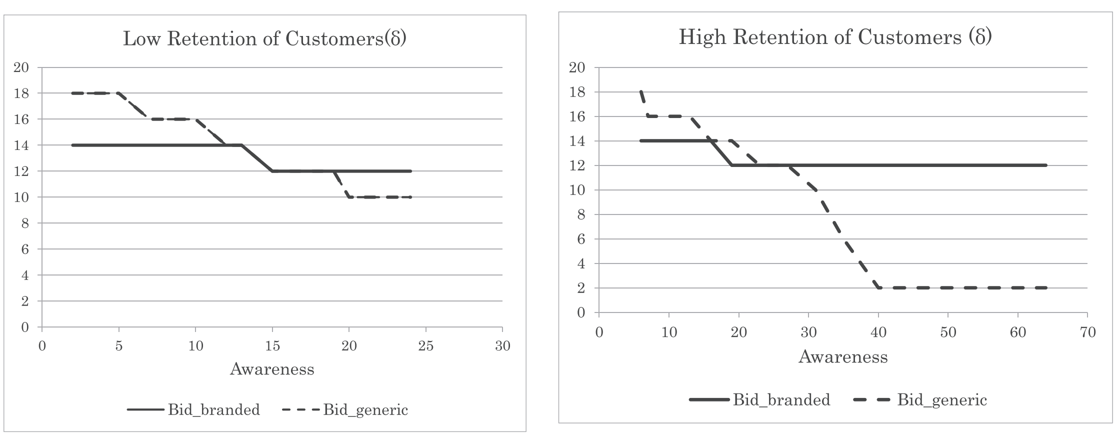


Fig. 5. Varying bid mix across consumer Awareness.



(a) For Low Retention consumers

(b) For high retention consumers

Fig. 6. Optimal bidding policies for consumers with differential retentions.

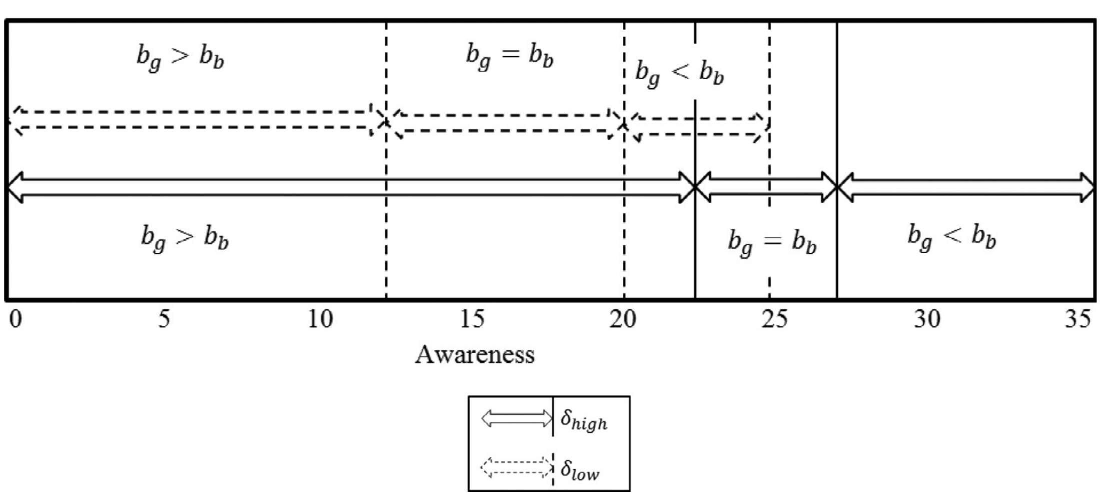
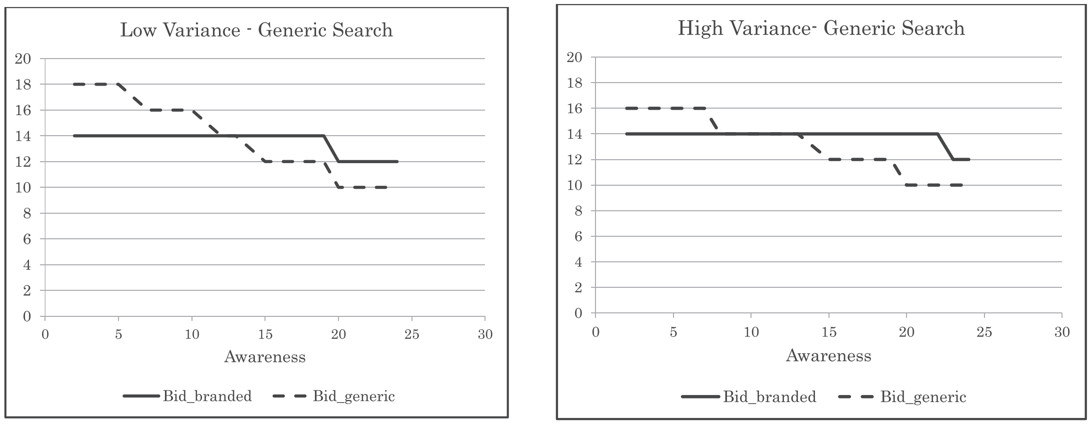


Fig. 7. Schematic diagram to depict the bid mix across Awareness levels for consumers with differing Recall.



(a) For Low variance (generic search) consumers (b) For high variance (generic search) consumers

Fig. 8. Consumers' Reservation Price variance impacting Optimal Bid allocation

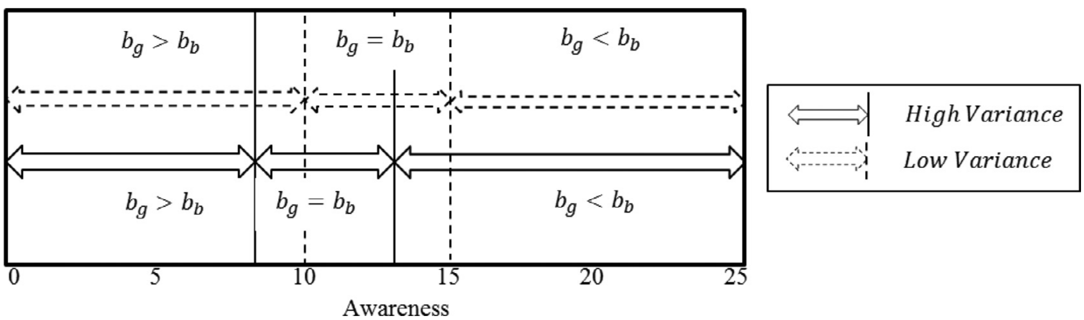


Fig. 9. Schematic Diagram showing Optimal Bid for variances in Consumers' Reservation Prices

category might have a wider price dispersion, the same would be reflected as far as the consumers' reservation prices are concerned. It is however possible that some consumers can have a relatively lower variance, while others have higher variances in their reservation prices due to various factors, e.g. previous experience with a limited range of brands, budget constraints, product usage context, etc. We test the model by considering two such groups, one with high reservation price variance and other with relatively lower variance and calculate the optimal mix of branded and generic bids from the retailer to generate awareness.

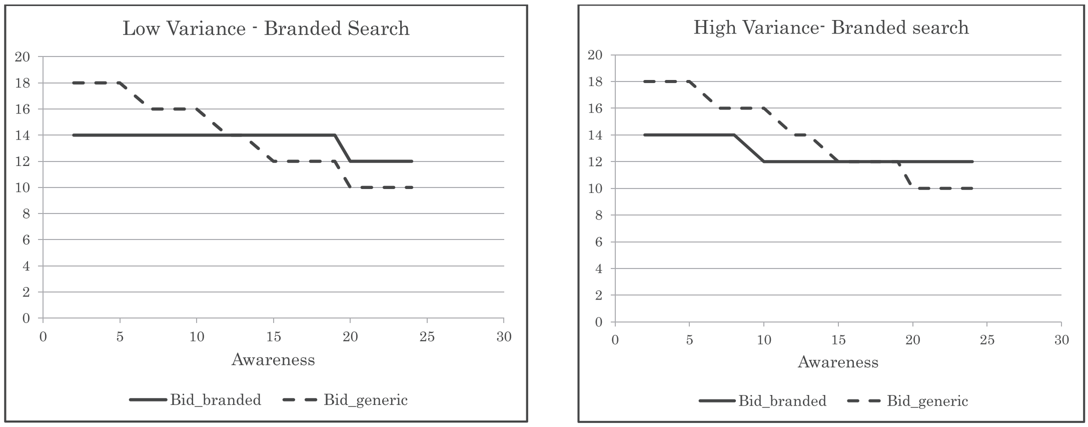
One of the key assumptions in our model is that the bid impacts the reservation price of the consumer; consumers are willing to pay a higher amount for brands, which appear higher on the search results. All other factors (e.g., Quality score of the sites) remaining constant, we know retailer's bid values determine their position on the search results. Therefore, we can assume that consumers with lower reservation price variances would largely focus on select results on the top and ignore the rest, while consumers with wider variances in reservation prices would look beyond just the topmost results. The retailer, in such cases does not have an incentive to bid higher to acquire a higher position on the search results; rather focusing on branded search by bidding high enough to ensure that consumers who come through branded search convert. Owing to this behavior, in case of consumers with high variance (reservation prices) generic bids converge earlier (i.e. at lower awareness levels) with branded bids rather than for consumers with lower variance in reservation prices.

This optimal bidding logic is shown in the schematic diagram in Fig. 9. At extremely low levels of awareness, the retailer obviously focuses on creating awareness and to build an association of the focal brand within the product category and therefore, invest on generic search to ensure that the brand definitely makes it to the search list. However, once a threshold level of awareness is generated, retailer reduces his investment in generic search and shifts his focus towards branded search. This is especially more apt for consumers with higher variance in reservation prices, as they are expected to be 'indifferent' to the search rank of the brand. For the retailer, the payoff would be more optimal if he retained his focus on consumers arriving through branded search by bidding higher, and ensure that they surely convert. This scenario of higher variance in reservation prices could be compared to consumers who have a large 'consideration set' and therefore, embracing a wider price dispersion.

5.4. Effect of variance in reservation prices for arrivals through branded search

In this section, we explore the optimal bidding problem for the retailer, who is evaluating his strategies for consumers who arrive based on the branded keyword search. For this situation, we do not essentially differentiate between consumers who arrive primarily through branded search (having prior information, or otherwise) and others, who could have been exposed to the brand through their generic search results and return in the next period through a branded search based on their recall/retention of the brand. Similar to the case for the generic search, we consider two groups of consumers who are aware of the brand (since they arrive through branded search), but have different reservation price variances, one with high variance and the other, having lower variance. We simulate the results to assess the nature of the optimal bid allocation by the retailer. The results are shown in Fig. 10.

The results from Fig. 10 corroborate our intuitive understanding. Since the arrivals are through branded search, the allocations of generic bids are agnostic of the type of consumers (i.e., varying



(a) For Low variance (branded search) consumers (b) For high variance (branded search) consumers

Fig. 10. Optimal bid allocation for Branded search arrivals

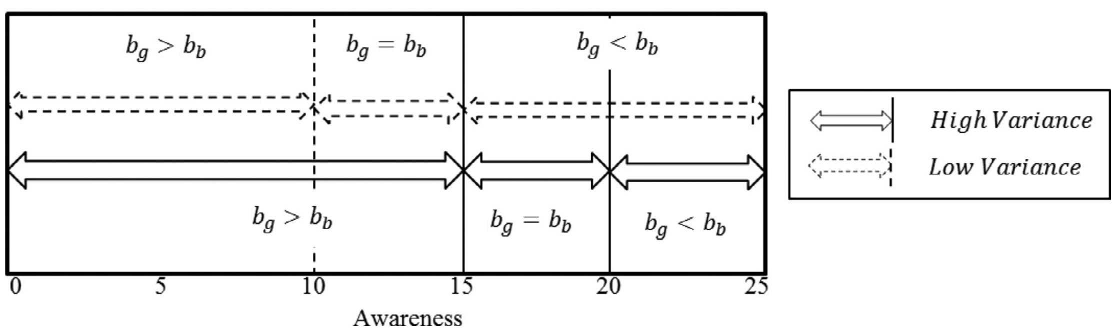


Fig. 11. Schematic Diagram in Consumers' Reservation Prices

on the basis of the reservation prices). At low levels of awareness, the retailer still invests in generic bids more than branded bids, however, the investment falls rather steeply for this case. Since the retailer is largely depending on arrivals through branded search, the investments are higher for branded keywords, and investments on generic bids are significantly lower in this case. The seemingly counter-intuitive results appear, however, for the nature of investments in branded search across the two groups differing in terms of the variance in reservation prices; the retailer maintaining a higher spend on branded keywords for the lower variance group, rather than focusing on the group with higher variance. The explanation for the selection of such an optimal policy is rooted in the similar logic of consumer behavior in case of generic search.

The dynamics of the optimal bids is shown in Fig. 11; for the high variance group, the retailer drops his level of investment in branded search in the medium awareness zone and has a relatively higher investment in generic bids. However, for the lower variance, the retailer maintains a relatively higher bid level even beyond the medium range (even higher than generic bids). Results indicate that for higher levels of awareness, the retailer's optimal allocation is higher for branded search compared to generic search. The retailer's apparent lack of interest towards higher variance group can be explained by the fact, that consumers belonging to this specific group may be aware of the brand, but are not particularly brand loyal. They might have a wide group of brands they maintain in their consideration set and freely substitute among these brands within the category. Therefore, the retailer has no particular incentive for maintaining a higher bid for branded search for this group, as he knows that a higher bid (and a higher rank) would not necessarily convert to a choice. The lower variance group is expected to be much more 'loyal' to a narrower set of brands in the category, and therefore, it would be optimal for the focal brand to invest higher to acquire a higher rank to ensure the conversion from such consumers. Other studies (Desai et al., 2014; Simonov et al., 2018) indicate that competition can attempt to 'poach' branded keywords to gain visibility, and well-known brands often fall prey to these strategies. Under such a condition, retailers would wish to bid higher for their own brands to protect their brands from getting 'poached' by a competitor. This would ensure consumers who arrive through branded search (and have lower variance in their reservation prices) do not get distracted by such 'poaching' strategies.

In the following section, we attempt to exploit our understanding in the preceding sections to investigate the bidding options for specific consumer purchase situations.

5.5. Optimal bidding choices for select purchase conditions

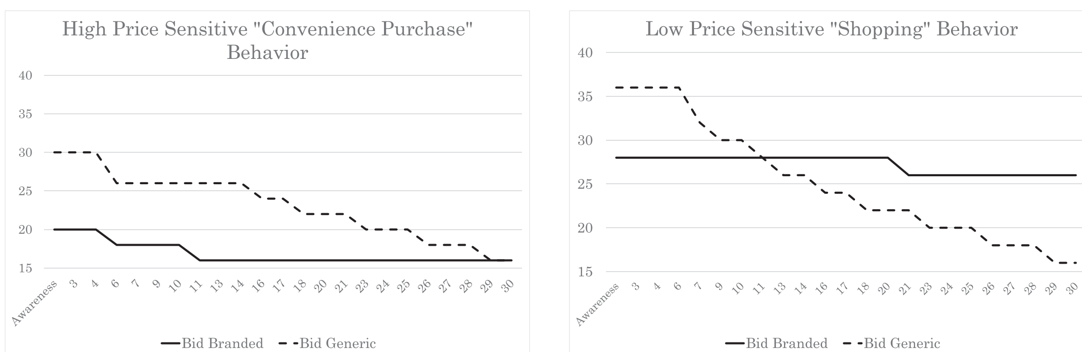
Following the seminal works by Copeland (1923), Holton (1958), Bucklin (1963) and Dommermuth & Cundiff (1967), we adopt the three distinct consumer purchase instances; (i) Convenience goods- where the consumer employs least effort in purchase-related decision making due to her inherent familiarity with the product category, usually price sensitive purchases with some degree of flexibility regarding brand choices, (ii) Shopping goods- where consumers 'shop' around, to select the 'best'. Consumers can engage in either a budget-constrained optimization across parameters, or, a non-budget constrained option depending on the nature of the product category, and the consumer disposition towards the product purchase situation, and finally (iii) Specialty goods- where consumers are relatively more knowledgeable about the product and the brands they are planning to purchase, therefore,

Table 3
Customer classification based on Mean price and variance considerations.

	Low variance (σ^2) in Price	High variance (σ^2) in Price
Low Willingness-to-pay (μ)	**'Convenience'** Purchase (Price Sensitive with narrow brand choices)	**Low-End 'Shopper'** (High Price Sensitive Wide range of brands)
High Willingness-to-pay (μ)	**'Specialty'** Purchase (Narrow set of brands)	**High-End 'Shopper'** (Less Price Sensitive Wide range of brands)

Table 4
Parameter values adopted for the numerical analyses.

	Low variance (σ^2) in Price	High variance (σ^2) in Price
Low Willingness -to-pay (Low μ)	**'Convenience' Purchase** $\gamma_{b\mu} = 1.2, \gamma_{b\sigma} = 0.08$ $k_g = 0.7, k_b = 0.4$	**Low-End 'Shopping' Behavior** $\gamma_{b\mu} = 1.2, \gamma_{b\sigma} = 0.1$ $k_g = 0.7, k_b = 0.4$
High Willingness -to-pay (High μ)	**'Specialty'** Purchase $\gamma_{b\mu} = 1.3, \gamma_{b\sigma} = 0.08$ $k_g = 0.7, k_b = 0.4$	**High-End 'Shopping' Behavior** $\gamma_{b\mu} = 1.3, \gamma_{b\sigma} = 0.1$ $k_g = 0.7, k_b = 0.4$



(a) Bidding strategy targeting Convenience purchasers (b) Bidding strategy targeting price sensitive 'Shoppers'

Fig. 12. Optimal bid allocation for customers with lower willingness-to-pay.

with higher willingness-to-pay and usually narrower, quite specific brand preferences (refer Table.3). Recent works by Thirumalai & Sinha (2005) and Thirumalai & Sinha (2009) highlight the relevance of the incorporation of similar customer classification in the e-retailing context.

For our analysis, we assume that for customers, who have a lower willingness-to-pay (therefore, high price sensitivity) demonstrate overall lower mean values of reservation prices. Further, for customers who are brand-agnostic will have a wider range of brands under consideration, leading to a concurrent higher variance in prices.

For the following numerical analysis, we took price, $p = 70$ and the maximum market potential $N_M = 80$. We varied the values of generic and branded bids such that $b_g \in [0, 60]$ and $b_b \in [0, 40]$. We assume customers have high/low willingness-to-pay (i.e., high/low values of μ) and we fixed variance at two levels, high and low, to represent customers' range of brand preferences, wide, or narrow, respectively. The parameters used to model the different kinds of shopping behaviours are presented in Table-4. We ensured the parameters fulfil the structural condition mentioned in Section 4. The parameter values chosen here are in line with the ones presented in Ye et al. (2015). To check the robustness of the results, we ran the models for other sets of parametric values and obtained similar insights (Please ref. Appendix E).

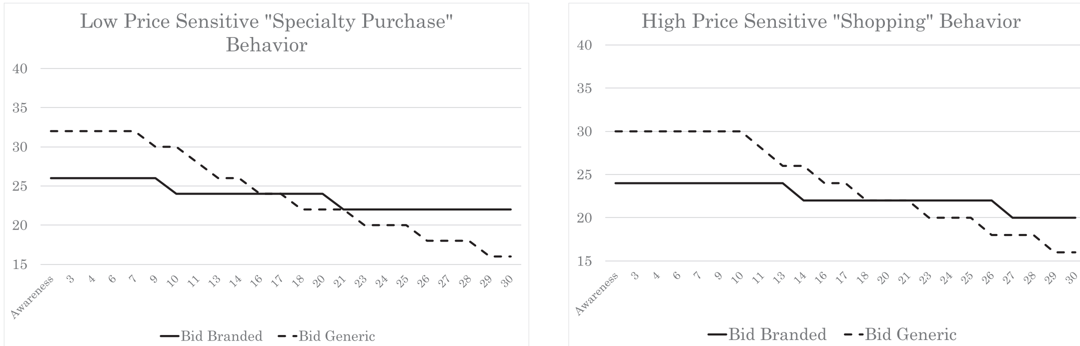
Based on this classification of customers, we ran a set of numerical analyses to identify the retailer's optimal bidding choices and shares of generic and branded bids. The following Figs. 12, 14 b & 14 provide useful insights (for a robustness check for the parametric values, please refer **Appendix**).

The overall insights from Fig. 12a & b are intuitive; in case of 'convenience' purchases, where customers are essentially engaged in a narrow brand search (suited to their more 'regular' purchases) and usually possess a higher price sensitivity to such categories of purchases (e.g., staples, toothpaste, household cleaning liquids, etc.), the retailer's optimal bidding strategy will be largely focused on investing in generic keywords.

The retailer's key objective would be to ensure that in category-related (i.e., generic keywords) search, to ensure that their brands acquire high ranks to ensure visibility and possible conversion. In most convenience purchases, branded keyword investment is kept at a relatively lower level, as there could be less instances of customers 'returning', since convenience purchases are mainly focused on more immediate consumption.

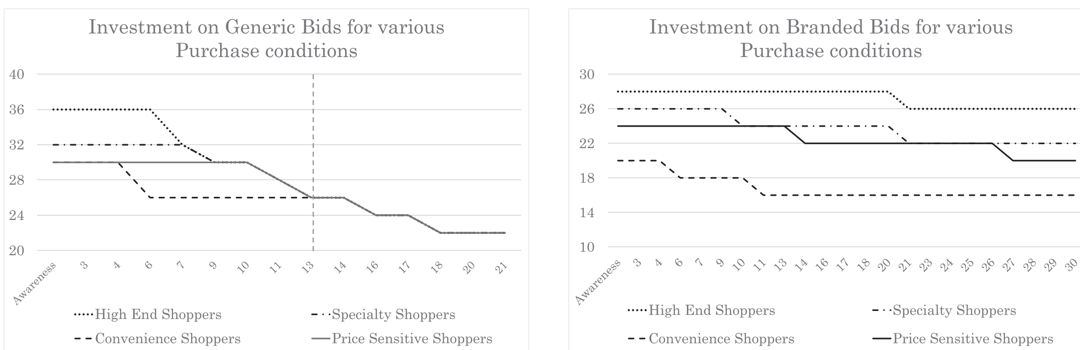
This particular behavior marks the difference between the convenience shoppers in Fig. 12a and price-sensitive 'Shoppers' shown in Fig. 12b. 'Shoppers' often may delay, or extend their purchase windows due to their penchant to compare across multiple brands for varieties' sake, which builds a stronger case for higher investments in branded keywords, as compared to 'Convenience' customers.

In the next set of figures, (Fig. 14a & b) we observe the bidding options considered optimal for the retailer. In relative terms, re-



(a) Bidding strategy targeting ‘Specialty’ purchasers (b) Bidding strategy targeting ‘High-end Shoppers’

Fig. 13. Optimal bid allocation for customers with higher willingness-to-pay.



(a) Generic Keyword bids for purchase situations (b) Branded Keyword bids for purchase situations

Fig. 14. Optimal bid investments across purchase situations.

tailer’s investment in branded keywords would be lower for ‘Specialty’ purchases, since these customers are the most discerning, and have stronger, more established brand preferences. Specialty purchasers are more likely to indulge in specific brand related searches. However, in case of high end (or, less price sensitive) ‘Shoppers’, retailers have to invest intensively in generic, as well as branded keywords to ensure visibility; such customers are anyway expected to vacillate between options, and would want to ‘try’ several brands before they can complete their purchase.

Fig. 14 a suggests that bidding on generic keywords would be extremely critical for ‘shoppers’, however, for high-end shoppers, it assumes greater importance as these customers would be interested in a wider range of brands to develop their ‘consideration sets’ and obtaining a higher rank to ensure visibility among competition would be critical.

The relative importance of branded bids is more evident for customers, who are high-end shoppers as well as Specialty shoppers (refer Fig. 14b). Since they are not constrained by budget, they often look out either for specific brands (specialty), or more quality choices (shoppers). In both cases, investment in branded keywords can be an optimal strategy for the retailer.

We observe that while for most purchase situations, the generic to branded bid ratio reduces over increase in awareness, retailers need to retain the focus on generic bids for Convenience shoppers, mainly due to their preference for generic solutions (not quite brand-specific) for their ‘daily’, regular purchases. Retailers would need to focus more on generic bids at moderate awareness levels, to ensure that customers (convenience shoppers) do not shift to their competitors as convenience shoppers might be more keen to focus on a ‘good deal’ due to their inherent price consciousness.

6. Concluding remarks, implications for managers and future research avenues

6.1. Model contributions

In this paper, we explore the decision making problem for the retailer who is trying to attract his consumers to his website through sponsored search advertising. The problem has been conceptualized as an optimal portfolio selection problem for the retailer consisting of generic and branded keywords. We develop a multi-period, dynamic programming logic to obtain the optimal bundles of investment in each category of keywords, depending on consumers’ characteristics.

Our model incorporates the following consumer characteristics in the model; consumers’ ability to retain awareness of product and brand and carry the awareness over to the next period, consumers’ awareness of products in a category (modeled through reservation price variance- generic search) and finally, consumers’ extent of preference for a brand (modeled by reservation price variance- branded search). When consumers have less awareness about a product category, it is logical that their price expectations

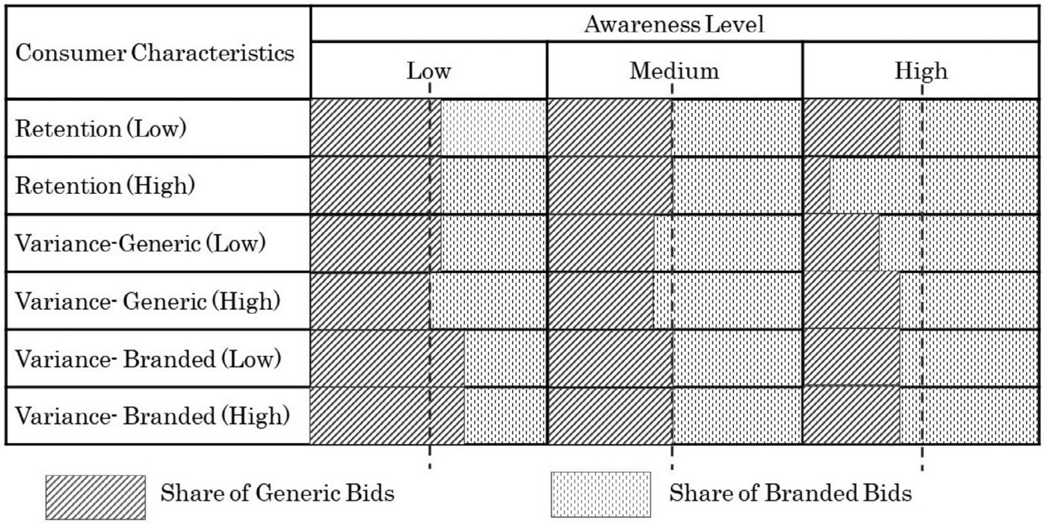


Fig. 15. Retailer's optimal share for Generic and Branded keywords.

would vary widely, as compared to other consumers who have greater awareness, or experience in that category. The reservation price expectation have large impact on conversions from consumers when they visit a retailer's site, and therefore, according to our model, retailer's optimal portfolio decisions can differ significantly across such consumer groups. Similarly, we postulate that if consumers have wider consideration set of brands, i.e. rather than one, or two brands, if consumers usually consider a larger set of brands for consideration for purchase, we expect consumers' reservation prices would also have higher variances. The retailer's optimal profit-maximizing portfolio of "keyword-mix" comprising of generic and branded keywords, is given by Fig. 15.

6.2. Implications for practice

The optimal bidding policies are depicted in Fig. 15; representative allocations for generic and branded keywords are presented. We find that as a policy, the retailer would obviously need to drive up awareness (at low initial levels) with significant investments in both generic and branded keywords. However, for low levels of awareness, generic bids' share would be higher than the branded, which reverses for higher levels of awareness, where share of branded keyword bids is higher. Intuitively, bids on generic keywords is crucial at the low levels of awareness to harness also the effects of consumer retention, to ensure that the 'spillover effect' takes over, resulting in significantly lower levels of generic bids to maintain awareness at high levels for consumer groups, which have higher retention. Retailers are able to capitalize on their investments in generic bids in earlier (lower levels of awareness) stages by significantly reducing their dependence (on generic bids) in later (higher awareness level) stages, by shifting their focus towards branded keywords, to maintain adequate residual awareness of the focal brand. The model tends to emphasize the importance of branded keywords across various awareness levels, which garners for the importance of branded keywords for the bidding strategy for a retailer.

We feel that the analyses provides some interesting and useful insights for the bid planning for retailers. Rather than relying merely on more expensive generic bids to generate brand visibility, retailer could focus on a mix of generic and branded bids to maintain the overall awareness levels among consumers. This model also allows retailers to customize their bid planning depending on the nature of the consumers; since online retailers often receive useful data regarding consumer characteristics from product searches, they could incorporate these parameters, e.g. prior product, or brand experiences, price sensitivities in their assessment of consumers and mix their bids accordingly to ensure optimal advertising spends.

Our analysis linking purchase situations (based on broad product type classifications) suggests that the focus on generic and branded bids obviously are different across these categories. Although intuitively the focus on generic bids fall over increase in awareness, the insights from the model suggests that for some product categories, viz., Convenience Shopping (i.e., for product types, which are frequently purchased and often of lower price points) requires retailers to maintain a significantly higher focus on generic bids and keep product category level push sustained even in higher awareness levels (refer Fig. 16). For the case of price-sensitive shopping, where consumers are expected to 'shop around' for their 'ideal' product choices. Since these shoppers are not necessarily brand conscious, branded bids investments may be lower. The optimal bid ratio spikes suggest that the generic bids remain steady (even when branded bids drop) at medium and high levels of awareness. These insights will benefit the retailers' decision making process for choosing appropriate bid mixes.

6.3. Future research in this area

Our work can be extended in different directions through future research endeavours. First and foremost, an empirical research project capturing the portfolio of generic and branded keyword searches in a dynamic setting to validate the memory effects for different kinds of products, as an extension of our current work may be an interesting future area of research. Second, linking inventory decisions with the optimal portfolio of generic and branded bids will be another useful research agenda. This work could be extended to the inventory decision-making process of a multi-product platform retailer. Our analysis suggests that retailers are better off maintaining a constant level of branded bid investment, even at high awareness levels. Consistent focus on branded keywords is expected to deter competing brands from potentially gaining from bidding on the focal brands to gain visibility through 'poaching' keywords. Although few studies (Desai et al., 2014; Simonov et al., 2018) have focused on this aspect, we believe the area of competitive bidding for branded keywords would raise interesting questions for future research. The impact of competitive

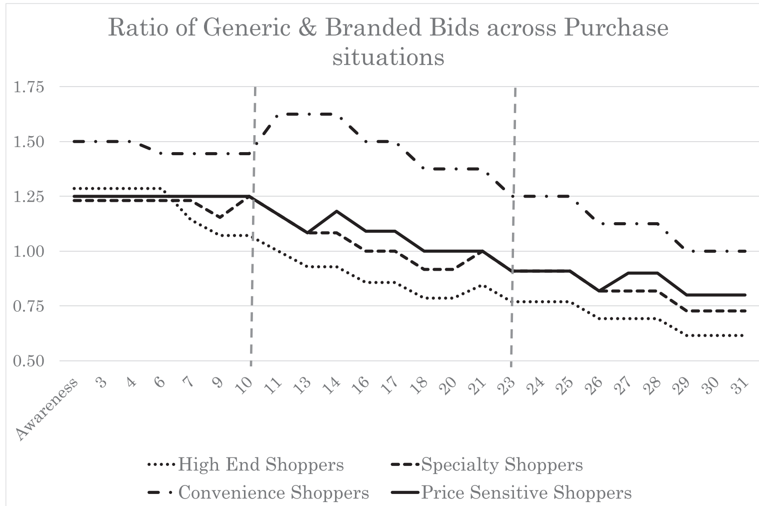


Fig. 16. Ratio of Generic to Branded Bids-Based on purchase situations.

bidding on the optimal portfolio of generic and branded bids may be the focus of future research in this area.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2022.10.021

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