The Interconnectedness between Green Finance Indices and Other Important Financial Variables

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Abstract

This study explores a novel approach to measuring the degree of interconnectedness between stock indices adjusted for green revenues of companies from major economies and various important macroeconomics and financial variables. Focusing on green finance indices, the authors propose graphical models to disentangle the complex connections between the variables of interest. The analysis illustrates the central role played by both United States and Europe in green finance. The selected graphical model identifies valuable dependence patterns: it indicates that the green revenues index for China is only directly related to the green revenues indices of the United States and Europe and technological stock, whilst the green revenues index for the United Kingdom is only linked with the green revenues indices of the United States and Europe.

Keywords: Climate Finance, Green Stock Indices, Graphical Models, Interconnectedness, Multivariate Correlation
THREE KEY HIGHLIGHTS

• There is a strong connection between the green revenues of Chinese companies on one side and U.S. and European companies on the other side. United States and Europe are central to the green financial system.

• The standard linear Pearson correlation coefficients can be misleading in describing the correlation structure among the variables under study. Partial correlation coefficients provide a better statistical measure for the analysis of complex interactions.

• Graphical models can illustrate directly multidimensional dependence structures underpinning international portfolios.
Climate change is acknowledged as an unprecedented environmental problem impacting the global economy. The European Commission indicated that in order to cut net greenhouse gas emissions to zero by the midcentury, green investments would have to rise to 2.8% of the European Union GDP. The Global Commission on the Economy and Climate estimated that $90 trillion of investment is needed by 2030 to reduce global warming by more than two Celsius degrees. Likewise, in November 2018, U.S. federal agencies estimated that the potential damage related to climate change could reduce U.S. GDP by 10% by 2100 (Hong et al., 2020). The Stern Review on the Economics of Climate Change (Stern, 2007) triggered a shift in focus on climate change policy and modelling. There are important gains from interactions between climate scientists and economists, as demonstrated by Hsiang and Kopp (2018). Until now economists were more focused on quantifying the social cost of carbon (see Auffhammer, 2018) or the cost of implementing greenhouse mitigation policies such as the standardization of automobile fuel economy, the quota of electricity production from renewables, subsidizing solar and wind power generation, expansion of biofuels (see Gillingham and Stock, 2018). At the same time, naive economic models may obscure the benefits of implementing policies related to climate change that may provide benefits to future and not current generations, and those benefits may be expressed not necessarily in monetary terms (Stern, 2013; Pindyck, 2013).

Companies pursue green activities and issue green finance instruments in different countries, with the US, China and the EU leading this market. The organizations reporting on green finance activities have balance sheets cumulating close to $120 trillion.1 With increased global demand for green financial products investors also need to understand better the linkages between these green indices and other important financial variables.

In this paper, the authors examine the multivariate dependencies of stock indices adjusted for green revenues of companies from major economies and other important macroeconomics and financial variables. The focus is on the connections between five main green finance indices produced by FTSE Russell, with the main objective being to understand the linkages between indices associated with various geographical areas and also their linkages with other important financial variables that may play a role in green finance such as oil prices (Oil), Treasury Bond 10-year prices (Bond), gold prices (Gold), Microsoft and Apple share prices (Microsoft, Apple) and the CBOE VIX series (VIX). The economies of United States, Europe, United Kingdom and China cover jointly a high level of climate finance activities (Buchner et al., 2019). If green finance is high on the agenda of the governments in all major economies then this should be reflected by the data. While there is a collective goal towards green finance, different countries may have different policies and legal systems in place that their companies must follow. Therefore, a priori it is difficult to establish how strong is the connection between green finance revenues of companies in China or Europe and those of companies in the US, for example. The common political efforts of reducing carbon emissions among these three economies may lead to a tripartite interaction which can be easily detected; however, standard association measures such as pairwise correlation coefficients would no longer adequately measure the level of this interaction.

In the preliminary analysis, the authors apply a new and innovative generalization of the

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1See Carney (2019) and https://www.fsb-tcfd.org/tcfd-supporters/ for a more detailed list. Four fifths of the top 1100 global companies are regularly reporting climate-related financial risks.
Pearson correlation coefficient that works beyond the bidimensional case. In particular, they calculate the generalized correlation coefficients for 3-dimensional and 4-dimensional subgroups of variables representing green finance indices. These less known generalized coefficients are easy to compute as descriptive statistics. If the understanding and the implementation of green finance issues are similar in different economic areas, then the various green finance indices will exhibit a strong interdependence. The main contribution of this analysis to the relevant literature is the application in a financial context, of multi-dimensional correlation coefficients among various subgroups of the variables under study.

Moreover, for the first time in this new strand of literature on green finance, the standard linear Pearson correlation coefficients are shown to be potentially misleading in describing the correlation structure among the variables, with the authors arguing that a more appropriate tool to capture the association structure in the data is the partial correlation coefficient.

The analytics of multivariate data would certainly benefit from more advanced techniques that go beyond standard measures such as correlation coefficients, linear regression models and principal components analysis. To this end, this study advocates employing graphical models as the main inferential tool to capture the interconnectedness between green finance indices and other important financial variables. This class of models is based on conditional independencies between pairs of variables and subgroups of variables, allowing a state-of-the-art analysis of the relationships among a group of variables, such as the green finance indices. The conditional independencies are measured together with the conditional interactions and the model can be visually expressed as a graph showing all interactions between groups or subgroups of variables, hence the name, graphical models. Based on the proposed methodology and its contribution to finance, the paper identifies and demonstrates the value of graphical modeling as an important addition to the financial economists’ tool set.

Graphical models are directly interpretable using graphs and therefore they are the natural statistical tools for depicting the architecture of networks of variables driving financial systems. The empirical results clearly indicate that green finance is pivotal to the current financial systems, but their relevance differs geographically. The inter-linkages between various financial markets has been investigated more intensively over the last decade, see Anton and Polk (2014), Reboredo (2018), Reboredo (2020), Raddant and Kenett (2021), Ando et al. (2022). In contrast with the network construction methodology presented by Diebold and Yilmaz (2014), which is based on volatility spillover effects, the graphical models approach tests for conditional independencies among subgroups of variables given the information on the remaining variables from the total set of variables under study. In addition, multivariate correlations that graphical models are naturally build on, are investigated, an aspect that is not developed for the technique in Diebold and Yilmaz (2014).²

²Although the literature on graphical models applied to financial problems is sparse, graphical models have been used as improved analytical tools for extracting information from financial datasets. Stanghellini et al. (1999) identified a discrete-variable chain graph model for selecting creditworthy retail credit applicants. Considering the relationships among financial ratio measures in the United Kingdom, Watson and Tunaru (2000) employed graphical modelling to disentangle the important versus less important financial measures. Chain graphical modeling has been applied in Fabozzi et al. (2007) to understand the complex relationships regarding the potential educational and work experience factors contributing to the performance and incentive satisfaction of fund managers.
One may argue that the rise of green finance and economics started with Georgescu-Roegen (1971) who was the first to advocate that any economic planning should take into account the limited biological resources of the planet. This was followed up by pioneering work on the economics impact of climate finance by William Nordhaus, the 2018 recipient of the Nobel Memorial Prize in Economic Sciences for his work in “integrating climate change into long-run macroeconomic analysis”, see Nordhaus (1977, 2019).

Environmentally responsible funds or green funds are an investment subclass of socially responsible investment (SRI) funds (Derwall et al., 2005; Konar and Cohen, 2001). On green investments, previous research looked mainly at environmental investing from a corporate finance perspective. Early research, such as Walley and Whitehead (1994), argued that if companies employ their financial resources to enhance environmental performance then that would lead to decreasing shareholder value because of higher product prices that translates into a lower profitability. Chava (2014) pointed out that investors require substantially higher expected returns on stocks that do not pass environmental filters compared to the stocks of firms not affected by these environmental concerns and furthermore, lenders also require a substantially higher interest rate charge on the loans issued to stocks of firms in the former category.

Studying positive corporate events defined by environmental prizes awarded to companies by third parties, Klassen and McLaughlin (1996) found evidence that those events are followed by positive subsequent abnormal returns while bad environmental events are followed by significantly negative returns. Heinkel et al. (2001) argued that the proportion of green investors must be above 20% in order to incent polluting firms to change their carbon-emission policy. Recent studies focused on how climate change may impact or be impacted by various asset markets. Andersson et al. (2016) suggest that a passive investment strategy focused on the stock of low-carbon emission companies may be able to hedge against climate risk. Choi et al. (2019) explain how investors may update their beliefs about climate risk, whilst Hong et al. (2019) analyze whether international stock markets incorporate in their price drought risk. Based on a global survey of institutional investors on their beliefs about climate risk, Krueger et al. (2020) find that institutional investors still consider climate risk as important although currently not as important as financial, legal, and operational risk.\(^3\) Focusing on data from the fund management sector, Alok et al. (2020) study whether the risk of climate disasters is mis-estimated in this industry, with managers overreacting to climate disaster events occurring near their offices and underweighting stocks of firms from disaster affected areas. Based on firm-level data, Addoum et al. (2020) estimate that extreme temperatures may impact negatively corporate earnings. Furthermore, their conclusions imply that companies in developed countries may be less likely on average to be affected by extreme temperature.

Pastor et al. (2020) demonstrated that in equilibrium, green assets produce low expected returns because investors have an ESG preference towards them and because green assets provide a hedge for climate risk. They also conclude that green assets outperform following positive shocks on the ESG factor, suggesting that the social benefits of sustainable investing leads to a shifting in real investment toward green firms. This is in line with Bolton and Kacperczyk

\(^3\)Note that operational risk does cover inappropriate actions related to extreme disaster events.
(2021) who provide evidence that investors require additional premium compensation for taking on carbon risk. At the same time Ilhan et al. (2021) show that the uncertainty of the impact of future climate regulation is already priced in the option markets and that firms which are less green have more tail risk and more variance risk. An ESG-efficient frontier framework is proposed in Pedersen et al. (2021). Proxing the environment impact with the firm’s carbon intensity impact the authors find no significant ex post improvement to the Sharpe ratio of an investor who takes into account the environment efficient frontier.

Lundgren et al. (2018) study how the stock market is influenced by renewable energy. They also find that, uncertainty variables such as VIX, stock market indices such as STOXX600, and Oil prices, play an important role in the network including renewable energy. Hammoudeh et al. (2020) focused on the relationship between green bonds and financial and environmental variables between 2016 and 2020, and they found evidence that U.S. 10-year Treasury bonds are strongly linked to green bonds, whereas clean energy index and CO2 emission are not. Furthermore, they could not find any evidence supporting a linkage between the green bonds index and other environmental and financial variables.

Hong et al. (2020) point out to several important recent papers in the climate finance literature. Climate econometrics is a new area of research that has emerged recently and an overview of this new discipline is provided by Castle and Hendry (2020). The correlation coefficients of stock return time series play an important role in the financial market. There are many studies in the finance literature that explore correlations and comovements and provide theoretical underpinnings such as the habitat-based framework of return comovement, consistent with observed empirical features, see Barberis et al. (2005), Green and Hwang (2009), Chen et al. (2016), Anton and Polk (2014). The majority of methodological innovations in those papers are based on univariate and bivariate regressions, restricting the multidimensional aspect of the analysis. More specifically, it is the inverse of the covariance matrix rather than the covariance matrix itself that will be used to describe relevant dependencies among a group of variables.

**Description of the Green Finance Indices Dataset**

A major challenge related to the green economy and green finance is the absence of consistent taxonomy and definitions of these concepts. FTSE Russell research points out that less than 30% of companies with green revenues provide disclosures that are granular enough to permit investors to systematically identify and quantify companies’ green business activities (see Kooroshy et al., 2020).

Green revenues data are collected from publicly available information by FTSE Russell analysts who carry out a quality control of all data including consistency checks over time, sector-relative checks, and knowledge checks, with any discrepancies verified on primary data sources. The FTSE Russell’s Green Revenues data model produces the green revenue exposure of more than 16,000 securities across 48 developed and emerging markets based on FTSE Russell’s Green Revenues Classification System — a comprehensive taxonomy for green products and services covering 10 subsectors, 64 subsectors and 133 micro sectors (see FTSE Russell, 2020; Kooroshy et al., 2020, for a full description and examples). Each green revenues
is mapped to one or more micro sectors and then aggregated at the company level. For all companies belonging to one or more of the green subsectors a total percentage(s) of revenue from green products is calculated. The overall methodology is based on the impact on climate change mitigation and adaptation, water, resource use, pollution, and agricultural efficiency.

The data comprise five FTSE Green Revenues Index series: FTSE Europe GR Index (Europe), FTSE China GR Index (China), FTSE All Share GR Index (UK), Russell 1000 GR Index (US), and FTSE All-World GR Index (All World). These indices have been constructed by FTSE Russell in London to capture changes in the revenues of companies during the transition period to improve climate change, depletion of resources and structural environmental damage. FTSE Green Revenues Index series are effectively the result of following companies that generate green revenues – which is an important component that has not being considered in current sustainability models.

Other data series used in this study include the Oil prices (Oil), U.S. Treasury bond 10-year prices (Bond), Gold prices (Gold), Microsoft and Apple share prices (Microsoft, Apple) and the CBOE Volatility index series (VIX). These variables are used to enhance the analysis of linkages between green indices and other important financial variables that have been found to drive returns in financial markets. The globalization of financial markets occurred for centuries (Lothian, 2002). There is a significant stream of literature linking stock returns to macroeconomic financial variables. The link between stock returns and the government bond yields has long been established, see Campbell (1987). There is evidence that changes in oil prices predict stock market returns worldwide (Driesprong et al., 2008). The implied volatility index VIX has been used as a measure of expectation of future returns uncertainty, playing a significant role in predicting stock returns (see Bollerslev et al., 2014, among others). The connection between gold prices and future stock returns is studied in Huang and Kilic (2019). Recent research shows that the returns of technology companies predict stock returns of firms using those technologies, emphasizing that there is a link between equity valuation and firms’ technological capabilities (Lee et al., 2019).

The data are daily observations between 28 June 2010 and 8 June 2018. Exhibit 1 reports the descriptive statistics of the logarithmic returns for all five green revenues indices. The largest mean return is observed for the Russell 1000 GR Index while the lowest is for FTSE Europe GR Index. The FTSE China GR Index has the largest uncertainty as measured by the standard deviation of the returns for this index, while the FTSE All-World GR Index has the lowest uncertainty, not surprising perhaps given the highest degree of diversification for this index.

**Hypotheses**

The purpose of this study is to understand the linkages or associations between green revenues indices and other important financial variables. The research questions are motivated by financial economics intuition. Gold is a unique investment asset with its value increasing when there is geo-political turmoil. A priori, one would not expect to see any strong relationship between gold price returns and green revenues indices returns. Furthermore, one would expect technol-
Exhibit 1. Descriptive statistics of the logarithmic return series for the five FTSE Green Revenues Indices

<table>
<thead>
<tr>
<th></th>
<th>All-World</th>
<th>Europe</th>
<th>China</th>
<th>US</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.00031</td>
<td>0.00017</td>
<td>0.00020</td>
<td>0.00047</td>
<td>0.00024</td>
</tr>
<tr>
<td>Median</td>
<td>0.00053</td>
<td>0.00040</td>
<td>0.00003</td>
<td>0.00044</td>
<td>0.00029</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>0.00806</td>
<td>0.01185</td>
<td>0.01281</td>
<td>0.00917</td>
<td>0.00898</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.92599</td>
<td>4.91024</td>
<td>3.27945</td>
<td>5.32313</td>
<td>2.68780</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.62544</td>
<td>-0.49673</td>
<td>-0.24252</td>
<td>-0.57107</td>
<td>-0.25935</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.05158</td>
<td>-0.09107</td>
<td>-0.06777</td>
<td>-0.07140</td>
<td>-0.04668</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.04133</td>
<td>0.05938</td>
<td>0.06639</td>
<td>0.04836</td>
<td>0.03774</td>
</tr>
</tbody>
</table>

Notes: The data used to produce these results are daily observations covering the period June 28, 2010 to June 8, 2018.

Technology companies to be directly involved with the greenification of the world economy. Apple is directly involved with green bond issuance, having raised USD 1.5 billion in 2016 and USD 1 billion in 2017 whilst at the end of 2019, it issued EUR 2 billion of six-year bonds with no coupon and 12-year bonds with offer a 0.5% coupon rate. Microsoft is also considered to be a relevant technology company that may be involved with green finance. Including the share prices of Tesla, the largest producer of electric cars, in the analysis would be another alternative for a company with green impact. However, the company’s net carbon footprint has never been disclosed. Furthermore, Tesla’s $1bn investment in bitcoin in 2021 caused concerns in terms of ESG-values to investors. Bank of America estimated that the bitcoin investment would produce the same carbon emissions as the annual output of more than one million cars due to energy usage associated with the bitcoin. Given the ambiguity surrounding its disclosure policies, Tesla has not been included in the current study.

Green finance is directly related to climate change and out of all macroeconomic variables, the oil prices stand out as highly relevant for green cash flows. The connection may be in the reverse direction in the sense that cash flows dependent on oil prices fall out of the green category. The mechanism to categorise cash flows as green and to adjust FTSE indices accordingly, is the same irrespective of geographical locations. Furthermore, due to global integration of major economies, many companies which are constituents of a particular index conduct business with companies that are constituents in another index. If a particular cash flow is green for one company, then it is green also for its counterparty. Hence, this double green effect will contribute to a high level of connection among green revenues indices.

Given the set of variables under investigation and based on graphical models identified to

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4Microsoft has been regarded as potentially carbon neutral since 2012 because they were offsetting polluting emissions with cash incentives to remove those emissions from the atmosphere. On January 2020 the Microsoft CEO announced the creation of a billion dollar climate innovation fund supporting the company’s commitment to become carbon negative and water positive by 2030 and to remove by 2050 all their historical carbon emissions since the company started in 1975.
fit the data well, the following hypotheses are tested:

**Hypothesis 1.** There is an association among all green indices in different parts of the world.

**Hypothesis 2.** There is an association between technological stocks and green revenues indices.

**Hypothesis 3.** There is an association between oil prices and green revenues indices.

If this methodology works well, then a financial variable such as Gold price should be only weakly associated and possibly not associated at all with any of the green variables. *A priori*, investors may assume a weak or no association between gold prices and green equity cash flows, so they may form expectations of rejecting the following hypothesis:

**Hypothesis 4.** There is an association between gold price and on green revenues indices.

All hypotheses are tested based on associations, or interactions as they are also referred to in this paper, based on the terminology in the field of graphical modeling. The associations can be of order two (i.e. common pairwise associations), but also of order three (i.e. groups of three indices all mutually associated), of order four (i.e. all four variables interacting as a group), and so on. The advantage of graphical models is their ability to reveal associations of any order on the conditional independence graph (also called graphical interaction graph), as described later in the paper. The next section presents a small scale investigation, involving only the green indices, in order to exemplify the steps of the construction of a graphical model for identifying conditional independencies.

**A Preliminary Correlation Analysis**

Many researchers when considering the association between the variables in a given study start with a preliminary analysis of the Pearson linear correlation coefficients. For the analysis of green revenues indices, the sample correlation matrix is reported in the top panel of Exhibit 2. All linear correlations are significant. The weakest link among this group seems to be between China and the US ($\rho = 0.22$), while the strongest link appears to be between All-World and the US ($\rho = 0.88$).

The limitation of the linear correlation coefficients is that they cannot capture the entire association structure of a pair of variables. The plot in Exhibit 3 illustrates the pairwise scatter plots and the individual histograms for daily green revenues indices logarithmic returns for All-World, Europe, China, the US and the UK, over the period 29 June 2010 to 8 June 2018.

The distributions of returns for all five indices show no sign of fat tails. This could be due to the fact that green equity finance is a market in nascency. There seems to be a high correlation of the returns on the green index for the All-World with the green revenue index returns for Europe and the US – almost identical plots– and with the UK but less with China. The bivariate plots suggests some weak correlation between the green index returns for China and the green revenue index returns for the UK but less so between China and Europe or US. A pairwise bivariate correlation analysis cannot tell the full story on the complex interconnectedness among variables driving the financial system. What is needed here to understand the network of financial variables under study is a set of tools that can deal with multivariate correlation or dependency among subsets of variables.
Exhibit 2. Correlation matrix and partial correlation matrix of green indices logarithmic returns for All-World, Europe, China, the US and the UK.

<table>
<thead>
<tr>
<th>Index</th>
<th>All-World</th>
<th>Europe</th>
<th>China</th>
<th>US</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-World</td>
<td>100.00</td>
<td>87.82</td>
<td>46.98</td>
<td>88.14</td>
<td>80.51</td>
</tr>
<tr>
<td>Europe</td>
<td>87.82</td>
<td>100.00</td>
<td>38.58</td>
<td>63.51</td>
<td>85.50</td>
</tr>
<tr>
<td>China</td>
<td>46.98</td>
<td>38.58</td>
<td>100.00</td>
<td>22.07</td>
<td>40.64</td>
</tr>
<tr>
<td>US</td>
<td>88.14</td>
<td>63.51</td>
<td>22.07</td>
<td>100.00</td>
<td>62.50</td>
</tr>
<tr>
<td>UK</td>
<td>80.51</td>
<td>85.50</td>
<td>40.64</td>
<td>62.50</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Partial correlation matrix of green indices.

<table>
<thead>
<tr>
<th>World</th>
<th>Europe</th>
<th>China</th>
<th>US</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>World</td>
<td>100.00</td>
<td>81.38</td>
<td>67.57</td>
<td>92.61</td>
</tr>
<tr>
<td>Europe</td>
<td>81.38</td>
<td>100.00</td>
<td>-50.89</td>
<td>-69.80</td>
</tr>
<tr>
<td>China</td>
<td>67.57</td>
<td>-50.89</td>
<td>100.00</td>
<td>-64.50</td>
</tr>
<tr>
<td>US</td>
<td>92.61</td>
<td>-69.80</td>
<td>-64.50</td>
<td>100.00</td>
</tr>
<tr>
<td>UK</td>
<td>-4.43</td>
<td>45.93</td>
<td>15.56</td>
<td>12.13</td>
</tr>
</tbody>
</table>

Notes: The data used to produce these results are daily returns covering the period June 29, 2010 to June 8, 2018. The numbers presented are percentages (%).

Exhibit 3. The plot matrix for Green Finance Indices logarithmic returns for All-World, Europe, China, the US and the UK, over the period June 29, 2010 to June 8, 2018, with daily frequency.
Decomposition of Multivariate Correlation Measures

Wang and Zheng (2016) generalized the Pearson coefficient of linear correlation to multiple variables. Consider now a set of variables \( \{Y_1, Y_2, \ldots, Y_d\} \) and let us denote by \( R \) the correlation matrix constructed from pairwise Pearson correlation coefficients

\[
R = \begin{pmatrix}
1 & \rho_{Y_1Y_2} & \cdots & \rho_{Y_1Y_d} \\
\rho_{Y_2Y_1} & 1 & \cdots & \rho_{Y_2Y_d} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{Y_dY_1} & \rho_{Y_dY_2} & \cdots & 1
\end{pmatrix}
\]  

Then the square of the multiple uncorrelation coefficient (MUC) is defined as

\[
\psi^2_{Y_1Y_2\ldots Y_d} = \det(R)
\]

It then follows by complementarity that the square of the multiple correlation coefficient (MCC) is

\[
\rho^2_{Y_1Y_2\ldots Y_d} = 1 - \psi^2_{Y_1Y_2\ldots Y_d}
\]

There are interesting properties discussed in detail by Wang and Zheng (2016). For the multiple correlation and uncorrelation coefficients

\[
0 \leq \rho^2_{Y_1Y_2\ldots Y_d} \leq 1 \quad \text{and} \quad 0 \leq \psi^2_{Y_1Y_2\ldots Y_d} \leq 1
\]

Moreover, \( \rho^2_{Y_1Y_2\ldots Y_d} = 1 \) if and only if the variables \( Y_1, Y_2, \ldots, Y_d \) are linearly dependent and \( \rho^2_{Y_1Y_2\ldots Y_d} = 0 \) if and only if the variables \( Y_1, Y_2, \ldots, Y_d \) are mutually orthogonal (uncorrelated). In addition, \( \rho^2_{Y_1Y_2\ldots Y_j} \geq \rho^2_{Y_1Y_2\ldots Y_{j-1}} \) and \( \psi^2_{Y_1Y_2\ldots Y_j} \leq \psi^2_{Y_1Y_2\ldots Y_{j-1}} \) for all \( j \in \{2, \ldots, d\} \).

The data series on green finance indices for All-World, Europe, China, the US and the UK are associated with the set of variables \( \{Y_1, Y_2, \ldots, Y_5\} \), respectively. To simplify the notation, for the multiple uncorrelation and correlation coefficients a subscript one-to-one notation is preserved. Therefore, \( \psi^2_{123} \) for example, will refer to the 3-dimensional multiple uncorrelation coefficient calculated from return series of All-World, Europe, and China, while \( \rho^2_{345} \) for example will refer to the 3-dimensional multiple correlation coefficient calculated from return series of China, the US and the UK.

Exhibit 4 reports the squared values of all multiple 3-dimensional uncorrelations and correlations for green indices logarithmic returns for All-World, Europe, China, the US and the UK for the studied period. The highest level of correlation occurs for the group of All-World, Europe and the US with \( \rho^2_{124} = 0.97 \), while the largest squared uncorrelation coefficient corresponds to two groups, the first group being Europe, China and the US and the second group being China, the US and the UK, both with \( \psi^2_{234} = \psi^2_{345} = 0.51 \). Equivalently, the two 3-dimensional (squared)correlations that include China are the smallest when combined with other single economies ( \( \rho^2_{234} = \rho^2_{345} = 0.49 \)). These findings, based on a simple graphical model with only five nodes, indicate that China’s green index returns are somewhat least correlated with the returns on the other four green indices All-World, Europe, the US and the UK.

The results for the 4-dimensional correlation and uncorrelation coefficients presented in Exhibit 5 reveal higher level of association than in the 3-dimensional case. The largest correlation occurs for the group All-World, Europe, US and UK as \( \rho^2_{1245} = 0.99 \), while the largest un-
correlation is obtained for the group Europe, China, the US and the UK, with $\psi_{1245}^2 = 0.13$. Finally, the squared value for the 5-dimensional correlation index for all variables is computed as 0.9963, implying that there is a very high overall association for the green finance indices under study.

Exhibit 4. Multiple 3-dimensional uncorrelations and correlations for green indices logarithmic returns for All-World, Europe, China, the US and the UK.

<table>
<thead>
<tr>
<th>3-dim correlation</th>
<th>MCC</th>
<th>3-dim uncorrelation</th>
<th>MUC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{123}^2$</td>
<td>82.24</td>
<td>$\psi_{123}^2$</td>
<td>17.76</td>
</tr>
<tr>
<td>$\rho_{124}^2$</td>
<td>96.82</td>
<td>$\psi_{124}^2$</td>
<td>3.18</td>
</tr>
<tr>
<td>$\rho_{125}^2$</td>
<td>94.14</td>
<td>$\psi_{125}^2$</td>
<td>5.86</td>
</tr>
<tr>
<td>$\rho_{134}^2$</td>
<td>86.35</td>
<td>$\psi_{134}^2$</td>
<td>13.65</td>
</tr>
<tr>
<td>$\rho_{135}^2$</td>
<td>72.66</td>
<td>$\psi_{135}^2$</td>
<td>27.34</td>
</tr>
<tr>
<td>$\rho_{145}^2$</td>
<td>92.87</td>
<td>$\psi_{145}^2$</td>
<td>7.13</td>
</tr>
<tr>
<td>$\rho_{234}^2$</td>
<td>49.27</td>
<td>$\psi_{234}^2$</td>
<td>50.73</td>
</tr>
<tr>
<td>$\rho_{235}^2$</td>
<td>77.68</td>
<td>$\psi_{235}^2$</td>
<td>22.32</td>
</tr>
<tr>
<td>$\rho_{245}^2$</td>
<td>84.62</td>
<td>$\psi_{245}^2$</td>
<td>15.38</td>
</tr>
<tr>
<td>$\rho_{345}^2$</td>
<td>49.24</td>
<td>$\psi_{345}^2$</td>
<td>50.76</td>
</tr>
</tbody>
</table>

Notes: The data used to produce these results are daily returns covering the period June 29, 2010 to June 8, 2018. The numbers presented are percentages (%).

Exhibit 5. Multiple 4-dimensional uncorrelations and correlations for green indices logarithmic returns for All-World, Europe, China, the US and the UK.

<table>
<thead>
<tr>
<th>4-dim correlation</th>
<th>MCC</th>
<th>MUC</th>
<th>4-dim uncorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{1234}^2$</td>
<td>98.54</td>
<td>$\psi_{1234}^2$</td>
<td>1.46</td>
</tr>
<tr>
<td>$\rho_{1235}^2$</td>
<td>95.50</td>
<td>$\psi_{1235}^2$</td>
<td>4.50</td>
</tr>
<tr>
<td>$\rho_{1245}^2$</td>
<td>99.19</td>
<td>$\psi_{1245}^2$</td>
<td>0.81</td>
</tr>
<tr>
<td>$\rho_{1345}^2$</td>
<td>95.68</td>
<td>$\psi_{1345}^2$</td>
<td>4.32</td>
</tr>
<tr>
<td>$\rho_{2345}^2$</td>
<td>87.31</td>
<td>$\psi_{2345}^2$</td>
<td>12.69</td>
</tr>
</tbody>
</table>

Notes: The data used to produce these results are daily returns covering the period June 29, 2010 to June 8, 2018. The numbers presented are percentages (%).

**Graphical Modelling of Green Revenue Indices**

The information on correlations revealed by the correlation matrix in the upper panel of Exhibit 2 can be misleading because it captures only the pairwise marginal linear correlations. A more informative measure is the partial correlation matrix that takes into account the whole structure of the data. The *partial correlation matrix* has non-significant off-diagonal elements.
when the corresponding variables are independent conditional on all the remaining variables in the entire study set. Hence, the partial correlation matrix reflects the conditional independencies structure within the network of variables.

The results for the computed partial correlation matrix are presented in the lower panel of Exhibit 2. There are several important observations coming out of this analysis. While the correlations among all possible pairs of green indices were all positive, the partial correlations are positive and negative. The negative ones that were not picked by the standard Pearson correlations are in pairs, World and the UK, Europe and China, Europe and the US, China and the US. Moreover, the partial correlation between World and the UK looks is negative and smaller than 5% in value whilst the partial correlations between the UK and China and the US are only marginally greater than 10%.

Better inference can be achieved with the help of appropriate models that permit formal testing of these associations measured by partial correlation coefficients. Graphical models are a relatively new class of models based on conditional independencies. The subclass of Gaussian graphical models is very flexible, covering many interesting conditional independence models and benefitting from inference being driven by the concentration matrix $K$ that is the inverse of the covariance matrix $\Sigma$. The off-diagonal elements of $K$, in the Gaussian special case, reveal directly the pairwise conditional independencies among the variables under the analysis. The methodology for constructing graphical models is described in the Online Appendix.

Model selection starts from the saturated model including all interactions among variables and continues testing and removing those links between variables that are not significant, leading to a simpler model that fits the data well and it is easier to interpret. The model selection is illustrated under both the AIC and BIC model selection yardsticks. The final hypotheses testing is carried out based on the model identified with BIC, since this procedure is consistent with selecting the true model (Yang, 2005).

Exhibit 6 illustrates the selected graphical model for the green finance indices. Starting from a saturated model that includes all possible pairwise interactions represented as edges on the graph, the inference process tests the elimination of each possible edge. Hence, the data implies that only the edge between the UK green index and the World green index should be removed, for all the other edges the p-value of the $\chi^2$ test being less than the 5% level, rejecting the null hypothesis of conditional independence between the variables represented by the corresponding nodes on the graph. The procedure is iterative, removing the edges with the largest significant p-values until no edge can be removed based on the individual test.

The graphical model described by Exhibit 6 shows that the nucleus of green finance seems to be spanned by China, US and Europe. Given the information in this set of variables, the green finance index for the UK is conditionally independent of the green finance index for All-World. There is no surprise in the central role played by the economic superpowers such as China, the US and Europe. This analysis reveals that those three economies are core to the interdependencies regarding green finance and there is clear evidence supporting Hypothesis 4.

Policymakers may use this information to focus more on connecting more the companies in the UK with those in the rest of the world with respect to green economy activities and revenues. Moreover, it is evident that a policy change disrupting the pathway to greener economies in China, Europe or the US, would decisively impact the course of green finance worldwide.
Notes: Calculations are based on logarithmic returns for All-World, Europe, China, the US and the UK, over the period June 29, 2010 to June 8, 2018, with daily frequency. The model is selected using stepwise searching and BIC criterion.

The Links between Green Revenue Indices and Other Financial Variables

In this section the graphical model is expanded to ten variables (nodes), with the aim to detect possible linkages among green revenue indices of the four major economies the US, the UK, Europe and China and various financial variables such as oil prices (Oil), Treasury Bond 10-year prices (Bond), gold prices (Gold), Microsoft and Apple share prices (Microsoft, Apple) and the CBOE VIX series (VIX). Oil and Bond prices are very important variables for all major economies, Microsoft and Apple are representing the new technologies sector driving innovation. Gold prices and VIX are variables that measure turbulences in the world economy; Gold prices increase when geo-political risk is elevating and are generally low during calmer times, whilst VIX is high when there is a general fear that equity markets may experience a negative period and low during normal times. The data used in the implementation of this graphical modelling analysis are daily logarithmic returns for all variables, except for VIX for which first differences are considered.

5We drop the All-World green index from the analysis since this index, by construction, is interlinked with all other variables worldwide. Other researchers may be interested in larger models by including other/more financial variables of their interest.
The model selection starts from the saturated model spanned by all ten variables of interest. A stepwise model selection algorithm is implemented using the Akaike Information Criterion (AIC) as the measure to discriminate among various models. This model will referred to as M-AIC, henceforth. Furthermore, given that models may become very complex in terms of the number of parameters, the Bayesian Information Criterion (BIC) that applies a higher penalty for model complexity is employed. This model will be referred to as M-BIC, henceforth. Exhibit 7 reports the main statistics which are relevant from a graphical modelling point of view when selecting a particular model. The comparative model fitting statistics for the two models applied to the ten variables of interest are presented in Exhibit 8 for convenience.

Exhibit 7. The estimated partial correlation matrix for green indices and other financial variables

<table>
<thead>
<tr>
<th></th>
<th>Europe</th>
<th>China</th>
<th>US</th>
<th>UK</th>
<th>Bond</th>
<th>Oil</th>
<th>Gold</th>
<th>Microsoft</th>
<th>Apple</th>
<th>VIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe</td>
<td>100</td>
<td>38</td>
<td>64</td>
<td>86</td>
<td>40</td>
<td>36</td>
<td>-3</td>
<td>40</td>
<td>29</td>
<td>-50</td>
</tr>
<tr>
<td>China</td>
<td>38</td>
<td>100</td>
<td>22</td>
<td>40</td>
<td>15</td>
<td>14</td>
<td>5</td>
<td>15</td>
<td>15</td>
<td>-17</td>
</tr>
<tr>
<td>US</td>
<td>64</td>
<td>22</td>
<td>100</td>
<td>63</td>
<td>44</td>
<td>37</td>
<td>-2</td>
<td>66</td>
<td>54</td>
<td>-83</td>
</tr>
<tr>
<td>UK</td>
<td>86</td>
<td>40</td>
<td>63</td>
<td>100</td>
<td>39</td>
<td>35</td>
<td>-3</td>
<td>40</td>
<td>29</td>
<td>-51</td>
</tr>
<tr>
<td>Bond</td>
<td>40</td>
<td>15</td>
<td>44</td>
<td>39</td>
<td>100</td>
<td>28</td>
<td>-2</td>
<td>25</td>
<td>22</td>
<td>-36</td>
</tr>
<tr>
<td>Oil</td>
<td>36</td>
<td>14</td>
<td>37</td>
<td>35</td>
<td>28</td>
<td>100</td>
<td>-2</td>
<td>20</td>
<td>17</td>
<td>-28</td>
</tr>
<tr>
<td>Gold</td>
<td>-3</td>
<td>5</td>
<td>-2</td>
<td>-3</td>
<td>-2</td>
<td>-2</td>
<td>100</td>
<td>-5</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Microsoft</td>
<td>40</td>
<td>15</td>
<td>66</td>
<td>40</td>
<td>25</td>
<td>20</td>
<td>-5</td>
<td>100</td>
<td>39</td>
<td>-54</td>
</tr>
<tr>
<td>Apple</td>
<td>29</td>
<td>15</td>
<td>54</td>
<td>29</td>
<td>22</td>
<td>17</td>
<td>3</td>
<td>39</td>
<td>100</td>
<td>-45</td>
</tr>
<tr>
<td>VIX</td>
<td>-50</td>
<td>-17</td>
<td>-83</td>
<td>-51</td>
<td>-36</td>
<td>-28</td>
<td>0</td>
<td>-54</td>
<td>-45</td>
<td>100</td>
</tr>
</tbody>
</table>

Notes: Calculations based on logarithmic returns for variables over the period June 29, 2010 to June 8, 2018, daily frequency.


<table>
<thead>
<tr>
<th>statistics</th>
<th>M-AIC</th>
<th>M-BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( det(K) )</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>( \log L )</td>
<td>-29656.91</td>
<td>-29682.34</td>
</tr>
<tr>
<td>( \log L - \text{saturate} )</td>
<td>-29653.70</td>
<td>-29653.70</td>
</tr>
<tr>
<td>( \text{deviance} )</td>
<td>66.42</td>
<td>57.27</td>
</tr>
<tr>
<td>AIC</td>
<td>59397.82</td>
<td>59420.67</td>
</tr>
<tr>
<td>BIC</td>
<td>59632.72</td>
<td>59577.27</td>
</tr>
</tbody>
</table>

Exhibit 7 implies that Gold has a very weak connection to all other variables. Most of the weight in the partial correlations is in the green indices zone, with China showing weaker
correlations in relative terms. The Bond variable has almost equal partial correlation with Europe, the US and UK and the same conclusion can be drawn for the Oil variable. The tech stocks display a stronger partial correlation to the US, while VIX has stronger links to Europe, the US and the UK green indices cash flows.

The model depicted by the conditional independence graph illustrated in Exhibit 9 allows us to identify directly the variables that are central in terms of associations and also those that are more peripheral. The model is selected with the AIC criterion and the overall picture is only slightly simplified. One straightforward way to measure the degree of interconnectedness among variables under study is to count the number of links (edges) coming out of each variable (node). Surprisingly, the U.K. green index has the highest degree of interconnectedness with nine edges being linked to it.

The U.S. green index, Europe green index and Microsoft share price returns follow very closely, each having eight edges. Microsoft is an example of a very green company given that its business model is focused on software and technology. Less connected than the other green indices is China green index, with only six edges linked to its node. Importantly, from this model, looking at its conditional independence graph, China green index appears independent of Bond, Oil and VIX conditional (given) the green indices for the UK, the US and Europe and Microsoft. Furthermore, the returns on the China green index are directly linked to returns on Gold price and Apple share price.

The variables added to the green indices play a peripheral role. Gold and VIX have the lowest degree of interconnectedness with only four edges. It is also interesting to see that Bond is conditionally independent of VIX given Oil, Europe, the UK and the US. Likewise, Bond, Oil and VIX are conditionally independent of Gold and Apple given the rest of the variables.

From the analysis of the graphical model illustrated in Exhibit 9, one can infer the decision for all four hypotheses tested in this study. Hypothesis 1 is accepted only for China and the UK, but rejected for the US and Europe. Regarding Hypothesis 2 this is accepted for China, the UK and the US in the case of Apple and for China, the UK and the US in the case of Microsoft. The same model implies that Hypotheses 3 and 4 are true for all four economies.
When the BIC criterion is used as the model criterion selector, a more simplified model is obtained. The selected model is illustrated in Exhibit 10. The model can be interpreted again directly on the conditional independence graph depicted in Exhibit 10. Gold is found to be totally independent of all the other variables, showing that Gold prices are not influenced by either green finance indices, or technological stocks, or macro-variables like Oil and Bond, or even by sentiment-based variables like VIX. VIX is linked only to the U.S. green index and the Europe green index and not to China green index and the U.K. green index. In other words, events strictly related to UK green index or China green index do not impact directly the volatility fear index VIX.

The most central variables (those with most links in the graphical model) are the U.S. green index and the Europe green index, with eight and seven edges, respectively. All four green indices are interconnected now. Other variables with a substantial degree of interconnectedness
are China and Bond, each with four edges. The UK, Oil, Bond and Apple have only three, while Microsoft and VIX have only two edges, implying that these variables are peripheral in explaining the structure generated by all 10 variables under investigation.

The graphical model in Exhibit 10 also exhibits some interesting conditional independence relationships. Conditional on the U.S. green index and Bond, Microsoft is independent of all the other variables. Likewise, given China, Europe and the U.S. green indices, Apple is independent of all the other variables. Oil is also conditionally independent of all the other variables, given Bond, Europe and the U.S. green indices, suggesting that it plays a role for greenification mainly through companies in the US and Europe.

Graphical models allow a flexible decomposition of the joint variables into a combination of marginal models with smaller dimensionality. By assessing the conditional independence graph in Exhibit 10 corresponding to the selected graphical model, one can observe that inference is determined by combining the subsets:

\[
\{\text{Gold}\}, \{\text{Apple, Europe, US, China}\}, \{\text{UK, Europe, US, China}\}, \\
\{\text{Oil, Bond, Europe, US}\}, \{\text{Microsoft, US, Bond}\}, \{\text{US, Europe, VIX}\}.
\]

Therefore, if some particular hypothesis is stated for the relationship between VIX and the green indices, further inference and modelling can be carried out looking only at the variables VIX, the US and Europe. Likewise, to investigate the relationship between Microsoft and green finance, according to the second graphical model selected with BIC, it is sufficient to look at Microsoft, the US and Bond variables.

Regarding the testing of association hypotheses, the final graphical model selected and de-
lected in Exhibit 10 provides the following results. The Hypothesis 1 is supported, confirming the same conclusion drawn from the graphical models with conditional independence graphs illustrated in Exhibit 6 and 9. The Hypothesis 2 is supported as follows: there is an association between Apple stocks and green revenues indices of the US, Europe and China, while Microsoft stock is associated only with the U.S. green index. The model also infers that the Hypothesis 3 of association between green indices and oil prices is accepted for the US and Europe but rejected for China and the UK. Last but not least, the Hypothesis 4 is rejected for all associations of gold with the green revenues indices, suggesting that gold is dissociated from economies with a stringent climate finance agenda. Finally, a strong green stock connectivity is found between the US and Europe and also between China and the UK, confirming that U.S. investors should focus more on diversifying their investments more internationally, as suggested in Becker and Schmidt (2015).

**Conclusion**

This study employs a novel approach to investigate in detail the links between the green finance indices for several major economies (Europe, China, the US and the UK), and other financial variables such as oil prices (Oil), Treasury Bond 10-year prices (Bond), gold prices (Gold), Microsoft and Apple share prices (Microsoft, Apple) and the CBOE VIX series (VIX). Implementing a stepwise model selection algorithm used in graphical Gaussian modelling, simplified models are identified to capture the true interactions in the data.

When the final model is selected using the BIC criterion, various association hypotheses can be interpreted directly on the graph. The green revenue index returns for the UK are conditionally independent of green index returns for the remaining worldwide companies, given green indices returns for Europe, China and the US. At the same time, there is evidence of a strong link between the green index returns of companies in the US and those of companies worldwide, establishing the key role that the U.S. economy can play to transform the other major economies into greener ones. Any progress towards a greener global economy is mainly driven by the US, Europe and China.

A more complex analysis augmenting the main green finance indices with technological stock variables, macro variables and volatility descriptors shows the central role played by the US and Europe green indices. It also reveals a more peripheral role played by Gold, VIX, Microsoft, Oil and Bond while Apple share price seems to be more involved with green finance. China and UK green indices are less interlinked with financial and macroeconomic variables compared to the US and Europe green indices.

Multivariate or group connections can be interpreted directly on the graph depicting the statistical model that is fitted. The green stock indices for the UK and China are more peripheral to the financial variables network than the green stock indices of the US and Europe, whilst Gold price exhibits total independence from the network.
References


Pindyck, R. (2013). Climate change policy: what do the models tell us? *Journal of Economic Literature* 51, 860–972.


