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Yin, Jiaqi, Wu, Shaomin and Spiegler, Virginia L.M. (2021) Optimisation of maintenance policies for a system with multiple deteriorating components. In: Castanier, Bruno and Cepin, Marko and Bigaud, David and Berenguer, Christophe, eds. Proceedings of the 31st European Safety and Reliability Conference Edited by Bruno Castanier, Marko Cepin, David Bigaud, and Christophe

### DOI

<https://doi.org/10.3850/978-981-18-2016-8>

### Link to record in KAR

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# Optimisation of maintenance policies for a system with multiple deteriorating components

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Condition-based maintenance (CbM) is a useful technique for scheduling maintenance policies aiming to reduce operating cost, improving the security of management, and ensuring the stable quality of the products. This paper models the deterioration process of a system composed of multiple components. Each deterioration process is modelled with the Wiener process. When a linear combination of the processes exceeds a pre-specified threshold, the block or age replacement policy will be carried out as the preventive maintenance for the system. Based on these two replacement policies, the optimized maintenance intervals are then sought. Besides, the paper also develops a cost process which considers the situation when the maintenance cost is higher than an expectation value, the decision-maker will prefer to replace the whole system but not repair it. Numerical examples are given to illustrate the optimisation process.

*Keywords:* Condition-based maintenance, Wiener process, Gamma distribution, Cost process

## 1. Introduction

Condition-based maintenance (CbM) is a useful technique for scheduling maintenance policies aiming to reduce operating cost, improving the security of management, and ensuring the stable quality of the products. In the CbM related literature, stochastic processes such as the gamma process Lawless and Crowder (2004); Cholette et al. (2019), the inverse Gaussian process Li et al. (2017); Hao et al. (2019), and the Wiener process Wen et al. (2018); Xie et al. (2019); Wang and Kang (2020) are widely used for different applications.

Basically, CbM is performed on the equipment once a parameter(s) related to the condition of the monitored system reaches a pre-specified value. Its purpose is to prevent the working efficiency of the system from reducing to an unacceptable condition or even if the system stops working completely, due to the ageing or deterioration of the system. It is therefore important to assess the status or remaining useful life of a system, which can further be used in deciding the future operation in order to maintain the system at a certain level of availability.

This paper intends to develop a deterioration process model, which is suitable for multi-component systems with multiple failure modes,

and its corresponding maintenance policies. This can help the company to develop a long-term maintenance plan to reduce maintenance costs and increase the effectiveness of a system.

### 1.1. Related work

Caballé et al. (2015) propose a condition-based maintenance strategy by combining the non-homogeneous Poisson process (NHPP) and the gamma process (GP). It models a multiple deterioration processes within dependent degradation-threshold-shock model. They also point out that the dependence analysis between the causes of failure is a potential development and the variability of the threshold should be considered in future.

Zhu et al. (2015) simulate a wear process with a non-stationary Gamma process and the random shock damage with a generalized Pareto distribution satisfying Poisson arrivals. It is worthwhile to noticing that this study does not consider the impact of shocks or inspection costs which may influence the result of a long-term optimized maintenance policy.

Liu et al. (2017) propose a new CbM model based on three-state degradation and the influence of external environmental shocks. The deterioration process of the system is modelled by a two-state Wiener process with a Doubly Stochastic Poisson Process (DSPP). It considers two different

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*Edited by yyy*

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ISBN: zzz :: doi: 10.3850/zzz\_esrel2021-paper-v04

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thresholds, namely normal threshold and defective threshold which is depending on the system state.

Wu and Castro (2020) investigate a CbM problem in which the deterioration process of a system is modelled by a weighted linear combination of multiple gamma processes for a pavement network. They also point that the degradation may follow different deterioration processes in one system. Besides, different failure modes can correspond to different thresholds which is a potential development as well.

### 1.2. Novelty and contributions

This paper models the deterioration process of a multi-component system, which is a linear combination of multiple Wiener processes. Specially, the paper investigates the cost process relating to the linear combination. Based on the cost process, it then formulates the expected cost of the lifecycle for the cases where the age replacement and the block replacement are applied.

The contribution of this paper includes

- development of a maintenance policy for a system whose deterioration process can be modelled by a linear combination of Wiener processes; and
- development of a cost process related to the linear combination of the deterioration processes.

### 1.3. Overview

The remainder of the paper is structured as follows. Section 2 lists notations and assumptions. Section 3 develops a linear combination of Wiener processes, derives the probability of the first time to exceed the pre-specified threshold, derives the cost process relating to the system deterioration process, and investigates the situation when the repair cost follows a probability distribution. Section 4 derives the maintenance policies based on the cost process. Section 5 shows some numerical examples. Section 6 concludes the paper, listed our findings, and proposes our future work.

## 2. Assumptions

### 2.1. Notation

Table 1 shows the notations used in this paper.

### 2.2. Assumption

- The system is new at time  $t = 0$ .
- Replacement is carried out every  $T$  time units.
- Degradations of different failure modes are Wiener processes, but have different parameters.
- There are  $n$  defects on the system, each defect develops from time  $t = 0$ . When a linear combination of the magnitudes of the defects exceeds a pre-specified value, the system needs replacement.

$k$	Types of failure modes.
$X_k(t)$	Degradation state of $k$ th failure modes at time $t$ .
$Y(t)$	Overall degradation of one system at time $t$ .
$\mu_k$	The drift of $k$ th failure modes.
$\sigma_k$	The infinitesimal variance of $k$ th failure modes.
$a_k$	The weight of failure mode $k$ .
$\mu_Y$	The drift of the overall degradation of one system.
$\sigma_Y$	The infinitesimal variance of the overall degradation of one system.
$T_a$	Interval time for the age replacement policy.
$T_b$	Interval time for the block replacement policy.
$C_i(T_a)$	Expected cost per unit time for the age replacement policy for maintenance policy $i$ .
$c_k$	PM cost for every unit of $k$ th failure modes.
$c_m$	Expected repair cost
$c_r$	Expected replacement cost
$C_k(t)$	Total cost of $k$ th failure modes at $t$ .
$U(t)$	Overall cost of one battery system at time $t$ .
$L$	The threshold of the degradation level for a system.
$L_c$	The threshold of the cost for a system.

- Failure modes are independent from each others.

## 3. Model development

### 3.1. Deterioration process

We assume that the system has  $k$  deterioration processes, each of which follows a Wiener process.

Let  $X_k(t)$  be the deterioration level of the  $k$ th deterioration process at time  $t$ . Then,  $X_k(t)$  have the following assumptions:

- $X_k(0) = 0$ , which also means that  $W_k(0) = 0$ ;
- $W_k(t)$  has independent increments that follows the normal distribution. That is, for  $0 < s < t$ ,  $W_k(t - s) - W_k(s)$  follows  $N(0, (t - s))$ .
- $W_k(t)$  is continuous in  $t$ .

$X_k(t)$  has said having drift coefficient  $\mu_k$  and variance parameter  $\sigma_k^2$ , the stochastic process of it is:

$$X_k(t) = \mu_k t + \sigma_k W_k(t), \quad (1)$$

where  $\mu_k$  and  $\sigma_k$  are the parameters of failure mode  $k$ , respectively,  $W_k(\cdot)$  is the standard Wiener process, which also can be called as the Brownian motion.

### 3.1.1. Basic Properties

The unconditional probability density function, which follows normal distribution with mean = 0 and variance =  $t$ , at a fixed time  $t$ :

$$f_{W_t}(x) = \frac{1}{\sqrt{2\pi t}} e^{-x^2/(2t)}.$$

We have  $E[W_k(t)] = 0$  and  $\text{Var}[W_k(t)] = t$ .

These results follow immediately from the definition that increments have a normal distribution, centred at zero.

Thus, the expected value and the variance of  $X_k(t)$  are given by:  $E(X_k(t)) = \mu_k t$ , and  $V(X_k(t)) = \sigma_k^2 t$ .

### 3.1.2. A linear combination of Wiener processes

Now let us assume  $Y(t)$  is a linear combination of  $n$  Wiener processes. The overall degradation  $Y(t)$  of the system is represented by

$$Y(t) = \sum_{k=1}^n a_k X_k(t), t \geq 0, a_k \geq 0, \quad (2)$$

where  $a_k$  is the weight of failure mode  $k$ . Fig. 1 shows the realisation of a linear combination of two Wiener processes.

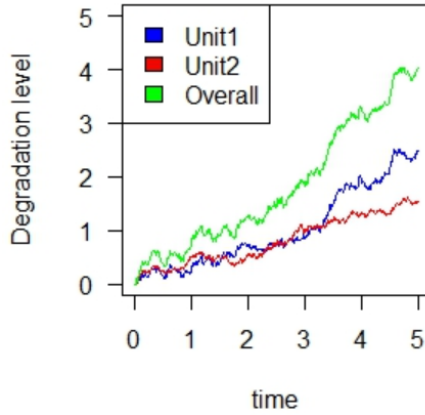


Fig. 1. Realisation of two deterioration processes and a linear combination

Furthermore, the overall deterioration process  $Y(t), t > 0$  is a stochastic process with the following properties (without the skew-normal random effects):

- $Y(0) = \sum_{k=1}^n a_k X_k(0) = 0$ ,
- $\Delta Y(t) = \sum_{k=1}^n a_k \Delta X_k(t)$  is an independent increment as well.

Thus,  $Y(t)$  would be:

$$Y(t) = t \sum_{k=1}^n a_k \mu_k + \sum_{k=1}^n a_k \sigma_k W_k(t). \quad (3)$$

Let  $\mu_Y = \sum_{k=1}^n a_k \mu_k$  and  $\sigma_Y^2 = \sum_{k=1}^n a_k^2 \sigma_k^2$ . Then  $Y(t)$  follows the normal distribution  $N(\mu_Y t, \sigma_Y^2 t)$ .

### 3.1.3. First time to exceed the pre-specified threshold $L$

The distribution of the first hitting time of the process  $\{Y(t), t \geq 0\}$ , which starts from  $Y(0) = 0$  should be obtained. The first hitting time  $\omega_{Y(t)}$  is defined when  $Y(t)$  reaches the degradation level  $L$ , according to the statistical characteristic of a Wiener process,  $\omega_{Y(t)}$  should follow the inverse Gaussian distribution Ross et al. (1996); Pan et al. (2017), then

$$\omega_L = \inf\{t > 0 : Y(t) \geq L\}, \quad (4)$$

Then, the pdf of  $\omega_L$  can be obtained as

$$\begin{aligned} f_{\omega_L}(t) &= \frac{L}{\sigma_Y \sqrt{2\pi t^3}} \exp\left(\frac{-(L - \mu_Y t)^2}{2\sigma_Y^2 t}\right) \\ &= \frac{L}{\sigma_Y \sqrt{\pi t^3}} \phi\left(\frac{-(L - \mu_Y t)}{\sigma_Y \sqrt{t}}\right), \end{aligned} \quad (5)$$

where  $\phi(\cdot)$  denotes the standard normal cdf. Then, the cdf of  $\omega_L$  is obtained by

$$\begin{aligned} F_{\omega_L}(t) &= P(Y(t) \geq L) \\ &= \Phi\left(\frac{-(L - \mu_Y t)}{\sigma_Y \sqrt{t}}\right) - \exp\left(\frac{2\mu_Y L}{\sigma_Y^2}\right) \end{aligned} \quad (6)$$

where  $\Phi(\cdot)$  denotes the standard normal cdf.

### 3.2. Repair cost process

The repair costs of different failure modes are normally different. Therefore,  $c_k$  denotes cost of repairing the  $k$ th failure mode. Besides, we consider that the actual cost is dependent on the deterioration level of the failure model. It is worth noticing that, according to Wu and Castro (2020), the total cost  $U(t)$ , which is associated to  $Y(t)$ , is also a stochastic process and does not have a linear relationship with  $Y(t)$ . As  $Y(t)$  is a Wiener process,  $U(t)$  is a Wiener process which is a sum of  $Y(t)$  with a drift.

Thus,

$$C_k(t) = a_k c_k X_k(t). \quad (7)$$

It is worth noticing that, when the cost of each inspection is considered, then the total repair cost  $U(t)$  can be represented as

$$U(t) = \sum_{k=1}^n C_k(t) = \sum_{k=1}^n a_k c_k X_k(t), \quad (8)$$

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$U(t)$  is a Wiener process with a linear drift.

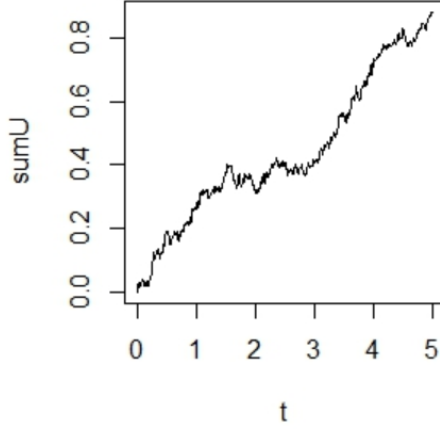


Fig. 2. Cost process of  $C(t)$

As  $X_k(t)$  follows the normal distribution with mean  $= \mu_k t$  and variance  $= \sigma_k^2 t$ , the expected value and the variance of  $C_k(t)$  are given by:  $E(C_k(t)) = a_k c_k \mu_k t$  and  $V(C_k(t)) = a_k c_k^2 \sigma_k^2 t$ . Then  $U(t)$  has expected value and variance,

$$E(U(t)) = \sum_{k=1}^n a_k c_k \mu_k t = \mu_U, \quad (9)$$

and

$$V(U(t)) = \sum_{k=1}^n a_k^2 c_k^2 \sigma_k^2 t = \sigma_U^2, \quad (10)$$

respectively.

Obviously, both of  $Y(t)$  and  $U(t)$  have the same values  $\mu_k$  and  $\sigma_k$ , respectively, so the covariance between  $Y(t)$  and  $U(t)$  is given by

$$\begin{aligned} \text{Cov}(Y(t), U(t)) &= \text{Cov}\left(\sum_{k=1}^n a_k X_k(t), \sum_{j=1}^n c_j X_j(t)\right) \\ &= \sum_{k=1}^n \sum_{j=1}^n a_k c_j \text{Cov}(X_k(t), X_j(t)) \\ &= \sum_{k=1}^n a_k c_k \mu_k^2 t. \end{aligned} \quad (11)$$

The characteristic function of the bivariate nor-

mal distribution is given by

$$\begin{aligned} \phi_{(Y(t), U(t))}(t_1, t_2) &= \mathbb{E}[\exp(it_1 Y(t) + it_2 U(t))] \\ &= \mathbb{E}[\exp(it_1 \sum_{k=1}^n a_k X_k(t) + it_2 \sum_{k=1}^n a_k c_k X_k(t))] \\ &= \mathbb{E}[\exp(i \sum_{k=1}^n (a_k t_1 + a_k c_k t_2) X_k(t))] \\ &= \mathbb{E}[\exp(i \sum_{k=1}^n (a_k t_1 + a_k c_k t_2) X_k(t))] \\ &= \prod_{k=1}^n \mathbb{E}[\exp(i(a_k t_1 + a_k c_k t_2) X_k(t))] \\ &= \prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + a_k c_k t_2), \end{aligned} \quad (12)$$

then we can obtain

$$\begin{aligned} f_{Y(t), U(t)}(y, u) & \quad (13) \\ &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{(Y(t), U(t))}(t_1, t_2) e^{-it_1 y - it_2 u} dt_1 dt_2 \\ &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + c_k t_2) \right) e^{-it_1 y - it_2 u} dt_1 dt_2, \end{aligned} \quad (14)$$

then conditional probability  $f_{U(t)|Y(t)}(y, u)$  is given by

$$\begin{aligned} f_{U(t)|Y(t)}(y, u) &= \frac{f_{U(t), Y(t)}(y, u)}{f_{Y(t)}(y)} \\ &= \frac{1}{4\pi^2 f_{Y(t)}(y)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \prod_{k=1}^n \phi_{X_k(t)}(a_k t_1 + c_k t_2) \right) e^{-it_1 y - it_2 u} dt_1 dt_2, \end{aligned} \quad (15)$$

where

$$\begin{aligned} &\phi_{X_k(t)}(a_k t_1 + c_k t_2) \\ &= \exp\left\{ \frac{\sigma_k [1 - (1 - 2i\mu_k^2 (a_k t_1 + a_k c_k t_2) \sigma_k^{-1})^{1/2}]}{\mu_k} \right\}, \end{aligned} \quad (16)$$

However, if we consider a real situation: after a period of time,  $U(t)$  becomes so high that using a new piece of equipment to replace the old one may be a better choice. Also, the owner of the equipment may have an expectation overall cost: when  $U(t)$  is larger than this expectation, they will buy a new piece of equipment. For example, we

assume this expectation cost is  $L_U$ , which will be described in the next section. Similarly, we define

$$\omega_U = \inf\{t > 0 : U(t) \geq L_U\}, \quad (17)$$

Then, the pdf of  $\omega_U$  can be obtained as

$$\begin{aligned} f_{\omega_U}(t) &= \frac{L_U}{\sigma_U \sqrt{2\pi t^3}} \exp\left(\frac{-(L_U - \mu_U t)^2}{2\sigma_U^2 t}\right) \\ &= \frac{L_U}{\sigma_U \sqrt{\pi t^3}} \phi\left(\frac{-(L_U - \mu_U t)}{\sigma_U \sqrt{t}}\right). \end{aligned} \quad (18)$$

Then, the cdf of  $\omega_U$  is obtained by

$$\begin{aligned} F_{\omega_{L_U}}(t) &= P(U(t) \geq L_U) \\ &= \Phi\left(\frac{-(L_U - \mu_U t)}{\sigma_U \sqrt{t}}\right) - \exp\left(\frac{2\mu_U L_U}{\sigma_U^2}\right). \end{aligned} \quad (19)$$

### 3.3. Random effect

Assuming that  $\mu_k$  is influenced by other factors due to various reasons such as different operating environment and usage intensity, which are referred to as random effect. We assume that this random effect follows a gamma distribution over time. Then  $\mu_k$  follows  $\Gamma(\alpha_k, \beta_k)$  where  $\alpha_k$  is the shape parameter and  $\beta_k$  is a scale parameter. Then  $\mu_k$  has pdf as following:

$$f_{\mu_k}(x; \alpha_k, \beta_k) = \frac{1}{\beta_k^{\alpha_k} \Gamma(\alpha_k)} x^{\alpha_k-1} e^{-x/\beta_k} \quad (20)$$

where  $\Gamma(\cdot)$  is a gamma function:

$$\Gamma(l) = \int_0^\infty y^{l-1} e^{-y} dy$$

As  $\mu_Y = \sum_{k=1}^n a_k \mu_k$  which have been described before, then the expected value and the variance of  $\mu_Y$  are given by:  $E(\mu_Y) = \sum_{k=1}^n a_k \beta_k \alpha_k$  and  $Var(\mu_Y) = \sum_{k=1}^n a_k^2 \beta_k^2 \alpha_k$ .

Then,

$$\mu_Y = \sum_{k=1}^n a_k \mu_k, \quad t \geq 0, \quad a_k \geq 0 \quad (21)$$

where  $\mu_k$  follows the gamma distribution.

According to Moschopoulos (1985), the density function of  $\mu_Y$  can be expressed by

$$z_{\mu_Y}(z) = B \sum_{k=0}^{\infty} \frac{\xi_k \beta_0^{-\tau-k}}{\Gamma(\tau+k)} z^{\tau+k-1} e^{-z/\beta_0} \quad (22)$$

where  $\beta_0 = \min a_k \beta_k$ , and  $B$  and  $\tau$  are

$$B = \prod_{k=1}^n \left(\frac{\beta_0}{a_k \beta_k}\right)^{\alpha_k} \quad (23)$$

and

$$\tau = \sum_{k=1}^n \alpha_k \quad (24)$$

It is worth noticed that  $\xi_{k+1}$  can be obtained as

$$\xi_{k+1} = \frac{1}{k+1} \sum_{j=1}^k j \eta_j \xi_{k+1-j} \quad (25)$$

where  $\xi_0 = 1$  and

$$\eta_k = \sum_{j=1}^n \alpha_j \left(1 - \frac{\beta_0}{a_k \beta_k}\right)^k / k \quad (26)$$

Using  $\mu'_Y$  to replace the original  $\mu_Y$ , then,  $Y(t)$  now can be replaced by  $Y'(t)$ , which can be presented by

$$Y'(t) = t\mu'_Y + \sigma_Y \sum_{k=1}^n W_k(t), \quad (27)$$

where  $\mu'_Y$  follows a gamma distribution .

## 4. Maintenance policies

In this section, we consider the age replacement policy.

We have discussed the degradation process and the cost process based on the age replacement before. Then we attend to discuss the possible situations under these two processes:

- **Maintenance Policy A:** Under the degradation process, when the degradation level achieves the pre-specified threshold  $L$ , then maintenance activities will be taken. We assume that this event as  $A_1$ .
- **Maintenance Policy B:** Under the cost process, when the cost level achieves the pre-specified threshold  $L_U$ , then maintenance activities will be taken. We assume that this event as  $A_2$ .
- **Maintenance Policy C:** Only if both  $A_1$  and  $A_2$  have occurred, the age replacement will be conducted. Denote this event as  $A_3 = A_1 \cap A_2$ .
- **Maintenance Policy D:** If one of the two events,  $A_1$  and  $A_2$ , occurs, the age replacement will be conducted. Denote this event as  $A_4 = A_1 \cup A_2$ .

Therefore,  $G_1(t) := P(A_1) = F_{\omega_L}(t)$  and  $G_2(t) := P(A_2) = F_{\omega_{L_U}}(t)$  and these functions can be obtained

$$\begin{aligned} G_3(t) &:= P(A_3) \\ &= P(A_1 \cap A_2) \\ &= P(A_1)P(A_2|A_1) \\ &= F_{\omega_L}(t)F_{\omega_{L_U}}(t|\omega_L) \\ &:= G_3(t), \end{aligned} \quad (28)$$

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and

$$\begin{aligned} G_4(t) &:= P(A_4) \\ &= P(A_1 \cup A_2) \\ &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\ &= P(A_1) + P(A_2) - P(A_3), \end{aligned} \quad (29)$$

where symbol  $:=$  is used to denote a definition.

**4.1. Age replacement policy**

For the age replacement policy, a preventive replacement is conducted after a continuous working time  $T_a$  when there is no failure occurs Barlow and Hunter (1960).

The expectation cost per time unit is given by

Let  $T_a$  be a replacement age, then the mean time between replacements  $M(T_a)$  will be

$$\begin{aligned} M(T_a) &= \int_0^{T_a} tf(t)dt + t_0P(X > T_a) \\ &= \int_0^{T_a} tf(t)dt + t_0(t - F(T_a)) \\ &= \int_0^{T_a} (1 - F(t))dt. \end{aligned} \quad (30)$$

Then, the mean cost per time unit is given by

$$C_i(T_a) = \frac{c_r + c_m G_i(T_a)}{\int_0^{T_a} (1 - G_i(t))dt}, \quad (31)$$

where  $i = 1, 2, 3, 4$ , corresponding to maintenance policies A, B, C, and D, respectively and  $T_a$  is the decision variable. Thus, the maintenance policy follows these principle.

- The inspection will be taken every  $T_a$ .
- Immediately after a preventive or corrective maintenance, the system rests its age to 0.
- Both  $c_r$  and  $c_m$  are constants.

By minimising  $C_i(T_a)$ , we can obtain the optimum  $T_a^*$  for the age replacement policy based on maintenance policies A, B, C, and D, respectively.

**5. Simulation study**

We use genetic algorithms to seek the optimal maintenance intervals.

We consider a system with two different failure modes. The deterioration process of the two defects is modelled with two Wiener process, each of which has different  $\mu$ ,  $\alpha$  and  $\beta$  parameters. We assume that two modes has weights as following  $a_1 = 0.3$  and  $a_2 = 0.7$ .  $\alpha_1$  and  $\beta_1$  are 0.4 and 0.5 for the first failure mode, respectively.  $\alpha_2$  and  $\beta_2$  are 0.3 and 0.6, respectively.

Thus, the linear combination of the two processes is given by

$$Y(t) = 0.3X_1 + 0.7X_2.$$

We assume that the system needs to be repaired when the degradation levels exceed the threshold  $L_{wL} = 5, 4$  and the threshold  $L_{wLc} = 3, 2$ , respectively. Replacement activities will be taken and the degradation level will be restored to zero when the component is completely replaced.

We assume that  $c_r = 100$  and  $c_m = 50$ , then we can obtain the following result:

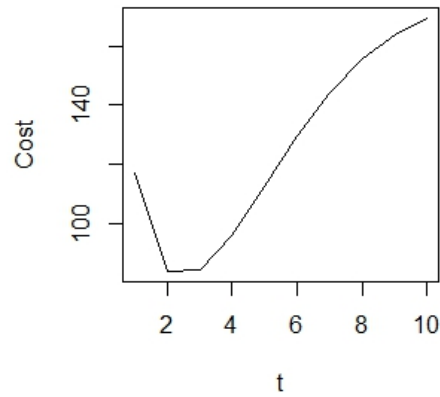


Fig. 3. Maintenance Policy A

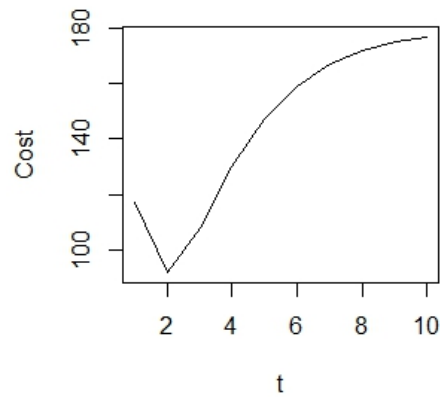


Fig. 4. Maintenance Policy B

Figures 3 and 4 show the expected cost per unit time when the thresholds for cost process are

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$L_{w_{L_c}} = 3, 2$ , respectively. Based on the genetic algorithms, we will obtain two optimized results as  $T_{opt} = 2.443284$  for  $L_{w_{L_c}} = 3$  (expected total cost is 82.12779) and  $T_{opt} = 1.846838$  for  $L_{w_{L_c}} = 2$  (expected total cost is 91.36122).

Besides, based on the age replacement policy, we can obtain the following results:

- When thresholds are  $L_{w_L} = 5$ , the optimized point is ( $T_{opt} = 3.552035$ ) and the expected unit cost per time is 79.85015.
- When thresholds are  $L_{w_L} = 4$ , the optimized point is ( $T_{opt} = 3.014603$ ) and the expected unit cost per time is 80.19919.

## 6. Conclusions

This paper discussed maintenance policies for a system with a linear combination of Wiener processes. When the degradation level of a linear combination of the processes exceeds a pre-specified threshold, the block or age replacement policy will be considered as the preventive maintenance for the system. Besides, we also develop a cost process which considers the situation that when the maintenance cost is higher than an expectation value, the decision-maker will prefer to replace the whole system than repair it.

However, there are several limitations in our research:

- The deterioration process of a system may be a non-linear combination of deterioration processes.
- Different component may follow different degradation models (e.g., one is the Wiener process and the others are gamma processes).
- The dependence among failure modes may exist and should therefore be considered.

Our future work aims to investigate the above limitations.

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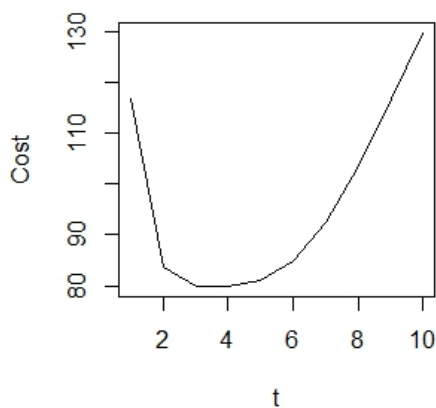


Fig. 5. Maintenance Policy C

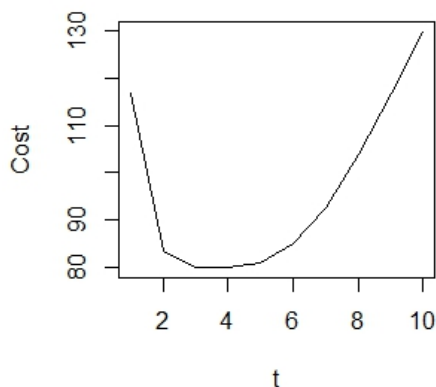


Fig. 6. Maintenance Policy D

Figures 5 and 6 show the expected cost unit for the age replacement policy when the thresholds are  $L_{w_L} = 5, 4$ , respectively.



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