Can We Break the Landauer Bound in Spin-Based Electronics?

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ABSTRACT We found that the minimum energy of reading or erasing a spin datum should be expressed by $E = 2 \mu_B B$, in contrast to $E = k_B T \ln(2)$ proposed by Landauer in 1961. The physics of using a spin’s orientation to represent a bit of information is fundamentally different from that of using a particle’s position in classical charge-based data storage: the former is quantum-dynamic (independent of temperature below the Curie point), whereas the latter is thermodynamic (dependent on temperature). Quantitatively, this new energy bound associated with a new information erasure protocol was estimated as $1.64 \times 10^{-36} J$, 15 orders of magnitude lower than the Landauer bound ($3 \times 10^{-21} J$), at no cost of angular momentum and increased total entropy. In this new information erasure protocol, there is no need to move the electron from one side of the potential well to the other side otherwise the energy used to retain the defined spin state still needs to be greater than the existing thermal fluctuation (the Landauer bound). We verified our new energy bound based on a number of experiments including the Rydberg atom and the spin-spin interaction.

INDEX TERMS Low power electronics, the Landauer bound, spintronics, irreversible computing, green computing, energy Internet, smart grid, quantum computing.

I. INTRODUCTION
Spin-based electronics or spintronics seeks to create new electronics that is based not solely on the charge of the electron, but on its spin as well. A generic impression is that spintronic devices consume much less energy due to a spin’s ‘agility’ in contrast to a charge’s ‘clumsiness’. In this work, we will use the manipulation of a bit of information as an example to demonstrate which (spintronics or electronics) is more energy-efficient.

As shown in Fig. 1(a), a trapped particle (e.g., electron) moving upwards and downwards between impenetrable barriers at either end of a one-dimensional nanotube can be treated as an ideal gas whose interactions with the ends are perfectly elastic collisions. It obeys the ideal gas law:

$$PV = k_B T,$$

where $P$ is the pressure, $V$ is the volume, $k_B$ is the Boltzmann constant and $T$ is the (absolute) temperature.

The work to push the info electron to the desired half is

$$W = \int_0^{L/2} PAdx = \int_{L/2}^L \frac{k_B TA}{Ax} dx = k_B T \ln(2),$$

in exchange for losing that written information (which side of the partition is occupied). Information is physical. The erasure of a bit of (classical) stored information requires the same amount of (heat) energy that is approximately $3 \times 10^{-21} J$ at room temperature (300 K). We called this energy limit the Landauer bound [1].

The generation of this heat inevitably results in both physical irreversibility and logical irreversibility. Such an irreversible operation can also be described by Clausius’s second law of thermodynamics [2] and Szilard’s minimal entropy production of $k_B \ln(2)$ [3].

In 2012, the Landauer bound ($3 \times 10^{-21} J$) was experimentally verified using a system of a single colloidal particle trapped in a modulated double-well potential [4]. The mean dissipated heat was observed to saturate at the Landauer bound in the limit of long erasure cycles [4]. This work defines the ultimate physical limit of irreversible computation [4]. Vaccaro and Barnett argued that there is no energy cost of erasing spin information $S = 1/2$ degenerated state,
which implies that, at a cost of increased total entropy, the cost of erasing a bit of information is less than the Landauer limit [5].

In 2014, Jun et al. demonstrated using small particles in traps and reducing the exerted work to the Landauer limit during the erasure at the expense of slow operation [6].

In 2016, Hong et al. used a laser probe to measure the amount of energy dissipation (4.2 zeptojoules) when a nanomagnetic bit was flipped from off to on [7].

In 2018, Gaudenzi et al. reported an array of giant-spin ($S = 10$) quantum molecular magnets [8]. They concluded that the erasure is still governed by the Landauer bound but can be performed at a high speed (as tunneling provides a high-speed path for spin reversal) even in the quantum realm [8].

II. READING OR ERASING A SPIN DATUM
An electron has a charge and a spin, which are inseparable. In classical information storage (Fig. 1), a charge is stored in order to save information. As a modern computing paradigm, a spin can replace a charge for the storage of information, allowing faster, low-energy operations [9]. As shown in Fig. 2, the energy of flipping a spin in a magnetic field $B$ can be expressed by:

$$\Delta E_{\uparrow \downarrow} = g \mu_B B \approx 2 \mu_B B,$$

(3)

where $\mu_B$ is the Bohr magneton and the value of the electron spin $g$-factor is roughly equal to 2.002319. The reason $g$ is not precisely 2 can be explained by quantum electrodynamics calculations of the anomalous magnetic dipole moment [10].

The minimum energy of reading or erasing a spin datum is $2\mu_B B$, regardless of which form of energy (electrical, magnetic, optical, chemical or even mechanical) is input. Analogous to the temperature $T$ (as an environmental parameter) in the Landauer bound, the magnetic field $B$ (as another environmental parameter) is essential to determine this new energy bound for “nonclassical” information. We will calculate the strength of the magnetic field in the following cases: 1. An internal magnetic field due to the electron orbit in an isolated atom or ion; 2. A magnetic field in the spin-spin interaction; and 3. Various ambient magnetic field sources outside matter.

According to the conservation of energy, the spin carrier’s spatial degrees of freedom are in thermal equilibrium with a much larger thermal reservoir at temperature $T$. In contrast, the spin degrees of freedom are in an independent equilibrium state [5]. The decoupling between the spatial degree of freedom (which is temperature-dependent) and the spin degree of freedom (which is temperature-independent) may be achieved by degenerating the states of each spin in energy [5]. The relationship between the temperature and spatial degrees of freedom is described in the Landauer bound [1], and such a decoupling means that the Landauer bound (which depends on temperature $T$) should not be used in the energy limit calculation for a spin datum at the erasure or destructive readout stage without necessarily moving the electron from one side of the potential well to the other side.

III. INTERNAL MAGNETIC FIELD IN AN ATOM/IION
An electron exists in two ways: it can be either bound to the nucleus of an atom by the attractive Coulomb force (in the
spin–spin interaction experiment to be illustrated in the next section, the ground state valence electron of a trapped $88\text{Sr}^+$ ion has only spin degrees of freedom) or can be promoted to free space (outside matter) by absorbing energy equal or greater than the work function. An electron in metals also behaves as if it was free. Let us first calculate the energy of flipping a spin in an atom.

As shown in Fig. 3, there is a magnetic field $B$ acting on the electron in the rest frame of the electron. The energy of flipping this spin in the $B$ that the electron experiences can be found in Eq. 3.

Directly, the energy of flipping the spin between two states (spin up and spin down) can be expressed in its quantum dynamical form [10], [11] as follows:

$$\Delta E = \frac{mc^2\alpha^4}{2} \cdot \frac{Z^4}{n^3ln(l+1)},$$

(4)

where $m$ is an electron’s mass, $c$ is the speed of light, $\alpha$ is the fine-structure constant, $Z$ is the effective central charge, $n$ is the principal quantum number and $\ell$ is the orbital angular momentum quantum number. These two states (spin up and spin down) exist inside the same “$n$”, and their total angular momentum number “$j$” is different by one. As one can see in Eq. 4, this value is proportional to $\frac{Z^4}{n}$.

For example, first, we obtain the energy difference between $2p_{3/2}$ and $2p_{1/2}$ of the hydrogen atom. The principal quantum number “$n$” is “2” in both states. $j = l + s = 3/2$ in $2p_{3/2}$ (the spin quantum number associated with the spin angular momentum $s = 1/2$), and $j = l - s = 1/2$ in $2p_{1/2}$. As shown in Fig. 4, the hydrogen fine structure (= doublet) between $2p_{3/2}$ and $2p_{1/2}$ is approximately 0.000045 eV, which is experimentally verified [8].

Some typical values of the internal magnetic field experienced by a hydrogen atom electron in different states are listed in Table 1. As seen in the table, there exists a strong magnetic field inside an atom [10], [11]. Since the energy is proportional to $\frac{Z^4}{n}$, the chemical elements ($Z > 1$) after hydrogen require more energy to flip their spins. In other words, a hydrogen atom tends to have the lowest energy to flip its spin. For the same chemical element, the larger the orbit is of an electron ($n > 2$), the less energy needed to flip its spin (note that the s state has no orbital angular momentum and that there is no spin-orbit splitting for this state).

The goal of this work is to find the minimum energy to flip a spin. Therefore, we need to calculate the energy values of a hydrogen atom electron with a very high principal quantum number $n$. In fact, an excited atom with one or more electrons that have a very high $n$ is called a Rydberg atom [10], [12]. While being excited into a high energy level, those outer electrons of an atom may enter a spatially-extended orbital (far outside the ones of the other electrons). From the standpoint of an excited electron, it sees an “equivalent” atomic core (consisting of the nucleus and all the inner electrons) orbiting around it. Hence, this core has a net charge of $+e$, which is the same as that of a hydrogen nucleus. Therefore, the excited electron behaves as if it belongs to a hydrogen atom. Indeed, in many respects, a Rydberg atom behaves like a hydrogen atom [12].

So far, a Rydberg atom with $n = 290$ has been achieved experimentally in the laboratory, and acre atoms with outer electrons in states with $n = 630$ are observed by radio astronomical methods in interstellar space [10]. Accordingly, we calculated both the energy required to flip the spin of its outermost electron and the internal magnetic field experienced by that electron, as presented in the bottom two rows of Table 1. Here, we chose $1.82 \times 10^{-12} \text{eV} (= 2.91 \times 10^{-31}J)$ as the (minimum) energy limit if one uses the electron spin in an atom to read or erase a bit of stored information in the human-made devices on Earth. Obviously, this energy limit is much lower than the Landauer bound ($3 \times 10^{-21}J$).

According to Newton’s second law, for an electron rotating about a hydrogen nucleus of charge $+e$, we obtain: $F = ma \Rightarrow \frac{k_e^2}{r^2} = m\alpha^2$, where $r$ is the radius of the circular orbit and $k = \frac{1}{4\pi\epsilon_0}$. Orbital momentum is quantized in units of the reduced Planck constant $\hbar$: $mv = nh$. Combining the above two equations, we obtain Bohr’s expression for the orbital radius in terms of the principal quantum number $n$: $r = \frac{\alpha^2\hbar^2}{m}\frac{1}{n^2}$. It is now apparent why the radius of the orbit of a Rydberg atom scales with $n^2$. As shown in Table 1,
TABLE 1. Some typical values of the internal magnetic field experienced by a hydrogen (or hydrogen-like) atom electron (a highly excited atom has an electronic structure roughly similar to that of atomic hydrogen) and the energy in \( Z^4/n^3 \).

<table>
<thead>
<tr>
<th>The principal quantum number ( n )</th>
<th>The orbital angular momentum quantum number ( l )</th>
<th>The orbital radius ( r = \frac{n^2 \hbar^2}{ke^2m} ) (Bohr radius)</th>
<th>( \Delta E_{1l} = E_{1} - E_{l} ) ( \propto \frac{Z^4}{n^3} )</th>
<th>( B = \frac{\Delta E_{1l}}{2\mu_B} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>( 5.3 \times 10^{-11} m )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>( 0.0021 \mu m )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>( 0.0021 \mu m )</td>
<td>( 4.5 \times 10^{-5} \text{ eV} )</td>
<td>( 0.388 \text{ T} )</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>( 0.00048 \mu m )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>( 0.00048 \mu m )</td>
<td>( 4.56 \times 10^{-5} \text{ eV} )</td>
<td>( 0.393 \text{ T} )</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>( 0.00048 \mu m )</td>
<td>( 4.48 \times 10^{-5} \text{ eV} )</td>
<td>( 0.386 \text{ T} )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>137 (Rydberg atom)</td>
<td>1</td>
<td>( 1 \mu m ) (close to the size of bacterial)</td>
<td>( 1.75 \times 10^{-11} \text{ eV} )</td>
<td>( 1.51 \times 10^{-7} \text{ T} )</td>
</tr>
<tr>
<td>290 (Rydberg atom, highest achieved in lab) [10]</td>
<td>1</td>
<td>( 4.5 \mu m )</td>
<td>( 1.82 \times 10^{-12} \text{ eV} )</td>
<td>( 1.58 \times 10^{-8} \text{ T} )</td>
</tr>
<tr>
<td>630 (acre atoms, highest observed in interstellar space) [10]</td>
<td>1</td>
<td>( 21.2 \mu m )</td>
<td>( 1.80 \times 10^{-13} \text{ eV} )</td>
<td>( 1.55 \times 10^{-9} \text{ T} )</td>
</tr>
</tbody>
</table>

\( n = 137 \) corresponds to an atomic radius \( r = \frac{n^2 \hbar^2}{ke^2m} = \frac{137^2 \times (1.05 \times 10^{-34} \text{ J} \cdot \text{s})^2}{1.60 \times 10^{-19} \text{ C}^2 \times 9.11 \times 10^{-33} \text{ kg}} \approx 1 \mu m \), which is even close to the size of bacteria!. Thus, Rydberg atoms are extremely large, with an extremely small internal magnetic field, and their spins can easily be flipped and ideally be used for information storage.

IV. MAGNETIC FIELD IN SPIN-SPIN INTERACTION

The importance of using a spin as an energy/angular momentum source to drive another spin (as a bit of information) is twofold: on the one hand, the magnetic interaction between two electronic spins can impose a change in their orientation and thereby alter the stored information if the spins are used for data storage; on the other hand, the key to calculating the minimum energy to flip a spin is to find a minimum magnetic field \( B \) in Eq. 3, and a spin is the smallest magnet (Bohr magneton) one can use to flip another spin. As shown in Fig. 5, the magnetic field of one Bohr magneton (\( \vec{\mu}_B \)) applied on the other magneton is given as

\[
\vec{B} = \frac{\mu_0}{4\pi r^3} \left[ \frac{3(\vec{\mu}_B \cdot \vec{r})\vec{r}}{r^2} - \vec{\mu}_B \right],
\]

(5)

where \( \mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \) is the vacuum permeability constant [10]. If \( \vec{\mu}_B \) is colinear with \( \vec{r} \), the generated magnetic field is also along \( \vec{r} \), and its strength is

\[
B = \frac{\mu_0}{4\pi} \frac{2\mu_B}{r^3}.
\]

(6)

\[B \quad \text{(in T)}\]

FIGURE 5. One Bohr magneton applies a magnetic field \( B \) on another magneton. Note that the magnetic field that is experienced by one magneton may not be the same as the field experienced by the other considering their possible orientations. In this work, we have covered five different energy-inputting means to manipulate the spin-spin interactions (see main text for details).

In 2014, the magnetic interaction between the two ground-state spin-1/2 valence electrons of two \(^{88}\text{Sr}^+\) ions across a separation (\( d = 2.18 \sim 2.76 \mu m \)) was reportedly observed and measured [13]. The experimental setup is shown in Fig. 6(a). For \(^{88}\text{Sr}^+\), the two spin states of the valence electron are \( | \uparrow \rangle = | 5s_{1/2}, J = \frac{1}{2}, M_J = 1/2 \rangle \) and \( | \downarrow \rangle = | 5s_{1/2}, J = \frac{1}{2}, M_J = -1/2 \rangle \). As mentioned above, an s-state electron has no orbital angular momentum, so it does not experience any internal magnetic field due to spin-orbit coupling since there is no spin-orbit splitting for this state.
As shown in Fig. 6(b), the four eigenstates of the two-ion Hamiltonian (the spin part) are \(|\uparrow\downarrow\rangle, |\downarrow\downarrow\rangle\) and the two entangled Bell states \(|\psi \pm \rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}\). The energy splitting between the last two is \(\mu_0 B \frac{4\mu_B}{4\pi d^3}\) (note that this is independent from the external magnetic field \(B_{ext}\)), where \(h\) is the Planck constant. The spin–spin interaction within the decoherence-free subspace can be described by the geometric Bloch sphere [Fig. 6(c)], in which, starting from the north pole \(|\uparrow\downarrow\rangle\), the system rotates through the fully entangled state \(|\chi^+\rangle = (|\uparrow\downarrow\rangle + i|\downarrow\uparrow\rangle)/\sqrt{2}\) and towards the south pole \(|\downarrow\uparrow\rangle\). The electronic spins can be placed at a controlled distance from one another (via the trap voltage against the Coulomb repulsion). It is also possible to initialize the system state to \(|\uparrow\downarrow\rangle\) or \(|\downarrow\uparrow\rangle\) (by optical pumping), manipulate the internal spin state (perform all possible collective spin rotations by pulsing a resonant radio-frequency magnetic field) and detect the internal spin state (by state-selective fluorescence) with high fidelity [13].

The measurements showed that the spin–spin interaction depends on the interelectron distance, with a \(d^{-3.04(4)}\) distance dependence [13], obeying the inverse-cube law as illustrated in Eq. 6. Therefore, as shown in Fig. 6(a), one electron applies a magnetic field \(B\) on the other (colinear with the line linking them), and the strength of this field can be calculated based on Eq. 6 as follows:

\[
B = 2 \mu_B / 4\pi d^3 = \frac{\mu_0 2 \mu_B}{4\pi d^3} = \frac{10^{-7\text{T}m}}{A} \times 9.27 \times 10^{-24} \text{A} \cdot \text{m}^2 \\
= 8.82 \times 10^{-14} \text{T},
\]

where \(d = 2.76 \text{ } \mu\text{m}\) is the largest separation for which the spin–spin interaction is still effective with a fidelity of above 98% [13]. Note that the used separation \((d = 2.76 \text{ } \mu\text{m})\) between the two spins is five orders of magnitude larger than the Bohr radius \((5.29 \times 10^{-11} \text{m})\), which is equal to the most likely distance between the nucleus and the electron in a hydrogen atom in its ground state. This separation is close to the radius \((4.5 \text{ } \mu\text{m})\) of the largest human-made Rydberg atom \((n = 290)\) considering the likelihood that the radius of the largest (human-made) Rydberg atom may represent the largest distance for which the attractive Coulomb force (between a proton and an electron) is effective on Earth.

According to Eq. 3, the energy to flip a spin via the spin–spin interaction in the presence of the magnetic field \(B\) is:

\[
\Delta E = 2 \mu_B B = 2 \mu_B \frac{\mu_0 2 \mu_B}{4\pi d^3} \approx \frac{\mu_0 4 \mu_B^3}{4\pi d^3} = 2 \times 9.27 \times 10^{-24} J/T \times 8.82 \times 10^{-14} T \\
= 1.64 \times 10^{-36} J.
\]

In this new energy bound (Eq. 8) for reading or erasing a stored datum in a spin, an environmental parameter (magnetic field \(B\), in which the spin immerses itself) is analogous to another environmental parameter (temperature \(T\)) in the Landauer bound (Eq. 2) in the sense that neither the magnetic field \(B\) nor the temperature \(T\) should be zero.

Although \(|\uparrow\downarrow\rangle\) and \(|\downarrow\uparrow\rangle\) are indistinguishable, as illustrated in Fig. 6(b), the energy limit of flipping a spin can still be found by the above equation since it is proportional to the aforementioned energy splitting in frequency \((\frac{\mu_0 4 \mu_B^3}{4\pi d^3} = 2 - 5\text{mHz})\) between the two entangled Bell states \(|\psi \pm \rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}\). In fact, our calculated new energy bound \((\Delta E = 1.64 \times 10^{-36} J)\) has been verified by this spin-spin interaction experiment since \(\Delta E = \frac{\mu_0 4 \mu_B^3}{4\pi d^3} = \frac{1.64 \times 10^{-36} J}{6.63 \times 10^{-34} J s} = 2.47 \text{mHz}\), which matches the lower end of the measured frequency range (2-5 mHz) [13].

This energy bound \((1.64 \times 10^{-36} J)\) is much lower than the Landauer bound \((3 \times 10^{-21} J)\). A good example taking advantage of such spin-spin interactions is the spin-transfer torque (STT) [14] and spin–orbit torque (SOT) [15], in which a spin-polarized current is directed into a magnetic layer, and the angular momentum can be transferred to change its spin orientation. STT/SOT can be used to flip the storage elements in magnetic random-access memory with no volatility and near-zero leakage power consumption; however, it always has a charge flow whose areal density is \(~3.4 \times 10^4 \text{A/cm}^2\) for SOT switching [15] (two orders of magnitude smaller than that for STT switching) and thereby suffers from substantial energy dissipation caused by Joule heating.

In 2019, it was reported that, without any moving electron, spins can even be flipped by a magnon current carrying spin angular momentum [16]. This experiment introduced another energy-efficient way to flip spins based on the spin-spin interaction and may facilitate future magnon-based memory/logic devices. Nevertheless, their antiferromagnet/insulator/ferromagnet interface (that is as thin as 25 nanometers) is much smaller than the spin-spin separation \((d = 2.76 \mu\text{m})\) that we used to estimate the minimum energy to flip a spin. That is to say, our energy limit is still the minimum so far.

Note that this new energy limit represents purely the (magnetostatic) potential energy in a magnetic field and does not include any heat dissipation. It is worth mentioning that, linking information and thermodynamics, the Landauer bound itself is actually the heat generated in erasing a (classical) bit of stored information, which inevitably results in physical and logical irreversibilities.

In 2011, an information erasure scheme was proposed, in which the energy cost can even be reduced to zero with a cost in angular momentum or another conserved quantity in principle [5]. The memory spin is put in spin-exchange contact with a reservoir of spins (if it contains only one spin, the setup is similar to the spin-spin interaction experiment), letting the combined reservoir-memory spin system come to equilibrium (conservation of angular momentum), and then separating the memory spin from the reservoir [5]. An extremely vital assumption of their claim is that there are no residual magnetic fields in the vicinity of the spins that would remove their energy degeneracy and produce an energy cost associated with the above operations. Obviously, this assumption is not affirmed at all by the spin-spin
ous sources, ordered by orders of magnitude [17], as well as Table 2 lists examples of magnetic field $B$.

**Different Magnetic Field Sources**

**elaborated above.**

inescapable in terms of effecting the spin-spin interaction as well.

interaction experiment [13], in which there exists a minimal magnetic field of $8 \times 10^{-14} T$. It was found that the two ions can be tightly confined and laser-cooled to their ground state. Note that an external magnetic field $B_{\text{ext}}$ was applied along the line (connecting the two ions during the measurements. In a quantum system, such a magnetic field promotes the quantum mixing of the ‘up’ and ‘down’ spin orientations. One can initialize the system to $|\uparrow \downarrow \rangle$ along that direction and measure the state projections on the $(|\uparrow \downarrow \rangle + |\downarrow \uparrow \rangle)/\sqrt{2}$ basis along $y$ and rotations of the system state around the $\hat{x}$ axis (the system dynamics are nearly ideal ($|\uparrow \downarrow \rangle \leftrightarrow |\downarrow \uparrow \rangle$ rotation). The energy splitting $\Delta E = hf = \hbar c$, where $h$ is the Planck constant, $f$ is the frequency, $c$ is the speed of light in vacuum and $\lambda$ is the wavelength.

interaction experiment [13], in which there exists a minimal magnetic field of $8.82 \times 10^{-14} T$, which is indispensable and inescapable in terms of effecting the spin-spin interaction as elaborated above.

**V. COMPARISON OF THE ENERGY LIMITS WITH DIFFERENT MAGNETIC FIELD SOURCES**

Table 2 lists examples of magnetic field $B$ produced by various sources, ordered by orders of magnitude [17], as well as the corresponding energies needed to flip a spin. Note that the magnetic field drops off as the cube of the distance from a dipole source, as illustrated in Eq. 5.

From this table, it can be seen that the energy ($1.64 \times 10^{-36} J$) in the spin-spin interaction is much lower (4-5 orders of magnitude) than that ($2.91 \times 10^{-31} J$ for $n = 290$ or $2.87 \times 10^{-32} J$ for $n = 630$) for a spin that is bound in an atom. Therefore, it is reasonable for us to take $1.64 \times 10^{-36} J$ as the minimum energy limit to flip a spin if we shield our data storage devices well from those external magnetic field noises from various sources (labeled “Outside Matter” in Table 2).

An s-state electron (bound in an atom) does not experience any internal magnetic field in the atom (as mentioned above) but still needs the minimum energy ($1.64 \times 10^{-36} J$) to flip it at no cost of angular momentum and increased total entropy. An s-orbital electron with zero orbital angular momentum has spherical orbitals, which does not mean it moves around on the boundary of that sphere (if it was a classical particle); rather, it means that it has some probability of being found anywhere in that sphere.

This new energy bound ($1.64 \times 10^{-36} Jn$) should be universal for a spin to be flipped regardless of which form of energy (electrical, magnetic, optical, chemical or even mechanical) is input unless new experimental evidence (e.g., the spin-spin interaction can still be observed while $d > 2.76 \mu m$) appears in the future proving that a lower energy is possible to flip a spin. This endeavor will no doubt be a major challenge since even the current measurement ($d \leq 2.76 \mu m$) was carried out in the presence of magnetic noise that was six orders of magnitude larger than the magnetic fields that the electrons apply on each other [13].

In 2010, the spin direction of an individual Co atom was manipulated by moving it along the in-plane antiferromagnetic Mn spin spiral (Mn magnetic moment up and down, alternatively) [22]. This is an example of using the
TABLE 2. Energy to flip a spin with different magnetic field sources, grouped by inside/outside matter, in contrast to the Landauer bound ($3 \times 10^{-21} J$) for classical data storage.

<table>
<thead>
<tr>
<th>Magnetic Field Sources</th>
<th>Magnetic Field $B$</th>
<th>Energy to Flip a Spin in $B$ ($\Delta E = 2\mu_B B$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inside matter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The spin is bound in an atom (s state, $l=0$ for all $n$).</td>
<td>0</td>
<td>$1.64 \times 10^{-36} J^*$</td>
</tr>
<tr>
<td>The spin is bound in an atom ($n=2$, $l=1$).</td>
<td>$0.388 , T$</td>
<td>$7.2 \times 10^{-24} J$</td>
</tr>
<tr>
<td>The spin is bound in an atom ($n=290$, $l=1$).</td>
<td>$1.58 \times 10^{-6} , T$</td>
<td>$2.91 \times 10^{-31} J$</td>
</tr>
<tr>
<td>The spin is bound in an atom ($n=630$, $l=1$).</td>
<td>$1.55 \times 10^{-9} , T$</td>
<td>$2.87 \times 10^{-52} J$</td>
</tr>
<tr>
<td>The spin interacts with another spin [13].</td>
<td>$8.82 \times 10^{-14} , T$</td>
<td>$1.64 \times 10^{-36} J$</td>
</tr>
<tr>
<td>Outside matter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Human brain magnetic field [17].</td>
<td>$100 , fT$ to $1 , pT$ [17]</td>
<td>$1.85 \times 10^{-36} \sim 1.85 \times 10^{-35} J$</td>
</tr>
<tr>
<td>Magnetic field around the solar system [18].</td>
<td>$10 , pT$ [18]</td>
<td>$1.85 \times 10^{-34} J$</td>
</tr>
<tr>
<td>Magnetic field near electric distribution lines [17].</td>
<td>$60-700 , nT$ [19]</td>
<td>$1.11 \times 10^{-33} \sim 1.30 \times 10^{-31} J$</td>
</tr>
<tr>
<td>Magnetic field near a toaster in use [19].</td>
<td>$100-500 , nT$ [17]</td>
<td>$1.85 \times 10^{-33} \sim 9.25 \times 10^{-33} J$</td>
</tr>
<tr>
<td>Magnetic field near a microwave oven in use [17].</td>
<td>$4 , \mu T$ to $8 , \mu T$ [17]</td>
<td>$7.4 \times 10^{-32} \sim 1.48 \times 10^{-31} J$</td>
</tr>
<tr>
<td>Strength of magnetic tape near tape head [17].</td>
<td>$24 , \mu T$ [17]</td>
<td>$4.44 \times 10^{-31} J$</td>
</tr>
<tr>
<td>Earth’s magnetic field at 0° latitude on equator [17].</td>
<td>$31 , \mu T$ [17]</td>
<td>$5.74 \times 10^{-31} J$</td>
</tr>
<tr>
<td>Earth’s magnetic field at 50° latitude [17].</td>
<td>$58 , \mu T$ [17]</td>
<td>$1.07 \times 10^{-30} J$</td>
</tr>
<tr>
<td>A typical refrigerator magnet [17].</td>
<td>$5 , mT$ [17]</td>
<td>$9.27 \times 10^{-26} J$</td>
</tr>
<tr>
<td>The magnetic field strength of a sunspot [17].</td>
<td>$150 , mT$ [17]</td>
<td>$2.78 \times 10^{-24} J$</td>
</tr>
<tr>
<td>Inside modern 50/60 Hz power transformer [20].</td>
<td>$1 , T$ to $2 , T$ [20]</td>
<td>$1.85 \times 10^{-23} \sim 3.71 \times 10^{-23} J$</td>
</tr>
<tr>
<td>Modern magnetic resonance imaging system [21].</td>
<td>$9.4 , T$ [21]</td>
<td>$1.74 \times 10^{-22} J$</td>
</tr>
</tbody>
</table>

mechanical energy (in spite of heat dissipation and inefficiency) to flip a spin. However, the distance between the Co and Mn atoms amounts to 2.286 Å, which is much smaller than the spin-spin separation ($d = 2.76 \, \mu m$) that we used to estimate the minimum energy to flip a spin. That is to say, our energy limit is still the minimum so far.

VI. CONCLUSION

In this work, we found that the minimum energy of reading or erasing a spin datum should be expressed by $\Delta E = 2\mu_B B$, in contrast to the well-known Landauer formula, $\Delta E = k_BT \ln(2)$, for classical data storage (Fig. 7). These two formulas are different because the physics of using a spin’s orientation to read or erase a bit of stored information is fundamentally different from that of using a particle’s position as a (classical) bit of information: the former is quantum-dynamic (independent of temperature below the Curie point), whereas the latter is thermodynamic (dependent on temperature). The new energy limit of flipping a spin at the readout or erasure stage (without necessarily moving the electron from one side of the potential well to the other side) was estimated as $1.64 \times 10^{-36} J$, 15 orders of magnitude lower than the Landauer bound ($3 \times 10^{-21} J$) at no cost of angular momentum and increased total entropy. We verified the above limits based on a number of experiments including the Rydberg atom [12] and the spin-spin interactions [13], [16], [22].

We stress that the decoupling between the (temperature-dependent) spatial degree of freedom and the (temperature-independent) spin degree of freedom does not mean that the Landauer bound (which depends on temperature $T$) should be abandoned at the writing/storing stage since the energy used to retain the defined spin state still needs to be greater than the existing thermal fluctuation (the Landauer bound).

Against the common wisdom that spintronic devices consume much less energy, spintronics may not be more energy-efficient than classical charge-based electronics at the ‘writing’ or ‘storing’ stage because we cannot separate the (internal, intrinsic) spin and charge of an electron. At the ‘erasure’ or ‘readout’ stage, the energy (of flipping a spin) is 15 orders of magnitude lower than the Landauer bound without any increase in total entropy.

This new energy limit may help design various low-power electronic and electrical systems to use less electric power than classic charge-based systems. For example, small-signal instability is always observed in batteries whose droop coefficients vary with their state-of-charge (SoC) and charge/discharge mode [23]. In the Energy Internet and the Smart Grid with the bidirectional power flow, in which various methods such as a multiagent-based consensus algorithm are utilised to increase the energy utilization [24]. Energy efficiency in charging base stations is always a concern to optimise the drone layout in complex pipeline networks [25].

With global energy consumption expected to almost double by 2050 [26], we need to focus on both the amount of energy we are consuming and the resulting emissions we generate [26]. New digital technologies, such as artificial intelligence, machine learning, deep learning and the Internet of Things (IoT), are paving the way for greater energy efficiency while ensuring sustainable operations. An example is that a generative adversarial network (tGAN) is used to detect the leakage with incomplete sensor data in the energy industry [27].
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REFERENCES

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