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Tomographic Reconstruction of Light Field PIV Based on Backward Ray-Tracing Technique

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Abstract

The calculation of the weight matrix is one of the key steps of the tomographic reconstruction in the light field particle image velocimetry (light field PIV) system. At present, the existing calculation method of the weight matrix in light field PIV based on the forward ray-tracing technique (named as Fahringer’s method) is very time-consuming. To improve the computational efficiency of the weight matrix, this paper presents a computational method for the weight matrix based on the backward ray-tracing technique in combination with Gaussian function (named as Gaussian function method). An Expectation-Maximization (EM) algorithm is employed for the reconstruction of the 3D particle field, and a summed line-of-sight (SLOS) estimation is further used to accelerate the reconstruction process. The computational accuracy and efficiency of the weight matrix, the reconstruction quality of the 3D particle field, and the velocity field accuracy by Gaussian function method are numerically investigated. Finally, experiments are carried out to verify the feasibility of the weight matrix by Gaussian function method. Numerical results illustrated that Gaussian function method can improve the computational efficiency of the weight matrix by more than 10 times. SLOS is capable of further accelerating the computational efficiency of the overall reconstruction process including the pre-determination, the calculation of the weight matrix and the reconstruction. The velocity field accuracy by Gaussian function method is almost the same as that by Fahringer’s method. The experimental results of the 3D-3C velocity field of a laminar flow further verify the feasibility of the computational method for the weight matrix based on Gaussian function.

Keywords: weight matrix, tomographic reconstruction, particle image velocimetry, backward ray-tracing technique, light field imaging

1. Introduction

Most problems in fluid dynamics are involved with complex, three-dimensional (3D) and unsteady flows such as the turbulent flow, flow in boundary layer and spray [1-3]. The 3D velocity field plays a crucial role in characterizing the 3D structure of various complex flows [4]. An accurate and efficient measurement of the 3D velocity field helps to reveal the topology and nature of various complex flows, which is useful for the optimized operation and design of fluid mechanics [5]. Three-dimensional and three-component (3D-3C) particle image velocimetry (PIV) technique is capable of achieving the measurement of 3D-3C velocity field and has the advantages of non-intrusiveness and instantaneity, and has become an important means for the
characterization of the topologies of various 3D unsteady and complicated flow structures [6].

The tomographic PIV (Tomo-PIV) is one of the most useful experimental methods for measurement and characterization of the 3D-3C velocity field due to its advantages of high spatial resolution and being suitable for the volumetric velocity measurement at different scales [7-10]. In Tomo-PIV, the measurement volume is captured by the conventional multi-cameras system (usually three or more conventional cameras) [11]. The Algebraic reconstruction technique (ART) and the multiplicative algebraic reconstruction technique (MART) are usually used for the reconstruction of the 3D particle field. The synthetic aperture PIV (SA-PIV) is a computationally cheap and efficient 3D-3C PIV technique, and its experimental setup is very similar to the Tomo-PIV [12-13]. The SA-PIV is implemented by a camera array similar to the light field imaging system distributed in the measurement volume from different views. The multiple 2D refocused planes at various depths throughout the entire measurement volume are reconstructed by the refocusing algorithm. The particles with lower intensities in the 2D refocused planes are removed when their intensities are lower than a set threshold. The SA-PIV is capable of obtaining the 3D particle field. Furthermore, the refocusing algorithm in the SA-PIV is more efficient than MART and ART in the Tomo-PIV. However, in the SA-PIV, eight or more conventional cameras are usually required to record the tracer particles seeded in the measurement volume from different views [14]. Thus, the multi-cameras system of the SA-PIV is more complex and expensive than that of the Tomo-PIV. This results in a complex coupling and synchronization of the multiple-cameras so that the operation and mounting of the multi-cameras system is inconvenient for the complex flow mechanics [15-16], especially for the space-constraint applications such as the 3D measurement of high-temperature and high-pressure turbulent flow in internal engine. As an alternative to the multi-cameras PIV, the light field PIV based on a single light field camera has received much attention recently [17-20]. Compared with the conventional camera, a microlens array (MLA) is mounted in front of the CCD sensor in the light field camera so that the direction and the position of the light field can be simultaneously recorded in a single exposure [21-22]. So a single light field camera is capable of capturing the tracer particles in the flow field instead of the multi-cameras system in the Tomo-PIV and SA-PIV. This uniqueness greatly simplifies the Tomo-PIV and SA-PIV system for the 3D flow measurement. Presently, many research works on the light field PIV have been performed for the measurement of the instantaneous velocity field. Skupsch et al. proposed the SA-PIV based on a single light field camera to measure the velocity field of the flow field [23]. In this work, the multi-light sheets with constant spacing were used to illuminate the flow field instead of the volumetric illumination. The refocused planes at the position of the multi-light sheets were reconstructed by the refocusing algorithm. Then, the deblurring algorithm was used to remove the particles with lower intensities in the 2D refocused planes. The 2D cross-correlation was used for the calculation of the velocity field in each refocused plane. Skupsch’s SA-PIV based on a single light field camera is a 2D-3C PIV technique.

Brian Thurow’s group of Auburn university [17-18] firstly proposed a Tomo-PIV technique based on a single light field camera and achieved the measurement of the 3D-3C velocity field. Since then, Brian Thurow’s group has carried out a lot of research works on the light field PIV for the measurement of the 3D-3C velocity field of the flow field, and many progresses have been made recently. Bolton et al. used a single light field camera to measure the shock wave-turbulent boundary layer interaction, and proven that the light field camera was capable of obtaining the 3D velocity information of the supersonic flows [24]. Thomason studied the calibration of the plenoptic camera based on the focal point method and the magnification method [25]. His thesis proved that the magnification method could provide a reasonable estimation for the image distance, and the focal point method is limited by the error when a complex lens is assumed to be a thin lens in the real application. The focal point method is capable of providing a comparable accuracy to the magnification method by using a correction equation. Munz et al. proposed a volumetric calibration based on a 3D polynomial mapping function to calibrate the light field camera, and corrected errors due to the lens distortion and thin lens assumption [26]. The volumetric calibration was capable of directly achieving the image position corresponding to the 3D position in the object space. Fahringer et al. used the dual light field cameras to reconstruct the 3D particle field. Their work indicated that the dual light field cameras were capable of mitigating the elongation of the reconstructed particle in the depth, and has a higher reconstruction quality than a single light field camera [27]. However, the computational cost of the weight matrix increases with the increase of the number of cameras.

Shi’s group of Shanghai Jiao Tong University has also conducted a lot of research works on the light field PIV. Shi et al. [28] studied the effects of the pixel-microlens ratio and MLA geometry on the tomographic reconstruction quality, and his work has verified that the light field PIV was capable of achieving the measurement accuracy level of the flow field similar to the multi-cameras Tomo-PIV when the plenoptic camera resolution is relatively high [29]. In 2019, Shi et al. proposed a calibration mode based on Gaussian optics. This method could determine the relationship between a voxel in the object space and its affected microlens and
pixels, and the center and diameter of confusion circle produced on microlens array [30]. The distortion caused by the main lens and the misalignments between the MLA and CCD sensor were taken into account. Thus, their calibration method was capable of providing an accurate weight coefficient for the 3D particle reconstruction. In 2020, they further proposed a flexible calibration method for the unfocused plenoptic camera based on the plenoptic type features [31]. They used a ‘plenoptic disk features’ to operate the raw light-field image. A centroid algorithm was used for the detection of the point-like features related to a point in 3-D space. Their method avoided some intermediate processing steps of generating sub-aperture images and detecting features on these sub-aperture images. Mei et al. studied a 3DPIV technique based on a dual light field camera framework [32]. Meanwhile, the influence of the view angle and tracer particle density on the reconstruction quality and spatial resolution were investigated. The comparison of the reconstruction results between a single light field camera and a dual light field camera was conducted. Their method showed that a dual light field camera could mitigate the elongation of reconstructed particles and improve the depth resolution. Meanwhile, Shi et al. used a single-camera light-field PIV to measure the velocity field of synthetic jet, self-similar adverse pressure gradient turbulent boundary layer, supersonic jet and turbomachinery blades [33-36].

Cao and Xu et al. of Southeast University [37] also studied the effects of the optical parameters of the light field camera on the tomographic reconstruction quality, and further optimized the configuration of the light field camera. Despite rapid progress and developments in the light field PIV for the measurement of the 3D-3C velocity field, some issues and challenges need to be addressed in the real applications. One of the major issues is the low calculation efficiency of the weight matrix, making the tomographic reconstruction of the light field PIV time-consuming. The weight matrix describes the relationship between the discretized voxels in the measurement volume and their corresponding imaging pixels in the light field camera. The computational efficiency of the weight matrix is closely dependent on the number of the elements of the weight matrix and the calculation method. In the light field PIV, the forward ray-tracing technique is usually used for the calculation of the weight matrix. Through the forward ray-tracing technique, the light rays emitted from the voxel are traced to the main lens, the MLA, and then onto the pixel on the CCD sensor. In Fahringer’s works [38], each light ray from the voxel center is assumed to have a certain width equal to a single microlens pitch instead of a point. Thus, for the orthogonal MLA (the center of all microlenses forms an orthogonal grid), each light ray from the voxel center passing through the main lens is always received by the 4 microlenses and the 16 pixels beneath these 4 microlenses. This means that the projection of each light ray on the MLA plane is divided into the 4 rectangles by the 4 microlenses, and the 16 rectangles on the pixel plane beneath these 4 microlenses by the 16 pixels. The area of the rectangle is the contribution coefficient. Because of the special geometric relationship, a linear interpolation method is easily used for the calculation of these rectangle areas. However, the linear method is limited to the orthogonal MLA. Additionally, the discretized voxel pitch in the measurement volume should be close to the microlens pitch to ensure the special geometric relationship. The calculation process of the weight matrix takes on the order of 10s of hours even by parallel computation [39]. In Shi’s studies [40], the dense light rays from a given voxel are traced onto the MLA plane and the CCD sensor, respectively. The weight coefficient is calculated as the multiplication of the overlap between the dense light rays and the MLA (w), and the overlap between the light rays and the pixels (w) [24]. It does not need to consider the geometry of the MLA. However, it is also time-consuming. Different from the forward ray-tracing technique, the backward ray-tracing technique is usually adopted by the conventional Tomo-PIV and traces the light rays from the pixel center to the measurement volume in the flow field [41-43]. The contribution of the voxel to the pixels is dependent on the distance between the voxel and the pixel’s line-of-sight, which is characterized by a Gaussian function [44-45]. The light field camera can better represent the light beam sampled by the pixel in direction than the conventional camera [46]. So the backward ray-tracing technique combined with Gaussian function has the potential to provide a simple computational method for the weight matrix in the light field camera to improve its calculation efficiency.

This paper aims to present a computational method based on the backward ray-tracing technique combined with Gaussian function (named as Gaussian function method) to improve the calculation efficiency of the weight matrix in light field PIV. The 3D particle field is reconstructed by an Expectation-Maximization (EM) algorithm. A summed line-of-sight (SLOS) estimation is further used for the optimization of the weight matrix to accelerate the reconstruction process. The computational accuracy and efficiency of the weight matrix, the reconstruction quality of the 3D particle field, and the velocity field accuracy by Gaussian function method and the forward ray-tracing technique (named as Fahringer’s method) are numerically compared. Finally, a light field PIV setup is built, and experiments are carried out to verify the feasibility of the weight matrix by Gaussian function method. Numerical and experimental results are presented and analyzed.

2. Principle of light field PIV

Fig. 1 shows the schematics of the focused light field camera (FL). From Fig. 1, for the light field camera, when
the MLA is mounted in front of the CCD sensor, the light rays are separated by the microlens based on their direction, and then imaged on the CCD sensor underneath the microlens [22]. As a consequence, the direction and position of the light field can be simultaneously recorded by a single light field camera in a single exposure. The light beam (also named as the sphere-cylinder, and marked as the yellow, blue and green part in Fig. 1) represents a collection of the rays collected by a single pixel on the CCD sensor. The angle \( \theta \) represents the apex angle of the light beam in the 3D object space, and is used to characterize the ability of a single pixel to collect the rays. The pixel’s line-of-sight (marked as the black dotted line in Fig. 1) is a ray that passes through the pixel center, the center of its corresponding microlens, the virtual image plane (VIP), the main lens and the virtual object plane (VOP).

For the light field camera, \( \theta \) is less than 0.95°, and much smaller than that (\( \theta \approx 6.9° \)) of the conventional camera at the same optical parameters [46]. This means that the light field camera has the better representative direction than the conventional camera, and the pixels’ line-of-sight is more representative of the direction of the light field. So the weight matrix can be calculated by the pixel’s line-of-sight in the object space, and then is used for the reconstruction of the 3D particle field.

The light field PIV is schematically shown in Fig. 2. The tracer particles are firstly seeded in the 3D flow region (named as the measurement volume), and then illuminated by a pulse volumetric laser light [9]. The scattered light by the tracer particles at 90° is collected by the light field camera from a single view. Thus, a pair of the light field images of the tracer particles is recorded by the light field camera at the time interval of \( \Delta t \), and the 3D tomogram of particle fields are then reconstructed from the light field images. From the 3D tomograms, the particle displacement and the velocity field in the measurement volume are then determined by a 3D cross-correlation technique.

3. Reconstruction of light field PIV

3.1 Tomographic reconstruction

The measurement volume is discretized into a 3D array of the cubic voxel with intensity \( E \) in the 3D object space. Then, the intensity of each pixel is the integration of the intensity along the pixel’s line-of-sight direction \( s \) in the volume, which is expressed as [47]

\[
P_i(x,y) = \int E(X,Y,Z) \, ds
\]

where \( P_i \) is the intensity recorded by the \( i \)th pixel, \( E \) is the intensity of the voxel in the measurement volume. \((x, y)\) is the coordinate of the pixel in the pixel coordinate system, and \((X, Y, Z)\) is the central coordinates of the voxel in the world coordinate system.

Equation (1) is discretized as

\[
\begin{bmatrix}
P_1 \\
P_2 \\
\vdots \\
P_m
\end{bmatrix} =
\begin{bmatrix}
W_{1,1} & W_{1,2} & L & W_{1,j} & L & W_{1,n} \\
W_{2,1} & W_{2,2} & L & W_{2,j} & L & W_{2,n} \\
M & M & M & M & M & E_1 \\
W_{j,1} & W_{j,2} & L & W_{j,j} & L & W_{j,n} \\
M & M & M & M & M & E_2 \\
W_{n,1} & W_{n,2} & L & W_{n,j} & L & W_{n,n}
\end{bmatrix}
\]

where \( m \) is the total number of pixels on CCD sensor, \( n \) is the total number of the discretized voxels in the measurement volume, and \( W_{i,j} \) is the contribution of the \( j \)th voxel to the \( i \)th pixel, which is named as the weight matrix.
The tomographic reconstruction is an inverse process of Equation (2). According to the principle of light field imaging, the light from an object point would affect multiple pixels on the CCD sensor, especially for those further away from the virtual object plane. When the light is spread into more pixels, the corresponding pixel intensity would decrease, and then the pixel intensity would fall below background noise. This is one of the limits of the light field camera. There are two ways to alleviate this limitation. Firstly, a high energy pulsed laser is usually used in 3D-PIV. For the experiments in previous publications and our work, the energy of 200mJ is sufficient for the light field imaging and the measurement of flow field in a certain depth range [18, 39, 41]. Secondly, the robust reconstruction algorithm is used to improve the background noise problem. Among the reconstruction algorithms, deconvolution is usually employed to remove the contributions from out-of-focus objects [48]. It can considerably improve image contrast and reduce noise of image. Expectation-Maximization (EM) algorithm is one of the deconvolution algorithms for a blurred image, which is based on the maximum-likelihood estimate to recover a blurred image. It has a simple iterative scheme, and can quickly converge to a satisfactory solution. In this paper, EM algorithm is used to reconstruct the intensity distribution of the 3D particle field, which is expressed as [49-51]

\[
E(X_j, Y_j, Z_j)^{t+1} = E(X_j, Y_j, Z_j)^t \left[ \frac{P}{\sum_{j \in N} E(X_j, Y_j, Z_j) \cdot W_{i,j}} \right] \tag{3}
\]

where \(k\) is the number of iterations.

The normalized correlation coefficient is used to evaluate the reconstruction quality of the 3D particle field [52],

\[
Q = \frac{\sum_j \left[ E_0(X_j, Y_j, Z_j) \cdot E_i(X_j, Y_j, Z_j) \right]}{\sqrt{\sum_j \sum_{j \neq i} E_0^2(X_j, Y_j, Z_j) \cdot \sum_j E_i^2(X_j, Y_j, Z_j)}} \tag{4}
\]

where \(E_0(X_j, Y_j, Z_j)\) and \(E_i(X_j, Y_j, Z_j)\) are the real 3D intensity distribution of the \(j^{th}\) voxel and the reconstructed 3D intensity distribution of the \(i^{th}\) voxel, respectively.

The effect of the particle concentration on the tomographic reconstruction is also studied. The concentration of the tracer particle is defined as the number of particles per each MLA (ppm) [18]

\[
p_{\text{con}} = \frac{N_{\text{particle}}}{N_{\text{microlens}}} \tag{5}
\]

where \(N_{\text{particle}}\) and \(N_{\text{microlens}}\) are the numbers of the tracer particles and the MLAs, respectively.

### 3.2 Calculation of the weight matrix based on the backward ray-tracing

A backward ray-tracing technique is used to trace the pixel’s line-of-sight from the pixel center to the object space. The schematics of the backward ray-tracing technique of the FL are shown in Fig. 3. As shown in Fig. 3, the pixel’s line-of-sight passes through the center of the MLA, and then reaches the main lens. The coordinate of the intersection point \(A\) between the pixel’s line-of-sight and the main lens is calculated by

\[
y_i = m_j - l_m \frac{s_j - m_j}{d_j} \tag{6}
\]

where \(m_j\) is the central coordinate of the MLA along the \(Y\)-axis in the world coordinate system, \(l_m\) is the distance between the main lens and the MLA, \(s_j\) is the central coordinate of the sub-image along the \(Y\)-axis in the world coordinate system, and \(d_j\) is the distance between the MLA and the CCD sensor.

![Fig. 3 Schematics of the backward ray-tracing technique of the FL](image)

Then, the pixel’s line-of-sight passes through the VOP, and the coordinate of intersection point \(B\) between the pixel’s line-of-sight and the VOP is calculated by

\[
y_2 = -l_i \frac{s_i d_i - m_i d_i + d_i m_\text{FL}}{d_2} \tag{7}
\]

where \(l_i\) is the distance between the VOP and the main lens, and \(d_i\) is the distance between the MLA and the VIP.

Finally, the coordinate of the intersection point \(C\) between the pixel’s line-of-sight and the arbitrary voxel plane (marked as the green dotted line in Fig. 3) is calculated by

\[
y_3 = y_2 - \frac{z_3 (y_3 - y_2)}{l_i} \tag{8}
\]

where \(z_3\) is the distance between the arbitrary voxel plane and the VOP in the \(Z\)-axis.

For a given \(j^{th}\) voxel, \(W_{i,j}\) is dependent on its size and the distance between its center and pixel’s line-of-sight, and is characterized by Gaussian function [26, 41-42].
\[ W_{i,j} = A e^{-\frac{d_{i,j}^2}{2\sigma^2}} \]  

where \( \sigma \) is the standard deviation which is used to characterize the width of Gaussian distribution, and \( d_{i,j} \) is the perpendicular distance between the center of the voxel and the pixel’s line-of-sight.

From the above analysis, the number of light rays traced by the backward ray-tracing technique is greatly less than that by the forward ray-tracing technique, which is greatly useful to improve compactional efficiency. Besides, the calculation of the weight matrix by Gaussian function method is suitable for both the orthogonal and hexagonal MLA.

### 4. Numerical simulations

#### 4.1 Validation of the weight matrix

In Elsinga’s research, the variable \( \sigma \) in equation (9) was considered to be a constant, which is very convenient for the calculation of the weight matrix [9]. Since the pixel’s line-of-sight in the light field camera has the better representative direction than that in the conventional camera, \( \sigma \) is also set as a constant to calculate the weight matrix in light field PIV. In the Tomo-PIV, the reconstructed particle is characterized by a 3D Gaussian-type blob. The size of the 3D Gaussian-type blob should be greater than 3 voxels, ensuring that the 3D cross-correlation algorithm reaches the sub-voxel accuracy level [9]. To achieve the 3×3 Gaussian distribution, \( \sigma \) is set as 0.05 in this paper.

The weight matrix by Gaussian function method is compared with those by the theoretical calculation and Fahringer’s method to validate its accuracy. Firstly, the theoretical contribution of the voxel to each pixel in the CCD sensor (named as the point spread function (PSF)) is calculated. Fig. 4 shows the schematics of the theoretical calculation of the PSF. In Fig. 4, the voxel is discretized into the point sources (marked as the black spots in Fig. 4). The dense rays (marked as the yellow lines in Fig. 4) from each point source are traced to the main lens, the MLA and the CCD plane, and then imaged on the CCD sensor, shown in Figs. 5(a) and 6(a).

Secondly, for the calculation of the PSF by Fahringer’s method and Gaussian function method, \( W_{i,j} \) (the \( j \)th column of \( W_{m,n} \)) is extracted from the whole weight matrix \( W \), and then converted to the 2D light field image. Figs. 5 and 6 show the PSF patterns of a single voxel by the theoretical calculation, Fahringer’s method and Gaussian function method at \( Z = -5 \) mm, respectively. From Figs. 5 and 6, the PSF patterns by Fahringer’s method and Gaussian function method are closely similar to that by the theoretical calculation at \( Z = -5 \) mm.

#### Table 1 Parameters of the light field camera

<table>
<thead>
<tr>
<th>Method</th>
<th>( d_1 ) (mm)</th>
<th>( d_2 ) (mm)</th>
<th>( f_m ) (mm)</th>
<th>( f ) (mm)</th>
<th>( l_1 ) (mm)</th>
<th>( l_m ) (mm)</th>
<th>( P_{m} ) (mm)</th>
<th>( P_\theta ) (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fahringer’s method</td>
<td>-</td>
<td>0.6</td>
<td>0.6</td>
<td>100</td>
<td>200</td>
<td>199.4</td>
<td>0.1045</td>
<td>5.5</td>
</tr>
<tr>
<td>Gaussian method</td>
<td>10.7996</td>
<td>0.5684</td>
<td>0.6</td>
<td>100</td>
<td>200</td>
<td>189.2004</td>
<td>0.1045</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Numerical simulations are performed to validate the accuracy and feasibility of the weight matrix by Gaussian function method.

A server with a 32 core Intel (R) Xeon(R) CPU E5-2696 V4@2.2GHz and 128GB RAW is used for the calculations of the weight matrix, the tomographic reconstruction and the 3D cross-correlation. The parameters of the light field camera in Fig. 3 are listed in Table 1. For the convenience, several terms are defined as:

- **FL-Gaussian-O**: Gaussian function method is used for the calculation of the weight matrix in the FL with the orthogonal MLA.
- **FL-Gaussian-H**: Gaussian function method is used for the calculation of the weight matrix in the FL with the hexagonal MLA.
- **FL-Gaussian-SLOS**: SLOS is used for the optimization of the weight matrix in FL-Gaussian-O.
The Structural Similarity (SSIM) is used to evaluate the similarity of the PSF between the theoretical calculation and the Gaussian function, which is expressed as [53]

$$\text{SSIM}(I_x, I_y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{\mu_x^2 + \mu_y^2 + (2\sigma_{xy} + c_2)(\sigma_x^2 + \sigma_y^2 + c_2)}$$

(10)

where $I_x$ and $I_y$ are the images, SSIM is the similarity between the images $I_x$ and $I_y$, $\mu_x$ and $\mu_y$ are the intensity averages of the images $I_x$ and $I_y$, respectively, $\sigma_x^2$ and $\sigma_y^2$ are the variances of the images $I_x$ and $I_y$, respectively, $\sigma_{xy}$ is the covariance of the images $I_x$ and $I_y$, $c_1 = (k_1L)$ and $c_2 = (k_2L)$ are the variables to stabilize the division with the weak denominator, $L$ is the dynamic range of the pixel value in the image, and $k_1=0.01$ and $k_2=0.03$. The larger SSIM, the better similarity between the two images. The maximum of SSIM is 1, meaning that the two images are completely the same.

Fig. 7 shows the comparisons of the SSIMs of the PSFs by the theoretical calculation, Fahringer’s method and Gaussian function method when the voxel position ranges from -7.5 mm to 7.5 mm. From Fig. 7, the SSIMs of Fahringer’s method are better than 0.997 and 0.998 at $Z=[-7.5, 7.5]$ mm, respectively, indicating that the PSFs by Fahringer’s method are very close to the theoretical PSFs. The SSIMs of the FL are better than 0.998 at $Z=[-7.5, 2.5]$ mm, and decreases when the voxel position ranges from 2.5 mm to 7.5 mm. But the SSIMs of the FL are still better than 0.992 at $Z=[2.5, 7.5]$ mm. This validates the feasibility of Gaussian function for the calculation of the weight matrix at $\sigma=0.05$. 

The calculation times of the weight matrix at the different numbers of voxels by Fahringer’s method and Gaussian function method are compared and summarized in Table 2. The calculated weight matrix needs to be saved in the computer hard disk for Fahringer’s method and Gaussian function method. Thus, the overall calculation time of the weight matrix includes the calculation and storage time of the weight matrix.

<table>
<thead>
<tr>
<th>The number of voxels</th>
<th>( W ) by Fahringer’s method</th>
<th>( W ) by Gaussian function</th>
</tr>
</thead>
<tbody>
<tr>
<td>117( \times )117( \times )117</td>
<td>19392 s</td>
<td>1747 s</td>
</tr>
<tr>
<td>207( \times )307( \times )167</td>
<td>250777 s</td>
<td>3012 s</td>
</tr>
<tr>
<td>207( \times )307( \times )207</td>
<td>302852 s</td>
<td>5214 s</td>
</tr>
</tbody>
</table>

From Table 2, the overall calculation times of the weight matrix by Fahringer’s method and Gaussian function method increase with the increase of the number of voxels. When the number of voxels ranges from 207\( \times \)307\( \times \)167 to 207\( \times \)307\( \times \)207, the overall calculation time of the weight matrix by Fahringer’s method exceeds 2 days, which is very time-consuming. However, Gaussian function method takes thousands of seconds to calculate the weight matrix. This is significantly meaningful for the improvement of the computational efficiency of the weight matrix.

**4.2 Comparisons of the reconstruction**

In this section, the tracer particles are randomly positioned in a 11.7\( \times \)11.7\( \times \)11.7 mm\(^3\) volume with 117\( \times \)117\( \times \)117 discretized voxels. Fig. 8 shows the comparisons of the reconstruction quality \( Q \) by Multiplicative Algebraic Reconstruction Technique (MART) and EM algorithm. From Fig. 8, EM algorithm can quickly converge to a stable value \( Q = 0.37 \) within 100 iterations while MART algorithm needs more than 600 iterations to reach a stable value \( Q = 0.35 \). The calculation times of EM algorithm and MART algorithm are 3050s and 157710s (43.8h), respectively. Thus, EM algorithm is used for the reconstruction of the 3D particle field.

In section 4.2.1, the comparisons of the reconstruction time by EM without SLOS between Gaussian function method and Fahringer’s method at different particle concentrations are conducted. In section 4.4.2, the comparisons of the calculation time of the weight matrix including SLOS and the reconstruction (EM including SLOS, SLOS-EM), reconstruction quality at different particle concentrations and velocity field accuracy are carried out.
In reconstruction process of EM without SLOS, the weight matrix is required to read from the computer hard disk. Thus, the overall reconstruction time includes the reading time of the weight matrix from the computer hard disk and reconstruction time of EM without SLOS. Fig. 11 shows the comparisons of the overall reconstruction times of EM without SLOS by Fahringer’s method and FL-Gaussian at different particle concentrations. From Fig. 11, the overall reconstruction time of Fahringer’s method and FL-Gaussian increases with increasing particle concentration. The overall reconstruction times of EM without SLOS by Gaussian function method are almost the same as that by Fahringer’s method. This is because the overall reconstruction process of EM without SLOS by Gaussian function method is exactly the same as that by Fahringer’s method. The calculation methods of weight matrix have little effect on the reconstruction time. From Table 2, the overall calculation time of the weight matrix is 1747s when the number of voxels is $117 \times 117 \times 117$. From Fig. 11, the overall reconstruction time by EM without SLOS is about 2750s when the particle concentration is 1 ppm. It takes a total of about 4497s ($1747+2750$) to complete all the process including the calculation, the storage and reading of the weight matrix, and the reconstruction. The whole weight matrix is required to calculate only once and saved in the computer hard disk in advance. For the complex flow, this time-consuming reconstruction is limited to the reconstruction of the particle field at the high concentration of tracer particles. The main reason for the time-consuming reconstruction is that the weight matrix is very large and usually takes up hundreds of GB disk storages [32].
Fig. 11 Comparisons of the overall reconstruction times of EM without SLOS by Fahringer’s method, and FL-Gaussian at different particle concentrations

4.2.2 Reconstruction by SLOS-EM

Research has shown that the tracer particles are sparsely distributed in the measurement volume [54]. Thus, only a small portion of the voxels in the measurement volume has tracer particles, which are defined as the non-zero voxel. For example, when the particle concentration is 1 ppm, there are 13689 tracer particles in the 1601613 discretized voxels, accounting for about 0.85% of the total voxels. As a consequence, the weight matrix is very sparse. So, if the non-zero voxels can be pre-determined, it is useful to further improve the calculation efficiency and the memory storage of the weight matrix and to accelerate the reconstruction process.

A Multiplied line-of-sight (MLOS) estimation has been proposed to pre-determine the non-zero voxels by Atkinson in the conventional Tomo-PIV, and then only the contribution of the non-zero voxels to the pixel is calculated and saved, greatly decreasing the number of the element of the weight matrix [54]. The method has been proved to be efficient to accelerate the reconstruction. In their research work, the pre-determination of the non-zero voxels is that if the intensity of one of the pixels which are affected by the same voxel is equal to 0, this voxel intensity is equal to 0. Thus, the non-zero voxel in the measurement volume is distinguished by multiplying the pixels affected by the same voxel (named as the multiplicative operator).

Fig. 12 shows the non-zero voxel’s line-of-sights distribution traced from the light field image of a single particle at (X, Y, Z)=(0, 0, 5) mm. As a consequence, when the sum of the number of the non-zero pixel’s line-of-sights (I_a) accepted by the same voxel is larger than a threshold (I_s>0.6I_t), this voxel is a non-zero voxel. This pre-determination of the non-zero voxel is named as the summed line-of-sight (SLOS) estimation.

The SLOS estimation includes 5 steps. Step 1: all the pixel’s line-of-sights including the zero and the non-zero pixels are calculated by the backward ray-tracing technique. Step 2: the pixels affected by the same voxel is determined, and then the sum of the number of the zero and the non-zero pixel’s line-of-sights accepted by the same voxel (I_t) is calculated. Step 3: According to the light field image, the sum of the number of the non-zero pixel’s line-of-sights affected by the same voxel (I_a) is calculated. When I_a>0.6I_t, each non-zero voxel in the discrete volume is distinguished. Step 4: Based on the non-zero voxels determined by SLOS, the contribution of the non-zero voxels to the pixels (W_i,j) is calculated. Step 5: Reconstruction. The flow chart of the reconstruction in combination with SLOS is shown in Fig. 13.
Fig. 13 Flow chart of the reconstruction including SLOS

Fig. 14 shows the calculation times of the pre-determination, the weight matrix including SLOS and the reconstruction (EM including SLOS, SLOS-EM). Compared with the reconstruction process of EM without SLOS, the weight matrix including SLOS is not saved in the computer hard disk. From Fig. 14, with increasing the particle concentration, the computation times of the pre-determination, the weight matrix including SLOS and SLOS-EM increase. At the particle concentration of 1 ppm, the calculation times of the pre-determination, the weight matrix including SLOS and SLOS-EM are within 75s, 650s and 275s, respectively. It takes a total of about 1000s (75+650+275) to complete the reconstruction process including the pre-determination, the calculation of the weight matrix and the reconstruction. Compared with Table 2 and Fig. 11, at the particle concentration of 1 ppm, the calculation time of the weight matrix including SLOS (650s) is shorter than that without SLOS (1747s). The reconstruction time of EM including SLOS (275s) is shorter than that without SLOS (2750s). The overall process time including SLOS (1000s) is shorter than that without SLOS (4497s). This indicates that SLOS for the pre-determination of the non-zero voxels is capable of accelerating the computational efficiency of the weight matrix and the reconstruction process, although the weight matrix needs to be recalculated in each reconstruction.

Fig. 14 Calculation times of the pre-determination, the weight matrix including SLOS and the reconstruction (EM including SLOS, SLOS-EM)

The effects of the MLA arrangement and SLOS on the reconstruction quality at different particle concentrations are also investigated. The reconstruction qualities of Fahringer’s method, FL-Gaussian-O, FL-Gaussian-H, and FL-Gaussian-SLOS at different particle concentrations are compared in Fig. 15. It can be seen that in all cases, the reconstruction quality decreases with the increase of the particle concentration for the light field PIV. The reconstruction qualities of Gaussian function method are almost consistent with those of Fahringer’s method. It can be concluded that the introduction of SLOS has little effect on the
reconstruction quality, but is capable of improving the computation efficiency of the reconstruction. In addition, Gaussian function method is also suitable for the calculation of the weight matrix in the MLA of the hexagonal arrangement.

The measurement accuracy of the velocity field is mostly concerned in the PIV. A simulation of the laminar flow is used to further validate the feasibility and accuracy of the weight matrix by Gaussian function method. The tracer particles with the concentration of 1 ppm are randomly distributed in a 11.7×11.7×11.7 mm³ volume. The laminar velocity is 0.02 m/s. In simulation, a pair of the synthetic light field images is firstly generated by the forward ray-tracing technique. The time interval Δt between the pairs of light field images is 16 ms. The 3D particle fields of the laminar flow are then reconstructed from the synthetic light field images, and finally the 3D velocity field of the laminar flow is calculated by the 3D cross-correlation algorithm. The interrogation window of 3D cross-correlation is set as 16×16×16 with 50% overlap of the interrogation windows.

Fig. 16 shows the theoretical and calculated 3D velocity fields of the laminar flow. The theoretical 3D velocity field of the laminar flow in Fig. 16 (a) is

\[ v(y) = v_m [1 - \left( \frac{y}{R} \right)^2] \]  

(11)

where \( v_m \) is the maximum velocity of the laminar flow, \( R \) is the size of the pipe, and \( y \) is the coordinate on the Y-axis.

Fig. 17 shows the comparisons of 1D velocity distributions of the laminar flow. It can be seen from Figs. 16 and 17 that the 3D velocity field by FL-Gaussian-O is consistent with the theoretical velocity field. The 3D velocity fields by FL-Gaussian-H and FL-Gaussian-SLOS are almost the same as the theoretical velocity field, and are not illustrated here. The 1D velocity distributions of the laminar flow by Fahringer’s method and Gaussian function method are parabolic and in good agreement with the theoretical velocity. Fig. 18 shows relative errors of the 1D velocity distributions. From Fig. 18, the relative errors of the 1D velocity distributions by all the cases are within 5%.
Fig. 16 Theoretical and calculated 3D velocity fields of the laminar flow

Fig. 17 Comparisons of 1D velocity distributions of the laminar flow

Fig. 18 Relative errors of 1D velocity distributions

Table 3 Results of the correlation coefficient

<table>
<thead>
<tr>
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<th>Fahringer's method</th>
<th>FL-Gaussian-O</th>
<th>FL-Gaussian-H</th>
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<tr>
<td>Z(mm)</td>
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<td>Y(mm)</td>
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<td>Relative errors</td>
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(a) XOY plane at Z=-0.13mm
(b) XOZ plane at Y=-0.13mm
5. Experimental verification

Experiments are further conducted to verify Gaussian function method for the calculation of the weight matrix. Fig. 19 shows the experimental rig of the light field PIV in our group. It is mainly comprised of the assembled light field PIV and the laminar flow rig. The assembled light field PIV, as shown in Fig. 19 (a), includes one double pulsed laser source, one focused light field camera (Raytrix R29) and one synchronous controller. The optical parameters of Raytrix R29 are calibrated using Sun’s and Strobl’s calibration method, and summarized in Table 4 [55-56]. From Table 4, the MLA of the Raytrix R29 has three microlenses with different focal lengths. The inaccurate optical parameters of light field camera such as the lens distortion, the MLA offset, the image distance, the distance between CCD and MLA etc. would affect the accuracy of the reconstructed particle position. The lens distortion has been considered in the Sun’s and Strobl’s calibration method. What’s more, the MLA offset is corrected by Raytrix R29 software. Thus, Raytrix R29 software is capable of providing the accurate light field image and total focused image for calibration. According to calibrated optical parameters of Raytrix R29, the depth accuracy has been verified by reconstructing the pinhole position in our previous work [37]. Our previous validation work showed that the optical parameters of Raytrix R29 by Sun’s and Strobl’s calibration method are capable of reconstructing an accurate particle position.

Fig. 19 (b) shows a schematic of the laminar flow rig, including two reservoirs, two valves, a flow meter and a submersible pump. The acrylic channel is a 15mm×15mm square with a length of 2000 mm. Purified water is pumped from the reservoir 1 into the reservoir 2 by the submersible pump, and flows into the reservoir 1 along the acrylic channel. To make the flow stable in measurement volume, the valves 1 and 2 are regulated to keep a constant water level in the reservoir 2. The flow field is seeded with polyamide particles with a mean diameter of 50 μm, a density of 1.03 g/cm$^3$ and a particle concentration of 0.89 ppm. Illumination is provided by a Vlite 200 double pulsed laser source with the maximum output energy of 200 mJ per pulse at 532 nm and a pulse duration of 7 ns.

The flowrate is 0.3 L/min in the experiment, and the corresponding average velocity and the Reynolds number (ReD) in the channel are 0.0222 m/s and 387, respectively. The measurement volume is 30mm (X-axis) × 15mm (Y-axis) × 15mm (Z-axis), and discretized into 300×150×150 voxels. The 3D cross-correlation window size is 64×64×64 with 85% overlap of interrogation windows.

| Table 4 Parameters of the Raytrix R29 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $d_1$ (mm)      | $d_2$ (mm)      | $f_m$ (mm)      | $f$ (mm)        | $l_1$ (mm)      | $l_m$ (mm)      | $P_m$ (mm)      | $P_x$ (μm)      |
| 10.2903         | 4.4447          | 1.1914          | 1.6277          | 100             | 193.7112        | 195.2292        | 0.1705          | 5.5             |
| 2.8496          | 0.9997          | 0.9998          | 0.9994          | 0.9998          | 0.9985          | 0.9983          |                 |                 |
Fig. 20 shows the raw light field images of the laminar flow with tracer particles. From Fig. 20, the pixels under each microlens are highlighted to form a series of sub-images that are the hexagonal arrangement. Fig. 21 shows the 3D-3C velocity field and 2D velocity distributions of the laminar flow. Figs. 21 (b), (c), and (d) show the 2D velocity distributions at five YOZ planes ($X=-10.95\text{mm}$, $-5.55\text{mm}$, $-0.15\text{mm}$, $5.25\text{mm}$ and $10.65\text{mm}$), one XOY plane ($Z=0.15\text{mm}$) and one XOZ plane ($Y=0.15\text{mm}$), respectively. From Fig. 21, the flow moves in the negative direction along the $X$-axis. The maximum velocity of the laminar flow is $0.042\text{ m/s}$ near the center of the channel. The corresponding average velocity is $0.028\text{ m/s}$, which is very close to the theoretical average velocity of $0.0222\text{ m/s}$.

Fig. 20 Raw light field images of the laminar flow with the tracer particles

(a) First frame

(b) Second frame

Fig. 21 3D-3C velocity field and 2D velocity distributions of the laminar flow

(a) 3D velocity field

(b) 2D velocity distribution at YOZ plane

(c) 2D velocity distribution at XOY plane

(d) 2D velocity distribution at XOZ plane
Fig. 22 (a) shows the 1D velocity distributions vary with the \( Y \)-axis at \( XOY \) plane corresponding to Fig. 21 (c). Fig. 22 (b) shows the 1D velocity distributions vary with the \( Z \)-axis at \( XOZ \) plane corresponding to Fig. 21 (d). From Fig. 22, all the 1D velocities along the \( Y \)-axis and the \( Z \)-axis have parabolic distributions, and their profiles are fairly good agreement with the theoretical velocity. The 1D velocity distributions in Fig. 22 (b) at \( Z = [-2, 2] \) mm is flat, which is caused by the low depth resolution of the Raytrix R29 in the \( Z \)-axis. The velocities in all the cases are slightly slower than the theoretical velocities. Note that the velocities near the wall of the acrylic channel are slower than those near the centre of the acrylic channel in all the 1D velocity distributions, which is caused by the viscosity of the fluid. What’s more, the velocities near the center of the acrylic channel is closer to the theoretical velocities than those near the wall of the acrylic channel, which is mainly attributable to the surface roughness of the wall of the acrylic channel. The surface roughness further slows down the velocity of the tracer particles near the wall of the acrylic channel. Fig. 23 shows relative errors of the 1D velocity distributions. It can be seen that the velocity errors near the center of the acrylic channel are within 8%, which is acceptable and consistent with the published research [57]. Table 5 shows results of the correlation coefficient corresponding to Fig. 22. The correlation coefficients of the 1D velocities along the \( Y \)-axis is better than that along the \( Z \)-axis. Overall, these experimental results further verify the feasibility of the weight matrix by the backward ray-tracing technique in combination with Gaussian function. The weight matrix by Gaussian function method is capable of measuring the instantaneous 3D-3C velocity field.
6. Conclusions

In this paper, a backward ray-tracing technique in combination with Gaussian function was proposed to calculate the weight matrix of the light field PIV and to improve its computational efficiency. An Expectation-Maximization (EM) algorithm in combination with SLOS estimation was further employed to accelerate the reconstruction of the tracer particle field. Numerical simulations were carried out to investigate the computational accuracy and efficiency of the weight matrix, the reconstruction quality of the tracer particles field, and the accuracy of the 3D velocity field. Experiments have been also conducted to measure the 3D-3C velocity field of a laminar flow. Simulation results showed that the calculation time of the weight matrix by Gaussian function method is better than that by Fahringer’s method. SLOS estimation is capable of improving the reconstruction efficiency, and has no effect on the reconstruction quality. Meanwhile, the reconstructed quality and the velocity field accuracy by Gaussian function method are almost similar to those by Fahringer’s method. Experiment results further proved the feasibility of the weight matrix by Gaussian function method.

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