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WenBin Liu

Kent Business School

DaQun Zhang

Institute of Policy & Management, Chinese Academy of Sciences

Li Qi

Kent Business School

XiaoXuan Li

School of Management, University of Science & Technology of China

University of Kent

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DEA Analysis Based on both Efficient and Anti-efficient Frontiers[1]

DaQun Zhang [a,b], Li Qi [c], XiaoXuan Li [a], WenBin Liu [c,*],

[a] Institute of Policy and Management, Chinese Academy of Sciences, China

[b] School of management, University of Science and Technology of China

[c] Kent Business School, University of Kent, UK

Abstract: In this work we developed a new approach to utilize both the efficient frontier and the anti-efficient frontier to enhance discrimination power of DEA analysis. The standard DEA models are used to identify the efficient frontier, while some DEA models with undesirable variables are used to identify the anti-efficient frontier. Then performance indexes are formulated to combine the information from both frontiers. Empirical study showed these performance indexes indeed have much more discrimination power.

Keyword: DEA, TOPSIS, efficient frontier, anti-efficient frontier.

1. Introduction

Data Envelopment Analysis (DEA) was first introduced by Charnes et al (1978), and has now been widely used in performance evaluation or productivity evaluation. The main idea of the classic DEA is to first identify the production frontier on which the decision making units (DMUs) will be regarded as efficient. Then those DMUs that are not on the frontier will be compared with their peers on the frontier to estimate their efficiency scores. All the DMUs on the frontier are regarded to have the same level of performance and to represent the best practice. One of the main advantages of DEA is to allow the DMUs to have the full freedom to select their weights, which are most favorable for their assessments to achieve the maximum efficiency score. This full flexibility of selecting weights is important in the identification of inefficient DMUs. However, weights with full flexibility may cause many serious problems in

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[*] Corresponding author.

E-mail addresses: daqun.zhang@googlemail.com (DaQun Zhang), qili@vip.163.com (Li Qi) , xiaoxuan@casipm.ac.cn (XiaoXuan Li), w.b.liu@kent.ac.uk (WenBin Liu)

practical applications. One of the problems is that this full flexibility may much reduce the discrimination power of DEA in the sense there often exist too many DMUs on the frontier, which cannot be further ranked in the standard DEA models. This is particularly so when there are many inputs and outputs variables. Decision makers may find all or most DMUs are efficient, and the results would have little use for decision making. Hence, Cooper et al. (2000, p. 252) proposed a rule of thumb for DEA, which demands

$$n \geq \max\{m \times s, 3(m+s)\},$$

where n is the number of DMUs, m and s are the number of inputs and outputs. However such conditions may not met in many applications.

To enhance the power of discrimination, some scholars have developed the weights restriction approach (see Allen et al 1997, Thanassoulis et al 2004) and preference change methods (see Liu, Meng and Zhang, 2006) to incorporate the prior information or value judgments of DMs into DEA models, such as the marginal rates of substitution between the inputs and/or outputs etc, although there are also various issues in applying these methods as well. In addition, Andersen and Petersen (1993) employed the super-efficiency DEA model to evaluate efficient DMUs, which excludes a DMU itself from the reference set. This model was first used to identify outliers of observations in Banker and Gifford (1988). It is clear that this model in fact uses different reference sets to evaluate the efficient DMUs and inefficient DMUs. Furthermore Banker and Chang (2006) reported that Andersen and Petersen's (1993) procedure using the super-efficiency scores for ranking efficient observations did not perform satisfactorily.

In real life, people often have more than one reference in judging DMUs, and may not just compare the DMUs with good references, but sometimes with bad references as well. For example, on one hand a DMU is better if it is closer to the good references (or efficient frontier); on the other hand, it is also good if it is far from the bad references (or anti-efficient frontier). Actually, TOPSIS (see below for the details) has used this idea in evaluating DMUs, although its best and worst peers may not be realizable and could be far from reality. In this sense the classical DEA models haven't fully taken the advantage of the information implied in the data. In fact, the classic DEA models have just employed the best practice DMUs to construct the efficient frontier, and compare each DMU with a peer located on the frontier.

In this paper, we will develop a new DEA approach based on the idea used in the TOPSIS approach to enhance discrimination power of DEA.

The paper is organized as follows: in Section 2, we present the DEA models that can find the anti-efficient frontier of DMUs; in Section 3, to combine the information from both best and worst viewpoints, we introduce two performance indexes. And then we give empirical studies to illustrate the features of different models in Section 4. Finally, conclusions and discussions are given in Section 5.

2. TOPSIS and DEA Analysis Based on Efficient and Anti-efficient Frontiers

2.1 TOPSIS method

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was first introduced by the Hwang and Yoon (1981), and has been widely applied in multi-attribute decision theory (MADT). Essentially, the steps of TOPSIS can be summarized as follows:

Step 1, normalize the decision matrix.

$x_{ij} = x_{ij} / \sqrt{\sum_{j=1}^{n} x_{ij}^2}$, and $y_{rj} = y_{rj} / \sqrt{\sum_{j=1}^{n} y_{rj}^2}$

And evaluators can also assign a weight for each criterion to have a weighted decision matrix.

Step 2, introduce two virtual ideal reference DMUs, that is the positive ideal reference point: $\{(x^*, y^*): x_i^* = \min_j \{x_{ij}\},\ y_r^* = \max_j \{y_{rj}\},\ j = 1,\dots,n\}$, and the negative ideal reference point: $\{(x^0, y^0): x_i^0 = \max_j \{x_{ij}\},\ y_r^0 = \min_j \{y_{rj}\},\ j = 1,\dots,n\}$, where x_{ij} and y_{rj} are the normalized input and output variables of DMU_j respectively.

Step 3, calculate the distance of each DMU to the ideal DMUs d_j^* and d_j^0, where

$$d_j^* = \sqrt{\sum_{i=1}^{m}(x_{ij} - x_i^*)^2 + \sum_{r=1}^{s}(y_{ij} - y_i^*)^2} \text{ and } d_j^0 = \sqrt{\sum_{i=1}^{m}(x_{ij} - x_i^0)^2 + \sum_{r=1}^{s}(y_{ij} - y_i^0)^2}.$$

For example, d_j^* and d_j^0 are the length of AC and AB respectively in Figure 1.

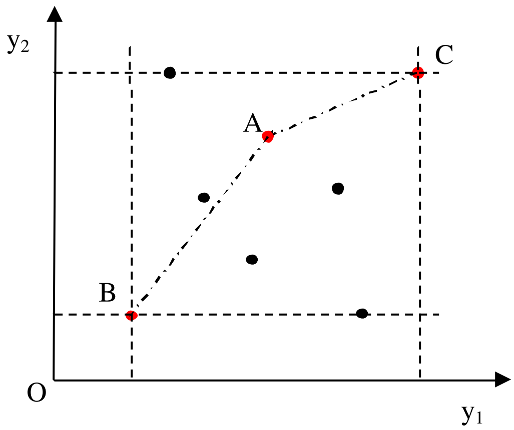

Figure 1: TOPSIS methods

Step 4, compute a comprehensive performance index,

$$S_j = \frac{d_j^0}{d_j^* + d_j^0}.$$

Finally, we can rank the alternatives by using the performance index.

As mentioned in Section 1, the best and worst ideal DMUs constructed in TOPSIS may not be realizable and could be far away from reality. Thus it may not be a good idea to just add the best and worst virtual DMUs constructed in TOPSIS into the existing DMU set and then to carry out DEA analysis using the extended data set as in Wang and Luo (2006) and Wu (2006) where the two virtual decision making units called ideal DMU and anti-ideal DMU were added into extended DMUs for further DEA analysis, since then the Possible Production Set (PPS) may have be greatly changed.

2.2 Anti-efficient frontier and DEA models with undesirable variables

Using the standard DEA, one can easily find the best practice reference – the efficient frontier that is realizable. To find the bad reference set, we simply treat the inputs and outputs of DMUs both as undesirable, and then use some kind of DEA models with undesirable inputs and outputs to find the bad reference set – to be called the anti-efficient frontier. The idea is simple: to find the anti-efficient frontier one should maximize the inputs and minimize the outputs, so this equivalently says the inputs and outputs are undesirable in DEA models. One widely used approach to deal with undesirable variables is to view the undesirable outputs as desirable inputs and the undesirable inputs as desirable outputs in classic DEA models, see for example, Liu et al (2006, 2007), and this leads to the following model (CCR type):

$$
\begin{aligned}
h_w^* = \text{Max} \quad & \theta \\
\text{Subject to:} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \geq \theta x_{i0}, i = 1,\ldots,m, \\
& \sum_{j=1}^{n} y_{rj}\lambda_j \leq y_{r0}, \ r = 1,\ldots,s, \\
& \lambda_j \geq 0, \ j = 1,\ldots,n.
\end{aligned}
\qquad (1)
$$

If h_w^* =1 in Model (1), then the DMU belongs to the anti-efficient group. Similarly, we can also have the following output oriented DEA model:

$$
\begin{aligned}
(h_w^*)^{-1} = \text{Min} \quad & \phi \\
\text{Subject to:} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \geq x_{i0}, i = 1,\ldots,m, \\
& \sum_{j=1}^{n} y_{rj}\lambda_j \leq \phi y_{r0}, \ r = 1,\ldots,s, \\
& \lambda_j \geq 0, \ j = 1,\ldots,n.
\end{aligned}
\qquad (2)
$$

If we wish to measure the slacks of inputs and outputs, we can simply introduce some variables s_i^+, s_r^- , and transform Model (1) to the following model:

$$
\begin{aligned}
h_w^* = \text{Max} \quad & \theta + \varepsilon\left(\sum_{i=1}^{m} s_i^+ + \sum_{r=1}^{s} s_r^-\right) \\
\text{Subject to:} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j - s_i^+ \geq \theta x_{i0}, i = 1,\ldots, m, \\
& \sum_{j=1}^{n} y_{rj}\lambda_j + s_r^- \leq y_{r0}, \ r = 1,\ldots,s, \\
& \lambda_j, s_i^+, s_r^- \geq 0, \ j = 1,\ldots, n.
\end{aligned}
\qquad (3)
$$

Let us emphasize that there are many different approaches to deal with undesirable variables in DEA, see, Liu et al (2007). However here we only explore this simpler approach.

Definition 1: DMU_0 is weakly anti-efficient, if and only if $h_w^* = 1$ in Model (1).

Definition 2: DMU_0 is anti-efficient, if and only if $h_w^* = 1$, and $s_i^{+*} = s_r^{-*} = 0$ in Model (3).

Theorem 1: The optimal value of Model (2) is the reciprocal of that of Model (1).

Proof: If we let $\lambda_j' = \lambda_j / \theta$, then we can rewrite Model (1) as follows:

$$\begin{aligned} &\text{Max} \quad \theta \\ &\text{Subject to:} \quad \sum_{j=1}^{n} x_{ij}\lambda_j' \geq x_{i0}, i = 1,\ldots,m, \\ &\qquad \sum_{j=1}^{n} y_{rj}\lambda_j' \leq \frac{1}{\theta} y_{r0},\ r = 1,\ldots,s, \\ &\qquad \lambda_j' \geq 0,\ j = 1,\ldots,n. \end{aligned} \tag{3}$$

By changing the object function θ with $1/\theta$, the Model (3) can be transformed to:

$$\begin{aligned} &\text{Min} \quad \frac{1}{\theta} \\ &\text{Subject to:} \quad \sum_{j=1}^{n} x_{ij}\lambda_j' \geq x_{i0}, i = 1,\ldots,m, \\ &\qquad \sum_{j=1}^{n} y_{rj}\lambda_j' \leq \frac{1}{\theta} y_{r0},\ r = 1,\ldots,s, \\ &\qquad \lambda_j' \geq 0,\ j = 1,\ldots,n. \end{aligned} \tag{4}$$

Hence if θ^* is the optimal solution of Model (4), it is also the optimal solution of Model (3). And then it is also the optimal solution of Model (1). Hence the optimal value of Model (4) is the reciprocal of that of Model (1).

If we let $\phi = 1/\theta$, then Model (4) can be written as Model (2), that is the optimal value of Model (2) is also the reciprocal of that of Model (1).

In the rest of the paper, we will simply refer to the above DEA models as anti-efficient DEA models. Now we continue to illustrate the meanings of the anti-efficient DEA models. For simplicity, we assume there are only two inputs and one output, and all of them are desirable. As shown in Fig. 2, contrary to the standard DEA models using the best practice DMUs to construct efficient frontier, these DEA models employ the worst practice DMUs to construct the anti-efficient frontier, and let the DMU being evaluated compare with a virtual or real DMUs located on the anti-efficient frontier. Suppose there are two DMU_1 and DMU_2 being evaluated. If the DMU_1 is relative farer from the anti-efficient frontier than DMU_2, that is $h_{1w}^* \geq h_{2w}^*$, then DMU_1 may be considered to have some advantages over DMU_2.

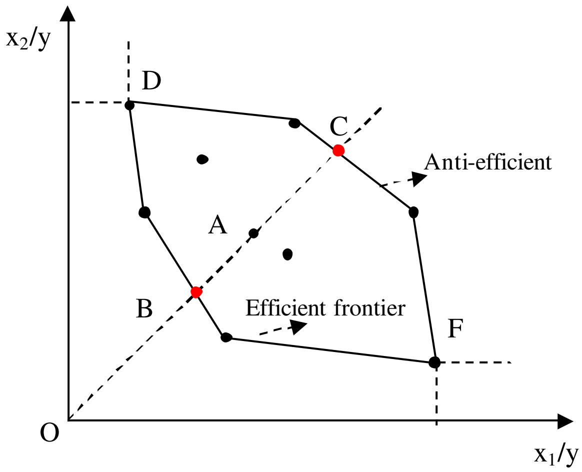

Figure 2: Best and Worst practice Frontier

And the dual model of Model (1) reads:

$$
\begin{aligned}
&\text{Min} && u^t y_0 \big/ v^t x_0 \\
&\text{Subject to:} && u^t y_j \big/ v^t x_j \geq 1, \\
& && u \geq 0,\ v \geq 0,\ j = 1, \ldots, n.
\end{aligned}
\qquad (5)
$$

where x_j, y_j, v, u are the inputs, outputs of DMU_j and the corresponding weights respectively. The standard DEA model is to evaluate the performance of DMUs from the perspective of optimism, and on the contrary, DEA Model (5) is to evaluate the performance of DMUs from the perspective of pessimism.

To deal with the problem of slacks in Model (1), we can use the Russell Measure. Then the model reads:

$$h_w^* = \text{Max} \quad \sum_{i=1}^{m} \frac{\theta_i}{m}$$

$$\begin{aligned} \text{Subject to:} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \ge \theta_i x_{i0}, \\ & \sum_{j=1}^{n} y_j \lambda_j \le y_0, \\ & \theta_i \ge 1, \lambda_j \ge 0. \end{aligned} \tag{6}$$

It should be noticed that Model (5) was first appeared in Yamada et al (1994), and was called "Inverted" DEA model. Recently, inverted DEA models have increasingly appeared in the literatures, such as in Entani et al (2002). Yoshiharu and Tone (2003) employed the DEA and inverted DEA with weights restriction to solve the problem of site selection for the relocation of several government agencies outside of Tokyo. Paradi, Asmild and Simak (2004) used similar ideas to identify the worst practices in banking credit analysis.

It is clear that by utilizing the anti-efficient frontier generated by the worst practice DMUs, we can obtain more information about performance, and then enhance power of discrimination for DEA analysis. For example, the DMUs D and F in Fig.2 may be regarded worse than the other efficient DMUs on the efficient frontier as D and F are also on the anti-efficient frontier. In the following section, we will examine some possible approaches to utilize this extra information in DEA analysis.

3. Combined Performance Indexes

It seems that there are many possible ways to utilize the worst frontier. Here we will propose two ways to aggregate the efficiency score h_b^*, h_w^* of the input-oriented CCR DEA and the anti-efficient CCR DEA models:

In the first approach, we define a 2-d performance index as $h_I^* = (h_b^*, h_w^*)$ using the lexicographic order, which is to be called "P-index I". Naturally people may argue it is more important to be close to the best frontier, and less important to be far from the worst frontier. Hence, we can use (h_b^*, h_w^*) score to rank the DMUs with the lexicographic order. For example, if we want to evaluate the performance of DMU_1 and DMU_2, we first use h_b^* to compare them, and if $h_{1b}^* > h_{2b}^*$, then DMU_1 is considered to perform better than DMU_2. Else if $h_{1b}^* = h_{2b}^*$, then we continue to use h_w^* to compare DMU_1 and DMU_2. If $h_{1w}^* > h_{2w}^*$, then DMU_1 performs better than DMU_2.

In the second approach, we treat the two scores more equally and define a composite performance index combing the two as $h_{II}^* = [h_b^* + (1-1/h_w^*)]/2$, which is to be called "P-index II" and its range is [0,1). Let us notice that it may not be a good idea to directly use $h_b^* + h_w^*$ because the ranges of h_b^* and h_w^* are different. The range of h_b^* is [0,1] and that of h_w^* is $[1,+\infty)$. Hence, we need to use some transformations before we can add the two scores. We may wish to keep the same orientation as h_b^* (i.e. larger is better). Let us notice that the range of the score h_w^* is $[1,+\infty)$, and $1/h_w^*$ is the optimal value of the output-oriented anti-efficient CCR DEA models according to Theorem 1. Thus it seems to be plausible to apply transformation $1-1/h_w^*$ as we have used in this paper. In this case if DMU_0 is on the anti-efficient frontier, then $h_w^* = 1$ and $h_{II}^* = h_b^*/2 \le 1/2$. If a DMU is on both the efficient and anti-efficient frontiers like D and F, then $h_b^* = 1$, $h_w^* = 1$ and $h_{II}^* = 1/2$. And if a DMU is on the efficient frontier but not on the anti-efficient frontier, then its score will be higher than 1/2, and thus has a better score than D and F.

4. Empirical Studies

Now we will apply the performance indexes to one empirical example. The data set comes from Zhu (2003), which is shown in Table 1.

Table 1: Fortune Global 500 Companies

Company	Assets	Equity	Employees	Revenue	Profit
Mitsubishi	91920.6	10950	36000	184365.2	346.2
Mitsui	68770.9	5553.9	80000	181518.7	314.8
Itochu	65708.9	4271.1	7182	169164.6	121.2
General Motors	217123.4	23345.5	709000	168828.6	6880.7
Sumitomo	50268.9	6681	6193	167530.7	210.5
Marubeni	71439.3	5239.1	6702	161057.4	156.6
Ford Motor	243283	24547	346990	137137	4139
Toyota Motor	106004.2	49691.6	146855	111052	2662.4
Exxon	91296	40436	82000	110009	6470
Royal Dutch/Shell Group	118011.6	58986.4	104000	109833.7	6904.6
Wal-Mart	37871	14762	675000	93627	2740
Hitachi	91620.9	29907.2	331852	84167.1	1468.8
Nippon Life Insurance	364762.5	2241.9	89690	83206.7	2426.6
Nippon Telegraph&Telephone	127077.3	42240.1	231400	81937.2	2209.1
AT&T	88884	17274	299300	79609	139

Then we employ the input-oriented CCR model, the anti-efficient CCR model (1), use both of the performance indexes. With the intention to find the characteristics and differences of these models, we rank the DMUs according to the scores from the different models, and the results are illustrated in Table 2.

Table 2: Results and ranks of companies using different models

Company	CCR		P-index I		P-index II		TOPSIS	
	h_b^*	Rank 1	h_w^*	Rank 2	h_{II}^*	Rank 3	S_j	Rank 4
Mitsui	1.00	1	2.93	1	0.83	1	0.646	5
Sumitomo	1.00	1	2.68	2	0.81	2	0.656	2
Exxon	1.00	1	1.45	3	0.65	3	0.690	1
General Motors	1.00	1	1.22	4	0.59	6	0.490	15
Itochu	1.00	1	1.18	5	0.58	7	0.652	3
Wal-Mart	1.00	1	1.00	6	0.50	8	0.515	12
Nippon Life	1.00	1	1.00	6	0.50	8	0.523	10
Marubeni	0.97	8	1.40	8	0.63	4	0.649	4
Royal Group	0.84	9	1.00	9	0.42	11	0.601	7
Ford Motor	0.74	10	1.31	10	0.49	10	0.514	13
Mitsubishi	0.66	11	2.27	11	0.61	5	0.638	6
Toyota Motor	0.52	12	1.07	12	0.30	12	0.545	8
Hitachi	0.39	13	1.00	13	0.19	13	0.520	11
Nippon T&T	0.35	14	1.00	14	0.17	14	0.510	14
AT&T	0.27	15	1.00	15	0.14	15	0.532	9

As shown in Table 2, the Ranks 1, 2, 3 and 4 are generated by the CCR efficiency score, P-index I, P-index II, and TOPSIS respectively.

It is clear that both the CCR and anti-efficient CCR models along have weaker power of discrimination in this empirical study. Among 15 companies, there are 7 companies are efficient in the CCR model, and 6 companies are anti-efficient.

As P-index I used the scores of CCR and anti-efficient CCR model ($h_I^* = (h_b^*, h_w^*)$) with the lexicographic order to rank DMUs, it is clear that the ranks of P-index I are not conflict with those from the CCR model. Furthermore it can discriminate these efficient DMUs, and those DMUs whose scores of CCR are equal.

It is interesting to see that the h_b^*, h_w^* scores of Wal-Mart and Nippon Life Insurance are both 1 in the CCR model and the anti-efficient CCR model, which implies that they are located on both the efficient and the anti-efficient frontiers. Hence, it is may not be reasonable if we only rank the DMUs according to the scores of the CCR model. On the other hand if we use TOPSIS to rank, the ranks of Wal-Mart and Nippon Life Insurance are 12, 10 respectively. These two ranks are far from those from the CCR model. Finally, if we apply the P-index I and II, the ranks of them are 6, 8 separately, which are much closer with those from the TOPSIS method.

There is another strong conflict between the CCR model and the TOPSIS method. General Motors is ranked the best in the CCR model, but on the contrary, it is ranked the worst in the TOPSIS method. However, it is ranked 4th and 6th using our P-index I and II respectively, which seem to be more reasonable. This result may come from the main characters of the CCR and TOPSIS. That is CCR just considers the best practice DMUs as the reference. Although TOPSIS considers the information both from the best and worst practice, the positive and negative ideal DMUs constructed in the method may be far from realizable.

It is also interesting to find the rank of Marubeni is 8th by the scores of CCR and P-index I, while it is ranked 4th by the score of P-index II and TOPSIS. This is because it is further away from the anti-efficient frontier than some other companies, although they are all on the efficient frontier. That is why it is also ranked high in the TOPSIS method. The following table shows correlation among these results.

Table 3: The Spearman's correlation Coefficient of four ranks

Coefficient	Rank 1	Rank 2	Rank 3	Rank 4
Rank 1	1	0.949**	0.768**	0.350

Rank 2	0.949**	1	0.871**	0.464
Rank 3	0.768**	0.871**	1	0.614*
Rank 4	0.350	0.464	0.614*	1

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

As shown in Table 3, the Spearman's correlation coefficient of rank 1 and rank 2 results is 0.949, which implies that the results form CCR and P-index I are closely correlated as they are. P-index II also shows higher correlations with CCR and P-index I, which are 0.768 and 0.871 respectively. On the other hand, it is interesting to notice that the ranks from CCR and TOPSIS are only slightly correlated and not even significant at 0.05 level. And only the ranks from P-index II and TOPSIS have shown stronger correlations, which is 0.614 and significant at 0.05 level.

Thus it seems that our P-indexes are more like a DEA approach in terms of the correlation and act like a bridge between the CCR model and the TOPSIS method.

5. Conclusions

It seems to be possible to consider both the efficient and anti-efficient frontiers in DEA analysis. This extra information may increase discrimination power of DEA analysis quite well. In our approach we employed the DEA models with undesirable inputs and outputs to identify the anti-efficient frontier. Two possible performance indexes have then been proposed and tested. The empirical results suggested that our approaches are closer to the DEA methods rather than the TOPSIS method. In addition, our approaches increased ability of discrimination for DEA analysis and acted like a bridge between the CCR model and the TOPSIS method.

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University of Kent