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# FISCAL POLICY IN THE PRESENCE OF INFORMAL LABOUR AND GOODS MARKETS

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Thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy (Ph.D) in Economics.

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### Abstract

This doctoral thesis consists of three chapters on the impacts of fiscal policy in the presence of informal labour and goods markets.

In developing countries, the presence of large informal sector as well as weak financial and economic institutions makes the conduct of the economy partially different from developed world. The structure of the goods and labour markets are different in both formal and informal sectors of the economy. For a better understanding of the response of an economy to external and internal shocks, it is important to have knowledge about the responses of the formal and informal sectors to these shocks. The first chapter "Macroeconomic effects of fiscal policy shocks in the presence of informal sector", develops a New-Keynesian dynamic stochastic general equilibrium model to analyse the impacts of five fiscal policy shocks in economies with large informal sector. The model shows that government expenditure and government consumption impact multipliers are larger than one in the presence of informal economy. The government expenditures shocks are more stimulating at increasing the level of formal sector output in the first few quarters, whereas decreases in capital and labour income taxes are more effective in the longer horizon. The analysis shows that, the increases in government expenditures and decrease in consumption tax, which are financed by increasing the labour and capital income taxes, increase the size of the informal economy. The size of the informal economy decreases on impact of a decrease in labour income tax, whereas a cut in capital income tax decreases the size of informal economy after ten quarters. This chapter also finds that an increase in labour mobility amplifies the impacts of fiscal policy shocks on both formal and informal segments of the economy. The government expenditures present value multipliers for aggregate and formal sector output increase when labour is highly mobile across both sectors. Additionally, the negative impacts of cuts in capital and labour income taxes are more pronounced on the output of informal sector, when there is a high degree of labour mobility.

The second chapter "Optimal capital taxation in the presence of informal labour and goods markets", contributes to the literature of optimal fiscal policy by incorporating the informal labour and goods markets in a neoclassical growth model. The objective of this study is to find the optimal tax rate on capital income in the long run. The model assumes that the set of the tax instruments is not complete. This chapter analyses the Ramsey problem in the context, where taxes cannot be collected from the consumption of informal goods and working in the informal labour market. According to Chamley (1986), the optimal tax rate on capital income is zero in a neoclassical growth model. There are several studies which show that the capital income tax rate is not zero if the modelling framework is modified in certain ways. In the lines of these papers we find that the optimal tax rate on capital income is different from zero in the presence of informal labour and good markets.

The third chapter "Fiscal multipliers and the choice of exchange rate in an open economy with informal sector", gives the impact and present value multipliers under fixed and flexible exchange rate regimes in a small open economy New-Keynesian DSGE model. This study shows that the fiscal policy is more effective at increasing the level of formal sector output in economies where the exchange rate is fixed. The informal sector responds strongly to flexible exchange rate regime. The difference between the size of impact multipliers under the both exchange rates is smaller than implied by the traditional Mundell-Fleming model. The results also reveal that, in an economy with segmented labour markets, the impact of fiscal policy becomes stronger and fiscal policy multipliers increase under the fixed exchange rate, when friction in the labour market decreases. An increase in the labour mobility does not have any significant impact on fiscal policy multipliers under flexible exchange rate regime. This implies that an increase in the labour mobility across the sectors widens the gap between the government expenditure multipliers associated with fixed and flexible exchange rates. We also analyse the effectiveness of the fiscal stimulus under different debt financing strategies. The fiscal multipliers are higher when debt is financed through an increase in the consumption tax under both exchange rate regimes, while capital tax financing scheme is found to be less effective. The difference of impact multipliers between the most and least effective schemes is larger under fixed exchange rate. We also report that the fiscal policy impact multipliers decrease with the increase in the size of informal economy under both exchange rate regimes but this effect is more pronounced when the exchange rate is fixed.

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### Chapter 1

# Macroeconomic effects of the Fiscal Policy Shocks in the Presence of Informal Sector

#### Abstract

How do aggregate economic activities respond to changes in different fiscal policy instruments in the presence of a large informal sector? In this study, we construct a New-Keynesian DSGE model with segmented labour and goods markets to address this question. We analyse the impacts of five fiscal policy instruments on the dynamics of key macroeconomic variables in the presence of an informal economy. This study also addresses the implications of the productive and non-productive government spending. We find that the government investment and government consumption impact multipliers for aggregate output are greater than one in an economy with large informal sector. The analysis shows that, the increases in government consumption and government investment increase the outputs of both formal and informal sectors. Capital income and consumption tax shocks also have positive impacts on the outputs of both sectors. However, a cut in labour income tax causes the informal sector output to fall. This dampening effect of a labour tax cut on informal sector output outweighs a positive impact on the output of formal sector. As a result of the movements of the formal and informal outputs in opposite directions, the aggregate output decreases on impact of a decrease in labour income tax. In the case of capital income tax, the informal sector output falls below the steady state after ten quarters. This implies that the size of the informal economy decreases on impact of a decrease in the labour income tax rate, whereas a cut in the capital tax rate decreases the size of the informal economy after a few quarters. Government consumption, government investment and consumption tax shocks increase the level of informal output for a longer period of time. In case of an increase in government expenditures and a cut in consumption tax, debt is financed through the increases in labour and capital income taxes, which lead to an increase in the size of the informal economy. We find that an increase in the labour mobility amplifies the impacts of government investment and government consumption shocks on the formal sector

output and increases the government expenditures present value multipliers. The impact of a decrease in the labour tax is also more pronounced on the outputs of both formal and informal sectors in case of higher labour mobility. Additionally, the informal sector output decreases on impact of a decrease in capital income tax when there is an increase in labour mobility between the sectors.

### **1.1** Introduction

The structure of the economy in emerging and developing countries is partially different as compared to the developed countries because of the presence of large informal economy. A large proportion of the labour force, up to 60 percent, is employed in informal sector in developing and emerging economies (Schneider, 2012). Jutting and de Laiglesia (2009) argue that there is a possibility of an increase in the size of informal sector employment due to the financial crisis of 2007 and it is expected that the informality may stay for many years to come. The OECD documents recommend "Governments should face this reality and incorporate informal employment into their policy making". This implies that to develop a better understanding of the response of an economy to external and internal shocks, it is important to have knowledge about the responses of the formal and informal sectors to these shocks. Moreover, the design of macroeconomic policies should carefully considers the implications of the presence of informal sector on the dynamics of the macroeconomic variables.

The research related to the flow between unemployment and employment is more relevant to developed economies as this explains most of the fluctuations in their labour markets, but for countries with a significant size of the informal sector, analysis of the flows between the formal and informal segments of the economies becomes more important. According to the literature, existence of the informal labour market influences the dynamics of the business cycle<sup>1</sup>. The studies show that the presence of an informal labour market increases the flexibility of labour market and conditions the impacts of transmission mechanism of different shocks<sup>2</sup>. Moreover, informal sector employment acts as a buffer stock for employment in the formal sector. However, there is a little agreement if any on the role of the informal sector in propagating the impacts of different shocks. According to the literature, the presence of the informal sector amplifies or mitigates the effects of different shocks<sup>3</sup>.

For more than three decades, there has been an exceptional progress in developing the dynamic stochastic general equilibrium (DSGE) approach to study and model macroeconomic linkages. However, mostly these models have typically been developed for the purposes of studying advanced economies. These models do not take into account the structural specifications which are relevant to the emerging and

 $<sup>^1\</sup>mathrm{Zenou}$  (2007), Orsi (2014) and Ngalawa and Viegi (2010)

<sup>&</sup>lt;sup>2</sup>For example, Bovi (2007).

<sup>&</sup>lt;sup>3</sup>See, Koreshkova (2006), Castillo and Montoro (2008) and Regassa (2013).

developing economies; for instance, weak financial sector and unstable economic and political institutions. Moreover, neglecting the informal sector in micro-founded DSGE framework is not justified, when it represents a non-trivial fraction of the developing economies.

The objective of this study is to construct a New-Keynesian DSGE model for a developing economy by incorporating the informal sector with a rich set of fiscal instruments. In this study, we consider the informality in labour and goods markets. On the basis of the constructed model, we examine the effects of the various tools of fiscal policy (government consumption, government investment, labour, capital income and consumption taxes) on the cyclical behaviour of both formal and informal sectors of the economy and also analyse the impacts of these tools on the size of informal economy. We also explore the role of the informal economy in determining the size of fiscal policy multipliers.

In the next section there is an overview of the literature on informal economy which is comprised of the definition of informal sector, importance of the informal sector for developing economies, its determinants and the methods of measuring the size of informal economy. The subsequent section provides a review of the literature related to the effects of the macroeconomic policies on the size and dynamics of the informal economy. In section four there is a brief review of the literature regarding the fiscal policy multipliers. Section five presents the research objectives. Section six consists of the model. In section seven, calibrated values of the parameters are given and section eight and nine include the results and conclusion of the study respectively.

### **1.2** Informal Economy

The structure of the economy in emerging and developing countries is partially different as compared to the developed countries because of the:

- presence of the large informal economy;
- variations in the endowments and constraints of the agents;
- vulnerability to external shocks;
- weak financial sector and unstable economic and political institutions.

A large proportion of the workers, up to 60 percent is employed in the informal sector in developing and emerging economies (Schneider, 2012). Figure (1.1) shows the size of informal employment as percentage of the non-agricultural employment in various regions of the developing and emerging countries. In Sub-Sahara region, 72.60 percent of the total labour force is employed in informal sector. The size of the informal employment in Pakistan and India is 70 and 68 percent respectively, quite higher than the other regions of the world. The size of the informal economy is also significant in Latin American countries. Jutting and de Laiglesia (2009) argue that

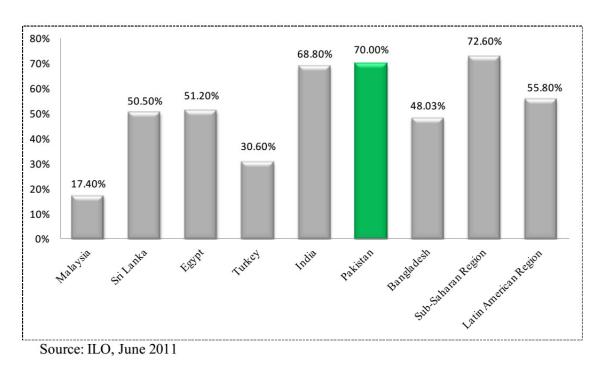


Figure 1.1: Informal Employment as percentage of Non Agricultural Employment

there is a possibility of an increase in the size of informal sector employment due to the recent financial crises and it is expected that informality may stay for many years to come. There are a lot of reasons for the governments to be concerned about the size and cyclical behaviour of the informality, for instance:

- the informal activities evade taxes and therefore these activities are damaging for the government efforts to collect the revenues;
- the dynamics of informal employment impact the cyclical behaviour of government revenues (Ihrig J. and Moe K.S. 2004);

- it is hard to reduce the size of informal economy (Boeri, 2006 and Bovi, 2007);
- in the presence of large informal economy, macroeconomic indicators become unreliable, so the planning and the policy making on the basis of these indicators prove to be fallacious;
- informal sector distorts the competition among the firms;
- labours in the underground sector mostly do not have any kind of social protection.

### **1.2.1** Definition of the Informal Economy

According to (Schneider, 2005) "Informal economy includes unreported income from all market-based legal production of goods and services, either from monetary or barter transactions and so includes all economic activities that would generally be taxable were they reported to the state (tax) authorities". The informal economy includes legal production of all market based goods and services that are intentionally hidden from the government authorities to avoid:

- the value added, income tax and social security contribution;
- meeting the market standards and legal requirement, such as maximum working hours and minimum wages;

• complying with administrative obligations like filling some administrative forms and replying statistical questionnaires.

Type of activity	Monetary transactions		Non-monetary transactions		
Illegal Activi- ties	Trade with stolen goods; drug dealing and manufacturing; prostitution; gambling; smuggling; fraud; etc.		Barter of drugs, stolen goods, smug- gling etc. Produce or growing drugs for own use. Theft for own use.		
	Tax Evasion	Tax Avoidance	Tax Evasion	Tax Avoidance	
Legal Activi- ties	Unreported in- come from self- employment; Wages, salaries and assets from unreported work related to legal services and goods	Employee dis- counts, fringe benefits	Barter of legal services and goods	All do-it-yourself work and neigh- bor help	

Table 1.1: A Taxonomy of Types of Informal Sector activities

Is the existence of the informality good or bad? The studies of costs and benefits of the informality provide mix evidences. Nikopour et al. (2008) surveys many empirical research papers which assess the relationship between the growth and informality for 21 OECD countries and conclude that the informality has positive impact on growth. Loayza (1997) develops endogenous growth model by assuming constant return to capital and predicts that the size of the shadow economy has negative effect on growth. Dell'Anno (2008) gives an interesting overview related to the positive and negative effects of the shadow economy. According to him, informal sector has ambiguous impacts on GDP but it acts as stabilizer. In the presences of multiple types of rigidities, the formal sector can be benefited from the informal sector as the presence of informal sector gives more flexibility to the system. According to his finding, informality reduces the productivity and hence negatively correlated with growth. Boeri and Garibaldi (2006) use the micro level date of Italy and find that the informal sector mainly consists of low educated and unskilled workers. This is one of the main reasons of the lower productivity in informal sector. The informal sector is also associated with low fiscal revenues and inferior working conditions. The cost of informality increases as in the presence of informal economy, it is difficult for the authorities to enforce workers' welfare, environmental and consumer protections law which may strict the integration of developing countries into the global market.

In the light of the analysis presented in this section, the understanding of the factors which drive the informality and analysing the reactions of the informal sector to different government policies are crucial to efficiently design and implement the macroeconomic policies. Following is a summary of some important factors which determine the size of informal sector.

#### **1.2.2** The Determinants of the Informal Economy

It is a widespread consensus that the main reason of the existence of informal sector is the higher burden of taxes. The larger is the difference between the wages of labour and after tax earning, larger is the incentive to work in informal sector (See, Giles 1999, Schneider, 2005, Dell'Anno 2007). Loayza (1997) argues that the economies with weak enforcement system and high tax burden generally have large informal sector. Kanniainen and Paakonen (2004) report that an increase in the taxes unambiguously increases the size of underground economy. The quality of the government sector institutions is another important factor that contributes in the development of shadow economy. According to many experts, efficiency in the applications and regulations of tax code plays more important role in the decisions of the agents to work informally than the actual burden of regulations and taxation. The authors who reported the evidence in favour of this are Friedman and Zoido (2000), Dreher, Kotsogiannis and Mccorriston (2005), Schneider (2012), Buehn (2012), Teobaldelli (2011) and Losby et al. (2002).

The increased intensity of the labour market regulations and trade barriers also increase the size of the informal labour market by decreasing the freedom of choice of agents working in the formal sector (Johnson and Shleifer, 1997). The efficiency of the public sector also affects the tax morale and in this way has an indirect impact on the size of informal economy. Schneider (2010) reports negative causality between tax morale and the size of informal sector. Frey (2007) discusses the psychological tax contract between the taxpayer and tax authorities. According to him, citizens are more willing to comply with their tax obligations, if they receive proper public services from government in exchange. Erard and Feinstein (1998) and Feld, Schmidt and Schneider (2007) and Feld et al. (2005) have consensus that the fine, punishment and audits do not play any significant role in decreasing the size of informal sector. According to Bosch and Maloney (2006, 2007), hiring behaviour of the firms working in the regulated sector is the main driving force which affects the share of informal employment in labour market. Antunes and Cavalcanti (2007) use a small open economy general equilibrium model to analyse the effects of financial contract enforcement and regulation cost on the size of shadow economy. They find the role of regulation cost more significant in accounting the informality. Aruoba (2010) uses the data set of 118 countries to analyse the relationship of the quality of institutions with inflation, taxation and informality. The study concludes that better institutions result in lower inflation, higher tax collection and relatively small informal sector.

Factors influencing the shadow economy	Influence on the shadow economy (in %)		
-	(a)	(b)	
(1) Increase of the Tax and Social Security Con- tribution Burdens	35-38	45-52	
(2) Quality of State Institutions	10-12	12-17	
(3) Transfers	5-7	7-9	
(4) Specific Labor Market Regulations	7-9	7-9	
(5) Public Sector Services	5-7	7-9	
(6) Tax Morale	22-25	-	
Influence of all Factors	84-98	78-96	
<ul><li>(a) Average values of 12 studies.</li><li>(b) Average values of empirical results of 22 studies.</li><li><i>Source:</i> Schneider (2009)</li></ul>		L	

Table 1.2: A Taxonomy of Types of Informal Sector activities

Table (1.2) summarizes the results of empirical papers which analyse the factors

influencing the size of informal sector. These studies use currency demand and MIMIC methods to measure the size of informal sector. The first column shows the influence of different factors on shadow economy without including the independent variable "tax morale" and the second column shows the influence of same factors with "tax morale" as an independent variable. According to the table, increases in taxes and social security contribution explain 35 to 38 percent and 45 to 52 percent of the variations in informal sector with and without tax morale respectively. The second most important factor which explains 22 to 25 percent of the variation is tax morale. The quality of institutions and labour market regulations account for 10 to 12 percent and 7 to 9 percent variations in the shadow economy in two different groups of studies. According to the table (1.2), three most important driving forces of the informal sector are taxes, tax morale and the quality of state regulations.

#### **1.2.3** Methods of Measuring the Size of Informal Economy

The estimation of the size of informal sector is quite difficult and challenging task. In the literature a number of direct and indirect methods have been developed to estimate the size of shadow economy. The direct methods are based on surveys and samples and the outcomes of these microeconomic methods depend on the voluntary responds, tax auditing and different compliance methods. It is very difficult to get the true informations through these surveys because people are not willing to declare what they are hiding from authorities. The results based on these surveys are most likely to underestimate the size of informal economy.

The indirect methods are mostly macroeconomic in nature and use different economic variables to estimate the size of shadow economy. The most commonly used indirect approaches are monetary approach, physical input approach and Multiple Indicator Multiple Cause (MIMIC) model. According to the currency demand approach, in informal economy, almost all the transactions are carried out through cash, so an increase in the demand of currency is an indicator of an increase in the size of shadow economy. The currency demand approach was introduced by Cagan (1958) and further developed by Tanzi (1983). Another widely used method is physical input approach. The elasticity of the consumption of electricity with respect to GDP is close to one. Kaufmann and Kaliberda (1996) use growth of the consumption of electricity as a proxy to measure the size of overall economy (formal and informal). According to this approach, the size of the informal economy can be obtained by subtracting the official estimate of GDP from the size of overall economy which is estimated by assuming electricity as a best physical indicator of overall economic activities. MIMIC approach assumes that there are multiple causes of the presence and growth of the informal economy and it also has multiple effects on other variables as well. As the size of the informal economy is not known, the latent (unknown) estimator approach applying MIMIC model is used which is based on the statistical theory of unobserved variables.

# 1.3 Informal Economy and Macroeconomic Policies

There is a large amount of literature in the field of economics which studies the responses of labour market to the changes in macroeconomic policies in the presence of different sort of frictions. For instance, Cheron and Langot (2000) study the effects of monetary policy shocks in the presence of search frictions and show the negative correlation of employment and vacancies with inflation. Trigari (2006) shows that the output elasticity of marginal cost is low in the presence of search frictions, which explains the output persistence and inflation sluggishness observed in the data. Sala and Trigari (2008), Trigari (2007), Blanchard et al. (2010), and Christiano and Mathias (2010, 2011) and Gertler and Trigari (2008) also introduce the labour market frictions in a New-Keynesian DSGE framework. These models incorporate various versions of the search and matching frictions of labour market by following the tradition of Diamond-Mortensen-Pissarides.

The work mentioned above considers the labour market as homogeneous. According to Fields (2009), the assumption of homogeneity in labour market is far from being realistic. Moreover, the focus of all these studies is on the developed countries. In the presence of labour market frictions, relatively little efforts have been made to understand the dynamics of the business cycle facts for developing and emerging economies. According to Batini (2010), data limitations is one of the important root causes of this. Moreover, the research related to the structure of labour market with an emphasis on the flow between unemployment and employment is more relevant to the developed economies, as it explains most of the fluctuations in labour market. In developed countries with relatively small size of informal sector, ignoring underground economy might be plausible as it has very limited effects on the dynamics of macroeconomic variables. In the presence of a significant size of informal sector, an analysis of the flows between formal and informal segments of the economies becomes more important. Neglecting the informal sector that represents a non-trivial fraction of the developing economies in micro-founded DSGE framework may not be justified. According to the empirical evidences, an existence of the informal labour market impacts the dynamics of the business cycle. The studies show that an existence of informal labour market increases the flexibility of labour market and conditions the impacts of transmission mechanisms of different shocks to the economy. The presence of informal labour market also acts as a buffer stock for the employment in formal sector.

Busato and Chiarini (2004) are among the first who model informal sector in DSGE framework. The papers of Bosch (2004, 2006) and Boeri and Garibaldi (2006) study, how the job are generated in informal labour market. Busato et al. (2012) analyse the equilibrium implications of the fiscal policies in the presence of informal sector and tax evasion. Bovi (2007) reports the pro-cyclical pattern of the informal

employment by using the labour market data of Italy. According to his finding, formal employment is a-cyclical. In a partial equilibrium analysis, this study considers formal firms with more adjustment cost and constant post tax wages. The firms hire informal labour because of the lower wages and higher flexibility. In his model, low and insufficient productivity prevents the firms from complete informalization. This study provides a theoretical as well as an empirical intuitions on the "buffer hypothesis" which states that, employment in the informal labour market increases flexibility of the labour market. He concludes "The shadow employment is more volatile than the regular one and following big shocks even the regular employment reacts to GDP, but with some unavoidable delay". Bowler and Morisi (2006), Carrillo and Pugno (2004) and Ihrig and Moe (2004) find the same evidence of pro-cyclical behaviour of the informal employment in different emerging economies. Bosch and Maloney (2006, 2007) study the flow of gross workers in Brazil and Mexico and find that the jobs in informal sector changed significantly due to the recessions and policy reforms. They conclude that the jobs finding rates are stable in informal sector, whereas in the regulated sector jobs finding rates are highly pro-cyclical. Busato and Chiarini (2004) and Galli and Kucera (2003) suggest that the informal sector might be counter-cyclical.

Zenou (2007) studies the movement of labour between the formal and informal markets as a result of different labour policies. He develops a two sector general equilibrium model by assuming search and matching frictions in formal labour market and perfect competitiveness in informal labour market. The results show that a decrease in the formal firms' entry cost and unemployment benefits, result in an increase in formal employment. The informal labour market is not directly affected by the policies, but it is not independent of the policies which are applicable to the regulated sector because of the interdependence of both formal and informal markets. Koreshkova (2006) uses the cross country data and concludes that the inflation, financing of government expenditures and the size of informal sector are positively related. The study develops a two-sector general equilibrium model and shows that when government is unable to collect taxes in the presence of large informal sector, the financing of expenditures through seigniorage is compatible with the solution of Ramsay problem. Orsi (2014) develops a two sector neoclassical stochastic growth model and finds that the relationship between the cyclical components of regulated and informal sectors is negative in Italy.

Castillo and Montoro (2008) examine the transmission of demand and supply shocks and the dynamics of inflation in the presence of informal labour market. They modify the DSGE model by incorporating the informal sector and labour market frictions following the Diamond-Mortensen-Pissarides model. The results of their study verify the buffer hypothesis. They find that, the effect of demand shock on inflation is relatively low in the presence of informal sector. This shows that the monetary policy is more effective at increasing the real output with less inflationary impact, when the size of informal sector is significantly large. Additionally, the model also shows cyclical movement of the employment form formal to informal sector which is consistent with the data. Batini et al.(2010) suggest the incorporation of informal sector in DSGE models to improve the understanding of costs and benefits of macroeconomic policies for developing countries. In a following study Batini et al. (2011a) evaluate the costs and benefits of informality in a two sector DSGE model. In their model, the informal sector is assumed as more labour intensive and lying outside of the tax regime. The burden of taxation is lying on the formal sector that produces all capital goods. They consider the informal sector with low productivity and without wage frictions. Following the tradition of Harris and Todero (1970), the model assumes the structure of formal labour market very simple and fix the wages of formal sector workers above the competitive equilibrium. Authors examine the costs and benefits of a more equal distribution of taxes which increases the size of formal market. They conclude that the net benefit from stabilization with tax smoothing outweighs the cost of less wage flexibility that exists in the formal sector. Gabriel et. al.(2010) find that the incorporation of different types of frictions and informal sector in DSGE model, improve the model fit for Indian economy.

Ngalawa and Viegi (2010) study the inter-dependence between two segmented sectors of the economy and analyse the effects of the presence of informality in the financial sector on economic activities. The model finds a complementarity between the formal and informal financial sectors and shows that an increase in the formal sector credit also increases the credit in informal sector. The study also finds that the rates of interest in formal and informal financial sectors move in opposite directions in response to a monetary policy shock. This implies that in the presence of informal financial sector, the conduct of monetary policy becomes hard. Baitni et al. (2011) compare the outcomes of different monetary policy rules and discretion along with a balanced budget fiscal regime in DSGE framework and emphasize on the importance of commitment in economies with a significant size of the informal sector. According to their finding, the presence of strong financial market frictions makes the time inconsistency problem worse in economies with large informal sector.

Regassa (2013), analyses the behaviour of macroeconomic variables in response to different domestic and external shocks. He develops a small open economy model with labour market segmentation. The results of this study do not verify the buffer hypothesis as the impulse responses of macroeconomic variables show more variability. The contradictory results of Castillo and Montoro (2008) and Regassa (2013) might be explained by the different modelling strategies. The model of Regassa (2013) adopts many features of the Castillo and Montoro (2008) and both studies extend the Blanchard and Gali (2010) model by incorporating the segmentation of labour markets. However, there are some important differences in the modelling strategies of both works. Castillo and Montoro (2008) model a closed economy that produces a single final good, whereas Regassa (2013) considers an open economy with multi-sectors in which two composite goods, tradable and non-tradable are produced. According to the modelling strategy of Regassa (2013), firms working in the formal and informal sectors are different and both types of firms produce tradable goods, while informal firms produce only non-tradable goods. Castillo and Montoro (2008) do not consider the informality as a nature of the economy as a whole. In their model the informality arises because of the hiring or employment decisions of the firms. They assume that all firms hire both formal and informal labours. The process of the determination of wages is also different in both studies. According to Castillo and Montoro (2008) wages are determined through bargain between the firms and workers in both formal and informal labour markets. Regassa (2013) assumes informal labour market as competitive in which wages are set at market clearing level. He follows the tradition of Harris-Todaro (1970) for formal sector firms and assumes that the wages in the formal sector adjust above the wages of informal sector due to the presence of different institutional factors like labor unions.

Samir Bandaogo (2015) develops a New-Keynesian small open economy model to analyse the conduct of monetary and fiscal policies. He finds that in the presence of informal sector, optimal tax rate reduces and macroeconomic volatility increases. Furthermore, the study finds that in the presence of informal sector, if macroeconomic policies can not credibly pre-commit then the fixed exchange rate is better than flexible exchange rate. Ahmad et al.(2012) suggest that the presence of informal sector mitigates the impacts of productivity, monetary and government spending shocks.

The review of the literature shows that there is a little agreement if any on the role of informal sector in propagating the impacts of different shocks. There are mixed evidences regarding the role of informal sector in amplifying or mitigating the effects of shocks. Furthermore, most of the studies discussed above focus only on the real economy and investigate the implications of different regulations and government expenditures which are financed by lump-sum taxes. These studies do not analyse the conduct of fiscal policy with a rich set of fiscal instruments like distortionary taxes, productive and non-productive government spending in the presence of informal sector. Given that changes in expenditures and distortionary tax instruments could potentially cause a reallocation of productive resources from both formal and informal sectors, it is of interest to explore the role of both sectors in propagating the impact of fiscal policy shocks and in determining the size of fiscal multipliers.

### 1.4 Fiscal Policy Multiplier

The global financial crisis that began in 2007 triggered the interest of researchers on the effects of fiscal stimulus as there was no room available for further easing of the monetary policy. Governments around the world announced massive fiscal stimuli packages of varied sizes. Burgeoning debt to GDP ratio and resulting increase in the budget deficit have also gained much attention after the great recession. Most of the studies conclude that the fiscal stimulus has positive effect on output but there is no consensus on the size of fiscal multiplier. This difference is reflected by the estimates of fiscal multipliers, which range from less than zero to greater than one in both theoretical and empirical literature. Old Keynesian theory suggests the size of government spending multiplier greater than one. The non-dynamic nature of the old Keynesian theory results in high multipliers. In traditional Keynesian analysis, consumption decision of the households solely based on current disposable income and they do not anticipate any future increase in taxes to finance the fiscal stimulus. The old Keynesian analysis also ignores the crowding out effect of an increase in government expenditure on consumption. The theory was also criticized for the lack of microeconomic foundations.

The neoclassical models are based on micro foundation and assume the optimizing behaviour of forward looking consumers and firms with flexible wages and prices. In these models, government spending multipliers are usually less than one. According to these models, any fiscal stimulus does not increase the level of output substantially as it crowds out private consumption and investment. An expansion in expenditure also results in a decrease in labour supply and drives down the wages (Hall, 2009). Woodford (2011) suggests that in neoclassical models fiscal multipliers are consistently positive but lower than one. According to Baxter and King (1993) and Burnside and Fisher (2000), in short run, the size of fiscal multipliers are ranging from -2.5 to 1.2, whereas in the long run they are positive but less than one. Aiyagari and Christiano (1992), Burnside and Fisher (2004), Shapiro (1998) and Ramey (2011) also find the same evidence of less than one fiscal expenditure multipliers in friction less business cycle models.

The New-Keynesian models are built by adding the features of traditional Keynesian model, such that the price and wage frictions in RBC framework of forward looking optimizing consumers and producers. In the New-Keynesian models, magnitudes of the fiscal spending multipliers depend on the specifications of models. In most of the studies magnitudes of the multipliers are less than one, similar to RBC models. In a New-Keynesian DSGE framework, Christiano and Rebelo (2011) find that the government spending multiplier is greater than one when the nominal interest rate is zero.

Baxter and King (1993) find that an expansion in government spending leads to an increase in the output if these spending are financed through lump-sum taxes and causes a decline in output when financed through distortionary taxation. Trostel (1993) explores that the consumption, investment, labour hours and output increase as a result of temporary substitution of debt for taxation after introducing distortionary taxation in place of lump-sum taxation. Jones (2002) in a model without debt, finds that an increase in labour income tax decreases the output and labour sup-

ply more as compared to a decrease in capital income tax. Bilbie and Straub (2004) emphasize on the importance of studying distortionary taxation in DSGE models and argue that the use of only lump-sum tax is unrealistic. They find that under distortionary taxation, it is more difficult to get a positive response of consumption expenditure in response to a government spending shock. Leeper et al.(2010a, 2010b) find that the dynamics of macroeconomic variables and magnitudes of fiscal multipliers differ significantly when a rich set of fiscal instruments is considered as compared to the case where only non-distortionary lump-sum tax is allowed to change in response to an increase in debt. Ludvigson (1996) concludes that the output and labour supply increase in response to an increase in purely deficit financing government spending when labour supply is highly elastic, whereas these variables show a declining trend in the case of tax financed spending schemes. Linnemann (2004) finds the crowding in effect of private consumption after a government spending shock when elasticity of labour supply is highl.

The studies discussed above recognize the importance of the structure of labour markets in conditioning the impacts of different fiscal policy tools but assume labour markets as homogeneous. These studies do not consider the presence of different kinds of frictions and heterogeneity such as, the presence of unionised labour and informal sector in shaping the effects of different fiscal strategies.

# **1.5** Research Objectives

In the literature, DSGE model has been modified in number of ways to assess the effects of different fiscal policy shocks. For instance, government spending is included in the utility function and public capital is used in Cobb-Douglas production function. Several authors examine the effects of fiscal policy shocks in DSGE models but none of them incorporates a rich set of fiscal policy instruments to measure the size of fiscal multipliers in the presence of informal economy. The objective of this study is to construct a New-Keynesian DSGE model by incorporating the informal sector to study the effects of different fiscal policy stimuli on key macroeconomic variables in the presence of informal economy. In this study we consider the informality in labour and goods markets. The introduction of the government investment, government consumption, and three distortionary taxes in the model with segmented labour and goods markets enables us to analyse the role of informal sector in conditioning the impacts of different types of fiscal stimuli on the dynamics of macroeconomic variables. The model with segmented formal and informal labour markets is used to assess:

- the effects of government expenditures and distortionary taxes such that consumption, capital and labour income taxation on the cyclical behaviour of the economy;
- the different implications of the productive and non-productive government

expenditures with large informal sector;

- the role of the informal sector in determining the size of fiscal policy multipliers in response to various types of fiscal stimuli;
- the role of different fiscal stimuli in controlling the debt dynamics in the presence of informal sector.

For best of my knowledge this study is the first attempt to measure the fiscal policy multipliers by introducing a rich set of fiscal instruments in New-Keynesian DSGE model with informal labour and goods markets. Fiscal policy instruments used in this research are government consumption, government investment and taxes on consumption, labour and capital income.

# 1.6 Model

The model consists of a representative household, representative firms of both formal and informal sectors, monetary and fiscal authorities. The economy is divided into formal (F) and informal (I) sectors with different technologies, producing different goods and selling them at different prices as in Ahmad et al. (2012), Batini et al. (2011b), Haider and Khan (2008) and Regassa (2013). The economy is inhabited by continuum of infinitely-lived households indexed by  $j \in [0, 1]$ . The households take the decision about the consumption of formal and informal goods and supply their labour to formal and informal sectors. Total labour supply is comprised of the labour supply to both formal and informal sectors and it is given by CES function. Firms operating in the formal sectors produce the output by hiring labour and capital. Informal sector firms use only labour as an input. In this study we relate the output of both sectors to public capital by making the distinction between productive and nonproductive government expenditures. This formulation of Cobb-Douglas production function with additional input of public capital considers the productivity enhancing role of public investment. According to this specification of the production function, public capital is *must* for producing both formal and informal goods. The fiscal authority issues risk-free bonds and levies taxes on capital and labour income of the formal sector and consumption of the formal goods to finance its expenditures. Fiscal authority determines a set of Taylor type rules for its instruments and responds to the cyclical changes in the output of formal sector and debt. This study assumes that the responses of government investment and government consumption to debt and to the deviation of formal sector output from steady-state are counter-cyclical, whereas the responses of three distortionary taxes to them are pro-cyclical. Setting of the fiscal instruments in this way keeps the dynamics of debt under control and acts as automatic stabilizer. The Central Bank sets the nominal rate of interest by following the Taylor rule.

## 1.6.1 The Households

The economy is inhabited by identical and infinitely-lived households which are represented by a household which is made up of continuum of family members. Each member of the household obtains utility by consuming the output of both formal and informal sectors, and supplies labour either to formal or informal sector of the economy. The life-time utility of the representative household is expressed as:

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$
(1.1)

where  $\beta$  is household's subjective discount factor,  $C_t$  represents the household's total consumption of goods and  $L_t$  is an index of the household aggregate labour supply. Functional specification of the objective function is given as:

$$U(C_t, L_t) = \frac{(C_t - hC_{t-1})^{1-\sigma_c}}{1 - \sigma_c} - \frac{(L_t)^{1+\sigma_l}}{1 + \sigma_l}$$
(1.2)

where  $\sigma_c$  is the elasticity of intertemporal substitution of consumption, h shows the degree of habit persistence and  $\sigma_l$  is the Frisch elasticity of labour supply. The household maximizes above objective function with respect to the following budget constraint:

$$P_t C_t + P_t I_t + B_t = B_{t-1} R_{t-1} +$$

$$(1.3)$$

$$(1 - \tau_t^k) R_t^k U_t K_{t-1} - P_t a(U_t) K_{t-1} + W_t L_t + Div$$

where,  $W_t L_t = (1 - \tau_t^l) W_{F,t} L_{F,t} + W_{I,t} L_{I,t}$ . Total expenditures  $(P_t C_t)$  comprised of

the expenditures on formal  $(P_{F,t}C_{F,t})$  and informal  $(P_{I,t}C_{I,t})$  goods.

$$P_t C_t = P_{I,t} C_{I,t} + (1 + \tau_t^c) (P_{F,t} C_{F,t})$$

The left side of the equation (1.3) shows the total expenditures of household which include the expenditure on consumption  $(C_t)$ , investment  $(I_t)$  and spending on one period bonds  $(B_t)$ .  $\tau^c$  represents the tax rate on the consumption of formal goods. The right hand side of the equation shows the total disposable earning of household which consists of the following:

- after tax, labour income from formal sector  $(1 \tau_t^l) W_{F,t} L_{F,t}$ , where  $W_{F,t}$  represents the wage rate in the formal sector and  $L_{F,t}$  represents the labour supply to the formal sector and  $\tau_t^l$  denotes the labour income tax rate;
- labour income from informal sector  $(W_{I,t}L_{I,t})$ , where  $W_{I,t}$  and  $L_{I,t}$  represent the wages in the informal sector and labour supply to the informal sector respectively;
- after tax return on capital income  $(1 \tau_t^k) R_t^k U_t K_{t-1}$ , where  $K_{t-1}$  denotes the physical stock of capital,  $R_t^k$  represents the rental rate of capital,  $\tau_t^k$  is the tax rate on capital income and  $U_t$  represents the rate of capital utilization. The cost of capital utilization is  $a(U_t)K_{t-1}^4$ ;
- dividends income (Div);

<sup>&</sup>lt;sup>4</sup>In steady-state U = 1

• interest income from bond holdings  $(R_t)$ .

The law of motion of the capital is:

$$K_t = (1 - \delta)K_{t-1} + F_t(I_t, I_{t-1})$$
(1.4)

where,  $F_t(I_t, I_{t-1}) = \left[1 - \chi(\frac{\epsilon_t^I I_t}{I_{t-1}})\right] I_t$ . Following Schmitt-Grohe and Uribe (2006), the cost of the adjustment function  $\left(\chi(\frac{\epsilon_t^I I_t}{I_{t-1}})\right)$  is assumed as  $\frac{\Psi}{2}\left(\left(\frac{\epsilon_t^I I_t}{I_{t-1}}\right) - 1\right)^2$ , where  $\epsilon_t^I$  is an investment specific efficiency shock. The properties of the cost of adjustment functions are:  $\chi(1) = \chi'(1) = 0$ , and  $\chi''(1) = \Psi > 0$ .

The Lagrangian takes following form;

$$\max_{C_{t},B_{t},L_{t},U_{t},K_{t},I_{t}} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{(C_{t}-hC_{t-1})^{1-\sigma_{c}}}{1-\sigma_{c}} - \frac{1}{1+\sigma_{l}} (L_{t})^{1+\sigma_{l}} - \lambda_{t} [P_{t}C_{t} + P_{t}I_{t} + B_{t} - B_{t-1}R_{t-1} - (1-\tau_{t}^{k})R_{t}^{k}U_{t}K_{t-1} + P_{t}a(U_{t})K_{t-1} - W_{t}L_{t} - Div] - \lambda_{t}Q_{t}[K_{t} - (1-\delta)K_{t-1} - F_{t}(I_{t},I_{t-1})] \right\}$$

$$(1.5)$$

The first order conditions of the households utility maximization problem with respect to  $C_t, B_t, L_t, U_t, K_t$  and  $I_t$  are:

$$\lambda_t = \frac{(C_t - hC_{t-1})^{-\sigma_c}}{(P_t)}$$
(1.6)

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta R_t \tag{1.7}$$

$$L_t^{\sigma_l} = \lambda_t W_t \tag{1.8}$$

$$(1 - \tau_t^k) R_t^k K_{t-1} = P_t a(U_t)' K_{t-1}$$
(1.9)

$$\lambda_t Q_t = \beta \lambda_{t+1} \left[ (1 - \tau_{t+1}^k) R_{t+1}^k U_{t+1} - P_t a(U_{t+1}) \right] + \beta \lambda_{t+1} Q_{t+1} (1 - \delta)$$
(1.10)

$$Q_{t}\left[1-\chi\left(\frac{\epsilon_{t}^{i}I_{t}}{I_{t-1}}\right)\right]-Q_{t}\chi'\left(\frac{\epsilon_{t}^{i}I_{t}}{I_{t-1}}\right)\frac{\epsilon_{t}^{i}}{I_{t-1}}I_{t} = \beta E_{t}\left[\frac{\lambda_{t+1}}{\lambda_{t}}Q_{t+1}\chi'\left(\frac{\epsilon_{t+1}^{i}I_{t+1}}{I_{t}}\right)\frac{\epsilon_{t+1}^{i}I_{t+1}}{I_{t}^{2}}I_{t+1}\right]+1$$
(1.11)

Above FOCs are solved simultaneously to obtain the following important results.

$$\beta R_t E_t \left[ \frac{1}{\pi_{t+1}} \frac{(C_{t+1} - hC_t)^{-\sigma_c}}{(C_t - hC_{t-1})^{-\sigma_c}} \right] = 1$$
(1.12)

$$\frac{(L_t^{\sigma_l})}{(C_t - hC_{t-1})^{-\sigma_c}} = W_t \tag{1.13}$$

$$(1 - \tau_t^k)r_t^k = a'(U_t) \tag{1.14}$$

$$Q_t = \frac{E_t \pi_{t+1}}{R_t} E_t [Q_{t+1}(1-\delta) + (1-\tau_{t+1}^k)(r_{t+1}^k U_{t+1}) - a(U_{t+1})]$$
(1.15)

Euler equation of consumption (1.12) is derived by combining (1.6) and (1.7). It shows that a loss in marginal utility from consuming less today is exactly the same as an increase in marginal utility from consuming more in some later period. Equation (1.13) equates the marginal rate of substitution between the consumption and leisure to real wages. The trade-off between the consumption and leisure shows how much income a household is willing to accept to sacrifice one hour of leisure time. Equation (1.14) represents the first order condition with respect to the rate of capital utilization and shows that the marginal cost of capital utilization is equal to the real rate of return on capital. A lower tax rate on capital income or a higher return on capital implies a higher utilization rate. Equation (1.15) shows that the price of capital is equal to the present value of future income from holding of the capital.

## 1.6.2 Household Consumption Decision

Aggregate consumption  $(C_t)$  is an index of the consumption of formal  $(C_{F,t})$  and informal  $(C_{I,t})$  sectors goods and it is given by the following CES aggregator:

$$C_t = \left[ (1-\gamma)^{\frac{1}{\nu}} C_{F,t}^{\frac{\nu-1}{\nu}} + (\gamma)^{\frac{1}{\nu}} C_{I,t}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}$$
(1.16)

where  $\nu$ , is the elasticity of substitution between the consumption of formal and informal goods,  $\gamma$  measures the proportions of the informal goods in the consumption basket of household. It can also be assumed as a proxy of the size of informal sector. The optimal allocation of expenditures gives the demand functions of the formal and informal goods. The demand functions of the formal and informal consumption are given by equations (1.17) and (1.18) respectively.

$$C_{F,t} = (1 - \gamma) \left(\frac{P_{F,t}(1 + \tau_t^c)}{P_t}\right)^{-\nu} C_t$$
(1.17)

$$C_{I,t} = \gamma \left(\frac{P_{I,t}}{P_t}\right)^{-\nu} C_t \tag{1.18}$$

where,  $P_{F,t}$  and  $P_{I,t}$  are the relative prices of the formal and informal goods. The overall price index is given as:

$$P_t = \left[ (1 - \gamma_1) ((1 + \tau_t^c)(P_{F,t}))^{1-\nu} + (\gamma_1)(P_{I,t})^{1-\nu} \right]^{\frac{1}{1-\nu}}$$
(1.19)

### 1.6.3 Household Labour Supply Decision

The informality in the labour market can arises from either optimizing behaviour of the household as well as from labour market segmentation (Isabel Gunther and Andrey Launov, 2006). According to (Fields, 2009 and Artuc et al. 2013), the labour markets in developing countries are fairly segmented and rigid due the institutional regulations and sector specific skills. In this study we assume that the members of each household supply the labour to both formal and informal sectors according to the CES function. The reason of the CES formulation of labour supply is to account for the limited labour mobility of workers across the sectors in economies with segmented formal and informal labour markets. (Bouakez et al. 2009). The total labour supply  $(L_t)$  is comprised of the supply of labour to both formal  $(L_{F,t})$ and and informal  $(L_{I,t})$  sectors and it is given by following CES function:

$$L_{t} = \left[\gamma_{l}^{-\theta_{l}}(L_{F,t})^{1+\theta_{l}} + (1-\gamma_{l})^{-\theta_{l}}(L_{I,t})^{1+\theta_{l}}\right]^{\frac{1}{1+\theta_{l}}}$$
(1.20)

where  $\gamma_l$  and  $(1 - \gamma_l)$  represent the share of the labour supply to both formal and informal sectors respectively.  $\theta_l$  is the inverse of the elasticity of substitution of labour supply between the formal and informal sectors. The higher value of  $\theta_l$  accounts for limited labour mobility between the two sectors and represents the labour market segmentation. Limited labour mobility across different sectors implies the different wage rate for the households working in formal and informal sectors. This formulation also permits the heterogeneity in hours worked across different sectors. Thus CES formulation of labour supply ensures that the members of the households are willing to supply labour to each sector even if wages are not equal in both sectors. (Bouakez et al., 2009, Dagher et al., 2012, Giovanni Melina, 2017, Mattesini and Rossi, 2009 and Regassa, 2013). In extreme case, if  $\theta_l$  approaches to zero, equation (1.20) becomes a linear aggregator implying perfect labour mobility and equal wages in both sectors. This study considers CES specification of the aggregate labour supply to analyse the impacts of different degree of labour mobility in conditioning the effects of fiscal policy shocks in the presence of informal sector. Household conditional labour supply to the formal and informal sectors are given as:

$$L_{F,t} = \gamma_l \left(\frac{(1-\tau_t^l)W_{F,t}}{W_t}\right)^{\frac{1}{\theta_l}} L_t$$
(1.21)

$$L_{I,t} = (1 - \gamma_l) \left(\frac{W_{I,t}}{W_t}\right)^{\frac{1}{\theta_l}} L_t$$
(1.22)

Conditional labour supplies to formal and informal sectors give the following composite wage index:

$$W_{t} = \left[\gamma_{l}\{(1-\tau_{t}^{l})(W_{F,t})\}^{\frac{1+\theta_{l}}{\theta_{l}}} + (1-\gamma_{l})(W_{I,t})^{\frac{1+\theta_{l}}{\theta_{l}}}\right]^{\frac{\theta_{l}}{1+\theta_{l}}}$$
(1.23)

## **1.6.4** Production in the Formal Sector

In the formal sector there are two types of firms. Perfectly competitive, final goods producers or retailers and monopolistic competitive intermediate firms.

#### Formal Sector Retailers

Retailers are the net buyers of different varieties of formal intermediate goods  $(Y_{F,t(f)})$ and combine them into one single consumption good  $(Y_{F,t})$  by using following CES technology.

$$Y_{F,t} = \left[\int_0^1 Y_{F,t(f)} \frac{\varphi_f^{-1}}{\varphi_f} df\right]^{\frac{\varphi_f}{\varphi_f^{-1}}}$$
(1.24)

where  $\varphi_f$  is the elasticity of substitution among different varieties of formal intermediate goods. Profit function of the formal goods retailers is given as follows:

$$\Pi_{F,t} = P_{F,t} Y_{F,t} - \int_0^1 P_{F,t(f)} Y_{F,t(f)} \, df \tag{1.25}$$

where  $P_{F,t(f)}$  is the price of the intermediate variety  $Y_{F,t(f)}$ . The zero profit condition results in a demand function for the variety, f, of the formal intermediate good, which is given as:

$$Y_{F,t(f)} = \left(\frac{P_{F,t(f)}}{P_{F,t}}\right)^{-\varphi_f} Y_{F,t}$$
(1.26)

The first order condition implies that the price index is represented by the following equation:

$$P_{F,t} = \left[ \int_0^1 P_{F,t(f)}^{1-\varphi_f} \, df \right]^{\frac{1}{1-\varphi_f}} \tag{1.27}$$

### Production of Intermediate Goods in the Formal Sector

The formal sector intermediate firms decide about the demand of capital and labour at given wages and capital rent. Formal sector intermediate firms employ the following Cobb-Douglas production function:

$$Y_{F,t} = A_{F,t} (U_t K_t)^{\alpha} L_{F,t}^{1-\alpha} (K_{q,t-1}^{\alpha_g})$$
(1.28)

where  $A_{F,t}$  is the exogenous level of technology,  $L_{F,t}$  is the amount of formal labour and  $K_t$  denotes the quantity of physical capital,  $U_t$  is the utilization rate of capital,  $K_g$  and  $\alpha_g$  represent the public capital and elasticity of output with respect to the public capital respectively. These formal sector firms hire the labours and capital at given wages  $(W_{F,t})$  and rental rate of capital  $(R_t^k)$  to maximize an expected discounted profit. The Monopolistic firms face the following cost-minimization problem:

$$\min_{W_{F,t},R_t^k} W_{F,t} L_{F,t} + R_t^k U_t K_{t-1} - \lambda_t P_{F,t} \big( Y_{F,t} - A_{F,t} (U_t K_t)^{\alpha} L_{F,t}^{1-\alpha} (K_{g,t-1}^{\alpha_g}) \big)$$

The solution of above problem yields the rental rate of capital and wages of formal labour which are given as:

$$R_t^k = M c_{F,t} P_{F,t} \alpha \frac{Y_{F,t}}{K_t}$$
(1.29)

$$W_{F,t} = M c_{F,t} P_{F,t} (1 - \alpha) \frac{Y_{F,t}}{L_{F,t}}$$
(1.30)

Capital to labour ratio across all monopolistic firms remains the same and given by the following equation:

$$\frac{K_t}{L_{F,t}} = \frac{\alpha}{1-\alpha} \frac{W_{F,t}}{U_t R_t^k} \tag{1.31}$$

Marginal cost of the formal sector production is then:

$$Mc_{F,t} = \frac{1}{A_{F,t}Kg_{t-1}^{\alpha g}} \left(\frac{R_t^k}{\alpha}\right)^{\alpha} \left(\frac{W_{F,t}}{1-\alpha}\right)^{1-\alpha}$$
(1.32)

The equation (1.32) shows that increases in wages and rental rate of capital increase the marginal cost. Marginal cost is negatively associated with public capital. An increase in the government investment and positive productivity shock decrease the marginal cost.

### **1.6.5** Price Setting in the Formal Sector

Formal sector intermediate firms maximize the profit by selling  $Y_{F,t(f)}$  at price  $P_{F,t(f)}$ subject to the demand function given in equation (1.26):

$$\pi_t = P_{F,t(f)} Y_{F,t(f)} - M c_{F,t} P_t Y_{F,t(f)}$$
(1.33)

$$\pi_t = \left[ \left( P_{F,t(f)} - Mc_{F,t}P_t \right) \left( \frac{P_{F,t(f)}}{P_t} \right)^{-\varphi_f} \right] Y_{F,t}$$
(1.34)

Intermediate goods producers set the prices by following a mechanism presented by Calvo (1983). A fraction of the firm,  $\omega_f$  can not re-optimize prices each period and follow the price indexation rule such that:

$$P_{F,t(f)} = \left(\frac{P_{F,t-1}}{P_{F,t-2}}\right)^{\omega_p} P_{F,t-1(f)}$$

where  $\omega_p$  measures the degree of indexation of formal firms. The remaining fraction of the firms choose the price to maximize the following objective function subject to the demand function given in (1.26):

$$\max E_{t} \sum_{k=0}^{\infty} (\beta \omega_{f})^{k} \left[ (P_{F,t+k(f)} - P_{t+k} M c_{F,t+k}) \left( \frac{P_{F,t+k(f)}}{P_{F,t+k}} \right)^{-\varphi_{f}} Y_{F,t+k} \right]$$
(1.35)

Aggregate price law of motion for formal firms is obtained from the price index given in (1.27) and expressed as:

$$P_{t} = \left[ (1 - \omega_{f}) (P_{F,t(f)}^{*})^{1 - \varphi_{f}} + \omega_{f} \left( \left( \frac{P_{F,t-1}}{P_{F,t-2}} \right)^{\omega_{p}} P_{F,t-1(f)} \right)^{1 - \varphi_{f}} \right]^{\frac{1}{1 - \varphi_{f}}}$$
(1.36)

## **1.6.6** Production in the Informal Sector

Informal sector firms produce informal goods by using only the labour as an input at given wages. This formulation of the production function is equivalent to more general specification, where production function of both formal and informal sectors use private and public capital and labour, for instance:

$$Y_{F,t} = A_{F,t}(U_t K_t)^{\alpha} L_{F,t}^{1-\alpha}(K_{g,t-1}^{\alpha_g}) \text{ and } Y_{I,t} = A_{I,t}(U_t K_t)^{\beta} L_{I,t}^{1-\beta} K_{g,t-1}^{\alpha_g}$$

The production in informal sector is more labour intensive and according to Uzawa (1965) and Lucas (1988) if  $\beta < \alpha$ , we can simplify the model and preserve the main economic intuition by assuming that the informal sector firms produce the output by using only labour and public capital.

Informal sector firms employ the following production function:

$$Y_{I,t} = L_{I,t} K_{g,t-1}^{\alpha_g}$$
(1.37)

We assume that the prices of informal goods are flexible. According to a survey of

informal firms conducted by ANSD (2013), informal firms face strong competition due to the non-differentiation of the output of informal sector. Choudhary et al. (2011) survey of the interaction of formal and informal firms also suggest that the degree of price stickiness of informal sector goods is low. The findings of these surveys provide the basis for the assumption of perfectly competitive informal sector. The instantaneous profit function of the informal sector firms is given by:

$$\Pi_{I,t} = P_{I,t}(L_{I,t}Kg_{t-1}^{\alpha_g}) - W_{I,t}L_{I,t}$$

Wages in the informal sector are given as:

$$W_{I,t} = P_{I,t} K_{g,t-1}^{\alpha_g} \tag{1.38}$$

### 1.6.7 Government Behaviour

The Central Bank sets the nominal rate of interest  $(R_t)$  by following a simple Taylor rule that links the rate of interest to its own lag term, output gap of the formal sector and inflation<sup>5</sup>.

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1-\rho)\rho_{\pi_f} \hat{\pi}_{F,t} + (1-\rho)\rho_y (\hat{Y}_{F,t} - \hat{Y}_{F,t-1})$$
(1.39)

The government budget constraint (1.40) shows that the government investment  $(GI_t)$ , government consumption  $(G_t)$ , and interest payment of debt  $(R_{t-1}) B_{t-1}$  is

<sup>&</sup>lt;sup>5</sup>Hats over the variables represent the deviation of the variables from steady-state.

equal to the taxes levies on formal sector consumption  $(\tau_t^c C_{F,t})$ , labour income of the formal sector  $(\tau_t^l W_{F,t} L_{F,t})$ , capital income  $(\tau_t^k R_t^k U_t K_t)$  and the purchase of new debt  $(B_t)$ .

$$(R_{t-1}) B_{t-1} + P_{F,t}G_t + P_{F,t}GI_t = \tau_t^c P_{F,t}C_{F,t} + \tau_t^l W_{F,t}L_{F,t} + \tau_t^k R_t^k U_t K_t + B_t \quad (1.40)$$

Equation of the law of motion of public capital is represented by:

$$K_{g,t} = (1 - \delta_g) K_{g,t-1} + G I_t \tag{1.41}$$

This study assumes that the responses of government consumption and investment to debt and to the deviation of formal sector output from steady-state are countercyclical, whereas the responses of three distortionary taxes to them are pro-cyclical. Setting of the fiscal instruments in this way keeps the dynamics of debt under control and acts as automatic stabilizers. According to Schmitt-Grohé and Uribe (2006), these type of fiscal policy rules can approximate optimal rule. Fiscal policy rules are set by following Lorenzo et al. (2009), Leeper et al. (2009,2010) and Coenen (2013) and these rules for all fiscal instruments are given as follows:

Public consumption:

$$\hat{G}_t = -\psi_{b,g}\hat{B}_{t-1} - \psi_{yf,g}\hat{Y}_{F,t} + g_t \tag{1.42}$$

Public investment:

$$\hat{GI}_t = -\psi_{b,gi}\hat{B}_{t-1} - \psi_{yf,gi}\hat{Y}_{F,t} + gi_t \tag{1.43}$$

Consumption tax rate:

$$\hat{\tau}_{t}^{c} = \psi_{b,\tau^{c}} \hat{B}_{t-1} + \psi_{yf,\tau^{c}} \hat{Y}_{F,t} - tc_{t}$$
(1.44)

capital income tax rate:

$$\hat{\tau}_{t}^{k} = \psi_{b,\tau^{k}} \hat{B}_{t-1} + \psi_{yf,\tau^{k}} \hat{Y}_{F,t} - tk_{t}$$
(1.45)

Labour income tax rate:

$$\hat{\tau}_{t}^{l} = \psi_{b,\tau^{l}} \hat{B}_{t-1} + \psi_{yf,\tau^{l}} \hat{Y}_{F,t} - t l_{t}$$
(1.46)

where  $g_t, g_i, tc_t, tk_t$  and  $tl_t$  are the fiscal policy shocks which affect the spending and revenue sides of the government. The fiscal policy shocks follow the first order auto-regressive process and constitute an unexpected change in the policy.

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \tag{1.47}$$

$$gi_t = \rho_{gi}gi_{t-1} + \epsilon_{gi,t} \tag{1.48}$$

$$tc_t = \rho_{tc} tc_{t-1} + \epsilon_{\tau^c, t} \tag{1.49}$$

$$tk_t = \rho_{tk}tk_{t-1} + \epsilon_{\tau^k,t} \tag{1.50}$$

$$tl_t = \rho_{tl} tl_{t-1} + \epsilon_{\tau^l, t} \tag{1.51}$$

## 1.6.8 Goods Market Equilibrium

Formal sector resource constraint is given as:

$$Y_{F,t} = C_{F,t} + I_t + G_t + GI_t + a(U_t)K_{t-1}$$
(1.52)

where  $C_{F,t}$  is the consumption of formal goods,  $I_t$  is for private investment,  $G_t$  and  $GI_t$  show the government consumption and investment respectively.

Following Batini et al. (2011b) and Vasco Gabriel et al. (2016), we assume that all output of the informal sector is consumed, so the resource constraint of the informal sector is given as:

$$Y_{I,t} = C_{I,t} \tag{1.53}$$

Aggregate output is the sum of the formal and informal sectors output and it is given by the following equation:

$$Y_t = Y_{F,t} P_{F,t} + Y_{I,t} P_{I,t} (1.54)$$

# 1.7 Calibration

All values of the parameters used in this analysis have already been calibrated and most of these are taken from the literature on emerging and developing countries feature with large size of informal market. The value of the discount factor ( $\beta$ ) is set as 0.991, as given by Ahmad et al. (2012). They estimated the discount factor for Pakistan economy by using the data of change in CPI and return on government bonds for the period of 1981 to 2011. Following the Fagan and Messina (2009) and Smets and Wouters (2007), the Frisch elasticity of labour supply ( $\sigma_l$ ) is taken as 1.5. Following Haider and Khan (2008), the value of the degree of habit persistence (h) is considered as 0.36. This is also in line of the finding of Lubik and Schorfeide (2005), according to them the degree of habit persistence is quite low in developing countries than the advanced and developed economies. The Inverse of the elasticity of intertemporal substitution in consumption  $(\sigma_c)$  is taken from Kose and Reizman (2001) and set as 2.61. The depreciation of the private capital ( $\delta$ ) is set at 0.03 following Ahmad et al. (2012) Bukhari and Khan (2008) and Haider and Khan (2008). The value of the inverse of the elasticity of substitution between formal and informal labour supply  $(\theta_l)$  is considered as 2, following Ahmad et al. (2012). The share of the formal sector employment in total employment  $(\gamma_l)$  is considered as 0.29. Choudhri and Malik (2012) also estimate the same value of this parameter. Elasticity of substitution between the consumption of formal and informal goods ( $\nu$ ) and share of the informal sector goods in total consumption ( $\gamma$ ) are also picked from Ahmad et al. (2012) and Khan and Khan (2011). These two parameters govern the distribution of formal and informal consumption of goods and taken as 0.70 and 0.45 respectively. Output elasticity of private capital ( $\alpha$ ) is assumed as 0.46 and it is taken from Bukhari and Khan (2008). Following Choudhri et al. (2012), degree of the price rigidity for the output of formal sector  $(\omega_f)$  is assumed as 0.24. This reflect the low degree of price stickings in developing countries. In the line of Haider and Khan (2008) and de-Castro (2011), degree of the price indexation for formal sector output  $(\omega_p)$  is set at 0.65. The persistence of shock in auto-regressive process is assumed as 0.9 which is consistent for developing countries following Aguiar and Gopinath (2007). The steady-state consumption, labour and capital income taxes  $(\tau^c, \tau^l \text{ and } \tau^k)$  are set at 0.17, 0.20 and 0.30 respectively.

In the literature there are diverse views related to the productivity of public capital and its impacts on private sector productivity. Aschauer (1989a, 1989b) find the large values of the output elasticities of public capital. According to his findings, the output elasticities of public capital with respect to the non-military public capital and core infrastructure are 0.39 and 0.24 respectively. Fernald (1999), Everaert at el. (2004) and Abdih (2008) also estimate the output elasticities of public capital as 0.35, 0.31 and 0.39 respectively. These surprisingly higher estimates imply that the public investment is more productive than private investment. Finn (1993), Kamps (2005) and Heintz (2010) found relatively smaller output elasticities of public capital in the range of 0.15 to 0.25. Holtz-Eakin (1994) concludes that the public sector capital has no impact on the productivity of private capital. Evans and Karras (1994) conclude that the public capital has negative impact on private sector productivity. Cutanda and Paricio (1992) find the estimate of output elasticity with respect to public capital as 0.1. Given these wide array of output elasticities, we are taking the value of output elasticity of public capital ( $\alpha_g$ ) as 0.1 by following Baxter and King (1993), Leeper (2010) and Iwata (2012). The deprecation of public capital  $(\delta_g)$  is taken as 0.02 following Baxter and King (1993) and Kamps (2004). The summary of all calibrated values of the structural and policy rule parameters is given in tables (1.3) and (1.4)respectively.

Table 1.3:	Structural	parameters
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Parameter	Symbol	Value	Source
Discount factor	β	0.991	Ahmad et al. (2012)
Habit persistence	h	0.36	Haider and Khan (2008) and Lubik and Schorfeide (2005)
Share of the private capital in production	α	0.46	Bukhari and Khan (2008) and Haider and Khan (2008)
Output elasticity of public capital	$\alpha_g$	0.01	Baxter and King (1993), Leeper (2010) and Iwata (2012)
Depreciation rate of the private capital	δ	0.03	Bukhari and Khan (2008) and Haider and Khan (2008)
Depreciation rate of the public capital	δ	0.02	Baxter and King (1993) and Kamps (2004)
Inverse Frisch elasticity of labour	$\sigma_l$	1.5	Fagan and Messina (2009) and Smets and Wouters (2007)
Inverse of elasticity of intertemporal sub- stitution in consumption	$\sigma_c$	2.61	Kose and Reizman (2001)
Share of the informal consumption in total consumption	γ	0.45	Ahmad et al. (2012) and Khan and Khan (2011)
Elasticity of substitution between the con- sumption of formal and informal goods	ν	0.7	Ahmad et al. (2012) and Khan and Khan (2011)
Elasticity of substitution among the differ- ent varieties of formal intermediate goods	$arphi_f$	6	Haider and Khan (2008) and Ahmad et al. (2012)
Share of the formal sector employment in total employment	$\gamma \iota$	0.29	Ahmad et al. (2012) and Choudhri and Ma- lik (2012)
Inverse of the elasticity of substitution be- tween formal and informal sector labour supply	$\theta_l$	2	Ahmad et al. (2012)
Price indexation of goods produced in the formal sector	$\omega_p$	0.65	Haider and Khan (2008) and de-Castro et al. (2011)
Degree of price stickiness in formal sector	$\omega_f$	0.24	Choudhri et al. (2012)

Parameter	Symbol	Value	Source
Relative weight assigned to formal infla- tion in Taylor rule	$ ho_{\pi_f}$	1.21	Ahmad, et al. (2012)
Relative weight assigned to formal sector output in Taylor rule	$ ho_y$	0.60	Ahmad, et al. (2012)
Weight of interest rate inertia in Taylor rule	ρ	0.63	Ahmad, et al. (2012)
Persistent of the shock in auto-regressive process	$ ho_x$	0.9	Aguiar and Gopinath (2007)
Response of fiscal instruments to debt	$\psi_b$	0.1	assumed*
Response of fiscal instruments to devia- tion of formal sector output from steady- state	$\psi_{y,f}$	0.1	assumed

Table 1.4: Policy rules and shock process parameters

\* For some of the model parameters for which references are not available for developing economies with large informal sector, unavoidably, we assign the values based on subjective judgment and pair these with sensitivity analysis.

# 1.8 Results

## 1.8.1 Impulses Responses and Present Value Multipliers

This section presents the impulse responses and present value multipliers implied by government investment, government consumption, labour and capital income and consumption tax shocks. For the five fiscal policy shocks, we provide the cumulative present value multipliers of aggregate, formal and informal sectors outputs for the first quarter (impact multiplier), two quarters, four quarters and twelve quarters. The present value multipliers are widely used in the literature for example Uhlig (2009), Leeper (2010) and Coenen (2013) and calculated by using the following formula:<sup>6</sup>

$$PV = \frac{\sum_{j=0}^{k} \left(\prod_{i=0}^{j} R_{t+i}^{-1}\right) \triangle Y_{t+k}}{\sum_{j=0}^{k} \left(\prod_{i=0}^{j} R_{t+i}^{-1}\right) \triangle X_{t+k}}$$

where, X=(G,GI,  $\tau^c, \tau^l$  and  $\tau^c)$ 

#### Government investment

Figures (1.2) and (1.3) show the impacts of transitory government investment and consumption shocks on the dynamics of macroeconomic variables<sup>7</sup>. A positive government investment shock results in an increase in the output of both formal (YF) and informal (YI) sectors. The output of the formal sector decreases below the steady-state after six quarters, whereas the informal sector output remains positive for a longer period of time before reverting to the steady-state. An increase in the demand of formal and informal goods leads to an increase in the demand of labour in both sectors of the economy which exerts an upward pressure on wages. Private investment (I) and the consumption of formal (CF) goods fall on impact of

$$\frac{\hat{Y}_t + \frac{Y_{t+1} - Y_t}{R(\hat{R}_t + 1)} + \frac{Y_{t+2} - Y_{t+1}}{R(\hat{R}_t + 1)R(\hat{R}_{t+1} + 1)}}{\hat{G}_{t+1} - \hat{G}_{t+1} - \hat{G}_{t+1} - \hat{G}_{t+1} - \hat{G}_{t+1}} \frac{Y}{G}$$

$$\hat{G}_t + \frac{\hat{G}_{t+1} - \hat{G}_t}{P(\hat{P}_{t+1})} + \frac{\hat{G}_{t+2} - \hat{G}_{t+1}}{P(\hat{P}_{t+1}) P(\hat{P}_{t+1})}$$

<sup>7</sup>Black line shows the impact of increase in government consumption, whereas red line is related to government investment shock

<sup>&</sup>lt;sup>6</sup>For example, for two periods the present value multiplier is:

an increase in government investment. A government investment shock affects the macroeconomic outcome through three different channels. These are, crowding out effect, wealth effect and change in the productivity of firms. The crowding out effect is induced by an increase in the demand of formal goods by the government. Formal firms increase the demand of labour which leads to higher wages (WF) in the formal labour market. The rise in the demand of formal goods exerts an upward pressure on the prices (FP) of the formal sector output. Central Bank increases the nominal rate of interest (R) in response to higher inflation (PI). This crowding out channel explains the sudden decline in private investment and decrease in the output of the formal sector after six quarters. In response to government investment shock, an increase in the demand of formal output by government makes the output of formal sector relatively expensive. Households substitute some of their expensive formal consumption with the consumption of informal goods. An increase in income of households also leads to an increase in the demand of informal goods. In this model, formal and informal sectors mainly interact through consumption. An increase in the income of households and the substitution of informal consumption for expensive formal consumption explain the spill over effect of an increase in government investment from formal to informal sector. Output and labour demand in the informal sector increase due to these aforementioned income and substitution effects.

A rise in government investment results in an increase in government debt which subsequently leads to a contraction in government consumption and taxes. The increase in taxes on labour income and consumption generate a negative wealth effect which further induces households to lower the consumption of formal sector output and leisure. Informal sector wages (WI) decline because of this increase in labour supply. Another reason of a decline in wages within the informal sector is the reallocation of formal sector workers toward the informal sector because of the higher labour income tax rate in formal sector. An increase in the capital tax rate has a strong negative impact on private investment. The crowding out effect and an increase in the capital tax rate to finance the debt explain a sharp decline in investment in response to a positive government investment shock.

Financing of an increase in government investment through the increase in taxation offsets some of the growth effects of public investment. As a result of this, the impact of an increase in government investment becomes negative after a few quarters. The increase in distortionary taxes and a decrease in government consumption do not have any direct impact on the output of informal sector, so an increase in the productivity enhancing government investment results in an increase in the output of informal sector for a longer period of time.

Table (1.5) gives the quantitative effects of government investment shock on aggregate, formal and informal sectors outputs. We report the present value multipliers for one (impact multiplier), two, four and twelve quarters. The government investment impact multiplier for aggregate level of output is higher than one. In the first quarter, the impact multipliers are 0.66 and 0.12 for the output of formal and informal sectors respectively. The present value multiplier for formal sector falls to 0.27 in second quarter and then further declines to -0.20 by the twelfth quarter.

#### Government consumption

The positive government consumption shock results in an increase in the demand of formal sector output. The formal sector output rises and remains above the steady state for almost seven quarters. The higher demand of the output and labour in the formal sector exert an upward pressure on wages and prices. Wages, rental rates of capital and inflation also rise sharply as a result of increase in government consumption. This increase derived from an increase in government consumption is larger than the increase implied by an increase in government investment. The Central Bank responds strongly by increasing the nominal interest rate in response to increases in inflation and formal sector output. This increase in the nominal interest rate is higher than the increase in rate of interest implied by the government investment shock. Some part of the expansion in government consumption is financed through the decrease in government investment. A decline in the productivity enhancing public capital reduces the productivity of both formal and informal sectors firms. As a result of a strong crowding out effect, negative wealth effect of increase in taxes and reduction in the public capital, the formal sector consumption and investment decline sharply. In the case of government consumption shock, the initial responses of aggregate and formal sector outputs are stronger. The impulse responses of these

variables decline at a faster rate and after a few quarters fall below the impulses responses of the aggregate and formal sector outputs implied by the government investment shock. Initially the informal sector remains unaffected by government consumption shock, because the government only consumes goods produced by the formal sector. The demand of informal sector output increases due to the income and substitution effects. Households substitute some part of their expensive formal consumption with the consumption from informal output which also contributes toward an increase in the demand for labour within the informal sector. The informal sector remains insulated from both crowding out and negative wealth effect, caused by the increases in government consumption and taxes and remain above the steady-state for a longer period of time. Table (1.6) shows that the government consumption multiplier for formal sector output is 0.82 in the first quarter which contract to -0.40by quarter twelve, whereas in the case of government investment shock the present value multiplier after twelve quarters is -0.20. This shows that, after a few quarters, impact of the crowding out and negative wealth effect of taxes becomes stronger in the case of government consumption shocks than implied by government investment shock.

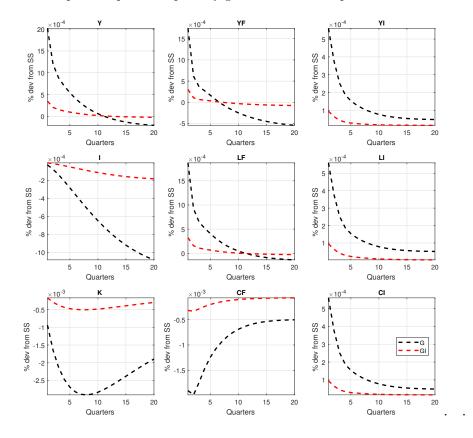


Figure 1.2: Impulse responses implied by government consumption and investment Shocks

Black line shows the impact of increase in government consumption, whereas red line is related to government investment shock

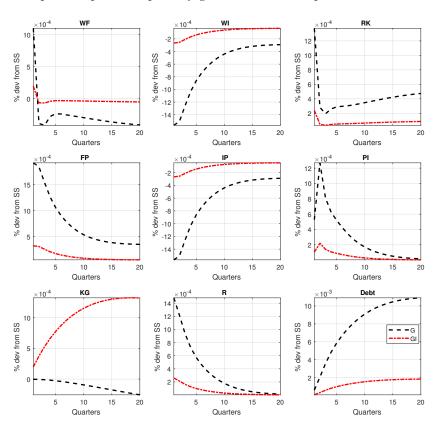


Figure 1.3: Impulse responses implied by government consumption and investment Shocks

Variable	1 quarter	2 quarter	4 quarter	12 quarter
$\frac{PV(\triangle Y)}{PV(\triangle GI)}$	1.2	0.90	0.68	0.20
$\frac{PV(\triangle YF)}{PV(\triangle GI)}$	0.66	0.27	0.15	-0.20
$\frac{PV(\triangle YI)}{PV(\triangle GI)}$	0.12	0.1	0.06	0.08

Table 1.5: Government investment present value multipliers

Table 1.6: Government consumption present value multipliers

Variable	1 quarter	2 quarter	4 quarter	12 quarter
$\frac{PV(\triangle Y)}{PV(\triangle G)}$	1.4	0.96	0.73	0.1
$\frac{PV(\triangle YF)}{PV(\triangle G)}$	0.82	0.35	0.20	-0.40
$\frac{PV(\triangle YI)}{PV(\triangle G)}$	0.14	0.13	0.1	0.09

#### Consumption tax

Figures (1.4) and (1.5) compare the impacts of consumption, capital and labour income tax shocks on the dynamics of macroeconomic variables<sup>8</sup>. In response to a reduction in consumption tax rate, the demand for formal sector output increases. Higher demand of the formal sector output leads to an increase in formal sector output, prices, formal sector labour demand and formal wages. The resulting increase in government debt is financed through the increases in capital and labour income taxes as well as a reduction in the government investment and consumption. Private investment and public capital fall immediately on impact of a decline in consumption tax rate, which causes formal sector output to contract after five quarters. Informal sector output remains above the steady-state for a longer period of time since it is not affected by the increases in both capital and labour income taxes. Income from the informal sector is not taxed so there is a movement of labour from formal to informal sector, which explains the decline in informal wages. Consumption tax present value multipliers are provided in table (1.7). The consumption tax impact multipliers are

<sup>&</sup>lt;sup>8</sup>black dotted line represents the consumption tax shock, blue line shows the impact of decrease in labour tax and red dotted line denote the capital income shock.

#### Capital tax

The instant effect of a cut in capital tax is an increase in investment and formal sector output. A capital tax shock is relatively more effective at increasing the level of aggregate and formal sector outputs than consumption and labour income tax shocks. Formal sector consumption decreases as households sacrifice consumption for investment (See, Baxter and King,1993, Braun,1994 and Leeper and Yang,2008). The increase in the demand of formal sector output and labour demand put an upward pressure on formal sector prices and wages respectively. The substitution and income effects cause the level of output and labour demand within the informal sector to increase.

In the case of capital tax shock the increase in informal sector output is less than that implied by consumption tax shocks and becomes negative after ten quarters. This suggests that the size of the informal sector contracts after ten quarters in response to a cut in capital income tax. Table (1.8) shows the present value multipliers for a capital tax shock. The magnitudes of the capital tax impact and present value multipliers for aggregate output are higher than the multipliers of consumption and labour income tax shocks. Despite having a negative impact on informal sector output, the size of capital tax present value multipliers for aggregate output become higher than the present value multipliers of all other fiscal shocks after a few quarters. This shows that, in the presence of informal economy the capital tax shock is most effective at increasing the output in the long-run.

#### Labour tax

Households reduce their supply of labour to the informal sector and increase labour supply to the formal sector in response to a decrease in labour income tax. An increase in labour supply to formal sector results in a decline of the wages in the formal sector. As a consequence of shifting of labour supply towards the formal sector, output of the informal sector falls and wages increase. An increase in the demand of labour in formal sector leads to an increase in the investment by the firms operating in the formal sector. This increase in investment and labour demand results in an increase in formal sector output. The positive impact of a decrease in the labour tax is relatively stronger on formal sector output than the impact of a decrease in consumption tax. However the negative effect of a decrease in the labour tax on the informal sector outweight its positive impact on the formal sector output. As a result of the movement of formal and informal outputs in opposite directions, the impact of a decrease in labour tax on aggregate output is relatively smaller in the first few quarters in comparison to the impact of a decrease in capital income and consumption taxes. The magnitudes of the present value multipliers for the formal sector output in the second, fourth and twelfth quarters are larger than the present value multipliers for a consumption tax shock and less than the present value multipliers for a capital tax shock. The labour tax impact and present value

multipliers for the aggregate output are insignificant (table 1.9).

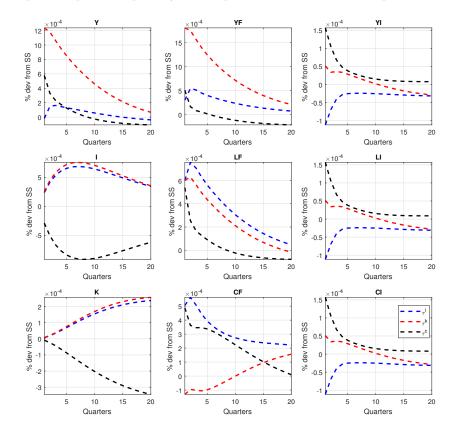


Figure 1.4: Impulse responses implied by consumption, labour income and capital income tax shocks

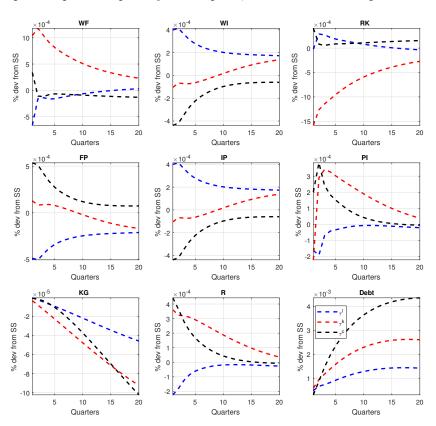


Figure 1.5: Impulse responses implied by consumption, labour income and capital income tax shocks

Variable	1 quarter	2 quarter	4 quarter	12 quarter
$\frac{PV(\triangle Y)}{PV(\triangle \tau^c)}$	- 0.34	-0.2	- 0.15	0.1
$\frac{PV(\triangle YF)}{PV(\triangle \tau^c)}$	- 0.24	-0.12	- 0.1	0.11
$\frac{PV(\triangle YI)}{PV(\triangle \tau^c)}$	- 0.03	-0.03	- 0.012	-0.01

Table 1.7: Consumption tax present value multipliers

Table 1.8: Capital income tax present value multipliers

Variable	1 quarter	2 quarter	4 quarter	12 quarter
$\frac{PV(\triangle Y)}{PV(\triangle \tau^k)}$	-0.40	-0.38	- 0.37	-0.34
$\frac{PV(\triangle YF)}{PV(\triangle \tau^k)}$	-0.35	-0.36	-0.30	-0.30
$\frac{PV(\triangle YI)}{PV(\triangle \tau^k)}$	-0.01	-0.01	-0.008	0.004

Table 1.9: Labour income tax present value multipliers

Variable	1 quarter	2 quarter	4 quarter	12 quarter
$\frac{PV(\triangle Y)}{PV(\triangle \tau^l)}$	0.01	-0.1	-0.1	-0.1
$\frac{PV(\triangle YF)}{PV(\triangle \tau^l)}$	- 0.1	-0.20	-0.17	-0.17
$\frac{PV(\triangle YI)}{PV(\triangle \tau^l)}$	0.03	0.02	0.01	0.14

#### **1.8.2** Sectoral Labour Mobility and Fiscal Policy Shocks

In this section, we analyse the role of labour mobility in conditioning the impacts of fiscal policy shocks in the presence of informal labour and goods markets. Figures (1.6-1.10) compare the impulse responses of macroeconomic variables in response to fiscal policy shocks with different levels of labour mobility across the sectors. When labour mobility is high<sup>9</sup> ( $\theta_l = 0.4$ ), an increase in formal sector output is relatively large in response to a government consumption shock (figure 1.6). Positive government spending shocks result in an increase in the demand for formal sector output. Labour demand increases in the formal sector and households reallocate their labour supply from the informal to the formal sector to a larger extent. This large movement of labour from the informal to the formal sector explains the weak response of informal sector output to government expenditure shocks in the case of high labour mobility. Table (1.10) provides the present value multipliers of the fiscal policy shocks with higher labour mobility. The impact multiplier of a government consumption shock for the aggregate level of output remains the same, whereas the present value multiplier for quarters two, four and twelve are slightly higher than implied by lower labour mobility. The government investment impact multipliers for the formal sector and the aggregate level of output increase when labour is highly mobile across the sectors.

Figure (1.8) compares the impacts of a decrease in labour tax for different levels of

<sup>&</sup>lt;sup>9</sup>In benchmark model, the value of  $\theta_l$  is set as 2.

labour mobility. The impact of a decrease in the labour tax is more pronounced on the outputs of both formal and informal sectors with higher labour mobility. The output of the informal sector declines more because of a large movement of labour from the informal to the formal sector which results in a large increase in the level of output of the formal sector. The large negative effect of a decrease in labour tax on informal sector, outweighs its positive impact on formal sector output and results in a negative response of the aggregate level of output on impact of a decrease in the labour tax. However after quarter one, the aggregate level of output increases at a faster rate in the case of higher labour mobility. Figure (1.9) shows that the positive response of the formal sector output is stronger and the output of the informal sector declines on impact of a decrease in capital tax when labour is highly mobile. In the case of lower labour mobility, informal sector output declines below the steady-state after 10 quarters in response to a cut in capital tax. Higher labour mobility also increases the response of formal sector output to the consumption tax shock, while the response of the informal sector to a decrease in consumption tax is small (figure 1.10).

The above analysis shows that an increase in labour mobility between the formal and informal sectors amplifies the impacts of fiscal policy shocks on aggregate, formal and informal sectors outputs. There is a large decline in the size of informal sector production in response to capital and labour tax shocks when labour is highly mobile.

Variable	1 quarter	2 quarter	4 quarter	12 quarter
$\frac{PV(\triangle Y)}{PV(\triangle G)}$	1.4	1.05	0.80	0.29
$\frac{PV(\triangle YF)}{PV(\triangle G)}$	1.16	0.62	0.41	-0.32
$PV(\triangle Y)$	1 45	1.00	0.79	0.22
$\frac{PV(\triangle Y)}{PV(\triangle GI)}$	1.45	1.02	0.78	0.33
$\frac{PV(\triangle YF)}{PV(\triangle GI)}$	1.13	0.61	0.40	-0.19
$\frac{PV(\triangle Y)}{PV(\triangle \tau^k)}$	-0.40	-0.41	-0.42	-0.39
$\frac{PV(\triangle YF)}{PV(\triangle \tau^k)}$	-0.37	-0.39	-0.39	-0.38
$\frac{PV(\triangle Y)}{PV(\triangle \tau^l)}$	0.04	-0.06	-0.1	-0.1
$\frac{PV(\triangle YF)}{PV(\triangle \tau^l)}$	-0.13	-0.27	- 0.29	-0.32
$\frac{PV(\triangle Y)}{PV(\triangle \tau^c)}$	-0.22	-0.15	-0.11	0.02
$\frac{PV(\triangle YF)}{PV(\triangle \tau^c)}$	-0.20	-0.11	- 0.1	0.11

Table 1.10: Present value multipliers with higher labour mobility

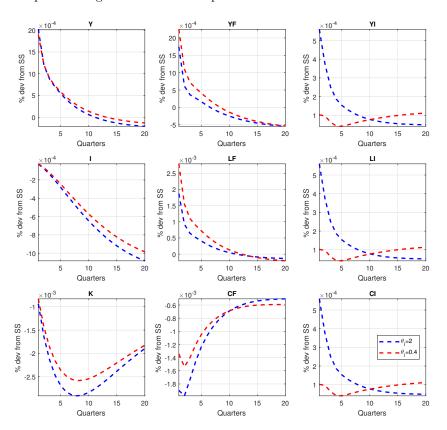


Figure 1.6: Comparison of government consumption shocks with different level of labour mobility

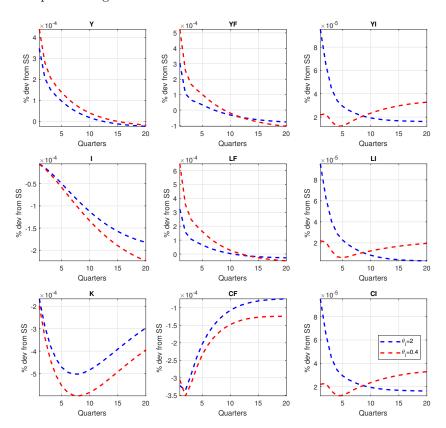


Figure 1.7: Comparison of government investment shocks with different level of labour mobility

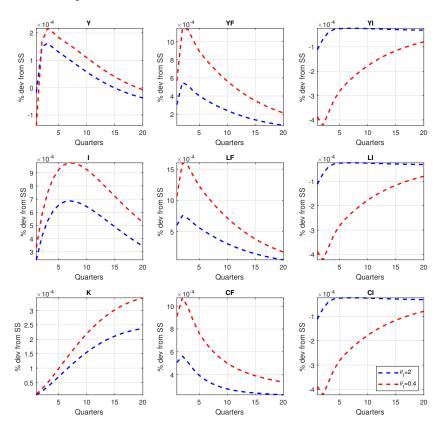


Figure 1.8: Comparison of labour tax shocks with different level of labour mobility

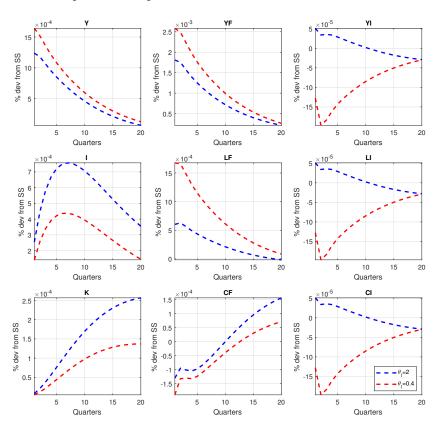


Figure 1.9: Comparison of capital tax shocks with different level of labour mobility

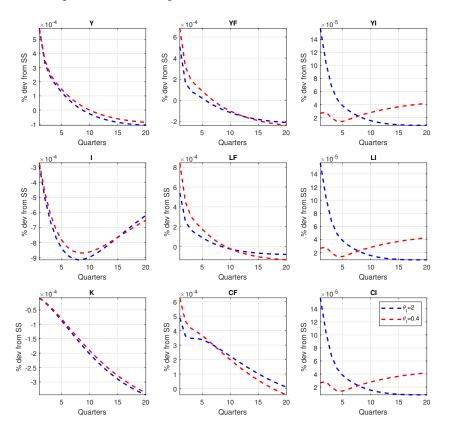


Figure 1.10: Comparison of consumption tax shocks with different level of labour mobility

#### 1.8.3 Robustness

In this section we check the robustness of the model with respect to the change in the values of some key parameters. Specifically we vary the value of a single parameter, holding all others constant at their benchmark values to assess the change in the behaviour of the model in response to the fiscal policy shocks. Our parameters of interest are elasticity of substitution between formal and informal consumption  $(\nu)$ , elasticity of intertemporal substitution in consumption  $(\sigma_c)$ , Frisch elasticity of labour supply  $(\sigma_l)$ , price stickings parameter  $(\omega_f)$  and output elasticity of public capital  $(\alpha_a)$ . The impulse responses of the macroeconomic variables do not show significant variations in response to the changes in values of these parameters. We found that the model is robust to the variations in the values of the parameters as most of the impulse responses remain the same qualitatively. The direction of the movement of most of the variables remain same. The impacts of the changes in the parameters are shown in figures (1.11-1.35) in appendix B. However, the output of informal sector shows relatively more variations in response to a change in elasticity of substitution of consumption between formal and informal goods. We also find small variations in the responses of the macroeconomic variables when there is a change in the value of the elasticity of intertemporal substitution of consumption.

## 1.9 Conclusion

In this paper we develop a DSGE model by incorporating the informal economy to study the role of segmented labour and goods markets in shaping the impacts of five fiscal policy shocks on aggregate economic activities. The analysis indicates that the increases in government consumption, government investment and cuts in consumption and capital income taxes results in the rise of the output of both formal and informal sectors, whereas a decrease in the labour tax causes the informal sector output to fall. The output of the informal sector increases and remains above the steady-state for longer period of time in response to government expenditures and consumption tax shocks. This shows that an increase in the government expenditures and decrease in the consumption tax when financed by increasing the labour and capital income taxes result in an increase in the size of informal economy. In the presence of informal economy, the government spending impact multipliers are greater than one. Among three taxes, the capital income tax shock is more stimulating at increasing the level output both in short and long run. Dampening effect of labour tax cut on the informal sector output outweight its positive impact on the formal sector output. As a result of the movement of formal and informal sectors outputs in opposite directions, the initial effect of labour tax cut on aggregate level of output is smallest than the decreases in capital income and consumption taxes. A cut in the capital tax also has a negative impact on the output of informal sector,

after a few quarters. This shows that cuts in capital and labour income taxes, decrease the size of informal economy, whereas a cut in consumption tax increases it. This study also analyses the role of labour mobility in conditioning the impacts of fiscal policy shocks in the presence of informal economy. An increase in the labour mobility amplifies the impacts of government investment and consumption shocks on the output of formal sector and results in higher government expenditure present value multipliers. The impact of a decrease in labour tax is more pronounced on both sectors when labour mobility across the two sector is high. The output of informal sector declines more because of the large movement of labour from informal to the formal sector. The large negative effect of a decrease in the labour tax on informal sector, outweighs its positive impact on the formal sector output and leads to a decline in the aggregate level of output on impact of a decrease in labour tax. A cut in capital income tax also has a strong negative impact on the production of informal sectors, when labour is highly mobile across the sectors.

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# 1.11 Appendices

### A.1: Model in Log-linearised form

The dynamics of the model are given by the set of 40 equations for 40 variables. In the log-linearised system of equation a hat over a variable represents log-deviation of that variable from steady-state and letters without subscript, t represent the steadystate value. The log-linearised system of equation is given as follows:

$$\begin{split} \hat{Q}_{t} &= -\hat{R}_{t} + E_{t}\pi_{t+1}^{-} + \frac{1-\delta}{(1-\delta) + (1-\tau^{k})R^{k}}E_{t}\left[\hat{Q}_{t+1} + R^{k}(1-\tau^{k})(\hat{R}^{k}_{t+1} - \frac{\tau^{k}}{1-\tau^{k}}\hat{\tau}^{k}_{t+1})\right] \\ \hat{W}_{t} &= \sigma_{l}L_{t} - \frac{\sigma_{c}}{1-h}\hat{C}_{t} + \frac{\sigma_{c}h}{1-h}\hat{C}_{t-1} \\ \hat{C}_{t} &= E_{t}\frac{\hat{C}_{t+1}}{1+h} + \frac{h\hat{C}_{t-1}}{1+h} - \frac{1}{\sigma_{c}}\frac{(1-h)}{(1+h)}E_{t}\left[\hat{R}_{t} - \pi_{t+1}\right] \\ \hat{I}_{t} &= \frac{\hat{Q}_{t}}{\psi(1+\beta)} + \frac{\hat{I}_{t-1}}{1+\beta} + \frac{\beta E_{t}\hat{I}_{t+1}}{1+\beta} + \frac{1}{1+\beta}E_{t}(\beta\epsilon_{t+1}^{I} - \epsilon_{t}^{I}) \\ \hat{U}_{t} &= \frac{1}{\zeta}\left[\hat{R}_{t}^{k} - \frac{\tau^{k}}{1-\tau^{k}}\hat{\tau}_{t}^{k}\right] \end{split}$$

$$\hat{K}_t = (1 - \delta)\hat{K}_{t-1} + \delta\hat{I}_t$$
$$\hat{C}_{F,t} = \hat{C}_t - \nu \left(\hat{P}_{F,t} + \frac{\tau^c}{1 + \tau^c}\hat{\tau}_t^c\right)$$
$$\hat{C}_{I,t} = \hat{C}_t - \nu(\hat{P}_{i,t})$$

$$\begin{split} \hat{Y}_{F,t} &= \hat{A}_{F,t} + \alpha \hat{K}_{t-1} + \alpha \hat{U}_t + (1-\alpha) \hat{L}_{F,t} + \alpha_g \hat{K}_{g,t-1} \\ \hat{M}c_{F,t} &= (1-\alpha) \hat{W}_{F,t} + \alpha \hat{R}_t^k - \hat{A}_{F,t} + \alpha_g \hat{K}_{g,t-1} \\ \hat{L}_{F,t} &= \hat{R}_{k,t} + \hat{K}_t - \hat{W}_{F,t} + \hat{U}_t \\ \hat{\pi}_{F,t} &= \frac{\beta}{1+\beta\omega_p} E_t \hat{\pi}_{t+1} + \frac{\omega_p}{1+\beta\omega_p} \hat{\pi}_{t-1} + \frac{(1-\beta\omega_f)(1-\omega_f)}{\omega_f(1+\beta\omega_p)} (\hat{M}c_{F,t} - \hat{\epsilon}_t^p) \\ \hat{Y}_{I,t} &= \hat{L}_{I,t} + \alpha_g \hat{K}_{g,t-1} \\ \hat{W}_{I,t} &= \alpha_g \hat{K}_{g,t-1} + \hat{P}_{I,t} \\ \hat{L}_{F,t} &= \frac{1}{\theta_l} \left( \hat{W}_{F,t} - \frac{\tau^l}{(1-\tau^l)} \hat{\tau}_t^l - \hat{W}_t \right) + L_t \\ \hat{L}_{I,t} &= \frac{1}{\theta_l} (\hat{W}_{I,t} - \hat{W}_t) + L_t \\ \hat{W}_t &= \frac{1}{W^{\frac{1+\theta_l}{\theta_l}}} \left[ \gamma_l \frac{1}{(1-\tau_l)} \hat{W}_{F,t} - \gamma_l \frac{\tau_l}{(1-\tau_l)W_F} \hat{\tau}_t^l + (1-\gamma_l) \hat{W}_{I,t} \right] \end{split}$$

$$\hat{K}_{g,t} = (1 - \delta_g)\hat{K}_{g,t-1} + \delta_g\hat{G}I_t$$

$$\frac{G}{Y_F}\hat{G}_t + \frac{GI}{Y_F}\hat{G}I_t + \frac{B}{Y_F}R[\hat{B}_{t-1} + \hat{R}_{t-1} - \hat{\pi}_{F,t}] = \frac{B}{Y_F}\hat{b}_{t-1} + \tau^c \frac{C_F}{Y_F}(\hat{C}_{F,t} + \hat{\tau}_t^c) + \tau^l \frac{W_F L_F}{Y_F}(\hat{W}_{F,t} + \hat{L}_{F,t} + \hat{\tau}_t^l) + \tau^k R^k \frac{K}{Y_F}(\hat{K}_t + \hat{R}_t^k + \hat{\tau}_t^k)$$

$$Y_{F,t} = \frac{C_F}{Y_F}\hat{C}_{F,t} + \frac{I}{Y_F}\hat{I}_t + \frac{G}{Y_F}\hat{G}_t + \frac{GI}{Y_F}\hat{G}I_t + (1-\tau^k)\frac{R^kK}{Y_F}U_t$$

$$\hat{Y}_{I,t} = \hat{C}_{I,t}$$

$$\hat{G}_{t} = -\psi_{b,g}\hat{B}_{t-1} - \psi_{y,g}\hat{Y}_{F,t} + g_{t}$$

$$\hat{G}I_{t} = -\psi_{b,gi}\hat{B}_{t-1} - \psi_{y,gi}\hat{Y}_{F,t} + gi_{t}$$

$$\hat{\tau}_{t}^{c} = \psi_{b,\tau^{c}}\hat{B}_{t-1} + \psi_{y,\tau^{c}}\hat{Y}_{F,t} - tc_{t}$$

$$\hat{\tau}_{t}^{k} = \psi_{b,\tau^{k}}\hat{B}_{t-1} + \psi_{y,\tau^{k}}\hat{Y}_{F,t} - tk_{t}$$

$$\hat{\tau}_{t}^{l} = \psi_{b,\tau^{l}}\hat{B}_{t-1} + \psi_{y,\tau^{l}}\hat{Y}_{F,t} - tl_{t}$$

### A.1.2: Model in Steady-state

Rate of interest (From Euler equation).

$$R = 1/\beta$$

Physical assets optimization equation.

$$R = \{ (1 - \delta) + (1 - \tau^k) r^k \}$$

Capital accumulation.

$$I = \delta K$$

Labour-leisure choice.

$$L^{\sigma_l} = \frac{W}{\left((1-h)C\right)^{\sigma_c}}$$

Marginal cost.

$$MC_F = P_F\left(\frac{\nu - 1}{\nu}\right)(1 - \beta\theta)$$

Labour supply to formal sector.

$$L_F = \gamma \left(\frac{(1-\tau^l)W_F}{W}\right)^{\frac{1}{\theta_l}} L$$

Labour supply to informal sector.

$$L_I = (1 - \gamma) \left(\frac{W_I}{W}\right)^{\frac{1}{\theta_l}} L$$

Composite wage rate.

$$W = \left[\gamma\{(1-\tau^l)(W_F)\}^{\frac{1+\theta_l}{\theta_l}} + (1-\gamma)(W_I)^{\frac{1+\theta_l}{\theta_l}}\right]^{\frac{\theta_l}{1+\theta_l}}$$

Composite price index.

$$P = \left[ (1 - \gamma_1)((1 + \tau^c)(P_F))^{1 - \nu_1} + (\gamma_1)(P_I)^{1 - \nu_1} \right]^{\frac{1}{1 - \nu_1}}$$

Consumption of the formal goods.

$$C_F = (1 - \gamma) \left( (1 + \tau^c) \frac{P_F}{P} \right)^{-\nu} C$$

Consumption of the informal goods.

$$C_I = (1 - \gamma_1) \left(\frac{P_I}{P}\right)^{-\nu_1} C$$

Production function of the formal sector.

$$Y_F = A_F K_t^{\alpha} L_F^{1-\alpha}(K_g^{\alpha_g})$$

Wage rate of the formal sector.

$$W_F = MC_F (1 - \alpha) \frac{Y_T^h}{L_F}$$

Rental rate of the capital.

$$R^k = MC_F \frac{\alpha Y_F}{K}$$

Production function of the informal sector.

$$Y_I = L_I K_g^{\alpha_g}$$

Wages in informal sector.

$$W_I = P_I k_g^{\alpha_g}$$

Government budget constraint.

$$G_t = \tau_c C_F + \tau^l W_F L_F + \tau^k R^k K + B(1-R) - GI$$

Public capital accumulation.

$$GI = \delta_g K_g$$

# B.1.1: Change in the Elasticity of Substitution of Consumption between Formal and Informal goods

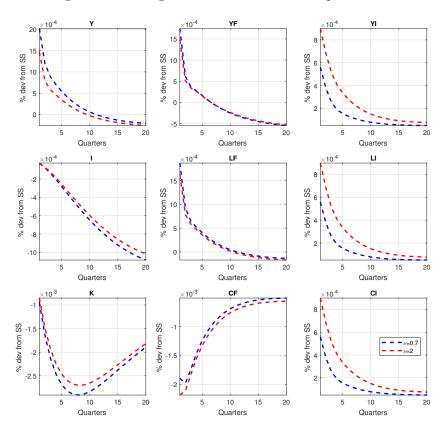


Figure 1.11: Change in  $\nu$ : Government consumption shock

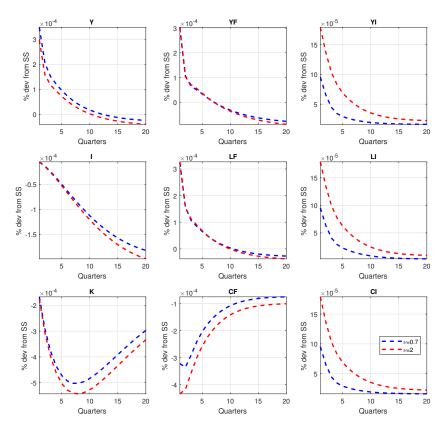


Figure 1.12: Change in  $\nu :$  Government investment shock

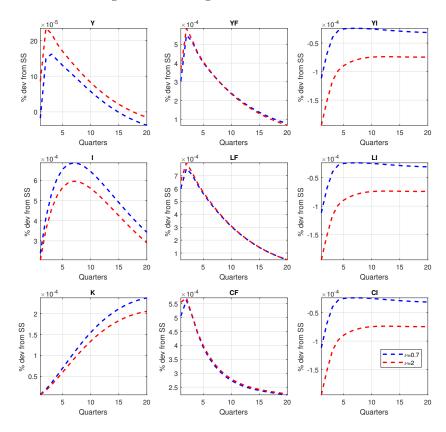


Figure 1.13: Change in  $\nu:$  labour tax shock

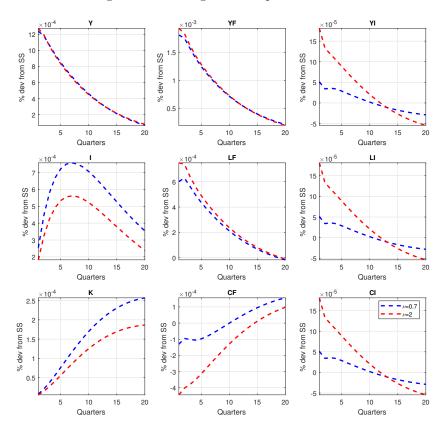


Figure 1.14: Change in  $\nu :$  Capital tax shock

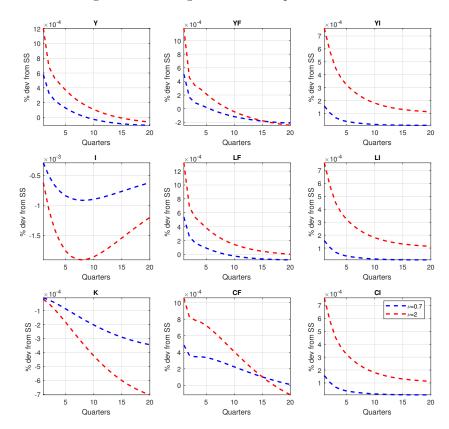


Figure 1.15: Change in  $\nu :$  Consumption tax shock

# **B.1.2:** Change in Intertemporal Elasticity of Substitution in Consumption

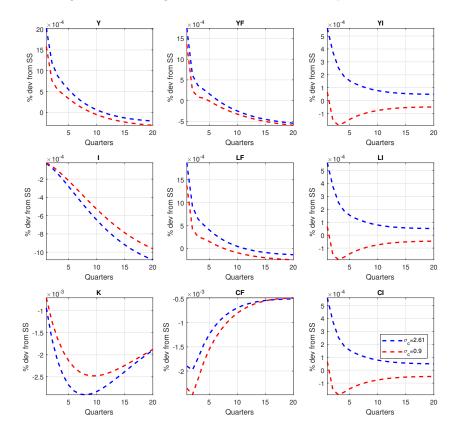


Figure 1.16: Change in  $\sigma_c$ : Government consumption shock

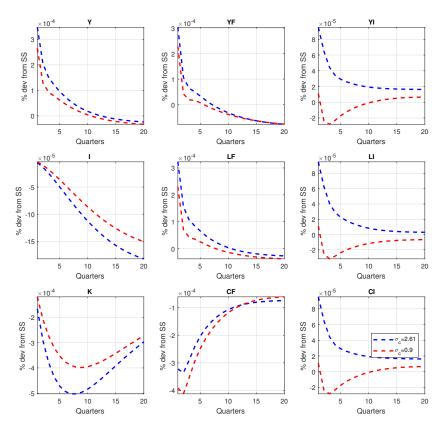


Figure 1.17: Change in  $\sigma_c$ : Government investment shock

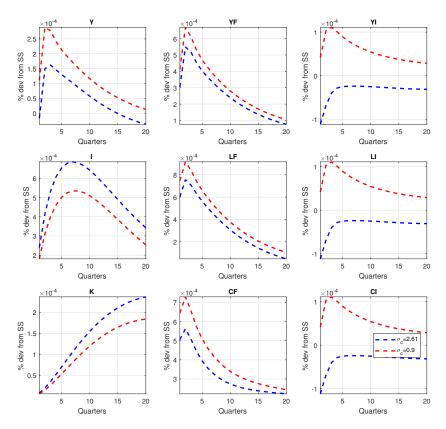


Figure 1.18: Change in  $\sigma_c$ : Labour tax shock

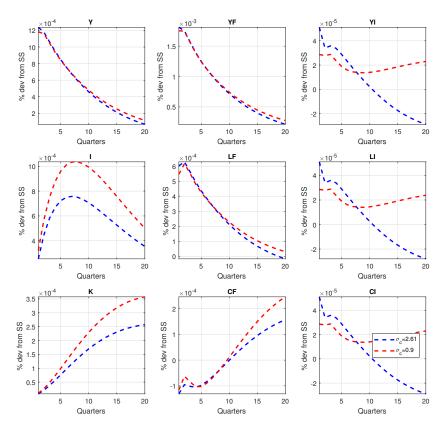


Figure 1.19: Change in  $\sigma_c$ : Capital tax shock

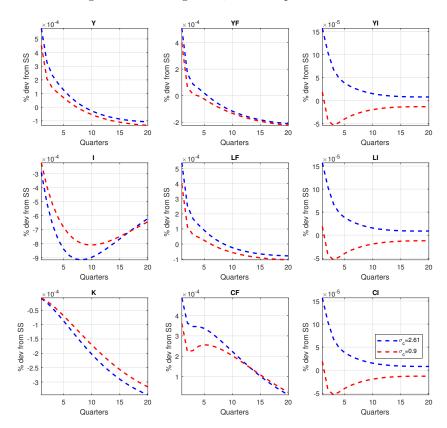


Figure 1.20: Change in  $\sigma_c$ : Consumption tax shock

# B.1.3: Change in Inverse Elasticity of Labour Supply

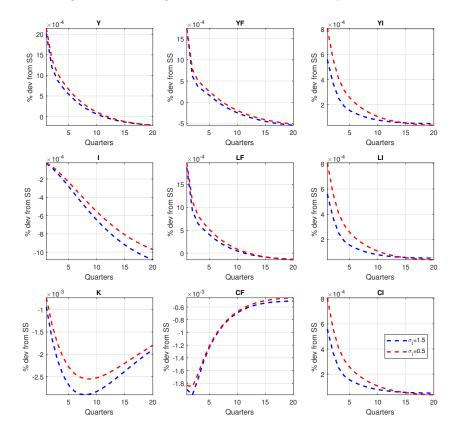


Figure 1.21: Change in  $\sigma_l$ : Government consumption shock

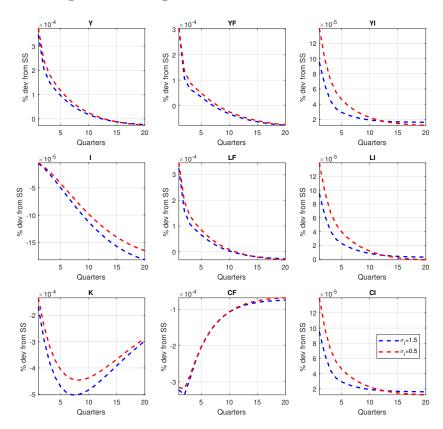


Figure 1.22: Change in  $\sigma_l$ : Government investment shock

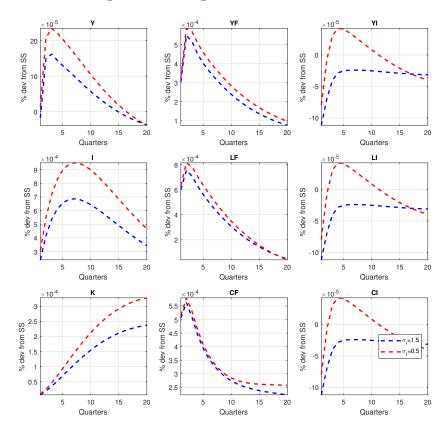


Figure 1.23: Change in  $\sigma_l$ : Labour tax shock

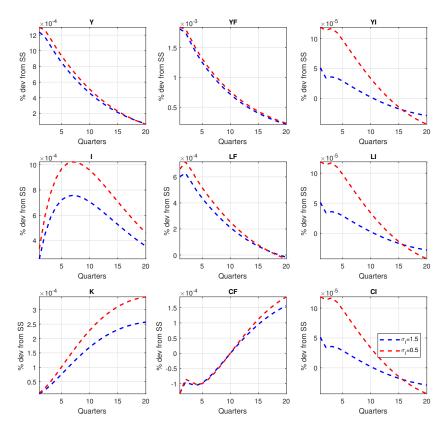


Figure 1.24: Change in  $\sigma_l$ : Capital tax shock

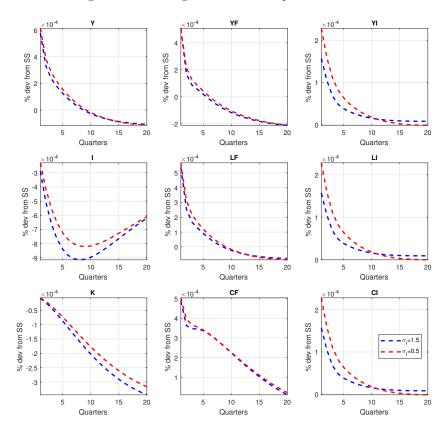


Figure 1.25: Change in  $\sigma_l$ : Consumption tax shock

# B.1.4: Change in the Price Rigidity of Formal goods

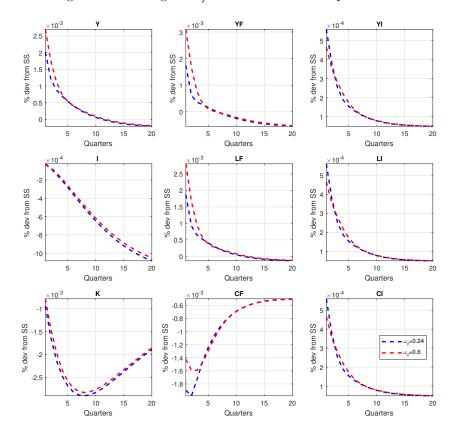


Figure 1.26: Change in  $\omega_f$ : Government consumption shock

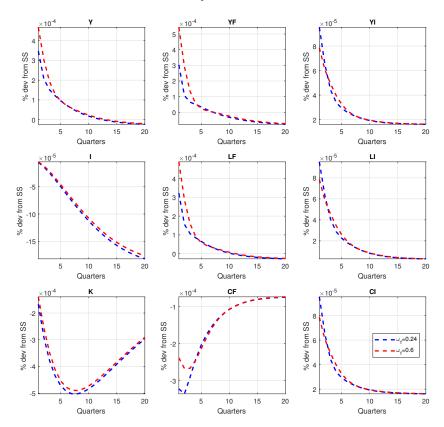


Figure 1.27: Change in  $\omega_f$ : Government investment shock

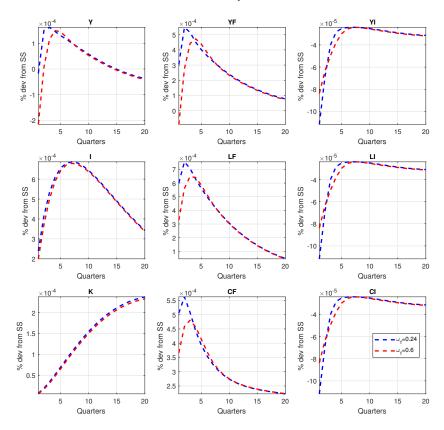


Figure 1.28: Change in  $\omega_f$ : Labour tax shock

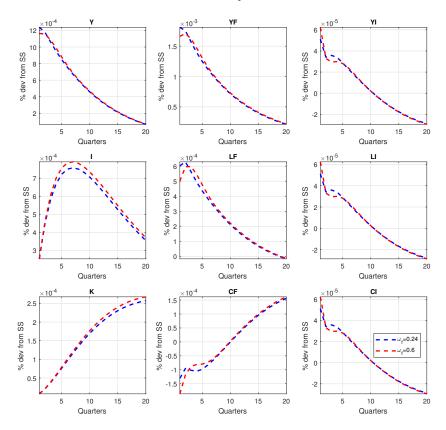


Figure 1.29: Change in  $\omega_f$ : Capital tax shock

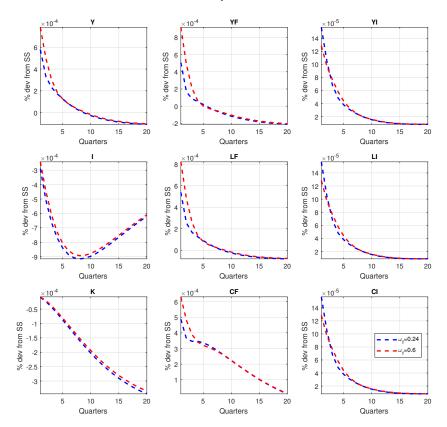


Figure 1.30: Change in  $\omega_f$ : Consumption tax shock

# B.1.5: Change in the Output Elasticity of Public Capital

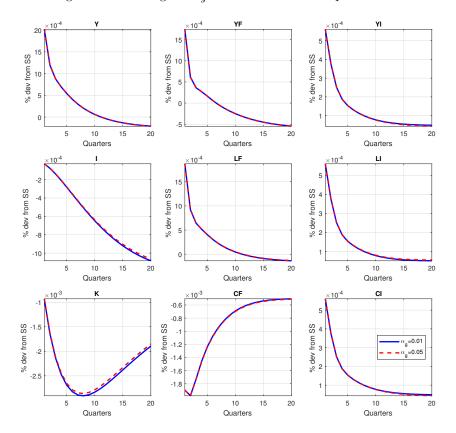


Figure 1.31: Change in  $\alpha_g$ : Government consumption shock

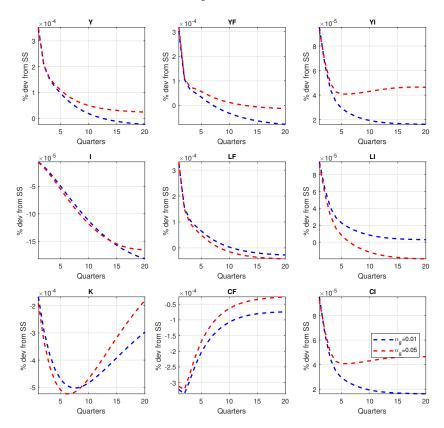


Figure 1.32: Change in  $\alpha_g$ : Government investment shock

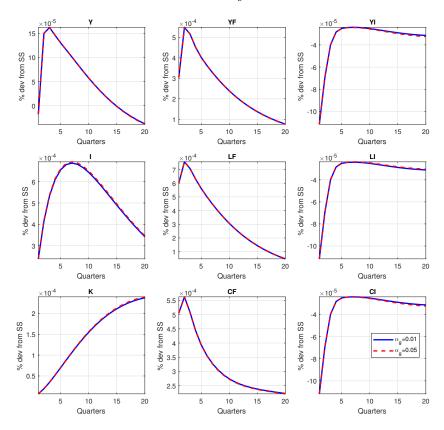


Figure 1.33: Change in  $\alpha_g$ : Labour tax shock

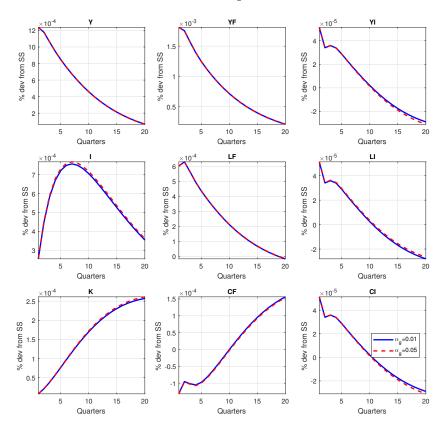


Figure 1.34: Change in  $\alpha_g$ : Capital tax shock

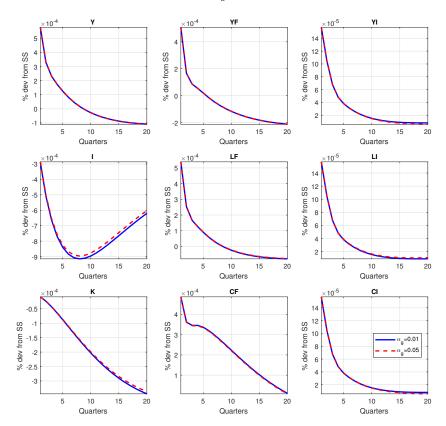


Figure 1.35: Change in  $\alpha_g$ : Consumption tax shock

# Chapter 2

# Optimal Capital Taxation in the presence of Informal Labour and Goods Markets

#### Abstract

Chamley (1986) in a neoclassical growth model shows that under optimal fiscal policy, capital income tax should be zero in the long run. In the line of this seminal paper, there has been a growing literature focusing on the optimal fiscal policy and setting of the optimal combination of distortionary taxes under different modelling assumptions. There are several studies which show that the capital income tax is not zero if the modelling framework is modified in some ways. Following Chamley (1986), this study contributes to the literature of the optimal fiscal policy by incorporating an informal economy in the neoclassical growth model. The Ramsey problem is analysed in a context where taxes can not be levied from informal sector. The set of tax instruments is incomplete such that, the consumption of informal goods and income from working in the informal labour market can not be taxed. The main finding of this study is that an optimal zero tax rate on capital income is not obtained in the model with informal labour and goods markets.

# 2.1 Introduction

Chamley (1986) uses a neoclassical growth model and shows that under optimal fiscal policy capital income tax should be zero in the long run. Judd (1985) assumes two different types of agents and arrives at the same conclusion. Contrary to these two seminal papers, there are several studies which show that the capital income tax is not zero if the modelling framework is modified in some ways. For instance, Judd (1997) shows that the optimal tax rate on capital is negative under imperfect product markets. Guo and Lansing (1999) consider tax allowance and depreciation of the physical capital and assume government expenditures as endogenous. They find that the optimal tax rate on capital can be positive or negative. Lansing (1999) also figures out that in the presence of imperfections in capital and labour markets the zero capital income tax is not obtained. Correia (1996) analyses the optimal fiscal policy when each factor input in the production process is not taxed. She concludes that in the long run capital income tax may be non-zero under an incomplete set of tax instruments. Jones et al.(1993) show that in the presence of government expenditures as productive inputs the optimal tax rate is not zero because of the incomplete taxation of inputs. Jones et al.(1997) find that the zero capital tax is not optimal when firms are making pure profits. According to their finding, capital tax is a way of taxing pure profits when they can not be taxed directly. Conesa et al. (2009) conclude that the presence of skills heterogeneity in the labour markets results in highly progressive labour income tax rates. Ludwig and Ivan (2020) revisited Chamley (1986) and Judd (1985) and showed that the long run tax on capital is not zero. They prove that in steady-state capital income tax is positive and significant, when the intertemporal elasticity of substitution is less than one.

Following Chamley (1986), this study contributes to the literature of optimal fiscal policy by incorporating an informal economy in the neoclassical growth model. Ramsey problem is analysed in the context where taxes can not be levied from informal sector consumption and income. The objective of this study is to find the steady-state optimal tax rate on capital income in the long run in general equilibrium model in the presence of informal labour and goods markets. In a representative agent model, this study follows Ramsey (1927) in which a benevolent planner chooses the structure of the optimal taxation. The planner has consumption tax from the consumption of formal goods, labour income tax from the earning of formal sector and capital income tax at her disposal. The set of tax instruments is incomplete because the consumption and income from informal sector can not be taxed. The finding of this study is that the optimal zero tax rate on capital income is not obtained in the model with informal labour and goods markets <sup>1</sup>.

The next section presents the model, section 3 consists of the explanation of decentralized competitive equilibrium, section 4 analyses the Ramsey problem and gives the optimal tax formula for capital income tax and section 5 provides the conclusion of the study.

# 2.2 Model

The model consists of a representative household, representative firms of both formal and informal sectors and fiscal authority. The economy is divided into formal (f) and informal (i) sectors with different technologies producing different goods and selling them at different prices as in Ahmad et al. (2012), Batini et al. (2011), Haider and Khan (2008) and Regessa (2013). We assume that the economy is inhabited by identical and infinitely-lived households that can be represented by a household which consists of continuum of family members. Total labour supply is comprised of

<sup>&</sup>lt;sup>1</sup>In Appendix C, we analyse the Ramsey problem in an economy without informal labour and goods market such that we assume the values of  $\gamma$  and  $\gamma_l$  equal to one.

the supply to both formal and informal sectors and it is given by CES function. Firms operating in the formal sector produce output by employing labour and capital. The informal sector firms use only labour as an input. The fiscal authority issues risk-free bonds to finance its expenditures and levies taxes on consumption, labour income and capital income of the formal sector.

#### 2.2.1 The Households

The economy is inhabited by identical and infinitely-lived households which can be represented by a household which is made up of continuum of family members. Each member of the household obtains utility by consuming both formal and informal goods and supplies labour either to formal or informal sector of the economy. The life-time utility of representative household is given as:

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$
(2.1)

where,  $\beta$  is the subjective discount factor of the household,  $C_t$  represents the total consumption of goods,  $L_t$  is an index of the household aggregate labour supply. Functional specification of the objective function is given as:

$$U(C_t, L_t) = \ln C_t - \frac{(L_t)^{1+\sigma}}{1+\sigma}$$
(2.2)

where,  $\sigma$  is the Frisch elasticity of labour supply. The household maximizes the objective function with respect to the following budget constraint:

$$C_{f,t}(1+\tau_t^c) + P_t C_{i,t} + K_t - (1-\delta)K_{t-1} + B_t =$$

$$(1-\tau_t^l)W_{f,t}l_{f,t} + W_{i,t}L_{i,t} + (1-\tau_t^k)R_t^kK_{t-1} + R_{t-1}B_{t-1} + Div$$
(2.3)

The left side of the equation shows the total expenditures of household which include the expenditure on formal consumption  $(C_{f,t})$ , expenditure on informal consumption  $(C_{i,t})$ , investment  $(I_t)$  and spending on one period bonds $(B_t)$ , while  $\tau_t^c$  represents the tax rate on formal consumption and  $P_t$  is the relative price of informal sector goods. The numeraire is the formal good. The right hand side of the equation shows the total disposable earning of the household which consists of:

- after tax labour income from formal sector  $(1 \tau_t^l)W_{f,t}L_{f,t}$ , where  $W_{f,t}$  represents the wage rate in the formal sector and  $L_{f,t}$  represents the labour supply to the formal sector and  $\tau_t^l$  denotes the labour income tax rate;
- labour income from informal sector  $(W_{i,t}L_{i,t})$ , where  $W_{i,t}$  represents the wage rate of informal sector and  $L_{i,t}$  represents the labour supply to informal sector;
- after tax return on capital  $(1 \tau_t^k) R_t^k K_{t-1}$ , where  $K_{t-1}$  represents the physical stock of capital,  $R_t^k$  denotes the rate of return on capital and  $\tau_t^k$  is the tax on capital;

- interest income from bond holdings,  $R_t$ .
- Dividend, Div

#### 2.2.2 Household Consumption Decision

Aggregate consumption,  $C_t$  is an index of the quantities of both formal,  $C_{f,t}$  and informal,  $C_{i,t}$  goods consumed and it is given by the following CES aggregator:

$$C_t = \left[ (\gamma)^{\frac{1}{\nu}} C_{f,t}^{\frac{\nu-1}{\nu}} + (1-\gamma)^{\frac{1}{\nu}} C_{i,t}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \qquad (2.4)$$

where,  $\nu$  is the elasticity of substitution between the consumption of formal and informal goods,  $\gamma$  measures the share of formal goods in the consumption basket of household,  $(1 - \gamma)$  can also be assumed as proxy of the size of informal sector.

#### 2.2.3 Household Labour Supply Decision

Total labour supply,  $L_t$  is comprised of the supply of labour to both formal  $(L_{f,t})$ and informal  $(L_{i,t})$  sectors. Members of each household supply the labour to both formal and informal sectors according to the following CES function:

$$L_t = \left[\gamma_l^{-\theta_l} (L_{f,t})^{1+\theta_l} + (1-\gamma_l)^{-\theta_l} (L_{i,t})^{1+\theta_l}\right]^{\frac{1}{1+\theta_l}}, \qquad (2.5)$$

where  $\gamma_l$  and  $(1 - \gamma_l)$  represent the proportion of the labour working in formal and informal sectors respectively and  $\theta_l$  is the inverse of the elasticity of substitution of labour supply between formal and informal sectors.

#### 2.2.4 Formal Sector Firms

In the formal sector, firms have to decide about their demand of capital and labour at given wages and rent of the capital. Formal sector producers employ the following Cobb Douglas production function:

$$Y_{f,t} = A_{f,t} K_t^{\alpha} L_{f,t}^{1-\alpha}$$
(2.6)

where  $A_{f,t}$  is the exogenous level of technology,  $L_{f,t}$  is the amount of formal labour and  $K_t$  is the quantity of physical capital. Formal sector firms maximize an expected discounted profit by making a choice on the level of employment of the labour and capital given the wages and rental rate of the capital. The instantaneous profit function of the formal firms is given by the following equation.

$$\Pi_{f,t} = (A_{f,t} K_t^{\alpha} L_{f,t}^{1-\alpha}) - W_{f,t} L_{f,t} - R_t^k K_t$$

Wages and rental rate of capital in the formal sector are given as:

$$W_{f,t} = (1 - \alpha) \frac{Y_{f,t}}{L_{f,t}}$$
(2.7)

$$R_t^k = \frac{\alpha Y_{f,t}}{K_t} \tag{2.8}$$

Marginal cost of the formal sector firms is given as:

$$MC_{f,t} = \frac{1}{A_{f,t}} \left(\frac{R_t^k}{\alpha}\right)^{\alpha} \left(\frac{W_{f,t}}{1-\alpha}\right)^{1-\alpha}$$
(2.9)

# 2.2.5 Production of Informal Sector

Informal sector firms produce informal goods by using only the labour as an input at given wages. Informal sector firms employ the following production function:

$$Y_{i,t} = L_{i,t} \tag{2.10}$$

The instantaneous profit function of the formal firm is given by the following equation:

$$\Pi_{i,t} = P_t L_{i,t} - W_{i,t} L_{i,t}$$

so , wages in the formal sector are obtained as:

$$W_{i,t} = P_t \tag{2.11}$$

#### 2.2.6 Government Behaviour

The government budget constraint (2.12) shows that the government expenditures  $(G_t)$  and interest payment of debt,  $(R_{t-1}) B_{t-1}$  is equal to the taxes levies on formal sector consumption  $(\tau_t^c C_{f,t})$ , labour income of formal sector  $(\tau_t^l W_{f,t} L_{f,t})$ , capital income  $(\tau_t^k r_t^k K_t)$  and purchase of new debt  $(B_t)$ .

$$G_t + R_{t-1}B_{t-1} = \tau_t^c C_{f,t} + \tau_t^l L_{f,t} W_{f,t} + \tau_t^k K_t R_t^k + B_t$$
(2.12)

#### 2.2.7 Goods Market Equilibrium

The formal sector resource constraint consist of the formal sector consumption, investment and government expenditures and given in equation (2.13)

$$Y_{f,t} = C_{f,t} + I_t + G_t \tag{2.13}$$

All output of the informal sector is consumed so the informal sector resource constraint is expressed as:

$$Y_{i,t} = C_{i,t} \tag{2.14}$$

# 2.3 Decentralized Competitive Equilibrium

Given the initial level of capital stock and debt  $(K_0, B_0)$  and three policy instruments  $\{\tau_t^c, \tau_t^l, \tau_t^k\}_{t=0}^{\infty}$  the competitive equilibrium is defined as a sequence of the allocation of  $\{C_{f,t}, C_{i,t}, L_{f,t}, L_{i,t}, K_t\}_{t=0}^{\infty}$  and prices  $\{W_{f,t}, W_{i,t}, R_t^k\}_{t=0}^{\infty}$  such that:

- households maximize utility;
- firms maximize their profits;
- the budget constraint of the government holds;
- all markets are clear.

The household maximizes utility (2.2) subject to budget constraint (2.3):

$$\max_{(C_{f,t},C_{i,t},L_{f,t}L_{i,t},K_{t})} \sum_{t=0}^{\infty} \beta^{t} \left\{ \left[ ln \left( (\gamma)^{\frac{1}{\nu}} C_{f,t}^{\frac{\nu-1}{\nu}} + (1-\gamma)^{\frac{1}{\nu}} C_{i,t}^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} - \frac{1}{(1+\sigma)} \left( \gamma_{l}^{-\theta_{l}} (L_{f,t})^{1+\theta_{l}} + (1-\gamma_{l})^{-\theta_{l}} (L_{i,t})^{1+\theta_{l}} \right)^{\frac{1+\sigma}{1+\theta_{l}}} \right] - \lambda_{t} \left[ C_{f,t} (1+\tau_{t}^{c}) + P_{t} C_{i,t} + K_{t} - (1-\delta) K_{t-1} + B_{t} + (1-\tau_{t}^{l}) W_{f,t} l_{f,t} - W_{i,t} L_{i,t} - (1-\tau_{t}^{k}) R_{t}^{k} K_{t-1} - R_{t-1} B_{t-1} - Div \right] \right\}$$
(2.15)

The first order conditions of the households utility maximization problem with respect to  $C_f, C_i, L_f, L_i, B_t$ , and  $K_t$  are:

$$U_{Cf,t} = \frac{\gamma^{\frac{1}{\nu}} C_{f,t}^{\frac{-1}{\nu}}}{(\gamma)^{\frac{1}{\nu}} C_{f,t}^{\frac{\nu-1}{\nu}} + (1-\gamma)^{\frac{1}{\nu}} C_{i,t}^{\frac{\nu-1}{\nu}}} = \lambda_t (1+\tau_t^c)$$
(2.16)

$$U_{Ci,t} = \frac{(1-\gamma)^{\frac{1}{\nu}} C_{i,t}^{\frac{-1}{\nu}}}{(\gamma)^{\frac{1}{\nu}} C_{f,t}^{\frac{\nu-1}{\nu}} + (1-\gamma)^{\frac{1}{\nu}} C_{i,t}^{\frac{\nu-1}{\nu}}} = \lambda_t P_t$$
(2.17)

$$U_{Lf,t} = \left[\gamma_l^{-\theta_l} (L_{f,t})^{1+\theta_l} + (1-\gamma_l)^{-\theta_l} (L_{i,t})^{1+\theta_l}\right]^{\frac{\sigma-\theta_l}{1+\theta_l}} \gamma_l^{-\theta_l} L_{f,t}^{\theta_l} = \lambda_t (1-\tau_t^l) W_{f,t} \quad (2.18)$$

$$U_{Li,t} = \left[\gamma_l^{-\theta_l} (L_{f,t})^{1+\theta_l} + (1-\gamma_l)^{-\theta_l} (L_{i,t})^{1+\theta_l}\right]^{\frac{\sigma-\theta_l}{1+\theta_l}} (1-\gamma_l)^{-\theta_l} L_{i,t}^{\theta_l} = \lambda_t W_{i,t} \quad (2.19)$$

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta R_t \tag{2.20}$$

$$\lambda_t = \lambda_{t+1} \beta \left[ (1 - \delta) + (1 - \tau_{t+1}^k) R_{t+1}^k) \right]$$
(2.21)

Above FOCs are solved simultaneously to obtain the following important results:

$$\frac{U_{Cf,t}}{U_{Ci,t}} = \frac{1 + \tau_t^c}{P_t}$$
(2.22)

$$\frac{U_{Lf,t}}{U_{Cf,t}} = \frac{(1 - \tau_t^l)W_{f,t}}{(1 + \tau_t^c)}$$
(2.23)

$$\frac{U_{Li,t}}{U_{Cf,t}} = \frac{W_{i,t}}{1 + \tau_t^c}$$
(2.24)

$$\frac{U_{Lf,t}}{U_{Li,t}} = \frac{(1 - \tau_t^l)W_{f,t}}{W_{i,t}}$$
(2.25)

$$R = [1 + (1 - \tau_{t+1}^k)R_{t+1}^k - \delta]$$
(2.26)

$$\beta R_t E_t \left[ \frac{C_{f,t} (1 + \tau_{t+1}^c)}{C_{f,t+1} (1 + \tau_t^c)} \right] = 1$$
(2.27)

Equation (2.22) shows the marginal rate of substitution between the formal and informal consumption and it is derived form equations (2.16) and (2.17). Equation (2.23) equates the marginal rate of substitution between formal consumption and leisure to real wages. The trade-off between formal consumption and leisure shows how much income a household is willing to accept to sacrifice one hour of leisure time in formal sector. It is obtained by combining equations (2.16) and (2.18). Equation (2.25) gives the marginal rate of substitution between formal and informal labour supply, it is derived by combining equations (2.18) and (2.19). Equation (2.26) is no arbitrage condition between the government bonds and capital and equates the rate of return of two assets. Euler equation of consumption (2.27) is derived by combining (2.16) and (2.20). It shows that the loss in the marginal utility from consuming less today is exactly the same as the increase in marginal utility from consuming more in some later period and an increase in the future tax rate results in an increase in current consumption. The decentralized competitive equilibrium consists of the household optimality conditions from equation (2.22) to (2.27), the first order conditions of both formal and informal firms such that equations (2.7), (2.8), (2.9) and (2.11), Budget constraint of the household (2.3), budget constraint of government (2.12) and aggregate resource constraints of both formal and informal sectors (2.13) and (2.14).

## 2.4 Optimal Policy: Ramsey Equilibrium

In this analysis, we follow Ramsey (1927), who studies the issue of choosing the optimal combination of distortionary taxes in the absence of lump-sum tax in a representative agent model. We assume that the social planner has access to commitment technology to avoid the problem of time inconsistency. Fiscal policy is restricted in the sense that only the consumption of formal sector goods and income from working in the formal sector is taxed. The social planner does not have access to the informal sector output and income. The tax rate on the capital income is constant for the first period. Initial capital stock is fixed in supply, this assumption of constant capital income tax rate rules out the possibility of levying lump-sum tax from the initial capital stock. The objective of the social planner is to find the sequence of optimal capital income tax  $\{\tau^k\}_{t=1}^{\infty}$  to maximize the welfare of the household subject to the optimality conditions of household such that, equations (2.22) to (2.27), their budget constraint (2.3) and resource constraints of formal (2.13) and informal (2.14) sectors. This study makes the use of primal approach to find the solution of Ramsey problem. The use of the primal approach implies choosing the path of formal and informal consumption, labour supply to both formal and informal sectors and capital stock which is consistent with the optimization of household and subject to the relative budget constraints. In this study wages and return on capital are expressed in term of quantities using the marginal product of the labour and capital respectively. In the primal approach prices and taxes can be substituted away and not need to be explicitly included in the planner's problem. After substituting equations (2.22), (2.23), (2.24) and (2.27) into the budget constraint of household (2.3), following implementability constraint is obtained:

$$\sum_{t=0}^{\infty} \beta \Big( U_{Cf,t} C_{f,t} + U_{Ci,t} C_{i,t} + U_{Lf,t} L_{f,t} + U_{Li,t} L_{i,t} \Big) = \frac{A_0 U_{Cf,0}}{1 + \tau_0^c}$$
(2.28)  
$$A_0 = (1 + (1 - \tau^k) F_{f,k0} - \delta) (B_0 + K_0).$$

The second best policy can be obtained by maximizing the equation (2.2) subject to the implementability constraint (2.28) and resource constraints of formal and inform

where<sup>2</sup>,

 $<sup>^2</sup> F_{f,k0}$  is the marginal productivity of the initial capital stock.

sectors such that equations (2.13) and (2.14) respectively.

$$\max_{(C_{f,t},C_{i,t},L_{f,t}L_{i,t},K_{t})} \sum_{t=0}^{\infty} \beta^{t} \left\{ U(C_{t},L_{t}) + \Psi_{t} \left[ U_{Cf,t}C_{f,t} + U_{Ci,t}C_{i,t} + U_{Lf,t}L_{f,t} + U_{Li,t}L_{i,t} \right] - \lambda_{t} \left[ C_{f,t} + G_{t} + K_{t} - (1-\delta)K_{t-1} - F(K_{t},L_{f,t}) \right] - \phi_{t} \left[ C_{i,t} - F(Li,t) \right] - \Psi \frac{A_{o}U_{Cf,0}}{1 + \tau_{o}^{c}} \right\}$$
(2.29)

where  $\Psi_t$  is the Lagrange multiplier of the implementability constraint given by equation (2.29)<sup>3</sup>. Optimization of the 2nd best problem with respect to the formal consumption and labour supply gives the following condition:

$$\frac{U_{Lf,t}}{U_{Cf,t}} = \frac{\Psi_t \Big[ U_{Lf,t} + L_{f,t} U_{Lf,tLf,t} + L_{i,t} U_{Li,tLf,t} \Big] + \lambda_t (1 - \tau_t^l) F_{Lf,t}}{\Psi_t \Big[ U_{Cf,t} + C_{f,t} U_{Cf,tCf,t} + C_{i,t} U_{Ci,tCf,t} \Big] + \lambda_t (1 + \tau_t^c)}$$
(2.30)

The above equation<sup>4</sup> shows that as long as the multiplier of the implementability constraint ( $\Psi$ ) is different from zero the condition of the competitive equilibrium (2.23) does not hold. In this case solution of the Ramsey problem can not be decentralized. This implies that the Condition (2.23) has to be explicitly included in the second best problem. Ramsey problem can now be represented as<sup>5</sup>:

 $<sup>{}^{3}</sup>Y_{f,t} = F(K_t, L_{f,t}) \text{ and } Y_{i,t} = F_{Li,t}.$ 

 $<sup>{}^{4}</sup>F_{Lf,t}$  is the marginal productivity of the labour, working in the formal sector.

<sup>&</sup>lt;sup>5</sup>Tax capital tax rate is obtained from equation (2.21), whereas we have 3 equations (2.22, 2.23) and (2.25) to find the tax rate on consumption and labour income. Therefore the equation (2.23) has to be explicitly included in the second best problem.

$$\max_{(C_{f,t},C_{i,t},L_{f,t}L_{i,t},K_{t})} \sum_{t=0}^{\infty} \beta^{t} \Biggl\{ U(C_{t},L_{t}) + \Psi_{t} \Biggl[ U_{Cf,t}C_{f,t} + U_{Ci,t}C_{i,t} + U_{Lf,t}L_{f,t} + U_{Li,t}L_{i,t} \Biggr] - \lambda_{t} \Biggl[ C_{f,t} + G_{t} + K_{t} - (1-\delta)K_{t-1} - F(K_{t},L_{f,t}) \Biggr] - \phi_{t} \Biggl[ C_{i,t} - F_{Li,t} \Biggr] + \varphi_{t} \Biggl[ - \frac{U_{Lf,t}}{U_{Cf,t}} + \frac{(1-\tau_{t}^{l})F_{Lf,t}}{(1+\tau_{t}^{c})} \Biggr] - \Psi \frac{A_{o}U_{Cf,0}}{1+\tau_{o}^{c}} \Biggr\}$$
(2.31)

The First order conditions of the Ramsey problem with respect to the  $C_{f,t}, C_{i,t}, L_{f,t}, L_{i,t}$ and  $K_t$  are:

$$U_{Cf,t} + \Psi_t \Big[ U_{Cf,t} + C_{f,t} U_{Cf,tCf,t} + C_{i,t} U_{Ci,tCf,t} \Big] - \lambda_t - \varphi_t \Big[ \frac{(U_{Lf,t}) U_{CftCft}}{U_{Cf,t}^2} \Big] = 0$$

$$(2.32)$$

$$U_{Ci,t} + \Psi_t \Big[ U_{Ci,t} + C_{i,t} U_{Ci,tCi,t} + C_{f,t} U_{Cf,tCi,t} \Big] - \phi_t - \varphi_t \Big[ \frac{(U_{Lf,t}) U_{CftCit}}{U_{Cf,t}^2} \Big] = 0$$
(2.33)

$$\Psi_t \Big[ U_{Lf,t} + L_{f,t} U_{Lf,tLf,t} + L_{i,t} U_{Li,tLf,t} \Big] + \lambda_t F_{Lf,t} + \varphi_t \Big[ -\frac{U_{Lf,tLf,t}}{U_{Cf,t}} + \frac{(1 - \tau_t^l) F_{Lf,tLf,t}}{(1 + \tau_t^c)} \Big] - U_{Lf,t} = 0$$
(2.34)

$$\Psi_t \Big[ U_{Li,t} + L_{i,t} U_{Li,tLi,t} + L_{f,t} U_{Lf,tLi,t} \Big] + \phi_t F_{Li,t} -\varphi_t \Big[ \frac{U_{Lf,tLi,t}}{U_{Cf,t}} \Big] - U_{Li,t} = 0$$

$$(2.35)$$

$$\lambda_t \beta^t = \lambda_{t+1} \beta^{t+1} \left[ (1-\delta) + F_{k,(t+1)} \right] -\varphi_{t+1} \beta^{t+1} \left[ \frac{(1-\tau_{t+1}^l) F_{Lf,t,k,(t+1)}}{(1+\tau_{t+1}^c)} \right]$$
(2.36)

### 2.4.1 Optimal Capital Income Tax Formula

### Proposition.

In the presence of the informal labour and goods markets when the consumption of the informal sector goods and income from working in the informal sector can not be taxed such that in the presence of incomplete set of tax instruments the optimal tax rate on capital income is not zero.

### Proof.

Asset optimization equation (2.26) and Euler equation (2.27) of competitive equilibrium in the steady-state are given as follows:

$$R = (1 - \delta) + (1 - \tau^k)F_k \tag{2.37}$$

$$R = \frac{1}{\beta} \tag{2.38}$$

First order condition of capital from second best problem can be written in steadystate as:

$$\lambda = \lambda \beta \left[ (1 - \delta) + F_k \right] - \varphi \beta \left[ \frac{(1 - \tau^l) F_{Lf,k}}{(1 + \tau^c)} \right]$$
(2.39)

The optimal tax formula for capital income in the presence of the informal labour and goods markets is obtained by combining equations (2.37), (2.38) and (2.39) and it is given as following:

$$\tau^{k} = \frac{\varphi}{\lambda F_{k}} \left[ \frac{(1 - \tau^{l}) F_{Lf,k}}{(1 + \tau^{c})} \right]$$
(2.40)

After simplification we get:

$$\tau^{k} = \frac{\varphi}{\lambda} \left[ \frac{(1-\tau^{l})}{(1+\tau^{c})} \frac{(1-\alpha)}{L_{f}} \right]$$
(2.41)

As in Ljungqvist and Sargent (2004),  $\lambda$  is interpreted as the marginal social value of goods and is strictly positive. In the presence of informal labour and goods market  $\varphi$ is positive and  $\frac{1-\tau^l}{1-\tau^c}$  is also greater than zero. It shows that in the presence of informal sector goods and labour markets, as long as the shadow price of the condition (2.31) such that  $\varphi$  is not zero, Chamley result does not hold and optimal tax rate on capital income in steady-state is positive.

## 2.5 Conclusion

This study contributes to the literature of the optimal fiscal policy for an economy that consists of formal and informal sectors. The Ramsey problem is analysed in the context where taxes can not be levied from informal sector. The set of tax instruments is incomplete because the consumption of informal goods and income from working in the informal sector can not be taxed. The introduction of the informal sector in neoclassical growth model implies that the optimal tax rate on capital income is not zero.

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## 2.7 Appendix

### C: Optimal Capital Taxation in One Sector Economy

In this part of the chapter, we assume an economy without informal labour and goods markets, such that we assume the values of  $\gamma$  and  $\gamma_l$  equal to one. In the case of one sector economy without informal labour supply the budget constraint of the household is given as:

$$C_{f,t}(1+\tau_t^c) + K_t - (1-\delta)K_{t-1} + B_t =$$

$$(1-\tau_t^l)W_{f,t}l_{f,t} + (1-\tau_t^k)R_t^kK_{t-1} + R_{t-1}B_{t-1} + Div$$
(2.42)

The household maximizes utility (2.2) subject to budget constraint (2.42):

$$\max_{(C_{f,t},L_{f,t},K_{t})} \sum_{t=0}^{\infty} \beta^{t} \left\{ \left[ lnC_{f,t} - \frac{L_{f,t}^{1+\sigma}}{(1+\sigma)} \right] -\lambda_{t} \left[ C_{f,t}(1+\tau_{t}^{c}) + K_{t} - (1-\delta)K_{t-1} + B_{t} - (1-\tau_{t}^{l})W_{f,t}l_{f,t} - (1-\tau_{t}^{k})R_{t}^{k}K_{t-1} - R_{t-1}B_{t-1} - Div \right] \right\}$$

$$(2.43)$$

The first order conditions of the households utility maximization problem with respect to  $C_f, L_f, B_t$ , and  $K_t$  are:

$$U_{Cf,t} = \frac{1}{C_{f,t}} = \lambda_t (1 + \tau_t^c)$$
(2.44)

$$U_{Lf,t} = L_{f,t}^{\sigma} = \lambda_t (1 - \tau_t^l) W_{f,t}$$
(2.45)

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta R_t \tag{2.46}$$

$$\lambda_t = \lambda_{t+1} \beta \left[ (1 - \delta) + (1 - \tau_{t+1}^k) R_{t+1}^k) \right]$$
(2.47)

Above FOCs are solved simultaneously to obtain the following important results:

$$\frac{U_{Lf,t}}{U_{Cf,t}} = \frac{(1 - \tau_t^l)W_{f,t}}{(1 + \tau_t^c)}$$
(2.48)

$$R = [1 + (1 - \tau_{t+1}^k)R_{t+1}^k - \delta]$$
(2.49)

$$\beta R_t E_t \left[ \frac{C_{f,t} (1 + \tau_{t+1}^c)}{C_{f,t+1} (1 + \tau_t^c)} \right] = 1$$
(2.50)

After substituting equations (2.48) and (2.50) into the budget constraint of household (2.42), following implementability constraint is obtained.

$$\sum_{t=0}^{\infty} \beta \left( U_{Cf,t} C_{f,t} + U_{Lf,t} L_{f,t} \right) = \frac{A_0 U_{Cf,0}}{1 + \tau_0^c}$$
(2.51)

The second best policy can be obtained by maximizing the equation (2.2) subject to the implementability constraint (2.51) and resource constraints (2.13).

$$\max_{(C_{f,t},L_{f,t},K_{t})} \sum_{t=0}^{\infty} \beta^{t} \left\{ U(C_{t},L_{t}) + \Psi_{t} \left[ U_{Cf,t}C_{f,t} + U_{Lf,t}L_{f,t} \right] - \lambda_{t} \left[ C_{f,t} + G_{t} + K_{t} - (1-\delta)K_{t-1} - F(K_{t},L_{f,t}) \right] - \Psi \frac{A_{o}U_{Cf,0}}{1+\tau_{o}^{c}} \right\}$$
(2.52)

The First order condition of the Ramsey problem with respect to  $K_t$  is:

$$\lambda_t \beta^t = \lambda_{t+1} \beta^{t+1} \left[ (1-\delta) + F_{k,(t+1)} \right]$$
(2.53)

Asset optimization equation (2.49) of competitive equilibrium and first order condition of capital from second best problem can be written in steady-state as:

$$R = (1 - \delta) + (1 - \tau^k) F_k \tag{2.54}$$

$$R = [(1 - \delta) + F_k]$$
(2.55)

By combining the equation (2.54) and (2.55), we get  $\tau^k = 0$  in steady state.

## Conclusion

In this part of the chapter we consider the values of  $\gamma$  and  $\gamma_l$  as one in order to find the optimal capital income tax without informal goods and labour markets respectively. We show that optimal capital income tax rate is zero in long run if we shut off the informality.

## Chapter 3

# Fiscal Multipliers and the Choice of Exchange rate in an Open Economy with Informal Sector

### Abstract

This chapter develops an open economy New-Keynesian DSGE model to analyse the role of exchange rate regime in determining the size of fiscal policy multipliers in an economy with large informal sector. The study finds that, in an economy with large informal sector, fixed exchange rate is more effective at increasing the level of aggregate and formal sector outputs. By contrast, the response of the informal sec-

tor output to government expenditure shock is stronger under flexible exchange rate regime. The difference between the multipliers under the two exchange rate regimes is smaller than suggested by the Mundell-Fleming model. An increase in the mobility of labour across the two sectors increases the fiscal policy multiplier under fixed exchange rate. We find that, there is no significant change in the fiscal multipliers when exchange rate is flexible. Therefore, this implies that a decrease in labour market frictions widens the gap between the government expenditure multipliers across the two exchange rate regimes. The study shows that the fiscal policy multipliers reduce with the increase in the size of informal economy under both exchange rate regimes. However, the fall in the size of fiscal multipliers is more pronounced under fixed exchange rate. We also explore that, the consumption tax financed scheme is most effective at increasing the level of formal sector output irrespective of the exchange rate regime. Moreover, among the four different debt financing schemes, the capital tax financed scheme gives the smallest fiscal policy multipliers. The difference between the fiscal multipliers of the most and least effective schemes is larger under fixed exchange rate, whereas in the case of flexible exchange rate regime, this difference is insignificant.

## 3.1 Introduction

The choice of the exchange rate regime, fixed or flexible, has remained a prominent subject in economic literature for years. Theoretical and empirical research has tried to figure out which regime is better in alleviating the financial crises and reducing the volatilities of trade, investment and output growth. The literature has been giving different policy prescriptions related to the optimal exchange rate regime. Tables (3.1) shows that the majority of the economies with large informal sector target the exchange rate<sup>1</sup>. This poses a puzzle in light of the conventional macroeconomic policy advice since Friedman (1953) that flexible exchange rate have superior stabilization properties.

In emerging and developing countries with large informal sector and weak economic as well as political institution, fluctuations in foreign inflows are more easily transmitted into the domestic economy and therefore targeting the exchange rate becomes more favourable than flexible exchange rate regime. Tara Iyer (2017) analyses the appropriate choice of exchange rate regime in economies with dual labour market. According to her findings, in countries with rigid labour market (when labour mobility is low across the sectors or labour market is not well developed) targeting the exchange rate, mitigates the relative wage and price fluctuations and increases the welfare relative to the flexible exchange rate regime. This study provides an argument

<sup>&</sup>lt;sup>1</sup>Sources: IMF Exchange Rates Report (IMF, 2014) and Shadow Economies All over the World, New Estimates for 162 Countries, World Bank Policy Research Working Paper 2010, No. 5356.

in favour of targeting the exchange rate in economies with underdeveloped and rigid segmented labour markets. Weak and fragile financial sector is another important characteristic of developing economies featuring with large informal sector. Philippe Aghion et al. (2006) use the data set of 83 countries and find that in economies with low financial development, flexible exchange rate reduces growth, whereas in financially developed countries, exchange rate regime does not have any significant effect on the productivity growth. Shu Lin and Haichun (2011) also show that countries which are not financially developed, more likely to adopt a fixed exchange rate.

Country	Size of Informal Economy	Exchange Rate Regime
Georgia	64.9	Floating exchange rate
Bolivia	62.3	Soft peg
Zimbabwe	60.6	Soft peg
Nigeria	56.3	Soft peg
Guatemala	54.7	Floating exchange rate
Benin	53.7	Soft peg
Haiti	53.3	Soft peg
Azerbaijan	52.5	Soft peg
		Continued on next page

Table 3.1: Exchange Rate Regimes and the Size of Informal Sector

Country	Size of Informal economy	Exchange Rate Regime
Gabon	52.4	Soft peg
Peru	52.4	Floating exchange rate
Tanzania	52.2	Floating exchange rate
Myanmar	51.4	soft peg
Thailand	50.6	Floating exchange rate
Gambia	46.9	Soft peg
Belize	46.8	Soft peg
Congo, Dem. Rep	46.4	Soft peg
Honduras	46.3	Soft peg
Cambodia	46	Soft peg
Uruguay	45.7	Floating exchange rate
EL Salvador	45.6	Hard peg
Srilanka	45.5	Soft peg
Zambia	45.3	Floating peg
Congo, Rep	45.1	Soft peg
Ukraine	44.8	Floating exchange rate
Belarus	44.5	Soft peg
Angola	44	Soft peg
		Continued on next page

Table 3.1 –continued from previous page

Country	Size of Informal economy	Exchange Rate Regime
Cote d'Ivoire	43.4	Soft peg
Moldova	43.4	Soft peg
Senegal	43.3	Soft peg
Liberia	43.2	Soft peg
Tajikistan	43	Soft peg
Ghana	42.9	Soft peg
Armenia	42.6	Soft peg
Madagascar	42.6	Soft peg
Nicaragua	42.6	Soft peg
Russian federation	42.6	Floating exchange rate
Central African Republic	41.9	Soft peg
Sierra-Leone	41.5	Floating exchange rate
Chad	40.1	Soft peg
Swaziland	40	Soft peg
Guinea	39.9	Soft peg
Niger	39.7	Soft peg
Eritrea	39.3	Soft peg
Philippines	39.3	Floating exchange rate
Continued on next page		

Table 3.1 –continued from previous page

Country	Size of Informal economy	Exchange Rate Regime
Comoros	39.1	Soft peg
Kazakhstan	38.9	Floating exchange rate
Mali	38.5	Soft peg
Uganda	38.7	Floating exchagne rate
Malawi	38.5	Floating exchange rate
Burkina Faso	38.4	Soft peg
Kyrgyz Republic	37.9	Soft peg
Brazil	37.6	Floating exchange rate
Nepal	37.5	Soft peg
Togo	37.3	Soft peg
Mozambique	37.2	Floating exchange rate
Burundi	36.7	Soft peg
Guinea-Bissau	36.4	Soft peg
Rwanda	36.3	Soft peg
Cate-verde		
Tunisia	35.3	Soft peg
Paraguay	34.5	Floating exchange rate
Ethiopia	34.3	
		Continued on next page

Table 3.1 –continued from previous page

Country	Size of Informal economy	Exchange Rate Regime
Soft peg Trinidad and Tobago	34.3	Soft peg
Bosnia and Harzegovina	34.2	Soft peg
Egypt	34.2	Soft peg
Jamaica	34.1	Soft peg
Morocco	34	Soft peg
Paua New Guinea	34	Soft peg
Bangladesh	33.6	Soft peg
Ecuador	33.6	Soft peg
Libya	33.6	Soft peg
Bahamas	33.5	Soft peg
Colombia	33.3	Floating exchange rate
Kenya	33.1	Floating exchange rate
Pakistan	33.1	soft peg
Albania	32.9	Floating exchange rate
Fiji	32.5	Soft peg
Cameron	32.4	Soft peg
Dominican Republic	32.3	Soft peg
Mauritania	32.3	Soft peg
		Continued on next page

Table 3.1 –continued from previous page

Country	Size of Informal economy	Exchange Rate Regime
Suriname	32.2	
Equatorial	31.8	Soft peg
Guyana	31.8	Soft peg
Mexico	31.7	Floating exchange rate
Lebanon	31.6	Soft peg
Malaysia	31.5	soft peg
Venezuela	31.4	Soft peg
Cyprus	31.3	Soft peg
Lesotho	31.3	soft peg
Turkey	31.3	Floating exchange rate
Algeria	30.9	Soft peg
Bulgaria	30.8	hard peg
Solomon Island	30.4	soft peg
Botswana	30.3	Soft peg
Croatia	31.3	Soft peg
Greece	30.3	Floating exchange rate
Lao PDR	30.3	Soft peg
Romania	30.1	Floating exchange rate
Continued on next page		

Table 3.1 –continued from previous page

Country	Size of Informal economy	Exchange Rate Regime
Brunei	29.8	hard peg
Malta	29.8	Floating exchange rate
Italy	29.6	Floating exchange rate
Estonia	28.8	Floating exchange rate
United Arab Emirates	28.7	Soft peg
Yemen	28.3	Soft peg
Namibia	28.1	Soft peg
Lithuania	27.7	Floating exchange rate
Maldives	27.4	Soft peg
Bhutan	26.9	Soft peg
Taiwan	26.9	Floating exchange rate
Costa-Rica	26.7	Soft peg
Poland	26.5	Floating exchange rate
Korea	26.4	Floating exchange rate
Latvia	26	Floating exchange rate
Slovenia	26	Floating exchange rate
South Africa	25.9	Soft peg
Hungry	25.2	Floating exchange rate
Continued on next page		

Table 3.1 –continued from previous page

Country	Size of Informal economy	Exchange Rate Regime
Spain	25.2	Floating exchange rate

Table 3.1 –continued from previous page

According to Mundell-Fleming model, the fiscal multiplier is zero under flexible exchange rate regime and countries with fixed exchange rate are characterized by large fiscal multipliers. Empirical evidence on the effects of the exchange rate regime on fiscal policy multipliers has been virtually non-existent. There are few exceptions, such as the study of Ilzetzki and Vegh (2010) and Karras (2011). Ilzetzki and Vegh (2010) find that the fiscal stimulus is effective under fixed exchange rate and it is completely ineffective when exchange rate is flexible. Karras (2011) uses the data set of 61 countries and find that the fiscal policy is more effective under fixed exchange rate than flexible exchange rate and the difference between the fiscal policy multipliers under the two regimes is substantial. Corsetti et al. (2011), in DSGE framework find the same evidence related to the effectiveness of fiscal policy under the two exchange rate but concludes that the difference between multipliers associated with fixed and flexible exchange rate regimes is smaller than implied by Mundell-Fleming model. Yunfang Hu and Kazuo Mino (2010) find that the introduction of home production plays an important role in altering the impacts of fiscal policy shocks in an open economy. Ester Faia and Christian Merkl (2010) introduce the labour market frictions in an open economy NK-DSGE framework and find that the changes in fiscal policies which reduce these frictions in labour market result in large multipliers. According to these studies the structure of labour market has strong implications for the effectiveness of fiscal policy and changes the fundamental results of Mundell-Fleming model in relation to the role of exchange rate regime in determining the size of fiscal multiplier in an open economy.

## 3.2 Research Objectives

In this study, we analyse the role of flexible and fixed exchange rates in determining the size of fiscal multiplier in the presence of dual labour and goods markets. This study also examines how different financing methods of public debt impact the effectiveness of fiscal stimulus in the presence of informal sector in a small open economy. We also evaluate, how the extent of labour mobility across the sectors and the size of informal goods market alter the results of above analysis. This study considers the following alternative financing schemes:

- all three tax instruments adjust in response to fiscal stimulus;
- only consumption tax adjusts;
- only capital income tax adjusts;

• only labour income tax adjust.

In line with the literature on open-economy, we assume that the Central Bank has following two options:

- Rate of interest is set to target the exchange rate;
- Inflation targeting under fully flexible exchange rate.

## 3.3 Model

Following Gali and Monacelli (2005), Gali (2008), Medina and Soto (2007), Smets and Wouters (2007), Walsh (2010) and Batini et al. (2011b), this section develops a New-Keynesian small open economy dynamic stochastic general equilibrium model for an economy with large informal labour and goods markets. The model consists of a representative household, representative firms of both formal and informal sectors, importers who operate in the formal sector, fiscal and monetary authorities and rest of the world. Formal firms produce tradeable goods and consist of intermediate and final goods producing firms. Intermediate formal firms produce differentiated goods and set prices similarly to the method proposed by Calvo (1983). Final sector firms assemble the intermediate goods and sell these final goods both in domestic and foreign markets. Importers assemble the different varieties of foreign goods into one composite good, distribute these composite goods domestically and set prices in a staggered fashion. Informal firms work in a competitive environment and produce non-tradeable goods. The fiscal authority issues risk-free bonds to finance its expenditures and levies taxes on consumption, labour and capital income of formal sector. It determines a set of Taylor type rules for its instruments and responds to the cyclical changes in formal sector output and debt. This study assumes that the response of government expenditure to debt and to deviation of the formal sector output from steady-state is counter-cyclical, whereas the responses of distortionary taxes to these variables are pro-cyclical. Finally, the rest of the world is assumed as exogenous to the small open economy. ROW imports the formal sector goods from the small open economy and exports intermediate goods to it. Moreover, the representative household of the small open economy also buys the bonds from rest of the world.

### 3.3.1 The Households

The economy is inhabited by identical and infinitely-lived households that can be represented by a household which is made up of continuum of family members. Each member of the household obtains utility by consuming formal, informal and imported goods and supplies labour either to formal or informal sectors of the economy. The consumers are assumed to maximize the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t - hC_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \frac{(L_t)^{1+\sigma_l}}{1+\sigma_l} \right]$$
(3.1)

where,  $\sigma_c$  is the elasticity of intertemporal substitution of consumption, h shows the habit persistence and  $\sigma_l$  is the Frisch elasticity of labour supply. Total labour supply,  $L_t$  is comprised of the supply of labour to both formal  $(L_{F,t})$  and and informal  $(L_{I,t})$  sectors. The household maximizes the objective function with respect to the following budget constraint:

$$P_{t}C_{t} + P_{t}I_{t} + \frac{B_{t}}{R_{t}} + \frac{e_{t}B_{t}^{f}}{\Phi(A_{t})R_{t}^{f}} = B_{t-1} + e_{t}B_{t-1}^{f}$$

$$+ (1 - \tau_{t}^{k})R_{t}^{k}K_{t-1} + W_{t}L_{t} + Div$$
(3.2)

where,  $W_t L_t = (1 - \tau_t^l) W_{F,t} L_{F,t} + W_{I,t} L_{I,t}$ . Total expenditures  $(P_t C_t)$  comprised of the expenditure on informal goods  $(P_{I,t} C_{I,t})$  domestically produced formal goods  $(P_{F,t}^h C_{F,t}^h)$  and imported goods  $(P_{F,t}^f C_{T,t}^f)$ :

$$P_t C_t = P_{I,t} C_{I,t} + (1 + \tau_t^c) (P_{F,t}^h C_{F,t}^h + P_{F,t}^f C_{F,t}^f)$$

The left side of the equation (3.2) shows the total expenditures of household, which include the expenditure on consumption  $(C_t)$ , investment $(I_t)$  and spending on one period domestic  $(B_t)$  and foreign  $(B_t^f)$  bonds.  $R_t$  and  $R_t^f$  are domestic and foreign one period gross interest rates respectively, whereas  $\tau^c$  represents the tax rate on the consumption of formal sector output and imports. The model follows Benigno (2001) and introduce the risk premium to ensure the stationary in foreign bond holding. The function  $\Phi(A_t) = e^{\Phi A_t + \zeta_{b,t}}$  represents the risk premium paid by the domestic resident to invest in foreign bonds, where  $A_t = \frac{e_t B_t^f}{P_t}$  and  $\zeta_{b,t}$  is the shock variable which accounts the uncertainty in risk premium<sup>2</sup>.

The right hand side of the equation shows total disposable earning of household which consists of:

- after tax labour income from formal sector  $(1 \tau_t^l) W_{F,t} L_{F,t}$ , where  $W_{F,t}$  represents the wage rate in formal sector,  $L_{F,t}$  represents the labour supply to formal sector and  $\tau_t^l$  denote the labour income tax rate;
- labour income from informal sector  $(W_{I,t}L_{I,t})$ , where  $W_{I,t}$  represents the wage rate in informal sector and  $L_{I,t}$  represents the labour supply to informal sector;
- after tax return on capital  $(1 \tau_t^k) R_t^k K_{t-1}$ , where  $K_{t-1}$  denotes the physical stock of capital,  $R_t^k$  represents the rental rate of capital and  $\tau_t^k$  is the tax rate on capital income;
- dividends income (Div);
- nominal exchange rate  $(e_t)$ , which is the price of the foreign currency in term of domestic currency.

<sup>&</sup>lt;sup>2</sup>The factor of proportionality  $(A_t)$  depends on the real holding of foreign bonds and it shows that domestic households take this function as given while deciding about the optimal holding of foreign bonds.

Household utility maximization problem is given as:

$$\max_{C_{t},B_{t},B_{t}^{f},L_{t},K_{t}} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{(C_{t}-hC_{t-1})^{1-\sigma_{c}}}{1-\sigma_{c}} - \frac{1}{1+\sigma_{l}} (L_{t})^{1+\sigma_{l}} - \lambda_{t} [P_{t}C_{t} + P_{t}I_{t} + \frac{B_{t}}{R_{t}} - B_{t-1} + \frac{e_{t}B_{t}^{f}}{\Phi(A_{t})R_{t}^{f}} - e_{t}B_{t-1}^{f} - (1-\tau_{t}^{k})R_{t}^{k}K_{t-1} - (W_{t}L_{t} - Div] \right\}$$

$$(3.3)$$

where,  $\lambda_t$  represents the marginal utility of income. The law of motion of the capital is:

$$K_t = (1 - \delta)K_{t-1} + I_t \tag{3.4}$$

The first order conditions of households utility maximization problem with respect to  $C_t, B_t, B_t^f, L_t$ , and  $K_t$  are:

$$\lambda_t = \frac{(C_t - hC_{t-1})^{-\sigma_c}}{P_t}$$
(3.5)

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta R_t \tag{3.6}$$

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta R_t^f \left(\frac{e_{t+1}}{e_t}\right) \Phi(A_t) \tag{3.7}$$

$$L_t^{\sigma_l} = \lambda_t W_t \tag{3.8}$$

$$\beta^{t}\lambda_{t}P_{t} = \beta^{t+1}\lambda_{t+1}P_{t+1}(1-\delta) + \lambda_{t+1}\beta^{t+1}(1-\tau_{t+1}^{k})R_{t+1}^{k}$$
(3.9)

Above FOCs are solved simultaneously to obtain the following important results:

$$\beta R_t E_t \left(\frac{1}{\pi_{t+1}}\right) \left[ \left(\frac{C_t - hC_{t-1}}{C_{t+1} - hC_t}\right)^{\sigma_c} \right] = 1$$
(3.10)

$$E_t(\pi_{t+1})\left[(1-\delta) + (1-\tau_{t+1}^k)r_{t+1}^k\right] = R_t$$
(3.11)

$$\frac{(L_t^{\sigma_l})}{(C_t - hC_{t-1})^{-\sigma_c}} = \frac{W_t}{P_t}$$
(3.12)

Euler equation of consumption (3.10) is derived by combining (3.5) and (3.6). It shows that the loss in marginal utility from consuming less today is exactly the same as an increase in marginal utility from consuming more in some later period. Equation (3.11) is no arbitrage condition between the government bonds and capital and equates the rate of return of two assets. Equation (3.12) equates the marginal rate of substitution between consumption and leisure to real wages. The trade-off between consumption and leisure shows how much income a household is willing to accept to sacrifice one hour of leisure time.

### 3.3.2 Household Consumption Decision

Aggregate consumption  $(C_t)$  is an index of quantities of formal  $(C_{F,t})$  and informal  $(C_{I,t})$  goods consumed and it is given by the following CES aggregator:

$$C_t = \left[ (1 - \gamma_1)^{\frac{1}{\mu_1}} (C_{F,t})^{\frac{\mu_1 - 1}{\mu_1}} + (\gamma_1)^{\frac{1}{\mu_1}} (C_{I,t})^{\frac{\mu_1 - 1}{\mu_1}} \right]^{\frac{\mu_1}{\mu_1 - 1}}$$
(3.13)

where,  $\mu_1$  is the elasticity of substitution between the consumption of formal and informal goods and  $\gamma_1$  measures the size of informal goods in the consumption basket of household. It is also a proxy of the size of informal economy. The optimal allocation of given expenditure gives the following demand functions of formal and informal goods:

$$C_{F,t} = (1 - \gamma_1) \left( (1 + \tau_t^c) \frac{P_{F,t}}{P_t} \right)^{-\mu_1} C_t$$
(3.14)

$$C_{I,t} = (\gamma_1) \left(\frac{P_{I,t}}{P_t}\right)^{-\mu_1} C_t \tag{3.15}$$

where,  $P_{F,t}$  and  $P_{I,t}$  are the relative prices of the formal and informal goods. Consumption of formal goods consists of domestically produced formal goods and imports and it is given by the following CES index:

$$C_{F,t} = \left[ (1 - \gamma_2)^{\frac{1}{\mu_2}} (C_{F,t}^h)^{\frac{\mu_2 - 1}{\mu_2}} + (\gamma_2)^{\frac{1}{\mu_2}} (C_{F,t}^f)^{\frac{\mu_2 - 1}{\mu_2}} \right]^{\frac{\mu_2}{\mu_2 - 1}}$$
(3.16)

where  $\mu_2$ , is the elasticity of substitution between consumption of domestically produced formal goods and imports and  $\gamma_2$  measures the degree of openness. The demand functions of domestic formal and imported good are expressed as:

$$C_{F,t}^{h} = (1 - \gamma_2) \left(\frac{P_{F,t}^{h}}{P_{F,t}}\right)^{-\mu_2} C_t^F$$
(3.17)

$$C_{F,t}^{f} = \gamma_2 \left(\frac{P_{F,t}^{f}}{P_{F,t}}\right)^{-\mu_2} C_t^F \tag{3.18}$$

where,  $P_{F,t}^h$  and  $P_{F,t}^f$  are the relative prices of the domestically produced formal goods and imports. Aggregate and formal goods price indexes are given as follows:

$$P_t = \left[ (1 - \gamma_1) ((1 + \tau_t^c)(P_{F,t}))^{1-\mu_1} + (\gamma_1)(P_{I,t})^{1-\mu_1} \right]^{\frac{1}{1-\mu_1}}$$
(3.19)

$$P_{F,t} = \left[ (1 - \gamma_2) (P_{F,t}^h)^{1-\mu_2} + (\gamma_2) (P_{F,t}^f)^{1-\mu_2} \right]^{\frac{1}{1-\mu_2}}$$
(3.20)

Domestic formal goods are sold domestically and exported to foreign households. We assume that the preferences of foreign consumers are identical as the preferences of domestic households for domestically produced formal goods. We also assume that the law of one price holds in the foreign economy. Foreign demand, the demand for export, for the domestically produced formal goods,  $C_{F,t}^{h*}$  is given as:

$$C_{F,t}^{h*} = \alpha_f \left(\frac{P_{F,t}^h}{e_t P_{F,t}^*}\right)^{-\mu_2} C_t^*$$
(3.21)

where  $P_{F,t}^h$  is the price of the formal goods in the domestic currency,  $C_t^*$  represents the international economic activity which is exogenous and  $P_{F,t}^*$  is the foreign price level of domestic goods converted into domestic currency by nominal exchange rate  $(e_t)$ .

### 3.3.3 Household Labour Supply Decision

Total labour supply  $(L_t)$  is comprised of the supply of labour to both formal  $(L_{F,t})$ and and informal  $(L_{I,t})$  sectors. Members of each household supply labour to both formal and informal sectors, according to the following CES function:

$$L_{t} = \left[\gamma^{-\theta_{l}} (L_{F,t})^{1+\theta_{l}} + (1-\gamma)^{-\theta_{l}} (L_{I,t})^{1+\theta_{l}}\right]^{\frac{1}{1+\theta_{l}}}$$
(3.22)

where  $\gamma$ , is a share parameter that can also be interpreted as the probability that a household member is employed in the formal sector (Conesa, Moreno and GaldonSanchez, 2002).  $\theta_l$ , is the inverse of elasticity of substitution of labour supply between formal and informal sectors. The higher value of  $\theta_l$  accounts for limited labour mobility between the two sectors and represents the labour market segmentation. This CES formulation of labour supply function limits the labour mobility across sectors in economies with dual labour markets. Limited labour mobility across different sectors implies a different wage rate for the household working in formal and informal sectors. This formulation also permits the heterogeneity in hours worked across different sectors. Thus, CES formulation of labour supply ensures that members of a household are willing to supply labour to each sector even if the wages are not equal (Bouakez et al., 2009, Dagher et al., 2012, Giovanni Melina, 2017, Mattesini and Rossi, 2009 and Regassa, 2013). In extreme case, where  $\theta_l$  approaches to 0, the equation (3.22) becomes a linear aggregator implying perfect mobility of labour and equal wages in both sectors. This study considers the CES specification of aggregate labour supply to analyse the effects of different degrees of labour mobility in conditioning the impacts of fiscal policy shocks in the presence of informal sector under different exchange rate regimes. Household conditional labour supply to formal and informal sector are given as:

$$L_{F,t} = \gamma_l \left(\frac{(1-\tau_t^l)W_{F,t}}{W_t}\right)^{\frac{1}{\theta_l}} L_t$$
(3.23)

$$L_{I,t} = (1 - \gamma_l) \left(\frac{W_{I,t}}{W_t}\right)^{\frac{1}{\theta_l}} L_t$$
(3.24)

where,  $W_t$  is the aggregate or composite wage index and it is given as:

$$W_{t} = \left[\gamma_{l}\{(1-\tau_{t}^{l})(W_{F,t})\}^{\frac{1+\theta_{l}}{\theta_{l}}} + (1-\gamma_{l})(W_{I,t})^{\frac{1+\theta_{l}}{\theta_{l}}}\right]^{\frac{\sigma_{l}}{1+\theta_{l}}}$$
(3.25)

### 3.3.4 Production of Formal Sector

In the formal sector there are two types of firm. Perfectly competitive final goods producers or retailers and monopolistically competitive intermediate firms.

#### Formal Sector Retailers

Retailers are the net buyers of different varieties of formal intermediate good,  $Y_{F,t(j)}^h$ and combine them into one single consumption good,  $Y_{F,t}^h$  by using the CES technology.

$$Y_{F,t}^{h} = \left[\int_{0}^{1} Y_{F,t(j)}^{h} \frac{\nu_{1}-1}{\nu_{1}} dx\right]^{\frac{\nu_{1}}{\nu_{1}-1}}$$
(3.26)

where,  $\nu_1$  is the elasticity of substitution among different varieties of formal intermediate goods. The Profit function of the formal goods retailers is given as follows:

$$\Pi_{F,t}^{h} = P_{F,t}^{h} Y_{F,t}^{h} - \int_{0}^{1} P_{F,t(j)}^{h} Y_{F,t(j)}^{h} dx$$
(3.27)

where,  $P_{F,t(j)}^h$  is the price of intermediate variety,  $Y_{F,t(j)}^h$ . The demand function for the variety j, of formal intermediate goods is given as:

$$Y_{F,t(j)}^{h} = \left(\frac{P_{F,t(j)}^{h}}{P_{F,t}^{h}}\right)^{-\nu_{1}} Y_{F,t}^{h}$$
(3.28)

The price index is represented by the following equation:

$$P_{F,t}^{h} = \left[\int_{0}^{1} P_{F,t(j)}^{h(1-\nu_{1})}\right]^{\frac{1}{1-\nu_{1}}}$$
(3.29)

#### Production of Intermediate Goods in the Formal Sector

In the formal sector, monopolistic competitive firms have to decide about their demand of capital and labour for given wages and rent. Formal sector intermediate producers employ the following Cobb Douglas production function:

$$Y_{F,t}^{h} = \zeta_{F,t} K_{t}^{\alpha} L_{F,t}^{1-\alpha}$$
(3.30)

where,  $\zeta_{F,t}$  is the exogenous level of technology,  $L_{F,t}$  is the quantity of formal labour and  $K_t$  is the quantity of physical capital used in the production process. The cost minimization problem yields the following results:

$$\min_{W_{F,t},R_t^k} W_{F,t} L_{F,t} - R_t^k K_t - \lambda_t P_{F,t}^h(\zeta_{F,t} K_t^{\alpha} L_{F,t}^{1-\alpha})$$
(3.31)

Wages in the formal sector are defined as:

$$W_{F,t} = M C^{h}_{F,t} P^{h}_{F,t} (1-\alpha) \frac{Y^{h}_{F,t}}{L_{F,t}}$$
(3.32)

The rental rate of capital is defined as:

$$R_{t}^{k} = M C_{F,t}^{h} P_{F,t}^{h} \frac{\alpha Y_{F,t}^{h}}{K_{t}}$$
(3.33)

The marginal cost of the formal sector production is then:

$$MC_{F,t}^{h} = \frac{1}{\zeta_{F,t}} \left(\frac{R_{t}^{k}}{\alpha}\right)^{\alpha} \left(\frac{W_{F,t}}{1-\alpha}\right)^{1-\alpha}$$
(3.34)

The marginal cost is positively associated with wages and rent of the capital and a positive total factor productivity shock decreases the marginal costs.

## 3.3.5 Price setting of Formal Sector

Formal sector intermediate firms maximize the profit by selling  $Y_{F,t(j)}^h$  at price  $P_{F,t(j)}^h$  subject to the demand function given in equation (3.28):

$$\pi_t = P^h_{F,t(j)} Y^h_{F,t(j)} - M C^h_{F,t} Y^h_{F,t(j)}$$
(3.35)

$$\pi_{t} = \left[ \left( P_{F,t(j)}^{h} - MC_{F,t}^{h} \right) \left( \frac{P_{F,t(j)}^{h}}{P_{F,t}^{h}} \right)^{-\nu_{1}} \right] Y_{F,t}^{h}$$
(3.36)

Intermediate goods producers set prices by following the mechanism presented by Calvo(1983). A fraction of the firm  $\omega_h$  can not re-optimize price each period and follow the partial indexation rule such that:

$$P_{F,t(j)}^{h} = \left(\frac{P_{F,t-1}^{h}}{P_{F,t-2}^{h}}\right)^{\gamma_{p}} P_{F,t-1(j)}^{h}$$

The optimal price  $P_{F,t(j)}^{h*}$  for rest of the firms  $(1 - \omega_h)$  is obtained by maximizing the objective:

$$\max_{P_{F,t(j)}^{h*}} E_t \sum_{k=0}^{\infty} (\beta \omega_h)^k \left[ (P_{F,t+k(j)}^{h*} - MC_{F,t+k}^h) \left( \frac{P_{F,t+k(j)}^{h*}}{P_{F,t+k}^h} \right)^{-\nu_1} Y_{F,t+k}^h \right]$$
(3.37)

subject to the demand function (3.28), the price index (3.29) can be written as:

$$P_{F,t}^{h} = \left[ (1 - \omega_{h}) (P_{F,t}^{h*})^{(1-\nu_{1})} + \omega_{h} \left( \left( \frac{P_{F,t-1}^{h}}{P_{F,t-2}^{h}} \right)^{\gamma_{p}} P_{F,t-1}^{h} \right)^{(1-\nu_{1})} \right]^{\frac{1}{1-\nu_{1}}}$$
(3.38)

### 3.3.6 Imports

Importers are the net buyers of different varieties of foreign goods  $(Y_{F,t(j)}^f)$  and combine these different varieties into one single consumption good  $(Y_{F,t}^f)$ , by using the CES technology:

$$Y_{F,t}^{f} = \left[\int_{0}^{1} Y_{F,t(j)}^{f} \frac{\nu_{2}-1}{\nu_{2}} dx\right]^{\frac{\nu_{2}}{\nu_{2}-1}}$$
(3.39)

where  $\nu_2$ , is the elasticity of substitution among different varieties of foreign goods. The profit function of importers is given as:

$$\Pi_{F,t}^{f} = P_{F,t}^{h} Y_{F,t}^{f} - \int_{0}^{1} P_{F,t(j)}^{f} Y_{F,t(j)}^{f} dx$$
(3.40)

The demand function for the variety j, of imported goods is:

$$Y_{F,t(j)}^{f} = \left(\frac{P_{F,t(j)}^{f}}{P_{F,t}^{f}}\right)^{-\nu_{2}} Y_{F,t}^{f}$$
(3.41)

The price index  $P_{F,t}^{f}$  is given by the following equation:

$$P_{F,t}^{f} = \left[\int_{0}^{1} P_{F,t(j)}^{f(1-\nu_{2})}\right]^{\frac{1}{1-\nu_{2}}}$$
(3.42)

Importers buy the foreign goods at given world market prices. We assume that the law of one price holds at the border for importers at wholesale level as in Gali and Monacelli (2005). Each Importer assembles imported goods to produce a unique brand and sell these differentiated products in the domestic market as final goods. The domestic market for the imports is characterized by monopolistic competition. The price-setting behaviour of importers is similar to the domestic intermediate firms. The only difference is the marginal cost of the imported goods, which is given by  $e_t P_{F,t}^*$ , where,  $e_t$  in the nominal exchange rate and  $P_{F,t}^*$  is the given world price of imports. A fraction of the importers  $\omega_f$  can not re-optimize prices each period and follow the partial indexation rule such that:

$$P_{F,t(j)}^{f} = \left(\frac{P_{F,t-1}^{f}}{P_{F,t-2}^{f}}\right)^{\gamma_{i}} P_{F,t-1(j)}^{f}$$

The optimal prices  $P_{F,t(j)}^{f*}$  for rest of the firms is obtained by maximizing the objective:

$$\max_{P_{F,t(j)}^{f*}} E_t \sum_{k=0}^{\infty} (\beta \omega_f)^k \left[ (P_{F,t+k(j)}^{f*} - e_{t+k} P_{F,t+k}^*) \left( \frac{P_{F,t+k(j)}^{f*}}{P_{F,t+k}^f} \right)^{-\nu_2} Y_{F,t+k}^f \right]$$
(3.43)

subject to the demand function (3.41), the price index for imports (3.42) can be written as:

$$P_{F,t}^{f} = \left[ (1 - \omega_{f}) (P_{F,t}^{f*})^{(1-\nu_{2})} + \omega_{f} \left( \left( \frac{P_{F,t-1}^{f}}{P_{F,t-2}^{h}} \right)^{\gamma_{i}} P_{F,t-1}^{f} \right)^{(1-\nu_{2})} \right]^{\frac{1}{1-\nu_{2}}}$$
(3.44)

## 3.3.7 Production of Informal Sector

Informal sector firms produce non-tradeable informal goods by using only the labour as an input at given wages. Informal sector firms employ the following production function:

$$Y_{I,t} = L_{I,t} \tag{3.45}$$

The instantaneous profit function of the informal sector firms is given by:

$$\Pi_{I,t} = P_{I,t}L_{I,t} - W_{I,t}L_{I,t}$$

Wages in the informal sector are then:

$$W_{I,t} = P_{I,t} \tag{3.46}$$

## 3.3.8 Rest of the world

#### The Term of Trade and Real Exchange Rate

The bilateral terms of trade is defined as the price of country's i's goods in term of domestic goods:

$$Z_{i,t} = \frac{P_{i,t}}{P_{F,t}^h} \tag{3.47}$$

The effective terms of trade are then:

$$Z_t = \frac{P_{F,t}^f}{P_{F,t}^h} \tag{3.48}$$

The effective terms of trade in log-linearisation forms is given as:

$$z_t = p_{F,t}^f - p_{F,t}^h (3.49)$$

CPI Inflation is defined as the weighted sum of formal and informal inflation:

$$\pi_t = (1 - \gamma_1)\pi_{F,t} + (\gamma_1)\pi_{I,t} \tag{3.50}$$

where,

$$\pi_t = p_t - p_{t-1}. \tag{3.51}$$

Formal inflation is the weighted sum of domestic and foreign formal inflation:

$$\pi_{F,t} = (1 - \gamma_2)\pi_{F,t}^h + (\gamma_2)\pi_{F,t}^f \tag{3.52}$$

where,

$$\pi_{F,t} = p_{F,t} - p_{F,t-1}. \tag{3.53}$$

Log-linearisation of tradeable price index (3.20) gives:

$$p_{F,t} = (1 - \gamma_2) p_{F,t}^h + \gamma_2 p_{F,t}^f.$$
(3.54)

Combining equation (3.49) and (3.54) we get:

$$p_{F,t} = p_{F,t}^h + \gamma_2 z_t. aga{3.55}$$

Substitution of equation (3.55) in the equation of formal goods inflation (3.53) then gives:

$$\pi_{F,t} = (p_{F,t}^h + \gamma_2 z_t) - (p_{F,t-1}^h + \gamma_2 z_{t-1})$$
  
$$\pi_{F,t} = \pi_{F,t}^h + \gamma_2 \Delta z_t.$$
 (3.56)

Combining equations (3.50) and (3.56) gives the following results:

$$\pi_t = (1 - \gamma_1)(\pi_{F,t}^h + \gamma_2 \Delta z_t) + \gamma_1 \pi_{I,t}$$
(3.57)

$$\pi_t = (1 - \gamma_1) \pi_{F,t}^h + \gamma_1 \pi_{I,t} + (1 - \gamma_1) \gamma_2 \Delta z_t.$$
(3.58)

According the to equation (3.58) domestic inflation is linked to the CPI. The difference between the two measures of inflation is proportional to the change in terms of trade. This proportion is given by the share of formal goods and index of openness. This study assumes that the law of one price holds which implies that the price of country's i's goods is:

$$P_{i,t} = e_{i,t} P_{i,t}^i \tag{3.59}$$

where  $e_{i,t}$  is the price of country's *i*'s currency in term of domestic currency and  $P_{i,t}^i$ is the price of country's *i*'s goods in term of its own currency. Log-linearisation of above equation around symmetric steady-state gives:

$$p_{F,t}^f = e_t + p_t^* \tag{3.60}$$

Combining equations (3.49) and equation (3.60) we get:

$$z_t = e_t + p_t^* - p_{F,t}^h \tag{3.61}$$

The real exchange rate is defined as the ratio between the CPI of two countries expressed in domestic currency:

$$S_i, t = \frac{e_{i,t} P_{i,t}^i}{P_t}$$

After log-linearisation of above expression we get:

$$s_t = e_t + p_t^* - p_t (3.62)$$

From equations (3.61) and (3.62) we obtain:

$$s_t = z_t + p_{F,t}^h - p_t (3.63)$$

and combining (3.55) and (3.63) then gives:

$$s_t = z_t - p_t + p_{F,t} - \gamma_2 z_t \tag{3.64}$$

Log-linearisation of CPI index gives:

$$p_t = (1 - \gamma_1) p_{F,t} + \gamma_1 p_{I,t} \tag{3.65}$$

After substituting the value of  $p_t$  from equation (3.65) in equation (3.64) we get:

$$s_t = (1 - \gamma_2)z_t + p_{F,t} - [(1 - \gamma_1)p_{F,t} + \gamma_1 p_{I,t}]$$
(3.66)

 $\mathrm{so},$ 

$$s_t = (1 - \gamma_2)z_t + \gamma_1(p_{F,t} - p_{I,t})$$
(3.67)

#### **Uncovered Interest Parity**

From equations (3.6) and (3.7) we get:

$$\frac{R_{t+1}}{R_{t+1}^f} = \frac{e_t}{e_{t+1}} \Phi(A_t)$$
(3.68)

Log-linearisation of equation (3.68) gives:

$$R_t = R_t^f + E_t s_{t+1} - s_t + E_t \pi_{t+1} - E_t \pi_{t+1}^f - \phi A_t + \zeta_{b,t}$$
(3.69)

where  $s_t$  is the real exchange rate and  $\zeta_{b,t}$  is the shock variable which accounts the uncertainty in risk premium and follows a first order auto-regressive process.

#### 3.3.9 Government Behaviour

The government budget constraint (3.70) shows that the government expenditure  $(G_t)$  and the interest payment of debt  $(R_{t-1}) B_{t-1}$  is equal to the sum of consumption tax on formal consumption  $(\tau_t^c C_{F,t})$ , labour income tax of formal sector  $(\tau_t^l W_{F,t} L_{F,t})$ , capital income tax  $(\tau_t^k r_t^k K_{t-1})$  and purchase of new debt  $(B_t)$ .

$$(R_{t-1}) B_{t-1} + P_{F,t}^h G_t = \tau_t^c P_{F,t}^h C_{F,t} + \tau_t^l W_{F,t} L_{F,t} + \tau_t^k r_t^k K_{t-1} + B_t$$
(3.70)

This study assumes that the response of government expenditure to debt and to the deviation of formal sector output from steady-state is counter-cyclical, whereas the responses of three distortionary taxes to them are pro-cyclical. Setting of the fiscal instruments in this way keeps the dynamics of debt under control and acts as automatic stabilizer. These fiscal policy rules are set by following Lorenzo et al. (2009) and Leeper et al. (2010a, 2010b). Fiscal policy rules for all fiscal instruments are given as follows:

Government expenditure rule:

$$\hat{G}_t = -\psi_{b,g}\hat{B}_{t-1} - \psi_{yf,g}\hat{Y}^h_{F,t} + \zeta_{g,t}$$
(3.71)

Consumption tax rate rule:

$$\hat{\tau}_{t}^{c} = \psi_{b,\tau^{c}} \hat{B}_{t-1} + \psi_{yf,\tau^{c}} \hat{Y}_{F,t}^{h} + \zeta_{\tau^{c},t}$$
(3.72)

Labour income tax rate rule:

$$\hat{\tau}_{t}^{l} = \psi_{b,\tau^{l}} B_{t-1} + \psi_{yf,\tau^{l}} Y_{F,t}^{h} + \zeta_{\tau^{l},t}$$
(3.73)

Capital income tax rate rule:

$$\hat{\tau}_{t}^{k} = \psi_{b,\tau^{k}} B_{t-1} + \psi_{yf,\tau^{k}} \hat{Y}_{F,t}^{h} + \zeta_{\tau^{k},t}$$
(3.74)

where  $\zeta_{g,t}$ ,  $\zeta_{\tau^c,t}$ ,  $\zeta_{\tau^l,t}$  and  $\zeta_{\tau^k,t}$  are the fiscal shocks which affect the spending and revenue sides of the government. These fiscal policy shocks constitute an unexpected change in the policy and follow the first order auto-regressive process.

$$\zeta_{g,t} = \rho_g \zeta_{g,t-1} + \epsilon_{g,t} \tag{3.75}$$

$$\zeta_{\tau^c,t} = \rho_{\tau^c} \zeta_{\tau^c,t-1} + \epsilon_{\tau^c,t} \tag{3.76}$$

$$\zeta_{\tau^k,t} = \rho_{\tau^k} \zeta_{\tau^k,t-1} + \epsilon_{\tau^k,t} \tag{3.77}$$

$$\zeta_{\tau^l,t} = \rho_{\tau^l} \zeta_{\tau^l,t-1} + \epsilon_{\tau^l,t} \tag{3.78}$$

## 3.3.10 Monetary Policy

Following the literature on open-economy (Ghironi, 2000 and Benigno and Ghironi, 2007), this study assumes that the Central Bank in economies with large informal sector has following two options for setting the rate of interest:

- inflation targeting under fully flexible exchange rate regime;
- the rate of interest is set to target the nominal exchange rate.

Under flexible exchange rate regime, the central bank set the nominal rate of interest following a Taylor-type rule. According to this specification, the exchange rate is free to move and adjusts according to the equilibrium conditions implied by the model.

$$\hat{R}_{t} = \rho \hat{R}_{t-1} + (1-\rho)\rho_{\pi}\hat{\pi}_{F,t} + \epsilon_{\pi,t}$$
(3.79)

Following Ghironi (2000), Benigno and Ghironi (2007), Ester Faia and Christian Merkl (2010), Corsetti et al. (2011) and Tara Iyer (2017), a feasible interest rate policy which ensures the exchange rate targeting and equilibrium determinacy is given by:

$$\hat{R}_{t} = \rho \hat{R}_{t-1} + (1-\rho)\rho_{e}\hat{e}_{t} + \epsilon_{e,t}$$
(3.80)

In equations (3.79) and (3.80)  $\rho_{\pi}$  and  $\rho_e$  represent the relative weight of inflation and nominal exchange rate respectively in monetary policy, whereas  $\epsilon_{\pi,t}$  and  $\epsilon_{e,t}$  denote an *i.i.d.* normal error terms on two interest rate rules.

#### 3.3.11 Equilibrium

Formal sector resource constraint is given as:

$$Y_{F,t}^h = C_{F,t}^h + C_{F,t}^{h*} + G_t + I_t aga{3.81}$$

where  $C_{F,t}^h$  is the consumption of domestically produced formal goods, consumed by domestic consumer,  $C_{F,t}^{h*}$  represents the foreign consumption of domestic formal tradeable goods,  $I_t$  represent investment and  $G_t$  shows the government expenditures. All the output of informal sector is consumed, so the resource constraint of the informal sector is given as:

$$Y_{I,t} = C_{I,t} \tag{3.82}$$

Aggregate output is the sum of the formal and informal sectors outputs and it is given by the following equation:

$$Y_t = Y_{F,t}^h P_{F,t}^h + Y_{I,t} P_{I,t}$$
(3.83)

The position of the domestic net foreign assets is given by:

$$\frac{e_t B_t^f}{R_t^f \Phi(A_t)} - e_t B_{t-1}^f = P_{F,t}^h C_{F,t}^{h*} - e_t P_{F,t}^* C_{F,t}^f$$
(3.84)

According the the equation (3.84), the change in the position of net foreign bond holding is equal to the net profit from international trade.

# 3.4 Calibration

All values of the parameters used in this analysis have already been calibrated and most of these are taken from the literature on emerging and developing countries features with large size of informal market. The value of the discount factor ( $\beta$ ) is set as 0.991, as given by Ahmad, et al. (2012). They estimated the discount factor for the Pakistan economy by using the data of change in CPI and return on government bonds for the period of 1981 to 2011. Following the Fagan and Messina (2009) and Smets and Wouters (2007), the Frisch elasticity of labour supply  $(\sigma_l)$  is set at 1.5. The value of the degree of habit persistence (h) is set as 0.36 as in Haider and Khan (2008). This is also in the line of the finding of Lubik and Schorfeide (2005). According to them the degree of habit persistence is quite low in developing countries than advanced and developed economies. The inverse of the elasticity of intertemporal substitution in consumption  $(\sigma_c)$  is taken from Kose and Reizman (2001) and set as 2.61. The elasticity of substitution between formal and informal goods  $(\mu_1)$  and share of the informal sector goods in the total consumption  $(\gamma_1)$  are also picked from Ahmad et al. (2012) and Khan and Khan (2011). These two parameters govern the distribution of the formal and informal consumption of goods and taken as 0.70 and 0.45 respectively.  $(\gamma_1)$  is also used as the proxy of the size of informal sector. The degree of openness  $(\gamma_2)$  is set at 0.3 which is the average import to GDP ratio for developing countries. Following Ahmad et al. (2012) the Armington (1963) elasticity of substitution between domestically produced formal goods and imports  $(\mu_2)$  is set at 1.01. The value of  $(\mu_2)$  is set at 1.01 because in developing countries, home goods are relatively lower in quality than imports, implying that the domestically produced goods are not a good substitute of imports (See, Hummels and Klenow, 2005, Henn et al. 2013). For developed world this parameter has been estimated well above 1 (Feenstra et al. 2014). The value of the inverse of the elasticity of substitution between formal and informal labour supply  $(\theta_l)$  is considered as 2, following Ahmad et al. (2012). They adopt the methodology given in Psacharopoulos and Hinchlise (1972) to estimate  $\theta_l$ . The probability of getting employed in the formal sector  $(\gamma)$ is considered as 0.29. This is also consistent with the estimates of Choudhri and Malik (2012). Output elasticity of private capital  $(\alpha)$  is assumed as 0.46 and it is taken from Bukhari and Khan (2008). The private capital depreciation  $(\delta)$  is set at 0.03 following Bukhari and Khan (2008), Haider and Khan (2008) and Ahmad et al. (2012).

According to a survey conducted by Choudhri et al. (2012), prices in developing countries are more flexible than in developed countries. Following Choudhri et al. (2012), Calvo degree of price stickiness for domestically produced formal sector output ( $\omega_h$ ) is assumed as 0.24. This reflects the low degree of price stickiness in developing countries. The value of price stickiness for imported goods ( $\omega_f$ ) is fixed at 0.70. In the line of Haider and Khan (2008) and de-Castro, et al. (2011), the degree of price indexation for domestic formal sector output ( $\gamma_p$ ) and imported goods  $(\gamma_i)$  are set at 0.65 and 0.45 respectively. The elasticity of substitution among the different varieties of intermediate formal goods  $(\nu_1)$  is calibrated at 6. We assume strict inflation and exchange rate targeting and set  $\rho_{\pi}$  and  $\rho_e$  equal to 1.5. The steady-state consumption, labour and capital income taxes  $(\tau^c, \tau^l \text{ and } \tau^k)$  are set at 0.17, 0.20 and 0.30 respectively. The persistence of shock  $(\rho_x)$  in auto-regressive process  $\zeta_t = \rho_x \zeta_{t-1} + \epsilon_{\zeta,t}$  is assumed as 0.9 which is consistent for developing countries following Aguiar and Gopinath (2007). The summary of all calibrated values of the structural and policy rule parameters are given in tables (3.2) and (3.3) respectively.

Table $3.2$ :	Structural	parameters
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Parameter	Symbol	Value	Source
Discount factor	β	0.991	Ahmad et al. (2012)
Habit persistence	h	0.36	Haider and Khan (2008) and Lubik and Schorfeide (2005)
Capital share in production	α	0.46	Bukhari and Khan (2008) and Haider and Khan (2008)
Depreciation rate of the private capital	δ	0.03	Bukhari and Khan (2008) and Haider and Khan (2008)
Inverse Frisch elasticity of labour	$\sigma_l$	1.5	Fagan and Messina (2009) and Smets and Wouters (2007)
Inverse of elasticity of intertemporal sub- stitution in consumption	σc	2.61	Kose and Reizman (2001)
Share of the informal consumption in total consumption	γ1	0.45	Ahmad et al. (2012) and Khan and Khan (2011)
Share of the imports in formal consumption	$\gamma_2$	0.3	Ahmad et al. (2012)
Elasticity of substitution between the con- sumption of formal and informal goods	$\mu_1$	0.7	Ahmad et al. (2012) and Khan and Khan (2011)
Elasticity of substitution between domestic and imported consumption	μ2	1.01	Ahmad et al. (2012)
Elasticity of substitution among the differ- ent varieties of formal intermediate goods	$\nu_1$	6	Haider and Khan (2008) and Ahmad et al. (2012)
Probability that a household member is employed in formal sector	γ	0.29	Ahmad et al. (2012) and Choudhri and Ma- lik (2012)
Inverse of the elasticity of substitution be- tween formal and informal sector labour supply	θι	2	Ahmad et al. (2012)
Price indexation for domestic firms	$\gamma_p$	0.65	Haider and Khan (2008) and de-Castro et al. (2011)
Degree of price stickiness in domestic for- mal sector	$\omega_h$	0.24	Choudhri et al. (2012)
Price indexation for importing firms	$\gamma_i$	0.45	Haider and Khan (2008) and de-Castro et al. (2011),
Calvo degree of price rigidity for imported goods	ω <sub>f</sub>	0.70	Haider and Khan (2008) and de-Castro et al. (2011)

Parameter	Symbol	Value	Source
Relative weight assigned to inflation in Taylor rule	$ ho_{\pi_f}$	1.5	Corsetti et al. (2011) and Tara Iyer (2017)
Relative weight assigned to nominal ex- change rate in monetary policy	$ ho_e$	1.5	Corsetti et al. (2011) and Tara Iyer (2017)
Weight of interest rate inertia in Taylor rule	ρ	0.63	Ahmad, et al. (2012)
Persistent of the shock in auto-regressive process	$ ho_x$	0.9	Aguiar and Gopinath (2007)
Response of fiscal instruments to debt	$\psi_b$	0.1	assumed*
Response of fiscal instruments to devia- tion of formal sector output from steady- state	$\psi_{y,f}$	0.1	assumed

Table 3.3: Policy rules and shock process parameters

\* For some of the model parameters for which references are not available for developing economies with large informal sector, unavoidably, we assign the values based on subjective judgment and pair these with sensitivity analysis.

# 3.5 Results

# 3.5.1 The Impact of Fiscal Policy Shocks and Sectoral Labour Mobility

Figure (3.1) compares the impacts of government expenditure shocks on aggregate output (Y), formal (YF) and informal (YI) sectors outputs, total consumption (C), consumption of domestically produced formal goods (CFH), informal (CI) consump-

tion, aggregate labour supply (L), labour supply to formal (LF) and informal (LI) sectors, composite wages (W), formal wages (WF), informal wages (WI), nominal exchange rate (E), real exchange rate (Q), prices of domestically produced formal goods (FPH) and prices of informal goods (IP) for two identical economies with different exchange rate regimes<sup>3</sup>. The responses of all variables are measured in percentage deviation from steady-state. The responses of aggregate and formal sector outputs to government expenditure shock remain above the steady-state for only seven and two quarters respectively under the both exchange rate regimes. The responses of informal sector output remain above the steady-state for longer period of time before reverting to steady-state. Our results are in line of the conventional Mundell-Fleming model in relative terms. In the case of fixed exchange rate regimes, the responses of aggregate and formal sector outputs are stronger and remain above the corresponding responses under flexible exchange rate regimes. The output of informal sector responds strongly to a government spending shock under flexible exchange rate regime. The Table (3.4) shows government expenditure impact and present value multipliers associated with fixed and floating exchange rate regimes. In the case of fixed exchange rate, the impact multiplier is 0.78, while the size of government impact multiplier is 0.37 under flexible exchange rate regime. The government expenditure impact multiplier is larger when exchange rate is fixed but the difference between the size of impact multipliers under both exchange rates regimes is smaller as compared

<sup>&</sup>lt;sup>3</sup>Solid black line relates to flexible exchange rate, whereas dotted red line is for fixed exchange rate regime.

to the finding of traditional Mundell-Fleming model. According to Mundell-Fleming model for an open economy, government expenditure multiplier is larger than one and zero under fixed and flexible exchange rate regimes respectively. Our results are confirming the finding of the Mundell-fleming model in relative terms. Corsetti et al. (2011) finds the similar evidence of a small difference in government expenditure multipliers under fixed and floating exchange rate regimes in an economy with homogeneous labour and goods markets.

Figure (3.2) depicts the impacts of government expenditure shocks under different exchange rate regimes with higher labour mobility across formal and informal sectors. As a result of positive government expenditure shock, the impulses responses of aggregate and formal sector outputs become stronger with the increase in labour mobility under fixed exchange rate regime. The initial responses of the informal sector output to fiscal expansion is weak but they increases afterword under both exchange rate regimes. The impact of government expenditure on the output of informal sector is stronger under flexible exchange rate. Table (3.5) gives the magnitudes of government expenditure impact and present value multipliers associated with both exchange rate regimes for an economy with higher labour mobility<sup>4</sup>. The government expenditure impact multiplier for aggregate level of output becomes close to 1 under fixed exchange rate when labour mobility is high. In the case of flexible exchange rate regime, the impact multiplier remains the same but the present value multipliers

<sup>&</sup>lt;sup>4</sup>The value of  $\theta_l$  is assumed as 0.4 to analyse the effects of increase in labour mobility. In benchmark model  $\theta_l$  is set at 2.

for second, fourth and twelfth quarters decrease slightly as a result of higher labour mobility. According to the Ester et al. (2010), fiscal policy that decreases labour market frictions results in a large fiscal policy multiplier. In this study we find that, in the economies with fixed exchange rate, fiscal policy becomes more effective if labour market is relatively developed or friction in the labour market decreases. The increase in labour mobility does not have any significant effect on the fiscal policy multiplier under flexible exchange rate regime. This shows that the an increase in the labour mobility, widen the gap between government expenditure multipliers under the two exchange rate regimes.

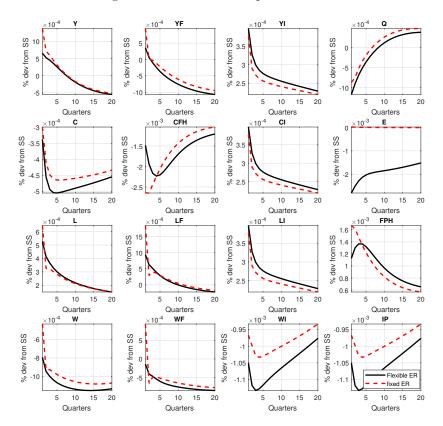


Figure 3.1: Government expenditure shock

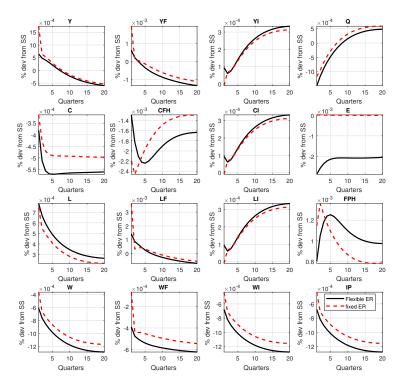


Figure 3.2: Impact of government expenditure shock with higher labour mobility

Variable	1 quarter	2 quarter	4 quarter	12 quarter
Fixed Exchange Rate	0.78	0.45	0.38	-0.59
Flexible Exchange rate	0.37	0.33	0.30	-0.70

Table 3.4: Fiscal policy present value multipliers

Table 3.5: Fiscal policy present value multipliers with higher labour mobility

Variable	1 quarter	2 quarter	4 quarter	12 quarter
Fixed Exchange Rate	0.99	0.41	0.36	-0.59
Flexible Exchange rate	0.37	0.30	0.26	-0.86

## 3.5.2 Fiscal policy Experiment

This section compares the impacts of different debt financing methods on the effectiveness of fiscal stimulus under alternative exchange rate regimes in an economy with dual labour and goods markets. In this policy experiment we consider the four alternative debt financing schemes: (a) one in which all taxes adjust to finance the debt; (b) only labour tax adjusts to finance the debt (labour tax financing scheme); (c) only consumption tax adjusts to stabilize the debt (consumption tax financing schemes); (d) the one in which only capital tax adjusts (capital tax financing scheme). The parameters values of all four different debt financing schemes are set as follows:

(a)
$$\psi_{yf,g} = \psi_{yf,\tau^{l}} = \psi_{yf,\tau^{k}} = \psi_{yf,\tau^{c}} = \psi_{b,g} = \psi_{b,\tau^{l}} = \psi_{b,\tau^{k}} = \psi_{b,\tau^{c}} = 0.1;$$
  
(b)  $\psi_{yf,g} = \psi_{yf,\tau^{l}} = \psi_{b,g} = \psi_{b,\tau^{l}} = 0.1, \psi_{yf,\tau^{k}} = \psi_{yf,\tau^{c}} = \psi_{b,\tau^{k}} = \psi_{b,\tau^{c}} = 0;$   
(c) $\psi_{yf,g} = \psi_{yf,\tau^{c}} = \psi_{b,g} = \psi_{b,\tau^{c}} = 0.1, \psi_{yf,\tau^{l}} = \psi_{yf,\tau^{k}} = \psi_{b,\tau^{l}} = \psi_{b,\tau^{k}} = 0;$   
(d) $\psi_{yf,g} = \psi_{yf,\tau^{k}} = \psi_{b,g} = \psi_{b,\tau^{k}} = 0.1, \psi_{yf,\tau^{l}} = \psi_{yf,\tau^{c}} = \psi_{b,\tau^{l}} = \psi_{b,\tau^{c}} = 0.$ 

Figures (3.3) and (3.4) show the impulse responses of macroeconomic variables to government expenditure shocks with different debt financing schemes under fixed and flexible exchange rate regimes respectively<sup>5</sup>. The response of the formal sector output to fiscal expansion is stronger when debt is financed through consumption tax under the both exchange rates. On the other hand adjustment to capital tax results in the weakest responses of aggregate and formal sectors outputs to fiscal expansion. The informal sector responds strongly to fiscal stimulus when all tax instruments adjust to finance the debt. The initial impact of the labour tax financing scheme on informal sector output is slightly stronger than capital and consumption tax financing schemes.

Tables (3.6) and (3.7) give the impact and present value multipliers for fixed and flexible exchange rates respectively under different financing schemes. The impact multipliers for consumption tax financed spending scheme are largest, with the magnitudes of 0.85 and 0.38 under fixed and flexible exchange rates respectively. In the case of fixed exchange rate regime, impact multiplier of capital tax financed scheme

<sup>&</sup>lt;sup>5</sup>Green line shows all tax financed scheme, dotted red line is for consumption tax financed scheme, doted black line represent the labour tax financed scheme whereas the blue line is for capital tax financed scheme.

is 0.46, which is lowest of all debt financing schemes. The difference between impact multiplies of the most and least effective schemes is very small under flexible exchange rate regimes.

Figure 3.3: Government expenditure shocks under different debt financing schemes: Fixed exchange rate.

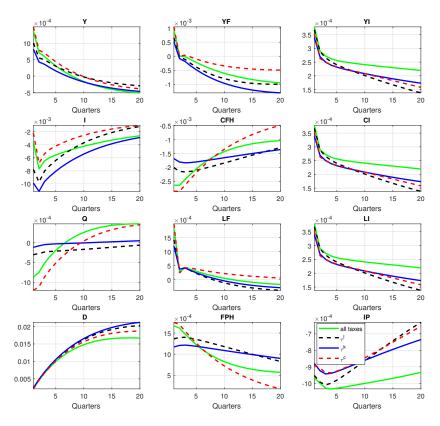
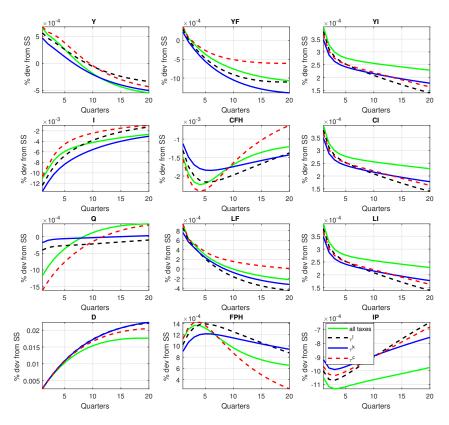


Figure 3.4: Government expenditure shocks under different debt financing schemes: Flexible exchange rate.



Fiscal financing scheme	1 quarter	2 quarter	4 quarter	12 quarter	
All taxes adjust to finance	0.78	0.45	0.38	-0.59	
fiscal stimulus	0.10	0.40	0.00	-0.33	
Only Consumption tax adjusts to finance	0.85	0.50	0.48	-0.22	
fiscal stimulus	0.05	0.00	0.40	-0.22	
Only labour tax adjusts to finance	0.56	0.35	0.33	-0.19	
fiscal stimulus	0.50	0.00	0.00	-0.15	
Only capital tax adjusts to finance	0.46	0.27	0.21	-0.65	
fiscal stimulus	0.40	0.21	0.21	-0.00	

Table 3.6: Fiscal policy multipliers with different financing schemes: Fixed exchange rate

Fiscal financing scheme	1 quarter	2 quarter	4 quarter	12 quarter	
All taxes adjust to finance	0.37	0.33	0.30	-0.70	
fiscal stimulus	0.01	0.00	0.50	-0.70	
Only Consumption tax adjusts to finance	0.38	0.37	0.30	-0.35	
fiscal stimulus	0.50	0.01	0.50	0.00	
Only labour tax adjusts to finance	0.32	0.29	0.27	-0.32	
fiscal stimulus	0.02	0.23	0.21	-0.52	
Only capital tax adjusts to finance	0.26	0.23	0.17	-0.79	
fiscal stimulus	0.20	0.20	0.17	-0.19	

Table 3.7: Fiscal policy multipliers with different financing schemes: Flexible exchange rate

#### **3.5.3** Size of the Informal Economy and Fiscal Multipliers

This section analyses, how does a change in the size of informal sector conditions the impacts of government expenditure shocks under different exchange rate regimes. An increase in the value of  $\gamma_1$ , represents an increase in the size of informal sector. Figures (3.5) and (3.6) depict the impulses responses of macroeconomic variables to government expenditure shocks, in economies with different size of informal sector under fixed and flexible exchange rate regimes. The impulse responses of aggregate, formal and informal outputs show that the stimulative effects of government expenditure shocks decrease with the size of informal economy irrespective of the exchange rate regime. In response to a government expenditure shock, the initial increase in aggregate, formal and informal sectors outputs is largest when the share of informal consumption is small in the consumption basket the households. Tables (3.8) and (3.9) show the government expenditure impact and present value multipliers associated with different values of  $\gamma_1$ , under fixed and flexible exchange rates respectively. The size of impact multipliers decline with an increase in the size of informal sector irrespective of the exchange rate regimes. In the case of a fixed exchange rate regime, when the value of  $\gamma_1$  changes from 0.35 to 0.55, the impact multiplier decreases from 0.88 to 0.60. By contrast, under a flexible exchange rate regime, the value of the impact multiplier declines from 0.39 to 0.33. This illustrates that as a consequence of an expanding informal economy, the decline in government expenditure impact multiplier is relatively larger under a fixed, rather than flexible exchange rate regime.

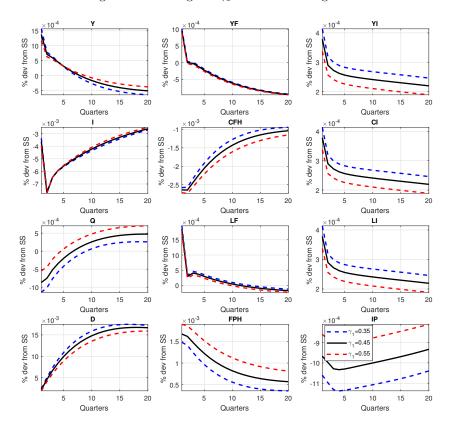


Figure 3.5: Change in  $\gamma_1$ : Fixed Exchange rate.

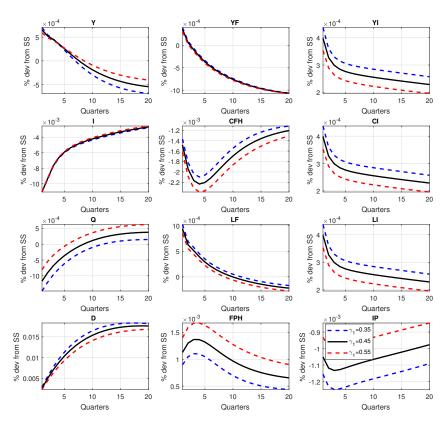


Figure 3.6: Change in  $\gamma_1$ : Flexible Exchange rate.

Table 3.8: Fiscal policy multiplier with different size of informal sector: Fixed exchange rate

size of informal sector	1 quarter	2 quarter	4 quarter	12 quarter
$\gamma_1 = 0.35$	0.88	0.50	0.40	-0.94
$\gamma_1 = 0.45$	0.78	0.45	0.38	-0.59
$\gamma_1 = 0.55$	0.60	0.40	0.35	-0.31

Table 3.9: Fiscal policy multiplier with different size of informal sector: Flexible exchange rate

size of informal sector	1 quarter	2 quarter	4 quarter	12 quarter
$\gamma_1 = 0.35$	0.39	0.35	0.30	-1.00
$\gamma_1 = 0.45$	0.37	0.33	0.30	-0.70
$\gamma_1 = 0.55$	0.33	0.31	0.29	-0.39

# 3.6 Sensitivity analysis

We analyse the sensitivity of the model to change in the values of calibrated parameters. The attempt of sensitivity analysis is constrained by the fact that there are very few empirical studies which estimate the parameters relevant to the labour and goods markets for developing and emerging economies featuring with large informal economy. However, the equilibrium response to positive government spending shock is assessed for the following range of the parameters: weight assigned to exchange rate and inflation in monetary policy rule,  $\rho_e, \rho_{\pi f} \in [1.5, 100]$ , inverse elasticity of labour supply,  $\sigma_l \in [0.8, 2]$ , elasticity of intertemporal substitution in consumption,  $\sigma_c \in [0.9, 4]$ , elasticity of substitution between the consumption of formal and informal goods,  $\mu_1 \in [0.7, 3]$ , price stickiness parameter of domestic firms,  $\omega_h \in [0.24, 0.75]$ and price stickiness parameter for imported goods,  $\omega_f \in [0.20, 0.75]$ . In response to the change in values of the parameters mentioned above, the dynamics of macroeconomic variables do not differ qualitatively, as the direction of the movement of all variables remain the same. The responses of macroeconomic variables with different values of parameters under the both exchange rates are given in Appendices D.1 to D.6.

# 3.7 Conclusion

In this chapter we develop an open economy New-Keynesian DSGE model to analyse the impacts of government expenditure shocks in a dual economy under alternative exchange rate regimes. Our results are in line of the of conventional Mundell-Fleming model in relative terms. Fiscal policy is more effective at increasing the level of output when exchange rate is fixed, but the difference between the multipliers under the two exchange rates is smaller than suggested by Mundell-Fleming model. The results also show that the government expenditure multipliers increase under fixed exchange rate when there is an increase in the labour mobility across the formal and informal sectors. An increase in the labour mobility between the sectors does not have any significant impact on the expenditure multipliers under flexible exchange rate regime. This implies that an increase in labour mobility widens the gap between government expenditure multipliers associated with the two exchange rate regimes. Another finding of the model is that the government expenditure impact multiplier is largest, when fiscal stimulus is financed by consumption tax under the both exchange rate regimes. The capital tax financed scheme is least effective in both formal and informal sectors. The informal sector responds strongly to the all tax financed scheme. The difference of impact multipliers between the most and least effective schemes is relatively big under the fixed exchange rate regime. We report an interesting finding which suggests that an expansion of the informal economy lowers fiscal policy multipliers under both exchange rate regimes. In addition, the decline in the magnitude of government expenditure impact multiplier is relatively larger under fixed, as opposed to flexible, exchange rate regime.

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# 3.9 Appendices

#### D.1: Change in the parameters of Monetary Policy rules

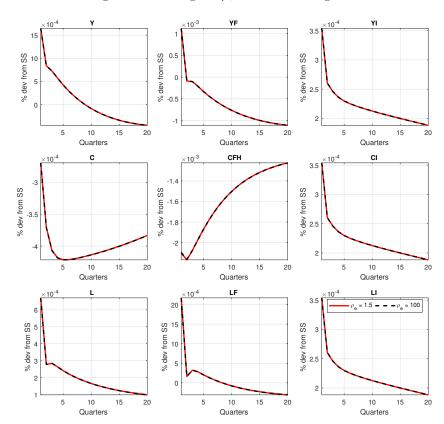


Figure 3.7: Change in  $\rho_e$ : Fixed Exchange rate

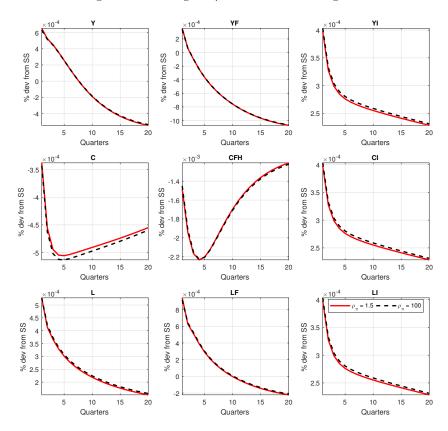


Figure 3.8: Change in  $\rho_{\pi}$ : Flexible Exchange rate

### D.2: Chang in the Inverse Elasticity of Labour Supply

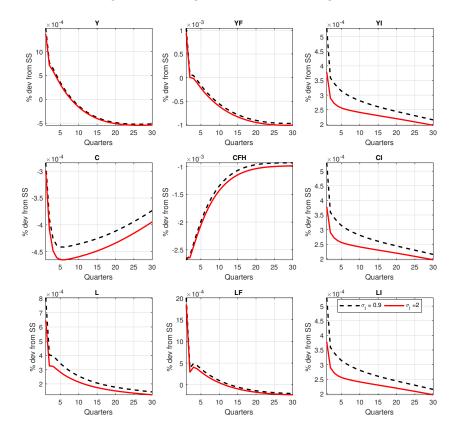


Figure 3.9: change in  $\sigma_l$ : Fixed Exchange rate

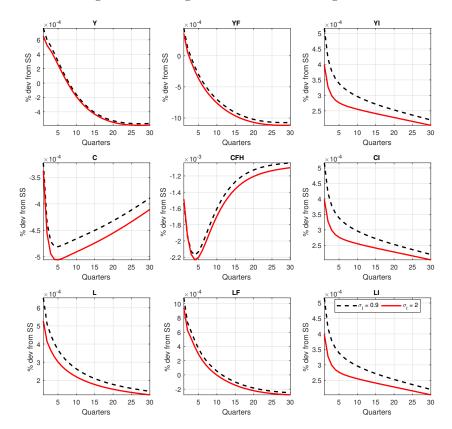


Figure 3.10: change in  $\sigma_l$ : Flexible Exchange rate

### D.3: Change in Intertemporal Elasticity of Substitution in Consumption

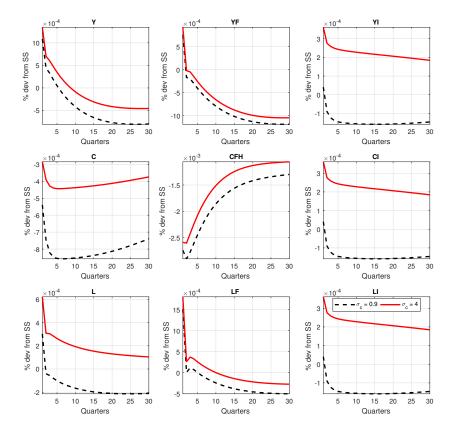


Figure 3.11: change in  $\sigma_c$ : Fixed Exchange rate

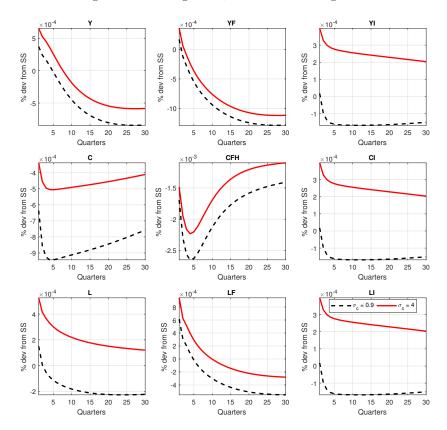


Figure 3.12: change in  $\sigma_c$ : Flexible Exchange rate

## D.4: Change in the Elasticity of Substitution of Consumption between Formal and Informal goods

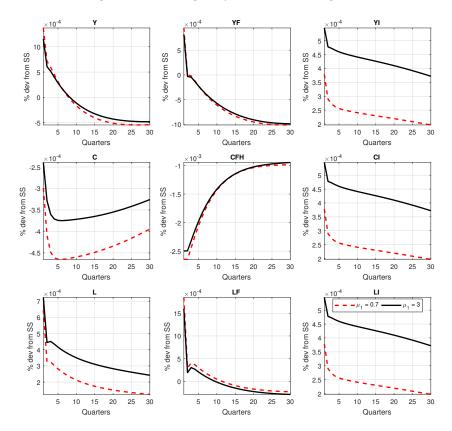


Figure 3.13: Change in  $\mu_1$ : Fixed Exchange rate

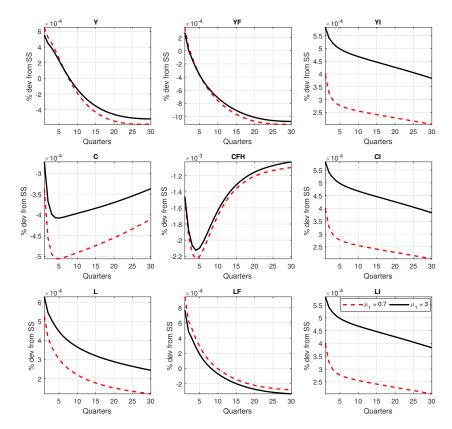


Figure 3.14: Change in  $\mu_1$ : Flexible Exchange rate

### D.5: Change in the Price Rigidity of Domestic goods

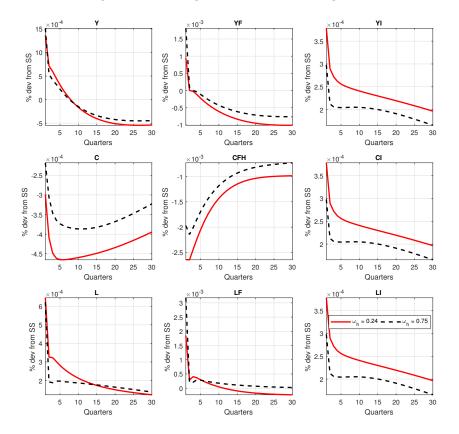


Figure 3.15: Change in  $\omega_h$ : Fixed Exchange rate

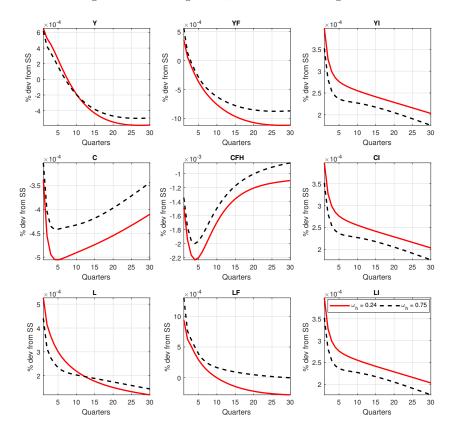


Figure 3.16: Change in  $\omega_h$ : Flexible Exchange rate

### D.6: Change in the Price Rigidity of Imported goods

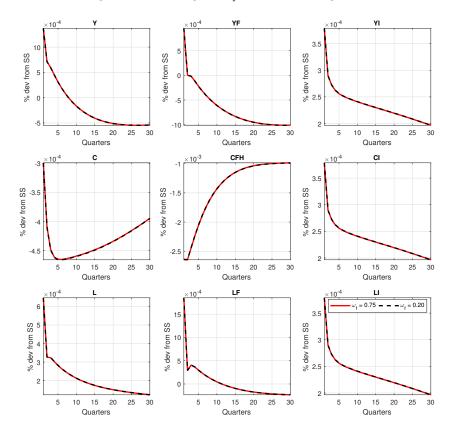


Figure 3.17: Change in  $\omega_f$ : Fixed Exchange rate

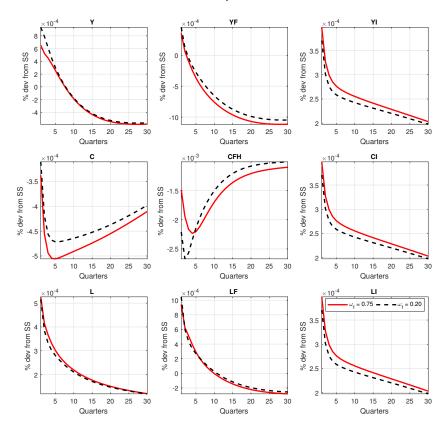


Figure 3.18: Change in  $\omega_f$ : Flexible Exchange rate