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IMPACT OF THE CHOICE OF RISK ASSESSMENT TIME HORIZONS ON DEFINED BENEFIT PENSION SCHEMES

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ABSTRACT

We examine the impact of asset allocation and contribution rates on the risk of defined benefit (DB) pension schemes, using both a run-off and a shorter 3-year time horizon. Using the 3-year horizon, which is typically preferred by regulators, a high bond allocation reduces the spread of the distribution of surplus. However, this result is reversed when examined on a run-off basis. Furthermore, under both the 3-year horizon and the run-off, the higher bond allocation reduces the median level of surplus. Pressure on the affordability of DB schemes has led to widespread implementation of so-called de-risking strategies, such as moving away from predominantly equity investments to greater bond investments. If the incentives produced by shorter term risk assessments are contributing to this shift, they might be harming the long term financial health of the schemes. Contribution rates have relatively lower impact on the risk.

KEYWORDS

Defined benefit pension schemes; Risk assessment; Economic capital; Time horizon; Asset allocation strategies; Contribution rates.

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1. INTRODUCTION

1.1 MOTIVATION

The main objective of a defined benefit (DB) pension scheme is to ensure that all of the promised benefits are paid. Whether the scheme has a surplus or deficit of assets is only known with certainty in the long run after the final benefit payment has been made. However, interim estimates of the scheme's finances are necessary in order to avoid runaway surpluses or deficits, and are also required for regulatory purposes. Is it possible that the incentives generated by interim estimates have an impact on pension funds' overall viability and cost? In this paper, we explore one channel through which this might occur.

Specifically, we examine the implications of using different time horizons for quantifying pension scheme risks on the long term financial health of a DB scheme. We use both a long term run-off approach and a short 3-year time horizon to analyse risk based on the distribution of surplus and compare the resulting estimates. In our analysis we consider different asset allocation strategies and contribution rates, the two main levers available to pension scheme managers and trustees to alter the risk profile of a DB scheme.

A long term run-off analysis involves projecting the scheme's cash flows, assets and liabilities over the entire term until the last of the current scheme members will leave the scheme through death or withdrawal. It is clearly a complex exercise. However, it arguably provides a truer reflection of the overall risk profile of a DB scheme, especially because the horizons faced by such schemes are typically much longer than the average investment fund. There is a large literature highlighting the differences between investing for the long term versus the short term (see, *e.g.* Campbell and Viceira, 2002; Ang and Kjaer, 2012), and it stands to reason that assessment of risks to portfolios with long horizons would be more accurate when assessed over a similar horizon.

On the other hand, interested parties like regulators, policymakers, pension scheme sponsors and scheme members, would justifiably require periodic updates on the financial health of the scheme. Typically, this can be simply obtained through a snapshot of current assets and liabilities, and the resulting funding surplus or deficit. However, obtaining point estimates of long term liabilities comes with its own challenges. In addition, if the estimate of liability is too prudent, there is a risk of overstating the underlying risk and any unnecessary follow-on remedial action can be detrimental for the scheme sponsor and can actually jeopardise the viability of an otherwise financially healthy scheme.

A slightly more sophisticated approach is to use a shorter time horizon for risk quantification *e.g.*, the UK regulator requires triennial valuations of DB schemes. Typical practice in the US is for large DB schemes to conduct annual valuations. Insurers, who also often face long term horizons, are required under the EU's Solvency II regulations to employ a 1-year horizon for risk reporting. Yet, any short horizon also suffers from the

same technical difficulty of arriving at a point estimate of the value of liabilities at the end of the period of measurement. To ensure consistent analysis, for the purposes of this paper, we have chosen 3 years as our shorter time horizon.

To address these challenges, we vary our analyses in two main ways. First, we allow for a number of possibilities with respect to the valuation actuary's choice in setting discount rates when risk is assessed on the intermediate horizon. We outline four possible approaches, solely for the purpose of allowing alternative views. We find that the valuation actuary's approach to setting the valuation discount rate at the end of 3 years has a material impact on the quantified risk for the 3-year risk assessment horizon.

Second, as we are considering the efficacy of changes to asset allocations and contribution rates, we consider two settings for the scheme environment. We model one scheme based on a large UK DB plan and the other on a stylised US one. These schemes differ in the benefit structures, such as the levels of guarantees, but also in terms of the economic scenarios required to assess their risk. As a result, we can explore how altering asset allocations or contribution rates affect scheme risk, while being as consistent as possible with regard to the interactions between promised benefits and economic conditions.

We find that regardless of the approach taken to setting the discount rate, the difference in time horizon generates contradictory conclusions regarding the best approach to manage risk through changes in asset allocation. Over the long term, increasing a scheme's allocation to long term bonds worsens the risk profile, both reducing the median level of surplus and increasing the spread of deficit. When examined over the shorter time horizon, increasing a scheme's allocation to long term bonds reduces the median surplus but also reduces the spread from the median. This particular finding suggests that there may be a significant distortion created by taking a short-term view of a scheme's financial status. It is important to clarify, however, that our goal is not to identify the optimal asset allocation, rather to highlight the impact of the risk assessment horizon on potential misjudgements in asset allocation and scheme management.

We also find that the impact on a scheme's financial status from changing contribution rates is much smaller than the impact of changes to the scheme's asset allocation.

1.2 LITERATURE REVIEW

Our contribution to the growing literature on risk quantification in DB pension schemes offers timely insight on the challenges faced by these schemes, particularly given the background of increasing life expectancy and a steady fall in interest rates over the last few decades. A large proportion of the current research is geared towards analysing the interactions between financial and mortality risks, and how pension scheme risk quantification can be formulated using a framework similar to existing insurance regulations, like Solvency II or the Swiss Solvency Test. However, as far as we are aware, there is limited evidence on the impact that a risk assessment approach based on a short time horizon can have on the long term financial health and viability of a DB scheme. Moreover, it is

important to explore to what extent the implications might be different for DB schemes in different countries with different benefit structures and economic conditions.

As a risk measurement framework, we adopt the economic capital approach, which is a useful tool to analyse the distribution of surplus. To be clear, we do not argue that pension schemes should hold economic capital in the same sense as some other financial services firms, rather we adopt it as a consistent framework with which to analyse the risk distribution of a DB scheme in isolation. Previous research on long term economic capital risk assessments of UK DB schemes can be found in Porteous et al. (2012) and Yang and Tapadar (2015). Porteous et al. (2012) assessed risk for UK's Universities Superannuation Scheme (USS) based on its 2008 valuation report and estimated its economic capital requirement to be 61% of liabilities. Yang and Tapadar (2015) estimated that the pooling benefits in the UK's Pension Protection Fund (PPF) could reduce its overall economic capital to just 10% of the aggregate economic capital of all individual UK DB schemes, although these schemes had economic capital estimates ranging between 66% and 134% of liabilities.

There are also a number of papers attempting to quantify the solvency capital requirement for DB schemes in a manner parallel to the Solvency II regulations applied to insurers. For example, Devolder and Piscopo (2014) modelled a simple final salary DB scheme with a single model point to find that a geometric Brownian motion model for asset returns gives a probability of default of 0%-40% and the solvency capital requirement between 0%-30% of liabilities. Ai et al. (2015) also used the Solvency II framework over a 1-year time horizon to arrive at a solvency capital charge of around 50% of estimated reserves. Butt (2012) used stochastic economic and mortality models to simulate assets and liabilities of a pension scheme over a 30 year time horizon to find that changes in discount rates, investment returns and demographic variables accounted for 46.4%, 33.3% and less than 1% of the funding risk respectively. Liu (2013) also found similar results for an annuity portfolio, where an equal 40% change in the interest rate model parameter and mortality model parameter separately, leads to approximately 18% and 5% changes to solvency capital respectively.

There is relatively little research on the impact of the time horizon on pension risk quantification. One recent attempt is Devolder and Lebegue (2016), who used ruin theory to estimate the solvency capital requirement for long term life insurance and pension products, arguing that the 1-year horizon of the Solvency II framework may not be appropriate for products with long terms. Using a simple model, the authors showed that under the Solvency II framework, solvency capital is understated at shorter durations and overstated at longer durations, citing inadequacy of the Solvency II framework to account for benefits of long term equity investments. Although not specifically aimed at pension schemes, Karabey et al. (2014) estimated that among the different risk factors involved, for a 25 year and a 45 year annuity, investment risk accounts for approximately, 89% and 63.5% of the total underlying risk, respectively.

This paper is related to recent work by some or all of the authors. In particular, we apply the stochastic economic model developed by Oberoi et al. (2020) for the UK economy. An extension of this model, for the US economy, is also presented in a technical report (Andrews et al., 2019) that is available online. The aim of the technical report was to develop an economic capital based risk management approach for DB pension schemes and to demonstrate this approach using the run-off horizon. In the current paper, our primary focus is to analyze the impact of the choice of time horizon.¹

In summary, our research builds on existing literature by looking at the impact of typical remedies for pension fund sustainability management, when considered in light of the incentives generated by the risk assessment and reporting horizon.

1.3 OUTLINE OF THE ARTICLE

The rest of this article is set out as follows. In Section 2, we describe the risk measurement framework that we employ. In Section 3, we describe our base case stochastic economic and demographic models. In Section 4, we describe the pension scheme provisions and valuation assumptions used. The results for the UK scheme are presented in Section 5. Section 6 contains the results for the stylised US scheme. Section 7 concludes.

2. RISK MEASUREMENT FRAMEWORK

The concept of economic capital is used extensively for the purpose of risk quantification of financial services firms and conglomerates, see for example Porteous and Tapadar (2005, 2008a,b). Porteous et al. (2012) and Yang and Tapadar (2015) extended the definition of economic capital to cover risk assessment of pension schemes. We adopt a modified version of the definition for the purposes of this article as stated below:

Definition: *The economic capital of a pension scheme is the proportion by which its existing assets would need to be augmented in order to meet net benefit obligations with a prescribed degree of confidence. A pension scheme's net benefit obligations are all obligations in respect of current scheme members, including future service, net of future contributions to the scheme.*

This definition is generic in nature and flexible in terms of time horizons and liability valuation methods. Note that we utilise this definition for purposes of measurement and quantification of the risk of a pension scheme. As such, we use this metric to compare

¹As expected, the run-off results presented in this paper for the representative UK pension scheme are similar to those given in Andrews et al. (2019); the differences are due to different simulation runs. Further, exact comparisons may not always be applicable because, in the current paper, we present analyses based on the choice of different time horizons. In addition, our model for the US pension scheme is different from the one presented in Andrews et al. (2019) to enable appropriate comparison of the UK and US results.

the risk profiles of different pension schemes, and to compare the risk profile of the same pension scheme under alternative circumstances (e.g., different time horizons and asset allocations). We are not necessarily suggesting that pension schemes must hold additional assets to cover economic capital. Instead, we recommend that pension scheme trustees use economic capital as a guide to formulate their risk management strategies in line with their risk appetite and risk limits. Additionally, our analysis is intended to apply to generic pension schemes so as to be broadly applicable.

Since we are employing this metric for comparing risk profiles of generic pension schemes, and not managing the risk of a particular pension scheme, we do not consider the size or effect of the employer covenant. For a full discussion of the valuation of the employer covenant, please see Turnbull (2014).

For a mathematical formulation of economic capital, we need the following notation:

A_t : Value of pension scheme assets at time t .

L_t : Value of pension scheme liabilities at time t , i.e. the point estimate as quantified by the valuation actuary at time t .

X_t : Net cash (in)flow of the pension scheme at time t (excluding investment returns).

$I_{(s,t)}$: Accumulation factor (accumulated value at time t , of a unit amount invested at time s , where $s \leq t$).

$D_{(s,t)}$: Discount factor, i.e. $D_{(s,t)} = I_{(s,t)}^{-1}$.

Given the history up to time 0, all the above entities ($A_t, L_t, X_t, I_{(s,t)}, D_{(s,t)}$) at future times s and t are random variables. In the first instance, we consider one single realisation, or simulation, of the future, over a specified time horizon $[0, T]$. For simplicity, we assume a discrete time framework with annual time intervals, i.e. all cash flows and valuation of assets and liabilities happen at annual intervals.

Based on this assumption, we define the profit vector, P_t for $t = 0, 1, 2, \dots, T$ as:

$$P_t = L_{t-1} I_{(t-1,t)} - X_t - L_t, \text{ for } t > 0 \text{ and } P_0 = A_0 - X_0 - L_0. \quad (1)$$

We then define the present value of future profits over time horizon $[0, T]$ as:

$$V_0^{(T)} = \sum_{t=0}^T P_t D_{(0,t)}. \quad (2)$$

This can be equivalently expressed as:

$$V_0^{(T)} = A_0 - \sum_{t=0}^T X_t D_{(0,t)} - L_T D_{(0,T)}. \quad (3)$$

Intuitively, $V_0^{(T)}$ represents the amount of excess assets, in excess of A_0 , that is required to meet net cash outflow over $[0, T]$ and also to set up reserves, L_T , as quantified by the valuation actuary at time T , to meet future outstanding obligations beyond time T . Note that $V_0^{(T)}$ can be defined directly using Equation (3), instead of defining it through the profit vector in Equation (2). However, the step from Equation (2) to Equation (3) is instructive because it shows that the intermediate liability values between times 0 and T do not play any role in $V_0^{(T)}$.

Under a run-off approach, the horizon T is set until the time when the last of the current scheme members leaves the scheme either through death or withdrawal. We denote the run-off approach by $T = \infty$. Clearly, $L_\infty = 0$, and hence:

$$V_0^{(\infty)} = A_0 - \sum_{t=0}^{\infty} X_t D_{(0,t)}. \quad (4)$$

If, instead, a shorter time horizon is used for economic capital calculation, Equation (3) needs to be used. To illustrate the implications of the choice of the time horizon for economic capital calculations, we use both the run-off approach and a shorter 3-year time horizon approach. We choose a time horizon of 3 years, because in the UK there is a regulatory requirement to carry out triennial valuations, so risk assessment over a 3-year time horizon might be a natural choice. For this horizon, the expression for present value of future profits becomes:

$$V_0^{(3)} = A_0 - \sum_{t=0}^3 X_t D_{(0,t)} - L_3 D_{(0,3)}. \quad (5)$$

Clearly, $V_0^{(3)}$ depends on the assumptions used by the valuation actuary at the end of year 3 to determine L_3 . For a prescient actuary, with perfect foresight, her future liability calculation for L_3 would be exact and so in that case $V_0^{(3)} = V_0^{(\infty)}$. However, in reality, a valuation actuary can only produce an estimate of L_3 , based on historical evidence up until that time and also the future economic and mortality outlook.

One of the key drivers in this calculation is the valuation discount rate assumption. The choice of discount rate has a material impact on the estimated value of L_3 , and consequently $V_0^{(3)}$. The code of practice, issued by The Pensions Regulator of the UK, provides practical guidance to trustees and employers of UK DB schemes on how to comply with the scheme funding requirements under the Pensions Act 2004. In terms of setting valuation discount rates, the code of practice stipulates in item 125²:

Discount rates used in setting technical provisions must be chosen prudently, taking into account either:

²<https://www.thepensionsregulator.gov.uk/en/document-library/codes-of-practice/code-3-funding-defined-benefits->

- *the yield on assets held by the scheme to fund future benefits and the anticipated future investment returns; and/or*
- *the market redemption yields on government or high quality bonds.*

In Sections 5 and 6, we discuss four different approaches that might possibly be taken to set valuation discount rates based on the principles set out in the code of practice and the implications of adopting these.

We use $V_0^{(T)}$ to measure the risk of a DB scheme. However, as we will be assessing risks of pension schemes in two different countries, with different currencies and different benefit structures, it will be useful to express $V_0^{(T)}$ as a percentage of initial assets A_0 , so that there is a standardisation in terms of currency and scale. This is also consistent with our economic capital definition and can be interpreted as the proportional change in assets required such that all future benefit obligations will be met with a certain degree of confidence.

$V_0^{(T)}$ is defined above for a single specimen realisation of the future. As the future is unknown at time 0, all future components in $V_0^{(T)}$ are random variables, and hence $V_0^{(T)}$ is itself a random variable. The actual economic capital risk measure can be defined in terms of the distribution of $V_0^{(T)}$. Two common measures used in the literature are:

- **Value-at-risk**, VaR , is defined as $\mathbb{P} \left[V_0^{(T)} \leq VaR \right] = p$, for a given probability p . VaR represents the amount of additional initial assets required at time 0 for the pension scheme to meet all its future obligations with probability, or confidence level, $(1 - p)$.
- **Expected shortfall**, ES , is defined as the average of all losses that are greater than or equal to the value of VaR for a given probability level p , i.e. $\mathbb{E} \left[V_0^{(T)} \mid V_0^{(T)} \geq VaR \right]$. In other words, ES provides an estimate of the expected value of losses in the worst p proportion of cases.

These definitions of VaR and ES are based on McNeil et al. (2015). Note that, in some academic literature, the above definition of ES is sometimes alternatively referred to as the conditional tail expectation, but to avoid confusion, we use McNeil et al. (2015) as our reference. For all our results, we present the entire distribution of $V_0^{(T)}$ in conjunction with VaR and ES at selected probability levels.

3. STOCHASTIC ECONOMIC AND DEMOGRAPHIC MODELS

To determine the distribution of $V_0^{(T)}$, we need to simulate future economic and demographic scenarios. In Section 3.1 we discuss the Economic Scenario Generators (ESGs) we have used for our analysis. Section 3.2 provides details of the stochastic mortality

models we have used to project mortality of UK and US pension scheme members. For generating the distributions of $V_0^{(T)}$ for different scenarios, we have used 10,000 simulation runs in each case.

3.1 ECONOMIC SCENARIO GENERATOR

Oberoi et al. (2020) propose a graphical model approach to develop an ESG suitable for use by a life insurance company or a pension scheme in the UK that invests in equities and bonds. Andrews et al. (2019) extend this approach to develop an ESG for the US economy. In this section, we provide a brief outline of the graphical model methodology and the relevant parameterisation for the UK and the US ESGs, which we will use in this article.

A minimal ESG, which can be used to analyse DB scheme risks, requires data for price inflation (I), salary inflation (J), stock returns and bond returns. Following the approach of Wilkie (1986), Oberoi et al. (2020) use dividend yield (Y), dividend growth (K) and Consols yield (C) to construct stock and bond returns. The dataset used by Oberoi et al. (2020) consists of annual values of these economic variables from 1926 to 2017 as at the end of June each year.

Andrews et al. (2019) obtain US data from two sources. The first is Robert Shiller, who provides online data for price inflation, S&P 500 Index, S&P 500 High Dividend Index and bond yields. The second source is Emmanuel Saez, who provides online data for average wages in the US. The US data extend from 1913 to 2015.

In a graphical ESG, involving a number of economic variables, first a univariate time series model, typically an AR(1) process, is fitted to each individual economic variable. If X_t denotes one of the economic variables to be modelled, with data available for times $t = 0, 1, 2, \dots, H$, the following model is fitted:

$$\mu_x = \mathbb{E}[X_t], \quad (\text{i.e. averaging over time}); \quad (6)$$

$$Z_t = X_t - \mu_x, \quad \text{for } t = 0, 1, 2, \dots, H; \quad (7)$$

$$Z_t = \beta_x Z_{t-1} + e_{x,t}, \quad \text{for } t = 1, 2, \dots, H; \text{ where } e_{x,t} \sim N(0, \sigma_x^2). \quad (8)$$

The parameter estimates of μ , β and σ for the chosen economic variables in the UK and the US are given in Table 1.

Once the residuals, $e_{x,t}$ are obtained for each variable X_t , a Gaussian graphical model is fitted to the multivariate residuals:

$$\mathbf{e}_t = (e_{I_t}, e_{J_t}, e_{Y_t}, e_{K_t}, e_{C_t}) \sim \mathcal{N}(\mathbf{0}, \Sigma), \quad (9)$$

separately for the UK and the US data. Estimation is carried out based on maximum likelihood and the resulting partial correlations are shown in Table 2.

Clearly, some of the partial correlations in the matrices are small. Graphical models provide a means to identify the minimum number of non-zero correlations that describe

Table 1: ESG time series parameter estimates.

	UK			US		
	μ	β	σ	μ	β	σ
I_t	0.0404	0.6102	0.0387	0.0328	0.6211	0.0392
J_t	0.0528	0.7801	0.0282	0.0464	0.4908	0.0643
Y_t	0.0468	0.6718	0.0085	0.0413	0.8293	0.0100
K_t	0.0527	0.4263	0.0852	0.0507	0.2746	0.1084
C_t	0.0617	0.9674	0.0083	0.0489	0.9346	0.0091

Table 2: Partial correlations of residuals.

	UK					US				
	I_t	J_t	Y_t	K_t	C_t	I_t	J_t	Y_t	K_t	C_t
I_t	1					1				
J_t	0.48	1				0.42	1			
Y_t	0.16	0.11	1			0.20	-0.47	1		
K_t	0.18	0.15	-0.06	1		0.17	0.10	0.28	1	
C_t	0.20	-0.09	0.37	0.06	1	0.19	0.04	0.12	-0.06	1

the underlying data adequately. In other words, it is a dimension reduction tool whereby correlations of all pairs of residuals need not be stipulated directly. Instead, in this approach, only the correlations between variables that are not conditionally independent are used.

Oberoi et al. (2020) use a number of statistical criteria, namely AIC, BIC and simultaneous p -values, to determine the optimal graphical model for the UK data. Model $E6$, shown in the top panel of Figure 1, is identified by Oberoi et al. (2020) as a model that satisfies certain desirable features and also possesses an intuitively appealing structure. For the purposes of this article, we use Model $E6$ as our ESG for the UK economy.

Andrews et al. (2019) use the same approach to determine the graphical structure for the US economic data. Interestingly, for the US data, all three statistical criteria, i.e. AIC, BIC and simultaneous p -values, produce the same graphical structure, which is shown in the bottom panel of Figure 1. We use this graphical ESG to model the US economy.

We have also checked that qualitatively, all the results and conclusions of our analysis are broadly very similar, regardless of the ESG employed. Specifically we have carried out the same analysis using the well-known Wilkie model (Wilkie, 1986, 1995; Wilkie et al.,

2011; Wilkie and Şahin, 2016, 2017a,b,c, 2018). We find that although, as expected, there are some quantitative differences between the numbers obtained from the Wilkie model and the graphical ESG, qualitatively the results point to very similar conclusions. The results for the Wilkie model are available from the authors on request.

3.2 STOCHASTIC MORTALITY MODELS

Future projections of pension scheme cash flows also depend on the mortality assumptions. Cairns et al. (2009) provide a quantitative comparison of eight stochastic mortality models using data from England and Wales and the US, of which Model M7 provides a good fit for both UK and US data. For the purposes of this article, we use model M7 to project forward stochastic mortality rates of the UK and the US.

The structure of model M7, which models $q(t, x)$, the probability that an individual aged x at time t will die between times t and $t + 1$, is as follows:

$$\text{logit } q(t, x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \kappa_t^{(3)} [(x - \bar{x})^2 - \sigma^2] + \gamma_{(t-x)}^{(4)}, \quad (10)$$

where x is the age, \bar{x} is average age, σ^2 is the average of $(x - \bar{x})^2$, $\kappa_t^{(i)}$ are the period effects and $\gamma_{t-x}^{(i)}$ is the cohort effect.

We parameterise model M7 using UK and US data from the Human Mortality Database for both males and females from 1961 to 2014 for ages 30–100.

Projecting future mortality rates involves projecting the time series $\kappa_t^{(i)}$ and $\gamma_{t-x}^{(i)}$ forward. Systematic risk arises from the uncertainty involved in projecting these time series. For example, if the mortality rates improve faster than expected then future $q(t, x)$ will be lower, which in turn will result in lower deaths. Cairns et al. (2011) suggest possible approaches to project mortality parameters forward based on the historical estimates of these parameters. For our purpose, we project $\kappa_t^{(i)}$ linearly over time. Given that we are not interested in future cohorts (but only in existing ones), we do not project $\gamma_{t-x}^{(i)}$ forward.

We do not model idiosyncratic (or non-systematic) mortality risk, because in this article, we model large pension schemes and this risk can be diversified away by pooling.

4. PENSION SCHEME MODELLING

As a representative UK pension scheme, we use the model of UK's USS. For the US, we use a stylised US scheme using the same membership profile as the UK scheme, to ensure a certain degree of comparability in our analysis. However, to capture US-specific pension scheme features, we have incorporated a number of changes to the benefit structure for the stylised US scheme, as discussed in Section 4.2.

4.1 UK PENSION SCHEME

In this section, we provide a brief overview of our modelling approach of the UK scheme. The UK scheme model is based on the USS, which is one of the largest open DB schemes operating in the UK, with more than 350 participating employers and approximately 400,000 scheme members. The approach presented is based on the actuarial valuation carried out for the scheme on March 31, 2014.

4.1.1 *Membership Profile*

The broad membership profile for the UK scheme is shown in Table 3. To capture the overall risk characteristics and the inter-generational risk dynamics of the scheme, we need a range of model points specifically for active members. So we use an age distribution for active members, as shown in Table 4. Table 4 also provides past service and salary assumptions for each model point, which have been set so that the average past service and average salary of active members broadly match the figures from Table 3. For deferred members and pensioners, we use single model points, based on the values given in Table 3, to represent each of these membership categories. We do not use a range of model points for deferred members and pensioners, as this does not have a material impact on our results and conclusions. We also assume an equal gender split and no salary differential between genders for all membership categories.

Table 3: UK scheme membership profile.

Active members	Number	167,545
	Average pensionable salary	£42,729
	Average age	43.8
	Average past service	12.5
Deferred members	Number	110,430
	Average deferred pension	£2,373
	Average age	45.1
Pensioners	Number	70,380
	Average pension	£17,079
	Average age	71.1

4.1.2 *Benefit Structure*

The pension benefits are comprised of an annual pension and a cash lump sum at retirement, calculated as follows:

$$\text{Annual pension} = \text{Pensionable salary} \times \text{Pensionable service} \times \text{Accrual rate};$$

$$\text{Lump sum} = 3 \times \text{Annual pension}.$$

Table 4: UK scheme model points, past service and salary of active members.

	Age	Proportion	Number	Past service	Salary
	30	30%	50,264	7	£25,500
	40	30%	50,264	11	£42,500
	50	20%	33,509	15	£52,500
	60	20%	33,509	19	£58,500
Total		100%	167,545		
Average	43			12.2	£42,600

To capture some of the USS-specific features of the benefit structure, while keeping the model simple, we assume that all members accrue benefits on a *final salary* basis up to March 31, 2014; and from April 1, 2014 onward all members move to a *career revalued* basis. The accrual rate for the final salary and career revalued schemes are 1/80 and 1/75 respectively.

Annual pension is assumed to increase in line with price inflation, subject to a 5% limit. Members' salaries increase in line with salary inflation in the economy. In addition to salary inflation, there is an explicit age-based promotional salary scale, based on the LG59/60 table which is widely used for actuarial valuation of UK DB schemes. Sample values of the scale are shown in Table 5. For our analysis, future projections of price inflation and salary inflation are generated by the ESG described in Section 3.1.

Table 5: UK pension scheme assumptions.

Age	Promotional salary scale		Withdrawal		Proportion married	
	Male (%)	Female (%)	Male (%)	Female (%)	Male (%)	Female (%)
25	3.8	3.1	14.42	19.28	10.90	10.90
35	3.8	3.1	9.19	11.40	53.41	53.41
45	2.0	1.8	3.79	3.83	69.76	69.79
55	1.1	1.4	3.79	3.83	69.76	69.79

For members who withdraw from the scheme, a deferred inflation-linked pension is provided based on accrued service. Inflation indexation of salary is provided between the date the member withdraws from the scheme and the date of retirement. A sample of the withdrawal rates for the USS taken from the 2014 actuarial valuation report, which

are 270% of the LG59/60 table for males and 113% of the LG59/60 table for females, is shown in Table 5.

On the death of an active member, a lump sum payment of three times the annual salary is paid at the time of death, along with a spouse's pension of half the amount of pension the member would have received if he or she had survived until normal retirement. On the death of a deferred pensioner, a lump sum equal to the present value of the deferred lump sum payable at normal retirement is provided along with a spouse's pension of half the amount of the deferred pension payable at normal retirement. On the death of a pensioner, a spouse's pension of half the amount of the member's pension is payable. All spousal pensions commence on the date of the scheme member's death. A sample of the married proportion for the USS taken from the 2014 actuarial valuation report, which is 109% of the 2008 Office of National Statistics table for both males and female, is shown in Table 5.

4.1.3 *Assets, Liabilities and Contributions*

On March 31, 2014, USS's total value of assets was £41.6 billion, of which approximately 70% was invested in UK and overseas equities, property and alternatives and 30% in fixed income assets. For the purposes of this article, we assume an asset allocation of 70% in equities and 30% in bonds.

The 2014 valuation report also provides an estimate of the scheme liabilities as calculated by the valuation actuary using the projected unit method, which is a prospective valuation method in which liabilities are estimated based on the past service accrued on the valuation date, taking into account future salary inflation. The estimated value of liabilities was £46.9 billion, giving an initial valuation deficit of £5.3 billion.³

We assume a contribution rate of 22.5% of salary.

4.2 STYLISTED US PENSION SCHEME

For our analysis, we use a hypothetical stylised US DB scheme with the same membership profile as for the UK scheme. In this section, we outline only those features which are specific to the stylised US scheme. Other aspects of scheme membership, benefit provisions and assumptions are the same as that for the UK scheme, except monetary amounts, which are in US dollars. The other key differences in the analysis of this scheme are the economic scenario generator and mortality projections, which are specific to the US.

³We note that this valuation of the liabilities is not necessarily consistent with the economic expectations arising from our model. However, the valuation of the liabilities at the beginning of the projection period has no bearing on our analysis, which is only concerned with the valuation of the liabilities at the end of year 3, and trivially at the end of the run-off period.

4.2.1 Benefit Structure

For the stylised US scheme, there is no lump sum paid on retirement, while pensionable salary is calculated on a *final salary* basis. The accrual rate is set at $1/66$ (or 1.5%). Crucially, there is no inflation indexation of pension during the payment period.

For members who withdraw from the scheme, a deferred pension is provided based on accrued service. Again, no inflation indexation of salary is provided between the date the member withdraws from the scheme and the date of retirement. There is also no inflation indexation during the payment period.

On the death of an active member, a lump sum equal to the present value of the pension the member would have received if he or she had survived until normal retirement is paid at the time of death. On the death of a deferred pensioner, a lump sum equal to the present value of the pension the member would have received if he or she had survived until normal retirement is paid at the time of death. On the death of a pensioner, a spouse's pension of half the amount of the member's pension is payable. The present value of any lump sum payments on death is calculated using the valuation basis.

4.2.2 Assets, Liabilities and Contributions

To ensure comparability, we assume the same asset allocation strategy of 70% equities and 30% fixed income assets for the stylised US scheme. On this assumption, we obtain a future contribution rate of 6.5% of salary, calculated using the projected unit standard contribution rate.

The projected unit method can also be used to obtain an estimate of the actuarial liability of the stylised US scheme. Using the prevailing long term US bond yield on the valuation date, we get an estimated value of liabilities of \$47.5 billion.

We further assume that the stylised US scheme has assets worth \$44.5 billion on March 31, 2014, to give an initial valuation deficit of \$3 billion. We will see in Section 6 that this particular assumption leads to the same median value for the distributions of $V_0^{(\infty)}$ (as a percentage of A_0) for both the UK scheme and the stylised US scheme, providing us with a common frame of reference.

5. UK PENSION SCHEME RESULTS

In this section, we examine the results for the UK scheme. We consider three different cases separately for the run-off and 3-year time horizon:

- the base case, in order to establish a baseline for the distributions of $V_0^{(\infty)}$ and $V_0^{(3)}$;
- the impact of changing the asset allocation to high bond content (30/70 in equity/bond) from the baseline allocation of high equity content (70/30 in equity/bond); and

- the impact of changing the contribution rate to 30% (high) and 15% (low), relative to the baseline contribution rate of 22.5% of salary.

5.1 BASE CASE

The top panel of Figure 2 shows the density of $V_0^{(\infty)}$ (as a percentage of A_0) for the UK scheme. The second and third panels show the sensitivity of this distribution to changes in the asset allocation and the contribution rate, which we discuss in detail in Sections 5.2 and 5.3, respectively. Table 6 provides all the summary statistics.

In this section, we first focus on the base case run-off result shown in the top panel of Figure 2 and the corresponding summary statistics in Table 6. We observe:

1. The distribution has a long left skew, indicating the possibility of a significant deficit, albeit with a diminishing probability.
2. The median of the distribution is 24% of A_0 , indicating that on *average* the scheme has more than sufficient assets to meet all its future obligations on a run-off basis.
3. The 10th percentile of the distribution is -36% of A_0 , which implies that if the scheme had 36% additional assets, it would meet all its future obligations with a 90% probability. The probability of a shortfall is 27%.
4. Going further into the left tail, we find that the scheme would require 148% additional assets to ensure a 99.5% chance of meeting all its future obligations.

Continuing with the base case, but now focusing on a shorter 3-year time horizon, Figure 3 shows the density of $V_0^{(3)}$ based on a number of alternative approaches that might be taken to value the liabilities⁴, L_3 , at the end of year 3. In any specific valuation, the valuation actuary will use their judgement to select a particular discount rate based on the code of practice, current data (such as on bond yields and inflation expectations), and long-term expected returns. In our stochastic projections, it is necessary to select an appropriate discount rate at several points in the future, and for 10,000 different scenarios. As such, whatever approach is taken will need to be mechanistic in nature and, perhaps, somewhat arbitrary.

We outline four illustrative approaches to select the valuation discount rate assumption and present the results based on these assumptions. We utilise these approaches to be able to contrast and compare them, not necessarily that they would be implemented by a valuation actuary. In each panel of Figure 3, we also include the density of $V_0^{(\infty)}$, as a grey curve, for reference.

⁴In the run-off illustrations there is no need to address this issue, since by definition L_∞ is nil.

Table 6: Summary statistics of the results for the UK scheme using the graphical ESG.

Scenarios	$P[V_0 \leq 0]$	50 th percentile		10 th percentile		0.5 th percentile	
		VaR	ES	VaR	ES	VaR	ES
$V_0^{(\infty)}$							
Base case	0.27	24	-14	-36	-74	-148	-192
High bond allocation	0.68	-23	-73	-103	-146	-230	-258
High contribution	0.19	35	-1	-22	-58	-129	-172
Low contribution	0.37	13	-27	-50	-89	-168	-211
No contribution	0.58	-9	-53	-79	-121	-205	-251
$V_0^{(3)}$ (Valuation rate based only on long term bond yields)							
Base case	0.92	-94	-197	-257	-364	-606	-734
High bond allocation	0.92	-99	-197	-252	-349	-550	-673
High contribution	0.91	-89	-193	-252	-359	-601	-729
Low contribution	0.94	-98	-202	-262	-368	-608	-739
$V_0^{(3)}$ (Valuation rate based on returns on backing assets)							
Base case	0.42	22	-153	-267	-468	-880	-1103
High bond allocation	0.59	-15	-127	-191	-326	-608	-755
High contribution	0.40	27	-148	-262	-463	-875	-1097
Low contribution	0.43	18	-157	-272	-473	-885	-1109
$V_0^{(3)}$ (Valuation rate converging to long term returns after 50 years)							
Base case	0.38	26	-70	-129	-227	-411	-508
High bond allocation	0.56	-9	-83	-128	-203	-353	-423
High contribution	0.36	31	-65	-124	-221	-406	-501
Low contribution	0.39	22	-74	-134	-232	-418	-514
$V_0^{(3)}$ (Valuation rate converging to long term returns after 15 years)							
Base case	0.30	30	-25	-58	-110	-206	-262
High bond allocation	0.52	-2	-55	-87	-133	-227	-264
High contribution	0.27	35	-20	-53	-104	-199	-255
Low contribution	0.32	26	-29	-63	-115	-211	-268

1. Firstly, we assume a valuation discount rate based solely on the prevailing bond yield at the end of 3 years. In the top-left panel of Figure 3, we show the density of $V_0^{(3)}$ (as a percentage of A_0), when L_3 is calculated using the prevailing (simulated) long-term bond yield at the end of 3 years plus a spread⁵ of 1.8%. Clearly, this approach produces a very different distribution of $V_0^{(3)}$, as compared to $V_0^{(\infty)}$, with a large median loss and a large negative 10th percentile.

One characteristic that stands out is that the left tail of the distribution is much fatter than the one for the full run-off. The reason for the fatter left tail of $V_0^{(3)}$, as compared to $V_0^{(\infty)}$, is the use of a constant valuation discount rate to calculate L_3 . If the long term bond yield in the third year is low, this low yield is assumed to continue as a constant, indefinitely into the future for calculating L_3 , which is unrealistic and is also unlikely to happen in the simulated realisations of the ESG, hence $V_0^{(\infty)}$ does not have such fat tails as $V_0^{(3)}$ in this case.

2. Another approach to calculate L_3 could be to use the returns earned on the backing pension scheme assets, reflecting the actual asset allocation strategy. The top-right panel of Figure 3 shows the result if the third year's returns (weighted by asset allocation) are used as the discount rate for all future years. Now the distribution of $V_0^{(3)}$ has approximately the same median as $V_0^{(\infty)}$, but has a very large dispersion (in both tails), as is expected, because any fluctuations in returns in the third year get magnified due to the use of a constant valuation discount rate to calculate L_3 .
3. However, using returns earned only in the third year as a discount rate for all future years is also unrealistic. It would be more appropriate to use a time varying (but deterministic) discount rate to calculate L_3 , where the discount rate increases (or decreases) from its value at the end of the third year to its long term mean over a certain period of time. The bottom-left and bottom-right panels of Figure 3 show the result if the time period of convergence to the long term mean is 50 years and 15 years, respectively⁶. If the convergence to the long term mean is slow (50 years), the dispersion is still large, but smaller than before. If the convergence to long term mean is quicker (15 years), the distributions of $V_0^{(3)}$ and $V_0^{(\infty)}$ get closer.

In Figure 4, we show the scatter plots, or joint densities, of $V_0^{(3)}$ and $V_0^{(\infty)}$, in the form of heatmaps, for all four approaches. The heatmaps represent densities that integrate to 1, with the same colour representing the same quantile level and darker shades representing higher density values.

⁵For the USS 2014 valuation, a discount rate of 5.2% was used, when the long-term bond yield was approximately 3.4%, i.e. a spread of 1.8%.

⁶The last three approaches to setting the discount rate show a range of results, from no convergence (Approach 2), to long-term convergence (Approach 3) and medium-term convergence (Approach 4).

If the valuation actuary could foresee the future, the observations would all fall on the 45° line. The scatter plots show how for 3-year time horizon results, using a constant long term bond yield as the valuation discount rate almost always under-estimates V_0 . This is a matter of concern because such under-estimation of V_0 could lead to a misleading perception of insufficient funding levels for an otherwise healthy scheme on a run-off basis.

The estimates improve when the backing assets are used to obtain the valuation discount rate, and they improve further if the rate is adjusted over 15 years to converge to the long-run mean.

5.2 SENSITIVITY TO HIGHER BOND ALLOCATION STRATEGY

The middle panel of Figure 2 shows the density of $V_0^{(\infty)}$ (as a percentage of A_0), when the bond allocation of the UK pension scheme is increased from 30% to 70%. For ease of reference, the density of the base case result is also included as a grey curve in the background. We make the following observations:

1. For increased bond investment, the distribution of $V_0^{(\infty)}$ has moved to the left and has greater dispersion.
2. The leftward shift of the distribution indicates a greater probability of larger deficits. This is reflected in the median of $V_0^{(\infty)}$, which becomes a deficit of 23% as compared to a surplus of 24% for the base case. The 10th and 0.5th percentiles have moved by even greater magnitudes (see Table 6).
3. The sensitivity patterns can be explained by the fact that the expected returns from bonds are lower in the long term compared to equities. So, a higher allocation to bonds can lead to potentially lower levels of cumulative assets, which is reflected in the leftward shift and greater dispersion in the distribution.
4. Moreover, fixed interest bonds are a poor match for real liabilities (the UK scheme benefits are inflation-protected). Hence, an increased allocation to nominal bonds has exacerbated the risk, producing a fatter-tailed distribution.

Alternatively, an allocation to real return bonds, such as Index Linked Gilts (ILGs), could provide some risk reduction benefits for this scheme's real liabilities, at a cost of a leftward shift in the risk distribution. As our ESG does not support ILGs, we do not consider ILGs further in this report.⁷

⁷Historical data on ILGs only dates back to 1981 when these were first issued in the UK. Wilkie et al. (2011) notes that although the demand in the UK is high from pension funds and insurance companies, the supply of ILGs is relatively limited, despite their low yield and high price. For the purposes of this article, we have not included ILGs in our ESG, as it would be inadvisable to make long term projections based on a model fitted using relatively short-term historical data. Moreover, as data for all other economic variables goes back for close to 100 years, incorporating ILGs in the ESG might make the modelling of the remaining variables less reliable.

Figure 5 shows the sensitivity of the results to a higher bond allocation strategy based on a 3-year time horizon. The most important observation here is that as the bond allocation of the scheme increases, a valuation basis based on bonds will produce results where the distributions of $V_0^{(3)}$ starts getting closer to that of $V_0^{(\infty)}$, as expected.

We summarise the main takeaways from the analysis so far as follows:

- It is natural for both pension regulators and scheme members to expect periodic assessments of the financial status of a pension scheme. For a triennial valuation, where the valuation discount rate is based only on long term bond yields (plus an arbitrary fixed spread to allow for excess equity returns), the UK scheme appears severely underfunded. Looking at the base case results for $V_0^{(3)}$ given in Table 6, the scheme appears to have a staggering 92% chance of insufficient assets at the end of 3 years, with an average deficit of 94% of the current value of assets.
- However, on a run-off approach, the chance of a shortfall is only 27%, and there is an average *surplus* of 24% of current assets. Note that the huge apparent deficit over a three year horizon is an artefact of using a valuation basis that is not consistent with the actual underlying asset allocation strategy. The true financial status is actually unaffected by how the valuation actuary is required to value future liabilities.
- Now, if in response to a requirement to use long term bond yield for liability valuation, the UK scheme tries to address the mismatch by changing its actual asset allocation strategy to involve greater bond investment, it is materially detrimental to the pension scheme over the long term. Note that the distribution of $V_0^{(3)}$ is broadly unaltered by the change in asset allocation as the valuation discount rate is unaffected by the change. However, as shown in Table 6, increasing the bond allocation from 30% to 70% now creates a *true* deficit for the scheme on a run-off basis. By increasing the bond allocation, the scheme has gone from 24% excess assets to 23% underfunded, on average. Moreover, now the chance of assets not being able to meet all future liabilities has increased from 27% to 68%.
- The impact of a 3-year assessment is somewhat less severe if the valuation discount rate is based on the returns on backing assets converging to the long term returns over an appropriate duration.

5.3 SENSITIVITY TO CONTRIBUTION RATES

The bottom panel of Figure 2 shows the distribution of $V_0^{(\infty)}$ (as a percentage of A_0) when the contribution rate is increased or decreased by 7.5% of salary; and also if there were no future contributions. For ease of reference, the density of the base case result with a contribution rate of 22.5% is also included as a grey curve in the background.

The main observation here is that increasing or decreasing contribution rates by $\pm 7.5\%$ has the effect of shifting the median of the distribution by $\pm 9\%$ (of A_0). The relative shifts in the left tail increase slightly as we move further into the tails of the distributions. However the impacts are small compared to the impact of a higher bond allocation. In fact, the impact at the median of moving the bond allocation from 30% to 70% is more severe than eliminating all future contributions as shown in Figure 2.

Figure 6 shows the sensitivity of the results towards a higher or lower contribution rate when using a 3-year time horizon. Firstly, the directions of the shifts in the distributions are as expected, but the magnitudes of the shifts are much smaller than for the run-off case. This is an artefact of the valuation methodology adopted, i.e. the projected unit method, to calculate L_3 , in which the actuarial liability is the discounted value of *accrued benefits* to date (i.e., ignoring future accruals based on future service and future contributions). Hence the liability calculations do not take into account future contributions. As a result, any contribution increase or decrease beyond year 3 has no impact on L_3 . So the shifts in the distributions in the 3-year time horizon results are solely due to the higher or lower contributions only over the time-period $[0, 3]$.

The main conclusions that can be drawn from these results are as follows:

- The impact of changes in contribution rates is smaller compared to changes in asset allocation strategy. In particular, if a short time horizon is employed, the contribution rate change needs to be extremely large in order to have any meaningful impact on pension scheme risk.
- If instead of changing asset allocation strategy, a pension scheme wants to address any apparent deficit in $V_0^{(3)}$ using higher contribution rates, the scheme will have to raise the contribution rates by enormous amounts to have a material impact on the distribution of $V_0^{(3)}$.

6. STYLISTED US PENSION SCHEME RESULTS

As with the UK scheme, for the stylised US scheme, we consider the following cases separately for the run-off and 3-year time horizon cases:

- the base case, in order to establish a baseline;
- the impact of changing the asset allocation to high bond content (30/70 in equity/bond) from the baseline allocation of high equity content (70/30 in equity/bond); and
- the impact of changing the contribution rate to 10% (high) and 3% (low), relative to the baseline contribution rate of 6.5% of salary.

6.1 BASE CASE

The top panel of Figure 7 shows the distribution, $V_0^{(\infty)}$ (as a percentage of A_0) for the stylised US scheme. The second and third panels show the sensitivity of this distribution to changes in the asset allocation and the contribution rate, which we discuss in detail in Sections 6.2 and 6.3, respectively. Table 7 provides all the summary statistics.

Focusing on the base case run-off result shown in the top panel of Figure 7 and the corresponding summary statistics in Table 7, we first observe its similarity with the corresponding result for the UK scheme as can be seen in Figure 2 and Table 6. The medians of the two distributions are the same, the probability of shortfall and the 10th percentile are also broadly similar. However, the 0.5th percentile for the stylised US scheme is farther to the left than that for the UK scheme, indicating that the distribution for the stylised US scheme has a fatter left tail.

The similarities between the base case run-off results for the UK and the stylised US schemes are partly by design to ensure a certain level of comparability. Recall that the membership profiles of the two schemes are the same. Also the starting asset value of the stylised US scheme is chosen specifically to produce the same median value of $V_0^{(\infty)}$. Although the benefit structures of the two schemes are different, with the stylised US scheme being less generous with no inflation indexation of benefits, this is compensated by a lower contribution rate.

However the fatter left tail of the distribution for the stylised US scheme is striking. This is the result of higher fixed guarantees involved in non-indexation of benefits. This feature plays a crucial role in the rest of the analysis for the stylised US scheme.

Continuing with the base case, but now focusing on a shorter 3-year time horizon, Figure 8 shows the same alternative approaches as for the UK scheme that a valuation actuary might take to value the liabilities, L_3 , at the end of year 3.

1. Firstly, we look at valuation discount rate based solely on the prevalent bond yield at the end of 3 years. Unlike for the UK scheme, where we added a spread to the long term bond yield for consistency with the USS 2014 valuation, for the US scheme we do not introduce this complexity of adding a fixed spread.

In the top-left panel of Figure 8, we show $V_0^{(3)}$ (as a percentage of A_0), when L_3 is calculated using the prevalent long term bond yield at the end of 3 years. Clearly, this approach produces a very different distribution of $V_0^{(3)}$, as compared to $V_0^{(\infty)}$, with a large median loss and a large negative 10th percentile.

However, it is instructive to observe that the difference between the distributions of $V_0^{(3)}$ and $V_0^{(\infty)}$ for the stylised US scheme is not as drastic as that for the UK scheme. This is due to the fact that for the UK scheme, fixed interest bonds are a poor match for real liabilities. However for the stylised US scheme with non-indexed benefits, liabilities calculated using long term bond yield are still poor, but less so.

Table 7: Summary of results for the stylised US scheme using the graphical ESG.

Scenarios	$P[V_0 \leq 0]$	50 th percentile		10 th percentile		0.5 th percentile	
		VaR	ES	VaR	ES	VaR	ES
$V_0^{(\infty)}$							
Base case	0.28	24	-17	-40	-86	-194	-267
High bond allocation	0.53	-3	-46	-69	-111	-205	-280
High contribution	0.22	31	-8	-28	-71	-170	-237
Low contribution	0.33	18	-25	-48	-95	-201	-270
No contribution	0.38	13	-33	-58	-108	-219	-298
$V_0^{(3)}$ (Valuation rate based only on long term bond yields)							
Base case	0.62	-17	-66	-95	-134	-209	-284
High bond allocation	0.66	-20	-58	-79	-108	-166	-240
High contribution	0.60	-14	-63	-91	-130	-202	-275
Low contribution	0.63	-19	-69	-97	-137	-209	-283
$V_0^{(3)}$ (Valuation rate based on returns on backing assets)							
Base case	0.34	34	-58	-120	-200	-330	-416
High bond allocation	0.43	10	-56	-99	-160	-275	-337
High contribution	0.33	36	-55	-116	-196	-326	-406
Low contribution	0.35	31	-60	-122	-202	-335	-416
$V_0^{(3)}$ (Valuation rate converging to long term returns after 50 years)							
Base case	0.28	38	-26	-67	-124	-224	-300
High bond allocation	0.39	15	-38	-71	-116	-205	-270
High contribution	0.27	40	-23	-63	-120	-219	-290
Low contribution	0.29	36	-29	-69	-126	-227	-299
$V_0^{(3)}$ (Valuation rate converging to long term returns after 15 years)							
Base case	0.21	41	-5	-33	-76	-156	-230
High bond allocation	0.34	19	-23	-47	-84	-153	-226
High contribution	0.20	43	-2	-30	-72	-150	-220
Low contribution	0.22	39	-8	-36	-78	-158	-229

2. The top-right panel of Figure 8 shows the result if the third year's returns (weighted by asset allocation) are used as the discount rate for all future years. The bottom-left and bottom-right panels of Figure 8 show the result if the time period of convergence to long term mean is 50 years and 15 years, respectively. Commentary on these panels is the same as for the UK scheme.

Similar to the UK scheme results, we show the scatter plots, or joint densities, of $V_0^{(3)}$ and $V_0^{(\infty)}$ for the stylised US scheme in Figure 9. Although using a constant long term bond yield as the valuation discount rate underestimates V_0 , it is less prominent for the stylised US scheme than that for the UK scheme. However, the estimates do improve when returns based on backing assets, in conjunction with convergence to the long-run mean, are used to obtain the valuation discount rate.

6.2 SENSITIVITY TO HIGHER BOND ALLOCATION STRATEGY

The middle panel of Figure 7 shows the distribution of $V_0^{(\infty)}$ (as a percentage of A_0), when the bond allocation of the stylised US scheme is increased from 30% to 70%. For ease of reference, the density of the base case result is also included as a grey curve in the background. We make the following observations:

1. Similar to the UK scheme with increased bond investment, the distribution of $V_0^{(\infty)}$ has moved to the left and has greater dispersion.
2. The leftward shift of the distribution indicates a greater probability of larger deficits. This is reflected in the median of $V_0^{(\infty)}$, which shows a deficit of 3% as compared to a surplus of 24% for the base case results.
3. The leftward shift can be explained by the fact that the expected returns from bonds are lower in the long term compared to equities, leading to potentially larger losses.
4. However, the leftward shift of the distribution for the stylised US scheme is less severe than that for the UK scheme. As the UK scheme benefits are fully inflation-protected, high fixed interest investment is far more detrimental for the UK scheme.

Figure 10 shows the sensitivity of the results to a higher bond allocation strategy when using a 3-year time horizon. The most important observation here is that as the bond allocation of the scheme increases, a valuation basis based on bonds will produce results where the distributions of $V_0^{(3)}$ starts getting closer to that of $V_0^{(\infty)}$, as expected.

As for the UK scheme, we provide a summary of the main takeaways from the analysis so far for the stylised US scheme:

- If we consider a triennial valuation, where the valuation discount rate is based only on long term bond yields, the scheme appears severely underfunded. Looking at

the results for $V_0^{(3)}$ given in Table 7, under the base case assumption, the scheme appears to have a 62% chance of insufficient assets at the end of 3 years, with an apparent median deficit of 17%. On the other hand, using the run-off basis, the chance of a deficit is only 28%, and the scheme has a true average surplus of 24% of current assets.

- As with the UK scheme, the apparent deficit is an artefact of using a valuation basis that is not consistent with the actual underlying asset allocation strategy. The true financial status is actually unaffected by how the valuation actuary is required to value future liabilities.
- However, if this valuation method is a prescribed requirement, the stylised US scheme might be obliged to address this mismatch by changing its asset allocation strategy to involve greater bond investment. This course of action is actually materially detrimental to the long term financial health of the pension scheme.
- As shown in Table 7, increasing the bond allocation from 30% to 70%, now creates a *true* deficit for the scheme, even on a run-off basis. By increasing its bond allocation, the fund has gone from 24% excess assets to 3% underfunded. Moreover, now the chance of assets not being able to meet all future liabilities has increased from 28% to 53%.
- We also note that the severity of the impact of the change in the asset allocation strategy is less severe for the stylised US scheme than that for the UK scheme.

6.3 SENSITIVITY TO CONTRIBUTION RATES

The bottom panel of Figure 7 shows the distribution of $V_0^{(\infty)}$ (as a percentage of A_0), when the contribution rate is increased or decreased by 3.5%; and also if there were no future contributions. For ease of reference, the density of the base case result with contribution rate of 6.5% is also included as a grey curve in the background.

Consistent with the results for the UK scheme, the main observation is that increasing or decreasing contribution rates by $\pm 3.5\%$ has the effect of shifting the median of the distribution by roughly $\pm 6\%$ (of A_0). The relative shifts in the left tail increase slightly as we move further into the tails of the distributions. However the impacts are small compared to the impact of a higher bond allocation. In fact, the impact at the median of moving the bond allocation from 30% to 70% has over twice the impact of eliminating all future contributions.

Figure 11 shows the sensitivity of the results towards a higher or lower contribution rate when using a 3-year time horizon. As with the UK scheme, the direction of the shifts in the distributions are as expected, and the magnitudes of the shifts are much smaller than for the run-off case.

As for the UK scheme, the main conclusion here is that the impact of changes in contribution rates is smaller compared to changes in asset allocation strategy. In particular, if a short time horizon is employed, the contribution rate change needs to be extremely large in order to have any meaningful impact on pension scheme risk. If instead of changing asset allocation strategy, a pension scheme wants to address any apparent deficit in $V_0^{(3)}$ using higher contribution rates, the scheme will have to raise the contribution rates by enormous amounts to have a material impact on the distribution of $V_0^{(3)}$.

We have attempted to account for significant and meaningful differences between the US scheme and UK scheme, including the benefit structure, ESG and mortality rates, thereby assessing a more representative range of DB pension schemes in general. Despite these differences and the resulting variation in the surplus distributions, our two main observations are valid for both schemes.

7. SUMMARY AND CONCLUSIONS

In this article we have examined the impact of employing different time horizons to evaluate the risk of two pension schemes with different benefit structures and in different economic settings.⁸ The aspiration of any pension scheme is to be able to provide for all promised benefits. This suggests a very long term planning horizon. However, members of the scheme and pension regulators have an interest in interim assessments of the financial status of a pension scheme, which suggests a short-term planning horizon.

Years of high inflation and good investment returns during the 1970s and 1980s created the perception that defined benefit (DB) pension schemes were easily affordable. Over the past few decades however, increasing life expectancy and steady fall in interest rates have meant that pension costs have increased. Increased pressure on the affordability of these schemes has led to the closure of many schemes and the widespread implementation of so-called de-risking strategies, such as moving away from predominantly equity investments to greater bond investments. Simultaneously, contribution rates have steadily increased.

We have shown that the risk assessment horizon has an impact on the quantification of the risk profile of pension schemes. Moreover, this conclusion is strikingly consistent across two different scheme structures under different (and internally consistent) assumptions about the economic model and mortality projections. This suggests that the assumptions used to determine the financial status of a scheme in the short term should minimise, to the extent possible, the differences between the risk profiles over the two risk assessment horizons.

In general, we find that the approach to setting the discount rate that generates the least distortion, relative to the discount rate that would be set by a fully prescient actuary,

⁸As noted above, these are generic pension schemes, with risk estimates abstracted from the effect of the employer covenant.

is one that uses a time-varying discount rate. That discount rate increases (or decreases) from the investment return actually realised in the third year to its long term mean over an intermediate period of time, say 15 years.

Regardless of the approach taken to setting the discount rate, the difference in time horizon generates different conclusions regarding the best approach to manage risk through changes in asset allocation. Over the long term, increasing a scheme's allocation to long term bonds worsens the risk profile – both reducing the median level of surplus (increasing the median level of deficit) and increasing the spread between the median and the 10th percentile level of deficit. When examined over the shorter time horizon, increasing a scheme's allocation to long bonds reduces the median to 10th percentile spread. However, this perceived short-term risk-reduction by shifting to bond investment might create true deficits in an otherwise financially healthy pension scheme, when viewed over the long run.

We also find that the choice of the risk assessment time horizon can have a differential impact on the quantified risk of the UK scheme and the stylised US scheme. The UK scheme with inflation-protected benefits is more adversely affected by a shift from equities to bonds. This result could potentially be moderated if we allowed for the scheme to invest in inflation-linked bonds, but we leave this question to future research.

Overall, we believe that it would be in the interest of all relevant parties to minimise any distortion created by taking a short-term view of a scheme's financial status. We also find that the impact on a scheme's financial status of changing contribution rates is much smaller than the impact of changes to the scheme's asset allocation.

All of our results remain qualitatively the same when using a different ESG.

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DISCLAIMER

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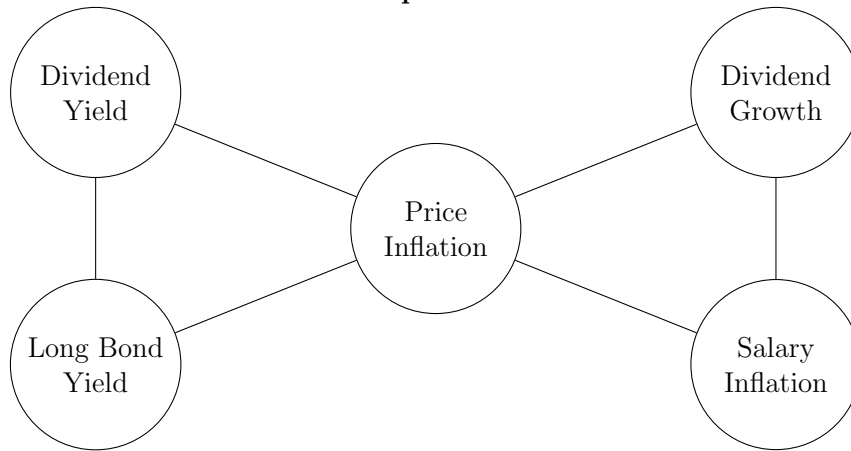
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UK Graphical Model



US Graphical Model

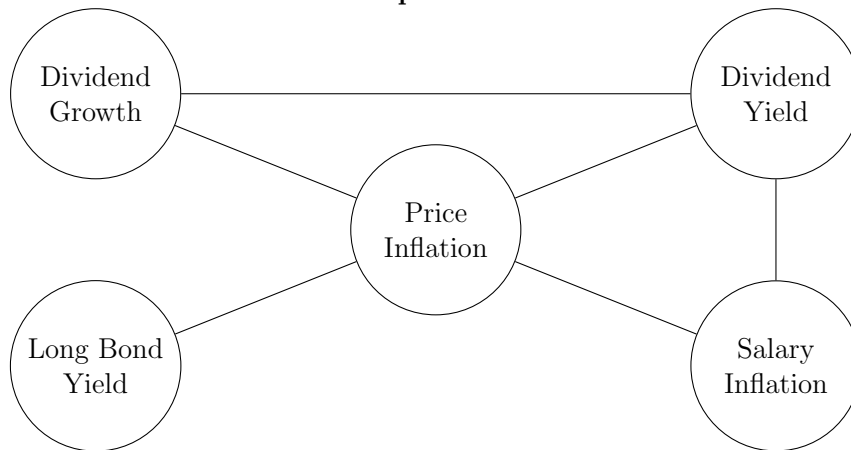


Figure 1: UK and US graphical models.

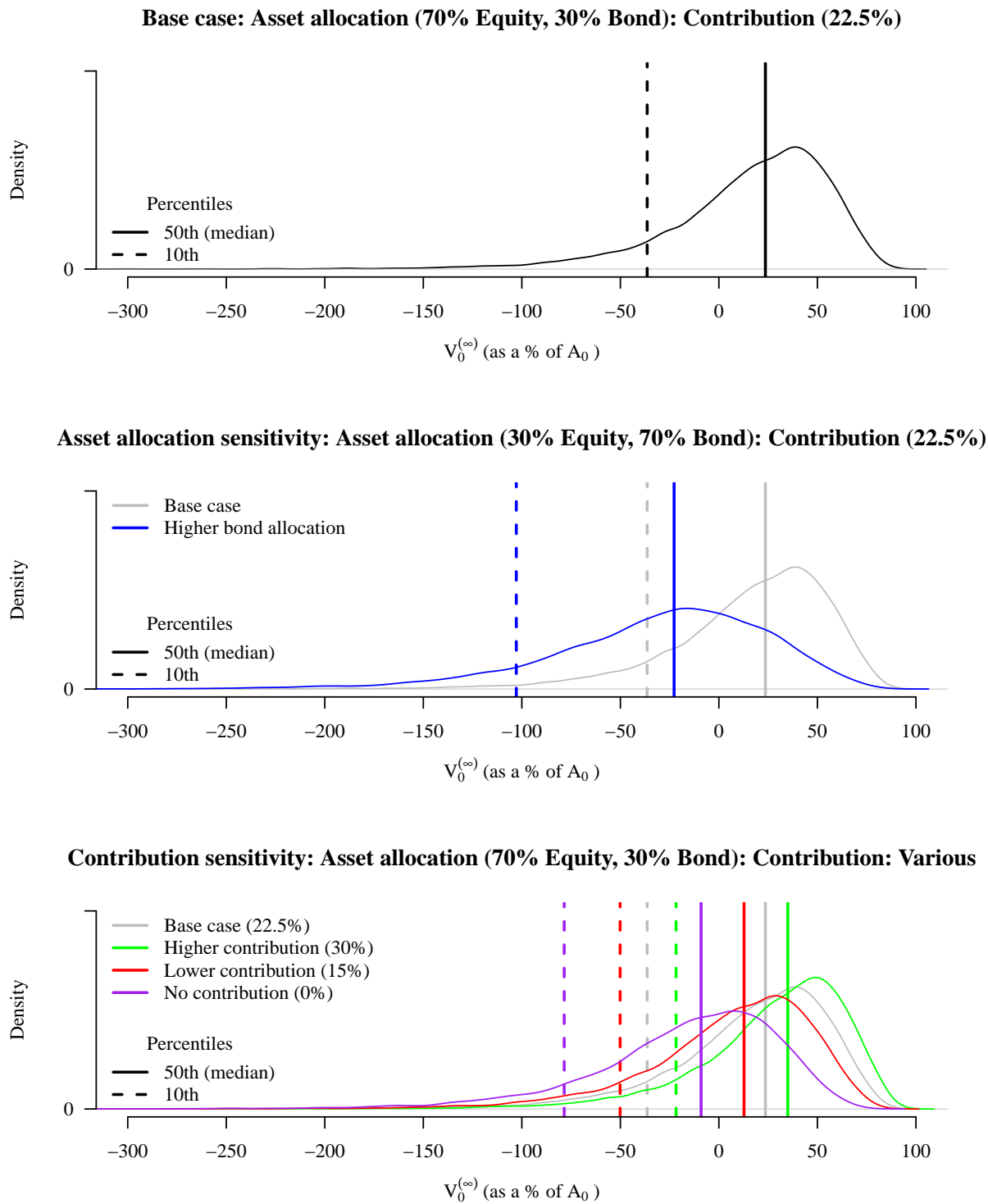


Figure 2: Run-off results for the UK scheme using the graphical ESG. Top panel shows base case. Middle panel shows sensitivity to asset allocation strategy with higher bond allocation. Bottom panel shows sensitivity to changing contribution rates.

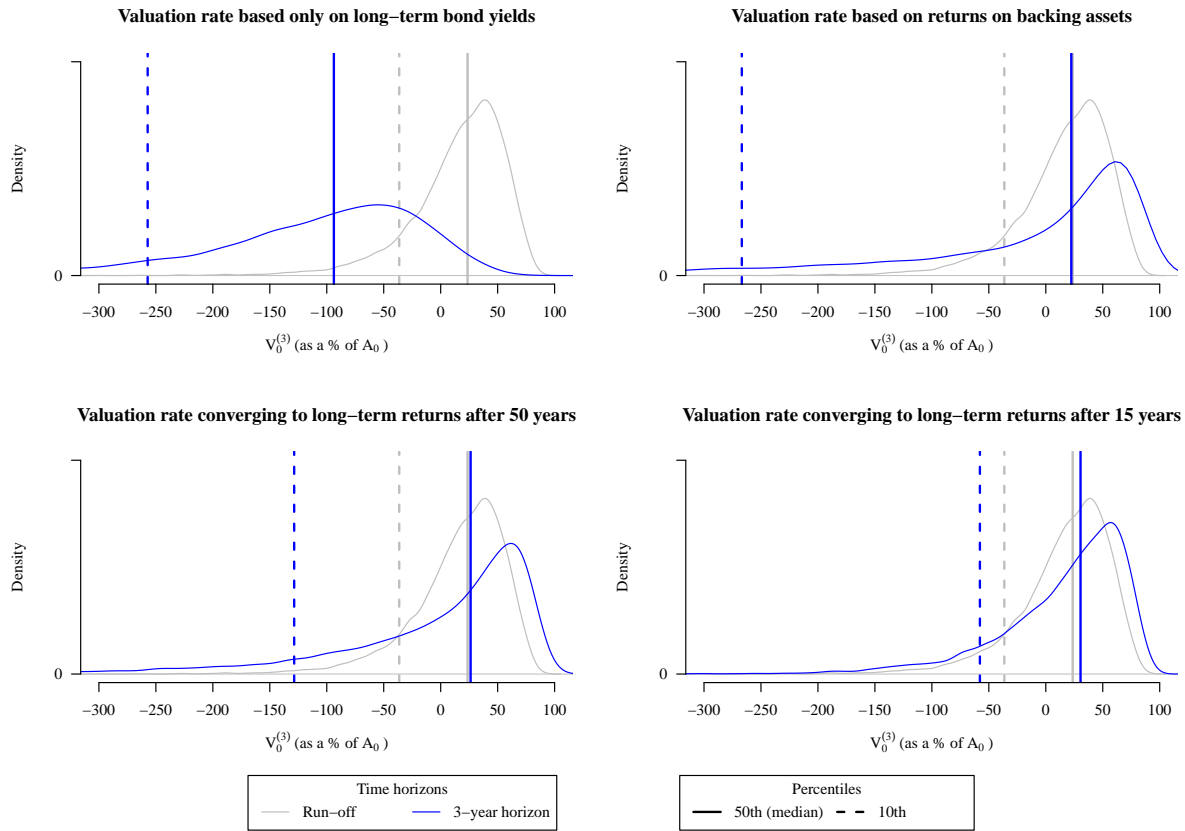


Figure 3: 3-year time horizon results for the UK scheme using base case assumptions and the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.

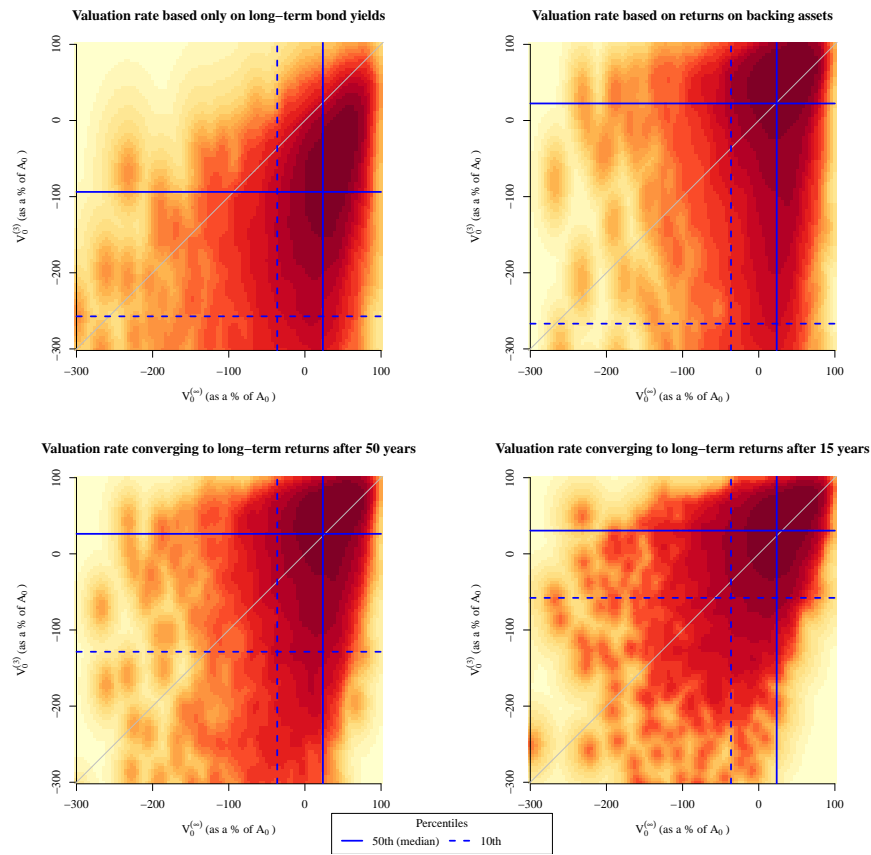


Figure 4: 3-year time horizon results for the UK scheme using base case assumptions and the graphical ESG. Each panel shows joint density of $V_0^{(\infty)}$ and $V_0^{(3)}$, as heatmaps, based on different discount rate approaches used in the valuation basis.

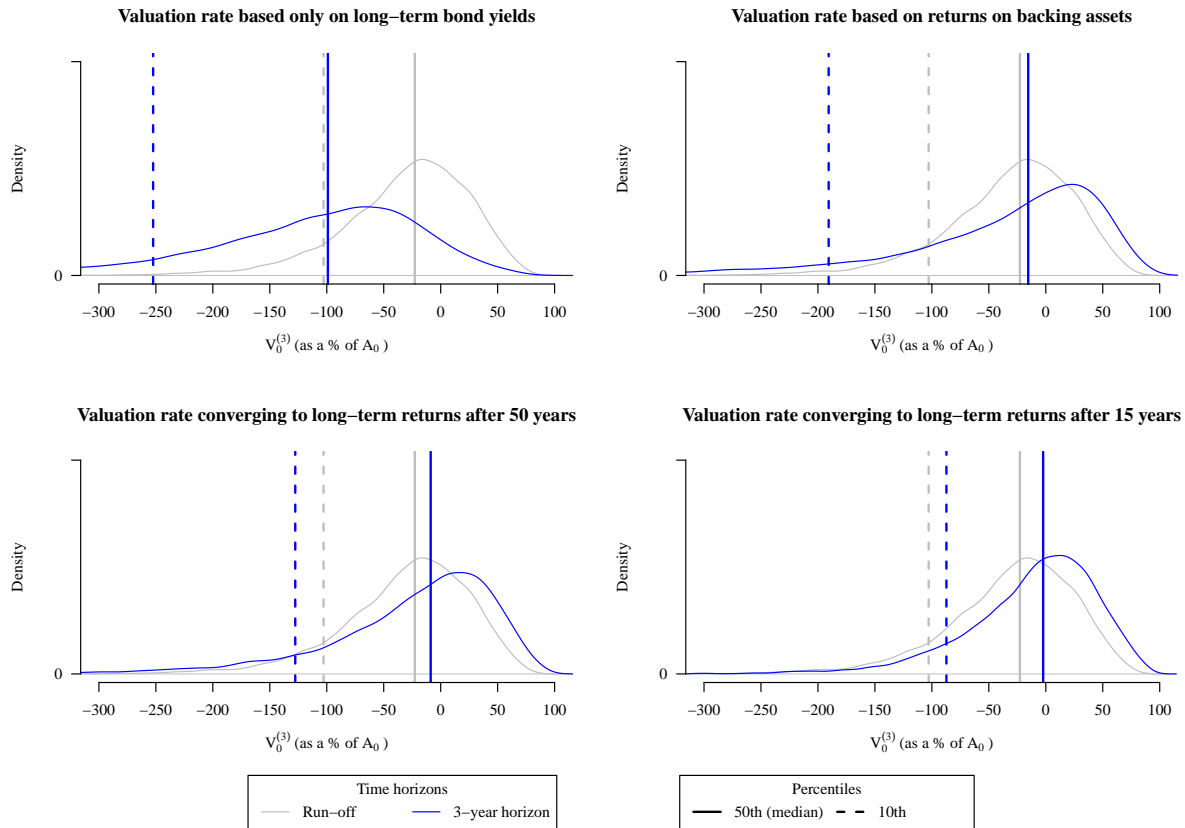


Figure 5: 3-year time horizon results showing sensitivity to higher bond allocation strategy for the UK scheme using the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.

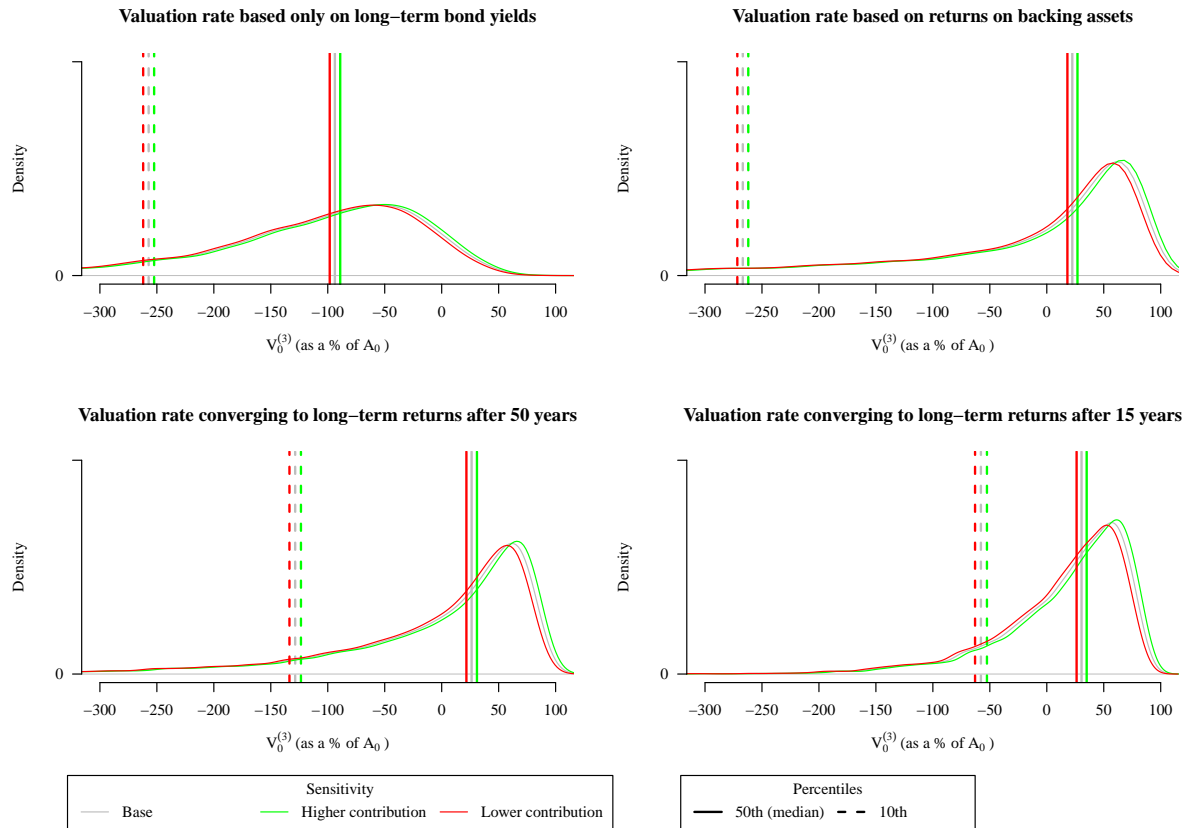


Figure 6: 3-year time horizon results showing sensitivity to higher and lower contribution rates for the UK scheme using the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the base case, $V_0^{(3)}$, is also shown, as a grey curve, for reference.

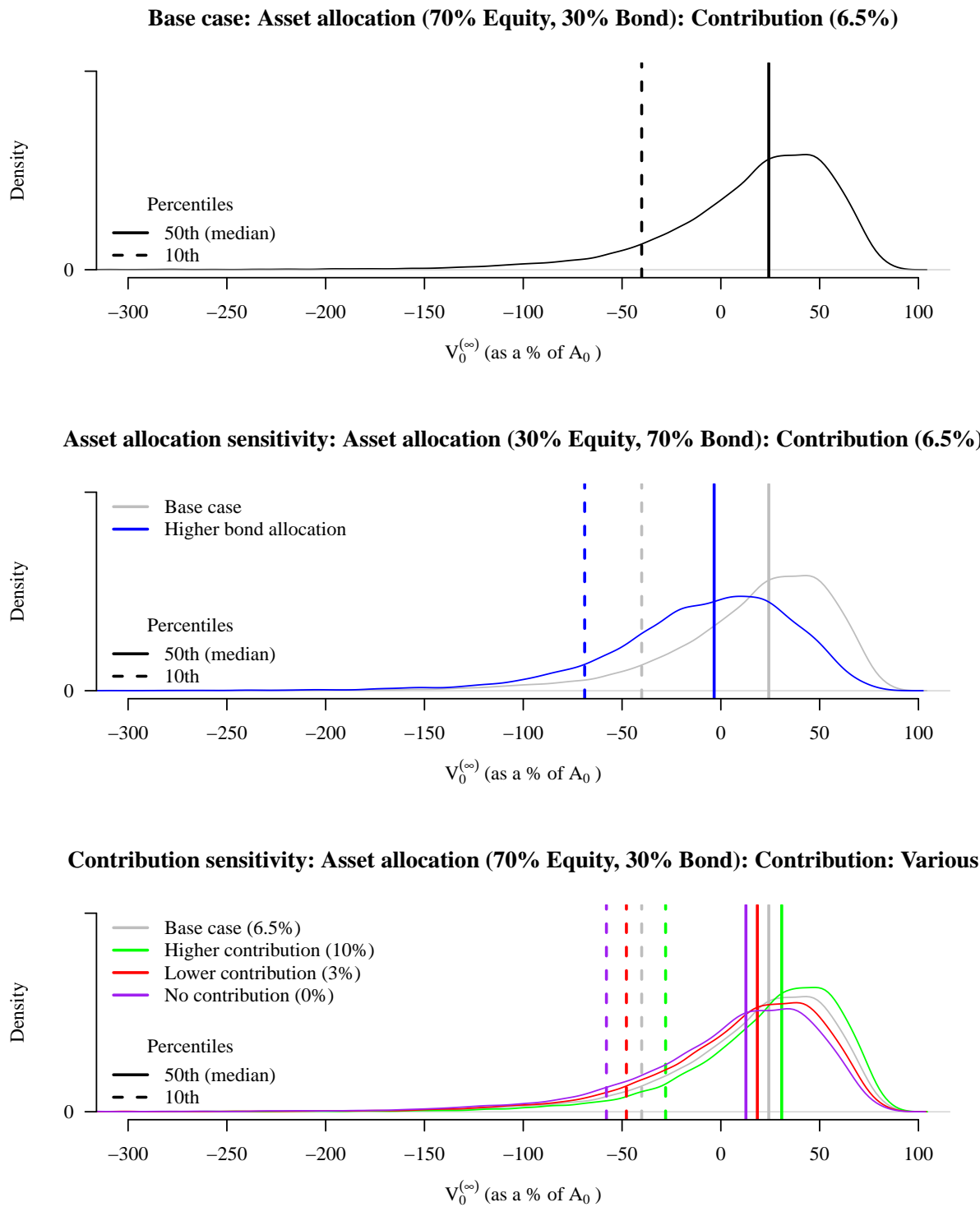


Figure 7: Run-off results for the stylised US scheme using the graphical ESG. Top panel shows base case. Middle panel shows sensitivity to asset allocation strategy with higher bond allocation. Bottom panel shows sensitivity to changing contribution rates.

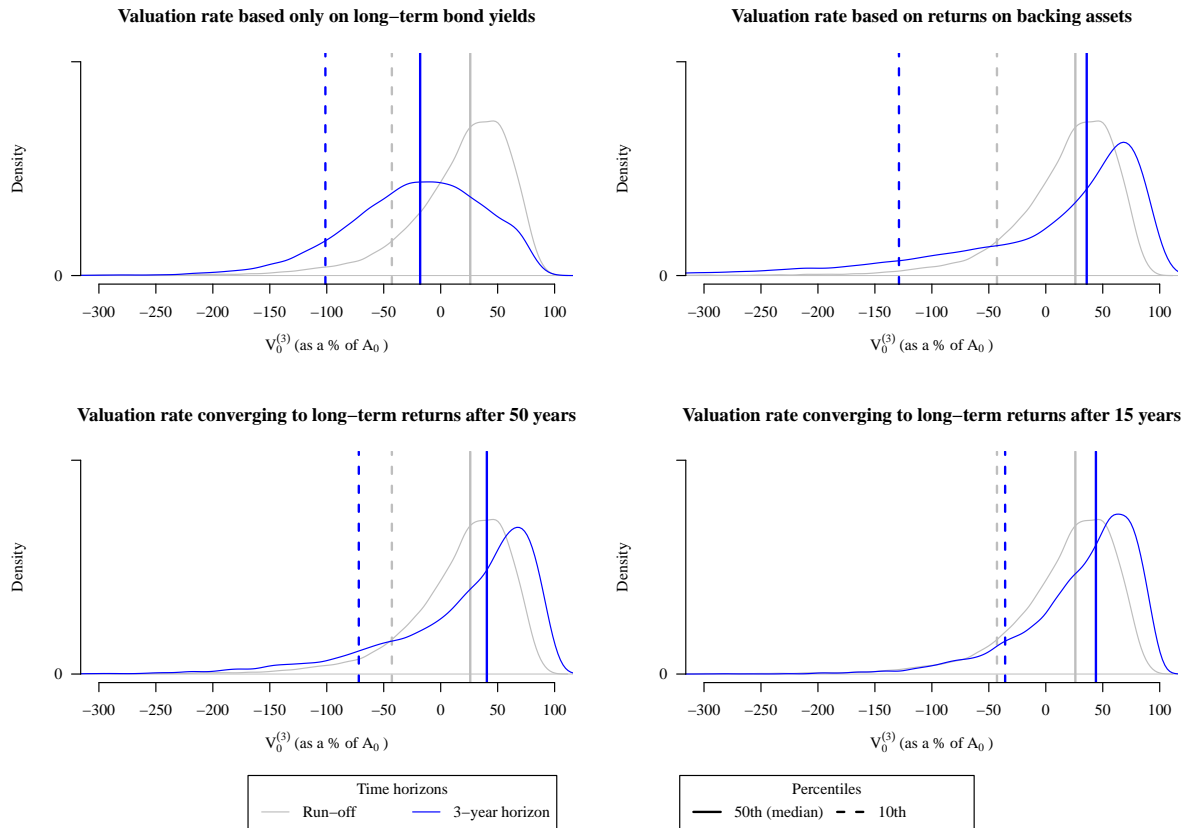


Figure 8: 3-year time horizon results for the stylised US scheme using base case assumptions and the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.

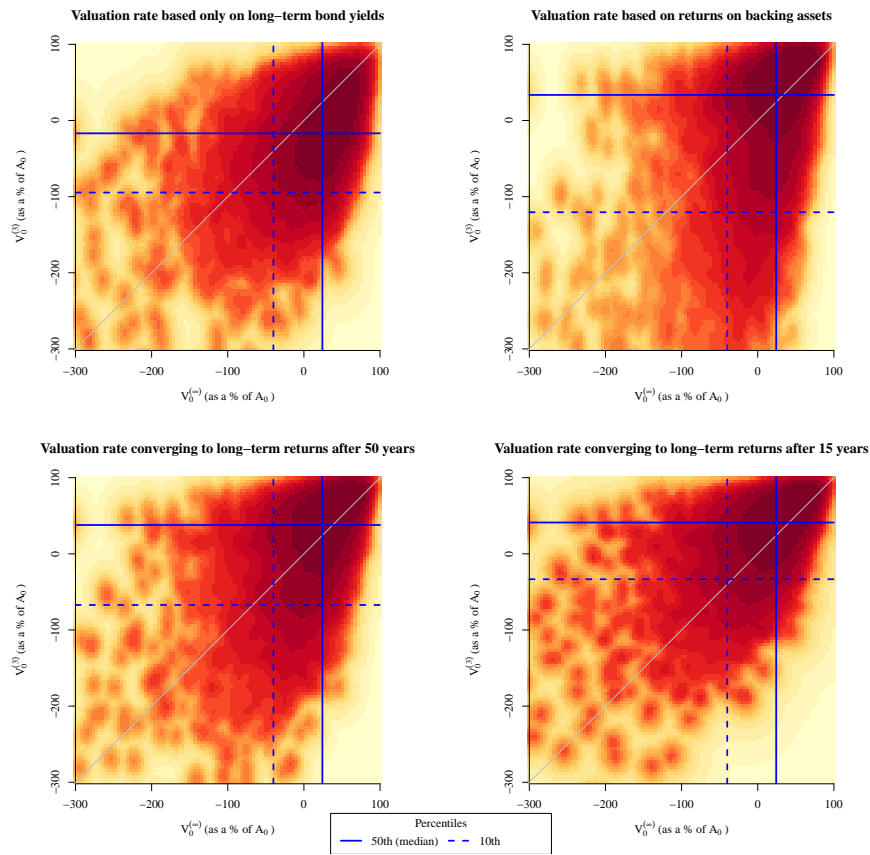


Figure 9: 3-year time horizon results for the stylised US scheme using base case assumptions and the graphical ESG. Each panel shows joint density of $V_0^{(\infty)}$ and $V_0^{(3)}$, as heatmaps, based on different discount rate approaches used in the valuation basis.

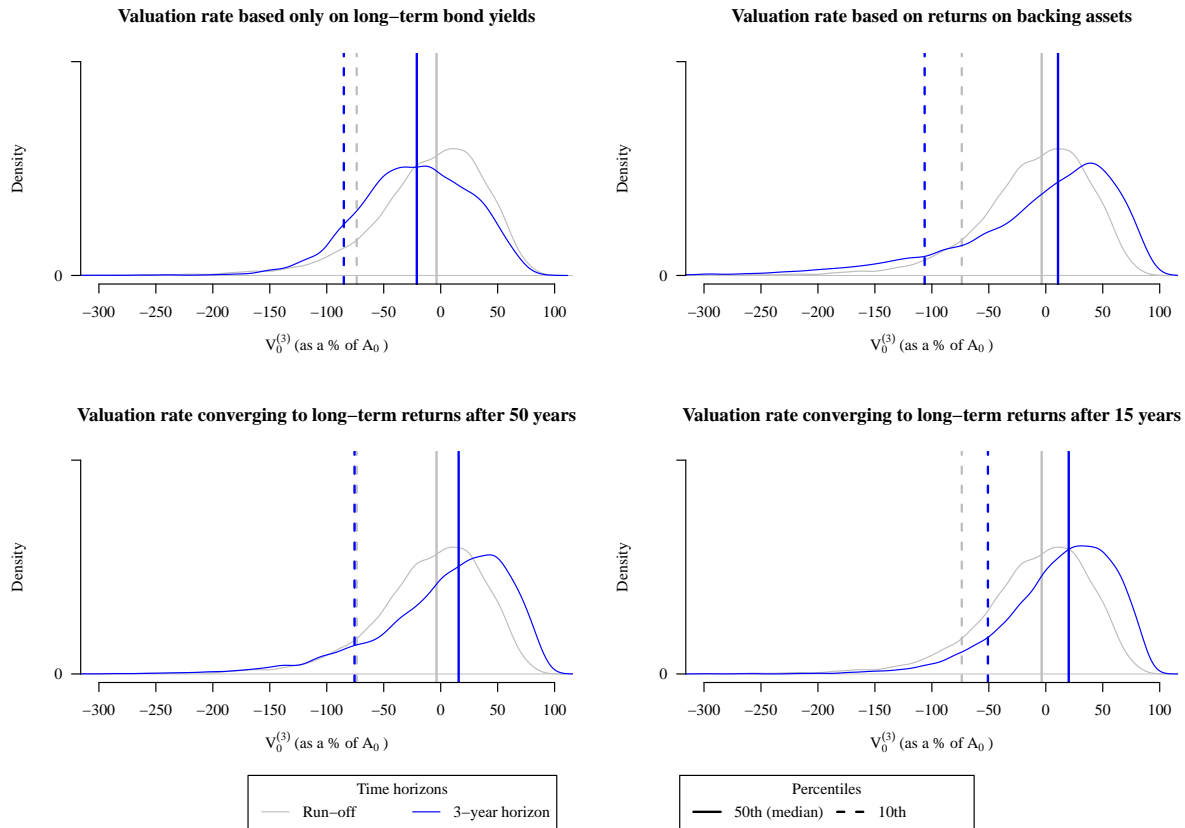


Figure 10: 3-year time horizon results showing sensitivity to higher bond allocation strategy for the stylised US scheme using the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the run-off result, $V_0^{(\infty)}$, is also shown, as a grey curve, for reference.

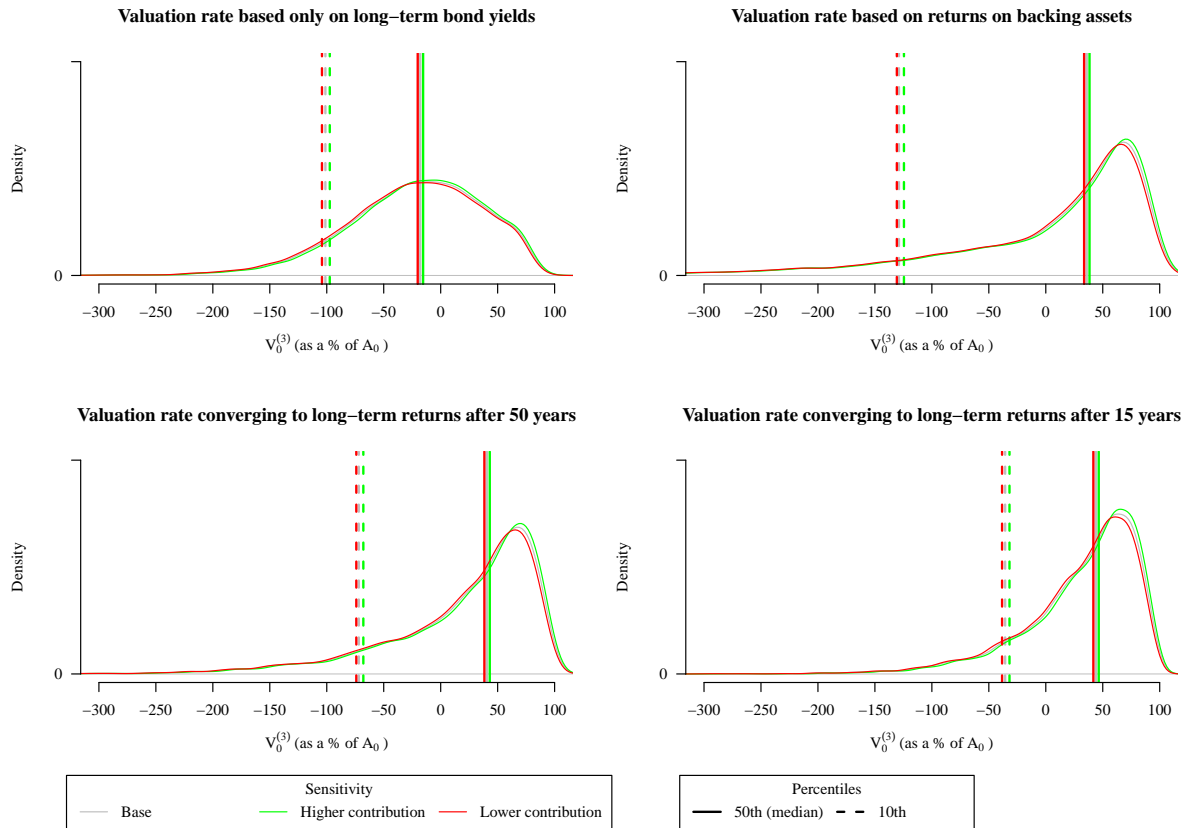


Figure 11: 3-year time horizon results showing sensitivity to higher and lower contribution rates for the stylised US scheme using the graphical ESG. Each panel shows the density of $V_0^{(3)}$ based on different discount rate approaches used in the valuation basis. In each panel, the density for the base case, $V_0^{(3)}$, is also shown, as a grey curve, for reference.