Optimal burn-in time and imperfect maintenance strategy for a warranted product with bathtub shaped failure rate

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Abstract: '*Burn-in/preventive maintenance*' programme is an efficient approach used to minimise the warranty servicing cost of a product with bathtub shaped failure rate. Burn-in is a widely used method to improve the quality of product during its 'infant mortality' period and preventive maintenance is a scheduled necessary activity carried out during its 'wear-out' period. In this paper, an optimisation model is developed to determine the optimal burn-in time and optimal imperfect preventive maintenance strategy that minimises the total mean servicing cost of a warranted product with an age-dependent repair cost. We provide a numerical study to illustrate our results.

Keywords: bathtub shaped failure rate; BTR; burn-in; preventive maintenance; PM; minimal repair; MR; warranty servicing cost.

Reference to this paper should be made as follows: Shafiee, M. and Asgharizadeh, E. (2011) 'Optimal burn-in time and imperfect maintenance strategy for a warranted product with bathtub shaped failure rate', *Int. J. Collaborative Enterprise*, Vol. 2, No. 4, pp.263–274.

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1 Introduction

In today's market, product warranty plays an increasingly important role. The use of warranty is widespread and serves many purposes, including protection for manufacturers, sellers, insurance, buyers and users. A warranty is a contract offered by a producer to a consumer to replace or repair a faulty item within a predetermined *warranty period* (Zuo et al., 2000). Due to advances in technology and the pressure of the competition in the market place, the length of the warranty period has been steadily increasing since the beginning of the 21st Century. Currently, some manufacturers are offering up to five years warranty for the products and six or more years warranty for some parts (Chattopadhyay and Rahman, 2008).

With extended warranties, items are covered for a significant part of their useful lives. This implies that failures due to manufacturing defects during *infant mortality* and degradation during *wear out* can occur within the warranty period. Although offering longer warranty increases customers' satisfaction, it incurs additional costs to the manufacturer. Along with increasing the warranty period, reducing warranty costs has become an issue of great importance to manufacturers.

Basically, there are two main strategies to reduce the warranty servicing cost of products with bathtub shaped failure rates:

a The *burn-in* procedure when products are operated for a reasonably short period of time prior to usage. This strategy can be effective when the initial failure rate is strictly decreasing (infant mortality).

b *Preventive maintenance (PM)* actions at discrete time instants over the warranty period that can effectively reduce the age of the item when the failure rate is strictly increasing (wear-out).

Pham and Wang (1996) mentioned that a maintenance action could be classified into *perfect* maintenance, *minimal* repair (MR) or *imperfect* maintenance. A perfect maintenance restores a product to be as good as new; an MR restores a product to have the same failure rate condition as it had just right before failure and an imperfect maintenance makes a product better than what it had before failure but not necessarily to be as good as new.

There are many interesting questions related to the link burn-in and PM for warranted products with bathtub shaped failure rate. For example: Should burn-in procedure be applied to products during infant mortality? If so, how long this procedure should be? Should PM action be used during wear-out period? If so, what type should it be – a minimal, imperfect or perfect? Moreover, what are the optimal number of PM actions and the corresponding maintenance degrees?

This paper develops a mathematical model to derive the optimal burn-in time, optimal number of PM actions and the corresponding maintenance level simultaneously such that the total mean servicing cost of the product with the bathtub shaped failure rate is minimised. In this study, we consider an imperfect PM action which is carried out periodically during the wear-out period until the warranty expires.

The rest of this paper is organised as follows. In Section 2, we carry out a review of the literature dealing with burn-in, warranty and maintenance. In Section 3, the model components, including the bathtub shaped failure rate, burn-in procedure and PM model are described in detail. In Section 4, we formulate our optimisation model and provide a numerical example to illustrate our results. Finally, concluding remarks are made in Section 5.

2 Literature review

Determining the optimal burn-in time has been the subject of numerous studies aimed at the mean residual life, the percentile residual life, the burn-in reliability and the total servicing costs. Restricting our attention only on publications within the last five years, we mention the following studies.

Wu and Clements-Croome (2007) developed two burn-in policies for products having dormant states and sold under warranty. Cha and Mi (2007) determined the optimal burn-in time for a periodically inspected system. Wu et al. (2007) developed a cost model to determine the optimal burn-in time and warranty policy for non-repairable products sold under the FRW/PRW policy. Bebbington et al. (2007) determined the optimal burn-in time to maximise mean residual life of a product with bathtub shaped failure rate. Cha et al. (2008) determined an upper bound for optimal burn-in time of a warranted product with two types of minor and catastrophic failures. Kim and Kuo (2009) determined the optimal burn-in time for repairable non-series systems to maximise reliability. Kwon et al. (2010) determined the optimal burn-in time and replacement policy for a product with bathtub shaped failure rate. Cha and Finkelstein (2010a) considered 'shocks' (as a method of burn-in) and determine the optimal severity levels of the shocks to minimise the expected costs. Cha and Finkelstein (2010b) defined two types of risks and investigate the optimal burn-in time, which minimises the mean number of repairs during the field operation. Shafiee and Chukova (2010) investigated the optimal burn-in/corrective maintenance strategy to minimise the total expected warranty servicing cost. Cha and Finkelstein (2011) developed a burn-in model for a population which is composed of two stochastically ordered subpopulations. Shafiee et al. (2011a) investigated the optimal burn-in time and periodic PM strategy for a warranted product. Also, as a good source of references for minimising a cost function that incorporates burn-in cost, repair cost and warranty cost, readers can refer to Sheu and Chien (2005).

There is an increasing amount of work on the optimisation of maintenance policies for equipment covered by warranty contracts. Papers published from year 2009 to year 2011 are briefly reviewed below.

Samatlı-Paç and Taner (2009) developed repair strategies for one- and two-dimensional warranties with the objective of minimising manufacturer's expected warranty cost. Huang and Yen (2009) determined the optimal warranty policy for products sold under two-dimensional warranties and preventive maintenance. Yeo and Yuan (2009) determined the optimal maintenance period and optimal level of repair based on the structures of the cost function and failure rate function. Chien (2010) proposes an age-replacement policy for products sold under a fully renewing FRW/PRW strategy. Jung et al. (2010) considered a system maintenance policy during the post-warranty period for a product sold under a renewing warranty policy. Shafiee et al. (2011b) determined the optimal maintenance level of warranted second-hand products. Wu et al. (2011) developed a general periodic PM policy for a repairable warranted system. Wu and Longhurst (2011) proposed an opportunity-based age replacement policy for equipment protected by both base and extended warranty policies. Tsai et al. (2011) developed an imperfect maintenance strategy within a finite time span of warranty for repairable products.

3 The components of the model

3.1 Bathtub shaped failure intensity

A variety of products have three phases in their lives, characterised by their failure-rate functions. Suppose the lifetime X of a product has density $f(t)$ and survival function $\bar{F}(t)=1-F(t)$ with $\bar{F}(0)=1,$ where $F(t)$ is the distribution of X. We further assume that the failure-rate function $h(t)$ given by $h(t)=f(t)/\bar{F}(t)$ is well defined for all $t \geq 0$.

Definition: A failure rate function $h(t)$ is said to be a bathtub-shaped failure rate (BTR) with two change points t_1 and t_2 If there exist change points $0 \leq t_1 \leq t_2 < \infty$ such that:

$$h(t)=\begin{cases} \text{strictly decreasing for } 0 \leq t \leq t_1 \\ \text{constant for } t_1 \leq t \leq t_2 \\ \text{strictly increasing for } t_2 \leq t \end{cases}$$

The time interval $[0, t_1]$ is called the infant mortality period; the interval $[t_1, t_2]$ is called the normal operating period or useful period and the interval $[t_2, \infty]$ is called the wear-out period.

3.2 Burn-in procedure

Consider a fixed burn-in time b and begin to burn-in a new item from time point 0. If the item fails at time t before burn-in time b, then it is repaired minimally and the burn-in procedure is continued with the repaired component. After the fixed burn-in time b, the burn-in stops and the product is released to the market. We assume that the cost of an MR of the item which fails at time age t during burn-in period $(0 \leq t \leq b)$, $c_1(t)$ is a continuous non-decreasing function of t. Since performing an MR becomes more expensive as the component ages, this assumption is more realistic than considering the MR cost as a constant value.

Let $E[c(b)]$ be the expected burn-in cost for a repairable product with burn-in time b. We assume the following cost structure:

$$E[c(b)]=c_s+c_b b+E[M_b], \tag{1}$$

where c_s is the fixed set up cost of burn-in per unit of product, c_b is the burn-in cost per unit time and $E[M_b]$ is the expected MR cost during the burn-in period.

Theorem 1: Let $N_x(y)$ be the random variable denoting the number of MRs for a component performed in the interval $[x, x + y]$. Let $C(t)$ be the cost of MR of the component which fails at time t. The expected MR cost in the interval $[x, x + y]$ is $\int_x^{x+y} C(t) \times h(t)dt.$

Proof: Denote by $H(t)$ the cumulative failure rate function of the product. Suppose that $N_x(y)= k$ and T_1, T_2, … T_k are the time of MRs. Then the total MR cost on the interval $[x, x+y]$ is $\sum_{i=1}^{k} C(T_i).$ Given $N_x(y) = k,$ it is well known that $U_1 = H(T_1),\ U_2 = H(T_2),\dots,\ U_k = H(T_k)$ are distributed as the order statistics of a random sample of size k from the *Uniform* distribution on $[H(x), H(x + y)]$. Hence, the expected MR cost in the interval $[x, x + y]$ is:

$$E\left[M_b\right] = E\left(E\left(\sum_{i=1}^{N_x(y)} C\left(T_i\right)\middle|N_x(y) = k\right)\right),$$

Here,

$$\begin{aligned} E\left(\sum_{i=1}^{N_x(y)} C\left(T_i\right)\middle|N_x(y) = k\right) &= E\left(\sum_{i=1}^{k} C\left(H^{-1}\left(U_i\right)\right)\right) = k \times E\left(C\left(H^{-1}\left(U_i\right)\right)\right) \\ &= \frac{k}{H(x+y) - H(x)} \int_{H(x)}^{H(x+y)} C\left(H^{-1}(t)\right)dt, \end{aligned}$$

Therefore,

$$\begin{aligned} E\left[M_b\right] &= \left(\frac{1}{H(x+y) - H(x)} \int_{H(x)}^{H(x+y)} C\left(H^{-1}(t)\right)dt\right) \times E\left(N_x(y)\right) \\ &= \int_{H(x)}^{H(x+y)} C\left(H^{-1}(t)\right) = \int_x^{x+y} C(t) \times h(t)dt. \end{aligned}$$

□

By using Theorem 1, the expected burn-in cost is as follows:

$$E[h(b)] = c_s + c_b b + \int_0^b c_1(t) \times h(t)\, dt. \tag{2}$$

It is obvious that the expected burn-in cost per unit is an increasing function of the burn-in time.

3.3 Maintenance model

Let t be the time in field use (after burn-in) and $h_b(t)$ and $H_b(t)$ denote the corresponding failure (hazard) rate function and the cumulative failure rate function, respectively. Since we have assumed that failures during burn-in are corrected by MRs, we have the following relationships:

$$r_b(t) = r(b+t),\ H_b(t) = H(b+t) - H(b). \tag{3}$$

Let w ($> t_2$) represent the length of warranty period and τ is the fixed time interval between two successive PM actions. We assume that n (≥ 1) periodic PM actions are carried out by the manufacturer at times $y_1 < y_2 < \ldots < y_n$ during the wear-our period with $y_j = t_2 + j\tau$ $(1 \le j \le n)$ and $y_{n+1} = w$.

Several linear and non-linear PM models have been proposed so far [for more see Wu and Zuo (2010)]. We assume that the effect of PM action is to reduce the age of the system proportional to time elapsed from the last PM action. The idea that maintenance action reduces the age of the system is due to Kijima's (1989) and Kijima et al. (1988) virtual age models.

Let v_j denote the virtual age of the system just after the j^{th} PM action. The virtual age just before the j^{th} PM is given by $v_{j-1} + \tau$ with $v_0 = t_2$. After performing the j^{th} PM action at time y_j, the age of a product is $k\tau$ units of time younger than before the maintenance. Then, the virtual age just after the j^{th} PM action is given by

$$v_j = v_{j-1} + (1-k)\tau, \tag{4}$$

where $0 < k < 1$ is the 'imperfect maintenance factor'. It is easy to see that the larger the value k, the more efficient the PM is.

As a result, the item's virtual age at time t during the field use is given by

$$v_t = \begin{cases} t & \text{for} \quad 0 \le t \le y_1 \\ v_j + t - y_j & \text{for} \quad y_j \le t < y_{j+1} \end{cases}, \tag{5}$$

where v_t denotes the virtual age of the item at time t. Taking into account that all failures are minimally repaired, the intensity function after the burn-in time b during the field use is:

$$h_b(v_t) = \begin{cases} h_b(t) & \text{for} \quad 0 \le t \le y_1 \\ h_b\left(v_j + t - y_j\right) & \text{for} \quad y_j \le t < y_{j+1} \end{cases}. \tag{6}$$

In general, the total cost of an imperfect PM action should be a non-negative and non-decreasing function of the maintenance degree, $k\tau$. Here, we assume that it is an increasing power function of the degree of maintenance $k\tau$. Then, the total cost of imperfect PM actions is

$$c_{PM}(n,k) = nc(k\tau)^{\psi}, 0 < k < 1 \tag{7}$$

where $C_{PM}(n, k)$ is the total cost of imperfect PM actions, the parameters c and ψ are greater than zero and $\tau = (w - t_2)/(n + 1)$. Expression (7) implies that the cost of imperfect PM actions increases as the number n of imperfect PM actions and/or the imperfect maintenance factor k increases.

3.4 Warranty policy

After the burn-in period, the product is released to the market with a *non-renewing free repair warranty (NFRW)* policy with a warranty period w. Under this policy, the manufacturer agrees to rectify, free of charge to the customer, any failures of the item only during the original warranty period w. We assume that the burned-in product is put into field operation immediately after purchase.

If the item fails at time t during warranty period, for $0 \le t \le w$ (t is a calendar time during field use) then it is repaired instantly by a MR with cost of $c_1(b + t)$. It means that the cost of a MR in field operation over the warranty period is higher than that of a MR during burn-in period.

Let $E[c_w(b)]$ be the expected warranty servicing cost in field operation with burn-in time b. Then from Theorem 1, the expected warranty servicing cost is:

$$E\left[c_w(b)\right]=\int_0^{y_1} c_1(b+t)\times h_b(t)dt+\sum_{j=1}^{n-1}\int_{y_j}^{y_{j+1}} c_1(b+t)\times h_b\left(v_j+t-y_j\right)dt \\ +\int_{y_n}^{w} c_1(b+t)\times h_b\left(v_n+t-y_n\right)dt. \tag{8}$$

4 The model

4.1 Optimisation problem

The total mean servicing cost of the product includes the expected burn-in cost, the total cost of imperfect PM actions and the expected warranty servicing cost. Let $E[c_w(b, n, k)]$ be the total expected cost per unit of product, subjected to a burn-in procedure with time b and n discrete imperfect PM actions with factor k and sold under warranty of length w. Therefore,

$$E\left[c_w(b,n,k)\right]=c_s+c_b b+nc(k\tau)^{\psi}+\int_0^{b+y_1} c_1(t)\times h(t)dt \\ +\sum_{j=1}^{n-1}\int_{b+y_j}^{b+y_{j+1}} c_1(t)\times h(v_j+t-y_j)dt+\int_{b+y_n}^{b+w} c_1(t)\times h\left(v_n+t-y_n\right)dt. \tag{9}$$

The optimisation problem is to select the optimal burn-in time, b^*, the optimal number of imperfect PM actions, n^*, and the optimal imperfect maintenance factor, k^*, to minimise the total expected cost given by equation (9) subject to $0 \le b^* \le t_1$, $n^* \in Z^+ + \{0\}$ and $0 \le k^* \le 1$. The length of the imperfect PM interval will then equal to $\tau^* = (w - t_2)/(n^* + 1)$.

4.2 When a burn-in procedure is beneficial?

It is beneficial for a manufacturer to carry out a burn-in procedure if the total expected cost with burn-in and imperfect PM actions is less than the total expected cost only with imperfect PM actions ($b = 0$). Therefore, the burn-in procedure is beneficial if:

$$E\left[c_w\left(b,n^*,k^*\right)\right]<E\left[c_w\left(0,n^*,k^*\right)\right], \tag{10}$$

or

$$c_s+c_b b+\int_w^{b+w} c_1(t)\times h\left(t-n^*k^*\tau^*\right)dt<\int_{t_2+\tau^*}^{b+t_2+\tau^*} c_1(t)\times\left[h\left(t-k^*\tau^*\right)-h(t)\right]dt.$$

4.3 When PM actions are beneficial?

It is beneficial for a manufacturer to carry out periodic PM actions if the total expected cost with burn-in and imperfect PM actions is less than the total expected cost only with burn-in ($n = 0$). Therefore, the PM actions are beneficial if:

$$E\left[c_w\left(b^*,n,k\right)\right] < E\left[c_w\left(b^*,0,0\right)\right], \tag{11}$$

or

$$nc(k\tau)^{\psi} < \sum_{j=1}^{n} \int_{b^*+t_2+j\tau}^{b^*+t_2+(j+1)\tau} c_1(t) \times \left[h(t) - h(t - jk\tau)\right] dt.$$

4.4 A numerical example

Suppose that the failure rate of a component has a Weibull-exponential-Weibull distribution, i.e.

$$r(t) = \begin{cases} \lambda + \alpha\beta\left(\dfrac{t-t_1}{\alpha}\right)^{\beta-1} & \text{for} \quad 0 \le t \le t_1 \\ \lambda & \text{for} \quad t_1 \le t \le t_2 \\ \lambda + \alpha\beta\left(\dfrac{t_2-t}{\alpha}\right)^{\beta-1} & \text{for} \quad t_2 \le t \end{cases}$$

Where $\beta \ge 1$ is the shape parameter and $\alpha \ge 0$ is the scale parameter and $\lambda \ge 0$ is a fixed constant. We take the MR cost $c_1(t)$ as an exponentially non-decreasing function of t as $c_1(t) = c_0 \exp\{aH(t)\}$, where the parameters $c_0 > 0$ and $a \ge 0$.

Table 1 presents the optimal burn-in time and the corresponding total mean servicing cost when $\lambda = 1$; $\alpha = 100$; $\beta = 3$; $t_1 = 10$; $t_2 = 30$; $c_s = 0.2$; $c_b = 5$; $c = 8$; $\psi = 0.7$; $c_0 = 1$; $a = 0.5$ and $w = 48$ for various choices of k and n.

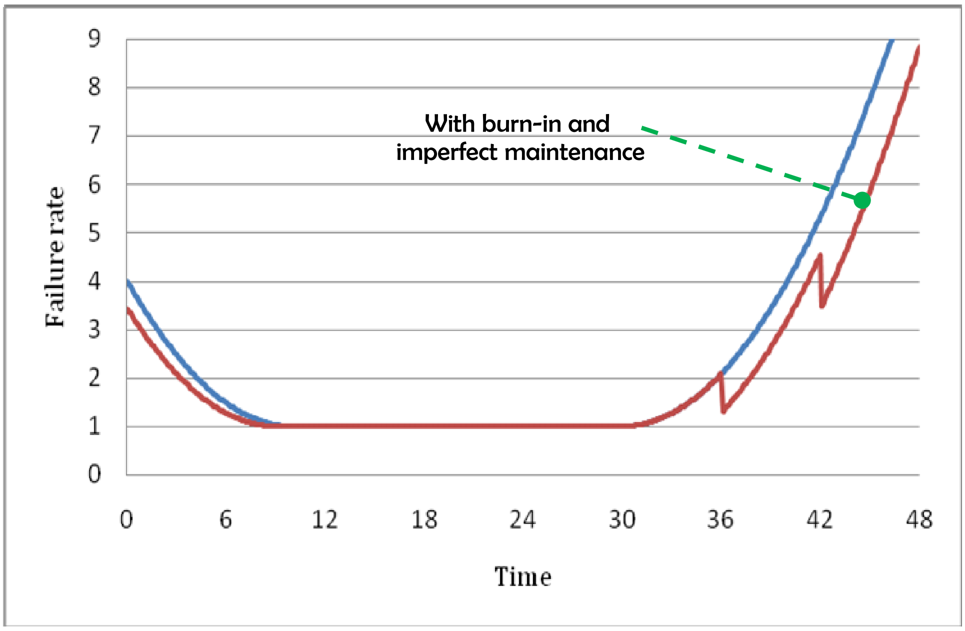
Table 1 $E[c_w(b^*, k, n)]$ and b^* for different combinations of k and n

k	$n = 1$		$n = 2$		$n = 3$		$n = 4$	
	$E[c_w(b^*, k, n)]$	b^*	$E[c_w(b^*, k, n)]$	b^*	$E[c_w(b^*, k, n)]$	b^*	$E[c_w(b^*, k, n)]$	b^*
0.1	18.66	2.1	18.75	1.8	19.08	1.5	19.65	1.0
0.3	18.42	1.5	18.28	1.3	18.76	1.1	18.84	0.8
0.5	17.78	1.2	17.46	1.0	18.02	0.8	18.39	0.3
0.7	17.53	0.9	*17.25*	0.6	17.82	0.4	18.63	0.0
0.9	17.34	0.7	17.39	0.3	18.04	0.0	18.98	0.0

As shown in Table 1, strategies with high values of (b, n, k) lead to unnecessary costs for the manufacturer. Also, if a strategy is worth low parameter values (b, n, k), it is inefficient, does not reduce the total manufacturing costs and it is not an optimal strategy. For example, when k is low, increasing number of PM actions results to additional manufacturing costs and, to offset this effect, the manufacturer should choose high burn-in time. Moreover, if the number of PM actions is low, Table 1 shows that the strategies with high value of the imperfect maintenance factor k are the most efficient.

Figure 1 depicts the failure rate function of the product lifetime with $n^* = 2$ imperfect PM actions, burn-in time $b^* \approx 0.62$ and imperfect maintenance factor $k^* \approx 0.68$.

Figure 1 Failure rate function under (b^*, n^*, k^*) = (0.62, 2, 0.68) (see online version for colours)



We now study the effect of the model parameters on the optimal burn-in time, optimal number of PM actions and the optimal imperfect maintenance factor.

4.4.1 *w and c_0 varying*

The optimal burn-in time b^*, optimal number of imperfect PM actions n^*, and the corresponding maintenance level k^* for various choices of w and c_0 are given in Table 2.

Table 2 w and c_0 varying

	$c_0 = 0.5$			$c_0 = 1.0$			$c_0 = 1.5$		
	b^*	n^*	k^*	b^*	n^*	k^*	b^*	n^*	k^*
$w = 36$	1.08	1.0	0.65	1.21	1.0	0.82	1.31	1.0	0.96
$w = 42$	0.44	2.0	0.53	0.52	2.0	0.60	0.64	2.0	0.69
$w = 48$	0.55	2.0	0.61	0.62	2.0	0.68	0.70	2.0	0.75
$w = 54$	0.66	2.0	0.71	0.73	2.0	0.81	0.79	2.0	0.88
$w = 60$	0.40	3.0	0.45	0.48	3.0	0.53	0.58	3.0	0.62

4.4.2 *t_1 and t_2 varying*

The optimal burn-in time b^*, optimal number of imperfect PM actions n^*, and the corresponding maintenance level k^* for various choices of t_1 and t_2 are given in Table 3.

Table 3 t_1 and t_2 varying

	$t_2 = 24$			$t_2 = 30$			$t_2 = 36$		
	b^*	n^*	k^*	b^*	n^*	k^*	b^*	n^*	k^*
$t_1 = 6$	0.28	3.0	0.52	0.28	2.0	0.68	0.28	1.0	0.93
$t_1 = 8$	0.43	3.0	0.52	0.43	2.0	0.68	0.43	1.0	0.93
$t_1 = 10$	0.62	3.0	0.52	0.62	2.0	0.68	0.62	1.0	0.93
$t_1 = 12$	0.87	3.0	0.52	0.87	2.0	0.68	0.87	1.0	0.93
$t_1 = 14$	1.16	3.0	0.52	1.16	2.0	0.68	1.16	1.0	0.93

Tables 2 and 3 show that the 'burn-in/preventive maintenance' programme is very important when warranty periods are long and MR costs in field use are high.

5 Conclusions

In this paper, a mathematical model is developed to derive the optimal burn-in time, optimal number of PM actions and the corresponding maintenance level simultaneously such that the total mean servicing cost per unit of a product. There is a wide scope for future research in the area of 'burn-in/preventive maintenance' for warranted products. Some of the possible extensions are:

a Introduce general repair strategies (additional to MR strategy) during the burn-in time.

b We have confined our analysis to the failure free policy. The analysis of other types of warranty policies for example, pro-rata, combination is yet to be carried out.

c In the current study, we assume that the PM actions are carried out at discrete time instants with fixed intervals over the warranty period. We can consider an aged-based PM action to reduce the virtual age of the item.

We have worked on some of these extensions and these finding will be reported elsewhere.

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