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Can adverse selection increase social welfare?

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**Background**

**Adverse selection:**

If insurers cannot charge **risk-differentiated** premiums, then:

- higher risks buy more insurance, lower risks buy less insurance,
- raising the **pooled** price of insurance,
- lowering the demand for insurance,

usually portrayed as a bad outcome, both for insurers and for society.

**In practice:**

Policymakers often see merit in restricting insurance risk classification

- EU ban on using gender in insurance underwriting.
- Moratoria on the use of genetic test results in underwriting.

**Question:**

How can we reconcile theory with practice?
Motivation: Two risk-groups $\mu_L = 0.01$ and $\mu_H = 0.04$

Scenario 1: No adverse selection: Risk-differentiated premiums: $\pi_L = 0.01$ and $\pi_H = 0.04$

Low risks →

Utility increase: $66.2 \times 10^{-4}$

High risks →

Scenario 2: Some adverse selection: Pooled premiums: $\pi_L = \pi_H = 0.028$

Low risks →

Utility increase: $71.2 \times 10^{-4}$

High risks →
Contents

- Introduction
- Insurance demand
- Insurance market
- Social welfare
- Conclusions
Why do people buy insurance?

Assumptions

Consider an individual with

- an initial wealth $W$,  
- exposed to the risk of loss $L$,  
- with probability $\mu$,  
- utility of wealth $u(w)$, with $u'(w) > 0$, and  
- an opportunity to insure at premium rate $\pi$.  

Insurance demand
Utility of wealth and insurance purchasing decision

Utility of wealth and insurance purchasing decision

Wealth

Utility

\[ u(W) \]
\[ u(W - \mu L) \]
\[ u(W - \pi_c L) \]

\[ (1 - \mu)u(W) + \mu u(W - L) \]

Fair premium

\[ \mu L \]

Maximum premium tolerated

\[ \pi_c L \]
Insurance demand

Utility of wealth and insurance purchasing decision

Heterogeneity

Simplest model:

If everybody has exactly the same $W$, $L$, $\mu$ and $u(\cdot)$, then:

- All will buy insurance if $\pi < \pi_c$.
- None will buy insurance if $\pi > \pi_c$.

**Reality:** Not all will buy insurance even at fair premium.

Heterogeneity:

- Even if individuals are **homogeneous** in terms of underlying risk,
- they can still be **heterogeneous** in terms of **risk-aversion** which is unobservable by insurers.

Source of randomness from insurers’ perspective:

Utility of insurance of an individual chosen at random, $u(W - \pi L)$, is a random variable, $U_I$. 
Standardisation

As certainty equivalent is invariant to positive affine transformations, we assume $u(W) = 1$ and $u(W - L) = 0$ for all individuals.

Insurance purchasing decision:

Given a premium $\pi$, an individual will purchase insurance if:

\[
\frac{u(W - \pi L)}{u(W) > (1 - \mu) u(W) + \mu u(W - L) = (1 - \mu)}.
\]

Utility with insurance

Utility without insurance

Demand as a function of premium:

Given a premium $\pi$, insurance demand, $d(\pi)$, is:

\[
d(\pi) = P[U_I > 1 - \mu].
\]
Demand for insurance

Small premium assumption

For small premium amounts $\pi L$ (compared to initial wealth $W$), the utility functions over $(W - \pi L, W)$ can be approximated by a straight line, i.e.:

$$u(W - \pi L) \approx u(W) - \pi L u'(W) = 1 - \pi L u'(W) = 1 - \pi \gamma,$$

where $\gamma = L u'(W)$ can be interpreted as a risk preferences index.

Insurance purchasing decision:

Under this assumption, an individual will purchase insurance if:

$$u(W - \pi L) > (1 - \mu) \iff 1 - \pi \gamma > 1 - \mu \iff \gamma < \frac{\mu}{\pi}.$$

Demand as a function of premium:

Given a premium $\pi$, insurance demand, $d(\pi)$, is:

$$d(\pi) = P[U_I > 1 - \mu] = P[\Gamma < \frac{\mu}{\pi}].$$

Note: Insurers cannot observe individual $\gamma$, so $\Gamma$ is a random variable.
Example: Iso-elastic demand

Constant demand elasticity

If demand for insurance can be modelled as\(^1\):

\[
d(\pi) = \tau \left( \frac{\mu}{\pi} \right)^\lambda,
\]

(subject to a cap of 1)

then elasticity of demand is a constant:

\[
\epsilon(\pi) = -\frac{\partial \log d(\pi)}{\partial \log \pi} = \lambda.
\]

---

\(^1\)Assumptions:

\[
u(w) = \left[ \frac{w - (W - L)}{L} \right]^\gamma,
\]

\[
F_{\Gamma}(\gamma) = P[\Gamma \leq \gamma] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau \gamma^\lambda & \text{if } 0 \leq \gamma \leq (1/\tau)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau)^{1/\lambda}. \end{cases}
\]
Example: Iso-elastic demand

Iso-elastic demand for insurance

\[ \lambda = 1 \quad \lambda = 2 \]

Fair-premium demand

\[ \tau \]

Demand

Premium
Risk-groups

Suppose a population can be divided into 2 risk-groups where:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: $p_1, p_2$;
- iso-elastic demand for a given premium, $\pi$:
  \[
  d_i(\pi) = \tau_i \left( \frac{\mu_i}{\pi} \right)^{\lambda_i}, \quad i = 1, 2;
  \]
- fair-premium demand: $\tau_i = d_i(\mu_i)$ for $i = 1, 2$;
- premiums offered: $\pi_1, \pi_2$.

Note: The framework can be generalised for $n > 2$ risk-groups.
Market equilibrium

For a randomly chosen individual, define:

\[ \begin{align*}
Q &= I \text{ [Individual is insured]} ; \\
X &= I \text{ [Individual incurs a loss]} ; \\
\Pi &= \text{Premium offered to the individual.}
\end{align*} \]

Simplifying assumption
The potential loss amount \( L \) is same for all individuals.

Expected premium, claim and market equilibrium

Market equilibrium: \[ E[Q\Pi] = E[QX], \quad \text{where,} \]

Expected premium: \[ E[Q\Pi] = p_1 d_1(\pi_1) \mu_1 + p_2 d_2(\pi_2) \mu_2, \]

Expected claim: \[ E[QX] = p_1 d_1(\pi_1) \mu_1 + p_2 d_2(\pi_2) \mu_2. \]
Risk-classification regimes

**Risk-differentiated premiums:** $\pi = (\mu_1, \mu_2)$

- Equilibrium is achieved when $\pi_1 = \mu_1$ and $\pi_2 = \mu_2$.
- No losses for insurers.
- No (actuarial/economic) adverse selection.

**Pooled premium:** $\pi = (\pi_0, \pi_0)$

If risk-classification is banned, insurers charge same premium $\pi_0$ to both risk-groups.

- Market equilibrium $\Rightarrow$ No losses for insurers! $\Rightarrow$ No (actuarial) adverse selection.
- Pooled premium is greater than average premium charged under full risk classification $\Rightarrow$ (Economic) adverse selection.
- Aggregate demand (cover) is lower than under full risk classification $\Rightarrow$ (Economic) adverse selection.
Contents

- Introduction
- Insurance demand
- Insurance market
- Social welfare
- Conclusions
Social welfare

Definition (Social welfare)
For any premium regime $\pi$, social welfare is the expected utility for an individual selected at random from the population:

$$S(\pi) = E \left[ \underbrace{Q U_I}_{\text{Insured population}} + (1 - Q) \left[ (1 - X) U_W + X U_{W-L} \right] \right].$$

$$= E \left[ Q U_I + (1 - Q) (1 - X) \right], \text{ using } U_W = 1 \text{ and } U_{W-L} = 0.$$

Social welfare under iso-elastic demand
For any premium regime $\pi = (\pi_1, \pi_2)$ satisfying market equilibrium:

$$S(\pi) = \sum_{i=1}^{2} p_i \tau_i \frac{1}{(\lambda_i + 1)} \left( \frac{\mu_i}{\pi_i} \right)^{\lambda_i+1} \pi_i + K,$$

where constant $K$ does not depend on the premium regime under consideration.
Iso-elastic demand with same demand elasticity

- \( \lambda < 1 \iff S(\pi_0) > S(\mu) \implies \text{Risk pooling is } better \text{ than full risk classification.} \\
- \( \lambda > 1 \iff S(\pi_0) < S(\mu) \implies \text{Risk pooling is } worse \text{ than full risk classification.} \\
- \textbf{Empirical evidence suggests } \lambda < 1 \text{ in many insurance markets.}
Iso-elastic demand with different demand elasticities

$S(\pi_0) > S(\mu)$
everywhere to left of boundary curve

$S(\pi_0) < S(\mu)$
everywhere to right of boundary curve

$S(\pi_0) = S(\mu)$

$\alpha_1 = 0.8$

$\alpha_1 = 0.99$
Iso-elastic demand with different demand elasticities

$$S(\pi_0) \geq S(\mu)$$
guaranteed in green shaded area
for all population structures

$$S(\pi_0) = S(\mu)$$

$$\alpha_1 = 0.8$$
$$\alpha_1 = 0.99$$

$$\lambda_1 \leq 1$$ and $$\lambda_1 \leq \lambda_2 \leq \frac{1}{\lambda_1} \Rightarrow S(\pi_0) \geq S(\mu).$$
Iso-elastic demand with different demand elasticities

\[ S(\pi_0) \geq S(\mu) \]

if \( \pi_0 > \pi^* \)

\[ S(\pi_0) = S(\mu) \]

\( \alpha_1 = 0.8 \)

\( \alpha_1 = 0.99 \)

\[ \exists \pi^* \ni \lambda_1 \leq 1 \text{ and } \lambda_2 > \frac{1}{\lambda_1} \text{ and } \pi_0 \geq \pi^* \Rightarrow S(\pi_0) \geq S(\mu). \]
The results can be generalised:

- For any number of risk-groups \( n \geq 2 \).
- For full take-up of insurance by the high risk-group.
- For general insurance demand function using arc elasticity of demand.
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- Introduction
- Insurance demand
- Insurance market
- Social welfare
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Adverse selection need not always be adverse.

Restricting risk classification increases social welfare if:
- \( \lambda \leq 1 \), when demand elasticity is the same for all risk-groups.
- \( \lambda_1 \leq 1 \) and \( \lambda_1 \leq \lambda_2 \leq 1 \), when demand elasticities are different.

Empirical evidence suggests \( \lambda < 1 \) in many insurance markets.
https://blogs.kent.ac.uk/loss-coverage/