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Security-Reliability Tradeoff Analysis for Underlay Cognitive Two-Way Relay Networks

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Abstract

We consider an underlay wiretap cognitive two-way relay network (CTWRN), where two secondary sources exchange their messages via multiple secondary decode-and-forward digital network coding relays in the presence of an eavesdropper by using a three-phase time division broadcast protocol and sharing the licensed spectrum of primary users. To mitigate eavesdropping attacks, an artificial noise (AN)-aided opportunistic relay selection scheme, called generalized max-min (GMM) relay selection is proposed to enhance physical layer security for the wiretap CTWRNs. The performance of the GMM scheme is analyzed, and evaluated by the exact closed-form outage probability and intercept probability. Additionally, we also provide asymptotic approximations for the outage probability and intercept probability at high signal-to-noise ratio. For comparison, we analyze the performance of the conventional max-min (MM) relay selection scheme as well. It is shown that the GMM scheme...
outperforms the MM scheme in terms of the security-reliability tradeoff (SRT), where the security and reliability are quantified by the intercept probability and outage probability, respectively. Moreover, the SRTs of the MM and GMM schemes can be substantially improved by increasing the number of secondary relays, while the improvement of the GMM scheme is more evident than that of the MM scheme.

**Index Terms**

Physical layer security, security-reliability tradeoff, cognitive two-way relay networks, artificial noise, relay selection.

**I. INTRODUCTION**

With the rapid development of wireless communications, wireless data traffic grows explosively, but spectrum resources are limited. To deal with the problem of spectrum shortage, cognitive radio has been proposed as a promising solution. The main idea of cognitive radio is that the secondary users (SUs) can opportunistically access the frequency bands allocated to primary users (PUs), as long as the interference caused by the SUs does not exceed a given threshold [1].

As an efficient relaying strategy in the half-duplex mode, two-way relaying can enhance the network throughput and the spectral efficiency significantly [2]. In recent years, the application of two-way relaying in cognitive radio networks has also attracted much attention. In [3], an opportunistic relay selection scheme, called max-min (MM) relay selection, was proposed for the cognitive two-way relay networks (CTWRNs) with a decode-and-forward digital network coding (DF-DNC) relaying strategy, and analyzed in terms of the outage probability. Based on the framework of [3], the authors of [4] took the interference from the primary transmitter (PT) to the SUs into consideration and studied the MM relay selection scheme for the underlay CTWRNs again. [5] adopted a decode-and-forward analog network coding (DF-ANC) relaying strategy in addition to the DF-DNC strategy, and studied a collaborative beamforming technique.
with the aim of reducing the mutual interference between PUs and SUs in an underlay CTWRN. Additionally, the performance of MM relay selection scheme for the full-duplex amplify-and-forward (AF) CTWRNs was investigated in [6].

Due to the broadcast nature of wireless medium, wireless communications are vulnerable to eavesdropping attacks, and thus secure communication in wireless networks has long been a challenging task. Traditionally, secure communication is achieved by cryptography techniques at the application layer. Recently, by exploiting the physical characteristics of wireless channels, physical layer security (PLS) has been emerged as a solution that can provide perfect secrecy in wiretap networks [7]-[8]. To improve PLS of wireless communications, the artificial noise (AN) (or cooperative jamming) [9]-[20] and relay selection [21]-[31] techniques have been widely investigated.

The AN technique is an effective approach for the implementation of PLS. The design and evaluation of the AN schemes for different wiretap networks have attracted a wide attention during the past several years. Generally, the AN schemes can be implemented in the following three ways. 1) Null space beamforming: The AN is generated at a legitimate terminal. It was firstly used to increase the secrecy capacity of multiple-input multiple-output (MIMO) wiretap channels [9]. Specifically, by generating a beamforming vector that is orthogonal to the channel state information (CSI) of legitimate links, the eavesdropper’s signal-to-interference-plus-noise ratio (SINR) can be degraded, whereas the interference caused by the AN at the legitimate terminal can be neglected. Recently, null space beamforming has been extended to multiple-input single-output (MISO) wiretap channels [10], [11], one-way and two-way relay wiretap networks [12], [13]. 2) External friendly jamming: Since null space beamforming is inapplicable when a transmitter only equips single antenna or only one relay node with one antenna is available in a cooperative relay network, an external friendly jammer can be introduced to emit AN to confuse the eavesdropper [14]-[17]. 3) Full-duplex aided jamming: The AN is generated at a full-duplex receiver. For a single-hop wiretap communication system, a full-duplex receiver
is able to simultaneously receive the source message and transmit AN on the same frequency band [18]-[20]. In this way, the loop-interference at the receiver can be significantly suppressed by the self-interference cancelation technique, while the signal quality of the eavesdropper is degraded by AN.

As another effective technique for secrecy performance enhancement, relay selection has been extensively applied for PLS implementation in cooperative wiretap networks. In [21], the authors investigated the performance of PLS-oriented relay selection schemes for the one-way cooperative relay networks where both AF and DF relaying strategies were considered. Afterwards, for the same cooperative wiretap networks as in [21], a joint relay and jammer selection scheme was proposed [22]. More recently, a buffer-aided relay selection scheme was developed for one-way two-hop wiretap networks in [23]. Relay selection technique can also be used to enhance PLS for two-way relay wiretap networks. In [24], several joint relay and jammer selection schemes for two-way AF relay wiretap networks were investigated. Taking the implementation complexity into consideration, a distributed relay selection scheme was employed to optimize the overall secrecy performance in two-way AF relay wiretap networks [25]. The authors of [26] studied the performance of MM relay selection scheme in two-way DF-DNC relay networks and derived a closed-form expression for the secrecy outage probability. In [27], a joint AN and relay selection technique was developed for two-way DF-DNC relay networks, and the security-reliability tradeoff (SRT) performance of the AN-aided MM relay selection scheme was analyzed.

Besides the traditional cooperative networks, relay selection has been adopted to improve the PLS in cognitive relaying systems as well. In [28], an opportunistic relay selection scheme was proposed for an overlay cognitive one-way DF relay network in the presence of multiple eavesdroppers. Later, for the underlay wiretap DF relay networks, the SRTs of several relay selection schemes were studied in [29], and the secrecy outage probabilities of different joint jammer and relay selection schemes were derived in [30]. Taking into account the channel
correlation between the legitimate and wiretap channels, [31] obtained a closed-form expression for the secrecy outage probability of a generalized relay selection scheme.

According to the existing researches, it is known that 1) the CTWRNs can significantly improve the spectral efficiency [1]-[6], however, it is vulnerable to eavesdropping attacks [7]-[8]; 2) the AN and relay selection techniques are quite effective for PLS enhancement in wireless networks [9]-[31]; and 3) the PLS-oriented relay selection for the underlay wiretap CTWRNs has not been studied so far. With these motivations, we consider an underlay wiretap CTWRN and investigate the AN-aided relay selection with the aim of enhancing the PLS. This is different from the existing works [21]-[31] in the following aspects. Firstly, we adopt the AN and relay selection techniques to improve PLS in the underlay wiretap CTWRNs, whereas the cognitive transmissions were not considered in [21]-[27] and the secondary cooperative networks in [28]-[31] are one-way. Thus, the communication system in this paper is fundamentally different from that of [21]-[31]. Secondly, the problem of how to enhance PLS in the underlay wiretap CTWRNs by using relay selection has not been investigated in [21]-[31]. Finally, in the underlay wiretap CTWRNs, we propose a new relay selection scheme which has not been studied in [21]-[31]. Moreover, the new relay selection achieves a better SRT performance than the conventional MM scheme.

The main contributions of the paper are summarized as follows:

- We propose a new relay selection scheme, called generalized max-min (GMM) relay selection against eavesdropping attacks and enhance the PLS in the underlay CTWRNs. Comparing to the conventional MM relay selection scheme, the GMM scheme takes into consideration the CSI of interference channels from the secondary relays to the primary receiver (PR) in addition to the CSI of the secondary legitimate links. Furthermore, the GMM scheme achieves a better SRT performance than the MM scheme.

- For the MM and GMM relay selection schemes, we derive closed-form expressions for the outage probability and intercept probability, which respectively characterize the reliability
and the security of the underlay CTWRNs. Moreover, asymptotic expressions for the outage probability and intercept probability at high signal-to-noise ratio (SNR) are presented to provide further insights.

- Based on the derived expressions of intercept probability and outage probability, the SRTs of the MM and GMM relay selection schemes are analyzed. It is shown that the SRT performance of the GMM scheme is better than that of the MM scheme. Moreover, when increasing the number of secondary relays, the SRT performance of the MM and GMM schemes can be substantially enhanced, while the improvement of the GMM scheme is more evident than that of the MM scheme.

The remainder of this paper is organized as follows. Section II describes the system model and introduces the AN-aided opportunistic relay selection schemes. The secrecy performance of the proposed relay selection schemes is analyzed in Section III. Simulation results are presented in Section IV, followed by the conclusions in Section V.

II. SYSTEM MODEL AND RELAY SELECTION SCHEMES

A. System Model

As shown in Fig.1, we consider an underlay wiretap CTWRN consisting of two secondary sources $S_a$, $S_b$, a set of $M$ secondary DF-DNC relays $R_i$ ($i \in \mathcal{R} = \{1, \cdots, M\}$), a passive eavesdropper $E$, a PT and a PR. It is assumed that all nodes are equipped with single antenna, and the eavesdropper can wiretap on all SUs. Due to the deep shadowing, there is no direct link between $S_a$ and $S_b$, which is a common assumption in literature (e.g., [3]-[6], [13], [15], [24]-[27]). Therefore, the secondary sources $S_a$ and $S_b$ exchange their confidential information via secondary relays with a three-phase time division broadcast protocol. The SUs are allowed to share the same frequency spectrum with the PUs as long as the interference received at PR does not exceed the maximum tolerable threshold.
It is assumed that all wireless links undergo quasi-static independent and nonidentical Rayleigh flat fading. Moreover, the PT is assumed to be far away from the SUs, implying that $S_a$, $S_b$, $R_i$ ($i \in \mathcal{R}$) and $E$ are not inflicted by the interference from primary system. Let $h_{ai}$, $h_{bi}$, $h_{ia}$ and $h_{ib}$, respectively, be the coefficients of the legitimate links $S_a \rightarrow R_i$, $S_b \rightarrow R_i$, $R_i \rightarrow S_a$ and $R_i \rightarrow S_b$. Denote by $h_{Ve} (V \in \{a, b, i\})$ the coefficients of wiretap links between SUs and eavesdropper, by $h_{Up} (U \in \{a, b, i\})$ the interference channel coefficients from SUs to PR. The channel gain $|h_{Up}|^2$ is an exponential random variable with a mean $\lambda_{Up}$. To simplify the theoretical analysis, we assume that the channel gains of each link during different time phases are independent and identically distributed random variables (e.g., [27]). In addition, the thermal noises at receiver nodes $S_a$, $S_b$, $R_i$ and $E$ are modeled as zero-mean additive white Gaussian noises (AWGNs) $n_a$, $n_b$, $n_i$ and $n_e$ with variances $\sigma_a^2$, $\sigma_b^2$, $\sigma_i^2$ and $\sigma_e^2$, respectively.

**B. Transmission Model**

As discussed earlier, with three-phase time division broadcast protocol, the bidirectional communication between $S_a$ and $S_b$ takes place in three phases. In the first phase, the secondary
source $S_a$ transmits its signal $x_a$ to the secondary relays with power $P_{a1}$, $S_b$ simultaneously emits an AN $x_{J1}$ to impair the eavesdropper with power $P_{b1}$. The AN lies in the null-space of $S_a$ to secondary relay channels. For the relay wiretap networks, several methods of designing such AN were detailedly introduced in [12]-[13], and the eavesdroppers can be greatly weakened. However, from a practical point of view, the estimation error of CSI cannot be negligible. Thus, it is assumed that the AN at secondary relays can be significantly suppressed, but cannot be completely eliminated. The eavesdropper knows nothing about the AN emitted by $S_b$. So that, the impact of the AN on $R_i$ and $E$ are quantified by $\epsilon_1$ and $\epsilon_2$, respectively. The received signals at $R_i$ and $E$ can be expressed, respectively, as

\[ y_{i1} = \sqrt{P_{a1}}h_{ai}^{(1)}x_a + \sqrt{\epsilon_1P_{b1}}h_{bi}^{(1)}x_{J1} + n_i, \quad y_{ae} = \sqrt{P_{a1}}h_{ae}^{(1)}x_a + \sqrt{\epsilon_2P_{b1}}h_{be}^{(1)}x_{J1} + n_e, \] (1)

where $h_{ai}^{(1)}, h_{bi}^{(1)}, h_{ae}^{(1)}$ and $h_{be}^{(1)}$ are the coefficients of channels $S_a \rightarrow R_i$, $S_b \rightarrow R_i$, $S_a \rightarrow E$ and $S_b \rightarrow E$, respectively, in the first phase. Due to above analysis, it can be seen that $\epsilon_1$ is much less than 1 and $\epsilon_2$ is approximate to 1.

In the underlay wiretap CTWRNs, the SUs are allowed to share the same spectrum with PUs, provided that the aggregate interference power at PR does not exceed the maximum tolerable threshold $P_I$. Thus, in the first phase, the interference power at PR from $S_a$ and $S_b$ satisfies the following inequality [32]

\[ P_{a1}|h_{ap}^{(1)}|^2 + P_{b1}|h_{bp}^{(1)}|^2 \leq P_I. \] (2)

Letting $P_{b1} = \alpha_1P_{a1}$, based on (2), we limit the transmit power $P_{a1}$ and $P_{b1}$ as

\[ P_{a1} = \frac{P_I}{|h_{ap}^{(1)}|^2 + \alpha_1|h_{bp}^{(1)}|^2}, \quad P_{b1} = \frac{\alpha_1P_I}{|h_{ap}^{(1)}|^2 + \alpha_1|h_{bp}^{(1)}|^2}. \] (3)

Moreover, the authors in [33] provided a detailed and comprehensive interpretation to show that $P_{a1}$ and $P_{b1}$ in form of equation (3) are not fundamentally different from that $P_{a1} = \min\{\frac{P_I}{|h_{ap}^{(1)}|^2 + \alpha_1|h_{bp}^{(1)}|^2}, P_{max}\}$ and $P_{b1} = \min\{\frac{\alpha_1P_I}{|h_{ap}^{(1)}|^2 + \alpha_1|h_{bp}^{(1)}|^2}, P_{max}\}$, where $P_{max}$ is the maximum.
transmit power constraint at SUs. Thus, at the end of the first phase, the SINRs at \( R_i \) and \( E \) can be given, respectively, by

\[
\Gamma_{ai} = \frac{\gamma_{ai}^{(1)}}{\epsilon_1 \alpha_1 |\gamma_{bi}^{(1)}| + |h_{ap}^{(1)}|^2 + \alpha_1 |h_{bp}^{(1)}|^2}, \quad \Gamma_{ae} = \frac{\gamma_{ae}^{(1)}}{\epsilon_2 \alpha_1 |\gamma_{be}^{(1)}| + |h_{ap}^{(1)}|^2 + \alpha_1 |h_{bp}^{(1)}|^2},
\]

where \( \gamma_{ai}^{(1)} = \frac{P_i}{\sigma_i^2} |h_{ai}^{(1)}|^2, \gamma_{bi}^{(1)} = \frac{P_i}{\sigma_i^2} |h_{bi}^{(1)}|^2, \gamma_{ae}^{(1)} = \frac{P_i}{\sigma_i^2} |h_{ae}^{(1)}|^2 \) and \( \gamma_{be}^{(1)} = \frac{P_i}{\sigma_i^2} |h_{be}^{(1)}|^2 \) are exponential random variables. The mathematical expectations of \( \gamma_{ai}^{(1)}, \gamma_{bi}^{(1)}, \gamma_{ae}^{(1)}, \gamma_{be}^{(1)}, |h_{ap}^{(1)}|^2 \) and \( |h_{bp}^{(1)}|^2 \) are denoted by \( \lambda_{ai}, \lambda_{bi}, \lambda_{ae}, \lambda_{be}, \lambda_{ap}, \lambda_{bp} \), respectively.

During the second phase, \( S_b \) transmits its signal \( x_b \) with power \( P_{b2} \), and meanwhile, the secondary source \( S_a \) sends an AN \( x_{r2} \) to confuse the eavesdropper with power \( P_{a2} \). Setting \( P_{a2} = \alpha_2 P_{b2} \), we can write the SINRs at \( R_i \) and \( E \) as \( \Gamma_{bi} = \frac{\gamma_{bi}^{(2)}}{\epsilon_1 \alpha_2 |\gamma_{ai}^{(2)}| + |h_{ap}^{(2)}|^2 + \alpha_2 |h_{bp}^{(2)}|^2}, \quad \Gamma_{be} = \frac{\gamma_{be}^{(2)}}{\epsilon_2 \alpha_2 |\gamma_{ai}^{(2)}| + |h_{ap}^{(2)}|^2 + \alpha_2 |h_{bp}^{(2)}|^2}, \) respectively, where \( \gamma_{bi}^{(2)} = \frac{P_i}{\sigma_i^2} |h_{bi}^{(2)}|^2, \gamma_{ai}^{(2)} = \frac{P_i}{\sigma_i^2} |h_{ai}^{(2)}|^2, \gamma_{be}^{(2)} = \frac{P_i}{\sigma_i^2} |h_{be}^{(2)}|^2 \) and \( \gamma_{ae}^{(2)} = \frac{P_i}{\sigma_i^2} |h_{ae}^{(2)}|^2 \) are exponential random variables. According to the assumption in Subsection II. A, the coefficients of the same link in different phases are identically distributed random variables. Thus, the mathematical expectations of \( \gamma_{ai}^{(2)}, \gamma_{bi}^{(2)}, \gamma_{ae}^{(2)}, \gamma_{be}^{(2)}, |h_{ap}^{(2)}|^2 \) and \( |h_{bp}^{(2)}|^2 \) are also denoted by \( \lambda_{ai}, \lambda_{bi}, \lambda_{ae}, \lambda_{be}, \lambda_{ap}, \lambda_{bp} \), respectively.

In the first and second phases, upon receiving the signals from the secondary sources, the secondary relays adopt the DF relaying strategy and aim to correctly decode the source messages. For simplicity, we assume that a secondary relay \( R_i \) can decode \( x_\omega \) successfully as long as the capacity of link \( S_\omega \rightarrow R_i \) is larger than a target rate \( r \), where \( \omega \in \{a, b\} \). Without loss of generality, the set \( \mathcal{D}_\omega = \{i_1, \cdots, i_{m_\omega}\} \) is denoted as the subscripts of the secondary relays who can decode the messages from \( S_\omega \) successfully, where \( \omega \in \{a, b\} \). Furthermore, we assume \( \mathcal{D}_{ab} = \mathcal{D}_a \cap \mathcal{D}_b = \{i_1, \cdots, i_k\} \), and \( u = m_a - k, v = m_b - k \). Applying a similar approach as in [3]-[5], [26]-[27], the probability \( \Pr\{|\mathcal{D}_{ab}| = k\} \) can be calculated as

\[
\Pr\{|\mathcal{D}_{ab}| = k\} = \sum_{|\mathcal{D}_{ab}|=k} \sum_{u=0}^{M-k} \sum_{v=0}^{M-k-u} \Pr\{\mathcal{D}_{ab}\} = \Pr\{\mathcal{D}_{ab}\}. \tag{5}
\]
Denote by $\tilde{\Pr}\{\cdot\}$ the asymptotic probability of $\Pr\{\cdot\}$. When $P_I \to \infty$, the asymptotic probability $\tilde{\Pr}\{|D_{ab}| = k\}$ can be derived by replacing $\Pr\{D_{ab}\}$ with $\tilde{\Pr}\{D_{ab}\}$ in (5). The closed-form expressions of $\Pr\{D_{ab}\}$ and $\tilde{\Pr}\{D_{ab}\}$ are derived in Appendix A.

In the third phase, the selected secondary relay $R_i$ combines the messages $x_a$ and $x_b$ by applying the XOR operation to generate a signal $x_i = x_a \oplus x_b$, and broadcasts it to the secondary sources with power $P_{\frac{I}{\left|h_{ip}\right|^2}}$. The SNRs at $S_a, S_b, E$ are written, respectively, as

$$
\Gamma_{\text{DNC}}^{ia} = \frac{\gamma_{ia}}{|h_{ip}|^2}, \Gamma_{\text{DNC}}^{ib} = \frac{\gamma_{ib}}{|h_{ip}|^2}, \Gamma_{\text{DNC}}^{ie} = \frac{\gamma_{ie}}{|h_{ip}|^2},
$$

(6)

where $\gamma_{ia} = \frac{P_I}{\sigma_a^2}|h_{ia}|^2$, $\gamma_{ib} = \frac{P_I}{\sigma_b^2}|h_{ib}|^2$, and $\gamma_{ie} = \frac{P_I}{\sigma_e^2}|h_{ie}|^2$ are exponential random variables. The expectations of $\gamma_{ia}, \gamma_{ib}$, and $\gamma_{ie}$ are denoted by $\lambda_{ia}, \lambda_{ib}$, and $\lambda_{ie}$, respectively.

C. Relay Selection Schemes

In this subsection, we assume $D_{ab}$ is not empty and employ two AN-aided opportunistic relay selection schemes to improve PLS in the underlay wiretap CTWRNs. Unlike [21]-[26], [28], [30], and [33]-[34], the CSI of wiretap links are not taken into account for the relay selection schemes.

1) Max-min Relay Selection: The conventional MM relay selection scheme has been widely studied (e.g., [3]-[6], [24] and [26]-[27]). Specifically, in the underlay wiretap CTWRNs, the MM scheme selects the optimal secondary relay $R_i$, with the following rule

$$
i^* = \arg \max_{j \in D_{ab}} \min\{\gamma_{ja}, \gamma_{jb}\}.
$$

(7)

2) Generalized Max-min Relay Selection: The MM relay selection scheme does not consider the interference from the SUs to the PR. Taking into account this interference, we propose the GMM relay selection scheme which maximizes the worse instantaneous end-to-end SNR at the secondary sources. Therefore, the GMM relay selection scheme is expressed as

$$
i^* = \arg \max_{j \in D_{ab}} \frac{\min\{\gamma_{ja}, \gamma_{jb}\}}{|h_{jp}|^2}.
$$

(8)
D. Outage Probability and Intercept Probability

As in [27] and [29], in order to examine the SRT performance of the MM and GMM schemes, we need to characterize the security and reliability of each relay selection scheme in the underlay wiretap CTWRNs by intercept probability and outage probability, respectively. Specifically, the outage event occurs when the capacity of a legitimate link falls below the transmission rate, and the intercept probability is defined as the SINR at E is higher than a given target value. Thus, for the data transmission of direction $S_a \rightarrow R_i \rightarrow S_b$, the outage probability at $S_b$ can be expressed as

$$P_{\text{outb}}^{\text{DNC} - D_{ab}} = \Pr \left\{ \Gamma_{i^*b}^{\text{DNC}} < \gamma \right\}, \quad (9)$$

where $\gamma = 2^{3r} - 1$.

According to the one-time pad encryption scheme, the eavesdropper E will not be able to obtain any information from $x_i = x_a \oplus x_b$ directly. Therefore, E is capable of intercepting the signal $x_a$ if and only if event $\{ \Gamma_{ae} > \gamma \}$ or event $\{ \Gamma_{ae} < \gamma, \Gamma_{be} > \gamma, \Gamma_{i^*e}^{\text{DNC}} > \gamma \}$ occurs. In this way, the intercept probability for the data transmission of direction $S_a \rightarrow R_i \rightarrow S_b$ can be formulated as

$$P_{\text{intb}}^{\text{DNC} - D_{ab}} = \Pr \left\{ \Gamma_{ae} < \gamma, \Gamma_{be} > \gamma, \Gamma_{i^*e}^{\text{DNC}} > \gamma \right\} + \Pr \left\{ \Gamma_{ae} > \gamma \right\}. \quad (10)$$

Resorting to the total probability formula [35], one has

$$P_A^{\text{DNC}} = \sum_{k=0}^{M} \Pr \{|D_{ab}| = k \} P_A^{\text{DNC} - D_{ab}}, \quad (11)$$

where $A \in \{ \text{outb}, \text{intb} \}$. Furthermore, the total outage probability and intercept probability are defined, respectively, as

$$P_{\text{out}}^{\text{DNC}} = \frac{P_{\text{outa}}^{\text{DNC}} + P_{\text{outb}}^{\text{DNC}}}{2}, \quad P_{\text{int}}^{\text{DNC}} = \frac{P_{\text{inta}}^{\text{DNC}} + P_{\text{intb}}^{\text{DNC}}}{2}, \quad (12)$$

where $P_{\text{outa}}^{\text{DNC}}$ and $P_{\text{inta}}^{\text{DNC}}$ can be derived by the same method of (11).

**Remark 1.** For (11), in case that $D_{ab}$ is an empty set, the assignment of outage probability $P_{\text{outb}}^{\text{DNC} - \emptyset}$ is 1. Nevertheless, the intercept probability $P_{\text{intb}}^{\text{DNC} - \emptyset}$ is evaluated by $\Pr \{ \Gamma_{ae} > \gamma \}$.  

March 12, 2019 DRAFT
III. PERFORMANCE ANALYSIS

In this section, we analyze the performance of the MM and GMM relay selection schemes in the underlay wiretap CTWRNs.

A. Performance of MM Scheme

1) Exact Performance: When the set $D_{ab}$ is not empty, from (9) and the total probability formula [35], the outage probability at $S_b$ of the MM relay selection scheme can be written as

$$P_{\text{outb-MM}} = \sum_{i \in D_{ab}} \left[ \Pr \left\{ \Gamma_{ib}^{\text{DNC}} < \gamma, \gamma_{ib} > Z_i \right\} + \Pr \left\{ \Gamma_{ib}^{\text{DNC}} < \gamma, \gamma_{ia} > Z_i \right\} \right], \quad (13)$$

where $Z_i = \max_{j \in D_{ab} - \{i\}} \min\{\gamma_{ja}, \gamma_{jb}\}$ and $\gamma = 2^{3r} - 1$.

Since links $R_i \rightarrow S_a$ and $R_i \rightarrow S_b$ are statistically independent, the cumulative distribution function (CDF) and probability density function (PDF) of $\min\{\gamma_{ia}, \gamma_{ib}\}$ can be calculated, respectively, as

$$F_{\min\{\gamma_{ia}, \gamma_{ib}\}}(x) = 1 - e^{-\left(\frac{\lambda_{ia}}{1} + \frac{\lambda_{ib}}{1}\right)x}, \quad f_{\min\{\gamma_{ia}, \gamma_{ib}\}}(x) = \left(\frac{\lambda_{ia}}{1} + \frac{\lambda_{ib}}{1}\right)e^{-\left(\frac{\lambda_{ia}}{1} + \frac{\lambda_{ib}}{1}\right)x}. \quad (14)$$

Owing to the assumption that the coefficients $h_{ia}$ and $h_{ib}$, for $i \in D_{ab}$, are independent of each other, we concluded that $\min\{\gamma_{1a}, \gamma_{1b}\}, \cdots, \min\{\gamma_{Ma}, \gamma_{Mb}\}$ are statistically independent. Therefore, the CDF of $Z_i$ can be calculated as

$$F_{Z_i}(x) = \left[ \prod_{j \in D_{ab} - \{i\}} F_{\min\{\gamma_{ja}, \gamma_{jb}\}}(x) \right] = \sum_{l=0}^{k-1} \sum_{A \subset D_{ab} - \{i\}, |A| = l} (-1)^l e^{-x \sum_{j \in A} (\frac{\lambda_{ja}}{1} + \frac{\lambda_{jb}}{1})}, \quad (15)$$

where the last equality is obtained by the multinomial expansion formula, which is given in Appendix A.

Noticing that random variables $|h_{ip}|^2, \gamma_{ia}, Z_i$ are independent of each other, and utilizing (15), we can calculated $P_{1i}$ as

$$P_{1i} = \int_0^{\infty} e^{-x(\frac{1}{\lambda_{ip}\gamma} + \frac{1}{\lambda_{ia}})} F_{Z_i}(x) f_{\gamma_{ib}}(x) dx = \sum_{l=0}^{k-1} \sum_{A \subset D_{ab} - \{i\}, |A| = l} \frac{(-1)^l \lambda_{ia} \lambda_{ib} \gamma}{\lambda_{ib} (\delta_{iA} \lambda_{ip} \gamma + 1)}, \quad (16)$$

where $\delta_{iA} = \lambda_{ia}^{-1} + \lambda_{ib}^{-1} + \sum_{j \in A} (\lambda_{ja}^{-1} + \lambda_{jb}^{-1})$. 

Applying the same approach as in the derivation of (16), we can compute $P_{2i}$ as
\[ P_{2i} = \int_0^\infty e^{-\lambda y} \int_0^x F_{Z_i}(y) f_{\gamma_{ia}}(y) dy f_{\gamma_{ib}}(x) dx \]
\[ = \sum_{l=0}^{k-1} \sum_{B \subseteq \mathcal{D}_{ab}-(1), |B|=l} \frac{(-1)^l \lambda_{ib}}{\lambda_{ia} (\delta_{lB} \lambda_{ib} - 1)} \left( \frac{\lambda_{ib}}{\lambda_{ip} \delta_{lB} \gamma + 1} \right), \]  
(17)
where $\delta_{lB} = \lambda_{ia}^{-1} + \lambda_{ib}^{-1} + \sum_{j \in B} (\lambda_{ja}^{-1} + \lambda_{jb}^{-1})$.

Finally, substituting (16) and (17) into (13), we can obtain $P_{\text{out}_{ab-MM}}$. In the same way, $P_{\text{out}_{ab-MM}}$ can be derived. According to (11) and (12), we can directly get $P_{\text{DNC}}$.

Next, from (10) and the total probability formula [35], the intercept probability at $S_b$ of the MM scheme can be written as
\[ P_{\text{DNC}_{ab}} = \sum_{i \in \mathcal{D}_{ab}} \left[ \Pr \{ \Gamma_{ae} > \gamma, \min \{ \gamma_{ia}, \gamma_{ib} \} > Z_i \} \right. \]
\[ + \Pr \{ \Gamma_{ae} < \gamma, \Gamma_{be} > \gamma, \Gamma^{\text{DNC}}_{ie} > \gamma, \min \{ \gamma_{ia}, \gamma_{ib} \} > Z_i \} \right]. \]  
(18)

Herein, the first and second terms in the bracket of (18) are defined as $Q_{1i}$ and $Q_{2i}$, respectively. Since the channel coefficients in an underlay wiretap CTWRN are statistically independent, we can see that events $\{ \min \{ \gamma_{ia}, \gamma_{ib} \} > Z_i \}$, $\{ \Gamma^{\text{DNC}}_{ie} > \gamma \}$, $\{ \Gamma_{ae} < \gamma \}$ and $\{ \Gamma_{be} > \gamma \}$ are independent. Accordingly, we have
\[ \left\{ \begin{array}{l}
Q_{1i} = \Pr \{ \min \{ \gamma_{ia}, \gamma_{ib} \} > Z_i \} \Pr \{ \Gamma_{ae} > \gamma \}, \\
Q_{2i} = \Pr \{ \min \{ \gamma_{ia}, \gamma_{ib} \} > Z_i \} \Pr \{ \Gamma^{\text{DNC}}_{ie} > \gamma \} \Pr \{ \Gamma_{ae} < \gamma \} \Pr \{ \Gamma_{be} > \gamma \}.
\end{array} \right. \]  
(19)

Using the result of [6, Eq.(6)], we can write the CDF of $Y = e_{2\alpha_1}\lambda_{be}^{(1)} + |h_{ap}^{(1)}| + \alpha_1 h_{bp}^{(1)}$ as
\[ F_Y(y) = 1 - \frac{1}{e_{2\alpha_1} \lambda_{be} - \lambda_{ap}} \left[ \frac{e_{2\alpha_1} \lambda_{be}}{e_{2\alpha_1} \lambda_{be} - \alpha_1 \lambda_{bp}} \left( e_{2\alpha_1} \lambda_{be}^{(1)} y - \frac{1}{e_{2\alpha_1} \lambda_{be}^{(1)}} - \frac{1}{e_{2\alpha_1} \lambda_{be}^{(1)}} y \right) \right. \]
\[ - \left. \frac{\lambda_{ap}}{\lambda_{ap} - \alpha_1 \lambda_{bp}} \left( \lambda_{ap} e^{\frac{1}{\lambda_{ap} y} - \alpha_1 \lambda_{bp} e^{\frac{1}{\lambda_{ap} y}}} \right) \right]. \]
\[ \]  
(20)

Thus, the CDF of $\Gamma_{ae}$ is calculated as
\[ F_{\Gamma_{ae}}(\gamma) = \int_0^\infty \left[ 1 - F_Y(y) \right] f_{\gamma_{ae}^{(1)}}(y) dy = \frac{1}{e_{2\alpha_1} \lambda_{be} - \lambda_{ap}} \left[ \frac{e_{2\alpha_1} \lambda_{be}}{e_{2\alpha_1} \lambda_{be} - \alpha_1 \lambda_{bp}} \left( \frac{e_{2\alpha_1} \lambda_{be}^{(2)\gamma}}{e_{2\alpha_1} \lambda_{be}^{(1)} + \lambda_{ae}} \right) \right. \]
\[ - \left. \frac{(\alpha_1 \lambda_{bp})^{2\gamma}}{\alpha_1 \lambda_{bp} \gamma + \lambda_{ae}} \right) - \frac{\lambda_{ap}}{\lambda_{ap} - \alpha_1 \lambda_{bp}} \left( \frac{\lambda_{ap}^{2\gamma}}{\lambda_{ap} \gamma + \lambda_{ae}} - \frac{(\alpha_1 \lambda_{bp})^{2\gamma}}{\alpha_1 \lambda_{bp} \gamma + \lambda_{ae}} \right) \].  
(21)

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By means of the same approach, \( F_{\Gamma_{be}}(\gamma) \) can be obtained. Then, adopting a similar method previously used for deriving (15), we can write \( \Pr\{\min\{\gamma_{ia}, \gamma_{ib}\} > Z_i\} \)

\[
\Pr\{\min\{\gamma_{ia}, \gamma_{ib}\} > Z_i\} = \int_0^\infty F_{z_i}(x)f_{\min\{\gamma_{ia}, \gamma_{ib}\}}(x)dx = \sum_{l=0}^{k-1} \sum_{A \subset D_{ab} - \{i\}, |A| = l} (-1)^l \frac{\lambda_{ia} + \lambda_{ib}}{\lambda_{ia} \lambda_{ib} \delta_i} \delta_i A. \tag{22}
\]

Finally, the CDF of \( \Gamma_{ie}^{DNC} \) is expressed as

\[
F_{\Gamma_{ie}^{DNC}}(\gamma) = \int_0^\infty \Pr\{\gamma_{ie} < \gamma\} f_{\|h_{ip}\|^2}(y)dy = 1 - \frac{\lambda_{ie}}{\lambda_{ip} \gamma + \lambda_{ie}} \tag{23}
\]

By summarizing the results of (21)-(23) and the CDF \( F_{\Gamma_{be}}(\gamma) \), \( Q_{1i} \) and \( Q_{2i} \) can be derived. After that, plugging \( Q_{1i} \) and \( Q_{2i} \) into (18) and using (11) and (12), \( P_{DNC_{int}}^{MM} \) can be obtained.

**Remark 2.** (21) is derived on the basis of the assumption that \( \epsilon_2 \alpha_1 \lambda_{be} \neq \lambda_{ap}, \alpha_1 \lambda_{bp} \neq \lambda_{ap} \) and \( \epsilon_2 \lambda_{be} \neq \lambda_{bp} \). The primary reason is that, when \( \epsilon_2 \alpha_1 \lambda_{be} = \lambda_{ap} \) or \( \alpha_1 \lambda_{bp} = \lambda_{ap} \) or \( \epsilon_2 \lambda_{be} = \lambda_{bp} \), the derivation of the CDF for \( \Gamma_{ae} \) is a special case of (21). Furthermore, from the probability theory point of view, we have \( \Pr\{\epsilon_2 \alpha_1 \lambda_{be} \neq \lambda_{ap}, \alpha_1 \lambda_{bp} \neq \lambda_{ap}, \epsilon_2 \lambda_{be} \neq \lambda_{bp}\} = 1 \), which was detailedly explained in [36, Note 5].

2) **Asymptotic Performance:** In the effort to gain further insights into the MM relay selection scheme, we look into the asymptotic performance of the outage probability and intercept probability in high-SNR region. As \( P_I \to +\infty \), we have

\[
e^{-\varepsilon x} \approx 1 - \varepsilon x \approx 1, \tag{24}
\]

where \( \varepsilon \in \{(\lambda_{ia}^{-1} + \lambda_{ib}^{-1}), \lambda_{ia}^{-1}, \lambda_{ib}^{-1}, \lambda_{ie}^{-1}\} \).

For (16), as \( P_I \to +\infty \), we make \( e^{-(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})x} \) approximate to \( 1 - (\lambda_{ia}^{-1} + \lambda_{ib}^{-1})x \) and solve the corresponding integral. Consequently, the asymptotic expression of \( P_{1i} \) is given by

\[
P_{1i} \approx \lambda_{ib}^{-1} \int_0^\infty \left[ \prod_{j \in D_{ab} - \{i\}} (\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right] x^{k-1} [1 - (\lambda_{ia}^{-1} + \lambda_{ib}^{-1})x] e^{-\frac{x}{\lambda_{ip} \gamma}} dx \approx (k - 1)! (\lambda_{ip} \gamma)^k \lambda_{ib}^{-1} \left[ \prod_{j \in D_{ab} - \{i\}} (\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right]. \tag{25}
\]
Following the same steps as in the derivation of (25), $P_{2i}$ is approximate to

$$P_{2i} \approx \frac{1}{\lambda_{ia}\lambda_{ib}} \int_{0}^{\infty} \int_{0}^{x} \left[ \prod_{j \in D_{ab}-\{i\}} (\lambda^{-1}_{ja} + \lambda^{-1}_{jb}) \right] y^{k-1} (1 - \lambda^{-1}_{ia}y)(1 - \lambda^{-1}_{ib}x) dy e^{-\lambda_{ia}\gamma} dx$$

$$\approx \lambda^{-1}_{ia} \lambda^{-1}_{ib} (\lambda_{ia}\gamma)^{k+1} \left[ \prod_{j \in D_{ab}-\{i\}} (\lambda^{-1}_{ja} + \lambda^{-1}_{jb}) \right]. \quad (26)$$

From (25), (26), formulas $P_{\text{DNC}-\text{outb}-\text{MM}} = \sum_{i \in D_{ab}} [\hat{P}_{1i} + \hat{P}_{2i}]$ and $P_{\text{DNC}-\text{outb}-\text{MM}} = \sum_{k=0}^{M} \hat{P}_{k}\{\mid D_{ab}\mid = k\} \hat{P}_{\text{DNC}-\text{outb}-\text{MM}}$, the asymptotic outage probability $P_{\text{DNC}-\text{out}-\text{MM}}$ can be obtained.

As $P_{I} \to +\infty$, we have $\Gamma_{ae} \approx \frac{\lambda^{-1}_{ae}}{\epsilon_{z_{2}}\gamma_{bc}}$. According to the probability theory [35, Chapters III, IV], the asymptotic CDF of $\Gamma_{ae}$ is given by

$$F_{\Gamma_{ae}}(\gamma) \approx \int_{0}^{\infty} \text{Pr}\{\gamma_{ae} < \epsilon_{z_{1}}\gamma_{x}\} f_{\gamma_{ae}}(x) dx = 1 - \frac{\lambda_{ae}}{\epsilon_{z_{1}}\gamma_{bc} + \lambda_{ae}}. \quad (27)$$

Following the same steps as in (27), $\hat{F}_{\Gamma_{be}}(\gamma)$ can be derived.

Letting $\beta_{D_{ab}} = \min_{j \in D_{ab}} (\lambda^{-1}_{ja} + \lambda^{-1}_{jb})^{-1}$, we have $\beta_{D_{ab}} \to +\infty$ as $P_{I} \to +\infty$. Based on this approximation, (22) and (24), we obtain the following asymptotic expression

$$\text{Pr}\{\min\{\gamma_{ia}, \gamma_{ib}\} > Z_{i}\} \approx \int_{0}^{\beta_{D_{ab}}} \left[ \prod_{j \in D_{ab}} (\lambda^{-1}_{ja} + \lambda^{-1}_{jb}) \right] y^{k-1} \left[ \lambda^{-1}_{ia} + \lambda^{-1}_{ib} \right]^{-x^{-k}} dx$$

$$\approx \left[ \prod_{j \in D_{ab}} (\lambda^{-1}_{ja} + \lambda^{-1}_{jb}) \right] \left( \frac{\beta_{D_{ab}}}{k} \right). \quad (28)$$

In light of the theory of mathematical analysis [37, Chapter III], we have $\frac{\lambda_{ae}}{\lambda_{ae} + \lambda_{ip}\gamma} \approx 1 - \lambda^{-1}_{ae}\lambda_{ip}\gamma$ as $P_{I} \to +\infty$. As such, using (23), the asymptotic expression of $F_{\Gamma_{\text{DNC}}}$ is given by

$$F_{\Gamma_{\text{DNC}}} (\gamma) \approx \lambda^{-1}_{ie}\lambda_{ip}\gamma. \quad (29)$$

Substituting (27)-(29) and $\hat{F}_{\Gamma_{be}}(\gamma)$ into (19), we can obtain $\hat{Q}_{1i}$ and $\hat{Q}_{2i}$. Subsequently, on the basis of the obtained results and (18), we can expressed the asymptotic probability as

$$\hat{P}_{\text{DNC}-\text{outb}-\text{MM}} = \sum_{i \in D_{ab}} [\frac{\lambda_{ae}}{\epsilon_{z_{2}}\lambda_{bc}\gamma + \lambda_{ae}} + \frac{\lambda_{ae}}{\epsilon_{z_{2}}\lambda_{bc}\gamma + \lambda_{ae}}][\frac{\lambda_{ae}}{\epsilon_{z_{2}}\lambda_{bc}\gamma + \lambda_{ae}}][\prod_{j \in D_{ab}} (\lambda^{-1}_{ja} + \lambda^{-1}_{jb}) \frac{\beta_{D_{ab}}^2}{k^2}].$$

According to (11), the asymptotic intercept probability at $S_{b}$ can be derived by formula

$$\hat{P}_{\text{DNC}-\text{intb}-\text{MM}} = \sum_{k=0}^{M} \hat{P}_{r}\{|D_{ab}| = k\} \hat{P}_{\text{DNC}-\text{outb}-\text{MM}}. \quad (29)$$

In a similar manner, we can get $\hat{P}_{\text{DNC}-\text{outa}-\text{MM}}$. Consequently, the asymptotic intercept probability $\hat{P}_{\text{DNC}-\text{int}-\text{MM}}$ can be derived.
Remark 3. (27) is obtained through probability theory [35, Chapters III and IV]. To be specific, if a random variable sequence \( \{\xi_n\} \) converges to the random variable \( \xi \), then \( \{\xi_n\} \) converges almost everywhere to \( \xi \). It follows that \( \{\xi_n\} \) converges in probability to \( \xi \). Moreover, writing the CDF of \( \xi_n \) as \( F_{\xi_n}(x) \), we have \( \{F_{\xi_n}(x)\} \) converges to \( F_{\xi}(x) \) if \( F_{\xi}(x) \) is a continuous function. Utilizing this conclusion, we can directly derive (27) from \( a_{e} \approx \frac{\gamma(1)}{e^{\alpha_1} e^{\alpha_1}}. \) Evidently, a lot of complex calculation is avoided by means of this method. Later on, the numerical results in Section IV verify the correctness of this method as well.

Remark 4. The value of \( \beta_{D_{ab}} \) is derived according to two rules. Firstly, the probability \( \Pr\{\min\{\gamma_{ia}, \gamma_{ib}\} > Z_i\} \) should not be larger than 1. Secondly, in some special case (e.g., the coefficients of links \( R_i \rightarrow S_a \) and \( R_i \rightarrow S_b \) for \( i \in D_{ab} \) are independent and identically distributed random variables), the probability \( \Pr\{\min\{\gamma_{ia}, \gamma_{ib}\} > Z_i\} \) is equal to 1. Moreover, \( \beta_{D_{ab}} \) is validated by numerical simulations.

B. Performance of GMM Scheme

1) Exact Performance: According to the total probability formula and (9), the outage probability at \( S_b \) of the GMM scheme can be represented as

\[
P_{DNC-D_{ab}}^{outb-GMM} = \sum_{i \in D_{ab}} \left[ \Pr\left\{ \Gamma_{ib}^{DNC} < \gamma, \frac{\gamma_{ia}}{|h_{ip}|^2} > \frac{\gamma_{ib}}{|h_{ip}|^2} > T_i \right\} \right]
+ \Pr\left\{ \Gamma_{ib}^{DNC} < \gamma, \frac{\gamma_{ib}}{|h_{ip}|^2} > \frac{\gamma_{ia}}{|h_{ip}|^2} > T_i \right\},
\]  

(30)

where \( T_i = \max_{j \in D_{ab}-\{i\}} \frac{\min\{\gamma_{ja}, \gamma_{jb}\}}{|h_{jp}|^2}. \) The closed-form expressions of \( P_{3i} \) and \( P_{4i} \) are given by (31) and (32), respectively. And the derivations of \( P_{3i} \) and \( P_{4i} \) are provided in Appendix B.

Substituting \( P_{3i} \) and \( P_{4i} \) into (30), \( P_{DNC-D_{ab}}^{D_{outb-GMM}} \) can be derived. Performing the same steps as in the derivation of \( P_{DNC-out-MM} \), we can obtain the outage probability \( P_{DNC-D_{out-GMM}} \).

Remark 5. In Appendix B, making use of the basic results in mathematical analysis [37,
\[ P_{3i} = \sum_{n=0}^{\infty} \frac{g_n^{i_Aq}}{(n+1)!} \left\{ \sum_{d=1}^{n-1} \frac{(d-1)!}{(-1)^d(n+1)!} \lambda_{ip} \right\} \frac{d^n}{(d-1)!} \frac{(n-d)!}{(\lambda_{ip} + \lambda_{ia} + \lambda_{ib})^n} \]
\[ + \frac{(n-d)!}{d_{i_Aq}^d \lambda_{ip}^{d-n-1}} \sum_{w=0}^{\infty} \left\{ 2F1[1, n+w+1; n+w+2; (1 + \lambda_{ip} a_{i_Aq} (\lambda_{ia} + \lambda_{ib}))^{-1}] \right\} \times \frac{\lambda_{ia}^{-1} a_{i_Aq}^{-1} w}{n^{-1}} \]
\[ \times 2F1[1, n+w+1; n+w+2; (1 + \lambda_{ip} (\gamma + a_{i_Aq}) (\lambda_{ia} + \lambda_{ib}))^{-1}] \times \frac{(n+1)!}{a_{i_Aq}^{-1} w!} \] (31)

\[ P_{4i} = \frac{\lambda_{ib}}{\lambda_{ia}} P_{3i} - \sum_{n=0}^{\infty} \frac{(-1)^n g_n^{i_Aq}}{(n+1)!} \lambda_{ia} \lambda_{ip} \frac{d^n}{(d-1)!} \frac{(n-d)!}{(\lambda_{ip} + \lambda_{ia} + \lambda_{ib})^n} \]
\[ + \frac{(n-d)! a_{i_Aq}^{-d}}{(\gamma \lambda_{ib} + \lambda_{ip})^{-n-d+1}} \sum_{w=0}^{\infty} \left\{ 2F1[1, n+w+1; n+w+2; (1 + \lambda_{ia}^{-1} a_{i_Aq}^{-1} \gamma \lambda_{ib}^{-1} + \lambda_{ip}^{-1})^{-1}] \right\} \times \frac{\lambda_{ia}^{-1} a_{i_Aq}^{-d} w}{n^{-d+1}} \]
\[ \times 2F1[1, n+w+1; n+w+2; (1 + (\gamma + a_{i_Aq}) \lambda_{ia}^{-1} (\gamma \lambda_{ib}^{-1} + \lambda_{ip}^{-1}))^{-1}] \times \frac{(n+1)!}{a_{i_Aq}^{-d} w!} \] (32)

Chapter V, the proper fraction \( \prod_{j \in A} \frac{1}{\lambda_{ja}^{-1} + \lambda_{jb}^{-1} x + 1} \) can be presented in the following form

\[
\left( \prod_{j \in A} \frac{1}{\lambda_{ja}^{-1} + \lambda_{jb}^{-1} x + 1} \right) = \sum_{p_1, \ldots, p_N} \sum_{q=1}^{p_q} \frac{1}{(x + a_{i_Aq})^n}. \tag{33}
\]

Substituting this result into (B.3) and replacing \( x \) with \( \frac{a}{x} \), we can obtain (B.4).

According to (10) and total probability formula, the intercept probability at \( S_o \) of the GMM
relay selection scheme is given by

$$P_{DNC-Dab-intb-GMM} = \sum_{i \in D_{ab}} \left[ \Pr \left\{ \Gamma_{ae} > \gamma, \min\left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\} > T_i \right\} \right]$$

$$+ \ Pr \left\{ \Gamma_{ae} < \gamma, \Gamma_{be} > \gamma, \Gamma_{DNC}^{ie} > \gamma, \min\left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\} > T_i \right\},$$  \hspace{1cm} (34)

where the first and second terms in the bracket of (34) are denoted as $Q_{3i}$ and $Q_{4i}$, respectively.

Since the events $\{\Gamma_{ae} > \gamma\}$ and $\{\min\left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\} > T_i\}$ are independent, the term $Q_{3i}$ can be written as

$$Q_{3i} = \Pr \{\Gamma_{ae} > \gamma\} \Pr \left\{ \min\left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\} > T_i \right\}.$$  \hspace{1cm} (35)

From (B.3), $\Pr \{T_i < \min\left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\}\}$ can be expressed as

$$\Pr \left\{ T_i < \min\left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\} \right\} = \sum_{l=0}^{k-1} \sum_{A \subset D_{ab}-\{i\}, |A|=l} \int_0^\infty f_{T_{2i}}(x) \left[ \prod_{j \in A} \lambda_{jp}^{-(\lambda_{ja}^{-1} + \lambda_{jb}^{-1})}x + 1 \right] dx,$$  \hspace{1cm} (36)

where $f_{T_{2i}}(x)$ is obtained by taking derivative of $F_{T_{2i}}(x)$ (B.2) with respect to $x$ and can be written as

$$f_{T_{2i}}(x) = \frac{\lambda_{ip}^{-1}(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})^{-1}}{[x + \lambda_{ip}^{-1}(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})^{-1}]^2}.$$  \hspace{1cm} (37)

Resorting to the Remark 5, equations (21), (36) and (37), the closed-form expression of $Q_{3i}$ is given by (38).

Noticing that the random variables $\Gamma_{DNC}^{ie}$ and $\min\left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\}$ have the common term $|h_{ip}|^2$, $Q_{4i}$ can be represented as

$$Q_{4i} = \Pr \{\Gamma_{ae} < \gamma\} \Pr \{\Gamma_{be} > \gamma\} \Pr \left\{ \Gamma_{DNC}^{ie} > \gamma, \min\left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\} > T_i \right\}.$$  \hspace{1cm} (39)

Similarly to the calculation of $P_{3i}$ and $P_{4i}$, applying the total probability theorem, [38, Eq.3.353.1] and [38, Eq.6.228.2], we can directly write the closed-form expression of $Q_{4i}$ as in (40).

Finally, plugging $Q_{3i}$ and $Q_{4i}$ into (34), we can get $P_{DNC-Dab-intb-GMM}$. Repeating the same processes as in the derivation of $P_{int-MM}^{DNC}$, we can obtain the intercept probability $P_{int-GMM}^{DNC}$. 

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\[ Q_{3i} = \frac{(1 - F_{\gamma}(\gamma))}{\lambda_{ip}(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})} \sum_{l=0}^{k-1} \sum_{A \subset D_{ab} \setminus \{i\}, |A| = l} \sum_{p_1 + \cdots + p_N = l} \sum_{q=1}^{N} (-1)^l \int_0^\infty \left[ \left( \sum_{n=2}^{p_q} \frac{f_{i}^{A_q}}{(x + a_{i}^{A_q})^n} \right) \right] \]

\[ + \lim_{Q_{4i} \to +\infty} \int_0^\infty F_{I}(\frac{y}{x}) f_{\min(\gamma_{ie}, \gamma_{ib})}(y) dy \Pr\{\gamma_{ie} > \gamma x\} f_{|h|_2}(x) dx \]

\[ = \frac{(1 - F_{\gamma}(\gamma))}{(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})^{n-1}} \sum_{d=1}^{n-1} \frac{(n-d)!}{(d-1)!} (\gamma + \lambda_{ia}^{-1})^{d-n-1} \]

\[ + \sum_{w=0}^{n+w+1} \frac{(a_{i}^{A_q}(\lambda_{ia}^{-1} + \lambda_{ib}^{-1}))^w \Gamma(n + w + 1)}{(n + w + 1)! (a_{i}^{A_q}(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})) + \lambda_{ie}^{-1}} \]

\[ \times_2 F_1 [1, n + w + 1; n + w + 2; (a_{i}^{A_q}(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})) + \lambda_{ie}^{-1} \gamma + \lambda_{ip}^{-1})^{-1} + 1]^{-1} \].

2) Asymptotic Performance: To provide further insights into the closed-form expressions for the outage probability and intercept probability, an asymptotic analysis (high-SNR regime) is carried out for the GMM relay selection scheme.

Evidently, as \( P_1 \to +\infty \), we have \( \frac{1}{1 + \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{ib}^{-1})} x \approx 1 - \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{ib}^{-1}) x \). Consequently, on the basis of (B.3), we obtain the following approximation

\[ F_{T_i}(\frac{y}{x}) \approx \left( \frac{y}{x} \right)^{k-1} \left[ \prod_{j \in D_{ab} \setminus \{i\}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right]. \]

Substituting (24) and (41) into (B.1) and (B.7), the asymptotic expressions of \( P_{3i} \) and \( P_{4i} \) are
expressed, respectively, as

\[
P_{3i} \approx \frac{1}{\lambda_{ib}\lambda_{ip}} \left[ \prod_{j \in \mathcal{D}_{ab} \setminus \{i\}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right] \int_{0}^{\infty} \int_{0}^{\gamma x} \frac{y^{k-1}}{x^{k-1}} (1 - (\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) y) dy e^{-\lambda_{ip}^{-1} x} dx
\]

\[
\approx (k \lambda_{ib})^{-1} \lambda_{ip}^{-1} \left[ \prod_{j \in \mathcal{D}_{ab} \setminus \{i\}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right]. \tag{42}
\]

\[
P_{4i} \approx \frac{\lambda_{ip}^{-1}}{\lambda_{ia}\lambda_{ib}} \int_{0}^{\infty} \int_{0}^{\gamma x} \left[ \prod_{j \in \mathcal{D}_{ab} \setminus \{i\}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right] \frac{y^{k-1}}{x^{k-1}} (\gamma x - y) dy e^{-\lambda_{ip}^{-1} x} dx
\]

\[
\approx 2 \gamma^{k+1} \lambda_{ip}^{2} (k+1) \lambda_{ia} \lambda_{ib}^{-1} \left[ \prod_{j \in \mathcal{D}_{ab} \setminus \{i\}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right]. \tag{43}
\]

Plugging (42) and (43) into (30), \(\hat{F}_{\text{DNC}_{out} - \text{MM}}\) can be derived. Similarly to the derivation of \(\hat{F}_{\text{DNC}_{out} - \text{GMM}}\), we can get \(\hat{F}_{\text{DNC}_{out} - \text{GMM}}\).

Next, we study the asymptotic intercept probability of the GMM relay selection scheme. Let

\[
\overline{\beta}_{D_{ab}} = \min_{j \in \mathcal{D}_{ab}} [\lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1})]^{-1}.
\]

As \(P_I \to +\infty\), using (24) and (41), we have

\[
\Pr\left\{ T_i < \min_{j \in \mathcal{D}_{ab}} \left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\} \right\} \approx \lambda_{ip}^{-2} \left[ \prod_{j \in \mathcal{D}_{ab}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right] \int_{0}^{\infty} \int_{0}^{\overline{\beta}_{D_{ab}} x} \left( \frac{y}{x} \right)^{k-1} dy e^{-\lambda_{ip}^{-1} x} dx
\]

\[
= k^{-1} (\overline{\beta}_{D_{ab}})^{-k} \left[ \prod_{j \in \mathcal{D}_{ab}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right]. \tag{44}
\]

In the same way, when \(P_I \to +\infty\), utilizing (24) and (41), we have

\[
\Pr\left\{ \Gamma_{ae}^{\text{DNC}} > \gamma, \min_{j \in \mathcal{D}_{ab}} \left\{ \frac{\gamma_{ia}, \gamma_{ib}}{|h_{ip}|^2} \right\} > T_i \right\} \approx \lambda_{ip}^{-2} \left[ \prod_{j \in \mathcal{D}_{ab}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right] \int_{0}^{\infty} \int_{0}^{\overline{\beta}_{D_{ab}} x} \left( \frac{y}{x} \right)^{k-1} dy
\]

\[
\times e^{-(\lambda_{ip}^{-1} + \lambda_{ae}^{-1}) x} dx \approx \frac{(1 - \lambda_{ip} \lambda_{ae}^{-1} \gamma)^2}{k (\overline{\beta}_{D_{ab}})^{-k}} \left[ \prod_{j \in \mathcal{D}_{ab}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right] \tag{45}
\]

Plugging (44), (45), and the asymptotic CDFs of \(\Gamma_{ae}\) and \(\Gamma_{be}\) into (34), we can obtain

\[
\hat{F}_{\text{DNC}_{int} - \text{GMM}} = \sum_{j \in \mathcal{D}_{ab}} \frac{(\overline{\beta}_{D_{ab}})^{j}}{k} \left[ \frac{\lambda_{ae}}{\varepsilon_2 \alpha_1 \lambda_{ae} \gamma + \lambda_{ae}} + \frac{(1 - \lambda_{ip} \lambda_{ae}^{-1})^2}{(\varepsilon_2 \alpha_2 \lambda_{e} \gamma + \lambda_{ae}) \left( \varepsilon_2 \alpha_1 \lambda_{ae} \gamma + \lambda_{ae} \right)} \right] \left[ \prod_{j \in \mathcal{D}_{ab}} \lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1}) \right].
\]

Then, employing the same method as in the derivation of \(\hat{F}_{\text{DNC}_{int} - \text{MM}}\), the asymptotic intercept probability \(\hat{F}_{\text{DNC}_{int} - \text{GMM}}\) can be derived.

IV. NUMERICAL RESULTS

In this section, Monte Carlo simulations are conducted to validate the theoretical results of the proposed AN-aided opportunistic relay selection schemes. The simulation environment follows
the model of Section II where the nodes $S_a$, $S_b$, PR, E and $R_i$ ($i \in \mathcal{R}$) are located at $(-1,0), (1,0), (0,1), (0,-1)$ and $(0.05 \cos \frac{2\pi i}{M}, 0.05 \sin \frac{2\pi i}{M})$, respectively. We assume all channels follow Rayleigh fading. The average channel gain between two nodes is denoted as $d^{-\beta}$, where $d$ is the Euclidean distance and $\beta$ is the path loss exponent. Throughout the performance evaluation, we set $\beta = 3$, $\alpha_1 = \alpha_2 = \epsilon_2 = 1$, $\epsilon_1 = 0.02$, and $\sigma_a^2 = \sigma_b^2 = \sigma_e^2 = \sigma_i^2 = \sigma^2$. For convenience, we denote $\frac{P_I}{\sigma^2}$ as the transmit SNR. In all figures, the theoretical curves of the outage probability and intercept probability for the MM and GMM relay selection schemes are obtained by the analytical results in Section III, and found to be perfectly matched with the Monte Carlo simulation results. This verifies the correctness of the theoretical analysis.

Fig. 2 depicts the outage probability versus SNR for the MM and GMM relay selection schemes. It is observed from Fig. 2 that the outage probability performance of the GMM scheme is obviously better than that of the MM scheme. As for the impact of the number of secondary relays on the outage probability performance, it can be seen that as $M$ increases from 6 to 9, the outage probability of the proposed relay selection schemes are significantly improved. In addition, we can observe from Fig. 2 that the asymptotic curves tightly approximate the analytical
Fig. 3. Intercept probability versus the threshold $\gamma$ of the MM and GMM relay selection schemes with SNR=10 dB.

Fig. 4. Intercept probability versus SNR of the MM and GMM relay selection schemes with $\gamma = 1$.

curves in high SNR region, which confirms the correctness of our analysis.

Fig. 3 plots the intercept probability as a function of $\gamma$ for the MM and GMM relay selection schemes, where the SNR is 10 dB. It can be seen that the intercept probability of each relay selection scheme almost has no change as the number of secondary relays increases. In addition, with the same number of secondary relays, the intercept probability of the MM scheme is almost
Fig. 5. Intercept probability versus SNR of the MM and GMM relay selection schemes with $\gamma = 7$.

equal to that of the GMM scheme. This is due to the fact that the proposed relay selection schemes do not take into account the CSI of wiretap links.

Figs. 4 and 5 show the exact and asymptotic intercept probability versus SNR of the proposed relay selection schemes with $\gamma = 1$ and $\gamma = 7$, respectively. Clearly, the MM relay selection scheme achieves almost the same intercept probability performance as the GMM relay selection scheme, given the same number of secondary relays. Moreover, increasing $M$ has a negligible impact on the intercept probability of the MM/GMM relay selection, which is consistent with our observation in Fig. 3.

It can be seen from Figs. 2, 4 and 5 that the outage probability and intercept probability are increasing and decreasing functions of SNR, respectively. This implies that there exists a tradeoff between the security (intercept probability) and reliability (outage probability) of the MM and GMM relay selection schemes for the given $M$ and $\gamma$. Based on this observation, we investigate the impact of $M$ on the SRT performance of the proposed relay selection schemes with $\gamma = 1$ and $\gamma = 7$ in Fig. 6. It can be seen that with the same number of secondary relays, the GMM scheme outperforms the MM scheme in terms of the SRT, indicating the advantage
Fig. 6. The SRTs of the MM and GMM relay selection schemes with $\gamma = 1$ and $\gamma = 7$.

Fig. 7. The SRTs of the MM and GMM relay selection schemes with $M = 6$.

of the GMM scheme. From Fig. 6, we can see that the SRT performance of the MM and GMM schemes is obviously improved as $M$ increases from 6 to 9. Moreover, it can also be seen that the gap between the SRT curves of $M = 6$ and $M = 9$ for the GMM scheme is larger than that for the MM scheme. This indicates that the GMM scheme can more efficiently enhance the SRT performance by increasing the number of secondary relays.
The SRT performance of the proposed relay selection schemes for different threshold values is studied in Fig. 7. Clearly, the SRT performance of the MM and GMM relay selection schemes is greatly improved as the threshold value $\gamma$ increases from 1 to 7. Additionally, it can be noticed that the SRT performance improvement of the GMM scheme is similar to that of the MM scheme.

The sum of the outage probability and intercept probability versus SNR of the MM and GMM relay selection schemes is studied in Fig. 8, for different number of secondary relays, i.e., $M = 6, 9$, and different threshold values, i.e., $\gamma = 1, 7$. It is shown that the sum probability performance of each relay selection scheme decreases as the number of secondary relays increases from 6 to 9. This observation essentially agrees with Fig. 6. Moreover, it is clear from this figure that, with the same threshold value, the sum probability performance of the GMM relay selection scheme is better than that of the MM scheme at the low/intermediate SNR. However, the MM and GMM schemes almost have the same sum probability performance at high SNR. Fig. 8 also indicates that for each relay selection scheme, the sum probability with $\gamma = 7$ outperforms the
case with $\gamma = 1$ at low/intermediate SNR, which turns out contrary to that in high SNR region.

V. CONCLUSION

In this paper, we proposed two AN-aided opportunistic relay selection schemes, namely the MM and GMM relay selection, to enhance the PLS in an underlay wiretap CTWRN. We investigated the performance of those two schemes, and derived exact closed-form expressions for the outage probability and intercept probability. To gain more insights, we further conduct asymptotic analysis of the proposed schemes at high SNR, and also derived closed-form approximate expressions for the outage probability and intercept probability. Based on the obtained closed-from outage probability and intercept probability, we studied the SRT performance of the MM and GMM relay selection schemes, and found that the GMM scheme outperforms the MM scheme in terms of the SRT for the given number of secondary relays and the threshold value $\gamma$. It was also illustrated that the SRT performance of both MM and GMM relay selection could be significantly improved by increasing $M$ or $\gamma$. Moreover, with the same threshold value $\gamma$, the improvement of SRT performance of the GMM relay selection is more evident than that of the MM scheme as the number of secondary relays increases.

Besides the DF-DNC relaying protocol, the DF-ANC is another important relaying protocol for the underlay wiretap CTWRNs. However, we found that the exact and asymptotic closed-form outage probability and intercept probability of the MM and GMM relay selection schemes for the DF-ANC relaying strategy can also be derived by the method adopted in this paper, and, in addition, the SRT performance of the GMM scheme remains better than that of the MM scheme. Finally, we only explored the SRT performance of the MM and GMM relay selection schemes, but examining other secrecy performance metrics including secrecy outage probability, secrecy diversity order, and secrecy capacity of the MM and GMM relay selection schemes for the underlay wiretap CRTWRs is also a promising extension.
**APPENDIX A**

**DERIVATION OF $\Pr\{\mathcal{D}_{ab}\}$ AND $\hat{\Pr}\{\mathcal{D}_{ab}\}$**

Since $\Gamma_{a_1}, \cdots, \Gamma_{a_M}$ have the common term $X = |h_{ap}^{(1)}|^2 + \alpha_1 |h_{bp}^{(1)}|^2$, the probability of $\mathcal{D}_a$ can be represented as

$$\Pr\{\mathcal{D}_a\} = \int_0^\infty f_X(x) \prod_{j_1 \in \mathcal{D}_a} \Pr\left\{ \frac{\gamma_{aj_1}}{\epsilon_1 \alpha_1 \gamma_{bj_1}} + x > \gamma \right\} \prod_{j_2 \notin \mathcal{D}_a} \Pr\left\{ \frac{\gamma_{aj_2}}{\epsilon_1 \alpha_1 \gamma_{bj_2}} + x < \gamma \right\} dx, \quad (A.1)$$

where $\gamma = 2^{3r} - 1$. Let $T_{ij} = \frac{\gamma_{aj}}{\epsilon_1 \alpha_1 \gamma_{bj} + x}$, whose cumulative distribution function (CDF) is expressed as

$$F_{T_{ij}}(\gamma) = \int_0^\infty \Pr\{\gamma_{aj} < (x + \epsilon_1 \alpha_1 y)\gamma\} f_{\gamma_{bj}(y)}(y)dy = 1 - \frac{\lambda_{aj}}{\epsilon_1 \alpha_1 \lambda_{bj} \gamma + \lambda_{aj}} e^{-\lambda_{aj} \gamma x}. \quad (A.2)$$

Substituting (A.2) into (A.1), and making use of the multinomial expansion formula $\prod_{j \in \mathcal{D}} (1 - x_j) = \sum_{l=0}^{||\mathcal{D}||} \sum_{E \subseteq \mathcal{D}, |E| = l} (-1)^l (\prod_{j \in E} x_j)$, the probability of $\mathcal{D}_a$ can be rewritten as

$$\Pr\{\mathcal{D}_a\} = \left[ \prod_{j_1 \in \mathcal{D}_a} \frac{\lambda_{aj_1}}{\epsilon_1 \alpha_1 \lambda_{bj_1} \gamma + \lambda_{aj_1}} \right] \sum_{l=0}^{M-m_a} \sum_{E \subseteq \mathcal{D}_a, |E| = l} \left[ \prod_{j_2 \notin \mathcal{E}} \frac{-\lambda_{aj_2}}{\epsilon_1 \alpha_1 \lambda_{bj_2} \gamma + \lambda_{aj_2}} \right] \int_0^\infty \frac{f_X(x)}{e^{\delta_{\mathcal{D}_a} x}} dx. \quad (A.3)$$

where $\mathcal{D}_a^c = \mathcal{R} - \mathcal{D}_a$ and $\delta_{\mathcal{D}_a, E} = \sum_{j_2 \notin \mathcal{E}} \lambda_{aj_2}^{-1} + \sum_{j_1 \in \mathcal{D}_a} \lambda_{aj_1}^{-1}$.

On the other hand, the CDF of $X$ can be calculated as

$$F_X(x) = \int_0^x \Pr\{|a_1|h_{ap}^{(1)}|^2 < x - y\} f_{|h_{ap}^{(1)}|^2}(y)dy = 1 - \frac{\lambda_{ap} e^{-\frac{1}{\lambda_{ap}} x}}{\lambda_{ap} - \alpha_1 \lambda_{bp}} - \frac{\alpha_1 \lambda_{bp} e^{-\frac{1}{\alpha_1 \lambda_{bp}} x}}{\alpha_1 \lambda_{bp} - \lambda_{ap}}. \quad (A.4)$$

Differentiating $F_X(x)$ with respect to $x$, the probability density function (PDF) of $X$ is given by

$$f_X(x) = (\lambda_{ap} - \alpha_1 \lambda_{bp})^{-1} (e^{-\frac{1}{\lambda_{ap}} x} - e^{-\frac{1}{\alpha_1 \lambda_{bp}} x}). \quad (A.5)$$

Substituting (A.5) into (A.3) and calculating the corresponding integral gives the probability of $\mathcal{D}_a$ as

$$\Pr\{\mathcal{D}_a\} = \sum_{l=0}^{M-m_a} \sum_{E \subseteq \mathcal{D}_a, |E| = l} \left[ \prod_{j_1 \in \mathcal{D}_a} \frac{\lambda_{aj_1}}{\epsilon_1 \alpha_1 \lambda_{bj_1} \gamma + \lambda_{aj_1}} \right] \left[ \prod_{j_2 \notin \mathcal{E}} \frac{-\lambda_{aj_2}}{\epsilon_1 \alpha_1 \lambda_{bj_2} \gamma + \lambda_{aj_2}} \right] \left( \delta_{\mathcal{D}_a, E} \lambda_{ap} + 1 \right)^{-1} \delta_{\mathcal{D}_a, E} \alpha_1 \lambda_{bp} + 1. \quad (A.6)$$
By means of the same method, the probability $\Pr\{D_b\}$ can be derived. Moreover, based on the assumption that coefficients of different wireless channels are statistically independent, the probability of $D_{ab}$ is expressed as $\Pr\{D_{ab}\} = \Pr\{D_a\} \Pr\{D_b\}$. Consequently, the closed-form expression of the probability $\Pr\{D_{ab}\}$ can be obtained.

According to the basic mathematical analysis theory, as $P_t \to +\infty$, we have $\Gamma_{ai} \approx \frac{\gamma_{ai}(1)}{\epsilon_1 \alpha_1 \gamma_{bi}^{(1)}}$. Therefore, the asymptotic probability of $\Gamma_{ai}$ can be written as $F_{\Gamma_{ai}}(\gamma) \approx 1 - \frac{\lambda_{ai}}{\epsilon_1 \alpha_1 \lambda_{ai} \gamma + \lambda_{ai}}$. The justification of this approximation is given in Remark 3. Making use of (A.1) and the asymptotic approximation of $F_{\Gamma_{ai}}(\gamma)$, the asymptotic probability of $D_a$ is given by

$$\hat{\Pr}\{D_a\} = \left[ \prod_{j \in D_a} \frac{\lambda_{aj_1}}{\epsilon_1 \alpha_1 \lambda_{bj_1} \gamma + \lambda_{aj_1}} \right] \left[ \prod_{j_2 \notin D_a} \frac{\epsilon_1 \alpha_1 \lambda_{bj_2} \gamma}{\epsilon_1 \alpha_1 \lambda_{bj_2} \gamma + \lambda_{aj_2}} \right]. \quad (A.7)$$

In the same way, the asymptotic probability $\hat{\Pr}\{D_b\}$ can be obtained. Thus, the asymptotic probability $\hat{\Pr}\{D_{ab}\}$ can be easily derived.

**APPENDIX B**

**DERIVATION OF $P_{3i}$ AND $P_{4i}$**

Since $\gamma_{ia}$ and $T_i$ are statistically independent, we can expressed $P_{3i}$ as follows:

$$P_{3i} = \int_{0}^{\gamma_x} \int_{0}^{x} \Pr\{\gamma_{ia} > y\} \Pr\{T_i < x\} f_{\gamma_{ia}}(y) f_{|h_{jp}|^2}(x) dy dx. \quad (B.1)$$

To obtain the closed-form expression of $P_{3i}$, we first calculate the CDF of $T_i$. Let $T_{2j} = \frac{\min\{\gamma_{ia}, \gamma_{ib}\}}{|h_{jp}|^2}$, whose CDF is given by

$$F_{T_{2j}}(x) = 1 - [\lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1})x + 1]^{-1}. \quad (B.2)$$

Note that, based on the assumption in subsection II. A, we can conclude that $T_{21}, \cdots, T_{2M}$ are statistically independent. Therefore, the CDF of $T_i$ can be evaluated as

$$F_{T_i}(x) = \left[ \prod_{j \in D_{ab} - \{i\}} F_{T_{2j}}(x) \right] = \sum_{l=0}^{k-1} \sum_{A \subset D_{ab} - \{i\}, |A| = l} (-1)^l \left[ \prod_{j \in A} (\lambda_{jp}(\lambda_{ja}^{-1} + \lambda_{jb}^{-1})x + 1)^{-1} \right]. \quad (B.3)$$
According to the basic theory of mathematical analysis [37, Chapter V], we can take \( F_{T_i}(\frac{y}{x}) \) as the summation of rational functions, and express \( F_{T_i}(\frac{y}{x}) \) as

\[
F_{T_i}(\frac{y}{x}) = \sum (-1)^{i} \frac{g_{n}^{A_{i}Q_n}}{(y + a_{i}A_{i}Q_n)^n}, \tag{B.4}
\]

where \( \sum = \sum_{l=0}^{k-1} \sum_{A \subset \mathcal{D}_{ab}, -\{i\}, |A|=l, p_1 + \cdots + p_N = l} \sum_{q=1}^{N} \sum_{p_q} \). A detailed interpretation for the derivation of (B.4) is given in Remark 5.

Substituting (B.4) into (B.1), we can rewrite \( P_{3i} \) as

\[
P_{3i} = \sum (-1)^{i} \frac{g_{n}^{A_{i}Q_n}}{\lambda_{ib}} \int_{0}^{\gamma x} \int_{0}^{y} \frac{e^{-(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})y}}{(y + a_{i}A_{i}Q_n)^n} dy x^n f_{[h_i, p_i]}(x) dx. \tag{B.5}
\]

To derive \( P_{3i} \), the integral \( I_1(x) \) needs to be solved. Applying [38, Eq.3.531.1], [38, Eq.3.532.1], and [38, Eq.1.211.1], \( I_1(x) \) is calculated as

\[
I_1(x) = \int_{0}^{\gamma x} \frac{e^{-(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})y}}{(y + a_{i}A_{i}Q_n)^n} dy - \int_{0}^{\gamma x} \frac{e^{-(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})y}}{(y + a_{i}A_{i}Q_n)^n} dy
\]

\[
= \frac{[-(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})]^{n-1}}{(n-1)!} \sum_{d=1}^{n} \frac{(d-1)!}{[-(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})]^d} \left[ \frac{x^{-d}}{\lambda_{ib}A_{i}Q_n} - \frac{x^{-d}e^{-(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})\gamma x}}{(\gamma + a_{i}A_{i}Q_n)^d} \right]
\]

\[
+ \sum_{w=0}^{\infty} \frac{[(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})a_{i}A_{i}Q_n]^w}{w!x^w} \left[ \text{Ei} \left( \frac{-(\gamma + a_{i}A_{i}Q_n)x}{(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})^{-1}} \right) - \text{Ei} \left( \frac{-a_{i}A_{i}Q_n x}{(\lambda_{ia}^{-1} + \lambda_{ib}^{-1})^{-1}} \right) \right], \tag{B.6}
\]

where \( \text{Ei}(\cdot) \) is the exponential integral \( \text{Ei}(x) = \int_{-\infty}^{x} e^{-t^{-1}} dt \). Then, plugging \( I_1(x) \) into (B.5), using [38, Eq.3.531.3], [38, Eq.6.228.2] and Gauss hypergeometric function [38, Eq.9.14.2], taking some algebraic manipulations, we can obtain the closed-form expression of \( P_{3i} \) in (31).

Following the same steps as outlined in the derivation of \( P_{3i} \), \( P_{4i} \) can be expressed as

\[
P_{4i} = \int_{0}^{\gamma x} \int_{0}^{y} \text{Pr}\{y < \gamma_{ib} < \gamma x\} \text{Pr}\left\{T_i < \frac{y}{x}\right\} f_{\gamma_{ia}}(y) dy f_{[h_i, p_i]}(x) dx. \tag{B.7}
\]

Substituting (B.4) into (B.7), \( P_{4i} \) can be rewritten as

\[
P_{4i} = \frac{\lambda_{ib}P_{3i}}{\lambda_{ia}} - \sum (-1)^{i} \frac{g_{n}^{A_{i}Q_n}}{\lambda_{ia}} \int_{0}^{\gamma x} \int_{0}^{y} x^n f_{[h_i, p_i]}(x) e^{-\lambda_{ia}^{-1}y} e^{\lambda_{ib}^{-1}x} \frac{dy}{(y + a_{i}A_{i}Q_n)^n} dx. \tag{B.8}
\]

Consequently, adopting the same approach as in the derivation of \( P_{3i} \), the closed-form expression of \( P_{4i} \) is given in (32).
REFERENCES


