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Joint Constellation-Labeling Optimization for VLC-CSK Systems

Linqiong Jia, Feng Shu, Ming Chen, Weibin Zhang, Jun Li, and Jiangzhou Wang, *Fellow, IEEE*

***Abstract*—In this paper, we have optimized the joint constellation-labeling for a finite number of signals in visible light communication (VLC) systems with Color-shift keying (CSK) modulation by maximizing the pragmatic mutual information (PMI). Firstly, an equivalent parallel channel model of the VLC-CSK system is constructed. Then the PMI is derived to be the objective function of the newly formulated joint constellation-labeling optimization problem of the VLC-CSK system. Simulated annealing (SA) algorithm and modified interior point methods (MIPM) are adopted to solve the joint constellation-labeling optimization problem. Simulation results show that the proposed methods improve the PMI performance compared to the existing constellation-labeling scheme of signals, especially in the low signal-to-noise ratio (SNR) region. Besides, it can been seen that the MIPM saves a large amount of computational time compared to the SA algorithm while achieving the same performance improvement.**

***Index Terms*—Visible light communication, color-shift keying, pragmatic mutual information, joint signal-labeling Optimization.**

I. Introduction

Visible light communication (VLC) using light-emitting diodes (LEDs) has become a promising candidate for indoor wireless communication due to its appealing advantages such as reusability of LED lighting infrastructures, freedom from hazardous electromagnetic radiation and costly frequency licensing [1]. Two kinds of LEDs can be used as the light source (i.e. the transmitter) for VLC system. One is the white phosphorescent LEDs, which utilize the optical spectrum inefficiently while preferred due to cost and complexity. The other one is the multi-color LEDs making use of the optical spectrum with high efficiency, of which, the red/green/blue (RGB) LEDs are the most widely used. If each color of RGB LEDs is modulated independently, it is possible to achieve a threefold increase in data throughput by wavelength division multiplexing. However, since undesirable light source that vary in both intensity and color over time may be created in this way, color-shift keying (CSK) is outlined in standard IEEE 802.15.7 as an intensity modulation scheme that transmits data imperceptibly through the variation of the light color emitted by RGB LEDs [2].

In [3] and [4], the constellation optimization problem of maximizing the minimum Euclidean distance between constellation points was proposed for CSK modulation system, which is solved by billiards algorithms and interior point methods, respectively. It was noticed that the labeling of constellations was not considered in [3] and [4], though it indeed had impact on the system performance. In fact, Gray mappings have been proved to be optimal labeling at high signal to noise ratios (SNRs) [5]. Unfortunately, Gray mapping is known for only several kinds of particular modulations such as QAM, QPSK, and PSK, but unsuitable for CSK modulation. Since, to the best of our knowledge, how to label the constellations is not exploited when the constellation positions are optimized. In this paper, we focus on the joint optimization of constellation-labeling for a finite number of signals. Our main contributions are summarized as follows:

1) Due to the fact that the encoder is separated from the modulator [2], an equivalent parallel channel model of the VLC-CSK system is established. The PMI expression for the VLC-CSK system is deduced as a function of the constellation positions and labelings based on the parallel channel model.
2) The joint constellation-labeling optimization problem maximizing the PMI is formulated under the constraints of the CSK modulation. As the PMI cannot be calculated in a closed form, we can resort to the simulated annealing (SA) algorithm to solve the jointly optimization problem [6]. Due to the fact that the PMI is differentiable, a modified interioir point methods (MIPM) is proposed to solve the joint optimization problem. Finally, simulation results show that MIPM can achieve the same PMI performance as the SA algorithm with significantly low-complexity.

The rest of the paper is organized as follows. In Section II, the system model of VLC-CSK system is introduced. In Section III, the expression for the PMI with a finite number of signal constellations is deduced to formalize the joint constellation-labeling optimization problem and SA and MIPM algorithms are used to solve the problem. In Section IV, simulation results obtained by SA algorithm and MIPM are presented and analyzed. Finally, the paper is concluded in Section V.

II. System Model

Fig. 1 illustrates a typical M-CSK modulated VLC-CSK system with RGB-LEDs, where $M = 2^m$ with m being a positive integer. In Fig. 1, a binary sequence of 0's or 1's with equal probability, representing the data source, is splitted into subsequences of m bits. Each subsequence corresponds

Linqiong Jia, Feng Shu, Weibin Zhang and Jun Li are with with the School of Electronic and Optical Engineering, Nanjing University of Science and Technology, 210094, China.

Ming Chen is with National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China.

Jiangzhou Wang is with the School of Engineering and Digital Arts, University of Kent, Canterbury CT2 7NT, U.K.

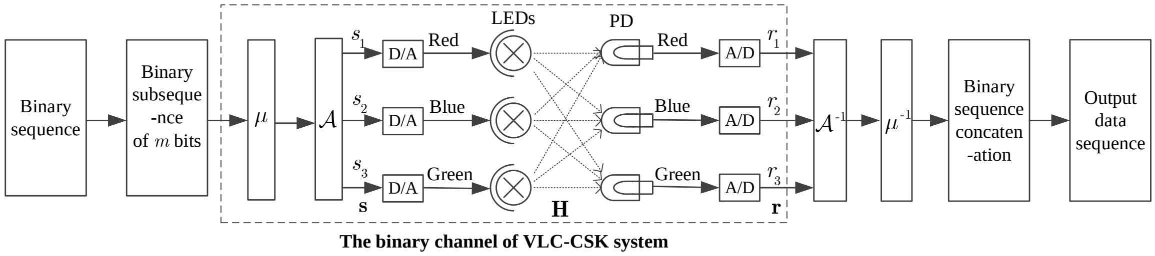

Fig. 1. Block diagram of transmission in VLC-CSK system.

to a labeling code $\boldsymbol{b} = (b_1, \cdots, b_m)$ in the binary labeling code set $\mathcal{B} = \{\boldsymbol{b}^{(1)}, \boldsymbol{b}^{(2)}, \cdots, \boldsymbol{b}^{(M)}\} = \{0,1\}^m$. Here, the probabilities $p(\boldsymbol{b}^{(1)}) = p(\boldsymbol{b}^{(2)}) = \cdots = p(\boldsymbol{b}^{(M)}) = \frac{1}{M}$. Let $\mathcal{A} = \{\boldsymbol{s}^{(1)}, \boldsymbol{s}^{(2)}, \cdots, \boldsymbol{s}^{(M)}\}$ be the signal set of CSK modulation with each signal vector $\boldsymbol{s}^{(i)} = (s_1, s_2, s_3)$ representing a specific constellation point in the signal space. Then, a one-to-one label mapping μ is applied to the subsequences,

$$\mu : \mathcal{B} \ni \boldsymbol{b} = (b_1, \cdots, b_m) \rightarrow \boldsymbol{s}^{(i)} = (s_1, s_2, s_3) \in \mathcal{A}. \tag{1}$$

Similarly, $p(\boldsymbol{s}^{(1)}) = \cdots = p(\boldsymbol{s}^{(M)}) = 1/M$. Finally, the resulting signal $\boldsymbol{s}^{(i)} = (s_1, s_2, s_3)$ is transmitted over the VLC channel by emitting light from red, green and blue LEDs with intensities s_1, s_2 and s_3, respectively.

The channel model of VLC-CSK system can be denoted as

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}, \tag{2}$$

where $\boldsymbol{r} \in \mathbb{R}^3$, $\boldsymbol{H} \in \mathbb{R}^{3\times 3}$ and $\boldsymbol{n} \in \mathbb{R}^3$ denote the receive vector, channel matrix and the noise vector, respectively. The light intensities emitted from the red/green/blue LEDs are measured by the photo-detectors (PDs) with red/green/blue optical filters, respectively, which constitute the received signal vector $\boldsymbol{r} = (r_1, r_2, r_3) \in \mathbb{R}^3$. The element of the channel matrix, $\boldsymbol{H} = [h_{ij}]_{3\times 3}$, h_{ij}, represents the channel gain between the i^{th} LED and the j^{th} PD. $\boldsymbol{H}$ is assumed to be deterministic and invertible generally [2]. $\boldsymbol{n} = (n_{\text{r}}, n_{\text{g}}, n_{\text{b}}) \in \mathbb{R}^3$ is the zero-mean Gaussian noise vector with covariance matrix $\sigma^2 \boldsymbol{I}_3$, i.e., $\boldsymbol{n} \sim \mathcal{N}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_3)$, where $\boldsymbol{I}_3$ is the identity matrix.

At the receiver, the received signal $\boldsymbol{r}$ is demodulated and then demapped to get a binary code in $\mathcal{B}$. Finally, the binary codes are concatenated to recover the orignal binary sequence.

Due to the illumination requirements in CSK systems, the symbol vector $\boldsymbol{s}$ should satisfy the non-negativity and peak intensity constraint, lighting intensity constraint and color constraint [2].

$$\boldsymbol{0} \leqslant \boldsymbol{s} \leqslant \boldsymbol{p} \tag{3}$$

$$\boldsymbol{1}^T \boldsymbol{s} = I > 0 \tag{4}$$

$$E[\boldsymbol{s}] = \frac{1}{M} \sum_{i=1}^{M} \boldsymbol{s}_i = \boldsymbol{s}_{avg} \tag{5}$$

where $\boldsymbol{0} = (0,0,0)$, $\boldsymbol{1} = (1,1,1)$, $\boldsymbol{p} = (P_r, P_g, P_b)$ with P_r, P_g and P_b denoting the peak intensity of the red, green and blue LEDs, respectively. I is a constant representing the total operating intensity. $\boldsymbol{s}_{avg}$ is the expected perceived color.

The signal space of $\boldsymbol{s}$ is a convex polygon in $\mathbb{R}^3$ according to the intensity constraints (3) and (4). If $\boldsymbol{H} = \boldsymbol{I}_3$ and

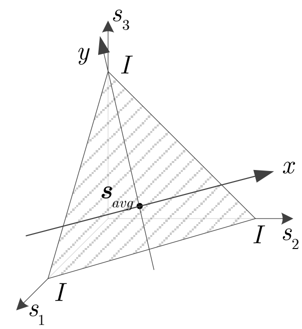

Fig. 2. Signal space of VLC-CSK modulation signals

$P_{r,g,b} \geq I$, the polygon is an equilateral triangle as shown in Fig. 2 and all the constellations should be distributed inside the shadowed triangle. Specifically, $\boldsymbol{s}_{avg}$, located in the center of the equilateral triangle, represents white perceived color. Hence, we construct a new 2-dimensional coordinate system $x\boldsymbol{s}_{avg}y$ as shown in Fig. 2 to conveniently present the constellation positions and labelings.

III. Optimization Problem Formulation

To jointly optimize the signal constellation and labeling, the PMI is taken as the optimization function since it is decided by not only the signal constellation set $\mathcal{A}$ but also the binary labeling $\mathcal{B}$. To derive the PMI of VLC-CSK system, an equivalent parallel channel model should be firstly constructed.

A. Equivalent Parallel Channel Model

As the encoder is separated from the modulator to reduce the complexity of the receiver [2], an equivalent parallel channel model corresponding to the dotted-squared part in Fig. 1 is derived following the approach as in [7]. To do this, the following notations are adopted. For any given signal $\boldsymbol{s}$, let $\mu^i(\boldsymbol{s})$ denote the i^{th} bit of the binary label associated to it, and $\mathcal{A}_b^i$ the subset of all the signals in $\mathcal{A}$ whose label takes on the value b in the i^{th} bit position (i.e., $\mathcal{A}_b^i = \{\boldsymbol{s} \in \mathcal{A} | \mu^i(\boldsymbol{s}) = b\}$). Further, let $\bar{b}$ denote the complement of b.

Given $\mu^i(\boldsymbol{s}) = b \in \{0,1\}$, the conditional probability density function (PDF) of $\boldsymbol{r}$ at the receiver is written as

$$\begin{aligned} p(\boldsymbol{r}|\mu^i(\boldsymbol{s}) = b) &= \sum_{\boldsymbol{s}\in\mathcal{A}} p(\boldsymbol{r}|\boldsymbol{s}, \mu^i(\boldsymbol{s}) = b) p(\boldsymbol{s}|\mu^i(\boldsymbol{s}) = b) \\ &= 2^{-(m-1)} \sum_{\boldsymbol{s}\in\mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s}) \end{aligned} . \tag{6}$$

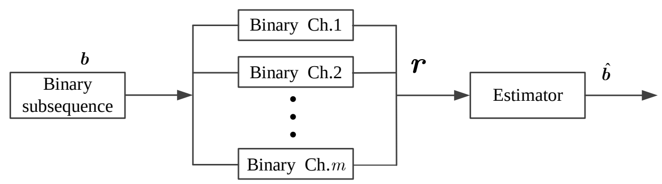

Fig. 3. Equivalent parallel channel model.

Suppose we have ideal channel state information (CSI) in the optimum receiver. The ML bit metrics of the demodulator per symbol should be $p(\boldsymbol{r}|\boldsymbol{s}, \mu^i(\boldsymbol{s}) = b)$. To simplify calculation, ignoring the constant multiplier and taking a logarithm of it, the ML bit metrics can be expressed as

$$\lambda^i(\boldsymbol{r}, b) = \log \sum_{\boldsymbol{s} \in \mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s}), \tag{7}$$

for $b \in \{0, 1\}$ and $i = 1, \cdots, m$. Therefore, the input binary sequence is estimated according to the rule

$$\hat{\boldsymbol{b}} = \arg\max_{\boldsymbol{b} \in \mathcal{B}} \sum_i \lambda^i(\boldsymbol{r}, b). \tag{8}$$

Given (8), the system model is reconstructed as an equivalent parallel binary channel model of m sub-channels as shown in Fig. 3. The parallel binary sub-channels are independent and memoryless. Each sub-channel corresponds to one bit in the label of the signals in $\mathcal{A}$.

B. Pragmatic Mutual Information

In accordance with the equivalent parallel channel model, let b denote the binary input for the i^{th} subchannel. Since b and $\boldsymbol{r}$ are conditionally jointly distributed as $p(b, \boldsymbol{r}|i) = 2^{-m} \sum_{\boldsymbol{s} \in \mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s})$, the conditional mutual information of b and $\boldsymbol{r}$ can be calculated by

$$I(b; \boldsymbol{r}|i) = 1 - E_{b,\boldsymbol{r}} \left[\log_2 \frac{\sum_{\boldsymbol{s} \in \mathcal{A}} p(\boldsymbol{r}|\boldsymbol{s})}{\sum_{\boldsymbol{s} \in \mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s})} \right]. \tag{9}$$

Then, the conditional average mutual information is obtained by averaging over all the subchannels, $I(b; \boldsymbol{r}) = \frac{1}{m} \sum_{i=1}^m I(b; \boldsymbol{r}|i)$. Because the m parallel channels are independent, the PMI can be expressed as m times $I(b; \boldsymbol{r})$, i.e.,

$$I(\mathcal{A}, \mu) = m - \sum_{i=1}^m E_{b,\boldsymbol{r}} \left[\log_2 \frac{\sum_{\boldsymbol{s} \in \mathcal{A}} p(\boldsymbol{r}|\boldsymbol{s})}{\sum_{\boldsymbol{s} \in \mathcal{A}_b^i} p(\boldsymbol{r}|\boldsymbol{s})} \right] \tag{10}$$

In general, expectations in (10) cannot be calculated in a closed form. It is normally obtained by numerical integral via the Monte Carlo method.

C. Joint Constellation-labeling Optimization Problem

The joint constellation-labeling optimization problem of maximizing the PMI subject to constraints (3), (4) and (5) can be formulated as

$$\max I(\mathcal{A}, \mu) \quad s.t. (3), (4), (5) \tag{11}$$

Since $I(\mathcal{A}, \mu)$ can not be expressed in a closed form, it is difficult to solve optimization problem (11) directly.

Actually, SA algorithm can be used to solve the joint constellation-labeling optimization problem [6]. Since SA is a well-developed heuristics algorithms, we briefly explain how the SA algorithm works. An initial consellation-labeling scheme and a cooling schedule have to be choosen first. If the logarithm cooling schedule $T_t = A/\log(t+1)$ is adopted, then the global optimal can be reached, provided A is large enough [8]. Whereas the linear cooling schedule $T_t = T_{t-1} - \alpha_t$ can not garantee the global optimality, it speeds up the SA algorithm. Then, at any step t, we randomly choose a constellation point and move it to a random new position inside the shadowed triangle in Fig. 2. If the PMI is increased by this movement, we accept the new position. On the other hand, if the PMI decreases, the new position is accepted with probability $e^{T_t \Delta \mathcal{I}}$, where $\Delta \mathcal{I}$ is the difference between the PMI of the constellation and labeling at time t and $t-1$.

Since the SA algorithm belongs to a global randomized searching algorithm in essence, it needs a large amount of computation which is really time consuming. A modified interior point method (M-IPM) with a tricky assumption is proposed to solve the joint optimization problem (11).

We realize that $I(\mathcal{A}, \mu)$ is differentiable if we apply the tricky assumption of fixing the labeling of the constellations when computing $I(\mathcal{A}, \mu)$. For instance, given a signal set $\mathcal{A} = \{\boldsymbol{s}^{(1)}, \boldsymbol{s}^{(2)}, \cdots, \boldsymbol{s}^{(M)}\}$, we can assume that the binary labeling for $\boldsymbol{s}^{(i)}$ is the binary expression of $i-1$. As a result, only the constellation position remains to be the optimization variable of the joint optimization problem and it is continuous and differentiable. Consequently, interior point method (IPM) can be used to solve the joint optimization problem under such assumption. However, since $I(\mathcal{A}, \mu)$ is not convex, the optimized result is locally optimal. Therefore, we can run the IPM several times with different start points and then choose the $(\mathcal{A}, \mu)$ that maximizes the PMI as the optimal constellation-labeling scheme. The concrete MIPM algorithm is stated in Algorithm 1.

Algorithm 1 Modified Interior Point Method

Input: maximum iteration time N; SNR γ
Initialize: iteration time $n = 1$; the PMI $\mathcal{I}^* = 0$;
While: $n \leq N$
  Choose an initial constellation points $\mathcal{A}_n^0$ randomly
  Calculate the optimum constellation points $\mathcal{A}_n^*$ by interior point method and record corresponding PMI $\mathcal{I}_n^*$
  If $\mathcal{I}_n^* > \mathcal{I}^*$
    $\mathcal{A}^* = \mathcal{A}_n^*$; $\mathcal{I}^* = \mathcal{I}_n^*$
  End If
  $n = n + 1$
End While
Return: the optimal constellation $\mathcal{A}^*$, $\mathcal{I}^*$

IV. Optimization Results

In the simulation, the SNR is defined as $\gamma = I^2/(3\sigma^2)$ since there are totally three PDs in VLC-CSK systems [9]. Without loss of generality, I is normalized to be 1. Therefore, different SNR implies different noise power σ^2. To observe the optimal constellation and labeling straightforward, the channel is assumed to be ideal, i.e. $\boldsymbol{H} = \boldsymbol{I}_3$ and the color constraint can be ignored temporarily.

The PMI lines calculated in accordance with the optimal constellations and labeling obtained by MIPM, SA algorithm the standard ones for 4-CSK, 8-CSK and 16-CSK are shown in Fig. 4 for comparison. Note that the signal constellation distribution in the standard was the same as the optimal constellation in [4] when the color constraint was not considered. It can be seen that the constellations and labelings after optimization improve the PMI, especially in the low SNR region. This is meaningful for VLC-CSK sytems as dimming must be considered to satisfying users' illumination demand thus the SNR may fall into a relatively low level [2]. Though the SA algorithm and the PMI achieve the almost same PMI performance in general, the SA algorithm performs a little bit worse occasionally. This is because the linear cooling schedule of SA algorithm is adopted in simulation which may lead to a local optimum. Besides, MIPM saves a lot of time compared with SA algorithm which would make sense when adaptive constellation and labeling is applied. For instance, when we run the simulation program in a personal computer configured to Inter Core I5 processor and 8G RAM to jointly optimize the constellations and labelings for 8-CSK when $\gamma = 6$dB, MIPM consumes only about 5 minutes whereas SA algorithm needs about 6 hours.

In addition, we present the optimization results of the pair $(\mathcal{A}, \mu)$ which maximize $I(\mathcal{A}, \mu)$ at given SNR for 4-CSK in the $x\boldsymbol{s}_{avg}y$ coordinate system in Fig. 5. The signal constellations and labeling in the standard are shown in the first subfigures for comparison. The optimal signal constellation and labeling for $\gamma = 4, 6, 8$ are represented by the red diomends and the binary labelings nearby in the rest sub-figures. It can be seen that if the constellation and labeling for 4-CSK in the standard is adopted, once the constellation labeled as 11 is demodulated to be the constellation labeled as 00, two-bits error occurs. Thus, it is better to enlarge the distance between the constellation labeled as 11 and the constellation labeled as 00 a little bit. Where to locate the constellation labeled as 11 explicitly is decided by the optimization problem (11). In particular, it is noticed that as γ increases, the optimal constellations and labelings approach the ones in the standard for 4-CSK modulation. As far as the authors concern, this is why the PMI improves inconspicuous in high SNR regimes.

V. Conclusion

We have made an investigation of the joint constellation-labeling optimization problem with a finite number of signals in VLC-CSK systems in this paper. The PMI was defined and proposed as the objective function of the joint constellation-labeling optimization problem. Due to the fact that PMI has no closed-form, we can resort to SA algorithm to solve the optimization problem. However, SA algorithm's computation complexity is very high and time-consuming. Then, we proposed the MIPM algorithm to solve the joint optimization problem, which can save a large amount of time and achieve an even better performance.

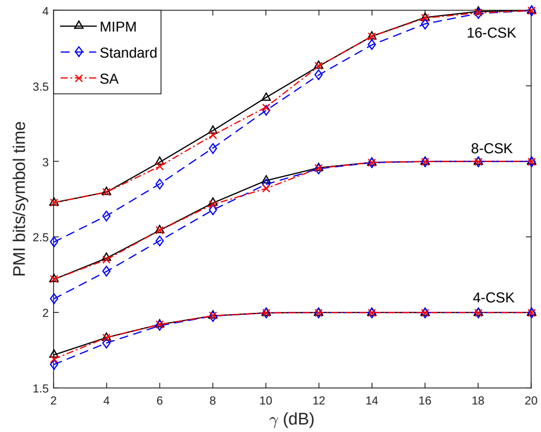

Fig. 4. PMI comparison between standard and optimal constellation and labeling

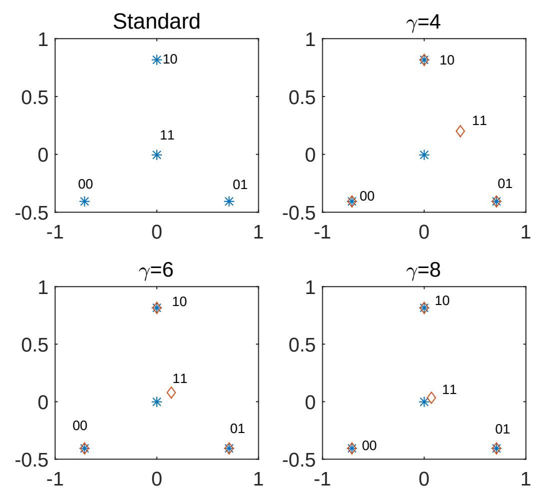

Fig. 5. Optimal constellation positions and labeling for 4-CSK modulation

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