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# Distributed Fault Estimation and Fault-Tolerant Control of Interconnected Systems

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and Xing-Gang Yan

**Abstract**—This paper studies distributed fault estimation and fault-tolerant control for continuous-time interconnected systems. Using associated information among subsystems to design the distributed fault estimation observer can improve the accuracy of fault estimation of interconnected systems. Based on static output feedback, the global outputs of interconnected systems are used to construct a distributed fault-tolerant control. The multi-constrained methods are proposed to enhance the transient performance and ability to suppress external disturbances simultaneously. The conditions of the presented design techniques are expressed in terms of linear matrix inequalities. Simulation results are illustrated to show the feasibility of the presented approaches.

**Index Terms**—Fault diagnosis, Robust fault estimation, Fault-tolerant control, Interconnected systems.

## I. INTRODUCTION

Over the past three decades, design and analysis of interconnected systems have received considerable attention and have been examined intensively. In essence, interconnected systems that include numerous coupled subsystems and a large number of state variables belong to large-scale systems. In general, interconnected systems have wide application scopes, such as industrial process systems, computer networks, mechanical systems, communication networks, and so on. Therefore, the study on interconnected systems is of both theoretical and practical significance [1]–[7].

On the other hand, due to higher security performance requirements for process control systems, the fault diagnosis and fault-tolerant control have become very important research topics in the last thirty years. The core technology is usually to quickly and accurately detect faults, determine the source of faults and the extent of faults, and further ensure the stability and reliability of the system by designing fault-tolerant controllers. The field of fault diagnosis and fault-tolerant control has achieved fruitful research results [8]–[10]. Different from fault detection and isolation, fault estimation can provide fault information, including fault time and fault magnitude, and is very important in fault accommodation [11]–[14]. Fault estimation-based fault accommodation belongs to active fault-tolerant control. This design method can make full use of

fault estimation information and design fault-tolerant controllers to compensate for the impact of faults on closed-loop systems, while passive fault-tolerant control does not require fault information, directly designing the robust controller to tolerate faults and making the closed-loop system insensitive to faults. Compared with passive fault-tolerant control, active fault-tolerant control is more flexible and is with better fault tolerance, so productive results have been achieved in the past three decades [15]–[17].

The studies of fault diagnosis and fault-tolerant control for interconnected systems has received considerable attention over the past decade. In [18], the problem of decentralised fault-tolerant finite-time stabilisation was investigated for a class of interconnected systems and each subsystem of the interconnected system is with a lower-triangular structure. In [3], a distributed technique for detecting and isolating sensor faults was considered for interconnected cyber-physical systems. The distributed sensor fault detection and isolation process is conducted in two levels. In [19], the fault estimation problem was studied for a class of interconnected nonlinear systems described by Takagi-Sugeno fuzzy models. The adaptive observer-based approach effectively estimated the actuator fault parameters at the presence of nonlinear interconnections. In [20], a distributed fault detection method was proposed to monitor the state of interconnected systems. The presented method allowed Plug & Play operations and the possibility to disconnect the faulty subsystem. In [21], a distributed fault detection and accommodation problem was addressed for uncertain nonlinear interconnected systems subject to faults. However, most of these previous considered only fault detection and isolation, and the relationship between fault estimation and fault-tolerant control was not considered from the overall system.

In this paper, a distributed fault estimation (DFE) and a distributed fault-tolerant control (DFTC) are presented for interconnected systems. The main contributions of this work are as follows. Firstly, based on associated terms among subsystems, a DFE design is proposed to estimate faults in all subsystems, which can be used for non-minimum phase systems. Secondly, from the perspective of the global system, a novel static output feedback (SOF)-based DFTC design is constructed using all outputs of subsystems to compensate for the fault impact on the interconnected system. Thirdly, the conditions of the proposed methods are expressed in terms of linear matrix inequalities, especially for SOF-based DFTC, such that design parameters can be calculated conveniently.

The rest of this paper is organised as follows. In Section 2,

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the system description of interconnected systems is presented. A DFE using multi-constrained approach is presented in Section 3. Section 4 proposes a SOF-based DFTC by the use of fault estimation. In Section 5, the simulation results are given to illustrate the feasibility and effectiveness of the proposed approaches. The conclusions of the manuscript are presented in Section 6.

## II. SYSTEM DESCRIPTION

We consider continuous-time interconnected systems composed of  $N$  subsystems as follows:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + E_i f_i(t) + D_i \omega_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N H_{ij} x_j(t) \\ y_i(t) = C_i x_i(t), \quad i = 1, 2, \dots, N, \end{cases} \quad (1)$$

where  $x_i(t) = [x_{i1}, x_{i2}, \dots, x_{in}]^T \in \mathbb{R}^n$ ,  $u_i(t) = [u_{i1}, u_{i2}, \dots, u_{im}]^T \in \mathbb{R}^m$ ,  $y_i(t) = [y_{i1}, y_{i2}, \dots, y_{ip}]^T \in \mathbb{R}^p$ , and  $\omega_i(t) = [\omega_{i1}, \omega_{i2}, \dots, \omega_{id}]^T \in \mathbb{R}^d$  are the state, the input, the output, and external disturbance vectors of the  $i$ th subsystem, respectively.  $f_i(t) = [f_{i1}, f_{i2}, \dots, f_{ir}]^T \in \mathbb{R}^r$  represents the system component or actuator fault, which are bounded. The derivative of the fault  $\dot{f}_i(t) \in L_2[0, +\infty)$  and  $\omega_i(t) \in L_2[0, +\infty)$ .  $p \geq m$  and  $m \geq r$ .  $A_i$ ,  $B_i$ ,  $C_i$ ,  $D_i$ ,  $E_i$ , and  $H_{ij}$  are constant real matrices with appropriate dimensions.  $H_{ij}$  is the interconnected matrix between subsystems  $i$  and  $j$ . We assume that pairs  $(A_i, B_i)$  and  $(A_i, C_i)$  are controllable and observable, respectively. Matrices  $C_i$  and  $E_i$  are of full rank.

**Remark 1.** In the interconnected system (1), matrices  $H_{ij}$  represent coupling association in subsystems. For the decentralized design, the coupling association is usually treated as a disturbance, which would affect the performance of controllers and observers. While our work is to put the coupling association between subsystems into the design of observers and controllers to improve system performances.

Before presenting main results of this work, the following lemmas are recalled.

**Lemma 1 [22].** The eigenvalues of a given matrix  $\mathcal{A} \in \mathbb{R}^{n \times n}$  belong to the circular region  $\mathcal{D}(\alpha, \tau)$  with center  $\alpha + j0$  and radius  $\tau$  if and only if a symmetric positive definite matrix  $\mathcal{P} \in \mathbb{R}^{n \times n}$  exists and satisfies:

$$\begin{bmatrix} -\mathcal{P} & \mathcal{P}(\mathcal{A} - \alpha I_n) \\ * & -\tau^2 \mathcal{P} \end{bmatrix} < 0. \quad (2)$$

**Lemma 2 [23].** For a continuous-time transfer function  $T(s) = \mathcal{C}(sI - \mathcal{A})^{-1}\mathcal{B} + \mathcal{D}$ , the following two statements are equivalent:

- (i)  $\|T(s) = \mathcal{C}(sI - \mathcal{A})^{-1}\mathcal{B} + \mathcal{D}\|_\infty < \gamma$  and matrix  $\mathcal{A}$  is stable;
- (ii) There exists a positive definite positive matrix  $\mathcal{P}$  satisfying

$$\begin{bmatrix} \mathcal{P}\mathcal{A} + \mathcal{A}^T\mathcal{P} & \mathcal{P}\mathcal{B} & \mathcal{C}^T \\ * & -\gamma I & \mathcal{D}^T \\ * & * & -\gamma I \end{bmatrix} < 0.$$

## III. DFE DESIGN

For the  $i$ th subsystem, the interconnected system (1) can be rewritten as the following form by putting state vector  $x_i(t)$  and fault vector  $f_i(t)$  together:

$$\begin{cases} \begin{bmatrix} \dot{x}_i(t) \\ \dot{f}_i(t) \end{bmatrix} = \begin{bmatrix} A_i & E_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_i(t) \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i(t) + \begin{bmatrix} D_i & 0 \\ 0 & I_r \end{bmatrix} \begin{bmatrix} \omega_i(t) \\ \dot{f}_i(t) \end{bmatrix} + \sum_{\substack{j=1 \\ j \neq i}}^N \begin{bmatrix} H_{ij} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_j(t) \\ f_j(t) \end{bmatrix} \\ y_i(t) = [C_i \quad 0] \begin{bmatrix} x_i(t) \\ f_i(t) \end{bmatrix}, \quad i = 1, 2, \dots, N, \end{cases} \quad (3)$$

where  $I_r$  is an  $r$ -dimension identity matrix.

Then we denote augmented vectors and matrices:

$$\begin{aligned} \bar{x}_i(t) &= \begin{bmatrix} x_i(t) \\ f_i(t) \end{bmatrix}, \quad \nu_i(t) = \begin{bmatrix} \omega_i(t) \\ \dot{f}_i(t) \end{bmatrix} \\ \bar{A}_i &= \begin{bmatrix} A_i & E_i \\ 0 & 0 \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \\ \bar{D}_i &= \begin{bmatrix} D_i & 0 \\ 0 & I_r \end{bmatrix}, \quad \bar{H}_{ij} = \begin{bmatrix} H_{ij} & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{C}_i &= [C_i \quad 0], \end{aligned}$$

and it derives

$$\begin{cases} \dot{\bar{x}}_i(t) = \bar{A}_i \bar{x}_i(t) + \bar{B}_i u_i(t) + \bar{D}_i \nu_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N \bar{H}_{ij} \bar{x}_j(t) \\ y_i(t) = \bar{C}_i \bar{x}_i(t), \quad i = 1, 2, \dots, N, \end{cases} \quad (4)$$

which is equivalent to system (1).

For the augmented interconnected system (4), the following DFE for the  $i$ th subsystem is constructed:

$$\begin{cases} \dot{\hat{x}}_i(t) = \bar{A}_i \hat{x}_i(t) + \bar{B}_i u_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N \bar{H}_{ij} \hat{x}_j(t) - \bar{L}_i (\hat{y}_i(t) - y_i(t)) \\ \hat{y}_i(t) = \bar{C}_i \hat{x}_i(t), \quad i = 1, 2, \dots, N, \end{cases} \quad (5)$$

where  $\hat{x}_i(t) \in \mathbb{R}^n$  and  $\hat{y}_i(t) \in \mathbb{R}^p$  are the state and output of the  $i$ th observer, respectively;  $\hat{f}_i(t) \in \mathbb{R}^r$  is the online fault estimate of the  $i$ th observer, and  $\bar{L}_i \in \mathbb{R}^{n \times p}$  are observer gain matrices to be designed.

**Remark 2.** Different from decentralized observer design, the presented DFE (5) adds the associated relationship among all subsystems, instead of treating the associated items as ‘‘disturbances’’. This design method can realize more accurate fault estimation.

For the  $i$ th subsystem, the following error vectors are defined:

$$\bar{e}_i(t) = \hat{x}_i(t) - \bar{x}_i(t), \quad e_{fi}(t) = \hat{f}_i(t) - f_i(t),$$

then from (4) and (5), we can get the local error dynamics

$$\begin{cases} \dot{\hat{e}}_i(t) = \hat{\dot{x}}_i(t) - \dot{\hat{x}}_i(t) \\ = (\bar{A}_i - \bar{L}_i \bar{C}_i) \bar{e}_i(t) - \bar{D}_i \nu_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N \bar{H}_{ij} e_j(t) \\ \dot{e}_{fi}(t) = \bar{I}_r^T \bar{e}_i(t), \quad i = 1, 2, \dots, N, \end{cases} \quad (6)$$

where  $\bar{I}_r = \begin{bmatrix} 0 \\ I_r \end{bmatrix}$ .

By denoting the following global augmented vectors and matrices

$$\begin{aligned} \bar{e}(t) &= [ \bar{e}_1^T(t) \quad \bar{e}_2^T(t) \quad \dots \quad \bar{e}_N^T(t) ]^T, \\ \bar{e}_f(t) &= [ \bar{e}_{f1}^T(t) \quad \bar{e}_{f2}^T(t) \quad \dots \quad \bar{e}_{fN}^T(t) ]^T, \\ \nu(t) &= [ \nu_1^T(t) \quad \nu_2^T(t) \quad \dots \quad \nu_N^T(t) ]^T, \\ \bar{A} &= \text{diag}(\bar{A}_1, \bar{A}_2, \dots, \bar{A}_N), \\ \bar{L} &= \text{diag}(\bar{L}_1, \bar{L}_2, \dots, \bar{L}_N), \\ \bar{C} &= \text{diag}(\bar{C}_1, \bar{C}_2, \dots, \bar{C}_N), \\ \bar{D} &= \text{diag}(\bar{D}_1, \bar{D}_2, \dots, \bar{D}_N), \\ \bar{H} &= \begin{bmatrix} 0 & \bar{H}_{12} & \dots & \bar{H}_{1N} \\ \bar{H}_{21} & 0 & \dots & \bar{H}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{H}_{N1} & \bar{H}_{N2} & \dots & 0 \end{bmatrix}, \end{aligned}$$

where  $\text{diag}()$  denotes the block diagonal matrix, the global error dynamics of interconnected systems is obtained

$$\begin{cases} \dot{\bar{e}}(t) = (\bar{A} + \bar{H} - \bar{L}\bar{C})\bar{e}(t) - \bar{D}\nu(t) \\ \bar{e}_f(t) = (I_N \otimes \bar{I}_r^T)\bar{e}(t) \end{cases} \quad (7)$$

For the error dynamics (7), we propose multi-constrained design including regional pole placement and  $H_\infty$  performance to calculate gain matrices  $\bar{L}_i$  of DFE.

**Theorem 1.** Given a circular region  $\mathcal{D}(\alpha, \tau)$  and a  $\mathcal{H}_\infty$  performance level  $\gamma$ . If there exists symmetric positive definite matrix  $\bar{P} \in \mathbb{R}^{(n+r)N \times (n+r)N}$  and matrix  $\bar{Y} \in \mathbb{R}^{(n+r)N \times pN}$ , where

$$\begin{aligned} \bar{P} &= \text{diag}(\bar{P}_1, \bar{P}_2, \dots, \bar{P}_N) \\ \bar{Y} &= \text{diag}(\bar{Y}_1, \bar{Y}_2, \dots, \bar{Y}_N) \end{aligned}$$

such that conditions (8) and (9) are satisfied,

$$\begin{bmatrix} -\bar{P} & \Pi_{12} \\ * & -\tau^2 \bar{P} \end{bmatrix} < 0, \quad (8)$$

where

$$\Pi_{12} = \begin{bmatrix} \bar{P}_1 \bar{A}_1 - \bar{Y}_1 \bar{C}_1 - \alpha \bar{P}_1 & \bar{P}_1 \bar{H}_{12} \\ \bar{P}_2 \bar{H}_{21} & \bar{P}_2 \bar{A}_2 - \bar{Y}_2 \bar{C}_2 - \alpha \bar{P}_2 \\ \vdots & \vdots \\ \bar{P}_N \bar{H}_{N1} & \bar{P}_N \bar{H}_{N2} \\ \dots & \bar{P}_1 \bar{H}_{1N} \\ \dots & \bar{P}_2 \bar{H}_{2N} \\ \vdots & \vdots \\ \dots & \bar{P}_N \bar{A}_N - \bar{Y}_N \bar{C}_N - \alpha \bar{P}_N \end{bmatrix},$$

and

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix} < 0, \quad (9)$$

where

$$\Xi_{11} = \begin{bmatrix} \eta_{11} & \bar{P}_1 \bar{H}_{12} + \bar{H}_{21}^T \bar{P}_2 \\ \bar{P}_2 \bar{H}_{21} + \bar{H}_{12}^T \bar{P}_1 & \eta_{22} \\ \vdots & \vdots \\ \bar{P}_N \bar{H}_{N1} + \bar{H}_{1N}^T \bar{P}_1 & \bar{P}_N \bar{H}_{N2} + \bar{H}_{2N}^T \bar{P}_2 \\ \dots & \bar{P}_1 \bar{H}_{1N} + \bar{H}_{N1}^T \bar{P}_N \\ \dots & \bar{P}_2 \bar{H}_{2N} + \bar{H}_{N2}^T \bar{P}_N \\ \vdots & \vdots \\ \dots & \eta_{NN} \end{bmatrix},$$

with  $\eta_{ii} = \bar{P}_i \bar{A}_i + \bar{A}_i^T \bar{P}_i - \bar{Y}_i \bar{C}_i - \bar{C}_i^T \bar{Y}_i^T$ ,

$$\Xi_{12} = \text{diag}(-\bar{P}_1 \bar{D}_1, -\bar{P}_2 \bar{D}_2, \dots, -\bar{P}_N \bar{D}_N),$$

$$\Xi_{13} = I_N \otimes (\bar{I}_r),$$

$$\Xi_{22} = I_N \otimes (-\gamma I_{d+r}),$$

$$\Xi_{33} = I_N \otimes (-\gamma I_r),$$

then the eigenvalues of  $(\bar{A} + \bar{H} - \bar{L}\bar{C})$  belong to  $\mathcal{D}(\alpha, \tau)$  and the error dynamics (7) satisfies the  $H_\infty$  performance  $\|-(I_N \otimes \bar{I}_r^T)(sI - (\bar{A} + \bar{H} - \bar{L}\bar{C}))^{-1} \bar{D}\|_\infty < \gamma$ . The gain matrices  $\bar{L}_i$  are calculated by  $\bar{L}_i = \bar{P}_i^{-1} \bar{Y}_i$ .

**Proof.** Condition (8): First, we denote a global matrix

$$\bar{Y} = \text{diag}(\bar{Y}_1, \bar{Y}_2, \dots, \bar{Y}_N),$$

and one gets  $\bar{Y} = \bar{P}\bar{L}$ , so condition (8) can be expressed as

$$\begin{bmatrix} -\bar{P} & \bar{P}(\bar{A} + \bar{H} - \bar{L}\bar{C}) - \alpha \bar{P} \\ * & -\tau^2 \bar{P} \end{bmatrix} < 0. \quad (10)$$

Then according to Lemma 1, if condition (8) is satisfied, then the eigenvalues of  $(\bar{A} + \bar{H} - \bar{L}\bar{C})$  belong to  $\mathcal{D}(\alpha, \tau)$ .

Condition (9): From the global view, condition (9) can be re-written as

$$\begin{bmatrix} \bar{P}(\bar{A} + \bar{H} - \bar{L}\bar{C}) + (\bar{A} + \bar{H} - \bar{L}\bar{C})^T \bar{P} \\ * \\ * \\ -\bar{P}\bar{D} & I_N \otimes \bar{I}_r \\ -\gamma I_{(d+r)N} & 0 \\ * & -\gamma I_{rN} \end{bmatrix} < 0 \quad (11)$$

Based on Lemma 2, if condition (9) is satisfied, then the error dynamics (7) satisfies the  $H_\infty$  performance  $\|-(I_N \otimes \bar{I}_r^T)(sI - (\bar{A} + \bar{H} - \bar{L}\bar{C}))^{-1} \bar{D}\|_\infty < \gamma$ .  $\square$

**Remark 3.** In Theorem 1, the regional pole placement can enhance the transient performance of fault estimation, while the  $H_\infty$  performance is used to restrain the effect of term  $\nu(t)$  with respect to fault estimation error  $e_f(t)$ . Meanwhile, conditions of Theorem 1 are expressed in terms of linear matrix inequalities, which are convenient to calculate gain matrices.

**Remark 4.** Based on DFE observer (5), the state of  $i$ th subsystem  $\hat{x}_i(t)$  can be estimated and the online fault estimation

can be obtained by

$$\hat{f}_i(t) = \bar{I}_r^T \hat{x}_i(t), \quad i = 1, 2, \dots, N. \quad (12)$$

The obtained fault information is used to design fault-tolerant control, which will be shown in section IV.

#### IV. SOF-DFTC DESIGN

Before expressing DFTC design, the following assumption and lemma are given.

**Assumption 1.**  $\text{rank}(B_i, E_i) = \text{rank}(B_i)$ ,  $i = 1, 2, \dots, N$ .

**Lemma 3 [15].** Under Assumption 1, there exist matrices  $B_i^* \in \mathbb{R}^{m \times n}$  such that

$$(I_n - B_i B_i^*) E_i = 0, \quad i = 1, 2, \dots, N. \quad (13)$$

Based on fault estimation obtained from DFE design, the SOF-based DFTC is constructed as follows:

$$u_i(t) = K_{ii} y_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N K_{ij} y_j(t) - B_i^* E_i \hat{f}_i(t), \quad i = 1, 2, \dots, N, \quad (14)$$

where  $K_{ii} \in \mathbb{R}^{m \times p}$  are local feedback matrices,  $K_{ij} \in \mathbb{R}^{m \times p}$  are interconnected feedback matrices, and  $B_i^*$  are generalized inverse matrices of  $B_i$ .

**Remark 5.** The proposed DFTC design is based on SOF. Compared with dynamic output feedback, the analysis and design of SOF are more challenging. Moreover, we consider distributed controller design and calculate all control gains  $K_{ij}$  together. The outputs of all subsystems are used to further the performance of fault-tolerant control.

Substituting DFTC (14) into the interconnected system (1), one derives

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i K_{ii} y_i(t) + B_i \sum_{\substack{j=1 \\ j \neq i}}^N K_{ij} y_j(t) - \\ & B_i B_i^* E_i \hat{f}_i(t) + E_i \hat{f}_i(t) - E_i \hat{f}_i(t) + E_i f_i(t) + \\ & D_i \omega_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N H_{ij} x_j(t) \\ &= (A_i + B_i K_{ii} C_i) x_i(t) + (I - B_i B_i^*) E_i \hat{f}_i(t) + \\ & \left( B_i \sum_{\substack{j=1 \\ j \neq i}}^N K_{ij} C_j + \sum_{\substack{j=1 \\ j \neq i}}^N H_{ij} \right) x_j(t) - E_i e_{fi}(t) + D_i \omega_i(t) \end{aligned}$$

And according to Lemma 3, we can get

$$(I - B_i B_i^*) E_i \hat{f}_i(t) = 0,$$

so it obtains

$$\begin{aligned} \dot{x}_i(t) &= (A_i + B_i K_{ii} C_i) x_i(t) - E_i e_{fi}(t) + D_i \omega_i(t) + \\ & \left( B_i \sum_{\substack{j=1 \\ j \neq i}}^N K_{ij} C_j + \sum_{\substack{j=1 \\ j \neq i}}^N H_{ij} \right) x_j(t) \\ &= (A_i + B_i K_{ii} C_i) x_i(t) + [ D_i \quad -E_i ] \begin{bmatrix} \omega_i(t) \\ e_{fi}(t) \end{bmatrix} + \\ & \left( B_i \sum_{\substack{j=1 \\ j \neq i}}^N K_{ij} C_j + \sum_{\substack{j=1 \\ j \neq i}}^N H_{ij} \right) x_j(t) \end{aligned} \quad (16)$$

Letting local augmented vectors and matrices

$$\mu_i = \begin{bmatrix} \omega_i(t) \\ e_{fi}(t) \end{bmatrix}, \quad \tilde{D}_i = [ D_i \quad -E_i ],$$

global vectors

$$\begin{aligned} x(t) &= [ x_1^T(t) \quad x_2^T(t) \quad \dots \quad x_N^T(t) ]^T, \\ \mu(t) &= [ \mu_1^T(t) \quad \mu_2^T(t) \quad \dots \quad \mu_N^T(t) ]^T, \\ y(t) &= [ y_1^T(t) \quad y_2^T(t) \quad \dots \quad y_N^T(t) ]^T, \end{aligned}$$

and global matrices

$$\begin{aligned} A &= \text{diag}(A_1, A_2, \dots, A_N), \\ B &= \text{diag}(B_1, B_2, \dots, B_N), \\ C &= \text{diag}(C_1, C_2, \dots, C_N), \\ \tilde{D} &= \text{diag}(\tilde{D}_1, \tilde{D}_2, \dots, \tilde{D}_N), \\ H &= \begin{bmatrix} 0 & H_{12} & \dots & H_{1N} \\ H_{21} & 0 & \dots & H_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1} & H_{N2} & \dots & H_N \end{bmatrix}, \\ K &= \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ K_{21} & K_{22} & \dots & K_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ K_{N1} & K_{N2} & \dots & K_{NN} \end{bmatrix}, \end{aligned}$$

Finally, the global dynamics under DFTC is

$$\begin{cases} \dot{x}(t) = (A + H + BKC)x(t) + \tilde{D}\mu(t) \\ y(t) = Cx(t) \end{cases} \quad (17)$$

Next, distributed output feedback matrices  $K_{ij}(i, j = 1, 2, \dots, N)$  in the global dynamics (17) are calculated by the following Theorem 2.

**Theorem 2.** Under the transformation matrices  $T_i = \begin{bmatrix} C_i^\perp \\ C_i \end{bmatrix}^{-1}$ , where  $C_i^\perp \in \mathbb{R}^{(n-p) \times n}$  are chosen in advance. Given a circular region  $\mathcal{D}(\sigma, \varsigma)$ , a positive scalar  $\rho$  and a  $\mathcal{H}_\infty$  performance level  $\gamma$ . If there exist symmetric positive definite matrices  $Q, S \in \mathbb{R}^{(nN) \times (nN)}$ , matrices  $W \in \mathbb{R}^{(nN) \times (nN)}$ ,

$G \in \mathbb{R}^{(mN) \times (nN)}$ , where

$$Q := \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1N} \\ * & Q_{22} & \cdots & Q_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & Q_{NN} \end{bmatrix},$$

$$S := \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ * & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & S_{NN} \end{bmatrix},$$

$$W := \text{diag}(W_1, W_2, \dots, W_N),$$

$$G := \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix},$$

with

$$W_i = \begin{bmatrix} W_i^{11} & W_i^{12} \\ 0_{p \times (n-p)} & W_i^{22} \end{bmatrix}, \quad G_{ij} = [0_{m \times (n-p)} \quad G_{ij}^2],$$

such that conditions (18) and (19) are satisfied:

$$\begin{bmatrix} -Q & \Phi_{12} \\ * & \Phi_{22} \end{bmatrix} < 0, \quad (18)$$

where

$$\Phi_{12} = \begin{bmatrix} \zeta_{11} & B_1 G_{12} + H_{12} T_2 W_2 \\ B_2 G_{21} + H_{21} T_1 W_1 & \zeta_{22} \\ \vdots & \vdots \\ B_N G_{N1} + H_{N1} T_1 W_1 & B_N G_{N2} + H_{N2} T_2 W_2 \\ \cdots & B_1 G_{1N} + H_{1N} T_N W_N \\ \cdots & B_2 G_{2N} + H_{2N} T_N W_N \\ \vdots & \vdots \\ \cdots & \zeta_{NN} \end{bmatrix},$$

with  $\zeta_{ii} = A_i T_i W_i + B_i G_{ii} - \sigma T_i W_i$ ,

$$\Phi_{22} = \begin{bmatrix} \phi_{11} & \zeta^2 Q_{12} & \cdots & \zeta^2 Q_{1N} \\ * & \phi_{22} & \cdots & \zeta^2 Q_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \phi_{NN} \end{bmatrix},$$

with  $\phi_{ii} = -\zeta^2 (T_i W_i + W_i^T T_i^T - Q_{ii})$ ,

and

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} \\ * & \Psi_{22} & 0 & \Psi_{24} \\ * & * & \Psi_{33} & 0 \\ * & * & * & \Psi_{44} \end{bmatrix} < 0, \quad (19)$$

where

$$\Psi_{11} = \begin{bmatrix} \varphi_{11} & B_1 G_{12} + \bar{H}_{12} T_2 W_2 + (B_2 G_{21} + \bar{H}_{21} T_1 W_1)^T \\ * & \varphi_{22} \\ \vdots & \vdots \\ * & * \\ \cdots & B_1 G_{1N} + \bar{H}_{1N} T_N W_N + (B_N G_{N1} + \bar{H}_{N1} T_1 W_1)^T \\ \cdots & B_2 G_{2N} + \bar{H}_{2N} T_N W_N + (B_N G_{N2} + \bar{H}_{N2} T_2 W_2)^T \\ \vdots & \vdots \\ \cdots & \varphi_{NN} \end{bmatrix},$$

with  $\varphi_{ii} = A_i T_i W_i + W_i^T T_i^T A_i^T + B_i G_i + G_i^T B_i^T$ ,

$$\Psi_{12} = \begin{bmatrix} S_{11} - (T_1 W_1)^T & S_{12} \\ S_{21} & S_{22} - (T_2 W_2)^T \\ \vdots & \vdots \\ S_{N1} & S_{N2} \\ \cdots & S_{1N} \\ \cdots & S_{2N} \\ \vdots & \vdots \\ \cdots & S_{NN} - (T_N W_N)^T \end{bmatrix} +$$

$$\rho \begin{bmatrix} A_1 T_1 W_1 + B_1 G_1 & B_1 G_{12} + H_{12} T_2 W_2 \\ B_2 G_{21} + H_{21} T_1 W_1 & A_2 T_2 W_2 + B_2 G_2 \\ \vdots & \vdots \\ S_{N1} & S_{N2} \\ \cdots & B_1 G_{1N} + H_{1N} T_N W_N \\ \cdots & B_2 G_{2N} + H_{2N} T_N W_N \\ \vdots & \vdots \\ \cdots & A_N T_N W_N + B_N G_N \end{bmatrix},$$

$$\Psi_{22} = \text{diag}(-\rho(T_1 W_1 + (T_1 W_1)^T), -\rho(T_2 W_2 + (T_2 W_2)^T), \dots, -\rho(T_N W_N + (T_N W_N)^T)),$$

$$\Psi_{13} = \text{diag}(\tilde{D}_1, \tilde{D}_2, \dots, \tilde{D}_N),$$

$$\Psi_{33} = I_N \otimes (-\gamma I_{d+r}),$$

$$\Psi_{14} = \text{diag}((C_1 T_1 W_1)^T, (C_2 T_2 W_2)^T, \dots, (C_2 T_2 W_2)^T),$$

$$\Psi_{24} = \text{diag}(\rho(C_1 T_1 W_1)^T, \rho(C_2 T_2 W_2)^T, \dots, \rho(C_2 T_2 W_2)^T),$$

$$\Psi_{44} = I_N \otimes (-\gamma I_p),$$

then the eigenvalues of  $(A+H+BKC)$  belong to  $\mathcal{D}(\sigma, \zeta)$ , the global dynamics (17) satisfies the  $H_\infty$  performance  $\|C(sI - (A+H+BK))^{-1} \tilde{D}\|_\infty < \gamma$ . The SOF-based DFTC gain matrices are given by  $K_{ij} = (G_{ij}^2)(W_i^{22})^{-1}$ .

**Proof.** Condition (18): Denote

$$T := \text{diag}(T_1, T_2, \dots, T_N),$$

then condition (18) can be expressed as the following global form

$$\begin{bmatrix} -Q & (A+H)TW + BG - \sigma TW \\ * & -\zeta^2(TW + W^T T^T - Q) \end{bmatrix} < 0 \quad (20)$$

Then according to the SOF-based DFTC gain matrices  $K_{ij} =$

$(G_{ij}^2)(W_i^{22})^{-1}$ , we get  $(G_{ij}^2) = K_{ij}(W_i^{22})$ .

Further, it derives

$$\begin{aligned} G_{ij} &= \begin{bmatrix} 0_{m \times (n-p)} & G_{ij}^2 \end{bmatrix} \\ &= \begin{bmatrix} 0_{m \times (n-p)} & K_{ij}(W_i^{22}) \end{bmatrix} \\ &= \begin{bmatrix} 0_{m \times (n-p)} & K_{ij} \end{bmatrix} \begin{bmatrix} W_i^{11} & W_i^{12} \\ 0_{p \times (n-p)} & W_i^{22} \end{bmatrix} \end{aligned} \quad (21)$$

where  $W_i^{11}$  and  $W_i^{12}$  are slack matrices that can add design freedom.

$$\begin{aligned} \text{Due to } W_i &= \begin{bmatrix} W_i^{11} & W_i^{12} \\ 0_{p \times (n-p)} & W_i^{22} \end{bmatrix} \text{ and } p \geq m, \text{ one gets} \\ G_{ij} &= \begin{bmatrix} 0_{m \times (n-p)} & K_{ij} \end{bmatrix} W_i \\ &= K_{ij} \begin{bmatrix} 0_{p \times (n-p)} & I_p \end{bmatrix} W_i \end{aligned} \quad (22)$$

Since matrices  $C_i$  are full-row rank, there always are matrices  $C_i^\perp$  such that  $\begin{bmatrix} C_i^\perp \\ C_i \end{bmatrix}$  are nonsingular. For transformation matrices  $T_i = \begin{bmatrix} C_i^\perp \\ C_i \end{bmatrix}^{-1}$ , one can obtain

$$C_i T_i = \begin{bmatrix} 0_{p \times (n-p)} & I_p \end{bmatrix} \quad (23)$$

then it follows

$$G_{ij} = K_{ij} C_i T_i W_i \quad (24)$$

Furthermore, from the global view, we have

$$G = KCTW \quad (25)$$

then (20) is equivalent to

$$\begin{bmatrix} -Q & (A + H + KC)TW - \sigma TW \\ * & -\zeta^2(TW + W^T T^T - Q) \end{bmatrix} < 0 \quad (26)$$

If (26) holds, one gets  $TW + W^T T^T > Q$ , which means that  $TW$  is nonsingular. Meanwhile, since  $Q$  is symmetric positive definite, the inequality  $(Q - TW)^T Q^{-1} (Q - TW) > 0$  holds, which can be re-written as  $-(TW)^T Q^{-1} (TW) < -TW - W^T T^T + Q$ . Further, it follows from (26) that

$$\begin{bmatrix} -Q & (A + H + KC)TW - \sigma TW \\ * & -\zeta^2(TW)^T Q^{-1} (TW) \end{bmatrix} < 0 \quad (27)$$

then pre- and post-multiplying by  $\text{diag}(I, Q(TW)^{-T})$  and its transpose, one obtains

$$\begin{bmatrix} -Q & (A + H + KC)Q - \sigma Q \\ * & -\zeta^2 Q \end{bmatrix} < 0 \quad (28)$$

which is equivalent to

$$\begin{bmatrix} -Q^{-1} & Q^{-1}(A + H + KC) - \sigma Q^{-1} \\ * & -\zeta^2 Q^{-1} \end{bmatrix} < 0 \quad (29)$$

So based on Lemma 1, it is concluded that if condition (18) holds, the eigenvalues of  $(A + H + KC)$  locate in  $\mathcal{D}(\sigma, \zeta)$ .

Condition (19): First, condition (19) can be expressed as the

following global form

$$\begin{bmatrix} \psi & S - (TW)^T + \rho((A + H)TW + BG) \\ * & -\rho(TW + (TW)^T) \\ * & * \\ * & * \\ & \tilde{D} & (CTW)^T \\ & 0 & \rho(CTW)^T \\ & -\gamma I_{(d+r)N} & 0 \\ & * & -\gamma I_{pN} \end{bmatrix} < 0 \quad (30)$$

where  $\psi = (A + H)TW + ((A + H)TW)^T + BG + (BG)^T$ .

According to  $BG = BKCTW$ , one gets

$$\begin{bmatrix} \xi & S - (TW)^T + \rho(A + H + BKC)TW \\ * & -\rho(TW + (TW)^T) \\ * & * \\ * & * \\ & \tilde{D} & (CTW)^T \\ & 0 & \rho(CTW)^T \\ & -\gamma I_{(d+r)N} & 0 \\ & * & -\gamma I_{pN} \end{bmatrix} < 0 \quad (31)$$

where  $\xi = (A + H + BKC)TW + ((A + H + BKC)TW)^T$ .

For (31), pre- and post-multiplying by

$$\begin{bmatrix} I_{nN} & (A + H + BKC) & 0 & 0 \\ 0 & 0 & I_{(d+r)N} & 0 \\ 0 & C & 0 & I_{pN} \end{bmatrix} < 0 \quad (32)$$

and its transpose, we have

$$\begin{bmatrix} (A + H + BKC)S + S(A + H + BKC)^T & \tilde{D} & SC^T \\ * & -\gamma I_{(d+r)N} & 0 \\ * & * & -\gamma I_{pN} \end{bmatrix} < 0 \quad (33)$$

which is equivalent to

$$\begin{bmatrix} S^{-1}(A + H + BKC) + (A + H + BKC)^T S^{-1} & S^{-1} \tilde{D} & C^T \\ * & -\gamma I_{(d+r)N} & 0 \\ * & * & -\gamma I_{pN} \end{bmatrix} < 0 \quad (34)$$

So if condition (19) holds, the global dynamics (17) satisfies the  $H_\infty$  performance  $\|C(SI - (A + H + BKC))^{-1} \tilde{D}\|_\infty < \gamma$  according to Lemma 2.  $\square$

**Remark 6.** From the proof of Theorem 2, we can see that through appropriate matrix transformation, conditions of Theorem 2 are given in terms of linear matrices inequalities.

## V. SIMULATION RESULTS

### A. System description

The following interconnected system with two subsystems to verify the feasibility of the proposed techniques:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + E_i f_i(t) + D_i \omega_i(t) + \\ \quad \sum_{\substack{j=1 \\ j \neq i}}^N H_{ij} x_j(t) \\ y_i(t) = C_i x_i(t), \quad i = 1, 2, \end{cases}$$

where

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -2 & 0 & 1 \\ -1 & -2 & 0 \\ 0 & 1 & 5 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \\
 D_1 &= \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix}, H_{12} = \begin{bmatrix} 0 & -0.1 & 0.1 \\ 0.1 & 0 & 0.1 \\ 0 & 0.1 & 0 \end{bmatrix}, \\
 C_1 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & -2 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
 D_2 &= \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix}, H_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.1 & 0.2 \\ -0.1 & 0 & 0.1 \end{bmatrix}, \\
 C_2 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
 \end{aligned}$$

For such interconnected system, it is assumed that the actuator fault of each subsystem is considered, i.e.,  $E_i = B_i$ . The system matrices  $A_1$  and  $A_2$  are unstable. It is verified that pairs  $(A_i, B_i)$  are controllable and  $(A_i, C_i)$  are observable, where  $i = 1, 2$ . Meanwhile, it is seen that  $(C_1 E_1)$  is not full column rank, but the DFE methods proposed in this paper is still feasible.

### B. DFE design

From (5) and (7), the local augmented error dynamics and global error dynamics can be constructed, and gain matrices  $\bar{L} = \text{diag}(\bar{L}_1, \bar{L}_2, \dots, \bar{L}_N)$  of DFE are calculated firstly.

Under regional pole placement constraint  $\mathcal{D}(-5.5, 5)$ , by solving the conditions of Theorem 1, we obtain  $H_\infty$  performance value  $\gamma = 0.3109$  and

$$\begin{aligned}
 \bar{P}_1 &= 10^3 \times \\
 &\begin{bmatrix} 0.1812 & 0.9060 & 0.0046 & -0.0197 & -0.0006 \\ 0.9060 & 7.6712 & 0.0406 & -0.0535 & -0.0044 \\ 0.0046 & 0.0406 & 0.2914 & -0.0003 & -0.0309 \\ -0.0197 & -0.0535 & -0.0003 & 0.0031 & 0.0001 \\ -0.0006 & -0.0044 & -0.0309 & 0.0001 & 0.0040 \end{bmatrix}, \\
 \bar{P}_2 &= \begin{bmatrix} 123.3117 & -0.8522 & -112.1247 \\ -0.8522 & 265.9286 & -9.8791 \\ -112.1247 & -9.8791 & 297.5672 \\ 3.2096 & -25.7559 & -2.0690 \\ -0.7507 & 1.3146 & -19.3378 \\ & 3.2096 & -0.7507 \\ & -25.7559 & 1.3146 \\ & -2.0690 & -19.3378 \\ & 3.0037 & -0.2208 \\ & -0.2208 & 2.9097 \end{bmatrix}, \\
 \bar{Y}_1 &= 10^4 \begin{bmatrix} 0.3660 & 0.0235 \\ 4.6960 & 0.1449 \\ 0.0575 & 0.3829 \\ -0.0140 & -0.0023 \\ -0.0055 & -0.0355 \end{bmatrix},
 \end{aligned}$$

$$\bar{Y}_2 = 10^3 \begin{bmatrix} 0.1040 & 0.2869 \\ 3.1673 & -0.3235 \\ -0.4263 & 0.9369 \\ -0.2669 & 0.0330 \\ 0.0358 & -0.0830 \end{bmatrix}.$$

Then the gain matrices are calculated as follows:

$$\begin{aligned}
 \bar{L}_1 &= \bar{P}_1^{-1} \bar{Y}_1 = \begin{bmatrix} -110.7994 & 0.9485 \\ 15.8958 & 0.0054 \\ 0.9934 & 20.2005 \\ -474.4152 & -0.4054 \\ 0.2775 & 66.8687 \end{bmatrix}, \\
 \bar{L}_2 &= \bar{P}_2^{-1} \bar{Y}_2 = \begin{bmatrix} -1.2461 & 12.6550 \\ 20.3476 & -0.9996 \\ -0.0071 & 11.0430 \\ 87.6388 & 0.0661 \\ 9.3916 & 48.6036 \end{bmatrix}.
 \end{aligned}$$

### C. DFTC design

Secondly, based on  $C_1$  and  $C_2$ , we can construct  $T_1 = T_2 = I_3$ . Meanwhile, from  $B_1$  and  $B_2$ , the following matrices  $B_1^*$  and  $B_2^*$  are derived

$$\begin{aligned}
 B_1^* &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\
 B_2^* &= \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix}.
 \end{aligned}$$

Setting regional pole placement constraint  $\mathcal{D}(-10, 10)$  and  $\rho = 0.04$ , by solving the conditions of Theorem 2, we obtain  $H_\infty$  performance value  $\gamma = 0.0500$  and symmetric positive definite matrices  $Q, S$ :

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix},$$

where

$$\begin{aligned}
 Q_{11} &= \begin{bmatrix} 7.1349 & 0.0229 & -0.0079 \\ 0.0229 & 0.2115 & 0.0002 \\ -0.0079 & 0.0002 & 0.9982 \end{bmatrix}, \\
 Q_{12} &= \begin{bmatrix} 1.2986 & -0.0018 & -0.0798 \\ 0.1327 & -0.0018 & -0.0223 \\ 0.0001 & 0.0033 & 0.0032 \end{bmatrix}, \\
 Q_{22} &= \begin{bmatrix} 26.5363 & -0.0469 & 0.9771 \\ -0.0469 & 0.9737 & 0.0023 \\ 0.9771 & 0.0023 & 1.0411 \end{bmatrix},
 \end{aligned}$$

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix},$$

where

$$\begin{aligned}
 S_{11} &= \begin{bmatrix} 7.2904 & 0.0662 & 0.0014 \\ 0.0662 & 0.2162 & -0.0013 \\ 0.0014 & -0.0013 & 0.9996 \end{bmatrix}, \\
 S_{12} &= \begin{bmatrix} 1.9088 & -0.0108 & -0.0062 \\ 0.0243 & 0.0035 & 0.0063 \\ 0.0285 & -0.0020 & -0.0015 \end{bmatrix},
 \end{aligned}$$



$$S_{22} = \begin{bmatrix} 32.0253 & 0.0336 & 0.9164 \\ 0.0336 & 1.0056 & 0.0019 \\ 0.9164 & 0.0019 & 0.9939 \end{bmatrix},$$

and matrices  $W, G$ :

$$W = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix},$$

where

$$\begin{aligned} W_1 &= \begin{bmatrix} W_1^{11} & W_1^{12} \\ 0 & W_1^{22} \end{bmatrix} \\ &= \begin{bmatrix} 7.2901 & 0.3063 & -0.0607 \\ 0 & 0.2092 & -0.0145 \\ 0 & 0.0002 & 0.9991 \end{bmatrix}, \\ W_2 &= \begin{bmatrix} W_2^{11} & W_2^{12} \\ 0 & W_2^{22} \end{bmatrix} \\ &= \begin{bmatrix} 28.8997 & 0.0389 & 0.9800 \\ 0 & 0.9922 & 0.0074 \\ 0 & 0.0068 & 1.0216 \end{bmatrix}, \end{aligned}$$

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix},$$

where

$$\begin{aligned} G_{11} &= \begin{bmatrix} 0 & G_{11}^2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 6.7670 & -1.2900 \\ 0 & -0.2106 & -24.9579 \end{bmatrix}, \\ G_{12} &= \begin{bmatrix} 0 & G_{12}^2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0.0665 & 0.7053 \\ 0 & -0.1194 & -0.0469 \end{bmatrix}, \\ G_{21} &= \begin{bmatrix} 0 & G_{21}^2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -0.0619 & -0.1798 \\ 0 & -0.2063 & -0.0571 \end{bmatrix}, \\ G_{22} &= \begin{bmatrix} 0 & G_{22}^2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -22.8072 & 1.0937 \\ 0 & 0.6194 & -18.3767 \end{bmatrix}. \end{aligned}$$

Finally, we obtain the following distributed fault-tolerant controller:

$$\begin{aligned} u_1(t) &= K_{11}y_1(t) + K_{12}y_2(t) - B_1^*E_1\hat{f}_1(t), \\ u_2(t) &= K_{22}y_2(t) + K_{21}y_1(t) - B_2^*E_2\hat{f}_2(t), \end{aligned}$$

where feedback matrices of SOF are calculated by

$$\begin{aligned} K_{11} &= G_{11}^2(W_1^{22})^{-1} = \begin{bmatrix} 32.3413 & -0.8203 \\ -0.9786 & -24.9934 \end{bmatrix}, \\ K_{12} &= G_{12}^2(W_1^{22})^{-1} = \begin{bmatrix} 0.3168 & 0.7106 \\ -0.5707 & -0.0553 \end{bmatrix}, \\ K_{21} &= G_{21}^2(W_2^{22})^{-1} = \begin{bmatrix} -0.0612 & -0.1756 \\ -0.2076 & -0.0544 \end{bmatrix}, \\ K_{22} &= G_{22}^2(W_2^{22})^{-1} = \begin{bmatrix} -22.9943 & 1.2373 \\ 0.7467 & -17.9944 \end{bmatrix}. \end{aligned}$$

#### D. Simulation

Initial conditions of two subsystems are  $[0 \ 0 \ -0.1]^T$  and  $[0 \ 0.1 \ 0]^T$ , respectively. In the simulation, we assume that actuator faults  $f_1(t) = \begin{bmatrix} f_{11}(t) \\ f_{12}(t) \end{bmatrix}$  and  $f_2(t) = \begin{bmatrix} f_{21}(t) \\ f_{22}(t) \end{bmatrix}$  simultaneously occur in the two subsystems as follows:

$$f_{11}(t) = \begin{cases} 0 & 0s \leq t < 6s \\ 2 & 6s \leq t \leq 20s \end{cases}, \quad f_{12}(t) = 0$$

and

$$\begin{aligned} f_{21}(t) &= 0, \\ f_{22}(t) &= \begin{cases} 0 & 0s \leq t < 4s \\ -(1 - e^{-2(t-4)}) & 4s \leq t \leq 20s \end{cases}. \end{aligned}$$

where  $f_1(t)$  is the abrupt fault and  $f_2(t)$  is the incipient fault.

The simulation results of the presented DFE design for the considered interconnected system are illustrated in Figure 1, which show accuracy fault estimation for each subsystems. Comparisons with the true fault vectors are shown in Figure 2 and Figure 3, respectively. According to the online obtained fault estimation, output responses of the interconnected system under DFTE are illustrated in Figure 4 and Figure 5.

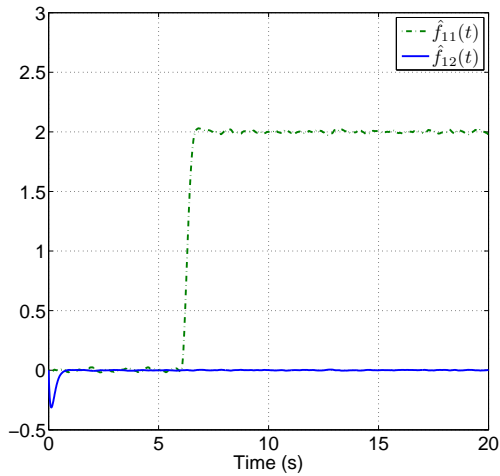
From simulation results, it is shown that after faults occur in interconnected system, the proposed DFTE can recover system performance and enhance system reliability.

## VI. CONCLUSIONS

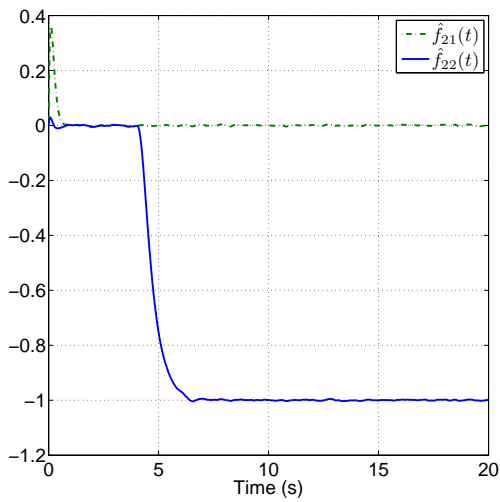
In this paper, we have proposed a novel SOF-based DFTE for interconnected systems. Based on coupling information among subsystems, a DFE is designed to provides accuracy fault estimation, while the DFTE guarantees reliability and safety of interconnected systems. The simulation results show that the presented design methods realize reliability improvement of interconnected systems.

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(a)



(b)

Fig. 1. Fault estimation of  $f_1(t)$  and  $f_2(t)$ .

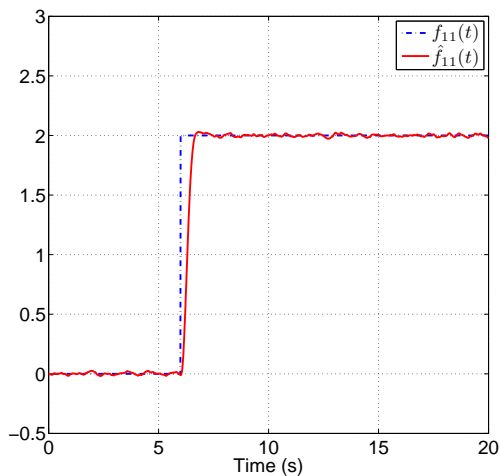


Fig. 2. Fault estimation of  $f_{11}(t)$ .

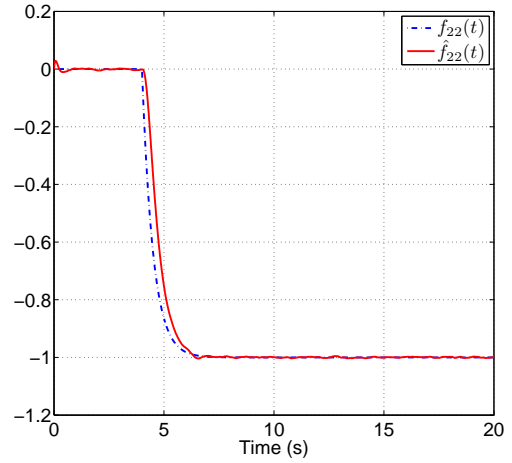


Fig. 3. Fault estimation of  $f_{22}(t)$ .

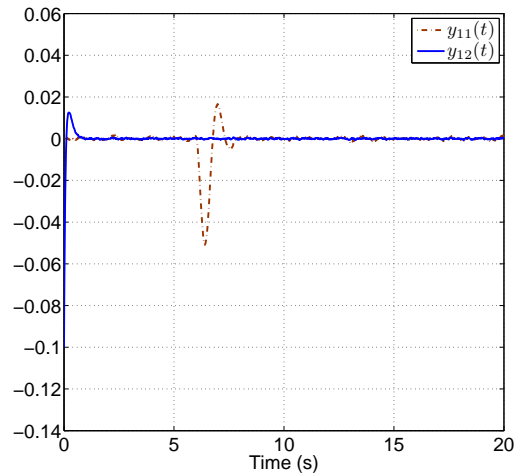


Fig. 4. Output responses of  $y_1(t)$  under fault-tolerant control.

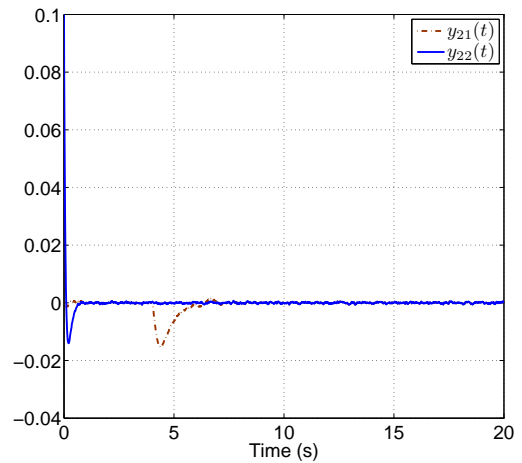


Fig. 5. Output responses of  $y_2(t)$  under fault-tolerant control.

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