Adaptive Fault-Tolerant Control of A Two-car High-speed Train Model with Inter-car Flexible Link and Traction Actuator Failures*

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***Abstract*— This paper studies the adaptive fault-tolerant tracking control problem for the high-speed trains with intercar flexible link and traction actuator failures. This study is focused on a benchmark model which, as a main dynamic unit of the CRH (China Railway High-speed) train, is a two-car dynamic system with a flexible link between two cars, for which the input acts on the second car and the output is the speed of the first car. This model is under parameter uncertainties and subject to uncertain actuator failures. For such an underactuated system, to ensure the first car tracking a desired speed trajectory, a coordinate transformation method is employed to decompose the system model into a control dynamics subsystem and a zero dynamics subsystem. Stability analysis is conducted to show that such a zero dynamic system is Lyapunov stable and is partially input-to-state stable. An adaptive fault-tolerant control scheme is developed which is able to ensure the closed-loop system signal boundedness and desired speed tracking, in the presence of the unknown system parameters and actuator failures. Simulation results from a realistic train dynamic model are presented to verify the desired adaptive control system performance.**

I. Introduction

High-speed trains with their high speeds and loading capacities, have become a main transportation tool, now. Speed tracking is the fundamental requirement for the punctuality of the operation of a train, which leads to the increasing of the automatic train operation control capabilities of high-speed trains. Great efforts have been devoted to the control design for high-speed trains ([1]-[5]).

In studies of train control problem, there are mainly two types of models used in the literatures, namely, the single mass point model and the cascade mass point model [6]. The former considers the whole train as one single mass point and ignores in-train dynamics of the train, see [7] and [8]. The latter models a train as individual mass points that are inter-connected via flexible links, see [9] and [10]. For the traditional trains, the traction force only acts on the head car, i.e., only the head car is the power car, so that the connections between each cars should be ensured to tolerant the traction force and do not break under the train operating. The single mass point model is enough to study the control problem.

Nowadays, to achieve the high speed for trains, the powers are distributed in a train, i.e., for a high-speed train, several cars are power cars and others are trailer cars, which makes the inter-force generated by the connections cannot be ignored in control design. This results in the cascade mass point model for controller design. Considering that the power and trailer cars are always distributed every other one, the two-car model with the input acting on the second car, is chosen as the benchmark model to study the control problem, in this paper.

On the other hand, the traction system treated as the actuator in high-speed trains, includes the rectifiers, inverters, PWMs (pulse width modulations), traction motors, and mechanical drives, etc., which always operates under the high temperature and vibration to cause the failure occurrences. It is necessary to utilize a fault-tolerant control scheme to guarantee the system stable and even asymptotic tracking. Although there are some results about the fault-tolerant control for high-speed train (see [11]-[13]), the fault-tolerant control for the unknown system parameters to achieve the speed tacking is not available. This motivates us to study the fault-tolerant control for the two-car high-speed model with the unknown system parameters.

The purpose of paper is to solve an adaptive fault-tolerant control problem for high-speed trains via the under-actuated two-car model with unknown system parameters and actuator failures. The main contributions of this paper are as follows: (i) For the under-actuated two-car high-speed train model, the stability study of the zero dynamics subsystem is presented. (ii) A stable adaptive fault-tolerant controller is proposed to ensure the closed-loop system signal boundedness and speed tracking, in the presence of the unknown system parameters and actuator failures.

The rest of the paper is organized as follows: Section II describes the benchmark two-car system dynamic model and the tracking control problem is formulated. Section III studies the stabilization condition for the zero dynamics subsystem. Section IV designs the adaptive fault-tolerant tracking controller. Section V includes the simulation study, followed by conclusions in Section VI.

II. System Description and Problem Formulation

For the high-speed trains, the power cars are always connected every other tailers, in which the two-car model with one car having power can be considered as the basic models. The link between two cars can be equivalent to

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the spring and damper. According to [10], [15] and [16], the motion dynamics of the two-car system with second car having control inputs, is described by

$$M_1\ddot{z}_1(t) = -k_1(z_1(t)-z_2(t)) - d_1(\dot{z}_1(t)-\dot{z}_2(t)) \\ -a_{r1} - b_{r1}\dot{z}_1(t) - c_{r1}\dot{z}_1^2(t), \quad (1)$$

$$M_2\ddot{z}_2(t) = F(t) - k_1(z_2(t)-z_1(t)) - d_1(\dot{z}_2(t)-\dot{z}_1(t)) \\ -a_{r2} - b_{r2}\dot{z}_2(t), \quad (2)$$

where $\dot{z}_1(t)$, $\dot{z}_2(t)$, $z_1(t)$ and $z_2(t)$ are the speed and the displacement of the 1st and 2nd bodies, respectively; M_1 and M_2 are the masses of the 1st and 2nd bodies, $F_2(t)$ is the control input acting on the 2nd car, k_1, k_2, d_1 and d_2 are the spring and damping constants; b_{r1} and b_{r2} are the car's resistance dependent on the speed.

Set $k_{11} = \frac{k_1}{M_1}$, $k_{12} = \frac{k_1}{M_2}$, $d_{11} = \frac{d_1}{M_1}$, $d_{12} = \frac{d_1}{M_2}$, $a_1 = \frac{a_{r1}}{M_1}$, $a_2 = \frac{a_{r2}}{M_2}$, $b_1 = \frac{b_{r1}}{M_1}$, $b_2 = \frac{b_{r2}}{M_2}$, $c_1 = \frac{c_{r1}}{M_1}$, $m_2 = \frac{1}{M_2}$, and choose $x(t) \in R^4 \triangleq [x_1(t),\ \ x_2(t),\ \ x_3(t),\ \ x_4(t)]^T = [z_1(t),\ \ \dot{z}_1(t),\ \ z_1(t)-z_2(t),\ \ \dot{z}_2(t)]^T$. The two-car dynamic equations (1)-(2) can be written as

$$\dot{x}(t) = Ax(t) + BF(t) - D_1x_2^2(t) - D_2, \quad (3)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -d_{11}-b_1 & -k_{11} & d_{11} \\ 0 & 1 & 0 & -1 \\ 0 & d_{12} & k_{12} & -d_{12}-b_2 \end{bmatrix}, \quad (4)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & m_2 \end{bmatrix}^T, \quad (5)$$

$$D_1 = \begin{bmatrix} 0 & c_1 & 0 & 0 \end{bmatrix}^T, \quad (6)$$

$$D_2 = \begin{bmatrix} 0 & a_1 & 0 & a_2 \end{bmatrix}^T, \quad (7)$$

with k_{11}, k_{12}, d_{11}, d_{12}, a_1, a_2, b_1, b_2, c_1, and m_2 being unknown parameters.

Actuator failure model. The actuator failures in traction system are always generated by the failed equipments. Consider n motors in a high-speed train. The failure model can be expressed as (see, e.g. [17])

$$F_j(t) = \bar{F}_j(t) = \bar{F}_{j0} + \sum\nolimits_{\rho=1}^{s_j} \bar{F}_{j\rho} f_{j\rho}(t), \quad (8) \\ t \geq t_j, \quad j \in \{1, 2, \ldots, n\},$$

where j is the failure index, t_j is the failure occurring time instant, $\bar{F}_{j0}$ and $\bar{F}_{j\rho}$ are unknown constants. The basis signals $f_{j\rho}(t)$ are known, with s_j being the number of the basis signals of the jth actuator failure.

Since there are n motors in the high-speed train, the resultant traction force $F(t)$ is the sum of the forces F_j, $j = 1, \ldots, n$, generated from the jth motor, given by:

$$F(t) = \sum\nolimits_{j=1}^{n} F_j(t). \quad (9)$$

From (8)-(9), the input of system (3) can be rewritten as

$$F(t) = k_\nu \nu(t) + \xi^T \varpi(t), \quad (10)$$

$$\xi = [\xi_1^T, \xi_2^T, \ldots, \xi_n^T]^T, \quad (11)$$

$$\xi_j = [\xi_{j0},\ \ \xi_{j1}, \ldots, \xi_{js_j}]^T \in R^{s_j+1}, \quad (12)$$

$$\varpi(t) = [1, f_{11}(t), \ldots, f_{1s_1}(t), \ldots, 1, f_{j1}(t), \ldots, f_{js_j}(t), \\ \ldots, 1, f_{n1}(t), \ldots, f_{ns_n}(t)]^T, \ \ \text{for}\ \ j = 1, \ldots, n, \quad (13)$$

where $\nu(t)$ is a designed control signal, and k_ν is the actuator failure pattern parameter with ξ and $\varpi(t)$ describing actuators and the types of failures.

For adaptive actuator fault-tolerant control design, an assumption is given as: (A1) for the case that any up to $n_0(n_0 < n)$ actuators fail, the remaining healthy actuators can still achieve the desired control objective. This assumption means that any n_0 of the n actuators may fail, and the parameter k_ν only takes one integer in the interval $[n-n_0, n]$ to reflect the different faults.

Control objective. From the structure of the input matrix B, it is clear to see that system (3) is an under-actuated system, for which the arbitrary state tracking is not achievable. Here, we choose the speeds of the first car is the controlled variables, i.e., $y(t) = x_2(t)$. The control objective of this paper can be summarized as: an adaptive fault-tolerant controller is designed for the two-car system (3) to make the output $y(t)$ tracking the desired speed signal $v_m(t)$, and simultaneously keep the states bounded, in the presence of unknown system parameters and actuator failures.

III. Feedback Control Design

It is straight to see that the considered two-car system (3) is a nonlinear system and the input does not act on the state $x_2(t)$ directly, on which the feedback linearization control method should be employed.

Set $f(x) = Ax(t) - D_1x_2^2(t)$, $g(x) = B$ and $h(x) = x_2(t)$. According to [14], the relative degree $\rho = 2$ for the system (3) can be calculated as:

$$L_g h(x) = 0, \quad L_g L_f h(x) = d_{11}m_2 \neq 0. \quad (14)$$

Then, we choose the diffeomorphsim coordination transformation to transform the system (3) into a normal form for the fault-tolerant controller design.

Normal form. For the uniform relative degree ρ, the system (3)-(4) can be transformed into two subsystems via a diffeomorphsim $T(x) = [\phi_1(x), \phi_2(x), \phi_3(x), \phi_4(x)]^T$, where $\phi_1(x) = h(x)$, $\phi_2(x) = L_f h(x)$, $\phi_3(x)$ and $\phi_4(x)$ satisfy $L_g\phi_3(x) = 0$ and $L_g\phi_4(x) = 0$.

Then, the coordination transformation is chosen as

$$\varphi_1(t) = \phi_1(x) = x_2(t), \quad (15)$$

$$\varphi_2(t) = \phi_2(x) = \dot{x}_2(t) = (-d_{11}-b_1)x_2(t) - k_{11}x_3(t) \\ +d_{11}x_4(t) - c_1x_2^2(t) - a_2, \quad (16)$$

$$\eta_1(t) = \phi_3(x) = x_1(t), \quad (17)$$

$$\eta_2(t) = \phi_4(x) = x_3(t). \quad (18)$$

to decompose the system (3) into the control dynamics subsystem

$$y(t) = \varphi_1(t), \tag{19}$$
$$\dot{\varphi}_1(t) = \varphi_2(t), \tag{20}$$
$$\dot{\varphi}_2(t) = R(x) + d_{11} m_2 (k_\nu \nu(t) + \xi^T \varpi(t)), \tag{21}$$

and the zero dynamics subsystem

$$\dot{\eta}_1(t) = \varphi_1(t), \tag{22}$$
$$\begin{aligned}\dot{\eta}_2(t) = & -\frac{k_{11}}{d_{11}}\eta_2(t) - \frac{b_1}{d_{11}}\varphi_1(t) - \frac{1}{d_{11}}\varphi_2(t) \\ & -\frac{c_1}{d_{11}}\varphi_1^2(t) - \frac{a_1}{d_{11}},\end{aligned} \tag{23}$$

where

$$\begin{aligned}R(x) = & -(d_{11} + b_1 + 2c_1 x_2(t))\big(- (d_{11} + b_1) x_2(t) \\ & -k_{11} x_3(t) + d_{11} x_4(t) - c_1 x_2^2(t) - a_1\big) \\ & -k_{11}\big(x_2(t) - x_4(t)\big) + d_{11}\big(d_{12} x_2(t) + k_{12} x_3(t) \\ & -(d_{12} + b_2) x_4(t) - a_2\big),\end{aligned} \tag{24}$$

$\varphi_1(t) = x_2(t) = \dot{z}_1(t)$ is the speeds of the first car, and $\varphi(t) \triangleq [\varphi_1(t), \varphi_2(t)]^T$ and $\eta(t) \triangleq [\eta_1(t), \eta_2(t)]^T$.

Feedback linearization control. If the parameters k_{11}, k_{12}, d_{11}, d_{12}, a_1, a_2, b_1, b_2, c_1, m_2, k_ν and ξ in the dynamics (19)-(21) with the actuator fault (10) are known, with $d_{12} m_1 > 0$, $\varphi_1(t) = x_2(t)$ and under assumption (A1), we can design the feedback linearization fault-tolerant control law,

$$\begin{aligned}\nu(t) = & -\frac{1}{d_{11} m_2 k_\nu}\Bigg(R(x) - \ddot{v}_m(t) + \alpha_1(\dot{x}_2(t) - \dot{v}_m(t)) \\ & +\alpha_2(x_2(t) - v_m(t))\Bigg) - \frac{1}{k_\nu}\xi^T \varpi(t),\end{aligned} \tag{25}$$

where $\alpha_1 > 0$ and $\alpha_2 > 0$ are design parameters such that $s^2 + \alpha_1 s + \alpha_2$ is Hurwitz polynomials, $R(x)$ is defined in (24). The desired speed $v_m(t)$, acceleration $\dot{v}_m(t)$ and its derivative $\ddot{v}_m(t)$ are bounded.

Submitting $\nu(t)$ into the system (20)-(21), it has

$$\begin{aligned}\ddot{x}_2(t) - \ddot{v}_m(t) = & -\alpha_1(\dot{x}_2(t) - \dot{v}_m(t)) \\ & -\alpha_2(x_2(t) - v_m(t)).\end{aligned} \tag{26}$$

With the tracking error $e(t) = x_2(t) - v_m(t) = \varphi_1(t) - v_m(t)$, (26) leads to

$$\ddot{e}(t) + \alpha_1 \dot{e}(t) + \alpha_2 e(t) = 0, \tag{27}$$

which implies that $\lim_{t\to\infty} e(t) = 0$ exponentially. With $e(t) = \varphi_1(t) - v_m(t)$ and $y(t) = \varphi_1(t)$, it has that the proposed nominal fault-tolerant control (25) can ensure that the output tracks the desired speed trajectory $v_m(t)$, and $\varphi(t)$ is bounded.

IV. Stability of Zero Dynamics

The proposed control $\nu(t)$ in (25) uses the signal $\eta(t)$ in the zero dynamics subsystem (22)-(23), which should be bounded. In this section, we will discuss the boundedness of the state $\eta(t)$, which is influenced by the sub-state vector $\varphi(t)$, to guarantee the effectiveness of the designed control signal $\nu(t)$.

Stability performance of $\eta_1(t)$. For the zero dynamics subsystem (22)-(23), both Lyapunov and input-to-state stability should be discussed.

If $\varphi_1(t) = 0$, from (22), it has

$$\dot{\eta}_1(t) = 0, \quad \text{i.e.,} \quad \dot{x}_1(t) = \dot{z}_1(t) = 0, \tag{28}$$

which implies

$$\eta_1(t) = z_1(t) = z_1(0). \tag{29}$$

From (29), $\eta_1(t)$ is Lyapunov stable. On the other hand, as $\varphi_1(t)$ and $\eta_1(t)$ represent the speed and displacement of the first car, respectively, the displacement trajectory $\eta_1(t)$ has a desired tracking property, if the speed $\varphi_1(t)$ tracks a desired speed trajectory by designing control. Then, the state $\eta_1(t) = z_1(t)$ satisfies the system performance, even if $\lim_{t\to\infty} \eta_1(t) = \infty$.

Stability performance of $\eta_2(t)$. As $\eta_1(t)$ satisfies the system performance with $\varphi_1(t)$ as input, we should analyze the stability performance of $\eta_2(t)$. Setting $\bar{\varphi}(t) = -\frac{b_1}{d_{11}}\varphi_1(t) - \frac{1}{d_{11}}\varphi_2(t) - \frac{c_1}{d_{11}}\varphi_1^2(t) - \frac{a_1}{d_{11}}$ as input, (23) can be rewritten as

$$\dot{\eta}_2(t) = -\frac{k_{11}}{d_{11}}\eta_2(t) + \bar{\varphi}(t), \tag{30}$$

where $\bar{\varphi}(t)$ is bounded if $\varphi_1(t)$ and $\varphi_2(t)$ are bounded.

As d_{11} and k_{11} are positive constants, $-\frac{k_{11}}{d_{11}} < 0$, which implies the system (30) is stable, i.e., (30) is Lyapunov stable and bounded-input-bounded-state stable with $\bar{\varphi}(t)$ as input.

Then, we have the following result.

Lemma 1: *The zero dynamic (22)-(23) is Lyapunov stable, that is, the solution $\eta(t)$ of $\dot{\eta} = A_1\eta$, $A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{k_{11}}{d_{11}} \end{bmatrix}$, is bounded for $\eta(0) \neq 0$.*

Lemma 2: *The dynamic system defined in (30) is bounded-input-bounded-state (BIBS) stable.*

We have studied that the zero dynamic system (22)-(23) is Lyapunov stable and partial bounded-input-bounded-state stable (see Lemmas 1 and 2). In the next section, we will design an adaptive fault-tolerant controller instead of the nominal control (25) to ensure that $\varphi_1(t)$ and $\varphi_2(t)$ are bounded and the closed-loop control system is stable and asymptotic output tracking is achieved.

V. Adaptive Controller Scheme

As the system parameters k_{11}, k_{12}, d_{11}, d_{12}, a_1, a_2, b_1, b_2, c_1, m_2 and ξ are unknown, an adaptive controller $\hat{\nu}(t)$ should be designed to replace the the nominal controller $\nu(t)$, such that $\lim_{t\to\infty}(\varphi_1(t) - v_m(t)) = 0$.

Adaptive controller structure. Under assumption (A1), the parameters of the nominal controller $\nu(t)$ in (25) are defined to design the adaptive controller $\hat{\nu}(t)$ as

$$\theta_1 = \frac{1}{d_{11}m_2k_\nu}, \tag{31}$$
$$\theta_2 = (d_{11}+b_1)^2 + 2c_1a_1 - k_{11} + d_{11}d_{12}, \tag{32}$$
$$\theta_3 = 3c_1(d_{11}+b_1),\quad \theta_4 = 2c_1^2,\quad \theta_5 = 2c_1k_{11}, \tag{33}$$
$$\theta_6 = 2c_1d_{11},\quad \theta_7 = (d_{11}+b_1)k_{11} + d_{11}k_{12}, \tag{34}$$
$$\theta_8 = k_{11} - (d_{11}+b_1)d_{11} - d_{11}(d_{12}+b_2), \tag{35}$$
$$\theta_9 = d_{11}a_2,\quad \theta_{10} = \frac{\xi}{k_\nu}, \tag{36}$$

which lead to the nominal controller $\nu(t)$ written as

$$\begin{aligned}\nu(t) = &-\theta_1\bigg(\theta_2x_2(t) + \theta_3x_2^2(t) + \theta_4x_2^3(t) + \theta_5x_2(t)x_3(t)\\ &-\theta_6x_2(t)x_4(t) + \theta_7x_3(t) + \theta_8x_4(t) - \theta_9 - \ddot{v}_m(t)\\ &+\alpha_1(\dot{x}_2(t) - \dot{v}_m(t)) + \alpha_2(x_2(t) - v_m(t))\bigg)\\ &-\theta_{10}\varpi(t).\end{aligned} \tag{37}$$

Design the adaptive controller $\hat{\nu}(t)$

$$\begin{aligned}\hat{\nu}(t) = &-\hat{\theta}_1(t)\bigg(\hat{\theta}_2(t)x_2(t) + \hat{\theta}_3(t)x_2^2(t) + \hat{\theta}_4(t)x_2^3(t)\\ &+\hat{\theta}_5(t)x_2(t)x_3(t) - \hat{\theta}_6(t)x_2(t)x_4(t) + \hat{\theta}_7(t)x_3(t)\\ &+\hat{\theta}_8(t)x_4(t) - \hat{\theta}_9(t) - \ddot{v}_m(t) + \alpha_1(\dot{x}_2(t) - \dot{v}_m(t))\\ &+\alpha_2(x_2(t) - v_m(t))\bigg) - \hat{\theta}_{10}(t)\varpi(t),\end{aligned} \tag{38}$$

where $\hat{\theta}_\varrho(t)$ are the estimations of θ_ϱ, for $\varrho = 1,\ldots,10$.

Closed-loop adaptive control system. To design the adaptive laws for $\hat{\theta}_\varrho(t)$, for $\varrho = 1,\ldots,10$, we define the parameter errors $\tilde{\theta}_\varrho(t) = \theta_\varrho - \hat{\theta}_\varrho(t)$ and use the control law (38) and the system (22)-(23) under the definition (31)-(36), to obtain

$$\begin{aligned}&\ddot{\varphi}_1(t) = \ddot{x}_2(t)\\ &= \ddot{v}_m(t) - \alpha_1(\dot{x}_2(t) - \dot{v}_m(t)) - \alpha_2(x_2(t) - v_m(t))\\ &\quad+\tilde{\theta}_1(t)\hat{\nu}(t) + \tilde{\theta}_2(t)x_2(t) + \tilde{\theta}_3(t)x_2^2(t) + \tilde{\theta}_4(t)x_2^3(t)\\ &\quad+\tilde{\theta}_5(t)x_2(t)x_3(t) - \tilde{\theta}_6(t)x_2(t)x_4(t) + \tilde{\theta}_7(t)x_3(t)\\ &\quad+\tilde{\theta}_8(t)x_4(t) - \tilde{\theta}_9(t) - \tilde{\theta}_{10}(t)\varpi(t),\end{aligned} \tag{39}$$

which can be rewritten as

$$\ddot{e}(t) + \alpha_1\dot{e}(t) + \alpha_2e(t) = \tilde{\Theta}^T(t)W(t), \tag{40}$$

with $e(t) = x_2(t) - v_m(t)$, $\tilde{\Theta}(t) = [\tilde{\theta}_1(t), \tilde{\theta}_2(t), \ldots, \tilde{\theta}_{10}(t)]^T$ and $W(t) = [\hat{\nu}(t), x_2(t), x_2^2(t), x_2^3(t), x_2(t)x_3(t), -x_2(t)x_4(t), x_3(t), x_4(t), -1, -\varpi(t)]^T$.

Ignoring the exponentially decaying effect of the initial conditions, the error dynamic (40) can be written as

$$e(t) = M(s)[\tilde{\Theta}^TW](t),\quad M(s) = \frac{1}{P(s)}, \tag{41}$$

where $P(s) = s^2 + \alpha_1s + \alpha_2$ is Hurwitz polynomial. We can also define the estimation error as

$$\epsilon(t) = e(t) + \hat{\Theta}^T(t)\zeta(t) - M(s)[\hat{\Theta}^TW](t), \tag{42}$$

where $\hat{\Theta}(t)$ is the estimations of Θ, and $\zeta(t) = M(s)[W](t)$.

Adaptive laws. The gradient adaptive update law for $\hat{\Theta}(t)$ is chosen as

$$\dot{\hat{\Theta}}(t) = -\frac{\Gamma\epsilon(t)\zeta(t)}{m^2(t)},\quad \hat{\Theta}(0) = \hat{\Theta}_0, \tag{43}$$

where $m(t) = \sqrt{1+\zeta^T(t)\zeta(t)}$, $\Gamma = \Gamma^T > 0$ is an adaptation gain, and $\hat{\Theta}_0$ is the initial estimates of Θ. This adaptive update law (43) has the following properties.

Lemma 3: *The adaptive law (43) guarantees that $\hat{\Theta}(t) \in L^\infty$, $\dot{\hat{\Theta}}(t) \in L^2 \cap L^\infty$, and $\frac{\epsilon(t)}{m(t)} \in L^2 \cap L^\infty$.*

Proof: Considering the positive definite function $V(\tilde{\Theta}(t)) = \frac{1}{2}\tilde{\Theta}^T(t)\Gamma^{-1}\tilde{\Theta}(t)$, we have the time derivative of $V(\tilde{\Theta}(t))$ along with (43) as

$$\dot{V}(\tilde{\Theta}(t)) = -\frac{\epsilon(t)\tilde{\Theta}^T(t)\zeta(t)}{m^2(t)}. \tag{44}$$

Form (42), we have

$$\begin{aligned}\epsilon(t) &= e(t) + \hat{\Theta}^T(t)M(s)[W](t) - M(s)[\hat{\Theta}^TW](t)\\ &= e(t) + \tilde{\Theta}^T(t)M(s)[W](t) - M(s)[\tilde{\Theta}^TW](t)\\ &= \tilde{\Theta}^T(t)M(s)[W](t) = \tilde{\Theta}^T(t)\zeta(t).\end{aligned} \tag{45}$$

From (44) and (45), we have

$$\dot{V}(\tilde{\Theta}(t)) = -\frac{\epsilon^2(t)}{m^2(t)}, \tag{46}$$

which implies that $V(\tilde{\Theta}(t))$ do not increase and $\int_0^\infty \frac{\epsilon^2(t)}{m^2(t)}dt < V(0) < \infty$. Thus, $\hat{\Theta}(t) = \tilde{\Theta}(t) + \Theta$ is bounded. With the boundedness of $\tilde{\Theta}(t)$, we have that $\frac{\epsilon(t)}{m(t)}$ is bounded, which implies that $\frac{\epsilon(t)}{m(t)} \in L^2 \cap L^\infty$. Therefore, from (43), we have $\dot{\hat{\Theta}}(t) \in L^2 \cap L^\infty$. ∇

Stability analysis. In this part, the closed-loop system stability and tracking properties are analysed when an overall adaptive control law (38) is applied to the system (3) in the presence of the unknown parameters k_{11}, k_{12}, d_{11}, d_{12}, a_1, a_2, b_1, b_2, c_1, m_2 and ξ.

Some lemmas that will be used in the adaptive fault-tolerant controller design from [18], are presented as follows.

Lemma 4 [18]: *If $H(s) = c(sI - A)^{-1}b$ is the minimal realization of a proper transfer function, then*

$$\begin{aligned}&\tilde{\Theta}^TH(s)[W](t) - H(s)[\tilde{\Theta}^TW](t)\\ &= c(sI - A)^{-1}[(sI - A)^{-1}b[W^T](t)\dot{\tilde{\Theta}}](t).\end{aligned} \tag{47}$$

Lemma 5[18]: *Let $y(t) = H(s)[u](t)$, where $H(s)$ is a proper stable transfer function. If $\|u\|_t \le \kappa\|q\|_t + \kappa$, then $\|y\|_t \le \kappa\|q\|_t+\kappa$. In addition, if $H(s)$ is strictly proper, then $\|y\|_t \le \tau\|q\|_t + \tau$, where $\kappa \in L^2 \cap L^\infty$ and $\tau \in L^2 \cap L^\infty$ with $\lim_{t\to\infty} \tau = 0$.*

Lemma 6[18]: *Let $y(t) = H(s)[u](t)$, where $H(s)$ is a proper and minimum phase transfer function. If u, $\dot{u} \in L^{\infty e}$ and $\|\dot{u}\|_t \le \mu\|u\|_t + \mu$, then $\|u\|_t \le \mu\|y\|_t + \mu$, where μ denotes a signal bound.*

With Lemma 3 and the results in [18], we have the following result.

Theorem 1: *The adaptive controllers (38) with the adaptive laws (43) applied to the system (3), guarantee that the corresponding closed-loop state signals $\dot{z}_1(t)$, $z_1(t) - z_2(t)$, $\dot{z}_2(t)$ are bounded, and the tracking error satisfies $\lim_{t\to\infty}(\dot{z}_1(t) - v_m(t)) = 0$.*

Proof: Using Lemma 4 with $M(s)$, the equations are obtained as

$$\begin{aligned}&\tilde{\Theta}^T M(s)[W](t) - M(s)[\tilde{\Theta}^T W](t)\\ &= c(sI - A)^{-1}[(sI - A)^{-1}b[W^T]\dot{\tilde{\Theta}}](t), \end{aligned}\tag{48}$$

with (c, A, b) being the minimal realisation of $M(s)$. Since $P(s) = \frac{1}{M(s)}$ is Hurwitz, $(sI - A)^{-1}b$ is stable. Further, $\dot{\tilde{\Theta}}(t) = \dot{\hat{\Theta}}(t) \in L^2 \cap L^\infty$, we have

$$\|(sI - A)^{-1}b[W^T]\dot{\tilde{\Theta}}\|_t \le \kappa\|W\|_t + \kappa. \tag{49}$$

Since $c(sI - A)^{-1}$ is strictly proper, using Lemma 5, we obtain that

$$\|\tilde{\Theta}^T M(s)[W](t) - M(s)[\tilde{\Theta}^T W](t)\|_t \le \tau\|W\|_t + \tau \tag{50}$$

Note that the differential equation for $\varphi = [\varphi_1, \varphi_2]^T = [x_2, \dot{x}_2]^T$ are

$$\varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = M(s) \begin{bmatrix} 1 \\ s \end{bmatrix} [\tilde{\Theta}^T W] + \begin{bmatrix} v_m \\ \dot{v}_m \end{bmatrix}. \tag{51}$$

Since $\tilde{\Theta}(t)$ is bounded, v_m and $\dot{v}_m$ are is bounded by hypothesis, $M(s)$ and $sM(s)$ are all proper stable, we have $\|\varphi\|_t \le \mu\|W\|_t + \mu$, $\|\varphi\|_t \le \mu\|\tilde{\Theta}^T W\|_t + \mu$. For the partial ISS of dynamics (22)-(23), we have $\|\eta_2\|_t \le \mu\|W\|_t + \mu$, $\|\eta_2\|_t \le \mu\|\Theta^T W\|_t + \mu$. Based on the property of the state transformation (15)-(18), we have

$$\|\bar{x}\|_t \le \mu\|W\|_t + \mu, \quad \|\bar{x}\|_t \le \mu\|\tilde{\Theta}^T W\|_t + \mu, \tag{52}$$

where $\bar{x} = [x_2, x_3, x_4]^T$.

From (38), we have $\|\hat{\nu}\|_t \le \mu\|W\|_t + \mu$. Recalling the definition of $W(t) = [\hat{\nu}(t), x_2(t), x_2^2(t), x_2^3(t), x_2(t)x_3(t), -x_2(t)x_4(t), x_3(t), x_4(t), -1, -\varpi(t)]^T$, $\|\partial W/\partial x\|$ and $\|\partial W/\partial\hat{\Theta}\|$ are bounded. Together with (52), we obtain

$$\|\dot{W}\|_t \le \mu\|W\|_t + \mu, \tag{53}$$

thus $W(t)$ is regular. Furthermore, since $M(s)$ is a stable polynomial, $\varphi(t) = M(s)[W](t)$ are also regular. A similar calculation yields $\tilde{\Theta}^T W$ to be regular as well. From $\frac{d}{dt}(\tilde{\Theta}^T W) = \dot{\tilde{\Theta}}^T W + \tilde{\Theta}^T\dot{W}$, using (53) and $\tilde{\Theta}, \dot{\tilde{\Theta}} \in L^\infty$, we obtain

$$\left\|\frac{d}{dt}(\tilde{\Theta}^T W)\right\|_t \le \mu\|W\|_t + \mu. \tag{54}$$

From (52), together with the construction of W, implies that

$$\|W\|_t \le \mu\|\tilde{\Theta}^T W\|_t + \mu. \tag{55}$$

Combining (55) with (54) yields the regularity of $\tilde{\Theta}^T W$,

$$\left\|\frac{d}{dt}(\tilde{\Theta}^T W)\right\|_t \le \mu\|W\|_t + \mu \le \mu\|\tilde{\Theta}^T W\|_t + \mu. \tag{56}$$

Calculating the derivative of $\frac{\epsilon(t)}{m(t)}$, we have

$$\begin{aligned}\left\|\frac{d}{dt}\left(\frac{\epsilon}{m}\right)\right\|_t &\le \left\|\frac{m(\dot{\tilde{\Theta}}^T\zeta + \tilde{\Theta}^T\dot{\zeta}) - \dot{m}(\tilde{\Theta}^T\zeta)}{m^2}\right\|_t\\ &\le \frac{\|\dot{\tilde{\Theta}}^T\zeta\|_t + \|\tilde{\Theta}^T\dot{\zeta}\|_t}{\|m\|_t} + \frac{\|\tilde{\Theta}^T\zeta\dot{\zeta}^T\zeta\|_t}{\|m^3\|_t}\\ &\le \frac{\|\dot{\tilde{\Theta}}^T\zeta\|_t + \|\tilde{\Theta}^T\dot{\zeta}\|_t}{\varepsilon} + \frac{\|\tilde{\Theta}^T\zeta\dot{\zeta}^T\zeta\|_t}{\varepsilon^3}\\ &\le \frac{\mu\|\zeta\|_t + \mu}{\varepsilon} + \frac{\mu\|\zeta\|_t^3 + \mu}{\varepsilon^3} \le \mu, \end{aligned}\tag{57}$$

where $\varepsilon = \max\{1, \|\zeta\|_t\}$. Therefore, we can see that $\frac{d}{dt}\left(\frac{\epsilon}{m}\right)$ is bounded. Using the fact $\frac{\epsilon}{m} \in L^2 \cap L^\infty$, we have $\lim_{t\to\infty}\frac{\epsilon}{m} = 0$, which implies that $\lim_{t\to\infty}\epsilon = 0$.

From (42), (50), and (55), we have

$$|e| \le \tau\|W\|_t + \tau + |\epsilon| \le \tau\|\tilde{\Theta}^T W\|_t + \tau + |\epsilon|. \tag{58}$$

Since $e = M(s)[\tilde{\Theta}^T W](t)$, using Lemma 6, we have

$$\|\tilde{\Theta}^T W\|_t \le \mu\|e\|_t + \mu. \tag{59}$$

From (58) and (59), we have $|e| \le \tau\|e\|_t + \tau + |\epsilon|$, which implies that $\lim_{t\to\infty} e = 0$, i.e., $\lim_{t\to\infty}(\dot{z}_1(t) - v_m(t)) = 0$. From (59), the boundedness of $e(t)$ implies that of $\tilde{\Theta}^T(t)W(t)$. From (55), the boundedness of $\tilde{\Theta}^T(t)W(t)$ implies that of $W(t)$. From (52) and $\|\hat{\nu}\|_t \le \mu\|W\|_t + \mu$, we have the boundedness of $\bar{x}(t)$ (i.e., $\dot{z}_1(t)$, $z_1(t) - z_2(t)$, $\dot{z}_2(t)$) and $\hat{\nu}(t)$. ∇

Because $x_1(t)$ ($z_1(t)$) is the position of the first car, which could go to finite as t goes to finite, the boundedness of $x_1(t)$ cannot be obtained and its performance is ensured by $x_2(t)$ ($\dot{z}_1(t)$).

VI. Simulation Study

To verify the proposed controller design method, a simulation study is presented in this section, in which the parameters are from a real train model of [19]. Two cars with input acting on second car are considered, for which, the adaptive controller proposed in (38) with the adaptive law (43) is used.

Simulation system. The parameters in the simulation are chosen as: $M_1 = 126000$kg, $M_2 = 101090$kg, $a_{r1} = 8.63$Ns/ton, $a_{r2} = 9.03$Ns/ton, $b_{r1} = 1.08e^{-4}$Ns/(m kg),

$b_{r2} = 1.08e^{-4}$ Ns/(m kg), $c_{r1} = 0.00112$ N s^2/(m^2 ton), $k_1 = 100e^6$N/m, $k_2 = 30e^6$N/m, $d_1 = 80e^4$Ns/m, $k_2 = 40e^4$Ns/m. The initial conditions are chosen as $x(0) = [0.05 \quad 0 \quad 0 \quad 0]^T$, and the initial parameter estimates as 95% of their nominal values. The gains of the adaptive law in (43) is chosen as $\Gamma = \mathrm{diag}[0.2 \;\; 0.002 \;\; 0.002 \;\; 0.002 \;\; 0.002 \;\; 0.002 \;\; 2 \;\; 0.2 \;\; 0.2 \;\; 0.2]$.

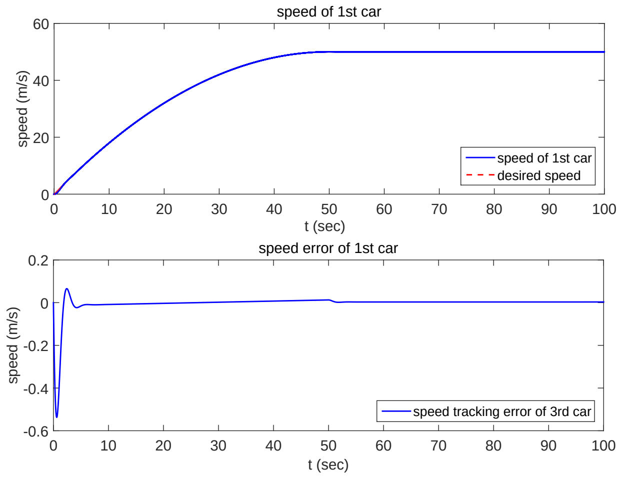

Fig. 1. Speed and speed tracking error of first car

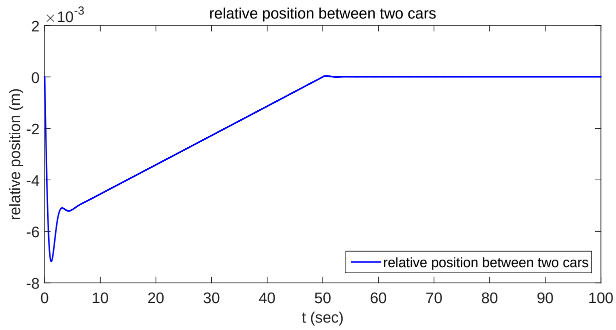

Fig. 2. Position error between two cars

Simulation results. Fig. 1 shows the simulation results of the speed and speed tracking errors for first car, from which, it can be seen that the tracking errors are close to 0. There are transit responses due to the adaptive laws and zero dynamics. Fig. 2 shows the position error between first and second cars ($z_1(t) - z_2(t)$). which becomes a constant in steady case. The simulation results show that the proposed stable adaptive control framework can achieve the close-loop stability even in the presence of unknown parameters.

VII. Conclusions

In this paper, the adaptive fault-tolerant controller design problem has been investigated for high-speed trains using a under-actuated two-car model even if the parameters are unknown. A coordinate transformation method is employed to decompose the system into a control dynamics subsystem and a zero dynamics subsystem. The stability analysis is conducted to show that such a zero dynamic system is Lyapunov stable and is also partially input-to-state stable. Then, the adaptive fault-tolerant controller is developed to ensure the closed-loop system signal boundedness and speed tracking, in the presence of the unknown system parameters and actuator failures.

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