Reheat Turbine LFC of Power Systems with Multiple Delays Based on Sliding Mode Techniques*

Adrian E. Onyeka [1], Xing-Gang Yan[1], Zehui Mao[2], Dongya Zhao[3], Bin Jiang[2]

Abstract— **This paper considers the growing effect of reheat turbine delays in a power system with multiple delays and nonlinear disturbance. In order to improve the operating condition of the systems and avoid the destabilizing effects of time delay, a model representation for reheat turbine delay is developed where a multiple delayed nonlinear term is used to describe disturbances. On this basis, an admissible upper bound is provided based on the Lyapunov Razumikhin theorem and an acceptable ultimate bound is calculated for the load disturbance. An improved load frequency sliding mode control (SMC) is synthesized such that the controlled system is uniformly untimately stable even in the presence of time delays and nonlinear disturbances. Effectiveness of the proposed method is tested by simulation via an isolated power system supplying a service load.**

I. INTRODUCTION

Load frequency control (LFC) is one of the major challenges in power systems [1], [2]. Due to growing industrial networks and consumers, power system stability is threatened as a result of fluctuations and growing uncertainties. Thus, load frequency studies, aimed at re-adjusting frequency to nominal values after a sustained disturbance, has been in the core centre of power system research. Result of this investigations [3], [4], [1] has shown that uncertainties in power systems arise due to multiple factors ranging from parameter variations, un-modelled dynamics, rate limits etc. In addition, time delays are natural issues which cannot be neglected especially with regards to improving system performance.

It is widely known that time delay is usually a source of instability and performance degradation in control systems. Many results have been produced for the stability problem of time delay systems (see, e.g. [5], [6], [7]), and has more recently been extended to multiple state delays and input delay. Lyapunov Krasovskii and Razumikhin techniques are most commonly used to study stability of time delay systems. State and parameter estimation for non-linear delay systems using sliding mode techniques was consider in [8], where Razumikhin technique was used to deal with delays. In [9], Delay-dependent stability control for power systems with multiple time-delays has been considered using Krasovskii theorem, but the rate of change of the time delay is bounded and may not be large enough to accommodate power system delays which may be large and fast varying. Delays in power systems may be classified into various forms as communications delay, reheaters delay (for reheat turbines), crossover delay etc., all of which can degrade the systems performance.

Sliding mode control, known for its strong robustness properties has proved very effective in dealing with non-linear uncertainties and delay effects [10], [11], [12]. However, power systems are highly characterized by multiple non-linearities associated with delays, un-modelled and fast varying parameters, various sliding mode techniques have been considered. Decentralized sliding mode LFC for multi-area system was proposed in [13], but did not consider time delay. In [14], [15], sliding mode LFC with state delay and load disturbances has also been considered. Integration of electric vehicles for load frequency output feedback $H\infty$ control of smart grids was proposed in [16], where both state and control input delays were considered. However, the input region of the LFC power system can be characterized by multiple non-linearities such as governor dead band (GDB), delays, all of which can degrade the performance of the system. In alternative to dealing with multiple input delays which may sometimes lead to increased oscillation and conservativeness of the system eigenvalues, it may be less restrictive to assume that the input region is affected by multiple delayed non-linearities. Thereby, taking considering GDB nonlinearities and delays both emanating from governor backlashes and communication signals respectively.

This work proposes a load frequency SMC for a reheat turbine power system with load disturbance and delayed non-linearities. Assumptions for turbine-end delays and multiple delayed uncertainties in the input region are considered. Since the load disturbance is non-vanishing, a acceptable ultimate bound is calculated based on the Lyapunov Razumikhin theorem, which is used as a main analysis tool to deal with the delay. Under the assumption that all states are accessible, a set of conditions is developed to guarantee that the designed sliding motion is ultimately bounded and can be achieved in finite time. A delay dependent sliding mode control which reduces chattering effects is synthesized such that the power system remains uniformly ultimately bounded in the presence of load disturbance and non-linearities. Compared with previous works, the mapped non-linearity

*This paper is partially supported by the National Natural Science Foundation of China via grants 61573180 and 61473312.

[1]Adrian E. Onyeka & Xing-Gang Yan are with the School of Engineering & Digital Arts, University of Kent, Canterbury CT2 7NT, United Kingdom aeo20@kent.ac.uk, X.Yan@kent.ac.uk

[2]Zehui Mao & Bin Jiang are with the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics Nanjing 210016, P.R. China zehuimao@nuaa.edu.cn, binjiang@nuaa.edu.cn

[3]Dongya Zhao is with the Department of Chemical Equipment and control Engineering, College of Chemical Engineering, China University of Petroleum (East China) dongyazhao@139.com, dyzhao@upc.edu.cn

associated with the valve position is flexible to adapt both input delay effect and possible non-linear growth and there is no limitation to the size of the delay. Robustness is enhanced by fully using the bounds on the uncertainties in this paper. A single area LFC model is simulated to show the feasibility of the developed results and the effectiveness of the proposed method.

II. Dynamic Model of Power System

This section describes the dynamic model of an isolated power system with reheat turbine as characterized by multiple time delays. The objective of LFC is to restore system frequency (f) to the nominal value by implementing adequate supplementary control action which adjusts the load reference set point. Depending on the turbine configuration, reheat turbines experience changes in high pressure (hp) turbine steam flow which are characterized by time delay, due to crossover pipings, re-heaters and turbine valve position adjustment [2], [17]. Therefore, it is suffice to say that multiple delays may abound as turbines grow larger. However, power system is highly nonlinear, and thus, more uncertainties arise due to parameter variations and un-modeled dynamics etc. In order to fully represent this outcomes, the modified dynamics of the reheat turbine system is given as follows (see, e.g. [16])

Governor model dynamics:

$$\begin{aligned}\Delta\dot{P}_v(t) &= -\frac{1}{RT_g}\Delta f - \frac{1}{T_g}\Delta P_v - \frac{1}{T_g}\Delta E(t) - \frac{1}{T_g}E(t-d_3) \\ &\quad + \frac{1}{T_g}u(t) \end{aligned} \tag{1}$$

where $\Delta P_v, \Delta f, \Delta E, R, T_g, u$ represent governor valve position, frequency deviation, integral control, governor speed regulation, governor time constant, supplementary control respectively. The control valves are naturally discontinuous relays. Due to adjustments in control valves position, delay is introduced into the system. In addition, speed governors are susceptible to time delay of about 3 to 5 secs and frequency control has been shown to decline due to increase in time delay [2], [17], [4]. Therefore, it is necessary to take into account these effects which are reflected on the time delay d_3 in the term $\frac{1}{T_g}E(t-d_3)$.

Turbine model:

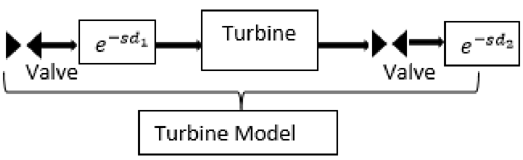

Fig. 1. Block of a reheat turbine unit

$$\Delta\dot{P}_r(t) = -\frac{1}{T_t}\Delta P_r + \frac{1}{T_t}\Delta P_v - \frac{1}{T_t}\Delta P_r(t-d_1) \tag{2}$$

$$\begin{aligned}\Delta\dot{P}_m(t) &= -\frac{1}{T_t}\Delta P_m + \frac{T_t - K_r}{T_t T_r}\Delta P_r + \frac{K_r}{T_t T_r}\Delta P_v \\ &\quad + \frac{K_p}{T_p}\Delta P_m(t-d_2) \end{aligned} \tag{3}$$

where $\Delta P_m, \Delta P_r, T_t, T_p, T_r, K_p, K_r$ represents the generator mechanical power deviation, intermediate power output deviation, turbine time constant, plant model time constant and reheat time constant, plant gain, re-heater gain respectively. The turbine model is characterized by multiple delays depending on its capacity. However, this delays are not represented in most studies. Delays can be introduced via various stop valves within the steam chest and inlet piping, re-heaters and crossover pipings [17], [3], [16]. In this paper, these are represented in Fig. 1 as turbine-end delays for the purpose of stability studies.

Frequency & Integral control dynamics:

$$\Delta\dot{f}(t) = -\frac{1}{T_p}\Delta f + \frac{K_p}{T_p}\Delta P_m - \frac{K_p}{T_p}\Delta P_L(t) \tag{4}$$

$$\Delta\dot{E}(t) = \beta\Delta f \tag{5}$$

where $\Delta P_L(t) = \Delta(t)$ is the time varying load deviation. β is the frequency bias factor.

Within the overall aim of designing a load frequency sliding mode control for the closed-loop system, the main objectives of this paper are

- To represent and analyze turbine delays within the affected LFC system for stability purpose.
- To determine an ultimate bound for the non-frequency sensitive load disturbance as well as create sufficient allowable bound for the multiple delayed nonlinearities.
- Design a load frequency sliding mode control to improve close-loop system performance and reduce the effect of chattering induced by relays and discontinuous control.

For convenience, the LFC dynamics system is described in the following form

$$\begin{aligned}\dot{x}(t) &= \bar{A}x(t) + \bar{E}_1 x(t-d_1) + \bar{E}_2 x(t-d_2) + \bar{B}u(t) \\ &\quad + \bar{H}_u(t, x, x(t-d_1), x(t-d_2), x(t-d_3)) \\ &\quad + \bar{H}\Delta(t) \end{aligned} \tag{6}$$

where $x(t) = [\Delta f \ \Delta P_m \ \Delta P_r \ \Delta P_v \ \Delta E]^T \in \Omega \subset \Re^5$, is the state vector, $u \in \Re$ is the control input at the governor terminal. The matrices $\bar{A}$, $\bar{E}_1$, $\bar{E}_2$, $\in \Re^{5\times 5}$, $\bar{B} \in \Re^{5\times 1}$ and $\bar{H} \in \Re^5$ in (6) are constant matrices of appropriate dimensions given by

$$\underbrace{\begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_t} & \frac{T_t - K_r}{T_t T_r} & \frac{K_r}{T_t T_r} & 0 \\ 0 & 0 & -\frac{1}{T_t} & \frac{1}{T_t} & 0 \\ -\frac{1}{RT_g} & 0 & 0 & -\frac{1}{T_g} & -\frac{1}{T_g} \\ \beta & 0 & 0 & 0 & 0 \end{bmatrix}}_{\bar{A}}, \underbrace{\begin{bmatrix} 0_{2\times 1} \\ \frac{1}{T_g} \\ 0_{2\times 1} \end{bmatrix}}_{\bar{B}} \tag{7}$$

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{M} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{T_t} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T_t} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\bar{E}_1,\ \bar{E}_2}, \underbrace{\begin{bmatrix} -\frac{1}{M} \\ 0_{4\times 1} \end{bmatrix}}_{\bar{H}} \quad (8)$$

with $\bar{B}$ being of full rank; $D = \frac{1}{K_p}$ is the damping coefficient, $M = \frac{T_p}{K_p}$ is the equivalent initial. The uncertain function $\bar{H}_u : \Re^5$ represents the multiple delayed nonlinearity affecting the input region. The time varying delay $d := d(t)$ are time delays for $i = 1,2,3$ which are assumed to be known, non-negative and bounded in $\Re^+ := \{t \mid t \geq 0\}$, satisfy

$$\bar{d} = \max\left\{ \sup_{t\in\Re^+} \{d_1(t)\}, \sup_{t\in\Re^+} \{d_2(t)\}, \sup_{t\in\Re^+} \{d_3(t)\} \right\} < \infty$$

The initial condition related to the delay is given by

$$x(t) = \phi(t), \quad t \in [-\bar{d}, 0] \quad (9)$$

where $\phi(\cdot)$ is continuous in $[-\bar{d}, 0]$. It is assumed that all the nonlinear functions are smooth enough such that the unforced system has a unique continuous solution.

Remark 1: Although this paper mainly explores the physical limitations such as delays and nonlinearities associated with a reheat turbine system, it should be emphasized that other physical constraints such as generation rate constraint (GRC) and governor deadband (GDB) nonlinearities exist in practice. Results of the impact of these nonlinearities are reported in [1], [18], [19]. In this paper, these nonlinearities are generalized in the lumped unstructured perturbation term $\bar{H}_u(\cdot)$ in (6).

III. Basic Assumptions and Lemma

In order to facilitate subsequent analysis, the following lemma is required.

Lemma 1: (see [20]). For the system,

$$\dot{x} = f(t,x) \quad (10)$$

Let $D \subset \Re^n$ be a domain that contains the origin and $V : [0,\infty) \times D \to R$ be a continuously differentiable function such that

$$\gamma_1(\|x\|) \leq V(t,x) \leq \gamma_2(\|x\|) \quad (11)$$
$$\frac{\delta V}{\delta t} + \frac{\delta V}{\delta x} f(t,x) \leq -W_3(x),\ \forall\ \|x\| \geq \varpi > 0 \quad (12)$$

$\forall\ t \geq 0$ and $\forall\ x \in D = \{x \in \Re^n : \|x\| < r\}$, where γ_1 and γ_2 are class $\mathscr{K}$ functions and $W_3(x)$ is a continuous positive definite function. Take $r > 0$ such that $Br \subset D$ and suppose that

$$\varpi < \gamma_2^{-1}(\gamma_1(r)) \quad (13)$$

Then, there exist a class $\mathscr{KL}$ function β and for every initial state $x(t_0)$, satisfying $\|x(t)\| \leq \gamma_2^{-1}(\gamma_1(r))$, there is $T \geq 0$ (dependent on $x(t_0)$ and ϖ) such that the solution of (10) satisfies

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0),\ \forall\ t_0 \leq t \leq t_0 + T \quad (14)$$
$$\|x(t)\| \leq \varpi < \gamma_2^{-1}(\gamma_1(r))\ \forall\ t \leq t_0 + T \quad (15)$$

Moreover, if $D = \Re^n$ and γ_1 belongs to class $\mathscr{K}_\infty$, then (14) and (15) holds for any initial state $x(t_0)$, with no restriction on how large ϖ is.

Assumption 1: The matrix pair $(\bar{A}, \bar{B})$ is controllable.

Thus, from [21] there exists a coordinate transformation $\hat{x} = \hat{T}x$ such that the corresponding matrices in system (6) in the new coordinate, can be described by

$$\hat{A} = \begin{bmatrix} \hat{A}_1 & \hat{A}_2 \\ \hat{A}_3 & \hat{A}_4 \end{bmatrix}, \hat{E}_1 = \begin{bmatrix} \hat{E}_{11} & \hat{E}_{12} \\ \hat{E}_{13} & \hat{E}_{14} \end{bmatrix} \quad (16)$$
$$\hat{B} = \begin{bmatrix} 0 \\ \hat{B}_2 \end{bmatrix}, \hat{E}_2 = \begin{bmatrix} \hat{E}_{21} & \hat{E}_{22} \\ \hat{E}_{23} & \hat{E}_{24} \end{bmatrix}, \hat{H} = \begin{bmatrix} \hat{H}_1 \\ \hat{H}_2 \end{bmatrix} \quad (17)$$
$$\begin{bmatrix} \hat{H}_{u1}(t,\hat{x},\hat{x}_d) \\ \hat{H}_{u2}(t,\hat{x},\hat{x}_d) \end{bmatrix} := \hat{T}\bar{H}_u(t,\hat{T}^{-1}x,\hat{T}^{-1}x_d) \quad (18)$$

where $\hat{A}_1 \in \Re^{4\times 4}$, and $\hat{B}_2 \in \Re$ is non-singular, $x_d := \mathrm{col}(x(t - d_1), x(t - d_2), x(t - d_3))$. It should be noted that such a transformation can be obtained systematically using matrix theory. Further from [21], since $(\bar{A}, \bar{B})$ is controllable implies that $(\hat{A}_1, \hat{A}_2)$ is controllable, and thus there exists a matrix $M \in \Re^{1\times 4}$ such that the matrix $\hat{A}_1 - \hat{A}_2 M$ is Hurwitz stable. Further, consider the transformation matrix $z = \tilde{T}\hat{x}$, with $\tilde{T}$ defined by

$$\tilde{T} = \begin{bmatrix} I_4 & 0 \\ M & I \end{bmatrix} \quad (19)$$

It follows from the analysis above that in the new coordinate $z = \tilde{T}\hat{x}$, the matrix triple $(\bar{A}, \bar{E}, \bar{B})$ in system (6) has the following form

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, E_1 = \begin{bmatrix} E_{11} & E_{12} \\ E_{13} & E_{14} \end{bmatrix} \quad (20)$$
$$B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, E_2 = \begin{bmatrix} E_{21} & E_{22} \\ E_{23} & E_{24} \end{bmatrix}, H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \quad (21)$$
$$\begin{bmatrix} \Theta_1(t,z,z_d) \\ \Theta_2(t,z,z_d) \end{bmatrix} := \tilde{T}\hat{H}_u(t,\tilde{T}^{-1}\hat{x},\tilde{T}^{-1}\hat{x}_d) \quad (22)$$

where $A_1 = \hat{A}_1 - \hat{A}_2 M$ is Hurwitz stable and $B_2 = \tilde{T}\hat{B}_2$ is a positive scalar.

IV. Sliding mode control analysis and design

Section III shows that there exists new coordinates $z = \tilde{T}\hat{x}$ such that in the new coordinate $z = \mathrm{col}(z_1, z_2)$, the system (6) can be described in (20)-(22). In this section, a sliding surface will be designed and stability of the corresponding sliding motion will be analysed. The matrix structure (20)-

(22) can be rewritten by

$$\begin{aligned}\dot{z}_1 &= A_1z_1 + A_2z_2 + E_{11}z_1(t-d_1) + E_{12}z_2(t-d_1)\\ &\quad + E_{21}z_1(t-d_2) + E_{22}z_2(t-d_2) + H_1\Delta(t)\\ &\quad + \Theta_1(t,z,z_d) \qquad (23)\\ \dot{z}_2 &= A_3z_1 + A_4z_2 + E_{13}z_1(t-d_1) + E_{14}z_2(t-d_1)\\ &\quad + E_{23}z_1(t-d_2) + E_{24}z_2(t-d_2) + B_2u + H_2\Delta(t)\\ &\quad + \Theta_2(t,z,z_d) \qquad (24)\end{aligned}$$

where $z := \mathrm{col}(z_1, z_2)$ with $z_1 := \mathrm{col}(z_{11}, z_{12}, z_{13}, z_{14})$ and $z_2 \in \Re$, A_1 is Hurwitz stable, and B_2 is a positive scalar.

Assumption 2: There exist a known continuous non-negative bounds such that

$$\begin{aligned}\|\Delta(t)\| &\leq \delta \qquad (25)\\ \|\Theta_1(t,z(t),z_d)\| &\leq \kappa\rho_1(t,z(t))\|z_d\| \qquad (26)\\ \|\Theta_2(t,z(t),z_d)\| &\leq \rho_2(t,z(t),z_d) \qquad (27)\end{aligned}$$

where $\delta > 0, \kappa > 0$ are positive constants, and the functions $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are nondecreasing.

A. Stability of sliding motion

Based on the above assumption, the main aim of this section is to design a sliding surface for the system (23)-(24) and then study the stability of the corresponding sliding motion in the presence of disturbances and delay. From the structure of system (23) and (24), define a switching function of the form

$$\sigma(z) = z_2 \qquad (28)$$

Therefore, the sliding surface for the isolated system (23)-(24) is described as

$$\sigma(z) = z_2 = 0 \qquad (29)$$

Since A_1 in (23) is stable, for any $Q > 0$, the following Lyapunov equation has a unique solution $P > 0$

$$A_1^TP + PA_1 = -Q \qquad (30)$$

From the structure of system (23)-(24), the sliding motion of system (6) associated with the sliding surface (29) is dominated by system (23). When dynamic system (23) is limited to the sliding surface (29), $z_2 = 0$. Thus, the reduced order dynamics can be described as

$$\begin{aligned}\dot{z}_1 &= A_1z_1 + E_{11}z_1(t-d_1) + E_{21}z_1(t-d_2)\\ &\quad + H_1\Delta(t) + \bar{\Theta}_1(t,z_1,z_{1d}) \qquad (31)\end{aligned}$$

where $\bar{\Theta}_1(t,z_1(t),z_{1d}) := \Theta_1(t,z(t),z_d)|_{z_2=0}$.

The following theorem is ready to be presented.

Theorem 1: Under Assumptions 1, the sliding motion of system (23)-(24) associated with the sliding surface (29), if governed by (31) is uniformly ultimately bounded with the ultimate bound

$$\bar{\delta} = \sqrt{\frac{\lambda_m(P)}{\lambda_M(P)}}\varpi \qquad (32)$$

where λ_m and λ_M are the minimum and maximum eigenvalue of matrix P if

$$\begin{aligned}k &= \Big(\lambda_m(Q) - \alpha_1^{-1}\lambda_M(PE_{11}P^{-1}E_{11}^TP)\\ &\quad - \alpha_2^{-1}\lambda_M(PE_{21}P^{-1}E_{21}^TP) - q\bar{\alpha}\lambda_M(P)(\alpha_1+\alpha_2)\\ &\quad - \kappa\rho_1(t,z_1)\lambda_M(P)(q\bar{\alpha}\varepsilon^{-1}+\varepsilon)\Big) > 0 \qquad (33)\end{aligned}$$

where $\delta, \kappa, \rho_1(\cdot)$ are given in assumption 2 and

$$\delta < \frac{k\Xi}{2\lambda_M(P)\|H_1\|}\sqrt{\frac{\lambda_m(P)}{\lambda_M(P)}}r \qquad (34)$$

where $r > 0$, $0 < \Xi < 1$ is a positive constant

Proof. For system (31), consider a Lyapunov function candidate

$$V(z_1) = z_1(t)^TPz_1(t) \qquad (35)$$

It follows from (30) that the time derivative of V along the trajectories of system (31) is given as

$$\begin{aligned}\dot{V}(z_1(t)) &= -z_1^T(t)Qz_1(t) + 2z_1^T(t)PE_{11}z_1(t-d_1)\\ &\quad + 2z_1^T(t)PE_{21}z_1(t-d_2) + 2z_1^T(t)PH_1\Delta(t)\\ &\quad + 2z_1^T(t)P\bar{\Theta}_1(t,z_1,z_{1d}) \qquad (36)\end{aligned}$$

it follows from inequality $2a^Tb \leq a^TN^{-1}a + b^TNb$, for any $a, b \in \Re^n$ that

$$\begin{aligned}\dot{V}(z_1(t)) &= -z_1^T(t)Qz_1(t) + z_1^T(t)PE_{11}N_1^{-1}E_{11}^TPz_1(t)\\ &\quad + z_1^T(t)PE_{21}N_2^{-1}E_{21}^TPz_1(t) + z_1^T(t-d_1)\\ &\quad \times N_1z_1(t-d_1) + z_1^T(t-d_2)N_2z_1(t-d_2)\\ &\quad + 2z_1^T(t)P\bar{\Theta}_1(t,z_1,z_{1d}) + 2z_1^T(t)PH_1\Delta(t)\\ &\leq -\lambda_m(Q)\|z_1(t)\|^2 + \alpha_1^{-1}\lambda_M(PE_{11}P^{-1}E_{11}^TP)\|z_1(t)\|^2\\ &\quad + \alpha_2^{-1}\lambda_M(PE_{21}P^{-1}E_{21}^TP)\|z_1(t)\|^2 + \alpha_1\lambda_M(P)\\ &\quad \times\|z_1(t-d_1)\|^2 + \alpha_2\lambda_M(P)\|z_1(t-d_2)\|^2 + 2\kappa\rho_1(t,z_1)\\ &\quad \times\lambda_M(P)\|z_{1d}\|\|z_1(t)\| + 2\delta\lambda_M(P)\|H_1\|\|z_1(t)\| \qquad (37)\end{aligned}$$

where $N_1 = \alpha_1P, N_2 = \alpha_2P$. $\alpha_1, \alpha_2, \kappa > 0$ and P is a positive definite matrix. Applying the inequality: $2ab \leq \varepsilon^{-1}a^2 + \varepsilon b^2$, it follows from (37) that

$$\begin{aligned}&2\kappa\rho_1(t,z_1)\lambda_M(P)\|z_{1d}\|\|z_1(t)\| \leq \varepsilon^{-1}\kappa\rho_1(t,z_1)\\ &\cdot\lambda_M(P)\|z_{1d}\|^2 + \varepsilon\kappa\rho_1(t,z_1)\lambda_M(P)\|z_1(t)\|^2 \qquad (38)\end{aligned}$$

where $\varepsilon > 0$ is arbitrary constant. Following Razumikhin theorem [6], assume that there exist constant $q > 1$ and $\bar{\alpha} > 0$, such that

$$\|z_{id}\|^2 \leq q\bar{\alpha}\|z_1(t)\|^2, \qquad (39)$$

Therefore,

$$\begin{aligned}\dot{V}(z_1(t)) &\leq -\Big(\lambda_m(Q) - \alpha_1^{-1}\lambda_M(PE_{11}P^{-1}E_{11}^TP)\\ &\quad - \alpha_2^{-1}\lambda_M(PE_{21}P^{-1}E_{21}^TP) - q\bar{\alpha}\lambda_M(P)(\alpha_1+\alpha_2)\\ &\quad - \kappa\rho_1(t,z_1)\lambda_M(P)(q\bar{\alpha}\varepsilon^{-1}+\varepsilon)\Big)\|z_1(t)\|^2\\ &\quad + 2\delta\lambda_M(P)\|H_1\|\|z_1(t)\|\\ &= -(1-\Xi)k\|z_1(t)\|^2\\ &\qquad - (k\Xi\|z_1\| - 2\delta\lambda_M(P)\|H_1\|)\|z_1(t)\|\\ &\qquad \forall \quad \|z_1(t)\| \geq \frac{2\delta\lambda_M(P)\|H_1\|}{k\Xi} = \varpi \qquad (40)\end{aligned}$$

where Ξ, k are defined in (34) and (33) respectively. Therefore, it follows from Lemma 1 that defining γ_1, γ_2: $[0, \infty) \to \Re^+$, by

$$\gamma_1(\|z_1\|) = \lambda_m(P)\|z_1\|^2, \quad \gamma_2(\|z_1\|) = \lambda_M(P)\|z_1\|^2$$
$$\|z_1(t)\| \leq \bar{\delta} \tag{41}$$

where $\bar{\delta}$ is defined in (32). Thus, the designed time delay system is globally uniformly ultimately bounded. Hence the result follows. ■

B. Sliding mode Control design

The objective now is to design a state feedback sliding mode control law such that the system state is driven to the sliding surface (29) in finite time. The following control is proposed:

$$u(t) \;=\; -B_2^{-1}\Big(\Gamma(z) + (\|H_2\|\delta + \rho_2(\cdot) + \eta)\mathrm{sgn}(\sigma(z))\Big) - \mu_l \sigma \tag{42}$$

where

$$\begin{aligned}\Gamma(z) \;=\;& A_3 z_1 + A_4 z_2 + E_{13} z_1(t-d_1) + E_{14} z_2(t-d_1)\\ &+ E_{23} z_1(t-d_2) + E_{24} z_2(t-d_2)\end{aligned} \tag{43}$$

where δ and $\rho_2(\cdot)$ are defined in (25), (27) respectively, μ_l is a positive design scaler and $\eta > 0$ is the reachability constant. The following result is ready to be presented.

Theorem 2: Consider the system (23)-(24). The control (42) drives the system (23)-(24) to the sliding surface (29) in finite time and maintains a sliding motion on it thereafter.
Proof:

From (28) and (24), it can be verified that

$$\begin{aligned}\dot{\sigma}(z) =\;& A_3 z_1 + A_4 z_2 + E_{13} z_1(t-d_1) + E_{14} z_2(t-d_1)\\ &+ E_{23} z_1(t-d_2) + E_{24} z_2(t-d_2) + Bu(t)\\ &+ \bar{\Theta}_2(t, z, z_d))\end{aligned} \tag{44}$$

Applying the control u in (42) to (44), it follows from (27) that,

$$\begin{aligned}\sigma^\tau \dot{\sigma} =\;& \sigma^\tau(z)\Big(A_3 z_1 + A_4 z_2 + E_{13} z_1(t-d_1) + E_{14} z_2(t-d_1)\\ &+ E_{23} z_1(t-d_2) + E_{24} z_2(t-d_2)\Big) - \sigma^\tau(z)\Bigg(\Gamma(z)\\ &+ \Big(\|H_2\|\delta + \rho_2(t, z(t), z_d) + \eta\Big)\mathrm{sgn}(\sigma(z)) - B_2\mu_l\sigma\Bigg)\\ &+ \sigma^\tau(z) H_2 \Delta(t) + \sigma^\tau(z)\bar{\Theta}_2(t, z(t), z_d)\\ =\;& \sigma^\tau(z)\bar{\Theta}_2(t, z(t), z_d) - \rho_2(t, z(t), z_d)\sigma^\tau(z)\mathrm{sgn}(\sigma(z))\\ & \sigma^\tau(z) H_2\Delta(t) - \delta\|H_2\|\sigma^\tau(z)\mathrm{sgn}(\sigma(z)) - \sigma^2 B_2 \mu_l\\ & -\sigma^\tau(z)\eta\,\mathrm{sgn}(\sigma(z))\\ \leq\;& \|\sigma(z)\|\|\bar{\Theta}_2(t, z(t), z_d)\| - \|\sigma(z)\|\rho_2(t, z(t), z_d)\\ & \|\sigma(z)\|\|H_2\|\|\Delta(t)\| - \|\sigma(z)\|\|H_2\|\delta - \mu_l\|B_2\|\sigma^2\\ & -\eta\|\sigma\|\\ \leq\;& -\mu_l\|B_2\|\sigma^2 - \eta\|\sigma(z)\|\end{aligned} \tag{45}$$

From [21], since $\mu_l\|B_2\|\sigma^2 > 0$, the η reachability condition holds and hence the conclusion follows. Theorems 1 and 2 together show that the corresponding closed-loop system is uniformly asymptotically stable. ■

Remark 2: It should be noted that by increasing the term η appearing in the control (42), chattering may be induced [8]. In order to revert this effect, μ_l in (42) can be chosen arbitrarily in order to increase the rate at which sliding is attained. Therefore, creating the possibility for η to be chosen small enough to reduce the switching amplitude.

V. Simulation Example

For simulation purposes, the following parameters $M = 10, T_t = 0.3s, T_g = 0.2s, D = 1.0, T_r = 7.0s, R = 0.05, K_r = 5$ as obtained from [2], [16] are used to obtain the simulation results. Consider the reheat steam turbine generating unit in

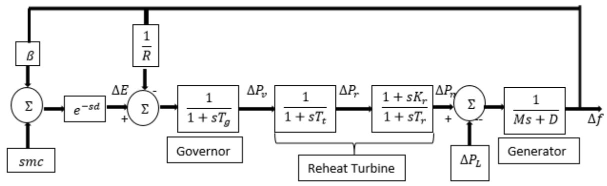

Fig. 2. Single reheat generating unit

Fig. 2, feeding an isolated load. By applying the algorithm in [21], the isolated LFC power system (20)-(22) in the new coordinates $z = \mathrm{col}(z_1, z_2)$ can be described by

$$\underbrace{\left[\begin{array}{cccc|c} -0.761 & 132.147 & 5.265 & -112.941 & -0.952\\ 0.10 & -0.10 & 0 & 0 & 0\\ -13.507 & -593.773 & -15.139 & 592.941 & 5\\ 0 & -0.425 & 0 & 0 & 0\\ \hline 38.10 & 186.70 & 38.40 & -186.6 & -15.9 \end{array}\right]}_{A}$$

$$\underbrace{\left[\begin{array}{cccc|c} 0.1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 5 & 0\\ 0 & 0 & 0 & 0 & 0\\ \hline -9.27 & -462.51 & -10.09 & 380.15 & 3.33 \end{array}\right]}_{E_1, E_2}$$

$$\underbrace{\begin{bmatrix} 0_{4\times 1}\\ -5 \end{bmatrix}}_{B}, \quad \underbrace{\left[\begin{array}{c} 0\\ 0.1\\ 0_{2\times 1}\\ \hline -13.87 \end{array}\right]}_{H}$$

The load deviation $\Delta(t) = 0.01\sin(2t)$ and the nonlinear delayed perturbation

$$\Theta(\cdot) = \begin{bmatrix} 0.2|z_{11}(t)|^{1/2} + \sin(z_{14}(t))\\ 0_{2\times 1}\\ 0.3\sin(|z_{11}(t-d_1) z_{14}(t-d_3)|)\\ 0.1\cos(|z_2(t) z_{14}(t-d_3))\end{bmatrix} \tag{46}$$

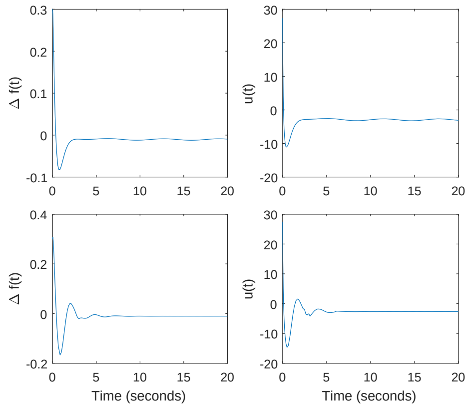

Fig. 3. Time response of $\Delta f(t)$ and control $u(t)$ respectively with nonlinearity $\bar{\Theta}(\cdot)$.
Upper: $\bar{\Theta}(\cdot)=0$
Bottom: $\bar{\Theta}(\cdot)$ defined in (46).

satisfy the conditions

$$\|\bar{\Theta}_1(t,z_1,z_{1d})\| \quad \le \quad \underbrace{2.2\sin^2(z_1)+0.5\cos^2\|z(t-d)\|}_{\rho_1(\cdot)}$$

choosing $\delta=0.03$. Therefore, on the sliding surface (29), when the sliding motion takes place. Since A_1 in (20) is stable, it follows that choosing $Q=I_4$, P is calculated based on the Lyapunov equation in (30) as

$$P=\begin{bmatrix} 0 & -5 & 0 & 1.177 \\ 0 & 0 & 0 & 1.177 \\ -0.210 & -7 & -0.040 & -7.765 \\ -0.005 & -0.293 & -0.002 & 1.007 \end{bmatrix}$$

By direct calculation, choosing $\alpha_1=\alpha_2=20, \varepsilon=0.1$ and $\bar{\alpha}=0.05$, it follows from Theorem 1 that for $\kappa<1$, $\rho_1(\cdot)\le 3.6\sin^2\|z_1\|$. Hence, k defined in (33) is positive definite. Furthermore, calculating $\lambda_m=0.05, \lambda_M=0.5$ and choosing $r=0.81, \Xi=0.6$. The ultimate bound $\bar{\delta}=0.05$. It is easy to verify that the conditions in Theorem 1 are satisfied. Thus, the sliding motion associated with the sliding surface is uniformly ultimately bounded. From Theorem 2, the supplementary control law $u(t)$ in (42) can be chosen to ensure that the system is driven to the surface in finite time. Simulation results as shown in Fig. 3, shows the satisfactory performance responses of the frequency deviation Δf and control under load change $\Delta(t)$ and nonlinearities $\bar{\Theta}(\cdot)$. However, it may be necessary to show the stability responses of the entire states. Due to space limitation, only Δf has been shown in this work which buttresses the main objective of LFC.

VI. Conclusion

An improved load frequency sliding mode control with reduced chattering effect has been proposed for an isolated reheat turbine power systems with multiple delayed nonlinearities. By employing the delay and exploiting the bounds on the uncertainties in both the sliding motion analysis and the control design, and based on Lyapunov Razumikhin technique, conservatism is reduced and the robustness is enhanced by achieving a suitable ultimate bound for the non-frequency sensitive load disturbance. The simulation results on a single area power systems has demonstrated the effectiveness of the obtained results and further illustrates the feasibility of the proposed methodology.

References

[1] W. Tan, S. Chang, and R. Zhou, "Load frequency control of power systems with non-linearities," *IET Generation, Transmission & Distribution*, vol. 11, no. 17, pp. 4307–4313, 2017.

[2] P. Kundur, N. J. Balu, and M. G. Lauby, *Power system stability and control*. McGraw-hill New York, 1994, vol. 7.

[3] B. Vahidi, M. R. B. Tavakoli, and W. Gawlik, "Determining parameters of turbine's model using heat balance data of steam power unit for educational purposes," *IEEE transactions on power systems*, vol. 22, no. 4, pp. 1547–1553, 2007.

[4] H. Bevrani, *Robust power system frequency control*. Springer, 2009, vol. 85.

[5] E. Fridman, "Tutorial on Lyapunov-based methods for time-delay systems," *European Journal of Control*, vol. 20, no. 6, pp. 271–283, 2014.

[6] K. Gu, J. Chen, and V. L. Kharitonov, *Stability of time-delay systems*. Springer Science & Business Media, 2003.

[7] A. E. Onyeka, X.-G. Yan, and J. Mu, "Sliding mode control of time-delay systems with delayed nonlinear uncertainties," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 2696–2701, 2017.

[8] X.-G. Yan, S. K. Spurgeon, and C. Edwards, "State and parameter estimation for nonlinear delay systems using sliding mode techniques," *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 1023–1029, 2013.

[9] J. Li, Z. Chen, D. Cai, W. Zhen, and Q. Huang, "Delay-dependent stability control for power system with multiple time-delays," *IEEE Transactions on Power Systems*, vol. 31, no. 3, pp. 2316–2326, 2016.

[10] A. E. Onyeka, X.-G. Yan, Z. Mao, and Q. Zhang, "Decentralized sliding mode Lfc for nonlinear interconnected power system with time delay," in *2018 Annual American Control Conference (ACC)*. IEEE, 2018, pp. 6666–6671.

[11] X.-G. Yan, S. K. Spurgeon, and C. Edwards, "Global decentralised static output feedback slidingmode control for interconnected time-delay systems," *IET control theory & applications*, vol. 6, no. 2, pp. 192–202, 2012.

[12] J. Zhang, M. Lyu, T. Shen, L. Liu, and Y. Bo, "Sliding mode control for a class of nonlinear multi-agent system with time delay and uncertainties," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 1, pp. 865–875, 2018.

[13] Y. Mi, Y. Fu, C. Wang, and P. Wang, "Decentralized sliding mode load frequency control for multi-area power systems," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4301–4309, 2013.

[14] S. Prasad, S. Purwar, and N. Kishor, "H-infinity based non-linear sliding mode controller for frequency regulation in interconnected power systems with constant and time-varying delays," *IET Generation, Transmission & Distribution*, vol. 10, no. 11, pp. 2771–2784, 2016.

[15] P. Ojaghi and M. Rahmani, "Lmi-based robust predictive load frequency control for power systems with communication delays," *IEEE Transactions on Power Systems*, 2017.

[16] T. N. Pham, H. Trinh, K. P. Wong *et al.*, "Integration of electric vehicles for load frequency output feedback h control of smart grids," *IET Generation, Transmission & Distribution*, vol. 10, no. 13, pp. 3341–3352, 2016.

[17] M. Eremia and M. Shahidehpour, *Handbook of electrical power system dynamics: modeling, stability, and control*. John Wiley & Sons, 2013, vol. 92.

[18] T.-S. Tsay, "Load–frequency control of interconnected power system with governor backlash nonlinearities," *International Journal of Electrical Power & Energy Systems*, vol. 33, no. 9, pp. 1542–1549, 2011.

[19] M. Kothari, P. Satsangi, and J. Nanda, "Sampled-data automatic generation control of interconnected reheat thermal systems considering generation rate constraints," *IEEE transactions on power apparatus and systems*, no. 5, pp. 2334–2342, 1981.

[20] H. K. Khalil, "Noninear systems," *Prentice-Hall, New Jersey*, vol. 2, no. 5, pp. 5–1, 1996.

[21] C. Edwards and S. Spurgeon, *Sliding mode control: theory and applications*. CRC Press, 1998.