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Document Version

Presentation
Insurance Risk Pooling, Loss Coverage and Social Welfare

When is adverse selection not adverse?

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Background

Adverse selection:

If insurers cannot charge risk-differentiated premiums, then:

- higher risks buy more insurance, lower risks buy less insurance,
- raising the pooled price of insurance,
- lowering the demand for insurance,

usually portrayed as a bad outcome, both for insurers and for society.

In practice:

Policymakers often see merit in restricting insurance risk classification

- EU ban on using gender in insurance underwriting.
- Moratoria on the use of genetic test results in underwriting.

Question:

How can we reconcile theory with practice?
Motivation: Two risk-groups $\mu_L = 0.01$ and $\mu_H = 0.04$

Scenario 1: No adverse selection: Risk-differentiated premiums: $\pi_L = 0.01$ and $\pi_H = 0.04$

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Loss coverage: 66.7%

Scenario 2: Some adverse selection: Pooled premiums: $\pi_L = \pi_H = 0.028$

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Loss coverage: 77.8%
Introduction

Agenda

We ask:

- **Why** do people buy insurance?
- **What** drives demand for insurance?
- **How much** of population losses is compensated by insurance?
- **Which** regime is most beneficial to society?

Definition (Loss coverage)

*Expected population losses compensated by insurance.*
Contents

- Introduction
- Why do people buy insurance?
- What drives demand for insurance?
- How much of population losses is compensated by insurance?
- Which regime is most beneficial to society?
- Conclusions
Why do people buy insurance?

Assumptions

Consider an individual with

- an initial wealth $W$,
- exposed to the risk of loss $L$,
- with probability $\mu$,
- utility of wealth $U(w)$, with $U'(w) > 0$ and $U'''(w) < 0$,
- an opportunity to insure at premium rate $\pi$. 
Utility of wealth

Utility of wealth as a function of wealth. The utility function $U(W)$ increases with wealth $W$, indicating a positive relationship between wealth and utility. The utility function $U(W-L)$ also increases with $W-L$, reflecting the utility of wealth minus a loss $L$. This suggests that individuals value their wealth more when they have a higher wealth level, even after accounting for potential losses.
Why do people buy insurance?

Expected utility: Without insurance

Wealth
Utility

Expected utility without insurance

$$(1 - \mu)U(W) + \mu U(W - L)$$
Expected utility: Insured at fair actuarial premium

Utility

Expected utility with insurance $U(W - \mu L)$

Fair premium $\mu L$

Wealth

$U(W)$

$U(W - L)$

$W - L$

$W - \mu L$

$W$
Maximum premium tolerated: $\pi_c$

Utility

$U(W)$
$U(W - \mu L)$
$U(W - \pi_c L)$
$U(W - L)$

Wealth

$W - L$
$W - \pi_c L$
$W - \mu L$
$W$

$(1 - \mu)U(W) + \mu U(W - L)$

$\pi_c L$

Maximum premium tolerated

$\mu L$

Fair premium

$\pi_c$

Maximum premium tolerated

Why do people buy insurance? Maximum premium tolerated
Contents

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Modelling demand for insurance

Simplest model:

If everybody has exactly the same $W, L, \mu$ and $U(\cdot)$, then:

- All will buy insurance if $\pi < \pi_c$.
- None will buy insurance if $\pi > \pi_c$.

**Reality:** Not all will buy insurance even at fair premium. Why?

Heterogeneity:

- Even if individuals are **homogeneous** in terms of underlying risk,
- they can still be **heterogeneous** in terms of **risk-aversion**.

Source of Randomness:

An individual’s utility function: $U_\gamma(w)$, where parameter $\gamma$ is drawn from random variable $\Gamma$ with distribution function $F_\Gamma(\gamma)$. 
Insurance demand

Standardisation

As certainty equivalent is invariant to positive affine transformations, we assume $U_\gamma(W) = 1$ and $U_\gamma(W - L) = 0$ for all $\gamma$.

Condition for buying insurance:

Given a premium $\pi$, an individual will buy insurance if:

$$U_\gamma(W - \pi L) > (1 - \mu) U_\gamma(W) + \mu U_\gamma(W - L) = (1 - \mu).$$

Demand as a function of premium:

Given a premium $\pi$, insurance demand, $d(\pi)$, is:

$$d(\pi) = P[U_\Gamma(W - \pi L) > 1 - \mu].$$
Insurance demand and heterogeneity in risk-aversion

Utility

\[ U(W) = (1 - \mu)U(W) + \mu U(W - L) \]

\[ U(W - L) \]

\[ W - L \]

\[ W \]

\[ W - \pi L \]

Density

\[ d(\pi) \]
Iso-elastic demand

Constant demand elasticity

If demand for insurance can be modelled as\(^1\):

\[
d(\pi) = \tau \left( \frac{\mu}{\pi} \right)^\lambda,
\]

then elasticity of demand is a constant:

\[
\varepsilon(\pi) = \left| \frac{\partial d(\pi)}{\partial \pi} \right| = \lambda.
\]

\(^1\) Assumptions: \(W = L = 1\), \(U_\gamma(w) = w^\gamma\) and \(\Gamma\) has the following distribution function:

\[
F_{\Gamma}(\gamma) = P[\Gamma \leq \gamma] = \begin{cases} 
0 & \text{if } \gamma < 0 \\
\tau \gamma^\lambda & \text{if } 0 \leq \gamma \leq (1/\tau)^{1/\lambda} \\
1 & \text{if } \gamma > (1/\tau)^{1/\lambda}.
\end{cases}
\]
Iso-elastic demand

Iso-elastic demand for insurance

\[ \lambda = 1 \quad \lambda = 2 \]

Fair-premium demand

Demand

Premium

\[ \mu \]

\[ \tau \]
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Risk classification

Suppose a population can be divided into 2 risk-groups where:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: $p_1, p_2$;
- premiums offered: $\pi_1, \pi_2$;
- iso-elastic demand:

$$d_i(\pi) = \tau_i \left(\frac{\mu_i}{\pi}\right)^\lambda, \quad i = 1, 2;$$

- fair-premium demand: $\tau_i = d_i(\mu_i)$ for $i = 1, 2$.

Assume $W = L = 1$ and constant demand elasticity $\lambda$ for all risk-groups.

Note: The framework can be generalised for $n > 2$ risk-groups.
Market equilibrium and loss coverage

For a randomly chosen individual, define:

\[ Q = I \] [Individual is insured];
\[ X = I \] [Individual incurs a loss];
\[ \Pi = \text{Premium offered to the individual}. \]

Expected premium, claim and market equilibrium

Expected premium:

\[ E[Q\Pi] = p_1 d_1(\pi_1) \pi_1 + p_2 d_1(\pi_2) \pi_2. \]

Expected claim:

\[ E[QX] = p_1 d_1(\pi_1) \mu_1 + p_2 d_1(\pi_2) \mu_2. \]

Market equilibrium:

\[ E[Q\Pi] = E[QX]. \]

Loss coverage (Population losses compensated by insurance)

Loss coverage: \( E[QX] \).
Scenario 1: Risk-differentiated premium

Market equilibrium

If risk-differentiated premiums are allowed,

- Equilibrium is achieved when $\pi_1 = \mu_1$ and $\pi_2 = \mu_2$.
- No losses for insurers.
- No (actuarial/economic) adverse selection.

Loss coverage (Population losses compensated by insurance)

\[
E[QX] = p_1 d_1(\mu_1) \mu_1 + p_2 d_1(\mu_2) \mu_2,
\]
\[
= p_1 \tau_1 \mu_1 + p_2 \tau_2 \mu_2.
\]
Scenario 2: Pooled premium

Market equilibrium
If risk-classification is banned, under iso-elastic demand pooled premium is:

\[ \pi_0 = \frac{p_1 \tau_1 \mu_1^{\lambda+1} + p_2 \tau_2 \mu_2^{\lambda+1}}{p_1 \tau_1 \mu_1^\lambda + p_2 \tau_2 \mu_2^\lambda}. \]

No losses for insurers! ⇒ No (actuarial) adverse selection.

Loss coverage (Population losses compensated by insurance)

\[ E[QX] = p_1 d_1(\pi_0) \mu_1 + p_2 d_1(\pi_0) \mu_2. \]
How much of population losses is compensated by insurance?

Pooled premium

Adverse selection under pooled premium

Pooled equilibrium premium

\[ \mu_2 \]

\[ \pi_0 \]

\[ \alpha_1 \mu_1 + \alpha_2 \mu_2 \]

\[ \lambda \text{ (Demand elasticity)} \]

Pooled equilibrium is greater than average premium charged under full risk classification: \( \pi_0 > \alpha_1 \mu_1 + \alpha_2 \mu_2 \Rightarrow \text{(Economic) adverse selection.} \)
Adverse selection under pooled premium

Aggregate demand (cover) is lower than under full risk classification $\Rightarrow$ (Economic) adverse selection.
How much of population losses is compensated by insurance?

**Loss coverage ratio**

\[
C = \frac{\text{Loss coverage under pooled premium}}{\text{Loss coverage under risk-differentiated premium}},
\]

\[
= \frac{p_1 d_1(\pi_0) \mu_1 + p_2 d_1(\pi_0) \mu_2}{p_1 \tau_1 \mu_1 + p_2 \tau_2 \mu_2}.
\]

**Comparison of risk-classification regimes**

- \( C > 1 \Rightarrow \) Risk pooling is *better* than full risk classification.
- \( C < 1 \Rightarrow \) Risk pooling is *worse* than full risk classification.
How much of population losses is compensated by insurance?

**Loss coverage ratio**

$\lambda$ (Demand elasticity)

- $\lambda < 1 \iff C > 1 \Rightarrow$ Risk pooling is *better* than full risk classification.
- $\lambda > 1 \iff C < 1 \Rightarrow$ Risk pooling is *worse* than full risk classification.
- **Empirical evidence suggests** $\lambda < 1$ *in many insurance markets.*
Which regime is most beneficial to society?
Which regime is most beneficial to society?

Social welfare

Definition (Social welfare)

Social welfare, $S$, is the expected utility for the whole population:

$$S = E\left[ Q U_\Gamma(W - \Pi L) + (1 - Q) \left[ (1 - X) U_\Gamma(W) + X U_\Gamma(W - L) \right] \right] .$$

Linking social welfare to loss coverage under iso-elastic demand

$$S = \frac{1}{\lambda + 1} \text{ Loss coverage + Constant}.$$ 

Result

- Maximising loss coverage maximises social welfare.
- $\lambda < 1 \Rightarrow$ Risk pooling is better than full risk classification.
Motivation: Two risk-groups $\mu_L = 0.01$ and $\mu_H = 0.04$

**Scenario 1: No adverse selection: Risk-differentiated premiums: $\pi_L = 0.01$ and $\pi_H = 0.04$**

Low risks →

Low risks →

High risks →

High risks →

Loss coverage: 66.7%

Utility increase: $66.2 \times 10^{-4}$

**Scenario 2: Some adverse selection: Pooled premiums: $\pi_L = \pi_H = 0.028$**

Low risks →

Low risks →

High risks →

High risks →

Loss coverage: 77.8%

Utility increase: $71.2 \times 10^{-4}$
Contents

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Adverse selection need not always be adverse.

Restricting risk classification increases loss coverage if $\lambda < 1$.

Maximising loss coverage maximises social welfare.

Restricting risk classification increases social welfare if $\lambda < 1$. 
Reference: Loss coverage blog

https://blogs.kent.ac.uk/loss-coverage/