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Inventories in Dynamic General Equilibrium

Katsuyuki Shibayama, University of Kent

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Katsuyuki Shibayama*†

University of Kent at Canterbury

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Abstract

This article investigates a dynamic general equilibrium model with a stockout constraint, which means that no seller can sell more than the inventories that she has. The model successfully explains two inventory facts; (i) inventory investment is procyclical, and (ii) production is more volatile than sales. The key intuition is that, since inventories and demand are complements in generating sales, the optimal level of inventories is increasing in expected demand. Thus, when demand is expected to be strong, firms increase their production not only to meet their demand but also to accumulate inventories. Also, our model shows that the inventory to sales ratio is persistent and countercyclical, while the (endogenous) markup is countercyclical. These are because a high interest rate in booms discourages firms to hold inventories.

KEYWORDS: Inventory investment, Inventory cycles, Stockout constraint, Dynamic stochastic general equilibrium model

JEL CLASSIFICATION: E32, C68

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[†]**Address for correspondence**: Department of Economics, University of Kent, Canterbury, Kent, CT2 7NP, U.K. Phone: +44(0)122082-4714. E-mail: k.shibayama@kent.ac.uk

1 Introduction

Inventories are important in understanding business cycles; inventory investment accounts for a large share of GDP fluctuations, especially during recessions.[1] In addition, some authors, such as Gertler and Gilchrist (1994) and Kashyap, Lamont, and Stein (1994), point out the importance of inventories as collateral for external finance, while Bernanke and Gertler (1995) shows that inventory investment responds to monetary policy shocks quickly and sharply. Also, Kahn, McConnell, and Perez-Quiros (2002) suggested that better inventory management (say, due to a better IT technology) may be a clue to explain the Great Moderation.[2]

Despite their importance, however, most existing theoretical studies of inventories focus only on firm/industry level analyses; only a few general equilibrium analyses exist.[3] The motivation of this article is to investigate a dynamic stochastic general equilibrium (DSGE) model with a stockout constraint, which means that *no seller can sell more products than the inventories she holds.* Especially, we show that the model quantitatively satisfies two stylised inventory facts: (i) production is more volatile than sales, and (ii) inventory investment is procyclical.[4]

In a sense, this article is a general equilibrium extension of Kahn (1987, 1992), who pioneered analyses of the stockout constraint. The key trade-off under the stockout constraint is that (a) having too much inventories is costly because unsold goods impose a carrying cost (Jorgenson's user cost), while (b) having too little inventories is also costly because the risk of losing sales opportunity due to stockout is too high. Balancing the

[1] For example, Fitzgerald (1997) reports that "changes in inventory investment are, on average, more than one-third the size of quarterly changes in real GDP over the post-war period." Also, Blinder and Maccini (1991). state "the drop in inventory investment has accounted for 87 percent of the drop in GNP during the average postwar recession in the United States."

[2] In Section 5.4, we investigate the effects of changing parameters that are related to inventory management. Previewing the results, however, around the plausible benchmark parameters, the effects are quantitatively small.

[3] To name a few, Hornstein and Sarte (2001), Boileau and Letendre (2004) and Jung and Yun (2005) for inventory analysis in a sticky price environment, and Fisher and Hornstein (2000) for the (S,s) model. See below for Khan and Thomas (2007a and 2007b).

[4] Theoretical studies of inventories started with production smoothing and buffer stock inventories (see Fitzgerald (1997) for the survey of earlier works). Because strong demand is expected to mean that many buyers take out inventories from sellers (thus, inventories decrease when demand is strong), these inventory facts have been considered to be puzzling.

carry cost against the stockout probability, sellers choose the optimal level of inventories.

On the one hand, (a) above implies that the optimal level of inventories (relative to sales) is governed by the real interest rate, as pointed out by Bernanke and Gertler (1995) among others. On the other hand, (b) above implies that *the optimal (target) level of inventories is an increasing function of expected demand*; if sellers did not increase inventories after a positive demand shock, the stockout probability would be too high. Hence, one unit of increase in expected demand is followed by a positive inventory investment. That is, after a positive demand shock, production must increase not only to accommodate an increase in sales but also to accumulate inventories.[5] This is the mechanism that explains above two inventory stylised facts in our model.

Note however that in our model inventories also work as buffer stocks to insulate production from demand shocks,[6] which implies that inventories decrease right after an *unexpected* positive demand shock. This contrasts to the above mechanism, in which the optimal level of inventories is increasing in *expected* demand; indeed, given the size of initial shock, as exogenous shocks become more persistent (and hence exogenous shock processes become more predictable), inventory investment followed by a positive demand shock becomes larger.

Perhaps, our research is most closely related to Khan and Thomas' (Khan and Thomas (2007a)) fully rational DSGE for inventories. While their (S,s) bunching order model are quite successful in replicating inventory facts qualitatively and quantitatively, their stockout constraint model generate almost no inventories; even adding large idiosyncratic shocks, there is only a too low level of inventories in their model. Perhaps, the main differences between their and our models are that we assume (i) a small degree of price stickiness and (ii) positive profit margin. We claim these two assumptions are essential in modelling the stockout constraint. If price is perfectly flexible, price

[5] Note that, with supply shocks (technology shocks), it is not surprising for above two inventory facts to hold; production is more volatile than sales simply because the source of shocks is on the production side, and hence inventory investment is procyclical simply because an increase in sales is not enough to absorb the increase in production (see Blinder (1986) for example). What is important in this article is that, even if the source of shocks lies on the demand side, still production is more volatile than sales.

[6] Note that this is a natural consequence of producers' optimisation (i.e., production smoothing due to a convex cost function), meaning that we do not need to add any additional assumption to model buffer stock inventories.

adjusts demand until demand is equal to supply (goods on shelf in this case). Also, if the net profit margin is zero, to avoid the carry cost of inventories, firms optimally choose zero inventories (unsold goods), unless the carry cost is negative. Certainly, if individual marginal costs change drastically, firms may want to have inventories to exploit the *negative cost of holding inventories*, which we guess is the mechanism that large idiosyncratic shocks generate a certain level of inventories in Khan and Thomas (2007a). We argue however that such inventories are held not because of the stock avoidance motive but because of the production smoothing motive; sellers/producers want to produce their products when their production cost is low and store them in the form of inventories. Note that we are not claiming that the stockout constraint model is superior to (S,s) ordering model; rather, we claim that (S,s) ordering model is suitable to explain *buyers'* inventory management, while the stockout constraint model fits to *sellers'* inventory management.[7]

The plan of this article is as follows. Sections 2 and 3 describe the model; while Section 2 derives the key assumptions and results specific to the stockout constraint, Section 3 formalises the equilibrium equations. Section 4 discusses some analytical findings and known empirical inventory facts. Section 5 simulates and evaluates the model in the light of these key empirical facts. The final section concludes.

2 Structure of M-goods Markets

This section describes the assumptions specific to our model and their implications. The formal optimisation problems are shown in Section 3.

[7] We do not believe that, as often claimed, the (S,s) and the stockout constraint models are limited to retailers' inventories and producers' final goods inventories, respectively. Certainly, some evidence such as Blinder and Maccini (1991) show that inventories of intermediate goods and row material explain the majority of inventory investment. But, it is plausible that, within a manufacturing company, a manager of a division, which is at the middle of the production line, employs an (S,s) rule to order intermediate goods to an upstream division, and at the same time he holds the output of his division to avoid stockout when he gets an order from the downstream division. The point is, as long as there are frictions and some degree of uncertainty in the demand for and/or the supply to a division at the timing of decision making, there is a non-trivial optimisation problem even in pipeline inventory management.

2.1 Overview

Before going on, as a means of exposition, we depict the model environment here. There are two types of firms and the representative household (HH); intermediate goods firms (M-firms) use labour and capital to produce intermediate goods (M-goods); final goods firms (F-firms) use M-goods to produce final goods (F-goods); and HH consumes, invests and supplies labour and capital to M-firms. In this article, F-firms are regarded as retailers; i.e., F-firms simply convert different types of M-goods into identical F-goods. The M-goods markets are subject to the stockout constraint; no seller (M-firm) can sell more than what she has on the shelf, even if she meets (too) many buyers (F-firms).[8]

There are four main assumptions in our model, two of which, we believe, are essential to understand the stockout constraint. First, *some degree of price rigidity* is necessary for the stockout constraint to be meaningful (Section 2.4); otherwise, stockout probability is always *zero* and there is no unsold goods remained (because all goods on shelf are sold out). This is simply because, if demand is strong, sellers raise their sales price, and vice versa, until demand equals supply (goods on shelf in our case). Second, sellers must earn *some positive net profit margin* to encourage them to hold inventories (Sections 2.5 and A.1); otherwise, stockout probability is always *one* and there is no unsold goods remained (because sellers have goods on shelf so that they meet the minimum possible demand). This is because, while sellers have no incentive to avoid stockout (anyway their net profit is zero), they definitely want to avoid having unsold goods because of the carry cost; i.e., the key trade-off does not work. The other two assumptions are the *infinitely many buyers and sellers*, which is necessary to exploit the law of large number (Section 2.2), and *idiosyncratic shocks*, which generate demand uncertainty. The following subsections discuss these four assumptions in order (Section 2.3).

Finally, note that (i) our model essentially falls into the class of representative agent models mainly because of the symmetricity of the equilibrium (see Sections 3.4 A.2), and (ii) our model falls into the class of flexible price models in aggregate intuitively

[8] Note that, in our model, a "buyer" and a "seller" are always an F-firm and an M-firm, referred to as he and she, respectively. Also, we use "inventories" to signify unsold goods and/or goods on shelf only when we do not need to distinguish these two concepts, while "inventory investment" always means the change in unsold goods.

because our price rigidity is confined within one period (see Section 2.4).

2.2 Agents Distribute over $[0,1] \times [0,1]$ ($\subset \mathbb{R}^2$)

We assume that sellers and buyers both distribute over *a rectangle*, which is similar to but somewhat different from the standard monopolistic competition models, in which agents distribute over a line segment. Specifically, there is a continuum of markets over $[0,1]$, and there is a continuum of sellers distributed over $[0,1]$ in each market. In total, both sellers and buyers have unit measure (i.e., their populations are normalised to be one). Since each buyer is assumed to visit all markets, each seller *on average* meets buyers with measure one.[9] In different markets, different varieties (types) of goods are traded; in each market, all sellers sell the same variety of goods (hence, there are one-to-one correspondences between markets and varieties). We also assume that varieties are differentiated from each other (see Section 2.5). In our notational convention, super/subscripts ji indicate the j-th seller in the i-th market.

This assumption facilitates aggregation. Having a continuum of sellers in each market, on the one hand, the law of large number (LLN) over j allows us to aggregate sellers' variables, such as sales, in each market. Having a continuum of markets that each buyer visits, on the other hand, LLN over i guarantees that all buyers enjoy the same number (measure) of varieties of goods.

2.3 Idiosyncratic Shocks

To motivate sellers to hold inventories, we introduce idiosyncratic demand shocks. More specifically, in our model, we assume that buyers do not distribute sellers evenly; that is, some sellers meet more buyers than others. In this subsection, we mainly focus on one

[9] Note that the (average) measures of sellers and buyers are one on a rectangle, but the measure of buyers who one seller meets is one on a line segment (i.e., its measure on rectangle is zero). To obtain a concrete image, consider the following discrete example. There are 1,000 markets and there are 1,000 sellers in each market. The number of total sellers are hence $1,000,000$ ($= 1,000 \times 1,000$). We assume that there is the same number of buyers. Since each buyers visit all markets, $1,000,000$ buyers appear in each market and hence each seller in a market on average sees $1,000$ buyers. If $1,000$ is replaced by 1, then $1,000 \times 1,000$ is also $1 = (1 \times 1)$, but the share of $1,000$ in $1,000,000$ is very small and is almost zero.

seller; i.e., the j-th seller in the i-th market. Though we keep superscript i for notational consistency, it is basically irrelevant in this subsection.

The stockout constraint means that sales is capped by goods on shelf. That is, on the one hand, even though demand M_t^{ji} is strong, if the seller does not have enough goods on shelf GoS_t^{ji}, her sales S_t^{ji} is equal to GoS_t^{ji}. In this case, she loses some of her sales opportunity. If M_t^{ji} is weak, on the other hand, the stockout constraint is not binding, and S_t^{ji} equals M_t^{ji}. In this case, some goods are unsold and they are carried to the next period as unsold goods U_{t+1}^{ji}. Simply put, the stockout constraint means that sales is the minimum of demand or goods on shelf.

$$S_t^{ji} = \min\{GoS_t^{ji},\ M_t^{ji}\} \tag{1}$$

Because we assume that all of today's output of M-goods $Y_t^{M,ji}$ can be placed in today's market, goods on shelf is the sum of today's output and unsold goods carried *from the previous period* U_t^{ji}.

$$GoS_t^{ji} = Y_t^{M,ji} + U_t^{ji} \tag{2}$$

As mentioned above, if demand for her goods is weak, a portion of GoS_t^{ji} is not sold and is carried *to the next period* as unsold goods U_{t+1}^{ji}.

$$U_{t+1}^{ji} = GoS_t^{ji} - S_t^{ji} \tag{3}$$

Roughly speaking, each seller chooses optimal GoS_t^{ji}, given the distribution of stochastic demand M_t^{ji}. The key trade-off is that, while *having too high level of GoS_t^{ji} is costly because carrying U_{t+1}^{ji} to the next period requires to too a high carry cost* (see (24b) or equivalently (30)), *having too low level of GoS_t^{ji} is also costly because it leads to too high stockout probability and the loss of profitable sales opportunity* (see (6)).

In our model, we assume that (i) the number of buyers who visit each seller N_t^{ji} is uncertain, and (ii) the seller chooses the optimal goods on shelf before observing N_t^{ji}. We refer to this uncertainty in the number of buyers as an *idiosyncratic shock*. Specifically, we assume that N_t^{ji} follows the log-normal distribution with mean $-\sigma_N^2/2$ and variance σ_N^2 so that the total number of buyers in this market is equal to one. Let $M_t^{b,ji}$ be the *demand per buyer* for this seller. Then, the total demand for the j-th seller M_t^{ji} is

$$\begin{aligned} M_t^{ji} &= M_t^{b,ji} N_t^{ji} \\ \ln N_t^{ji} &\sim N\left(-\sigma_N^2/2, \sigma_N\right) \end{aligned}$$

For notational simplicity, define the following two variables.

$$n_t^{ji} = \ln N_t^{ji} \tag{4a}$$

$$g_t^{ji} = \ln(GoS_t^{ji}/M_t^{b,ji}) \tag{4b}$$

Then, we can rewrite the stockout constraint as

$$S_t^{ji} = \begin{cases} GoS_t^{ji} & \text{if } n_t^{ji} > g_t^{ji} \\ M_t^{b,ji} e^{n_t^{ji}} & \text{otherwise} \end{cases} \tag{5}$$

From the log-normality assumption, we can directly derive the following four key numbers. The first two of them are *stockout probability* π_t^{ji} and *cost of stockout* $\tilde{\pi}_t^{ji}$.[10]

$$\pi_t^{ji}(g_t^{ji}) = \frac{1}{\sqrt{2\pi}\sigma_N} \int_{g_t^{ji}}^{\infty} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji}+\frac{\sigma_N^2}{2}\right)^2} dn_t^{ji} = 1 - \Phi\left(g_t^{ji};\ -\frac{\sigma_N^2}{2}, \sigma_N\right) \tag{6a}$$

$$\begin{aligned} \tilde{\pi}_t^{ji}(g_t^{ji}) &= \frac{1}{\sqrt{2\pi}\sigma_N} \int_{g_t^{ji}}^{-\infty} N_t^{ji} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji}+\frac{\sigma_N^2}{2}\right)^2} dn_t^{ji} = \hat{E}\left[N_t^{ji} | n_t^{ji} > g_t^{ji}\right] \pi_t^{ji}(g_t^{ji}) \\ &= \frac{1}{\sqrt{2\pi}\sigma_N} \int_{g_t^{ji}}^{-\infty} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji}-\frac{\sigma_N^2}{2}\right)^2} dn_t^{ji} = 1 - \Phi\left(g_t^{ji};\ \frac{\sigma_N^2}{2}, \sigma_N\right) \end{aligned} \tag{6b}$$

where, for example, $\Phi(n_t^{ji};\ -\sigma_N^2/2, \sigma_N)$ is the *cdf* of the normal distribution with mean $-\sigma_N^2/2$ and variance σ_N^2. Both π_t^{ji} and $\tilde{\pi}_t^{ji}$ are strictly decreasing in g_t^{ji}, and they move

[10] Exactly speaking, $\tilde{\pi}_t^{ji}$ is the expected number of buyers who seller j meets conditional that stockout takes place times the stockout probability. The cost of stockout is our interpretation, because $\hat{E}[N_t^{ji}|n_t^{ji} > g_t^{ji}] - g_t^{ji}$ is the number of buyers who the seller loses due to stockout, if it is binding. Also, note that, to derive $\tilde{\pi}_t^{ji}$, we use the completion of squares: $\exp\{n_t^{ji}\}\exp\{\frac{-1}{2\sigma_N^2}(n_t^{ji} + \sigma_N^2/2)^2\} = \exp\{\frac{-1}{2\sigma_N^2}(n_t^{ji} - \sigma_N^2/2)^2\}$.

closely to each other.

The third variable is expected sales $\hat{E}_t[S_t^{ji}]$, which equals aggregate sales in the i-th market S_t^i due to LLN over j (which, it turns out, equals aggregate sales S_t in all market due to LLN over i).

$$\begin{aligned}\hat{E}_t\left[S_t^{ji}\right] &= \frac{1}{\sqrt{2\pi}\sigma_N}\left[\int_{-\infty}^{g_t^{ji}} M_t^{b,ji} e^{n_t^{ji}} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji}+\frac{\sigma_N^2}{2}\right)^2} dn_t^{ji} + GoS_t^{ji}\int_{g_t^{ji}}^{\infty} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji}+\frac{\sigma_N^2}{2}\right)^2} dn_t^{ji}\right] \\ &= \hat{E}\left[M_t^{b,ji}N_t^{ji}|\text{ not stockout}\right](1-\Pr[\text{stockout}]) + GoS_t^{ji}\Pr[\text{stockout}] \\ &= M_t^{b,ji}\left(1-\tilde{\pi}_t^{ji}\right) + GoS_t^{ji}\pi_t^{ji} \end{aligned} \tag{7}$$

where hat notation ˆon $\hat{E}_t[\ ]$ indicates that the information set includes all information up to time t except for idiosyncratic shock N_t^{ji} at t.

The last one is the probability that a buyer *does not* face stockout Q_t (see also Section 2.5.1 for the interpretation of Q_t). Because not only sellers but also buyers suffer from stockout, as a logical consequence, we have to take into account buyers' stockout as well. To derive Q_t, let N_t^{ji*} be the number of buyers who can buy the i-th variety without facing stockout at the j-th seller's shop.

$$N_t^{ji*} = \begin{cases} GoS_t^{ji}/M_t^b & \text{if } n_t^{ji} > g_t^{ji} \quad \text{(stockout)} \\ N_t^{ji} & \text{otherwise} \quad \text{(not stockout)} \end{cases}$$

Due to LLN, the buyers' probability of not facing stockout Q_t equals the number of buyers who can buy the variety in each market (divided by the number of total buyers 1), which is obviously the aggregation of N_t^{ji*} over j.

$$Q_t = \frac{1}{\sqrt{2\pi}\sigma_N}\left[\int_{-\infty}^{g_t^{ji}} e^{n_t^{ji}} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji}+\frac{\sigma_N^2}{2}\right)^2} dn_t^{ji} + \int_{g_t^{ji}}^{\infty} \frac{GoS_t^{ji}}{M_t^{b,ji}} e^{\frac{-1}{2\sigma_N^2}\left(n_t^{ji}+\frac{\sigma_N^2}{2}\right)^2} dn_t^{ji}\right] \tag{8}$$

Because (i) demand per buyer is the same for all buyers in equilibrium; $M_t^{b,ji} = M_t^b$ (see Section A.2), and (ii) aggregate sales equals expected sales by LLN; $S_t = \hat{E}_t[S_t^{ji}]$, (8) and (7) show

$$Q_t = S_t/M_t^b \tag{9}$$

2.4 Price Posting and Information

Note that M-markets do not clear in our model. An important consequence of non-Walrasian M-markets is that we cannot use the market clearing condition as a pricing mechanism. Hence, we assume the following price posting rule as an alternative. The rule follows a simple extensive game, in which each seller first sets her sales price (M-price $P_t^{M,ji}$ of the j-th seller in the i-th market), then buyers are distributed among sellers *unevenly* (idiosyncratic shocks), and finally each buyer chooses optimal quantity $M_t^{b,ji}$ conditional that he is not subject to stockout. This extensive game is played in every M-market in every period.

0. All aggregate shocks are revealed.

1. Anticipating buyers' action, sellers choose their sales price $P_t^{M,ji}$ and GoS_t^{ji} before observing the realisation of idiosyncratic shocks N_t^{ji} (price posting).

2. The idiosyncratic shocks are revealed; some sellers meet many buyers, while others meet only a few.

3. All buyers stand in a queue, and then buyers choose an optimum amount to buy in order (see (15)). The order in the queue is stochastic for buyers, and a buyer cannot buy that type of goods if GoS_t^{ji} run out before his turn; in this case, he simply loses one variety.

To make the stockout constraint sensible, we assume that each buyer visits only one seller for each variety, and, even if he fails to buy a variety due to stockout, he cannot visit other shops in that market.[11] Also, we claim that *some degree of price rigidity is necessary for modelling the stockout constraint.* This is simply because, unless sellers expect extremely high production cost in the next period,[12] there is no incentive for them to carry unsold goods to the next period. That is, if demand is weak, they simply sell their goods by discounting their sales price, and there are no unsold goods remained.

[11] If we allow revisiting, our M-goods markets reduce to Walrasian markets; i.e., there is no unsold goods remained.

[12] More precisely, unless sellers expect the carry cost of inventories is negative.

Contrary, if demand is strong, they raise their sales price to the point that demand equals their GoS_t^{ji}; again there are no unsold goods. Hence, if price is totally flexible, the stockout constraint does not make sense.

There are three additional remarks in order. First, demand function is derived from the FOC of buyers (15), and it is used as a sellers' constraint; since demand per buyer $M_t^{b,ji}$ is deceasing in $P_t^{M,ji}$, the seller sets $P_t^{M,ji}$ by anticipating buyers optimal choice. Price posting defines the information in sellers' price setting (see Section 3.2.1). Second, related to the first point, because prices are posted before observing idiosyncratic shocks, to obtain sellers' FOCs, we need to differentiate $\hat{E}[S_t^{ji}]$ but not S_t^{ji}; we can find the FOCs because, while S_t^{ji} is a kinked function, its expectation $\hat{E}[S_t^{ji}]$ is smooth (see Section 3.2.1 for further analytical details).[13] Third, M-price $P_t^{M,ji}$ cannot react to the idiosyncratic shocks, but can react to all aggregate shocks (and state variables). Hence, in aggregate, our model falls into the class of flexible price models, because idiosyncratic shocks are aggregated out at the end of the day.

2.5 Monopolistic Competition and Cost of Losing Varieties

As mentioned above, we also assume monopolistic competition à la Dixit and Stiglitz (1977). A positive net profit margin is necessary for inducing sellers to hold inventories.[14] On the one hand, having inventories is costly because of the carry cost (see (24b) or equivalently (30)). On the other hand, however, if the net profit margin is zero, sellers do not care about the losing sales opportunity; i.e., stockout is not painful. Hence, under perfect substitution, there is no unsold goods; $U_{t+1}^{ji} = 0$. Indeed, we can show that as $\theta \to \infty$ our model effectively reduces to the standard RBC model (see Appendix A.1), where θ is the elasticity of substitution among varieties.

In our environment, two-stage budgeting with quantity and price indices still holds,[15]

[13] This technique is originally applied to the stockout constraint by Kahn (1987), and is rather commonly used in the analyses of voting. Also, consider the fact that, though option payoffs are kinked, it is possible to calculate option deltas before their expiration date (see Section 4.3).

[14] We define the *gross* profit margin as sales price $P_t^{M,ji}$ minus cost of sales $\lambda_t^{U,ji}$, and the *net* profit margin as the gross profit margin minus the carry cost of unsold goods $\lambda_t^{U,ji} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,ji}]$ (see Sections 3.1.1 and 3.2 for notations).

[15] Interestingly, one of the main motivations of Dixit and Stiglitz (1977) is to analyse firms' entry and

but we need a slight modification. This is because the number of available varieties Q_t fluctuates over time, and hence we need to consider the cost effect of losing varieties.[16]

Note that, in this section, we discuss *buyers' decisions*. This implies that, although we keep superscript j for notational consistency, it is basically irrelevant in this subsection, because buyers do not care about the identity of a seller.

2.5.1 Number of Available Varieties

The cost effect of losing varieties in itself is not of interest, and quantitatively its effect seems weak under the plausible parameter range. It is a logical consequence of the combination of Dixit-Stiglitz monopolistic competition and stockout.

First, let Q_t^i be an indicator function that is 1 if a buyer can buy the i-th good, and 0 otherwise. Then, the measure of available varieties Q_t is:

$$Q_t = \int_0^1 Q_t^i di \tag{10a}$$

$$Q_t^i = \begin{cases} 1 & \text{if } i\text{-th variety is available} \\ 0 & \text{otherwise} \end{cases} \tag{10b}$$

Due to LLN, Q_t has two meanings: (i) the number (measure) of available varieties (10a) and (ii) the probability that a *buyer* can buy a variety without encountering stockout (see (8)). Note that Q_t is a distinct concept from $1 - \pi_t^{ji}$, the probability that a *seller* does not face stockout.

exit, explicitly addressing the effect of a changing number of firms (or varieties, in our language).

[16] The intuition of the cost of losing variety is as follows. Let us consider a familiar example, say, ice cream. Suppose a consumer prefers vanilla and chocolate ice creams equally, but vanilla and chocolate ice creams are not perfect substitutes for one another. Also, suppose that their costs are the same. Then, one vanilla and one chocolate give higher utility than two vanillas, because they are differentiated from each other. However, the costs of vanilla + vanilla and vanilla + chocolate are the same. Thus, given the level of expenditures, having fewer varieties provides lower utility, and vice versa. Or, equivalently, with fewer varieties available, the pecuniary cost of achieving a certain level of utility is higher.

2.5.2 Quality-Adjusted Quantity Index

Next, we *define* the quantity index of M-goods as:

$$M_t^F = \left[\int_0^1 Q_t^i \left(M_t^{b,ji}\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \tag{11}$$

where $M_t^{b,ji}$ is the *physical quantity* of the i-th variety of goods purchased by an F-firm (buyer), and M_t^F is the quantity index of M-goods. Note that (11) represents F-firms' *production* as retailers (see Section 3.3), while, from the viewpoint of M-firms, $M_t^{b,ji}$ is the demand per buyer (see Section 2.3). Compared to (13), the multiplication of Q_t^i inside the integral implies that unavailable varieties are not taken into account, while, *not* divided by Q_t, the index is the "summation" of $M_t^{b,ji}$ over i.

2.5.3 Price Index

To *derive*[17] the price index and the demand curve, consider the following F-firms' cost minimisation problem (see also Section 3.3).

$$\min \int_0^1 Q_t^i P_t^{M,ji} M_t^{b,ji} di \qquad \text{s.t.} \quad \left[\int_0^1 Q_t^i \left(M_t^{b,ji}\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \geq M_t^F$$

Let $\lambda_t^{MC,F}$ be the Lagrange multiplier for the constraint. Then, for i such that $Q_t^i = 1$ (buyers do not care about the optimal quantity of the goods that they cannot buy), the FOC with respect to $M_t^{b,ji}$ is

$$P_t^{M,ji} = \lambda_t^{MC,F} \left[\int_0^1 Q_t^i \left(M_t^{b,ji}\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}-1} \left(M_t^{b,ji}\right)^{\frac{\theta-1}{\theta}-1} \tag{12}$$

To determine the price index, raise both sides of (12) to the power of $1-\theta$, multiply them by Q_t^i/Q_t, integrate them over i and raise them to the power of $1/(1-\theta)$.

$$P_t^M := \left[\int_0^1 \frac{Q_t^i}{Q_t} \left(P_t^{M,ji}\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}} = Q_t^{\frac{1}{\theta-1}} \lambda_t^{MC,F} \tag{13}$$

[17]Note that many combinations of definitions of price and quantity indices are logically consistent, but we have chosen our definitions so that $P_t^{M,ji} = P_t^M$.

Again, compared to (11), the multiplication of Q_t^i implies that unavailable varieties are not taken into account, while, divided by Q_t, the index is the "average" of individual prices.

2.5.4 Two-Stage Budgeting

To show the two-stage budgeting, first multiply both sides of (12) by $Q_t^j M_t^{b,ji}$ and integrate them over i, we find

$$\int_0^1 Q_t^i P_t^{M,ji} M_t^{b,ji} di = Q_t^{\frac{-1}{\theta-1}} P_t^M M_t^F \quad \left(= Q_t P_t^M M_t^b\right) \tag{14}$$

The multiplicative term $Q_t^{\frac{-1}{\theta-1}} > 1$ in (14) represents the cost of losing varieties;[18] i.e., the actual cost $\int_0^1 Q_t^i P_t^{M,ji} M_t^{b,ji} di$, which is equal to price P_t^M times total physical purchases $Q_t M_t^b$ in the symmetric equilibrium, is larger than $P_t^M M_t^F$. As explained above, this is because goods are *not* perfect substitutes for each other.

2.5.5 Demand Curve

From (12) and (13), a buyer's physical demand schedule for $M_t^{b,ji}$ is

$$M_t^{b,ji} = Q_t^{\frac{-\theta}{\theta-1}} \left(\frac{P_t^{M,ji}}{P_t^M}\right)^{-\theta} M_t^F \tag{15}$$

This is also used in the M-firms' optimisation as a constraint.

Note that demand per buyer $M_t^{b,ji}$ is not an index, but instead is measured *in physical unit*. To obtain some intuition, consider the equilibrium, in which $M_t^{b,ji} = M_t^b$ for all ji (see Appendix A.2). Then,

$$M_t^b = Q_t^{\frac{-\theta}{\theta-1}} M_t^F \tag{16}$$

Since $Q_t < 1$ and $\theta > 1$, $Q_t M_t^b > M_t^F$; i.e., physical demand is greater than the index, where $Q_t M_t^b$ is the number of available varieties times physical demand for each variety. This difference becomes larger as Q_t becomes smaller. This implies that, for given

[18] Note that $Q_t^{\frac{-1}{\theta-1}} P_t^M M_t^F = Q_t P_t^M M_t^b$ in the symmetric equilibrium, where $M_t^b = M_t^{b,ji}$ (see (16)).

prices, to achieve a certain level of index M_t^F, buyers have to buy more (hence pay more), as Q_t becomes smaller. This is another representation of what we call the cost of losing varieties. Finally, note that, since sellers exploit the slope of the demand curve as monopolists, the quantity traded is not socially optimal.[19]

3 The Model: Agents' Optimisation

This section formulates the optimisation problem of each type of agents and closes the model. The key features of our model appear in M-firms' optimisation, while HH's and F-firms' optimisations are rather standard.

3.1 Household

The infinitely-lived representative household (HH) maximises its expected lifetime utility. HH supplies capital and labour, while it demands F-goods for consumption and investment.

$$\max_{\{C_t,H_t,B_t,I_t\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ U\left[C_t, 1-H_t; \xi_t^U\right] \right\} \right]$$
$$\text{where } U\left[C_t, 1-H_t\right] = (1-\psi) \frac{C_t^{1-\gamma}}{1-\gamma} \xi_t^U + \psi (1-H_t) \tag{17}$$

s.t.

$$\begin{aligned} C_t + I_t + B_{t+1} &= R_t^B B_t + \left(R_t^K - 1\right) K_t + W_t H_t + D_t^{iv} - G_t && \text{(18a)} \\ K_{t+1} &= (1-\delta) K_t + I_t && \text{(18b)} \end{aligned}$$

The period utility $U[.,.]$ is time additive, which is discounted by subjective discount factor β^t. For the period utility, we assume that the cross partial is zero $\partial^2 U / \partial C_t \partial H_t = 0$. Parameter ψ governs the relative importance of leisure $1 - H_t$, while γ is the coefficient of relative risk aversion (or, the reciprocal of elasticity of intertemporal substitution of

[19]There is another reason for the socially not optimal allocation – search externalities (see Appendix A.3).

consumption). We follow Khan and Thomas (2004b) for the preference shock ξ_t^U.

The first constraint shows the period budget constraint. The period expenditure is the sum of consumption C_t, capital investment I_t and bond purchase B_{t+1}. The period revenue is the sum of the *gross* return on one-period discount bonds purchased in the previous period $R_t^B B_t$, the *net* return on capital $\left(R_t^K - 1\right) K_t$, labour income $W_t H_t$ and dividends D_t^{iv}, where R_t^B, R_t^K and W_t are gross bond return, gross capital return and wage, respectively. The second constraint shows the evolution of capital, where δ is the depreciation rate.

3.1.1 FOCs

The first order conditions (FOCs) are as follows.

$$E_t\left[\Lambda_{t,t+1} R_{t+1}^B\right] = 1 \tag{19a}$$

$$\frac{\partial U_t / \partial L_t}{\partial U_t / \partial C_t} = W_t \tag{19b}$$

$$E_t\left[\Lambda_{t,t+1}\left\{R_{t+1}^K - \delta\right\}\right] = 1 \tag{19c}$$

where $\Lambda_{s,\tau} = \beta \frac{\partial U_\tau / \partial C_\tau}{\partial U_s / \partial C_s}$ for $\tau \geq s \geq 0$ is the stochastic discount factor (SDF).

3.2 M-firms

M-firms are subject to the stockout constraint. They take the F-firms' demand curve as a constraint (see (15) and (20d)).

$$\max_{\left\{P_t^{M,ji}, S_t^{ji}, H_t^{ji}, K_t^{ji}, U_{t+1}^{ji}, Y_t^{M,ji}\right\}_{t=0}^{\infty}} E_0\left[\ \sum_{t=0}^{\infty} \Lambda_{0,t}\left\{\ P_t^{M,ji} S_t^{ji} - W_t H_t^{ji} - \left(R_t^K - 1\right) K_t^{ji}\ \right\}\ \right]$$

s.t.

$$S_t^{ji} = \min\left\{GoS_t^{ji}, M_t^{ji}\right\} \tag{20a}$$

$$U_{t+1}^{ji} = GoS_t^{ji} - S_t^{ji} \tag{20b}$$

$$GoS_t^{ji} = U_t^{ji} + Y_t^{M,ji} \tag{20c}$$

$$M_t^{ji} = M_t^{b,ji} N_t^{ji} = Q_t^{\frac{-\theta}{\theta-1}} \left(\frac{P_t^{M,ji}}{P_t^M}\right)^{-\theta} M_t^F N_t^{ji} \tag{20d}$$

$$Y_t^{M,ji} = \xi_t^M \left(K_t^{ji}\right)^{\alpha} \left(H_t^{ji}\right)^{1-\alpha} \tag{20e}$$

The j-th M-firm in the i-th market maximises the present value (PV) of its current and future net cash inflow. It optimally chooses sales price $P_t^{M,ji}$, labour input H_t^{ji} and capital input K_t^{ji} in every period. It takes wage W_t and return on capital R_t^K as given (hence no ji superscripts). See Section 3.1.1 for the definition of stochastic discount factor $\Lambda_{0,t}$.

The first three constraints shows the evolution of unsold goods U_{t+1}^{ji}, the definition of goods on shelf GoS_t^{ji} and the stockout constraint, respectively. The fourth constraint shows that total demand M_t^{ji} is demand per buyer $M_t^{b,ji}$ times number of buyers N_t^{ji}. Due to monopolistic competition, demand per buyer $M_t^{b,ji}$ is decreasing in its relative sales price $P_t^{M,ji}/P_t^M$ (see Section 2.5.5). Remember that $Q_t^{\frac{-\theta}{\theta-1}}$ represents the cost of losing variety, P_t^M is aggregate price index of M-goods and M_t^F is the *quantity index* of M-goods (see (11)). The last constraint shows that the production function is Cobb-Douglas with capital share α, and ξ_t^M is the productivity shock.

3.2.1 Implication of Price Posting

Before deriving the FOCs, we need to discuss the implication of price posting: timing of the price setting (or equivalently the information available at the price setting). In relation to the stockout constraint $S_t^{ji} = \min\{U_t^{ji} + Y_t^{M,ji}, M_t^{ji}\}$, M-firms decide both sales price $P_t^{M,ji}$ and goods on shelf $GoS_t^{ji} = U_t^{ji} + Y_t^{M,j}$ before observing idiosyncratic shock N_t^{ji}. Hence, the proper FOCs involve

$$\frac{\partial}{\partial GoS_t^{ji}}\hat{E}\left[\min\left\{GoS_t^{ji}, M_t^{ji}\right\}\right] \quad \text{and}$$
$$\frac{\partial}{\partial P_t^{M,ji}}\hat{E}\left[\min\left\{GoS_t^{ji}, M_t^{ji}\right\}\right] = \frac{\partial M_t^{b,ji}}{\partial P_t^{M,ji}}\frac{\partial}{\partial M_t^{b,ji}}\hat{E}\left[\min\left\{GoS_t^{ji}, M_t^{ji}\right\}\right]$$

where expectation $\hat{E}[\quad]$ is over the idiosyncratic shock. Note again that $\hat{E}[S_t^{ji}]$ is differentiable though S_t^{ji} is not. Remember

$$S_t^{ji} = \min\left\{GoS_t^{ji}, M_t^{ji}\right\} = \begin{cases} GoS_t^{ji} & \text{if } n_t^{ji} < g_t^{ji} \\ M_t^{b,ji}e^{n_t^{ji}} & \text{otherwise} \end{cases}$$

Thus, *for any continuously differentiable distribution function* $f(n_t^{ji})$,[20]

$$\begin{aligned}
&\frac{\partial}{\partial GoS_t^{ji}}\hat{E}\left[\min\left\{GoS_t^{ji}, M_t^{ji}\right\}\right] = \frac{\partial}{\partial GoS_t^{ji}}\int_{-\infty}^{\infty}\min\left\{GoS_t^{ji}, M_t^{b,ji}e^{n_t^{ji}}\right\}f(dn_t^{ji}) \\
= &\int_{g_t^{ji}}^{\infty}\frac{\partial}{\partial GoS_t^{ji}}GoS_t^{ji}f(dn_t^{ji}) + \int_{-\infty}^{g_t^{ji}}\frac{\partial}{\partial GoS_t^{ji}}\left(M_t^{b,ji}e^{n_t^{ji}}\right)f(dn_t^{ji}) \\
= &\int_{g_t^{ji}}^{\infty}f(dn_t^{ji}) = 1 - \Pr[GoS_t^{ji} < M_t^{ji}] = \pi_t^{ji}
\end{aligned}$$

Similarly,

$$\frac{\partial}{\partial M_t^{b,ji}}\hat{E}\left[\min\left\{GoS_t^{ji}, M_t^{ji}\right\}\right] = \int_{-\infty}^{g_t^{ji}}e^{n_t^{ji}}f(dn_t^{ji}) = 1 - \hat{E}\left[N_t^{ji}|n_t^{ji} > g_t^{ji}\right]\pi_t^{ji}(g_t^{ji}) = 1 - \tilde{\pi}_t^{ji}$$

Since $\partial GoS_t^{ji}/\partial U_t^{ji} = \partial GoS_t^{ji}/\partial Y_t^{M,ji} = 1$ and $\partial M_t^{b,ji}/\partial P_t^{M,ji} = -\theta M_t^{ji}/P_t^{M,ji} = -\theta Q_t^{\frac{-\theta}{1-\theta}}$ $(P_t^{M,ji}/P_t^M)^{-\theta}M_t^F/P_t^{M,ji}$, by chain rule,

$$\begin{aligned}
\frac{\partial\hat{E}[S_t^{ji}]}{\partial U_t^{ji}} &= \frac{\partial\hat{E}[S_t^{ji}]}{\partial Y_t^{M,ji}} = \pi_t^{ji} \\
\frac{\partial\hat{E}[S_t^{ji}]}{\partial P_t^{M,ji}} &= -\theta\left(1 - \tilde{\pi}_t^{ji}\right)Q_t^{\frac{-\theta}{1-\theta}}\left(\frac{P_t^{M,ji}}{P_t^M}\right)^{-\theta}\frac{M_t^F}{P_t^{M,ji}}
\end{aligned}$$

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[20] Some careful readers may wonder if GoS_t^{ji} also affects $\partial\hat{E}_t[S_t^{ji}]/\partial GoS_t^{ji}$ through g_t^{ji}. However, since $GoS_t^{ji} = M_t^{b,ji}e^{g_t^{ji}}$, $\frac{\partial}{\partial g_t^{ji}}\left(\int_{g_t^{ji}}^{\infty}GoS_t^{ji}f(dn_t^{ji}) + \int_{-\infty}^{g_t^{ji}}M_t^{b,ji}e^{n_t^{ji}}f(dn_t^{ji})\right) = 0.$
Also, see the last line of (7) to obtain a further intuition.

These results are used in Sections 3.2.2 and 3.2.3.

3.2.2 Time-Varying Markup

We first want to derive the FOC with respect to sales price $P_t^{M,ji}$. Let $\lambda_t^{U,ji}$ and $\lambda_t^{MC,ji}$ be the Lagrange multipliers on (20b) and (20e), respectively. We can interpret $\lambda_t^{U,ji}$ and $\lambda_t^{MC,ji}$ as the shadow price of U_{t+1}^{ji} at the end of t and marginal cost of production at t, respectively. Following the accounting terminology, we also refer to $\lambda_t^{U,ji}$ as *the cost of sales*. Noting the result of the previous subsection, we obtain

$$0 = \hat{E}_t[S_t^{ji}] - \theta\left(P_t^{M,ji} - \lambda_t^{U,ji}\right)\left(1 - \tilde{\pi}_t^{ji}\right) Q_t^{\frac{-\theta}{\theta-1}} \left(\frac{P_t^{M,ji}}{P_t^M}\right)^{-\theta} \frac{M_t^F}{P_t^{M,ji}} \tag{21}$$

Hence, a natural definition of markup is $P_t^{M,ji}/\lambda_t^{U,ji}$ rather than $P_t^{M,ji}/\lambda_t^{MC,ji}$ in our model. This is intuitively because, when sellers decide their sales price, they weigh up two possibilities, i.e., the stockout constraint is binding or not binding. If the demand is strong and the constraint is binding, the marginal GoS_t^{ji} is sold at price $P_t^{M,ji}$, while if not the marginal GoS_t^{ji} is carried to the next period as U_{t+1}^{ji} and its value is $\lambda_t^{U,ji}$.

Our model generates time-varying markup.[21] Since (i) $\hat{E}_t[S_t^{ji}] = Q_t M_t^{b,ji}$ because Q_t can be interpreted by the probability of stockout for buyers (see (9)), and (ii) $M_t^{b,ji} = Q_t^{\frac{-\theta}{\theta-1}}(P_t^{M,ji}/P_t^M)^{-\theta} M_t^F$ from the demand curve (see (15)), (21) can be rewritten as

$$P_t^{M,ji} = \theta\frac{1-\tilde{\pi}_t^{ji}}{Q_t}\left(P_t^{M,ji} - \lambda_t^{U,ji}\right) \tag{22}$$

Rearranging it, we obtain the time-varying markup formula.

$$P_t^{M,ji} = \frac{\tilde{\theta}_t^{ji}}{\tilde{\theta}_t^{ji} - 1}\lambda_t^{U,ji} \quad , \text{ where } \quad \tilde{\theta}_t^{ji} = \theta\frac{1-\tilde{\pi}_t^{ji}}{Q_t} \tag{23}$$

Previewing our simulation results, our model exhibits that, for both demand and supply shocks, the movements of P_t^M/λ_t^U and P_t^M/λ_t^{MC} are in parallel, and they are both

[21] Clearly, if buyers' and sellers' stockout probabilities are both zero, i.e., $1 - Q_t = \pi_t^{ji} = \tilde{\pi}_t^{ji} = 0$, then $S_t^{ji} = M_t^F$ (see (9) and (16)) and $\lambda_t^{U,ji} = \lambda_t^{MC,ji}$ (see (24c)). Hence, we obtain the standard constant markup formula in symmetric equilibrium; $P_t^{M,ji} = \frac{\theta}{\theta-1}\lambda_t^{MC,ji}$.

positively correlated with GDP,[22] while $\tilde{\theta}_t$ is countercyclical.

3.2.3 FOCs

In (24), the last two FOCs are rather standard; wage equals marginal product of labour, and net rental rate of capital equals the marginal product of capital. Note that the values of marginal products are evaluated in terms of marginal cost λ_t^{MC}. The first three FOCs are however peculiar to our model.

$$\begin{aligned}
P_t^{M,ji} &= \theta \frac{1-\tilde{\pi}_t^{ji}}{Q_t} \left(P_t^{M,ji} - \lambda_t^{U,ji} \right) && \text{(24a)} \\
0 &= E_t \left[\Lambda_{t,t+1} \left\{ P_{t+1}^{M,ji} \pi_t^{ji} + \lambda_{t+1}^{U,ji} \left(1 - \pi_t^{ji} \right) \right\} \right] - \lambda_t^{U,ji} && \text{(24b)} \\
0 &= P_t^{M,ji} \pi_t^{ji} + \lambda_t^{U,ji} \left(1 - \pi_t^{ji} \right) - \lambda_t^{MC,ji} && \text{(24c)} \\
0 &= \lambda_t^{MC,ji} \frac{\partial Y_t^{M,ji}}{\partial H_t^{ji}} - W_t && \text{(24d)} \\
0 &= \lambda_t^{MC,ji} \frac{\partial Y_t^{M,ji}}{\partial K_t^{ji}} - \left(R_t^K - 1 \right) && \text{(24e)}
\end{aligned}$$

The first FOC (24a) is with respect to unsold goods $P_t^{M,ji}$, which is detailed in Sections 3.2.1 and 3.2.2.

The second FOC (24b) is with respect to unsold goods U_{t+1}^{ji}. Note that this makes clear that $\lambda_t^{U,ji}$ is the shadow price of U_{t+1}^{ji} not U_t^{ji}. The inside of the curly bracket of (24b) means that *marginal* U_{t+1}^{ji} may be sold with probability π_{t+1}^{ji} or may be unsold with probability $1-\pi_{t+1}^{ji}$ at $t+1$; if it is sold it generates revenue $P_{t+1}^{M,ji}$, but if not it is carried to the next period $t+2$ and its value is $\lambda_{t+1}^{U,ji}$ at the end of $t+1$. Hence, the first constraint says that, at optimum, today's shadow price $\lambda_t^{U,ji}$ is equal to the present value (PV) of U_{t+1}^{ji}.

The third FOC (24c) is with respect to output $Y_t^{M,j}$. Similarly to (24b), if it is sold it generates revenue $P_t^{M,ji}$, but if not it is carried to the next period and its value is $\lambda_t^{U,ji}$. Hence, the second constraint says that, at optimum, marginal cost $\lambda_t^{MC,ji}$ is equal

[22] In this relation, (24c) below implies that the ratio of two profit margin concepts is $(P_t^{M,ji} - \lambda_t^{MC,ji})/(P_t^{M,ji} - \lambda_t^{U,ji}) = 1 - \pi_t^{ji}$. Also, we can show that $E_t[\lambda_{t+1}^{MC,ji}] = \lambda_t^{U,ji}$ and $\lambda_t^{MC,ji} > \lambda_t^{U,ji}$ (see (24b) and (24c)).

to the value of the marginal unit of $Y_t^{M,ji}$.

3.3 F-firms' Optimization, etc.

The role of F-firms is just to convert M-goods into F-goods; the output of F-firms $Y_t^F = M_t^F$. Also, we assume competitive F-goods market, and hence, from the zero profit condition, $\lambda_t^{F,MC} = 1$ (because F-goods price is normalised to be 1). Hence, from (13), effective M-price $Q_t^{\frac{-1}{\theta-1}} P_t^M$ must be one.

$$Q_t^{\frac{-1}{\theta-1}} P_t^M = 1 \tag{25}$$

In the following, we use M_t^F to indicate Y_t^F. See also Section 2.5.3 for F-firms' cost minimisation problem.

Finally, the supply of F-goods $M_t^F \left(= Y_t^F\right)$ is equal to consumption C_t plus investment I_t.

$$C_t + I_t = M_t^F \left(= Y_t^F\right) \tag{26}$$

3.4 Aggregation

In our model, aggregation is not trivial, because individual state variable U_t^{ji} differs among sellers due to idiosyncratic shock N_{t-1}^{ji}. In Appendix A.2, however, we show that, given different U_t^{ji}, M-firms choose different $Y_t^{M,ji}$ *so that* they have the same $GoS_t^{ji} = GoS_t$; in other words, all sellers choose the identical GoS_t^{ji} regardless of U_t^{ji}. This is mainly guaranteed by price posting and constant returns to scale (CRS) in production. Intuitively, CRS guarantees that all M-firms face same marginal cost λ_t^{MC}, while price posting makes M-firms' FOCs depend on $\hat{E}_t[S_t^{ji}]$, which is also common to all M-firms.

The only variables that differ among M-firms are U_{t+1}^{ji}, S_t^{ji}, K_t^{ji}, H_t^{ji} and $Y_t^{M,ji}$. It is straight forward to aggregate K_t^{ji}, H_t^{ji} and $Y_t^{M,ji}$ under CRS. Due to LLN over j (and trivially over i), aggregate sales S_t is equal to the expected sales of a seller $\hat{E}_t[S_t^{ji}]$ (see (7)).

$$S_t = M_t^b\,(1 - \tilde{\pi}_t) + GoS_t\pi_t \;(= \hat{E}_t[S_t^{ji}]) \tag{27}$$

where we use $M_t^{b,ji} = M_t^b$, which is true because $P_t^{M,ji} = P_t^M$ (see (15)) as shown in Appendix A.2. Knowing aggregate GoS_t, aggregate U_{t+1} is simply given by $U_{t+1} = GoS_t - S_t$.

For the other variables, due to the symmetricity of the equilibrium, we can obtain aggregating variables simply by dropping off superscripts j and i.

3.5 Equilibrium

The core part of the model has 19 endogenous variables and 19 equations. Given initial condition $\{U_0, K_0\}$, proper TVCs and exogenous shocks $\{\xi_t^M, \xi_t^g\}_{t=0}^{\infty}$, the equilibrium in our model is defined as the set of variables $\{R_t^B, R_t^K, W_t, P_t^M, Q_t, \pi_t, \tilde{\pi}_t, \lambda_t^U, \lambda_t^{MC}, C_t, H_t, M_t^F, Y_t^M, I_t, S_t, M_t^b, GoS_t, U_{t+1}, K_{t+1}\}_{t=0}^{\infty}$ that satisfy the following equations.

◊ HH's constraint (18b) and FOCs (19).

◊ M-firms' constraints (20b-e) and FOCs (24).

◊ F-firms' FOC (25) and F-market clearing (26).

◊ M-market equations (27), (6a), (6b) and (9).

Because all agents are effectively symmetric in our equilibrium, all superscripts ji should be dropped off from the above equilibrium equations. In addition to this core part, we also calibrate the behaviour of time-varying markup, I/S ratio, etc. in Section 5.

4 Analytical Results and Key Empirical Findings

Before showing numerical results, this section briefly discusses some analytical implications. Also, we briefly review the known key empirical findings, which we use to evaluate the model performance.

4.1 Inventory Facts

This subsection makes clear that two inventory stylised facts are just two different descriptions of a single fact. Typically, two facts are referred to as:[23]

Fact 1: Inventory investment is procyclical.

Fact 2: Output is more volatile than sales.

To see why they are identical, consider the law of motion of inventories: $U_{t+1} - U_t = Y_t^M - S_t$. Then, it is quite straightforward to show

$$Var\left(S_t\right) = Var\left(Y_t\right) + Var\left(U_{t+1} - U_t\right) - 2Cov\left(Y_t, U_{t+1} - U_t\right)$$

which means that $Cov\left(Y_t, U_{t+1} - U_t\right) > 0$ is a *necessary* condition of $Var\left(S_t\right) < Var\left(Y_t\right)$. With a similar manipulation, we can easily show that $Cov\left(S_t, U_{t+1} - U_t\right) > 0$ is a *sufficient* condition of $Var\left(S_t\right) < Var\left(Y_t\right)$. In sum,

a. If *procyclical* in Fact 1 means $Cov\left(Y_t, U_{t+1} - U_t\right) > 0$, then Fact 1 is a necessary condition for the Fact 2.

b. If *procyclical* in Fact 1 means $Cov\left(S_t, U_{t+1} - U_t\right) > 0$, then Fact 1 is a sufficient condition for Fact 2.

However, in data, sales and production move very closely to one another; thus, $Cov\left(Y_t, U_{t+1} - U_t\right) > 0$ and $Cov\left(S_t, U_{t+1} - U_t\right) > 0$ are nearly interchangeable. Hence, stylised Facts 1 and 2 are almost equivalent.

Also, many authors such as Ramey and West (1997) and Wen (2002) have pointed out the following fact.

Fact 3: Inventory to sales ratio is countercyclical and persistent.

[23] See also Wen (2002) for more detailed description of inventory facts.

4.2 Target Inventories

Consider the following general class of sales function.

$$S_t^{ji} = \left(\left(M_t^{ji}(P_t^{ji})\right)^{\psi} + \phi \left(GoS_t^{ji}\right)^{\psi} \right)^{\frac{1}{\psi}} \tag{28}$$

where $M_t^{ji}(.)$ is demand as a function of price P_t^{ji}, GoS_t^{ji} is goods on shelf (inventories), and ψ and ϕ are parameters. In this subsection, we drop off superscript M that indicates M-firms. The model reduces to the inventories as sales facilities model in Bils and Kahn (2000) if $\psi = 0$,[24] while it reduces to the stockout avoidance model when $\psi = -\infty$.

This class of sales functions generate the *target level of inventories*; the optimal level of inventories depends on the demand. Since $\partial^2 S_t^{ji}/\partial GoS_t^{ji}\partial M_t^{ji} > 0$, goods on shelf GoS_t^{ji} and demand M_t^{ji} are complements to each other in generating S_t^{ji}. That is, all other things being equal, stronger M_t^{ji} implies higher GoS_t^{ji} at optimum, and vice versa. In our case, however, exactly speaking, GoS_t^{ji} *and* M_t^{ji} *are complements in* $\hat{E}[S_t^{ji}]$ not in S_t^{ji}; $\partial \hat{E}[S_t^{ji}]/\partial GoS_t^{ji} = \pi_t^{ji}$ and stockout probability π_t^{ji} is clearly increasing in M_t^{ji} (see Section 3.2.1).

4.2.1 Distributors' Demand

The above discussion reveals an important implication, which we call *distributors' demand.*

$$Y_t^M = (U_{t+1} - U_t) + S_t \tag{29}$$

Consider the above law of motion of unsold goods, which explicitly shows that the supply/output of M-goods Y_t^M must be equal to sales S_t (or *final demand* in the parlance of business economists) and inventory investment $U_{t+1} - U_t$. In this subsection, for simplicity we assume that none of today's output can be placed on today's market so that $GoS_t = U_t$.

Clearly, if there are no inventories, then a unit increase in S_t is translated to a unit increase in production Y_t^M. Under the target level of inventories, however, inventory

[24]See also Jung and Yun (2005) as the general equilibrium extension of Bils and Kahn (2000).

investment $U_{t+1} - U_t$ is positive when sales *is expected to be* strong, because the target level of inventories is now higher. That is, when a positive demand shock hits an economy and it is persistent, not only does S_t increase but also $U_{t+1} - U_t$ must be positive. The point is, in the target inventory models, a demand shock is followed by additional demand $U_{t+1} - U_t$, which makes Y_t^M increase more than sales.

We call this additional demand *distributors' demand*, because sellers hold inventories to distribute their products (i.e., to overcome the stockout constraint). Although the exact economic interpretation differs among models, the common implication of (28) is that inventories GoS_t and demand M_t^F are complementary to each other in generating sales S_t (or its expectation). This is the essence of (28), and hence basically all the models in the class of (28) should have procyclical distributors' demand. Clearly, distributors' demand amplifies demand shocks; in target inventory models, $x\%$ increase in S_t leads to more than $x\%$ increase in Y_t^M.

Note that *expected* future demand is important in distributors' demand. That is, *the size of distributors' demand depends on the persistence of the demand shock*; if it is *iid*, for example, high S_t does not imply high S_{t+1} and hence the target level of GoS_{t+1} does not change (see Section 5.4.3 for the numerical results of *iid* shocks). Since distributors' demand reacts to the forecastable components of future (final) demand, it typically materialises at business cycle frequencies.

Finally, note that the discussion in this section ignores the feedback mechanism through the price system in the general equilibrium; prices change after an aggregate shock, which may suppress or may magnify fluctuations in final demand. Indeed, previewing the numerical results, under a plausible parameter range, distributors' demand suppresses the fluctuations of sales, and hence its total effect on output volatility is very small (see Section 5.4).

4.2.2 Stockout Avoidance vs. Buffer Stock Motives

In addition, inventories also work as buffers against unexpected demand shocks. That is, since firms do not want to adjust their production quickly,[25] they let inventories decrease right after a positive demand shock (countercyclical inventory investment). Note that both mechanisms – buffer stocks and distributors' demand – do not contradict each other, and they indeed coexist in our model. In our model, the production smoothing motive is evident in the FOC with respect to U_{t+1}^{ji} (24b); taking into account stockout probability and future profit margin, M-firms balance $\lambda_t^{U,ji}$ against $\lambda_{t+1}^{U,ji}$.[26]

Of course, our model also embeds the cost shock mechanism; in our model, productivity shock ξ_t^M directly affects marginal cost λ_t^{MC}. In sum, our model embraces the three leading mechanisms: stockout avoidance (distributors' demand), production smoothing and cost shock models.[27]

4.3 Inventories as Options to Sell

This subsection compares the FOC with respect to unsold goods U_{t+1}^{ji} with the Black-Scholes option pricing formula. That is, we claim that *having inventories is having options to sell.*

Note first that (24b) can be rewritten in Jorgenson's user cost representation.

$$E_t\left[\Lambda_{t,t+1}\left(P_{t+1}^{M,ji}-\lambda_{t+1}^{U,ji}\right)\Pr[M_{t+1}^{ji}>GoS_{t+1}^{ji}]\right]=\lambda_t^{U,ji}-E_t\left[\Lambda_{t,t+1}\lambda_{t+1}^{U,ji}\right] \tag{30}$$

Next, remember that Black-Scholes formula[28] of a call option can be rewritten as

$$E_t^{\mathbf{Q}}\left[e^{-r(T-t)}\left(\quad S_T-K\quad\right)\mathbf{1}\left(S_T>K\right)\right]=V_t^{call}$$

[25] Even if production technology exhibits CRS, as long as labour supply is convex (due to the concave utility function), this mechanism works.

[26] However, quantitatively, this effect is very weak under our CRS production technology and linear utility in leisure. In our model, inventories work as buffers mainly because we assume that labour supply cannot react to the current period aggregation shocks (see Sections 5.1 and 5.4), although the implications are not affected very much.

[27] For further discussions, see also Wen (2002) and Fitzgerald (1997) among others.

[28] For this representation, see equation (12.7) (and p.90 for notation) in Bjork (2004) among others.

where S_T is the price of underlying stock at expiration date T and K is the strike price. Also, indicator function $\mathbf{1}(S_T > K)$ is 1 if $S_T > K$ but is 0 otherwise, and $E_t^{\mathbf{Q}}[\ \ ]$ is the expectation operator under the risk-neutral probability. This expression simply says that the cost of purchasing a call option V_t^{call} is equal to the PV of $S_T - K$ conditional that the call is in-the-money (i.e., $S_T > K$) under the risk-neutral measure with respect to S_T.

There are clear one-to-one relationships.[29]

◊ cost of holding option: $\lambda_t^{U,ji} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,ji}]$ vs. V_t^{call}

◊ discount factor: $\Lambda_{t,t+1}$ vs. $e^{-r(T-t)}$

◊ profit margin: $P_{t+1}^{M,ji} - \lambda_{t+1}^{U,ji}$ if $GoS_{t+1}^{ji} > M_{t+1}^{ji}$ vs. $S_T - K$ if $S_T > K$

Note that, since M-firms are risk neutral by CRS, the difference between $E_t[\ \ ]$ and $E_t^{\mathbf{Q}}[\ \ ]$ does not really matter in this comparison. Finally, note that the option payoff is kinked on the maturity date but is differentiable (and hence an option delta exists) before the maturity date, because of the uncertainty in the stock price S_T, which is exactly parallel to the reason why we can differentiate the expected sales, but not sales itself.

5 Numerical Results

This section describes the quantitative properties of the model. The model developed in Sections 2 and 3 are numerically examined by linearising the equilibrium equations around the non-stochastic steady state (see Section 3.5). Note that aggregate sales S_t is a smooth function (though individual sales S_t^{ji} is not) and hence we can linearise it. Remember that we have two sources of shocks; productivity and preference shocks. We interpret the former as a supply shock and the latter as a demand shock, although we must be cautious about such labelling; for example, even the technology shock stimulates demands through wage and capital return.

[29] In addition, the difference between $N(d_2)$ and $N(d_1)$ in the standard Black-Scholes formula (see any textbook for these notations) is almost exactly the same as the difference between π_t and $\tilde{\pi}_t$. This is not by chance; we can interpret $N(d_2)$ as the probability that $S_T > K$ under the risk-neutral measure, while it can be shown that $N(d_1) = E_t^Q[S_T/S_t\ |S_T > K]N(d_2)$ (compare this with (6b)).

We compare our model performance with the U.S. data and the *no-inventory model*, which is identical to our main model except for $\sigma_N = 0$ (no idiosyncratic shock). Though there are several minor differences, such as imperfect substitution among varieties, the no-inventory model behaves in the almost same way as the standard RBC model.

Finally, note that, as mentioned above, in aggregate, our model falls into the class of the models with representative agents and flexible prices. One period in our model is assumed to be one quarter, and all variables are HP filtered with penalty parameter $\lambda_{HP} = 1600$ to obtain the second moments.

Table 1: Parameters

symbol	note	No-Inventories	Benchmark
β	subjective discount factor	0.986	0.986
γ	relative risk aversion	1.000	1.000
ψ	relative weight for leisure	0.650	0.650
θ	elasticity of substitution of Mgoods	7.500	7.500
σ_N	range parameter of the idio shock	**0.000**	0.400
α	capital share in production	0.350	0.350
δ	depreciation rate of capital	0.015	0.015
ρ_U	AR(1) coef of preference shock	0.990	0.990
ρ_M	AR(1) coef of productivity shock	0.990	0.990
σ_G	sd of innov to preference shock	0.01/0.00	0.01/0.00
σ_M	sd of innov to productivity shock	0.00/0.01	0.00/0.01
η	share of unobservable component in current period innovation for labour supply decision	1.000	1.000
λ_{HP}	HPfilter penality parameter	1,600	1,600

5.1 Parameter Selection and Information

We do not employ any optimal criteria for parameter selection. Rather, for RBC parameters, we follow the conventional values for comparison sake (see Table 1). For other parameters, elasticity of substitution among varieties θ is set to be 7.5, which is somewhat lower than the typical Dixit-Stigliz monopolistic competition models assume. Steady state stockout probability π_{ss} is mainly affected by θ, and setting $\theta = 7.5$ generates a plausible stockout probability 8.1% (see Bils (2004)).[30] For the standard deviation of the idiosyncratic shock σ_N, we choose $\sigma_N = 0.4$ so that inventory-to-sales ratio $U_{ss}/S_{ss} = 0.66$ (around two months).

[30] Exactly speaking, the key determinant of π_{ss} is the net profit margin. Indeed, we can have the same stockout probability by instead adding an annual convenience yield 1.3% with $\theta = 10.0$, which generates the almost same quantitative results as our benchmark model.

Also, we assume that *labour supply cannot respond to the current period aggregate shocks*; otherwise, we find that the behaviour of our model is quite similar to that of the standard RBC model. That is, if production can fully react to the aggregate shocks, then M-firms do not need to use inventories as buffers against aggregate shocks and effectively the aggregate shocks have no impact on inventories.[31] In Section 5.4, we investigate the effects of different information assumptions.

Table 2: Steady State Values

symbol	note	No-Inventories	Benchmark
R^B	gross return on bonds	1.015	1.015
R^K	net return on capital	0.030	0.030
W	wage	1.970	1.928
P^M	price index of Mgoods	1.000	0.996
Q	measure of available variety	1.000	0.972
π	Pr[Stockout] for sellers	0.000	0.081
π_{tilde}	cost of losing sales opportunity	0.000	0.158
λ^U	shadow price of unsold goods	0.867	0.842
λ^{MC}	marginal cost of Mproduction	0.867	0.855
C	consumption	1.061	1.038
H	hours	0.358	0.354
M^M	quantity Index (= Fgoods supply)	1.253	1.224
Y^M	output of Mgoods	1.253	1.229
I	capital investment	0.192	0.186
S	sales of Mgoods	1.253	1.229
U	unsold goods	0.000	0.816
K	capital	12.804	12.392

5.2 Steady State

In the steady state, consumption and investment are 85% and 15% of M-production Y_{ss}^M, respectively. Working hours are roughly 1/3 of time endowment, which is normalised to be 1. Capital/annual GDP ratio (K_{ss}/Y_{ss}^M) is around 2.5.

As mentioned above, sellers' stockout probability is 8.1%, and I/S ratio is 0.66 (= 0.816/1.229). Because of the cost of losing varieties (see Section 2.5 and equation (25)), M-price is strictly lower than the final goods price in the steady state; $P_{ss}^M < 1$. However, as seen in Table 2, it is almost one 0.996, because $Q_{ss} = 0.972$ is very close to 1 (i.e., the buyers' stockout probability is 2.8%).

[31] Even if all information is available for labour supply decision, inventory facts are qualitatively satisfied, but the fluctuation of inventories becomes very small. Note also that information assumption does not affect the steady state.

Table 3: Second Moments (HP-filtered, 1600)

	US Data		RBC		No-Inventory Model (σ_N = 0.0)				Benchmark Model (σ_N = 0.4)			
					Supply Shock		Demand Shock		Supply Shock		Demand Shock	
	rel sd	corr	rel sd	corr	rel sd	corr	rel sd	corr	rel sd	corr	rel sd	corr
Y^M	1.353%	1.000	1.351%	1.000	1.905%	1.000	1.076%	1.000	1.904%	1.000	1.080%	1.000
S^M	**0.848**	0.934	-	-	**1.000**	1.000	**1.000**	1.000	**0.931**	0.993	**0.820**	0.945
H	**1.241**	0.884	**0.769**	0.986	**0.597**	0.892	**1.563**	0.998	**0.599**	0.892	**1.559**	0.999
C	**0.823**	0.839	**0.329**	0.843	**0.466**	0.959	**0.618**	0.754	**0.448**	0.957	**0.601**	0.753
I	**4.776**	0.899	**5.954**	0.992	**4.115**	0.984	**4.541**	0.870	**3.679**	0.991	**2.820**	0.900
U/S^M	-	-	-	-	-	-	-	-	0.920	**-0.941**	0.991	**-0.900**
dU_{t+1}/Y^M	**0.366**	**0.561**	-	-	-	-	-	-	**0.133**	**0.571**	**0.350**	**0.644**
π	-	-	-	-	-	-	-	-	1.750	0.917	1.751	0.673
π_{tilde}	-	-	-	-	-	-	-	-	1.440	0.917	1.441	0.673
P^M/λ^U	-	-	-	-	-	-	-	-	0.040	**0.917**	0.040	**0.673**
P^M/λ^{MC}	-	-	-	-	-	-	-	-	0.011	**0.917**	0.011	**0.673**
θ_{tilde}	-	-	-	-	-	-	-	-	0.220	-0.917	0.221	-0.673
Y^M/H	0.589	**-0.164**	0.606	**0.978**	0.540	**0.866**	0.568	**-0.987**	0.539	**0.864**	0.562	**-0.990**

Notes

1. "rel sd" and "corr" are standard deviation relative to that of GDP (M-production) and correlation with GDP, respectively. But, rel sd of Y^M shows the sd of GDP. "-" stands for zero or n.a.
2. The sources of US data are US national economic accounts and current employment statistics from 1975Q1 to 2008Q4. The numbers of RBC are taken from Cooley and Prescott (1995).

5.3 Second Moments and Impulse Response Functions

5.3.1 RBC Facts

Our model inherits many from the standard RBC model, though its investment behaviour is somewhat deteriorated.

In terms of labour behaviour, our model performs in a quite similar way to the standard RBC model. Hours are less volatile than output for the supply shock and vice versa for the demand shock. Labour productivity is almost perfectly positively correlated to output Y_t^M for the supply shock. Having the demand shock, this is overturned, as found in Christiano and Eichenbaum (1992) for government expenditure shock and Bencivenga (1992) for preference shock, but not it is almost perfectly negatively correlated to Y_t^M.

For both shocks, investment is less volatile than the data. Its volatility is too low even compared with the no-inventory model. This shows that *inventories are competing with capital* in the sense that the former generates sales, while the latter generates output.

In a sense, inventory and capital investments crowd out each other.[32]

Note that, under our parameter setting, investment initially slightly decreases after a positive preference shock (see Figure 1). This is partly because of our information assumption; since both labour and capital inputs are determined before observing unexpectedly strong consumption, other demand components (i.e., inventory and capital investments) must decrease due to the resource constraint. It is not difficult to eliminate this sharp drop in investment by, e.g., setting $\gamma = 0.5$, the model has no initial sharp decrease in investment at the cost of lower consumption volatility. Because it is not our intention to discuss the value of the controversial parameter, we follow the standard value $\gamma = 1.0$ for the following discussions, although the model performs much better with $\gamma = 0.5$ in inventory facts.

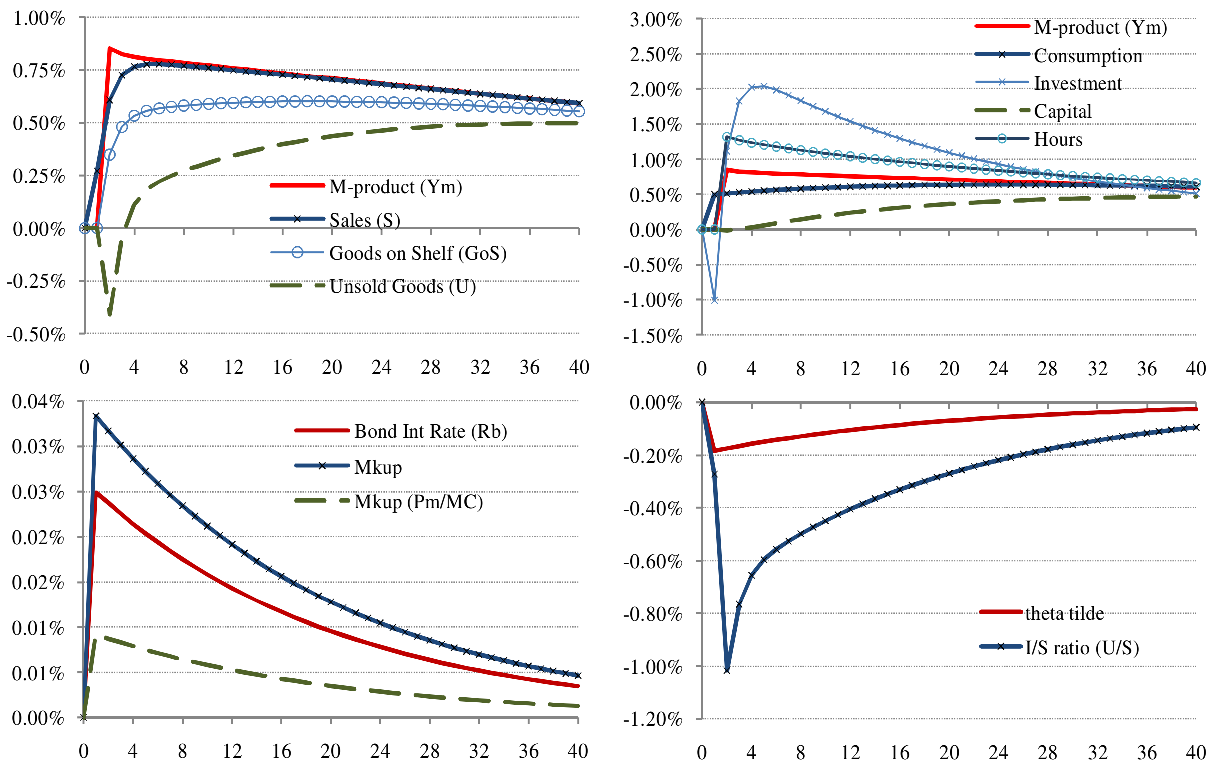

Figure 1: Selected impulse response functions to 1% positive preference shock.

[32]Khan and Thomas (2007a) also find a similar observation in their stockout constraint model.

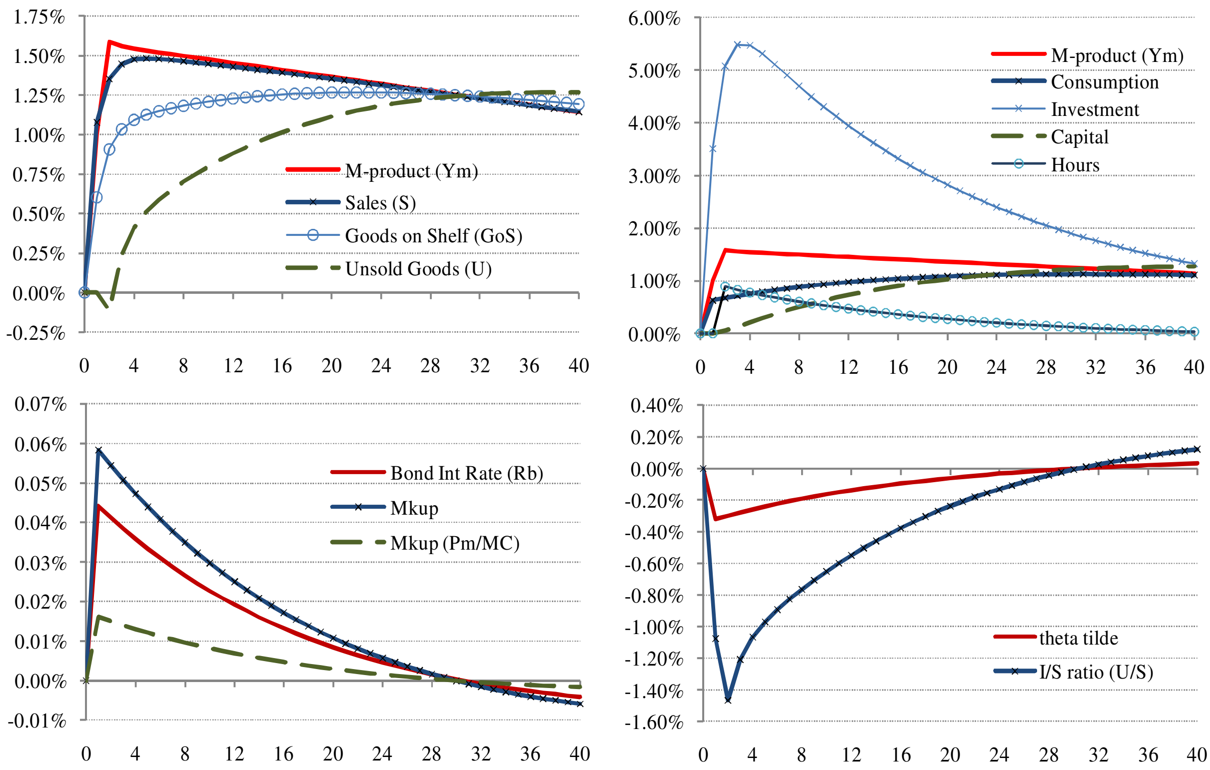

Figure 2: Selected impulse response functions to 1% positive productivity shock.

5.3.2 Inventory Facts

For both supply and demand shocks, production is more volatile than sales, and inventory investment is procyclical (see Table 3). With supply shocks, it is not surprising because the source of shocks lies on the side of production. In our model, however, even with demand shocks, production is more volatile than sales. The key intuition is *distributors' demand* as discussed in Section 4.2.1. Since the optimal level of inventories is increasing in demand, when a positive demand shock hits the economy, M-firms increase their production not only to accommodate the increase in demand but also to accumulate inventories (distributors' demand). This basic mechanism works even for a positive productivity shock, as long as demand increases.

Interestingly, however, the volatility of sales relative to that of production is lower for demand shocks (0.820) than for supply shocks (0.931), which may look strange at first glance, because the relative volatility of sales is lower when the source of shock lies on the demand side. The key is on the fact that *inventories also work as buffer stocks* *right after a shock*. On the one hand, the initial reaction of inventories to a positive supply shock is very small, simply because production as well as sales increases; as a result, there is only a little initial impact on inventories. In the subsequent periods, F-firms do not need to recover their inventories hence they do not need to increase their production very much. On the other hand, for a positive demand shock, initially inventories sharply decrease, simply because buyers take out inventories more than usual from sellers' shops, while supply does not change very much. In this case, M-firms must replenish reduced inventories before accumulating them. Hence, in subsequent periods, production increases sharper for the demand shock than for the supply shock. Due to the same reason, inventory investment for supply shocks is much less volatile than for demand shocks.[33]

5.3.3 Inventory to Sales Ratio

Inventory to sales ratio U_t/S_t (I/S ratio) is countercyclical and persistent in our model (see Table 3). Under our parameter setting, quantitatively the behaviour of I/S ratio is mainly governed by the carry cost of unsold goods $\lambda_t^{U,j} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,j}]$ (see (30)), and hence is governed by interest rate R_t^B (through SDF $\Lambda_{t,t+1}$). Intuitively, when the economy is in boom, high R_t^B discourages M-firms to have inventories (relative to expected sales), just because having inventories is more costly. Hence, roughly speaking, *countercyclicality and persistence of I/S ratio is due to the procyclicality and persistence of interest rate* in our model.

Table 4: Persistence of I/S ratio (HP-filtered, 1600)

	Data	**Benchmark Model (σ_N = 0.4)**	
		Demand Shock	Supply Shock
Cor(U_t/S_t, U_{t-1}/S_{t-1})	0.88 to 0.97	0.700	0.779
Cor(U_t/S_t, U_{t-2}/S_{t-2})	0.80 to 0.91	0.372	0.450

Note
The numbers under label "Data" are taken from Ramey and West (1997).

[33] While we cannot calculate the deviation of the *logarithm* of inventory investment $U_{t+1} - U_t$, the deviation of $U_{t+1} - U_t$ *itself* has the problem of the unit of measurement. To avoid this problem, instead we use $(U_{t+1} - U_t)/Y_t^M$ in our numerical results, following Khan and Thomas (2007a).

Quantitatively, our model generates less persistent I/S ratio than the numbers reported in Ramey and West (1997); especially, the second autocorrelation (0.372 and 0.450 for demand and supply shocks, respectively) is much lower than the data (see Table 4). Under our parameter range, the persistence of the exogenous shocks is the most dominating factor to account for the persistence of I/S ratio through R_t^B. However, see also Section 5.4.3 for the effects of inventories as buffer stocks in generating persistent output from *iid* demand shocks.

5.3.4 Time-Varying Markup and $\tilde{\theta}$

Our model exhibits that, for both demand and supply shocks, two markup concepts $P_t^M/\lambda_t^{U,j}$ and P_t^M/λ_t^{MC} are positively correlated with output (see Table 3), and their impulse response functions have almost identical shapes with different magnitude (see Figures 1 and 2). This is because $\tilde{\theta}_t$ is countercyclical for both shocks (see (23) for the definition of $\tilde{\theta}_t$). Intuitively, interest rate R_t^B tends to be high in boom, which leads to high carry cost of unsold goods U_{t+1}, and as a result M-firms accept high stockout probability π_t and a large loss of sales opportunity $\tilde{\pi}_t$. Since the effects of Q_t are quantitatively small under our parameter setting, the behaviour of $\tilde{\theta}_t$ is dominated by $\tilde{\pi}_t$.

A close investigation of the derivation of (23) reveals an implication of high $\tilde{\pi}_t$ on procyclical markup. That is, facing stockout, firms *a posteriori* regret that their sales price should have been higher; the decrease in demand per buyer does not matter, because stockout implies that there are still unsatisfied buyers in front of their shops (this is essentially what large $\tilde{\pi}_t$ means), and such a situation is more likely in booms. Intuitively, *when demand is strong, sellers care about their profit margin per sales but do not care about the reduction of demand per buyer, because it is likely that they can find unsatisfied buyers waiting outside their shops.* Hence, when π_t and $\tilde{\pi}_t$ are larger, the optimal M-price P_t^M is also higher relative to the cost of sales λ_t^U.

In the data, there seems to be mixed evidence. For example, Martins and Scarpetta

(1999) are supportive of a procyclical markup,[34] while Small (1997) and Nishimura, Ohkusa, and Ariga (1999) find some evidence of a countercyclical markup; others such as Marchetti (2002) draw an indefinite conclusion. See also Rotemberg and Woodford (1999) among others for the importance of the cyclicality of markup. Note that, although we find the procyclical markup *as a result* of the stockout constraint, it is quite possible to consider an exogenous stochastic markup *as a cause* of business cycle fluctuations. In such a case, a high markup may (or may not) lead to a recession.

5.4 Effects of Changing Parameters

In this subsection, we investigate the effects of changing three parameters: the standard deviation of idiosyncratic shocks σ_N, the share of observable component in the aggregate shocks η and the persistence of the exogenous shocks ρ_U and ρ_M.

We investigate the effects of changing σ_N and η, not only to check the robustness but also to draw some implications on the Great Moderation. In addition to the two leading explanations – good monetary policy and good luck –, Kahn, McConnell, and Perez-Quiros (2002) suggest that the increase in output stability observed since around 1980 in the U.S. may be due to the improvement in inventory management, which is perhaps induced by new IT technologies. In our model, good inventory management can be interpreted as a lower σ_N and more information available at the timing of labour supply decision. Previewing our numerical results, we find that there are only little effects of changing σ_N and η on output volatility. Remember that in our benchmark parameter setting, $\sigma_N = 0.4$ and we assume no current aggregation shocks can be observed for the labour supply decision.

In light of the importance of the durable goods sector in terms of the reduction in the volatility of total output, Kahn, McConnell, and Perez-Quiros (2002) reports the following key observations started at around early 1980s; (i) output volatility has decreased, which can be partly explained by the reduction in sales volatility; (ii) in light of the evolution of inventories (29), the reduction of output volatility relative to

[34] See also Bils and Kahn (2000).

sales volatility is mainly accounted for by the decreases in both inventory investment volatility and correlation between inventory investment and sales; (iii) the level and the fluctuation of I/S ratio have declined.

5.4.1 Size of Idiosyncratic shocks σ_N

First, for both shocks, changes in σ_N have little effect on output volatility in our model around $\sigma_N = 0.4$ (see Figure 3). What is suggestive in this exercise is that our model mimics most of above observations reasonably well, *except for sales volatility.* That is, for both shocks, as σ_N decreases, inventory investment volatility decreases, while its correlation with sales does not change very much around $\sigma_N = 0.4$ (see (ii) above); the level of I/S ratio decreases, because the option value of inventories decreases; and the volatility of I/S ratio also decreases for supply shocks, though it increases for demand shocks[35] (see (iii) above). Hence, for both shocks, *output volatility relative to sales volatility decrease as σ_N decreases.* However, *since sales volatility increases as σ_N decreases, output volatility itself decreases only a little.* The increase in sales volatility is mainly caused by the increase in investment volatility.

If the demand components such as investment and consumption were exogenous, perhaps the story would be simple; smaller σ_N causes a lower volatility in inventory investment, which reduces output volatility, as Kahn, McConnell, and Perez-Quiros (2002) discuss. However, in our model, both investment and consumption are endogenous; it seems that a kind of feedback mechanism through price system works too strongly; that is, *as σ_N decreases, inventory investment crowds out capital investment less severely (through a lower volatility in M-goods price), which increases sales volatility.* In this sense, while inventories destabilise the economy by creating distributors' demand, they also stabilise demand by crowding out capital investment.

[35] From the definition of I/S ratio (U_{t+1}/S_t),

$$Var\left(\ln U_{t+1}/S_t\right) = Var\left(\ln U_{t+1}\right) - 2Cov\left(\ln U_{t+1}, \ln S_t\right) + Var\left(\ln S_t\right)$$

we find that the volatility of I/S ratio for demand shocks increases due to the increase in sales volatility (the other two components contribute to it negatively).

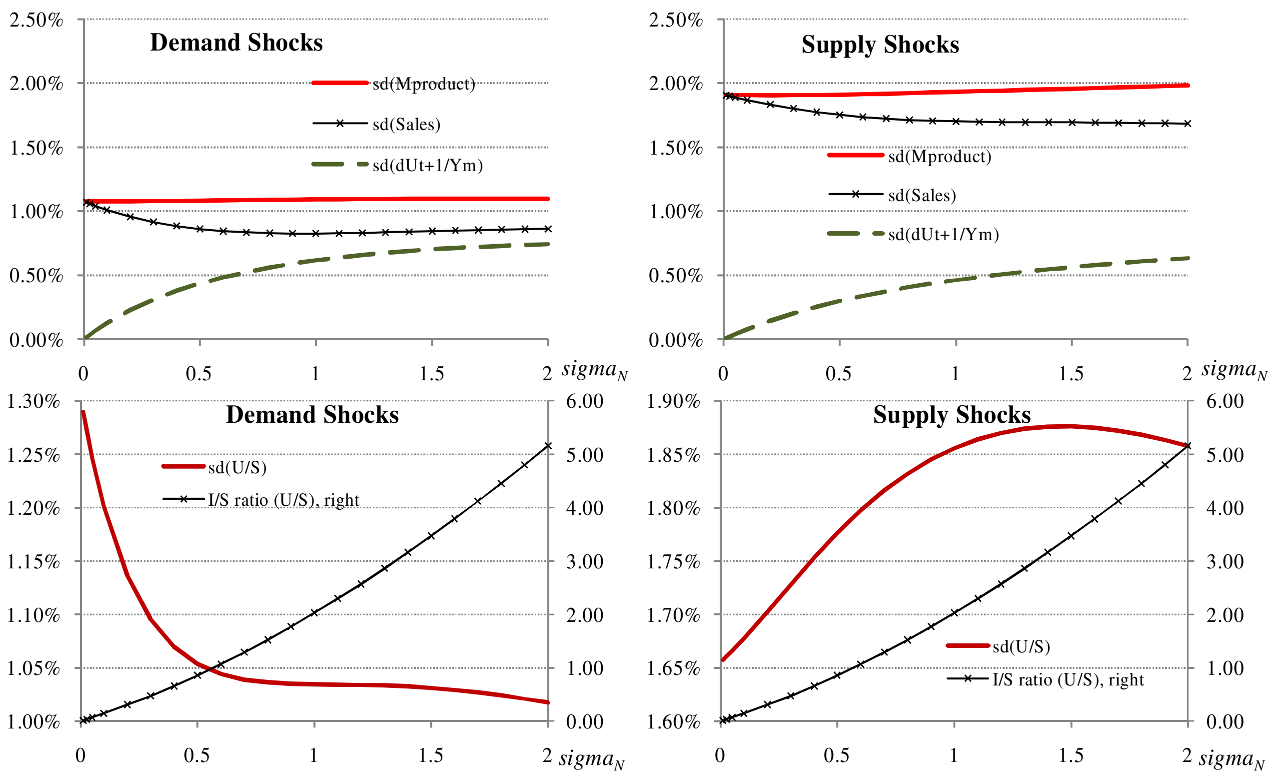

Figure 3: Second Moments of Different Sizes of Idiosyncratic Shocks (σ_N).

5.4.2 Information Available to Labour Supply Decision

To consider information available at the timing of the labour supply decision, we impose two restrictions to solve our model; if an aggregate shock ξ_t is not observable when HH decides labour supply H_t, then (i) H_t cannot react to ξ_t, and (ii) FOC with respect to labour supply (19b) holds only if $\xi_t = 0$.[36] Note that it is straightforward to decompose a shock ξ_t into observable component ξ_t^{ob} and unobservable component ξ_t^{un}; if for example $(1-\eta)\,\%$ of ξ_t is observable, let $\xi_t = \sqrt{1-\eta}\xi_t^{ob} + \sqrt{\eta}\xi_t^{un}$ for $0 \leq \eta \leq 1$[37] and apply the above two restrictions to ξ_t^{un}. In Figure 4, for example, $\eta = 0.5$ means that 50% of the current aggregate shocks are unobservable when HH decides its labour supply. It is straightforward to extend this idea; e.g., $\eta = 1.5$ means that all of the current aggregate shocks are unobservable and 50% of the aggregate shocks at time $t-1$ is also unobservable.

[36] See Shibayama (forthcoming) for the solution method used in this article. The matlab codes are available at:
http://www.kent.ac.uk/economics/research/papers/2007/0703.html

[37] Assume also that ξ_t^{ob}, ξ_t^{un} and ξ_t all follow a normal distribution with the same variance.

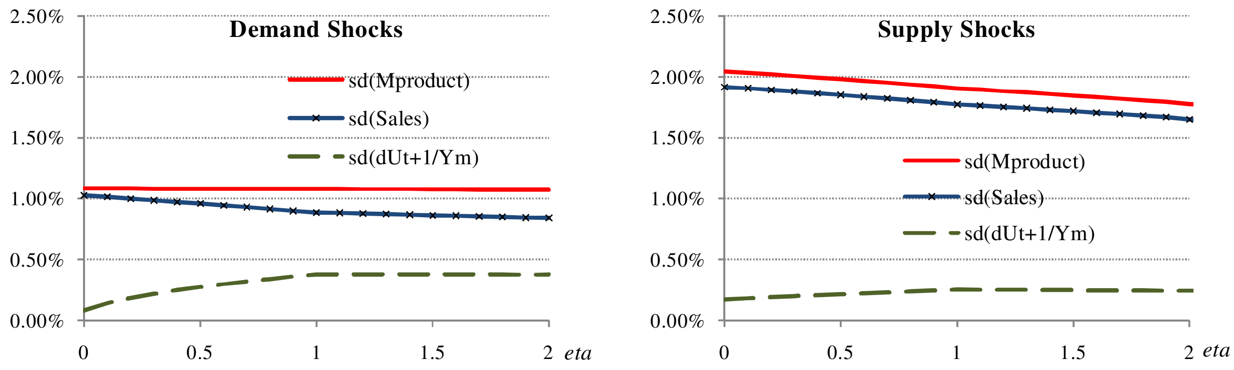

Figure 4: Second Moments for Different Information Available for Labour Supply Decision (η).

Figure 4 shows that, for both shocks, again output volatility does not change very much. The intuition is quite simple. On the one hand, a lack of information has a direct effect on output volatility; since labour supply cannot react to unobservable shocks, the lack of information directly suppresses output volatility. On the other hand, right after a positive shock, less responsive output causes the gap between sales and inventories. Hence, in the subsequent periods, production must increase so that inventories catch up with sales (distributors' demand). *These two effects offset each other.* Hence, for both shocks, as more information becomes available, output volatility relative to sales volatility decreases as η decreases but it is only very slightly, while sales volatility increases. Note that, since the information assumption is irrelevant to the *non-stochastic* steady state, I/S ratio at the steady state is not affected by η.

5.4.3 *iid* Shocks

This subsection examines the model behaviour with *iid* aggregate shocks. All in all, the model performance is quite poor with *iid* shocks and there is little empirical relevance. However, having *iid* shocks, we can eliminate the effects of distributors' demand, and inventories play only the role of buffer stocks. Table 5 summarises the results of *iid* shocks, in which (i) we fix capital investment at its steady state level to avoid nuisance crowding-out effects and (ii) we assume that labour supply is decided after observing all information up to the current period (perfect information) so that production smoothing is solely due to convex cost function. The main finding is that *inventories as buffer stocks generate some persistence.*

Table 5: Effects of Aggregate Shock Persistence (HP-filtered, 1600)

	Data	**No Inventories (σ_N = 0.0)**				**With Inventories (σ_N = 0.4)**			
		Supply Shock		Demand Shock		Supply Shock		Demand Shock	
		persistent	*iid*	persistent	*iid*	persistent	*iid*	persistent	*iid*
$\mathrm{Cor}(Y^M_t, Y^M_{t-1})$	0.850	0.722	-0.075	0.722	-0.076	0.684	-0.153	0.697	**0.406**
$\mathrm{Cor}(Y^M_t, Y^M_{t-2})$	0.669	0.485	-0.069	0.484	-0.072	0.432	-0.108	0.450	**0.170**
$\mathrm{Cor}(S_t, S_{t-1})$	0.843	0.722	-0.075	0.722	-0.076	0.741	**0.543**	0.734	-0.115
$\mathrm{Cor}(S_t, S_{t-2})$	0.717	0.485	-0.069	0.484	-0.072	0.511	**0.239**	0.501	-0.092
$\mathrm{Cor}(U_t/S_t, U_{t-1}/S_{t-1})$	0.88 to 0.97	-	-	-	-	0.565	**0.473**	0.551	**0.685**
$\mathrm{Cor}(U_t/S_t, U_{t-2}/S_{t-2})$	0.80 to 0.91	-	-	-	-	0.269	**0.204**	0.249	**0.310**
$\mathrm{Cor}(Y^M_t, S_t)$	0.934	1.000	1.000	1.000	1.000	0.989	0.584	0.995	0.783

Notes

1. A persistent shock means AR(1) coefficient on an exogenous shock is 0.990.
2. In this table, we assume (i) capital investment does not fluctuate, and (ii) HH can observe all information up to time t when it decides labour supply at t.
3. The data of autocorrelations of I/S ratio (U/S) are taken from Ramey and West (1997). The data sources of other figures are US national economic accounts and current employment statistics from 1975Q1 to 2008Q4.

For *iid* demand shocks, on the one hand, output is persistent, because of production smoothing. M-firms optimally choose to accommodate a suddenly strong demand by reducing inventories right after a positive demand shock, not by increasing production. In subsequent periods, M-firms increase their production to recover their lost inventories. Hence, inventories as buffer stocks generate persistent output from *iid* demand shocks; in this version of the model, the first autocorrelation of Y_t^M is 0.406. For *iid* supply shocks, on the other hand, sales is persistent, because of consumption smoothing. Rather than consuming a sudden increase in output at one time, such an increase in output is stored in the form of inventories. Hence, inventories as buffer stocks generate persistent sales from *iid* supply shocks. These exercises show that buffer stock inventories not only insulate production from demand shocks but also insulate demand from supply shocks in general equilibrium. Finally, with *iid* shocks, the correlation between output and sales is much lower, and, because inventories gradually return back to the steady state level, I/S ratio is persistent.

5.5 Summary of Numerical Results

In terms of RBC facts, our model inherits most features from the standard RBC model, though its performance in capital investment is not as good as the standard RBC model; this is mainly because inventory investment competes with capital investment and hence they crowd out each other.

In terms of inventory facts, the model performs fairly well. The key intuition is distributors' demand. Since the optimal level of inventories is increasing in demand, if there is one unit of increase in demand, the target level of inventories becomes higher. Hence, after a positive demand shock, inventory investment becomes positive (procyclical inventory investment), and output must increase more than sales to accumulate inventories (otherwise, inventory investment becomes negative). Interestingly, output volatility relative to sales volatility is larger when the source of shocks lies on the demand side. This is because of the interaction between buffer stock and stockout avoidance motives; initially, inventories decrease right after a positive demand shock (to insulate production from demand shock) but not decrease very much right after a positive supply shock. Hence, in the subsequent periods, production must increase much more sharply for the demand shock, because production must not only accumulate inventories but also replenish the reduced inventories.

In our discussion, the existence of inventories (due to the stockout constraint) may seem to amplify shocks at first glance; it would be true if demand were exogenous. In our general equilibrium model, however, although production is more volatile *than sales* because of the distributors' demand (procyclical inventory investment), inventories suppresses the fluctuations of capital investment (and hence that of sales). In this relation, perhaps it is fair to say that our model is not a strong support for the hypothesis that improvements in inventory management have reduced the output volatility in the data; the author conjectures that this might be because the general equilibrium feedback through price changes is too strong in our model.

Quantitatively, the behaviour of inventories relative to sales is most strongly affected by interest rate (through the carry cost of inventories). Hence, in booms, sellers optimally choose a lower I/S ratio by accepting high stockout probability, which leads to both countercyclical and persistent I/S ratio and procyclical markup. Note that a high stockout probability leads to a thick profit margin; in booms (when the stockout probability is high), sellers have an incentive to set a higher price (relative to the cost of sales) because, while a higher price gives them a better profit margin per demand, sellers do not care about the reduction of *demand per buyer* due to a higher price; a high stockout probability implies that it is likely that they can sell their goods to unsatisfied buyers and their *total sales* does not decrease.

6 Conclusion

This article investigates a rational dynamic general equilibrium model with the stockout constraint. The stockout constraint means that, even if demand is strong, sellers cannot sell more than the goods on shelf that they have; that is, sellers hold inventories to avoid stockout. The key trade-off is that, while (a) having too much inventories is costly because it leads to too high carry cost, (b) having too few inventories is also costly because it leads to too high stockout probability. While the former implies that the optimal level (relative to sales) is strongly affected by interest rate, the latter implies that the optimal level of inventories is increasing in demand.

Since the optimal level of inventories is increasing in demand, inventory investment is positive when sales is strong. This implies that one unit of increase in sales leads to more than one unit of increase in production; otherwise the law of motion of inventories implies that inventory investment becomes negative. In the parlance of business economists, under stockout constraint, one unit of final demand (i.e., sales) is followed by distributors demand (i.e., inventory investment). This is the main mechanism that our model generates the two inventory stylised facts: (i) inventory investment is procyclical, and (ii) production is more volatile than sales. Also, in our model, I/S ratio is countercyclical and persistent, and markup is procyclical; these are mainly because the optimal level of inventories (relative to sales) is decreasing in interest rate (through the carry cost of inventories).

The most closely related work to our model is Khan and Thomas (2007a), in which, while their (S,s) ordering model is quite successful in explaining inventory stylised facts, they find that their version of stockout constraint model performs poorly. Especially, their stockout constraint model fails to generate plausible average inventory level; even adding large idiosyncratic shocks, there is only a too low level of inventories in their model. Perhaps, the main differences between their model and ours are that we assume (i) a small degree of price stickiness and (ii) positive net profit margin. As discussed in Section 2, if there is no price stickiness at all, prices adjusts demand so that demand equals to supply (goods on shelf). Also, if the net profit margin is zero, due to the carry cost of inventories, firms optimally choose zero inventories (unsold goods), unless the carry cost becomes negative. Certainly, if individual marginal costs change drastically, firms may want to have inventories to exploit the *negative cost of holding inventories*, which we guess is the mechanism that large idiosyncratic shocks generate a certain level of inventories in Khan and Thomas (2007a). We argue however that such inventories are held not because of the stock avoidance motive but because of the production smoothing motive. Anyhow, one of our main analytical claims is that (i) and (ii) above are essential in modelling the stockout constraint.

Finally, as shown in our analytical and numerical results, the interest rate plays a key roll in determining the behaviour of inventories and markups (through the user cost of inventories). Although monetary policy is absent in our model, as Bernanke and Gertler (1995) suggest, inventories may have an important interaction with monetary policy. Though this article studies inventories to investigate their effects on real business cycles, it may be also interesting to investigate the role of inventories in monetary policy transmission.

A Appendix

A.1 Perfect Substitution

This appendix sketches the proof that the model economy reduces to the standard RBC model when $\theta \to \infty$ (perfect substitute).

First, dividing (21) by θ, it is clear that, as $\theta \to \infty$, $P_t^{M,ji} - \lambda_t^{U,ji} \to 0$ and/or $\tilde{\pi}_t^{ji} \to 1$ must hold. However, only the latter holds because, if the former were true, (24b) would not satisfy a TVC; indeed, it can be shown that, at the limit, (24b) reduces to $E_t[\Lambda_{t,t+1}\{P_{t+1}^{M,ji} - \lambda_{t+1}^{U,ji}\}] = \lambda_t^{U,ji} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,ji}]$ (the expected gross profit margin just covers the carry cost of inventories) and $\lambda_t^{U,ji} = E_t[\lambda_{t+1}^{MC,ji}] \neq 0$ (we can calculate the shadow price of U_{t+1}^{ji}, even though U_{t+1}^{ji} does not exist). Also, because $P_t^{M,ji} \to \lambda_t^{MC,ji}$ as $\theta \to \infty$ and CRS guarantees $\lambda_t^{MC,ji}$ is the same for all ji, (24c) directly shows that $P_t^{M,ji} \to P_t^M$. Furthermore, (25) implies $P_t^M \to 1$ (= F-goods price).

Second, (6a) implies that $g_t^{ji} \to g_l$ and hence $\pi_t^{ji} \to 1$, as $\tilde{\pi}_t^{ji} \to 1$, where g_l is the lower support of g_t^{ji} (under our log-normal distribution assumption, $g_l = -\infty$). Intuitively, as $\theta \to \infty$, the net profit margin (gross profit margin minus the carry cost) becomes zero, meaning that (i) even if the marginal U_{t+1}^{ji} is sold in the next period, sellers does not appreciate such a sales very much since the net profit is zero, but (ii) if U_{t+1}^{ji} is unsold, sellers simply have to pay its carry cost $\lambda_t^{U,j} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,j}]$. That is, having unsold goods is a one-sided unfair betting; if you will you get zero, but if you lose you have to pay. Because sellers do not care about the loss of sales opportunity, sellers optimally choose GoS_t^{ji} as if they will surely see the minimum possible number of buyers N_l ($N_t^{ji} = N_l$ for all ji). Hence, *the stockout always takes place,* and effectively there is no demand uncertainty. Thus, $U_{t+1}^{ji} \to 0$ for all ji, unless the carry cost is negative; i.e., unless sellers expect a sharp drop in the marginal cost of production in the future (in such a case, sellers want to hold inventories due to the capital gain on inventories, not due to stockout avoidance motive).

Third, (15) implies that $M_t^{b,ji} \to M_t^F/Q_t$, which means that, if a buyer has an access to Q_t% of varieties, he buy each available variety by M_t^F/Q_t, and his total purchase is just M_t^F. Since $Q_t \to N_l$ (intuitively because $1 - Q_t$ is the stockout probability for buyers and such probability must be $(1 - N_l)/1$, where 1 is the total number (measure) of buyers), $M_t^{ji} \to M_t^F$. Neither N_l nor g_l affects M_t^{ji}. Hence, $GoS_t^{ji} \to M_t^F$ for all ji. At the limit, $M_t^F = Y_t^M = GoS_t^{ji}$ since $U_{t+1}^{ji} = 0$ (again, unless $\lambda_t^{U,j} - E_t[\Lambda_{t,t+1}\lambda_{t+1}^{U,j}] < 0$).

Note that, in our log-normality assumption, $N_l = 0$, and hence $Q_t \to 0$ and $M_t^{b,ji} \to \infty$, which may sound strange. But, the above results still hold, because they approach to 0 and ∞ at balanced speeds; $Q_t M_t^{b,ji} = M_t^F$. Roughly speaking, this is the situation in which each buyer can buy an infinitesimally small number of varieties, say, one out of a million, and buy a huge amount of this single variety of goods. This does not cause any problem; since all goods are perfect substitute, buyers do not need to visit more than one market.

In sum, unless the carry cost becomes negative, at the limit that $\theta \to \infty$, (i) there is no unsold goods $U_{t+1}^{ji} = 0$ and hence no inventory investment $U_{t+1}^{ji} - U_{t+1}^{ji} = 0$, (ii) sales equals output; $GoS_t^{ji} = Y_t^{M,ji} = S_t^{ji} = M_t^F$, (iii) buyers can achieve their purchasing index M_t^F without suffering from the cost of losing varieties, (iv) sellers choose the marginal cost pricing; $P_t^{M,ji} = \lambda_t^{MC} = 1$ (= F-goods price), and (v) although stockout always takes place, stockout does not have any importance for both sellers and buyers. Since all other parts of the equilibrium are the same as the standard RBC model, this completes the sketch of the proof. These results hold for a general class of distribution functions of the idiosyncratic shock.

A.2 Symmetricity in Equilibrium

This subsection shows our equilibrium is symmetric by proving that FOCs (24a-c) imply that $\lambda_t^{U,ji}$, $P_t^{M,ji}$, π_t^{ji} and GoS_t^{ji} are the same for all ji at optimum. The following results crucially depend on the assumption of CRS, which guarantees that λ_t^{MC} is common to all sellers.

Because $\lambda_{t+1}^{MC,ji} = \lambda_{t+1}^{MC}$, (24b) and (24c) imply that $\lambda_t^{U,ji} = \lambda_t^U$ (Note that SDF $\Lambda_{t,t+1}$

is common to all firms). From (24a) and (24c),

$$\pi_t^{ji}\lambda_t^U + \left(\lambda_t^{MC} - \lambda_t^U\right) = \theta\frac{1-\tilde{\pi}_t^{ji}}{Q_t}\left(\lambda_t^{MC} - \lambda_t^U\right)$$

Because π_t^{ji} and $\tilde{\pi}_t^{ji}$ are both *strictly* increasing in g_t^{ji} under log-normal N_t^{ji} and because LHS and RHS are strictly increasing and decreasing in g_t^{ji}, respectively ($\lambda_t^{MC} - \lambda_t^U > 0$), this equation uniquely pins down g_t^{ji} as a function of variables that are independent from ji. Hence, $\pi_t^{ji} = \pi_t$ and $\tilde{\pi}_t^{ji} = \tilde{\pi}_t$ for all ji. Since g_t^{ji} does not depend on ji, GoS_t^{ji} is the same for all ji. See Section 3.4 for the remaining discussions.

A.3 Search Externalities

This subsection documents a kind of search externalities in M-markets.

On the buyers' side, each buyer ignores the negative effect of *congestion*. Intuitively, if buyers buy more, then available varieties ($Q_t = \Pr[\text{can buy}]$) become fewer because stockout arises more often, but infinitesimal buyers ignore such an effect. Consider the same cost minimisation in Section 2.5.3 with two-stage budgeting (14).

$$\min Q_t^{\frac{-1}{\theta-1}} P_t^M M_t^F \qquad \text{s.t.} \quad Y_t^F \geq M_t^F$$

If Q_t is given, clearly we obtain (13)

$$Q_t^{\frac{-1}{\theta-1}} P_t^M = \lambda_t^{MC} \tag{31}$$

However, if there is, say, a strong union of purchasing managers that coordinates buyers' decisions, it is

$$Q_t^{\frac{-1}{\theta-1}} P_t^M \left(1 + \frac{-1}{\theta-1}\frac{\partial Q_t/Q_t}{\partial M_t^F/M_t^F}\right) = \lambda_t^{MC} \tag{32}$$

which implies that the social cost (RHS of (32)) is larger than the private cost (RHS of (31)). The additional term shows the effect of congestion, which infinitesimal buyers ignore.

On the sellers' side, similarly, if there is a powerful union of sellers that coordinates sellers, the FOC with respect to unsold goods U_{t+1} is

$$E_t\left[\Lambda_{t,t+1}\left\{\begin{array}{c}\left(P_{t+1}^{M,i}-\lambda_{t+1}^{U,j}\right)Pr_{t+1}\\-\frac{\theta}{\theta-1}\left(1-Pr_{t+1}\right)\frac{M_{t+1}^{b}}{U_{t+1}}\frac{\partial Q_{t+1}/Q_{t+1}}{\partial U_{t+1}/U_{t+1}}\end{array}\right\}\right]=\lambda_t^{U,j}-E_t\left[\Lambda_{t,t+1}\lambda_{t+1}^{U,j}\right]$$

where the additional term $-\frac{\theta}{\theta-1}\left(1-Pr_{t+1}\right)\frac{M_{t+1}^{b}}{U_{t+1}}\frac{\partial Q_{t+1}/Q_{t+1}}{\partial U_{t+1}/U_{t+1}}<0$ shows the effect through *cost effect of losing variety* (compare this with (24b)). That is, having less GoS_t, the sellers' union wants to squeeze buyers, because by doing so physical demands (i.e., actual sales quantities) become larger due to lower Q_t. Exactly the same externality appears in the FOC with respect to M-production Y_t^M.

However, since the fluctuation of Q_t is very small in most cases, it seems that the effects of these externalities are rather small.

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