Low-Complexity Hybrid Digital-to-Analog Beamforming for Millimeter-Wave Systems with High User Density

Manish Nair¹, Qasim Zeeshan Ahmed², Junyuan Wang¹ and Huiling Zhu¹
¹School of Engineering and Digital Arts, University of Kent, Canterbury, CT2 7NT, United Kingdom.
²University of Huddersfield, Queensgate, Huddersfield, HD1 3DH, United Kingdom.
Email: {mn307, jw712 and h.zhu}@kent.ac.uk, q.ahmed@hud.ac.uk

Abstract—Supporting high user density and improving millimeter-wave (mm-Wave) spectral-efficiency (SE) is imperative in 5G systems. Current hybrid digital-to-analog beamforming (D-A BF) base stations (BS) can only support a particular user per radio frequency (RF) chain, which severely restricts mm-Wave SE. In this paper a novel low-complexity selection combining (LC-SC) is proposed for supporting high user density for mm-Wave BS. When compared with the current state of the art hybrid D-A BF, simulations show that LC-SC can support high user density and attain higher SE.

I. INTRODUCTION

Millimeter-wave (mm-Wave) cellular bands can significantly enhance spectral efficiency (SE) in 5G cellular systems [1]. A complete digital system having radio frequency (RF) transceiver chain per antenna (ANT) element cannot be implemented in base stations (BS) at mm-Wave due to the cost and complexity [2]. A practical 5G BS deploys a small number of RF chains with each RF chain supporting a massive number of transmit (Tx) antennas, resulting in hybrid digital-to analog beamforming (D-A BF) [3].

For the hybrid D-A BF BS schemes proposed in [4], [5], the digital beamformer is identity and the analog beamformer is the channel hermitian. However, the major drawback in this type of hybrid D-A BF structure is that each RF chain can only support a particular user, and the maximum number of users that can be supported by the BS cannot exceed the number of RF chains [4]. This will severely limit the SE of the future mm-Wave 5G environments such as train stations, stadiums or shopping malls. Therefore, it is of paramount necessity to design new hybrid D-A BF schemes which can support multiple users by employing a single RF chain and achieve similar SE as in the hybrid D-A BF techniques proposed in [4], [5]. Superposition coding can be applied to the Tx symbols on a single stream to support multiple users through a RF chain. However, it cannot serve multiple users simultaneously as only a single 3 dimensional (3D) beam is formed [6].

In this paper, a new hybrid D-A BF algorithm for supporting high user density is proposed, where each user will have its own separate 3D beam. This algorithm is the low-complexity selection combining (LC-SC). Our proposed hybrid D-A BF algorithm also accounts for the 3D mm-Wave channel for a high user density mm-Wave system which is generated when planar antenna (ANT) arrays are employed [7]. LC-SC is a space-time analog beamforming (A-BF) technique which modifies the A-BF matrix by designating a set of antenna elements to each user. The users and antennas are selected depending upon their instantaneous channel state information (CSI).

Simulation results corroborate that the proposed hybrid D-A BFs using LC-SC algorithm achieves superior SE compared to other hybrid D-A BF algorithms as proposed in [4], [5].

The reminder of this paper is organized as follows. Section II describes the system model. In Section III, the hybrid D-A BF LC-SC algorithm is proposed. Section IV presents the simulation results. The paper is concluded in Section VI. Throughout this paper, upper case and lower case boldfaces are used for matrices and vectors, respectively. \(X, X^T, X^H\) denote a matrix transpose and hermitian respectively. \(|| \cdot ||\) and \(|| \cdot ||_F\) represents the norm and Frobenius norm, respectively. Lastly, \(\mathbf{I}\) is the identity matrix.

II. DESCRIPTION OF THE HYBRID D-A BF SYSTEM

The block diagram of the current hybrid D-A BF BS system is shown in Fig. 1 [4], [5]. Each of the \(N\) RF chains is connected to a large-scale array of \(M\) identical antennas. In this paper, the analysis is initially carried out considering a downlink scenario for an \(i\)-th RF chain supporting only a particular \(k\)-th user, where \(0 < i \leq N - 1\) and \(0 < k \leq K - 1\). Subsequently, two hybrid D-A BF algorithms, LC-SC and PC, are considered for scaling-up the number of users supported by the \(i\)-th RF chain by \(K\), where \(K \leq M\). Furthermore, for the \(i\)-th RF chain, the A-BF is performed over \(M\) antennas and \(L\) time-slots by the space-time analog beamformer \(\mathbf{a}_i(t)\). The complete digital beamformer \(\mathbf{D} = \text{diag}[d_1, d_2, \ldots, d_N]\) is an \(N \times N\) matrix accounting for all \(N\) RF chains of the BS.
A. 3D mm-Wave Modified SV Channel Model

The 3D mm-wave modified Saleh-Valenzuela (SV) channel modeled by L-path coefficients [8–14] is given by

\[ h_{i,m,k}(t) = \sum_{\nu=0}^{\nu-1} \sum_{u=0}^{u-1} \beta_{\nu,m,uv,k} h_{i,m,uv,k}(t - \tau_{uv}) \]

\[ = \sum_{\tau=0}^{L-1} \beta_{\nu,m,i,k} h_{\nu,m,i,k}(t - l \tau), \quad (1) \]

where \( h_{\nu,m,i,k} \) is the k-th user convolutional impulse response (CIR) of l-th resolvable multi-path for the m-th Tx antenna in the i-th RF chain. \( V \) denotes the number of clusters, \( U \) the number of resolvable multi-path in one cluster, and \( L = UV \) is the total number of resolvable multi-paths at the receiver, \( l \) is related to \( u \) and \( v \) by \( l = l(U + u) \), \( h_{\nu,m,uv,k} = |h_{\nu,m,uv,k}|e^{j\theta_{uv}} \) represents the fading gain of the u-th resolvable multi-path in the v-th cluster connecting the m-th antenna in the i-th RF chain to the k-th user. \( \tau_{uv} \) is the time-of-arrival (ToA) of the v-th cluster and \( \tau_{uv} = u \tau \) denotes the ToA of the u-th resolvable multi-path in the v-th cluster. In our mm-Wave channel, it is assumed that the average power of a multi-path at a given delay is related to the power of the first resolvable multi-path of the first cluster through the following relationship [9], [10]

\[ P_{uv}^k = P_{00}^k \exp\left(-\frac{\tau_{uv}}{\Psi}\right) \exp\left(-\frac{\tau_{uv}}{\psi}\right), \quad (2) \]

where \( P_{uv}^k = |h_{\nu,m,uv,k}|^2 \) represents the expected power of the u-th resolvable multi-path in the v-th cluster connecting the k-th user to the m-th antenna in the i-th RF chain. \( \Psi \) and \( \psi \) are the corresponding power delay constants of the cluster and the resolvable multi-path respectively. For the channel model to be generic, we assume that the delay spread, which is \((L - 1)\tau\) for the mm-Wave channel, spans \( g \geq 1 \) data bits, satisfying \((g - 1)N_T \leq (L - 1)\tau \leq gN_T\), where \( N_T \) is the number of time slots per symbol [9], [10]. Secondly, we assume that the L resolvable multi-path components are randomly distributed and does not change over each symbol. Due to the wide bandwidth at mm-wave, all the L multi-path components can be potentially resolved at the receiver (Rx) side [15]. Lastly, the k-th user’s 3D BF gain \( \beta_{\nu,m,uv,k} = \beta_{\nu,m,i,k} \) for the m-th Tx antenna of the i-th RF chain is given in (3) shown at the bottom of this page. In (3), \( F_{Rx,V} \) and \( F_{Rx,H} \) are the Rx antenna radiation patterns for the vertical (V) and horizontal (H) polarizations, respectively. \( F_{Tx,i,V} \) and \( F_{Tx,i,H} \) are the corresponding vertical (V) and horizontal (H) polarizations for the i-th RF chain. \( \phi_{V}^i, V, \phi_{H}^i, V \) and \( \phi_{H}^i, H \) are the initial random phases for vertical (VV), cross (VH, HV), and horizontal polarizations (HH) for the i-th multi-path. \( \kappa_m \) is the intra-cluster Rician K-factor associated with the m-th Tx antenna cluster. \( \phi_1 \) and \( \phi_2 \) are the elevation and azimuth angle-of-arrival (AoA), respectively, at the k-th user. Finally, \( \beta_{1,m} \) and \( \phi_{1,m} \) are the elevation and azimuth angle-of-departure (AoD) of the l-th resolvable multi-path from the m-th Tx antenna in the i-th RF chain.

B. Received Symbols of the Hybrid D-A BF System

The k-th user \( L \times 1 \) received symbol vector is given by

\[ y_{i,k}(t) = H_{i,k}(t)z_i(t) + n_{i,k}(t), \quad (4) \]

where \( H_{i,k}(t) = L \times (2L - 1) \) space-time channel matrix associating the i-th RF chain having M Tx antennas with k-th user given by (6) in the following page. In (6), \( H_{i,m,k}(t) = L \times (2L - 1) \) Block-Toeplitz space-time CIR matrix associating the m-th Tx antenna of the i-th RF chain with the k-th user, and is given by (7). In (4)

\[ z_i(t) = |z_0(t), z_1(t - 1), \ldots, z_{L-1}(t - L + 1)|^T \]

is the \( M(2L - 1) \times 1 \) beamformed data symbol vector associated with the i-th RF chain. \( n_{i,k}(t) = L \times 1 \) complex Gaussian channel vector with a co-variance of \( 2\alpha_i^2 I \) for the k-th user.

C. Space-Time Analog Beamformer for the mm-Wave Hybrid D-A BF System

The beamformed data vector \( z_i(t) \) in (4) is generated by the \( M(2L - 1) \times 1 \) space-time analog beamformer \( a_i(t) \) operating over the \( L \times 2(2L - 1) \) space-time channel matrix \( H_{i,k}(t) \) as well as the information bearing symbol \( x_{i,k}(t) \) with \( E[x_{i,k}(t)^j x_{i,k}^H(t)^l] = \gamma_0 \), where \( \gamma_0 \) is the expected transmitted symbol power. \( x_{i,k}(t) \) is given by

\[ z_i(t) = a_i(t)d_i x_{i,k}(t), \quad (5) \]

where \( d_i \) is the i-th element of the D-BF matrix in Fig. 1 which corresponds to the i-th RF chain of the BS. The D-BF matrix is initially taken as identity [4], [5]. Therefore in (5), \( d_i = 1 \).

\[ a_i(t) = M(2L - 1) \times 1 \) our proposed novel space-time analog beamformer given by (8a), where the m-th element of \( a_i(t) \), denoted by \( a_{i,m}(t) \), is a normalized \( (2L - 1) \times 1 \) vector given in (8b). The \( L \times 1 \) k-th user’s Rx signal from the i-th RF chain is denoted by the vector \( H_{i,k}(t) a_i(t) \) in (9).

Lastly, the dimensions of (6) – (9) have been indicated by their respective under-braces.

D. Receive SNR and Spectral Efficiency for the Hybrid D-A BF System

The signal to noise ratio (SNR) of the i-th RF chain is denoted by \( \gamma_i \) and is given as [16]

\[ \gamma_i(d_i, a_i(t), H_{i,k}(t)) = \frac{70}{L^2} \sum_{l=0}^{L-1} |H_{i,k}(t)a_i(t)d_i|^2 \]

The SE (bps/Hz) for the k-th user associated with the i-th RF chain can be obtained as [16]

\[ \eta_{i,k} = \log_2 \left[1 + \gamma_i(d_i, a_i(t), H_{i,k}(t)) \right] \]

\[ \beta_{i,m,l,k} = \sqrt{T_{i,k}} \left[ F_{Rx,V}(\phi_2, \theta_i) F_{Rx,H}(\phi_1, \theta_i) \right]^T \left[ e^{j\phi_H^{i, V}} e^{j\phi_H^{i, H}} \right] \]

\[ \begin{bmatrix} e^{j\phi_V^{i, V}} \sqrt{\kappa_m} \sqrt{\kappa_m} \end{bmatrix} \begin{bmatrix} F_{Tx,i,V}(\phi_{1,m}, \theta_{1,m}) \ F_{Tx,i,H}(\phi_{1,m}, \theta_{1,m}) \end{bmatrix} \]

(3)
This process is repeated maximum power, i.e., associated with the SV channel effects and a set of simple Rx BF weights at every antenna elements that are allocated to other users. MUI can be interference (MUI) from the beamformed signals generated from users will be discussed in further detail in Section IV and Section V using the \( i \)-th RF chain, \( i \)-th antenna in the \( m \)-th antenna of the \( M \) with \( i \)-th symbol stream onto the \( M \) antennas where \( i \)-th user by the combination of mm-Wave users are selected for \( s \)-th user. It will experience multi-user interference multi-user inter-
ference

\[
H_{i,k}(t) = \left[ H_{i,0,k}(t), H_{i,1,k}(t), \ldots, H_{i,M-1,k}(t) \right]
\]

where

\[
a_i(t) = \left[ a_{i,0}(t), a_{i,1}(t), \ldots, a_{i,M-1}(t) \right]^T
\]

or

\[
a_i(t) = \left[ a_{i,0}(t), a_{i,1}(t), \ldots, a_{i,m,k}(t) \right]^T
\]

where

\[
H_{i,k}(t) a_i(t) = \sum_{m=0}^{M-1} H_{i,m,k}(t) a_{i,m}(t)
\]

or

\[
H_{i,k}(t) a_i(t) = \sum_{m=0}^{M-1} H_{i,m,k}(t) a_{i,m}(t)
\]

with

\[
\begin{bmatrix}
|h_{i,m,0,k}(t)|^2 & \cdots & |h_{i,m,L-1,k}(t)|^2 \\
|h_{i,m,0,k}(t)|^2 & \cdots & |h_{i,m,L-1,k}(t)|^2 \\
\vdots & \ddots & \vdots \\
|h_{i,m,0,k}(t)|^2 & \cdots & |h_{i,m,L-1,k}(t)|^2 \\
\end{bmatrix}^T
\]

III. BEAMFORMER DESIGN FOR HYBRID D-A BF

Suppose \( K \) users are to be supported by the \( i \)-th RF chain with \( M \) Tx antennas, where \( K \leq M \). The allocation of antennas is based on the calculation of instantaneous power of the 3D mm-Wave modified SV channel for every user. For the \( m \)-th antenna in the \( i \)-th RF chain, the channel power associated with the \( k \)-th user is calculated as

\[
p_{i,m,k} = \sum_{l=0}^{L-1} |h_{i,m,l,k}|^2, k = 1, \ldots, K
\]

The \( m \)-th antenna is then assigned to that user which has the maximum power, i.e.,

\[
k_m = \arg\max_k \{p_{i,m,0}, p_{i,m,1}, \ldots, p_{i,m,K-1}\}
\]

This process is repeated \( M \) times until all the \( M \) antennas are allocated to the \( S \) users where \( S \leq K \). The remaining \( (K - S) \) users are not supported. Since \( S \) users are selected for the \( i \)-th RF chain, \( S \) continuous symbols from the \( i \)-th symbol stream have to be multiplexed. In this paper, we propose using \( S \) orthogonal time slots from the \( i \)-th symbol stream, to create \( S \) continuous symbol-streams for the selected \( s \)-th user. The resulting trade-offs in complexity and performance will be discussed in further detail in Section IV and Section V respectively.

As the LC-SC algorithm allocates non-contiguous antenna elements to the \( s \)-th user, it will experience multi-user interference (MUI) from the beamformed signals generated from antenna elements that are allocated to other users. MUI can be eliminated at every \( s \)-th user by the combination of mm-Wave SV channel effects and a set of simple Rx BF weights at every \( s \)-th user. It is assumed that the LC-SC antenna allocation information is available at each user. For example, consider a scenario depicted in Fig. 2 in which:

- The number of antenna elements in the \( i \)-th RF chain is \( M = 4 \).
- The total number of single antenna users to be supported by this \( i \)-th RF chain is \( S = 3 \).
- The SC antenna allocation for the \( i \)-th RF chain that follows the pattern as shown in Fig. 2. In this scenario, antenna \( m = 0 \) is allocated to user \( s = 0 \); \( m = 1 \) and \( m = 3 \) to \( s = 2 \); and \( m = 2 \) to \( s = 1 \).

As shown in the Fig. 2, in order to implement this scenario, \( S = 3 \) orthogonal time slots from the \( i \)-th symbol stream corresponding to the \( i \)-th RF chain will be selected to create \( S \) continuous symbol-streams for each of the \( s \)-th user. LC-SC space-time A-BF matrix \( a_{i,SC}(t) \) is then performed over each of the \( M \) antennas over \( L \) time slots. The LC-SC space-time A-BF matrix \( a_{i,SC}(t) \), specific to the system scenario depicted in Fig. 2, can be derived using a similar approach as previously applied in (8). It is given by (14), where \( h_{i,m,l,s}(t) \) is the conjugate of the complex channel coefficient connecting the

\[\text{Figure 2. LC-SC antenna allocation for the } i \text{-th RF chain. } t_0, t_1, t_2 \text{ and } t_3 \text{ indicates the 0-th, 1-st, 2-nd and the 3-rd orthogonal time slots, or any 4 contiguous orthogonal time slots of the data stream } \mathbf{x}, \text{ which maps the } i \text{-th symbol stream onto the } i \text{-th RF chain.} \]
As an example, let us examine specific the case for the 2-
d-th RF chain is \( s \) from user \( i \)-th RF chain to the
\( \gamma \)-th Tx ANT allocated to \( i \)-th RF chain by LC-SC algorithm

\[
\begin{align*}
\gamma_{i,SC}(\mathbf{w}, d_i, a_i, SC(t), H_{i,s}(t)) &= \frac{M}{SM} \sum_{l=0}^{L-1} \left( \left( \sigma_i^2, \mathbf{w}^H \right)^{-1} \times \mathbf{w}^H H_{i,s}(t) a_i, SC(t) d_i^H \times d_i a_i, SC(t) H_{i,s}(t) \mathbf{w} \right) \\
\eta_{i,SC} &= \sum_{s=0}^{S-1} \log_2 [1 + \gamma_{i,SC}(\mathbf{w}, d_i, a_i, SC(t), H_{i,s}(t))] 
\end{align*}
\]

Lastly, the digital beamformer \( \mathbf{D} = \mathbf{I} \) is an identity matrix of
size \( N \).

IV. SIMULATION RESULTS AND DISCUSSION

In this section, sum SE performance of two different kinds of hybrid D-A BF algorithms are investigated. The LC-SC based hybrid D-A BF is compared with the separate hybrid D-A BF [4], [5]. Perfect channel state information [17–28] is assumed. Two different environments are considered in our simulations. In the first environment perfect line-of-sight (LoS) is available. While, in the second environment, multi-path are present, and the number of resolvable multi-path is assumed to be 100 which accounts for a wideband mm-Wave channel. A uniform planar array of \( M = 16 \times 16 \) antennas is considered.

Fig. 3 shows the sum SE of this hybrid D-A BF system when using \( i \)-th RF chain. Fig. 3 indicates that by using the LC-SC algorithm to design the hybrid D-A BF system, the SE increases when the number of users per RF chain increases.

This is because with a larger number of users, the number of resolvable multi-path in the mm-Wave channel increases which are combined using A-BF to improve the SNR at the respective users. In this way, multi-path diversity has been exploited in our mm-Wave system. This is observed in the
from SNR of single LoS channel and Figure 4. Normalized beam pattern for reported for a downlink mm-Wave system with Hybrid Analog and Digital Beamforming for Millimeter Wave SG, IEEE Communications Magazine, vol. 53, no. 1, pp. 186–194, January 2015.


