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The Determinants of the Model-Free Positive and Negative Volatilities

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Abstract

In this paper we analyze the role of macroeconomic and financial determinants in explaining stock market volatilities in the U.S. market. Both implied and realized volatility are computed model-free and decomposed into positive and negative components, thereby allowing us to compute directional volatility risk premia. We capture the behaviour of each component of implied volatility and risk premium in relation to their different determinants. The negative implied volatility appears to be linked more towards financial conditions variables such as uncertainty and geopolitical risk indexes, whereas positive implied volatility is driven more by macro variables such as inflation and GDP. There is a clear shift in importance from macro towards financial determinants moving from the pre towards the post financial crisis. A mixed frequency Granger causality approach uncovers causality relationships between volatilities and risk premia and macro variables and vice versa, a finding which is not detected with a conventional low frequency VAR model.

Keywords: Implied Volatility, Risk Premia, Macro Variables, Financial Variables, Granger Causality, Mixed Frequency

G14, G15, C12

Declarations of interest: none
1. Introduction

In recent years there has been much interest in the asymmetric behaviour of volatility and the different roles associated with its positive and negative components within different areas of finance, such as asset pricing, macro finance and volatility spillovers (e.g. Ang et al., 2006; Barndorff-Nielsen et al., 2010; Segal et al., 2015; Barunik et al., 2016, 2017; Feunou et al., 2017; Kilic and Shaliastovich, 2018). Given this recent interest, the question arises as to what are the determinants of volatility and, more specifically, its positive and negative components. This study aims to contribute to the financial volatility literature and to the better understanding of its relationship with macroeconomics and financial conditions variables by answering the following questions - Are the decomposed positive and negative components of volatility driven by the same variables? Are they, in turn, carrying similar information useful to predict the financial or macroeconomic activity? This paper applies a model-free approach to compute both implied and realized stock market volatility measures in the U.S. and to combine them into volatility risk premium measures, considering them in a comparative framework in which the determinants of their positive and negative components are investigated.

This paper aims to shed new light on the identification of the potential determinants of asymmetric volatility and risk premium, and to contribute to different strands of literature by taking into account a new set of variables, which include both macroeconomic and financial conditions variables. Macroeconomic variables have often been looked upon as possible determinants of volatility in many empirical studies (see Schwert, 1989; Cutler et al., 1989; Kandel and Stambaugh, 1990; Whitelaw, 1994; Lettau et al., 2007; Diebold and Yilmaz, 2008; Engle and Rangel, 2008). Among them, Schwert (1989) examined the relationship between macroeconomics and stock market volatility and found no significant evidence in the U.S. stock market. Cutler et al. (1989) argued that macroeconomics explains only a fraction of volatility movements. These studies opened up a new strand of subsequent research on the relationship between macroeconomic and stock market volatility. Focusing on the role of macroeconomic variables, Engle and Rangel (2008) found inflation and industrial production impacting on the stock market volatility, while Diebold and Yilmaz (2008) provided evidence of a relationship between stock market volatility and gross domestic product (GDP). However, the overall conclusion reached on the relationship between stock market volatility and macroeconomic activity is by no means clear cut.

The opacity of this relationship between stock market volatility and macroeconomic activity might be due to at least two reasons. Firstly, it may simply be that macroeconomics is not central in explaining the key determinants of volatility (Cutler et al., 1989). Our paper also considers other financial conditions variables that appear to better detect and track the volatility trends and behaviour alongside the macro variables in line with Paye (2012) and Christiansen et al. (2012). We expand our set of variables by including market sentiment, credit and liquidity proxies, the political and economic uncertainty index by Baker et al. (2016) and the geopolitical risk index by Caldara and Iacoviello (2018), to test the role, if any, such variables play in driving volatility. Macroeconomic factors are able to capture the state of the economy, but less able to capture investors’ expectations, whereas
these financial conditions proxies may be better able to reflect such beliefs and also contribute to volatility over a shorter time frame. In addition, market sentiment cannot be ignored for understanding investors’ future beliefs and expectations.\(^1\) For model-free implied volatility, in particular, the main determinants are more likely to be placed mostly among the contemporaneous time variables related to investors’ sentiment, such as exuberance and fear driving options trading. Secondly, many different models have been applied in an attempt to best measure volatility. Engle and Rangel (2008) employed a Spline-GARCH model which smooths out the high data frequency of volatility so as to allow a better comparison and linkage with low frequency macroeconomic data. Engle et al. (2013) subsequently provided a comparison between the different volatility models and economic fundamentals as inputs using a mixed data sample approach (GARCH-MIDAS) to study the same macro volatility link. The identification of the determinants of volatility is highly sensitive to the method used to measure volatility, often referred to as the volatility modelling problem.\(^2\) This is a problem that is well recognized in the model risk literature (see Engle and Rangel, 2008; Jokivuolle and Tunaru, 2017). The model choice, variables and market period selection increases the uncertainty and subjectivity of volatility determinant analysis (Beltratti and Morana, 2006). In order to circumvent this volatility modelling problem, we consider in this paper model-free volatility measures, namely, the implied volatility index, VIX, extracted from a bunch of S&P500 options by following the CBOE methodology, and the realized volatility computed from stock market returns.

Our paper is also motivated by recent studies which have begun to examine volatility in its different shapes and components. For instance, Beltratti and Morana (2006) decomposed volatility into one part associated with structural break and one associated with long memory dynamics, Engle et al. (2013) decomposed volatility into short and secular run components and Bekaert and Hoerova (2014) decomposed VIX into a proxy for risk aversion and a proxy for uncertainty, suggesting that both components have a different relationship to macroeconomics. We continue this line of research through the decomposition of the volatility index into its positive and negative components, with positive volatility computed only from call options, referred to in this paper as VIX\(^+\) and negative volatility computed only from put options, referred to as VIX\(^−\). In implied volatility terms, we recognize that investors are more willing to buy equity index put options for hedging purposes during negative times and crises (see Bakshi et al., 2003; Bollen and Whaley, 2004; Bondarenko, 2014). Following Barndorff-Nielsen et al. (2010), we decompose realized volatility into its positive and negative components, RVOL\(^+\) and RVOL\(^−\). We disentangle the good uncertainty associated with potential profits, representing what investors like, from the bad uncertainty associated with potential losses, representing what investors dislike (see Segal et al., 2015; Feunou et al., 2017).

\(^1\)Shiller (1989) posed the question: “Can we trace the source of movements back in a logical manner to fundamental shocks affecting the economy, the shocks to technology, to consumer preferences, to demographics, to natural resources, to monetary policy or other instruments of government control? Or are price movements due to changes in opinion or psychology, that is, changes in confidence, speculative enthusiasm, or other aspects of the world-view of investors, shocks that are best thought of as coming ultimately from people’s mind?” These questions are still unanswered.

\(^2\)“The number of models that have been developed to predict volatility based on time series information is astronomical, but the models that incorporate economic variables are hard to find.” (Engle and Rangel, 2008).
In addition, by considering model-free calculations we further extend our analysis to include volatility risk premium, VRP, and its components by following the definition as in Carr and Wu (2008), namely, as the difference between physical and risk neutral expectations of return variation. The same definition is applied in Kilic and Shaliastovich (2018), whereas other studies computed risk premia as a short position in a variance swap, namely, as the difference between risk neutral and physical expectations of returns (e.g. Bollerslev et al., 2009; Bekaert and Hoerova, 2014; Feunou et al., 2017). This recent strand of literature has begun to investigate the explanatory ability of the risk premia and its components with relation to the stock market. For instance, according to Bollerslev et al. (2009), the variance risk premium has predictive powers for short-term stock returns (from three to six months), a finding also confirmed by Bekaert and Hoerova (2014). However, according to Feunou et al. (2017), considering only the aggregate VRP measure is restrictive given that this imposes the same coefficient on both the asymmetric views of investors in relation to the two components of VRP related to good uncertainty (VRP⁺) and bad uncertainty (VRP⁻). Indeed, they found that the downside VRP (which would correspond to our VRP⁺) is the main component of the variance risk premium, finding it to be significant with a positive relationship with the equity premium, and showing superior ability in explaining future excess returns compared to the aggregate and upside VRP (which would correspond to our VRP⁻). Amengual and Xiu (2017) linked upward and downward volatility jumps together with policy measures, finding that resolutions to policy uncertainty leads to a downward volatility movement. In addition, more recently, Kilic and Shaliastovich (2018) measured good and bad variance risk premia which help predict assets returns in the long-term horizon. The good variance risk premium predicts future assets returns with a positive sign, whereas the bad variance risk premium with a negative sign, thus, both components of the variance risk premium should be considered in order to obtain a higher return predictability.

However, the literature on the determinants of implied volatility is quite sparse: Corradi et al. (2013) found that VIX and the business cycle are related to industrial production growth and Bekaert et al. (2013) assessed that VIX is also linked to monetary policy, highlighting that lax monetary policy decreases risk aversion. Furthermore, previous literature is silent on the potential information content of volatility risk premium and its components with relation to macro and financial conditions variables. Thus, we aim in this paper, not only to further investigate the linkage between decomposed volatilities and equity, but also to expand this linkage to other selected macro and financial conditions variables in order to determine which are the main variables driving their two components separately. As far as we are aware, this is the first paper looking at the impact of macro and financial factors on implied volatility within a framework separating information contained in call options from that contained in corresponding put options. In addition, we focus on risk premium, since it uses final market information which naturally cleanses option implied volatility from the effect of physical volatility (realized), resulting in a measure correlated with risk aversion (see Bekaert and Hoerova, 2014). Also, the predictability power of the variance risk premia is mainly driven by the implied volatility, which contributes more than realized volatility. Firstly we conduct an empirical analysis based upon a temporal aggregation in which all the macroeconomic and financial
conditions variables are considered at their lowest common frequency, namely monthly, and are tested in an OLS stepwise framework and in a single LF-VAR model. Subsequently, the variables are divided according to their frequency, namely, low and high frequency, modelling the low frequency variables in a mixed frequency VAR model with respect to the daily volatility series, and the high frequency variables in a high frequency daily VAR model. This is undertaken with the objective of testing for Granger causality relationships at the most accurate frequency for our selected variables following the mixed frequency Granger causality methodology as in Ghysels et al. (2016) and Ghysels (2016). Diebold and Yilmaz (2008) advocated that one-way causality from macro variable volatility and stock market volatility deserves further research, especially in the case of implied volatility. A lead-lag relationship is examined through different VAR models at different variable frequencies to capture, not only unilateral feedback from the variables to the volatility measures, but also vice versa, with the aim of identifying any potential bilateral feedback (e.g. Jermann and Quadrini, 2006; Bansal et al., 2014).

We find evidence of different determinants dependent upon the volatility components considered, for both implied volatility and risk premium. There is evidence that the macro variables impact more on the positive implied volatility component, $VIX^+$, especially in the case of GDP and inflation, which are variables more attached to the investors’ consumption sphere. On the other hand, the financial conditions variables such as credit, liquidity, EPU and GPR indexes impact more on the negative implied volatility component, $VIX^-$. The global financial crisis has generated a shift in importance from macroeconomic to financial conditions variables, both for implied volatilities and also for risk premia. We uncover Granger causality relationships by applying a mixed frequency VAR model, especially from macro variables to volatility and vice versa, which would, otherwise, be hidden at lower frequency. We detect and confirm implied volatility as a good predictor of economic activity, whereas the volatility risk premium a good predictor of future stock returns. However, we find that different components contain a separate set of information useful for future financial and economic activity predictability.

The remainder of this paper is organized as follows. Section 2 summarizes the model-free approach to compute and decompose our volatility measures. Section 3 describes the volatility series and the selected macroeconomic and financial conditions variables. Section 4 discusses the empirical methodology of the paper, namely, stepwise backward regression, high frequency, low frequency and mixed frequency VAR models and Granger causality tests. Sections 5 and 6 report the empirical results for both the implied volatility and volatility risk premia with regards to the stepwise regression and Granger analysis, respectively. Section 7 concludes the paper.

2. Model-Free Volatilities Calculation and Decomposition

In this section, we describe in detail the measures and the decomposition of implied volatility (subsection 2.1), realized volatility (subsection 2.2) and volatility risk premia (subsection 2.3), relating it to previous literature. We rely entirely on a model-free approach to compute the implied and realized volatility measures and their positive and negative components in order to be able to compute the positive and negative volatility risk premia

5
accordingly. Aiming to understand how macroeconomic and financial conditions variables impact on the aggregate implied volatility along with its components, and likewise on the volatility risk premium and its components, the following hypothesis is considered: **Hypothesis 1**: Implied volatility and risk premium components - positive and negative - are related to macroeconomic and financial conditions variables in a different way. We thus attempt to investigate whether or not macroeconomic and financial conditions variables impact in the same way on both the negative and positive components of volatility and on the respective aggregate measures.

2.1. Decomposition of Implied Volatility

The implied volatility measure, VIX, is computed model-free from a set of out of the money (OTM) S&P500 options, being an interpolation between the near term and far term option maturities for each day in which it is calculated. It is, therefore, a forward-looking volatility measure based on the changes over the next 30 days in the S&P500 options price (see CBOE, 2009). The following formula is used to calculate the implied variance:

\[
\sigma^2_{VIX_j} = \frac{2}{T} \sum_{i=1}^{n} \frac{\Delta K_i}{K_i} e^{rT} Q_t(K_i) - \frac{1}{T} \left[ \frac{F_t}{K_0} - 1 \right]^2
\]

where \(i = 1, \ldots, n\) marks the options strike price available on that specific date, \(T\) is the expiration date, \(j\) is either (1) or (2), representing the near or far term, respectively, and \(F_t\) is the forward price of S&P500 calculated from the Put-Call parity as \(F_t = e^{rT}[c(K, T) - p(K, T)] + K\). Moreover, \(K_0\) (Reference Price) is the first exercise price less or equal to the forward level \(F_0\) and \(K_i\) is the strike price of \(i\)-OTM option, which would be a call option if \(K_i > K_0\), a put option if \(K_i < K_0\) and the average between call and put options if \(K_i = K_0\). \(r\) is the risk free rate with expiration \(T\), and \(\Delta(K_i)\) is the sum divided by two of the two nearest prices to the exercise price \(K_0\). Equation (1) is based on the variance swap approximation as shown by equation (2):

\[
\sum_{i=1}^{n} \frac{\Delta K_i}{K_i} e^{rT} Q_t(K_i)
\]

where \(Q_t(K_i)\) is the price of a European call or put with a strike price respectively above or below \(K_0\), the first strike price below \(F_0\). In the case \(K_i = K_0\), \(Q_t(K_i)\) is equal to the average between an ATM call and an ATM put, relative to that strike price. To calculate the expected variance, an adjustment term is added to the expression in (2). This adjustment is required to convert in the money (ITM) calls to out of the money (OTM) puts: \(\frac{1}{T} \left[ \frac{F_0}{K_0} - 1 \right]^2\). The VIX is calculated by interpolating the near term variance and the far term variance\(^3\), \(\sigma^2_{VIX_i}(T_1)\) and \(\sigma^2_{VIX_i}(T_2)\) computed through equation (1):

\(^3\)These are the closest expirations to a 30 days average target in which monthly or weekly S&P500 options are traded. The aim of the VIX calculation is to better track the 30-days implied volatility in the equity market, an aim easily achieved with the introduction of Weekly S&P500 options since 2014. Weekly S&P500 options selected must have an expiration of \(\geq 23\) days, \(\leq 37\) days. When monthly S&P500 options are selected, the first 3-months expirations are considered. VIX is calculated through the interpolation of the first two months expirations, 1M and 2M. Where the first month is not available or less than 3 days are left for its expiration, the selected month is rolled onto the next expiration, taking the 3M, since if shorter the impact of volatility and volume can misdirect the computation.
In order to compute the positive and negative components of the VIX, an adjustment is made to equation (1), applying filters on the $K_i$ term. For VIX$^+$ only S&P500 call options are considered when $K_i \geq K_0$, and for VIX$^-$ only put options are considered when $K_i \leq K_0$. We define the first options sub-sample with strike prices above the reference price as $K_i^+$ and the sub-sample below the reference price as $K_i^-$. Substituting $K_i$ in equation (1) with both $K_i^+$ and $K_i^-$ provides the two respective near and far term positive and negative variances:

$$\sigma^2_{VIX_i} = \frac{2}{T} \sum_{i=1}^{n} \frac{\Delta K_i^j}{(K_i^j)^2} e^{r_T Q_t(K_i^j)} - \frac{1}{T} \left[ \frac{F_i}{K_0} - 1 \right]^2 \text{ with } j = + \text{ or } - .$$

Resultantly, the two implied volatility components VIX$^+$ and VIX$^-$ are:

$$VIX_t^+ = 100 \sqrt{\frac{365}{30} \left[ T_1 \sigma^2_{VIX_1} \frac{N_2 - 30}{N_2 - N_1} + T_2 \sigma^2_{VIX} \frac{30 - N_1}{N_2 - N_1} \right]}$$

(5)

$$VIX_t^- = 100 \sqrt{\frac{365}{30} \left[ T_1 \sigma^2_{VIX_1} \frac{N_2 - 30}{N_2 - N_1} + T_2 \sigma^2_{VIX} \frac{30 - N_1}{N_2 - N_1} \right]}$$

(6)

Extracting volatility only from call options provides us with a proxy for positive implied volatility, whereas extracting volatility only from put options provides a proxy for the negative implied measure.

2.2. Decomposition of Realized Volatility

The importance of identifying the downside risk in the volatility, as shown in Ang et al. (2006), brings a decomposition of the realized variance measures in an attempt to better understand the two different risk components separately (see Barndorff-Nielsen et al., 2010; Patton and Sheppard, 2015; Segal et al., 2015). In our paper, the realized volatility (RVOL) is calculated starting from the historical S&P500 index returns, thus, using close to close price realized volatility measures consistent with the model-free approach. This is an end-of-the-month monthly volatility, computed from daily log-returns (see Schwert, 1989). The formula used in this paper for the annualized realized volatility is $RVOL_t = \sqrt{\frac{252}{n} \sum_{i=1}^{n} r^2_i}$, where $r_i = ln(P_i/P_{i-1})$ representing daily log returns computed from the price difference, with $P_i$ representing the S&P500 daily index levels with $i \in \{1, \ldots, n\}$. The decomposition into the positive and negative components for the realized volatility is achieved by taking only sums over positive returns or sums over negative returns, indicated as RVOL$^+$ and RVOL$^-$, respectively. We further follow the methodology in Barndorff-Nielsen et al. (2010) to get:

$$RVOL_t^+ = \sqrt{\frac{252}{n} \sum_{i=1}^{n} r^2_i 1_{(r_i>0)}} \text{ and } RVOL_t^- = \sqrt{\frac{252}{n} \sum_{i=1}^{n} r^2_i 1_{(r_i\leq0)}}$$

(7)
where \( r_i = \ln\left(\frac{P_i}{P_{i-1}}\right) \) represents daily log returns computed from the price difference, with \( P_i \) representing the S&P500 daily index levels with \( i \in \{1, \ldots, n\} \). The positive semi-realized volatility considers only positive returns while the negative semi-realized volatility considers only negative returns.

2.3. Decomposition of Volatility Risk Premium

We are, now, able to combine these different volatility measures and their components according to their respective positive and negative binaries in order to obtain the volatility risk premium series. The importance of the risk premium for explaining stock market expected returns has been well documented in the literature (see Bollerslev et al., 2009; Kelly and Jiang, 2014; Feunou et al., 2017; Kilic and Shaliastovich, 2018). In this section, following the definition in Carr and Wu (2008), we compute the volatility risk premium by taking the difference between the physical measure of volatility (realized) and the risk neutral expectation of return variation extracted from options (implied). It represents the return of buying volatility in a volatility swap contract (see Carr and Wu, 2008), where the VIX replaces the conditional return volatility using a risk neutral probability measure and the realized volatility is given by the actual physical probability measure (see Bekaert and Hoerova, 2014; Feunou et al., 2017). Thus, by following Kilic and Shaliastovich (2018), we decompose the volatility risk premium into its positive and negative components, the first as the difference between \( RVOL^+ \) and \( VIX^+ \) and the latter as the difference between \( RVOL^- \) and \( VIX^- \) as follows:

\[
VRP_{V}^{q} = RVOL_{V}^{q} - VIX_{V}^{q} \quad \text{where} \quad q = Tot, +, -. \tag{8}
\]

3. Data: Volatility Series and Selected Variables

In subsection 3.1, we illustrate the options data and stock market index (S&P500) prices used to compute our volatility and risk premia series, and provide a discussion of our findings presenting plots and correlation analysis in relation to the volatility measures. Subsection 3.2 describes all the variables used in the empirical analysis of the paper, namely, the macroeconomic and financial conditions variables.

3.1. Decomposed Volatility Series

Daily S&P500 options and index prices are collected from OptionMetrics and Bloomberg over a total time period ranging from 04-01-1996 to 29-09-2016. Daily observations total 5222, while when monthly observations are taken, end-of-the-month, they total 250 for each volatility and risk premium series in the study. The following Figure 1 illustrates the relationship between the decomposed model-free implied and realized volatilities as well as risk premia, at monthly frequency, during the total period.\(^5\)

\(^{4}\)Other papers, such as, Bollerslev et al. (2009), Bekaert and Hoerova (2014) and Feunou et al. (2017) defined the variance risk premium as the difference between the risk neutral and physical expectations of return variation, finding a measure which is, most of the time, positive. In our paper, we find a measure of risk premium which is, most of the time, negative due to the way it is calculated. Feunou et al. (2017), because
Figure 1: Decomposed Volatility Series

The upper panel in Figure 1 compares the VIX together with its positive and negative components. The spikes in the indexes correspond to all the main financial events during our time period. For instance, we notice a peak corresponding to the Asian financial crisis at the end of 1997, to the Russian Financial Crisis and to the Long-Term Capital Management (LTCM) collapse in 1998, to the dot-com bubble period and the 2001-2002 NBER recession period (highlighted in gray). In response to the Russian financial crisis in August 1998, the VIX index reached its all time high before the global financial crisis. It then spiked massively during the global financial crisis, especially in response to the Lehman Brother collapse in September 2008. Subsequently, the implied volatility indexes reacted to the two stages of the European sovereign debt crisis, to Grexit and the Chinese Yuan crisis in mid 2015 and, finally, to Brexit in June 2016. The negative and positive implied volatilities, VIX− and VIX+, closely track the aggregate measure VIX, especially during turbulent times and VIX− is, most of the time, higher than VIX+ (see Fu et al., 2016; Kilic and Shaliastovich, 2018). There are times, be it rare, when VIX+ is higher than VIX−, but only during calm and optimistic periods characterized by positive investors’ expectations and a more active call options trading such as around the dot-com bubble. Post global financial crisis, VIX− is always of the same reason, also obtained opposite signs compared to us when decomposing risk premium in its positive and negative components.

Notes: This figure shows a comparison between the VIX, VIX− and VIX+ indexes (upper panel), RVOL, RVOL− and RVOL+ (mid panel) and VRP, VRP− and VRP+ (bottom panel) during the period from 04-01-1996 to 29-04-2016, at monthly frequency. The NBER recession periods are highlighted in gray.

Events such as the Asian financial crisis, the dot-com bubble, the 9/11 terrorist attack, the Iraq invasion, the global financial crisis and the Lehman Brother crash, the European sovereign debt crisis, the tension between Russia and Ukraine, the Chinese Yuan collapse and the Brexit vote, are only some of the various political, economic and financial events in the U.S. and worldwide which are included within our time period spanning from 1996 to 2016.
found to be higher than VIX+ emphasizing the puts hedging role and investors’ concerns regarding the possibility of another similar event occurring. We recognize that there exists an asymmetry in the volatility indexes possibly due to the fact that investors are more willing to buy put options for hedging purposes, especially during negative times, which, in turn, inflates the negative volatility component (Bollen and Whaley, 2004; Bondarenko, 2014). The mid panel of Figure 1 depicts almost the same pattern for the realized volatilities which reacted and spiked to the same events. All the realized series move closely to each other. RVOL− and RVOL+ are more intertwined without a clear predominance of one over the other showing how both are equally important for the aggregate measure, RVOL. In turbulent market times and periods of increased volatility, RVOL− is found to be above RVOL+ , having one of the highest spread around the 9/11 terrorist attack due to the stock exchanges closing, whereas during calmer periods and especially in bullish periods such as the dot-com bubble, RVOL+ is found to be higher than RVOL− (see Kilic and Shaliastovich, 2018). The last panel of Figure 1 shows the trend for the volatility risk premia. We observe that the aggregate VRP oscillates between positive and negative values, being, on average, negative due to the fact that for most of the time VIX is higher than RVOL+. VRP− is also negative, whereas VRP+ is positive for most of the time period.

The correlation analysis in Table 1 shows that positive and negative implied volatility are highly correlated in levels, while less correlated in first differences. The same is also found for the realized volatility measures. These results are in line with many studies which have decomposed variance measures (see Barndorff-Nielsen et al., 2010; Fu et al., 2016; Feunou et al., 2017; Kilic and Shaliastovich, 2018).

<table>
<thead>
<tr>
<th>Volatility Series: Levels</th>
<th>VIX</th>
<th>VIX−</th>
<th>VIX+</th>
<th>RVOL</th>
<th>RVOL−</th>
<th>RVOL+</th>
<th>VRP</th>
<th>VRP−</th>
<th>VRP+</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>VIX−</td>
<td>0.98</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VIX+</td>
<td>0.96</td>
<td>0.92</td>
<td>1.00</td>
<td></td>
<td></td>
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<tr>
<td>RVOL</td>
<td>0.86</td>
<td>0.85</td>
<td>0.82</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>RVOL−</td>
<td>0.84</td>
<td>0.84</td>
<td>0.80</td>
<td>0.95</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>RVOL+</td>
<td>0.79</td>
<td>0.79</td>
<td>0.76</td>
<td>0.95</td>
<td>0.82</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>VRP</td>
<td>0.17</td>
<td>0.18</td>
<td>0.14</td>
<td>0.64</td>
<td>0.59</td>
<td>0.65</td>
<td>1.00</td>
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<td></td>
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<tr>
<td>VRP−</td>
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<td>0.06</td>
<td>0.10</td>
<td>0.49</td>
<td>0.58</td>
<td>0.34</td>
<td>0.82</td>
<td>1.00</td>
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<tr>
<td>VRP+</td>
<td>0.27</td>
<td>0.31</td>
<td>0.20</td>
<td>0.65</td>
<td>0.48</td>
<td>0.78</td>
<td>0.85</td>
<td>0.42</td>
<td>1.00</td>
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<table>
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<tr>
<th>Volatility Series: First Differences</th>
<th>VIX</th>
<th>VIX−</th>
<th>VIX+</th>
<th>RVOL</th>
<th>RVOL−</th>
<th>RVOL+</th>
<th>VRP</th>
<th>VRP−</th>
<th>VRP+</th>
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<tbody>
<tr>
<td>VIX</td>
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<tr>
<td>VIX−</td>
<td>0.92</td>
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<tr>
<td>VIX+</td>
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<td>0.79</td>
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<tr>
<td>RVOL</td>
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<td>0.42</td>
<td>0.32</td>
<td>1.00</td>
<td></td>
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<tr>
<td>RVOL−</td>
<td>0.48</td>
<td>0.49</td>
<td>0.40</td>
<td>0.81</td>
<td>1.00</td>
<td></td>
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<tr>
<td>RVOL+</td>
<td>0.08</td>
<td>0.12</td>
<td>0.02</td>
<td>0.77</td>
<td>0.32</td>
<td>1.00</td>
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<tr>
<td>VRP</td>
<td>-0.20</td>
<td>-0.15</td>
<td>-0.26</td>
<td>0.71</td>
<td>0.44</td>
<td>0.73</td>
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<td>VRP−</td>
<td>-0.04</td>
<td>-0.05</td>
<td>-0.03</td>
<td>0.67</td>
<td>0.70</td>
<td>0.35</td>
<td>0.78</td>
<td>1.00</td>
<td></td>
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<tr>
<td>VRP+</td>
<td>-0.23</td>
<td>-0.14</td>
<td>-0.34</td>
<td>0.51</td>
<td>0.11</td>
<td>0.82</td>
<td>0.83</td>
<td>0.32</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: This table reports the correlation analysis for the implied volatility, realized volatility and volatility risk premium series during the period from 04-01-1996 to 29-04-2016, at monthly frequency.

There is a high positive correlation between the implied and realized series decreasing from the aggregate to
the positive. Still positive, but smaller is the correlation between VRP$^-$ and VRP$^+$. Correlations among our volatility series decrease when first differences are taken and we observe that the correlations between positive and negative VRP are smaller than those for positive or negative volatilities (see Kilic and Shaliastovich, 2018). For this reason, in this paper, we aim to show, first of all, how different options portfolios, namely, calls and puts, might contain different information compared to the VIX alone and, then, how the smaller correlation found between risk premia might suggest that these measures contain separate information and may be driven by different variables.

Our analysis is undertaken considering, not only the total time period spanning from January 1996 to September 2016, but also the pre-crisis and post-crisis periods. This is done with the aim of checking for potential differences in our dependent variables and covariates’ behaviour. Upon applying a Bai and Perron (2003) break-point test on the daily S&P500 and VIX series, August 2007 is selected as the pre-crisis ending month, having then a January 1996 - August 2007 pre-crisis sub-period with April 2009 as the month in which the global financial crisis turbulence vanished. Thus the post-crisis sub-period spans from April 2009 to September 2016.

3.2. Macroeconomic and Financial Variables

The variables in this study are divided into two main groups, namely, the macroeconomic variables and the financial conditions variables. The following macroeconomic variables are collected from FRED (Federal Reserve Economic Data) for the U.S.: the consumer price index (CPI) as a proxy for inflation, the industrial production (IP) as a proxy for the real activity, the unemployment rate (UR), the money supply (M1) and the real gross domestic product (GDP) as a variable accounting for changes in real economic activity. The quarterly GDP series is interpolated into a monthly series. The term structure component (TS) is computed as the difference between long-term government bond rates (10 years) and short-term government bond rates (2 years). Among the macroeconomic variables we also include crude oil price (OIL), gold price (GOLD) and the JPY-USD exchange rate (ER), however these variables are extremely close to the financial market activity and can be considered a hybrid group.\footnote{We adopted the JPY-USD exchange rate because the introduction of the EURO occurred after the beginning of our time period and we decided to select a exchange rate that was available for the entire time period. In addition, U.S. and Japan are two of the largest global economies, heavily linked through both imports and exports and are both mixed economies resulting in common news which might impact on both stock markets (e.g. Karolyi and Stulz, 1996; Ng, 2000).}

Among the financial conditions variables, we select those that better track the markets’ reaction to financial, economic and political events, investors’ sentiment and future expectations. However, they are included in the financial conditions category for simplicity. From FRED we collect the S&P500 (SPX) as the stock market index proxy, the credit spread (CRE) computed as the difference between Moody’s BAA and AAA corporate bonds yields (see Christiansen et al., 2012), the market sentiment (SENT) identified by the Consumer Sentiment Index from the University of Michigan which tracks consumers’ attitudes and market expectations, and the TED spread (TED), computed as the spread between the 3-Month USD LIBOR and the 3-Month Treasury Bill. The TED spread has commonly been recognized in the financial literature as a liquidity proxy (Brunnermeier et al., 2008;
Christiansen et al., 2012). We also select variables that mostly track economic, political and geopolitical uncertainty, namely the U.S. Economic Policy Uncertainty (EPU) index by Baker et al. (2016) and the Geopolitical Risk (GPR) index by Caldara and Iacoviello (2018). The variables are collected at their highest available frequency which is monthly for CPI, IP, UR, M1, GDP (interpolated) and SENT, while all the other variables used in the main empirical analysis (TS, OIL, GOLD, ER, SPX, CRE, TED, EPU, GPR) are collected at daily frequency. As in Schwert (1987), all the variables are expressed in log-differences except those which are already expressed in percentage rates. Following the augmented Dickey Fuller (ADF) unit root test, all the selected variables are first difference stationary I(1).

4. Identifying Determinants of Volatilities

The first part of our empirical section consists of a regression analysis, stepwise backward approach, in order to detect the main variables driving the aggregate and decomposed implied volatilities and the risk premia with the aim of testing Hypothesis 1, as mentioned in Section 2. This empirical analysis is conducted over the total time period as well as the pre and post global financial crisis sub-periods, in order to further test the following hypothesis - Hypothesis 2: A difference in significance among the selected variables in explaining the volatility series might be found between the total time period (1996-2016) and the two pre-crisis and post-crisis sub-periods. Static results together with rolling p-values for those variables found to be significant in explaining the volatility series are reported in Section 5 for both the implied volatility and volatility risk premium. In addition, Granger causality tests are performed through Vector Autoregressive (VAR) models in order to test whether or not the macro and financial conditions variables have informative power in explaining the implied volatility and risk premium measures and vice versa. In other words, we examine the presence of unilateral or bilateral relationships between the volatility series and their components and the independent variables by testing the following hypothesis - Hypothesis 3: There are unilateral (or bilateral) interactions at different frequencies depending on the two volatility components and characteristics. Results of the Granger analysis are reported in Section 6.

4.1. Relationship between Volatilities and Selected Variables

Because of stationarity issues, the first difference of the volatility series is taken to avoid problems of spurious regressions. The covariates we consider in the analysis are the macroeconomic and financial conditions variables

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7The role of economic and political uncertainty as one of the main determinants of stock market volatility is not a new concept in the financial literature. Political factors and episodes have been recognized as a cause of change in stock market returns, outputs and volatility (Bloom, 2009; Pastor and Veronesi, 2012, 2013). The EPU index is computed from news associated with the ten most important American newspapers, reflecting the concerns and uncertainty in the news surrounding specific economic or political global events. The words that the newspapers’ articles should contain in order to be relevant include, in brief, “uncertainty”, “economics”, “congress”, “deficit”, “Federal Reserve”, “legislation” along with other policy related words. The GPR index, by applying a similar methodology, measures geopolitical events and news such as wars, tension among countries and terrorist attacks worldwide. These two variables are collected from the following websites, http://www.policyuncertainty.com/ and https://www2.bc.edu/matteo-iacoviello/gpr.htm, respectively.

8We also undertake a correlation analysis between all the differences or log-differences of our determinants, finding that there is no evidence of multi-collinearity between the selected variables. We expected multi-collinearity between EPU and GPR though we found no evidence and, consequently, we can consider the two together in the same regression model. Results are available from the authors upon request.
discussed in Section 3.2. The regression analysis is conducted through the following equation:

\[ \Delta VOL_q^t = \alpha + \sum_{j=1}^{n} \beta_j (\Delta X_{Macro, Fin})_{t,j} + \sum_{j=1}^{n-1} \gamma_j (\Delta X_{Macro, Fin})_{t-1,j} + \epsilon^q_t. \]  

(9)

where \( VOL \) is either the VIX variable or the VRP variable with \( q = Tot, +, - \) and \( X \) represents the matrix containing both the macroeconomic or financial conditions independent variables with \( j \) varying from 1 to \( n = 15. \)

4.2. Low Frequency and High Frequency Granger Causality Test

The Granger causality approach conducted through different frequency VAR models is undertaken so as to obtain an improved understanding of the lead-lag relationships between the volatility measures and the financial conditions and macroeconomic determinants. We assess the significance of the impact of the determinants on the various implied volatility and risk premium measures, and furthermore, we also investigate whether these volatility measures or their positive or negative components contain useful information in predicting the economic and financial activity. More so the implied volatilities, which are well recognized in the literature to anticipate the financial and economic conditions through investors’ expectations and options trading. The information they carry is already projected ahead since they are 30-days forward looking information containers. Thus, from the Granger causality test it should emerge whether or not a set of variables contains useful information in predicting another set of variables, especially when the implied volatility indexes are considered (e.g. Diebold and Yilmaz, 2008).

The results for each pair of variables considering differences or log-differences of the series is tested through the following VAR models for the low frequency (LF) and high frequency (HF) variables. The LF-VAR is expressed as follows:

\[ \Delta Y_j^L = \omega^L + \sum_{i=1}^{\tau_L-l_L} \alpha_i \Delta Y_i^L + \sum_{i=1}^{\tau_L-l_L} \beta_i \Delta \chi_i^L + \epsilon_j^L \]  

(10)

where \( L \) denotes the low frequency domain, \( j = 1, \ldots, 6 \) is an indicator for the variables available only at monthly frequency and included in \( \chi^L \), \( l_L \) is the lag indicator, in this case monthly with \( \tau_L \) the number of observations in the sample, at monthly frequency. In the case of high frequency, we keep the implied volatility and risk premium measures as daily and model them in relation to the other variables available at daily frequency. The HF-VAR model is expressed as follow:

\[ \Delta Y_k^H = \omega^H + \sum_{i=1}^{\tau_H-l_H} a_i \Delta Y_i^H + \sum_{i=1}^{\tau_H-l_H} b_i \Delta \chi_i^H + \epsilon_k^H \]  

(11)

where \( H \) denotes the high frequency domain, \( k = 1, \ldots, 9 \) is an indicator for the variables available at daily frequency, \( l_H \) is the daily lag indicator, \( \tau_H \) is the number of observations of the daily sample. The regressors are the lagged \( Y \) dependent variables and the lagged \( \chi^H \) independent variables and \( \epsilon \) is distributed as \( N(0, \sigma^2) \). The null hypothesis we test is \( H_0: X \ does \ not \ Granger \ cause \ (GC) \ Y \), abbreviated to \( H_0: X \not\Rightarrow_{LF} Y \) for the low frequency
case, and to $H_0$: $X \not\Rightarrow Y$ for the high frequency case and vice versa from $Y$ to $X$.

### 4.3. Mixed Frequency Granger Causality Test

We check, in addition, whether or not temporal aggregation, in our case from high daily frequency to lower monthly frequency, end-of-the-month, may hide causality links among our covariates. According to Ghysels et al. (2016), a mixed frequency (MF) approach is able to recover more causal relationships compared to the standard LF approach which, in turn, might not capture causality even in simple cases. Given that our dependent variables are available daily and temporal aggregation would result in a loss of information, we test whether or not in our framework MF approach recovers underlying patterns better than the traditional LF approach.

We compare the analysis we have undertaken through a LF-VAR model considering the temporal aggregated volatility series, end-of-the-month, and common frequency with our macroeconomic variables, with the MF-VAR model which is run taking the volatility series at the highest frequency at which they are available. We then compare the interaction between our daily dependent variables VIX, VIX$^-$, VIX$^+$ and VRP, VRP$^-$ and VRP$^+$ and the six explanatory variables that are available only at monthly frequency.\(^9\) The following hypothesis is tested - **Hypothesis 4**: Mixed frequency (MF) analysis should uncover additional causality relationships among our covariates compared to the conventional low frequency (LF) approach.

The following simplifying assumptions are applied for estimating the MF model: \(m\) is implicitly fixed equal to 20 and the total time period re-scaled accordingly\(^10\) and only two frequencies are selected, namely, daily and monthly. We discuss in details, the mixed-frequency VAR model in Appendix A together with the mixed frequency Granger causality definition. According to Ghysels et al. (2016), the latter relies on and is an extension of Dufour and Renault (1998) definition (see Appendix A for MF-VAR definition and formulas). In our case, the MF-VAR model is constructed by including one dependent HF variable, \(\chi_H\), which can be, in turn, VIX or VRP (or their sub-components) and six LF explanatory variables \(\chi_{L,1...6}\) with \(m = 20\) being the frequencies ratio. Expanding equation (15) in Appendix A, we have, in this case, a \(26 \times 1\) vector as follow:

\[
\chi_{\tau_L} = [\chi_H(j(\tau_L,j,1), \chi_H(j(\tau_L,j,2), \ldots, \chi_H(j(\tau_L,j,20), \chi_{L,1}(\tau_L), \chi_{L,2}(\tau_L), \ldots, \chi_{L,6}(\tau_L)] (12)
\]

where the two concatenated mixed frequency sub-vectors are \([\chi_H(j(\tau_L,j,1), \chi_H(j(\tau_L,j,2), \ldots, \chi_H(j(\tau_L,j,20)])\] at HF and \(\chi_{L}(\tau_L)\) at LF with \(j = 1...6\) indexed for the two set of model-free volatility measures considered in the paper, namely, VIX, VIX$^-$, VIX$^+$, VRP, VRP$^-$ and VRP$^+$. As an example, for the first variable, aggregate

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\(^9\)The macroeconomic variables include CPI, IP, UR, M1, GDP plus market sentiment, SENT. We consider the interpolated GDP growth rate at monthly frequency so as not to complicate the analysis even further by having only a single variable at a different lower frequency.

\(^10\)More specifically, since not every month contains 20 daily observations, but can vary from 19 to 23, we consider a common and fixed number of observations equal to 20. When more than 20 observations are observed the days exceeding 20 are averaged with the 20th observation, while when only less than 20 observations are available the interpolation between end-of-the-month and beginning of the next month is considered. This allows us to still have a large sample totalling 4980 and 259 observations for HF and LF, respectively.
VIX and the six LF variables, the concatenated vector would be as follow:

\[ \chi_{\tau L} = [VIX_H(\tau_L, 1), VIX_H(\tau_L, 2), ..., VIX_H(\tau_L, 20), CPI(\tau_L), GDP(\tau_L), UR(\tau_L), IP(\tau_L), M1(\tau_L), SENT(\tau_L)] \]  

(13)

Following Ghysels et al. (2016) and by applying definition A.1, \( \chi_H \) does not Granger cause \( \chi_L \) at horizon \( h \) given \( l = \chi_H \Rightarrow \chi_L | l \) if \( P[\chi_L(\tau_L + h) | \chi_H(-\infty, \tau_L) + \chi_L(-\infty, \tau_L)] = P[\chi_L(\tau_L + h) | l(\tau_L)] \forall \tau_L \in \mathbb{Z} \). The same definition applies for the reverse, \( \chi_L \) does not Granger cause \( \chi_H \) at horizon \( h \) given \( l = \chi_L \Rightarrow \chi_H | l \).

To sum up, we test all three possible Granger causality cases according to the frequency of our variables:

- **I Case: LF to LF** - Granger causality from the \( \chi_{L,i_1} \) to the \( \chi_{L,i_2} \) low frequency variable at horizon \( h \) through model (10) considering only the variables at monthly frequency. \( H_0^1 : \chi_{L,i_1} \not\Rightarrow_{LF} \chi_{L,i_2} | l \).

- **II Case: HF to HF** - Granger causality from the \( \chi_{H,i_1} \) to the \( \chi_{H,i_2} \) high frequency variable at horizon \( h \) through model (11) considering only the variables available at daily frequency. \( H_0^2 : \chi_{H,i_1} \not\Rightarrow_{HF} \chi_{H,i_2} | l \).

- **III Case: Mixed Frequency (MF)** - Granger causality from the \( \chi_{L,i_1} \) low frequency to the \( \chi_{H,i_2} \) high frequency variable at horizon \( h \) (and vice versa from the \( \chi_{H,i_1} \) to the \( \chi_{L,i_2} \)) through model 13 considering our set of variables at the available frequency \( H_0^3 : \chi_{H,i_1} \not\Rightarrow_{MF} \chi_{L,i_2} | l(\chi_{H,i_2} \not\Rightarrow_{MF} \chi_{L,i_1} | l) \).

Lags are selected in accordance with the minimum value between Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC) for the LF-VAR and HF-VAR, while the MF-VAR lag is chosen equal to one, \( l = 1 \). The prediction horizon \( h \) is set between one and four, \( h \in \{1, \ldots, 4\} \).

The groups of variables in the MF-VAR are the LF-monthly variables (CPI, GDP, IP, UR, M1, SENT) and the HF-daily dependent implied volatility and risk premium series. The frequency ratio is set to \( m = 20 \) such that the LF observations in our MF-VAR model are equal to \( T_L = 249 \) (\( T_H = 4980 \) HF observations divided by 20). \( K_H = 1 \), is the dependent variable, whereas \( K_L = 6 \), are the low frequency variables having a total number of variables in the MF-VAR which is \( K = 26 \). The analysis is run following Ghysels et al. (2016), considering Newey (1987) HAC covariance estimator and Newey and West (1994) automatic lag selection.

5. **Relationship between Volatilities and Selected Variables: Empirical Results**

In this section we report the results of the stepwise regression analysis of the aggregate and decomposed model-free implied volatility indexes, VIX, VIX− and VIX+, and of the aggregate and decomposed volatility risk premia, VRP, VRP− and VRP+, onto the macroeconomic and financial conditions variables performed by running equation (9). The results of the stepwise backward regression are presented in Table 2 for the implied volatilities and risk premia in the first panel and second panel, respectively.

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11As discussed in Ghysels et al. (2016) redundant lags might have an adverse impact on power especially when \( h \) increases.

12Due to the large number of covariates, a stepwise backward regression approach with stopping threshold equal to 0.1% is undertaken. One period lagged variables are also included in the analysis to check for any possible interactions of the lagged variables.
We observe, in general, that the stock market proxy, S&P500, impacts significantly on the implied volatility indexes, with a negative sign in the contemporaneous relationship, whereas positive and decreasing in the first lag. Volatility characteristics such as the leverage effect and mean reversion explain these findings. The S&P500 index appears to be impacting more on the VIX\(^-\) during the post financial crisis, showing investors’ concerns about market downturns and the possibility of another crisis which is reflected in the puts hedging strategies.\(^{13}\) Some of the pure macroeconomic variables, namely inflation (CPI), industrial production (IP), gross domestic product (GDP), money supply (M1) and term structure (TS) are weakly significant in explaining the VIX indexes, with significance levels, where significant, never exceeding 5%. No relevance at all is found for unemployment rate (UR). Other macroeconomic variables, such as CPI, IP and GDP appear to be also more significant in explaining the VIX\(^+\) component, the agents consumption willingness or a country’s production ability being linked to the good volatility proxy. Inflation shows a weak effect with regards to volatility in line with previous studies, and the sign of this relationship in contemporaneous time is found to be negative, the opposite to previous studies looking at realized volatility (e.g. Schwert, 1989; Paye, 2012; Engle et al., 2013). This interesting negative relationship between changes in implied volatilities and changes in inflation might be justified by the fact that the time period of this study has been characterized by a relatively low level of inflation in the U.S. Given this, investors react in a positive way when, starting from a very low level, inflation increases, since this is considered good news for the stability of the financial system, generating an economic stimulus, thus implying less uncertainty on investors’ consumptions and trades which, eventually, leads to a decrease in volatility (see Coibion et al., 2012). On the other hand, when inflation moves towards disinflation or, eventually, deflation, this is actually bad news for the economy and the volatility may suddenly increase.

During the total and pre-crisis periods, we find that the impact of the macroeconomic variables is skewed towards the positive implied volatility, VIX\(^+\), with their role appearing to be placed mostly in the call options, whereas financial conditions variables appear to impact more on VIX and VIX\(^-\), the aggregate and negative implied volatilities. It is found that an increase in the credit spread drives an increase in the implied volatility. This is due to the variation in the credit default conditions underlying the bonds which reflects the credit risk perception and changes in the financial market, thereby being related to financial volatility in a positive way, and this link appears to pass, mainly, through the puts market channel.

The JPY-USD exchange rate is found to be a significant driver of the implied volatility indexes impacting with a negative sign in contemporaneous time. One must recognize that the exchange rate is quoted as number of yens per one dollar and the U.S. is a net importer. Hence, when JPY-USD drops, the dollar depreciates against the yen, 16

\(^{13}\)The S&P500 has a sizeable contribution to the adjusted \(R^2\), explaining on its own about half of this goodness-of-fit statistic. This variable plays an important role in our analysis and controlling for it, allows us to compare our results with previous studies in this area (see Paye, 2012; Christiansen et al., 2012). Furthermore, retaining the S&P500 variable also permits us to investigate volatility properties in relation to the equity variable such as leverage effect, as well as to study its asymmetric impact on the decomposed volatility series. Considering the equity levels in the Granger analysis allows us to anchor our paper to previous literature that has studied the predictability power of volatility and risk premia towards equity and vice versa (see Feunou et al., 2017; Kilic and Shaliastovich, 2018).
resulting in U.S. companies experiencing more expensive imports which, in turn, increases their costs and impacts negatively on their revenues and on their stock prices, thus, resulting in an increase in stock market volatility. The opposite chain applies when JPY-USD rises and the dollar appreciates against the yen. This relationship might also be discussed with a more market sentiment explanation, such as, when the dollar appreciates against a foreign currency this is seen as a stabilization and strengthening signal for the U.S. economy, thus, might reflect in a drop in the U.S. uncertainty level and so in a drop in the “fear” index, VIX, and vice versa when the dollar depreciates. We believe that while the first more market driven explanation might apply with relation to the VIX+, more related to investments and consumptions, the second market sentiment explanation might be more related to the VIX−, more related to investors fear and uncertainty. Overall, both volatility components are negatively related to the exchange rate, thus, resulting in an overall negative relationship between JPY-USD and volatility.

The variables EPU and GPR present higher coefficients and stronger significance with respect to VIX−, impacting more on the put options side, whereas they appear to never impact on the VIX+. This reflects the investors’ fear regarding economic, political and geopolitical uncertainty, and consequently the investors’ willingness to hedge themselves against it. The positive relationship between the EPU index and the implied market volatility is consistent with Pastor and Veronesi (2012, 2013) who advocated that equity volatility is affected by changes in government policy, and therefore, when new policies are introduced uncertainty and risk premia will increase, leading to more volatile stock returns. On the other hand, the geopolitical risk index by Caldara and Iacoviello (2018) is found to show a negative relationship mainly with VIX− which is justified by the composition of the index which does not appear to respond to financial events in the same way as VIX and EPU. Thus, while EPU index has a positive impact on volatilities given that this reacts to most of the economic downturn and financial crisis, there is no such expectation with respect to the GPR index where a negative sign is detected. Moreover, the relationship between GRP and implied volatility is found to emerge mainly in the period before the crisis due to the presence of events, such as, 9/11 and Iraq invasion as confirmed also from Figure 2.

Interestingly, in the pre-crisis period, we find no role for commodities, exchange rate and credit in driving the implied volatilities. We find, instead that the implied volatilities are driven by variables, such as, market sentiment, liquidity, EPU and GPR indexes as well as the stock market, findings that might be justified by the events which occurred during this time period. We find that the liquidity proxy, TED, is positively related, in contemporaneous time, to stock markets falls, and, consequently, to an increase in financial market volatility in line with Christiansen et al. (2012). This is due to the fact that an increase in the TED spread is seen as a warning

14This relationship is already described in Baker et al. (2016) with respect to the aggregate VIX which is found to have a correlation of 58% with the EPU index. During the time period we adopt for our study we find the EPU index and VIX to be positively correlated at 42%.

15This index is computed in a similar way to Baker et al. (2016). Caldara and Iacoviello (2018) found the GPR index to spike in response to events such as the Gulf War, the 9/11 terrorist attack, the 2003 invasion of Iraq and the Ukraine-Russia conflict. The only peak the GPR index shares with the VIX index is the recession period in 2000-2001 and after the 9/11 terrorist attack, while it appears quite neutral to financial turbulence and crucial financial events such as the Asian financial crisis, the LTCM, the global financial crisis, the Lehman Brother failure, periods in which both VIX and EPU index reacted instead. The GPR index captures events such as wars, terrorist attacks and global conflicts and appears to carry an additional source of risk compared to the EPU index, thereby allowing us to test both together in a common model avoiding problems of multi-collinearity.

16This time period includes events such as the Asian financial crisis, the Russian financial crisis and the LTCM collapse in 1998, the 9/11 terrorist attack and the recession period between 2001-2002, all events which spread uncertainty for U.S. economic stability.
sign, namely that liquidity might be withdrawn due to the fact that lenders expect an increase in counterparty risk, which in turn will increase the LIBOR component of the TED spread, a mechanism which reached its extreme during the global financial crisis (see Cornett et al., 2011).
Table 2: Stepwise Backward Regression between Implied Volatilities, Risk Premia and Selected Variables

<table>
<thead>
<tr>
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<th>Total Period</th>
<th>Pre-Crisis Period</th>
<th>Post-Crisis Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VIX</td>
<td>VIX−</td>
<td>VIX+</td>
</tr>
<tr>
<td>CPI</td>
<td>-0.064**</td>
<td>-0.080*</td>
<td>-0.114**</td>
</tr>
<tr>
<td>IP</td>
<td>0.112*</td>
<td>0.277*</td>
<td>0.299*</td>
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<tr>
<td>GDP</td>
<td>0.125*</td>
<td>-0.117**</td>
<td>0.058*</td>
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<tr>
<td>M1</td>
<td>-0.010*</td>
<td>0.150*</td>
<td>-0.520**</td>
</tr>
<tr>
<td>TS</td>
<td>-0.064**</td>
<td>-0.078*</td>
<td>-0.062*</td>
</tr>
<tr>
<td>OIL</td>
<td>0.095*</td>
<td>0.100*</td>
<td>0.137*</td>
</tr>
<tr>
<td>ER</td>
<td>0.096**</td>
<td>0.107**</td>
<td>0.148**</td>
</tr>
<tr>
<td>M2</td>
<td>-0.328**</td>
<td>-0.402**</td>
<td>0.274**</td>
</tr>
<tr>
<td>CRE</td>
<td>0.196**</td>
<td>0.219**</td>
<td>0.276**</td>
</tr>
<tr>
<td>TED</td>
<td>-0.142**</td>
<td>-0.150**</td>
<td>-0.142**</td>
</tr>
<tr>
<td>EPU</td>
<td>0.127**</td>
<td>0.144*</td>
<td>0.205**</td>
</tr>
<tr>
<td>GPR</td>
<td>-0.040*</td>
<td>-0.055**</td>
<td>-0.067**</td>
</tr>
<tr>
<td>Adj R²</td>
<td>0.558</td>
<td>0.466</td>
<td>0.550</td>
</tr>
</tbody>
</table>

Notes: This table presents the output of the stepwise backward regression analysis between our dependent variables, both implied volatilities (VIX, VIX− and VIX+) and also the volatility risk premia (VRP, VRP− and VRP+) and the 15 selected macroeconomic and financial conditions variables, namely, Inflation (CPI), Industrial Production (IP), Gross Domestic Product (GDP), Unemployment Rate (UR), Money Supply (M1), Term Structure (TS), Oil Price (OIL), JPY-USD Exchange Rate (ER), Gold Price (GOLD), S&P500 Index (SPX), Credit Spread (CRE), Market Sentiment (SENT), TED Spread (TED), Economic and Policy Uncertainty (EPU) Index and GeoPolitical Risk (GPR) Index in the first and second panel, respectively. The regressions as shown in equation (9), for implied volatility: \( \Delta IV_q t = \alpha + \sum_{n j = 1}^{n} \beta_j (\Delta X_{Macro,Fin})_{t,j} + \sum_{n - 1 j = 1}^{n - 1} \gamma_j (\Delta X_{Macro,Fin})_{t-1,j} + \epsilon_q t \) and for risk premium, \( \Delta VRP_q t = \alpha + \sum_{n j = 1}^{n} \beta_j (\Delta X_{Macro,Fin})_{t,j} + \sum_{n - 1 j = 1}^{n - 1} \gamma_j (\Delta X_{Macro,Fin})_{t-1,j} + \epsilon_q t \) are run for contemporaneous variables (t) and one period lag (t-1) variables. The table reports the regression coefficients only for the variables that passed the stepwise regression test. Selection method is stepwise backwards with stopping threshold p-values higher than 10%. All the variables are taken with difference or log-difference and re-scaled accordingly. Significance levels: * p ≤ 0.1, ** p ≤ 0.05, *** p ≤ 0.01. The regression is run over the total period, from 04-1996 to 09-2016, over the Pre-Crisis period from 01-1996 to 08-2007 and over the Post-Crisis period from 04-2009 to 09-2016, at monthly frequency.
The post financial crisis bullish period and the exuberance resulting from the dot-com bubble period might, instead, be identified as possible causes for the fact that market sentiment impacts mainly on the positive volatility proxy, \( VIX^+ \). The sentiment index has a lagged and inverse relationship with implied volatilities. This relationship is interesting knowing that the sentiment index seems to reduce the level of the next period implied volatilities, which itself makes sense in that it reduces the investor’s uncertainty about the future spending behaviour of consumers and general future states of the economy.

In the post-crisis period, we observe a weak and minimal effect of macro determinants on the implied volatilities. Table 2 shows a clear shift from a mixed macro-financial variables effect detected in the pre-crisis period towards a financial oriented determinants effect in the post-crisis period. Iconic is the role of industrial production, a variable which illustrates changes in the structure of the economy and may be an indicator of future inflation, thereby possibly impacting on financial markets. While in the post-crisis period the level of IP was very low thus having no impact on financial market volatility, during the pre-crisis period we observe how higher level of IP signals a stronger economy, future inflation outlook and, thus, it plays a role on the financial market. Basically, during the post-crisis period, the concerns of another event such as the global financial crisis had moved the attention of investors trading S&P500 options underlying \( VIX \) towards other variables, such as credit, TED spread, EPU index and market sentiment, more related to the investors’ expectations, financial risk and uncertainty sphere.

In general, we can assess that the variables identified as playing a more important role in influencing both the aggregate \( VIX \) and \( VIX^- \) are more attached to the financial market conditions group of variables, namely credit spread, market sentiment and the EPU index. These variables are the ones found to be significant at the 1% level at least once either over the total time period or over the two sub-periods. On the other hand, macroeconomic variables play a smaller role showing significance at the 5% level and their effect is placed, most of the time, on the positive volatility component, \( VIX^+ \). We find no relevance at all for commodities in explaining the implied volatility indexes. For instance, gold, which is mainly seen as a hedge against inflation, shows a negligible effect in relation to volatility due to the overall low period of inflation in the U.S. It is notable that the lag of the credit proxy appears to increase in importance in the post financial crisis, whereas the EPU effect is found to be, in general, stronger in a contemporaneous framework, with coefficients decreasing in significance when the first lag is considered. These two findings are in line with Amengual and Xiu (2017), confirming that policy news is more relevant in the short-term while credit default spread is important in the long-run.

5.2. Volatility Risk Premia Stepwise Regression Analysis

The second panel in Table 2 shows how the aggregate volatility risk premia are largely impacted by S&P500, credit, market sentiment, and economic and policy uncertainty during the total period. Among the macroeconomic variables, money supply has the largest and most significant impact on the all the premium series before the crisis, but it becomes non-significant after the crisis, possibly due to important changes in monetary policies. In the pre-crisis period, we find that inflation is still driving the positive component, this time being \( VRP^+ \), while over
this period the term structure drives VRP−. The unemployment rate shows its major impact on VRP−. The post-crisis period shows a poor role for the macroeconomic variables in impacting the VRP series, instead finding that the VRP series are mostly impacted by commodities, such as, oil and gold, and the exchange rate (except for VRP+ ) and also the financial conditions variables.

In the total period, the JPY-USD exchange rate is also found to be highly significant in influencing all the VRP series with its first lag. A similar predominant role is found for S&P500 and credit, EPU index and market sentiment, Commodities such as, oil and gold are shown to be significant in explaining the volatility risk premia, in contrast to the implied volatility indexes where they were found to have difficulty showing significance. In particular, oil is significant during the two sub-periods, with a positive sign on the risk premia coherent with the well established negative relationship between oil price and stock market confirmed. The pre-crisis period being stressed by turbulence due to the Iraq war and the post-crisis period by OPEC cutting oil production and the ongoing tension in the Middle East.

Regarding the equity market, we detect a stronger impact of S&P500 on the VRP+ in the total and pre-crisis periods, whereas a smaller impact on the VRP+ in the post-crisis period compared to the aggregate. The predominant role of the VRP+ as a volatility risk premium component confirms the results in Kilic and Shaliastovich (2018) who found that the VRP+ is more related to the aggregate risk premium, and also in line with Feunou et al. (2017) who found that the VRP+ is the main component of the aggregate VRP. We find a positive and significant (1%) relationship between VRP+ and the stock market, whereas a negative and barely significant (10%) relationship between VRP− and the stock market during the total period. We observe that when S&P500 increases, VRP+ increases as well, VRP− decreases, while VRP shares, most of the time, the same sign as for the VRP+ implying it increases. However, while it is evident that the VRP− (VRP+ ) shows a negative (positive) relationship with the equity market, the relationship sign between the aggregate VRP and equity can sometimes be masked since it is a mixture of information emanating from the two components, VRP+ (positive sign) and VRP− (negative sign), as pointed by Kilic and Shaliastovich (2018). In addition, we also notice that the VRP+ is influenced by more variables compared to VRP and VRP−, resulting in a higher adjusted R², in both the total sample and sub-samples, with the difference reducing during the post-crisis period.

We find a positive relationship between the EPU index and risk premia, regardless of their nature, implying that the higher the economic and policy uncertainty the higher the premia that the investors are willing to pay in order to be hedged against it, in line with Pastor and Veronesi (2012, 2013). The relationship between EPU and the negative component (VRP− ) is confirmed to be stronger when compared to the positive component (VRP+ ) risk premia case. We detect no role for EPU in the pre-crisis period. Interestingly, we find, again, a shift in the

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17In Feunou et al. (2017) however, VRP+ corresponds to the downside VRP since they compute the risk premia as a difference between implied volatilities and realized volatilities, opposite to us.
18By running a regression rolling coefficient (β) analysis with regards to the relationship of VRP and equity we confirmed this finding showing that the sign of the equity β in relation to S&P500 changes according to the period, whereas the signs of S&P500 β in relation to VRP− and VRP+ remain positive and negative, respectively. Results on this brief exercise are available from the authors upon request.
role of the variables when moving from the pre-crisis to the post-crisis period during which we notice that there is no role left for the macroeconomic variables in explaining the volatility risk premia. On the other hand, financial conditions variables, such as, equity, credit, liquidity and the EPU index, but also commodities, strengthen in their role in the post-crisis period, with increased β coefficients as compared to the pre-crisis or total period.

In Table 2 it can be seen that the signs of the coefficients for some variables change when lags are introduced. Thus, we examine the coefficients of the second difference, \( \Delta X_{t-l} - \Delta X_{t-l-1} \) with \( l = 0, 1 \) over the total time period by running the following regression model for both the implied volatility and volatility risk premium:

\[
\Delta VOL_q^t = \alpha + \sum_{j=1}^{n} \beta_j (\Delta^2 X_{\text{Macro,Fin}})_{t,j} + \sum_{j=1}^{n-1} \gamma_j (\Delta^2 X_{\text{Macro,Fin}})_{t-1,j} + \epsilon_q^t. \tag{14}
\]

where \( q = \text{Tot}, +, - \). In relation to the implied volatility, we find that despite the change in sign, the speed of the rate of change coefficient is found to be positive for inflation, credit, liquidity and term structure (\( \Delta^2 CPI, \Delta^2 CRE, \Delta^2 TED, \Delta^2 TS \)), whereas it is found to be negative for S&P500 and GDP (\( \Delta^2 S&P500, \Delta^2 GDP \)) confirming the leverage effect for S&P500 and the GDP negative relationship with volatility as suggested in the literature (e.g. Engle and Rangel, 2008). Stock return volatility behaves counter-cyclically (e.g. Schwert, 1989; Paye, 2012) moving counter-cyclically with respect to GDP (see Campbell and Diebold, 2009). We also check the speed of the rate of change coefficient with respect to those variables where the sign of the coefficient changes between time periods \( t \) and \( t-1 \) in relation to the volatility risk premia (\( \Delta^2 M1, \Delta^2 S&P500, \Delta^2 CRE, \Delta^2 SENT \)), finding that the coefficients sign detected at time 0 holds for all the variables.

5.3. Implied Volatilities and Risk Premia Rolling Regressions

Overall, from Table 2, we detect an asymmetric impact of the selected variables according to the different volatility components studied. In order to further test both Hypothesis 1 and Hypothesis 2 in a dynamic framework, we conduct a rolling regression over the total time period. Figures 2 and 3 show the selected variables’ rolling p-values for the VIX and VRP series, respectively.\(^{19}\)

For the implied volatility, interesting differences in behaviour of the selected variables emerge from the pre-crisis to the post-crisis periods and especially in the midst of the two recession periods. For instance, before the first 2001-2002 recession period resulting from the dot-com bubble and optimistic investors’ expectations, many of the variables (CPI, TS, S&P500, SENT) are shown to be impacting mainly on the VIX\(^+\), a proxy for positive implied volatility, rather than on the negative component. Market sentiment actually only affects the VIX\(^+\) component during the dot-com bubble, reflecting the investors’ exuberance at that time. Inflation is found to be significant mainly during the pre-crisis period, while during the post financial crisis inflation is hardly seen as a problem in the U.S. as also reflected in Table 2. Industrial production alternates, showing periods in which it

\(^{19}\)The rolling p-values are reported only for those variables found to be significant for at least one of the VIX series or VRP series from the stepwise backward regression in Equation 9 as reported in Table 2 over the total time period. Rolling window length is selected equal to 30 months and the regression is rolled every month.
impacts more on \( VIX^- \), while others on \( VIX^+ \). GDP is found mainly to be significant in the period surrounding the financial crisis and the 2008-2009 recession, but also at the end of 2014 and beginning of 2015. After the dot-com bubble term structure is found to be significant for \( VIX^+ \) while relevant for \( VIX^- \) in the immediate pre-crisis period. The JPY-USD exchange rate is associated mainly with the \( VIX^+ \), especially in the post global financial crisis. For instance, in October 2010 the dollar value dropped below 84 yen for the first time in almost two decades. It was seen as negative news for U.S. companies facing more expensive imports and reflected on American consumers and, in turn, reflected on \( VIX^+ \), more associated to investors’ consumption sphere. By the end of 2014, the dollar increased above 110 yens and JPY-USD is found, again, to mainly impact on \( VIX^+ \).

Figure 2: Rolling P-Values for the Implied Volatility Regression

Notes: This figure shows the rolling p-values for the variables selected by the stepwise backward regression in equation (9) to explain at least one of the implied volatility components over the total time period (see Table 2): Inflation (CPI), Industrial Production (IP), Gross Domestic Product (GDP), Term Structure (TS), JPY-USD Exchange Rate (ER), S&P500 Index (SPX), Credit Spread (CRE), Market Sentiment (SENT), Economic and Policy Uncertainty (EPU) Index and GeoPolitical Risk (GPR) Index. The reported rolling p-values are associated with the different volatility series, namely, \( VIX^- \) (blue line), \( VIX^+ \) (red line) and \( VIX^- \) (green line). 10% significance threshold is shown. Selected window size is 30 months and the regression is rolled every month. The NBER recession periods are highlighted in gray. The rolling regression analysis is run over the total time period from 01-1996 to 09-2016, at monthly frequency.

The credit measure, as shown also in Table 2, emerges as significant for \( VIX^- \) mainly in the post-crisis period. Market sentiment appears to drive the two implied volatility components according to the market period and investors’ beliefs. It impacts on the \( VIX^+ \) component during the dot-com bubble and it translates into carrying fear and concerns to the \( VIX^- \) in the immediate pre and post financial crisis periods. The EPU index impacts mainly on the \( VIX^- \) responding to concerns due to events such as; the Russian financial crisis, the 2001-2002 recession, the
9/11 terrorist attack, the Lehman Brother failure, the European sovereign debt crisis, the Russia-Ukraine conflict and the Brexit vote. The GPR index mainly spikes in relation to events which are not related to economic and financial activity as discussed in subsection 5.1. However, economic uncertainty derived from geopolitical risk might turn into depression in economic activity and stock prices, thus, into pessimistic and negative expectations about future market conditions (see Caldara and Iacoviello, 2018), a reason why the GPR index seems to be mostly related and priced in the put options.

Figure 3: Rolling P-Values for the Volatility Risk Premia Regression

Notes: This figure shows the rolling p-values for the variables selected by the stepwise backward regression in equation (9) to explain at least one of the volatility risk premium components over the total time period (see Table 2): Inflation (CPI), Money Supply (M1), Unemployment Rate (UR), Term Structure (TS), JPY-USD Exchange Rate (ER), S&P500 Index (SPX), Credit Spread (CRE), Market Sentiment (SENT), Economic and Policy Uncertainty (EPU) Index and GeoPolitical Risk (GPR) Index. The reported rolling p-values are associated with the different volatility series, namely, \( \text{VRP} \), \( \text{VRP}^- \) and \( \text{VRP}^+ \) (green line). 10% significance threshold is shown. Selected window size is 30 months and the regression is rolled every month. The NBER recession periods are highlighted in gray. The rolling regression analysis is run over the total time period from 01-1996 to 09-2016, at monthly frequency.

In relation to the volatility risk premia, we observe that money supply is found to explain the \( \text{VRP} \) series especially during the global financial crisis and more recent years, whereas it is found to impact, mainly, \( \text{VRP}^- \) in the post 2000-2001 recession period. A similar pattern appears to be confirmed for unemployment rate, whereas term structure is found to affect the \( \text{VRP} \) series mainly during the global financial crisis and its aftermath. The JPY-USD exchange rate is found to be significant mainly during the 2000-2001 recession period, financial crisis and Brexit. We detect a major role for the S&P500 in driving the positive volatility risk premium, \( \text{VRP}^+ \), as also shown in Table 2, in line with Feunou et al. (2017) and Kilic and Shaliastovich (2018), while affecting the \( \text{VRP}^- \).
only during the global financial crisis. Credit is found to impact on VRP\(^+\) in the pre financial crisis and during the crisis, while it is found to invert its role from 2012 onwards with a clear breakpoint, becoming significant in explaining VRP\(^-\). Market sentiment is mainly detected as significantly driving the VRP\(^+\) during the dot-com bubble and during the post global financial crisis, thereby reflecting periods of investors’ optimism. Only for a few years, between 2012 and 2014 does it significantly impact the VRP\(^-\). The opposite trend is shown by the EPU index which is found to to be mainly related to VRP\(^-\). Inflation and geopolitical risk index appear to be rarely significant in impacting on the VRP series as shown in Table 2.

Overall, from both Table 2 and Figures 2 and 3, we can confirm Hypothesis 1 and Hypothesis 2: different selected variables appear to impact on the different implied volatility and volatility risk premium components and, in addition, variables’ behaviour and effect vary, for the majority of them, according to the selected time period, namely, the pre and the post global financial crisis.

6. Granger Causality at Different Frequencies: Empirical Results

In this Section, we report the Granger causality (GC) analysis results from the different frequency VAR models as shown by equations (10), (11) and (13) with the aim of testing Hypothesis 3 and Hypothesis 4. In all the models we test for all the casual patterns from the explanatory variables to the implied volatility and volatility risk premium series and vice versa. Table 3 reports the summary of these relationships for implied volatility, while Table 4 for volatility risk premium. The low frequency GC columns show the causality relationship at monthly frequency in which temporal aggregation, end-of-the-month, is applied. The mixed frequency GC columns show the results obtained by running a MF-VAR with the six low frequency variables used in this paper. In the MF case, one lag is selected and we control for the forecasting horizon \(h\), where \(h \in \{1, \ldots, 4\}\). The high frequency GC columns show the results for the HF-VAR run only for those variables available at daily frequency, which in turn include mostly financial conditions variables.

6.1. Granger Causality at Different Frequencies: Implied Volatilities

Regarding the implied volatilities, among the LF variables, market sentiment is found to be caused by all the VIX indexes, whereas a unilateral Granger causality is detected only from VIX\(^+\) to inflation and only from VIX\(^-\) to unemployment rate. Among the other variables, we find VIX, VIX\(^-\) and VIX\(^+\) to Granger cause both TED and EPU. Actually, a bidirectional relationship is found between VIX\(^-\) and the EPU index confirming the importance of VIX\(^-\) as a channel for transmitting the economic and policy uncertainty into the volatility market. We find that credit market is informative, in turn, in predicting all the VIX series, whereas the equity market predicts VIX and VIX\(^-\). There is no Granger causality between the S&P500 index and VIX\(^+\) at low frequency which is, instead, uncovered by HF Granger causality. In general, for the low frequency, we find that some lagged macroeconomic variables are, most of the time, unable to predict implied volatility indexes, especially when lagged beyond the first month, but, in turn, also lagged implied volatility indexes are rarely found to be informative for predicting
macroeconomics. While we interpret the first as a mismatch in information containers between macro variables and implied volatilities, one attached to a slower economic state and the other to faster and more contemporaneous investors’ beliefs, we believe that for the forward looking implied volatilities the informative power in predicting financial conditions and macroeconomic variables should emerge more strongly (e.g. Diebold and Yilmaz, 2008).

We correct this low frequency limitation identified in the literature (see Ghysels et al., 2016; Ghysels, 2016) by performing a mixed frequency Granger causality test, with the main aim being to uncover possible causality relationships which we are unable to detect when using temporal aggregation. The MF analysis sheds light on several causality chains among low frequency macro variables and daily implied volatility indexes that we find hidden in the low frequency state, aligning the results more with those we detected in the stepwise regression analysis in the previous section. For instance, VIX is now found to be able to predict inflation, money supply and unemployment rate, relationships robust for three out of four forecasting horizons, while VIX Granger causes GDP and IP for $h = 2$ and $h = 3$. Beltratti and Morana (2006) observed the existence of a causal linkage running from stock market volatility to macroeconomics, however with short lived effects, a reason as to why LF Granger is found not to uncover these relationships. They are also found to be more in line with previous studies, such as Paye (2012), who found that lagged volatility provides an efficient indicator of the economic state due to the relationship between volatility and business conditions, Vu (2015) who found that past innovations in stock market volatility contain significant information about future changes in output growth, and Bekaert and Hoerova (2014) who found that implied volatility is able to predict future economic activity. In the other direction, we detect causality chains going from IP, M1 and SENT to VIX for the majority of forecasting horizons.

With respect to $\text{VIX}^-$, we mainly confirm the unidirectional causality from $\text{VIX}^-$ to unemployment rate and market sentiment as for the low frequency, and we uncover robust relationships from industrial production and market sentiment towards $\text{VIX}^-$. With regards to $\text{VIX}^+$, we show a bilateral relationship with market sentiment and unilateral relationships from $\text{VIX}^+$ to unemployment rate and from industrial production to $\text{VIX}^+$. Market sentiment confirms its simultaneous role next to the volatility indexes (maximum one month lag selected), but a bilateral relationship is found between the two only with the MF-VAR approach. This might be interpreted as a mismatch in the market sentiment information frequency which needs a higher frequency in order to be detected.

The high frequency Granger causality also captures linkages which we are unable to capture with monthly aggregation. Interestingly, for all the VIX series, we find evidence of a bilateral relationship with the JPY-USD exchange rate which reflects the currency trading activity impacting on the options trading and vice versa. Relationships from VIX to credit and liquidity proxies and to uncertainty and geopolitical risk indexes are also detected for $\text{VIX}$ and $\text{VIX}^-$, while for $\text{VIX}^+$ there is no causality towards the GPR index. There is evidence of a significant two way feedback between the EPU index and volatilities, however this causality chain being stronger when going from the VIX series towards the EPU index. This can be explained by the way the EPU index is computed from newspaper articles which have a minimum lag of one day compared to the options market.
Table 3: Pairwise Granger Causality Test for Mixed Frequencies: Implied Volatilities

<table>
<thead>
<tr>
<th>Aggregate Implied Volatility: VIX</th>
<th>Low Frequency GC</th>
<th>High Frequency GC</th>
<th>Mixed Frequency GC</th>
<th>Low Frequency GC</th>
<th>High Frequency GC</th>
<th>Mixed Frequency GC</th>
</tr>
</thead>
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<td>HP</td>
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<td>P Value</td>
<td>t</td>
<td>P Value</td>
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<td></td>
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<td></td>
<td>H = 2</td>
<td></td>
<td>H = 3</td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
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<td>0.012</td>
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<td>0.000</td>
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<table>
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<td>0.000</td>
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<td>0.000</td>
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<tr>
<td>△ VIX ⇒ △ EPU</td>
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<td>0.000</td>
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<tr>
<td>△ VIX ⇒ △ GPR</td>
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<td>0.356</td>
<td>0.134</td>
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<table>
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</tbody>
</table>

Notes: This table shows the VAR Granger causality tests performed through equation (10), (11) and (13) for low, high and mixed frequency variables, respectively. The models are run between the macroeconomic and financial conditions variables and the implied volatility series. VIX VIX− and VIX−. Lags are selected according to the minimum value between AIC and SIC tests for LF and HF, while fixed to one for MF. MF-VAR controls for forecasting horizon h = 1, 2, 3, 4. Null hypotheses: X ⇒LP Y, X ⇒HP Y, and X ⇒MF Y. The total time period is from 04-01-1996 to 29-09-2016. Frequency is according to the model and variables frequency availability.
Investors react to an increase in “fear” and uncertainty by trading equity put options for the month ahead based on their expectations at time $t - 1$. This uncertainty is also captured in the EPU index through economic and financial news. Indeed, this relationship is quite contemporaneous and, most of the time, does not last beyond the first lag (see Amengual and Xiu, 2017). On the other hand, credit spread requires a little more time, namely more than ten days, to impact on the volatility indexes, thus the causality relationship which is detected monthly is lost at a daily frequency. This is in line with the results from Table 2 in which the lagged credit variable emerged significant at its first lag in the post-crisis period. Also, credit spread is found to be caused, rather than a cause, by the volatility changes at daily frequency (e.g. Zhang et al., 2009).

Overall, this analysis also highlights how more refined information carried by the options trading behind the VIX might be able to reflect investors’ expectations regarding daily frequency variables, such as exchange rate, liquidity and the EPU index, which are in turn connected to several tradable assets echoing the actual market participants beliefs, both exuberance and fear. Lastly, we detect a poor predictive power of the implied volatility for future stock market returns, S&P500, regardless of the volatility component and the VAR frequency selected, a finding in line with Bekaert and Hoerova (2014). To conclude, for the implied volatility series, we can confirm Hypothesis 3 and Hypothesis 4. Lag selection for financial conditions variables shows how their impact on volatility, or vice versa, most of the time cannot be captured at monthly frequency given that it dissolves within a few days or weeks. For the macroeconomic variables, we show how some of the relationships they have with implied volatilities can be detected only with MF VAR models, whereas forward looking volatility indexes are scarcely responsive to lagged macro variables at low frequency.

6.2. Granger Causality at Different Frequencies: Volatility Risk Premia

Table 4 shows that macroeconomic variables barely cause VRP series in low frequency with the exception of market sentiment towards the negative and positive risk premia. For instance, inflation growth rate is found to have no effect on realized volatility (see Engle and Rangel, 2008) which is reflected, in our case, in the risk premia given that the risk premia contain a mixture of information between implied and realized volatilities. All the VRP series are able to Granger cause inflation, VRP is also found to Granger cause money supply, while VRP+ Granger causes inflation, GDP, money supply and market sentiment, being connected to future levels of macro variables.

When MF-VAR is applied the picture changes, and we are now able to detect a more informative role for the risk premia in predicting the future levels of macro variables. Not only can we confirm the Granger causality relationships already detected at low frequency, but we also uncover variables which were hidden at low frequency. For instance, all the VRP series are found to Granger cause inflation and market sentiment, VRP and VRP+ are found to have a unidirectional relationship towards money supply, while only VRP− is found to Granger cause unemployment rate and only VRP+ industrial production.
Table 4: Pairwise Granger Causality Test for Mixed Frequencies: Volatility Risk Premia

<table>
<thead>
<tr>
<th>Aggregate Volatility Risk Premium: VRP*</th>
<th>Low Frequency GC</th>
<th>High Frequency GC</th>
<th>Mixed Frequency GC</th>
<th>Low Frequency GC</th>
<th>High Frequency GC</th>
<th>Mixed Frequency GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP*</td>
<td>3</td>
<td>0.000</td>
<td>3</td>
<td>0.000</td>
<td>3</td>
<td>0.000</td>
</tr>
<tr>
<td>t-1</td>
<td>P-Value</td>
<td>t-2</td>
<td>P-Value</td>
<td>t-3</td>
<td>P-Value</td>
<td>t-4</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ CPI</td>
<td>3</td>
<td>0.000</td>
<td>3</td>
<td>0.084</td>
<td>0.124</td>
<td>0.028</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ GDP</td>
<td>3</td>
<td>0.198</td>
<td>3</td>
<td>0.128</td>
<td>0.110</td>
<td>0.164</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ IP</td>
<td>3</td>
<td>0.343</td>
<td>3</td>
<td>0.206</td>
<td>0.102</td>
<td>0.000</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ UR</td>
<td>3</td>
<td>0.982</td>
<td>3</td>
<td>0.122</td>
<td>0.000</td>
<td>0.201</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ M1</td>
<td>4</td>
<td>0.021</td>
<td>4</td>
<td>0.000</td>
<td>0.038</td>
<td>0.000</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ SENT</td>
<td>3</td>
<td>0.406</td>
<td>3</td>
<td>0.002</td>
<td>0.030</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Negative Volatility Risk Premium: VRP*</th>
<th>Low Frequency GC</th>
<th>High Frequency GC</th>
<th>Mixed Frequency GC</th>
<th>Low Frequency GC</th>
<th>High Frequency GC</th>
<th>Mixed Frequency GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP*</td>
<td>3</td>
<td>0.000</td>
<td>3</td>
<td>0.003</td>
<td>3</td>
<td>0.039</td>
</tr>
<tr>
<td>t-1</td>
<td>P-Value</td>
<td>t-2</td>
<td>P-Value</td>
<td>t-3</td>
<td>P-Value</td>
<td>t-4</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ CPI</td>
<td>3</td>
<td>0.008</td>
<td>3</td>
<td>0.053</td>
<td>0.513</td>
<td>0.000</td>
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<tr>
<td>Δ VRP* ✇ Δ GDP</td>
<td>3</td>
<td>0.577</td>
<td>3</td>
<td>0.115</td>
<td>0.211</td>
<td>0.305</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ IP</td>
<td>3</td>
<td>0.166</td>
<td>3</td>
<td>0.202</td>
<td>0.053</td>
<td>0.200</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ UR</td>
<td>3</td>
<td>0.870</td>
<td>3</td>
<td>0.000</td>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ M1</td>
<td>4</td>
<td>0.289</td>
<td>4</td>
<td>0.120</td>
<td>0.073</td>
<td>0.040</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ SENT</td>
<td>2</td>
<td>0.571</td>
<td>2</td>
<td>0.035</td>
<td>0.249</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Positive Volatility Risk Premium: VRP*</th>
<th>Low Frequency GC</th>
<th>High Frequency GC</th>
<th>Mixed Frequency GC</th>
<th>Low Frequency GC</th>
<th>High Frequency GC</th>
<th>Mixed Frequency GC</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP*</td>
<td>3</td>
<td>0.000</td>
<td>3</td>
<td>0.000</td>
<td>3</td>
<td>0.000</td>
</tr>
<tr>
<td>t-1</td>
<td>P-Value</td>
<td>t-2</td>
<td>P-Value</td>
<td>t-3</td>
<td>P-Value</td>
<td>t-4</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ CPI</td>
<td>3</td>
<td>0.000</td>
<td>3</td>
<td>0.072</td>
<td>0.000</td>
<td>0.048</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ GDP</td>
<td>3</td>
<td>0.010</td>
<td>3</td>
<td>0.320</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ IP</td>
<td>3</td>
<td>0.198</td>
<td>3</td>
<td>0.002</td>
<td>0.244</td>
<td>0.080</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ UR</td>
<td>3</td>
<td>0.907</td>
<td>3</td>
<td>0.299</td>
<td>0.342</td>
<td>0.000</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ M1</td>
<td>4</td>
<td>0.000</td>
<td>4</td>
<td>0.000</td>
<td>0.000</td>
<td>0.012</td>
</tr>
<tr>
<td>Δ VRP* ✇ Δ SENT</td>
<td>2</td>
<td>0.088</td>
<td>2</td>
<td>0.003</td>
<td>0.000</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Notes: This table shows the VAR Granger causality tests performed through equation (10), (11) and (13) for low, high and mixed frequency variables, respectively. The models are run between the macroeconomic and financial conditions variables and the volatility risk premium series, VRP*. Lags are selected according to the minimum value between AIC and SC tests for LF and HF, while fixed to one for MF. MF-VAR controls for forecasting horizon h = 1, 2, 3, 4. Null hypotheses: X ✇ Y, X ✇ Y and X ✇ Y. The total time period is from 04-01-1996 to 29-09-2016. Frequency is according to the model and variables frequency availability.
In the other direction, with MF-VAR model, we uncover unilateral relationships from industrial production and market sentiment towards all the VRP series, while money supply is able to Granger cause only VRP$^+$. Looking at the variables available at daily frequency, we find unilateral Granger causality from VRP towards term structure, oil price, gold price and liquidity, both when these relationships are studied at low frequency and also at high frequency. However, the causality linkage expands also to S&P500 and the EPU index when high frequency VAR is considered. On the other hand, the financial conditions variables which are more informative for future VRP levels are S&P500, credit and exchange rate at both low frequency and high frequency and also EPU when high frequency is selected. For the VRP$^-$, we detect an unilateral Granger causality towards gold at both high and low frequency, whereas towards credit and liquidity proxies only at the first frequency and towards term structure, oil and exchange rate only at the second frequency. Exchange rate, S&P500 and EPU are also able to Granger cause the VRP$^-$ at both low and high frequencies, while we uncover causality relationships from oil and credit towards VRP$^-$ at high frequency. Lastly, VRP$^+$ is found to Granger cause term structure and TED spread, while it is Granger caused by S&P500 index, credit, TED spread and the EPU index, at low frequency. However, with high frequency Granger causality, VRP$^+$ is found to have a bilateral causality link with many of the financial conditions variables, such as, exchange rate, S&P500, EPU index and GPR index, while VRP$^+$ is predicted by variables, such as, term structure and credit.

In general, this comparative frequency Granger causality test further confirms Hypothesis 3 and Hypothesis 4 also for the volatility risk premia. Our findings are in part in line with Bekaert and Hoerova (2014) with regards to finding the aggregate VRP unable to Granger cause economic activity (IP), neither at lower frequency nor in mixed frequency. However, by decomposing the VRP measure, we detect some predictive power for the VRP$^+$ in explaining future economic activity in mixed frequency. We also confirm the findings in Bekaert and Hoerova (2014) in relation to stock market returns by also detecting VRP as a significant predictor of stock market returns, opposite to the implied volatility, and also the findings in Feunou et al. (2017) who found superior ability of the VRP$^+$ in explaining future excess market returns when compared to VRP$^-$. 

7. Conclusion

This paper considered the relationship between model-free stock market volatilities and a renewed set of variables that included not only macro variables, but also variables which are able to track financial market conditions, market sentiment and economic and geopolitical uncertainty, variables which have often been overlooked in the literature. Given the progress over the years in the volatility and risk premium literature we took this opportunity to further develop the research in the field of macro finance. Our paper contributes to the existing literature by presenting a new volatility point of view, decomposing volatility associated with positive stock market movements from volatility generated by negative stock market movements, for both the forward looking volatility (implied) and, by combining them with the backward looking volatilities (realized), also for the risk premia, all computed
model-free. The empirical analysis has produced different results depending upon the volatility components, upon the time period and upon the data frequency considered.

Overall, we found that variables that are more closely related with financial conditions, in particular those with of a more financial market implication, such as, equity, credit, market sentiment, liquidity and economic policy uncertainty, are found to be robust and stronger determinants of implied volatility and risk premia. In contrast, more macroeconomic variables are found to be less informative in driving volatility. We find evidence of a different behaviour with respect to the positive and negative components of the volatility series in the U.S. financial market when they are decomposed and related to this new set of potential driving factors. Positive implied volatility, $VIX^+$, is affected more by macroeconomic variables, especially those linked with investments and consumption, such as inflation and GDP. Changes in volatility due to economic and geopolitical uncertainty have been found to be placed mainly in the puts activity given that $VIX^-$ mirrors the fears and concerns perceived from investors related to negative stock market returns.

A comparative exercise between pre-crisis and post-crisis sub-periods has shown a shift from the information related to the two implied volatility components. There was a shift from calls to puts going from pre to post financial crisis explained by the fact that investors were more concerned about financial market losses, thereby began to actively hedge their equity portfolios by trading put options. Furthermore, while macroeconomic variables appeared to impact more strongly on the volatility and risk premium series in the pre-crisis period, we observe a shift in favor of financial conditions variables emerging more significant in the post financial crisis.

Lastly, a better structured mixed frequency VAR model allowed us to answer further research questions still open in the literature. For instance, by aligning the frequency of the macroeconomic information to the volatilities and risk premia we were able to uncover precious information contained in the latter which is actually able to predict macroeconomic variable changes. Vice versa, we uncovered several variables which are found to be able to predict future level of implied volatility and risk premia. This picture is even more refined and improved when we looked at the positive and negative components of our volatility series. Forward looking implied volatilities are found to predict future levels of economic activity, output growth and inflation rate, whereas volatility risk premia is found to be an informative predictors of future levels of stock returns.

Acknowledgments

We would like to thank two anonymous referees for their comments and suggestions which certainly improved the paper.

Appendix A  Mixed Frequency Granger Causality Test

In this appendix we explain how the mixed frequency VAR (MF-VAR) model is constructed and specified following the notation in Ghysels et al. (2016) and Ghysels (2016). The high frequency (HF) process includes
\[ \{\chi_H(\tau_L, \kappa)\}_{k=1}^m \{\chi_L(\tau_L, \kappa)\}_{k=1}^m \tau_L \text{ and } \{\chi_L(\tau_L, \kappa)\}_{k=1}^m \tau_L, \] where \( \tau_L \in \{0, \ldots, T_L\} \) is the LF indicator (monthly), \( k \in \{1, \ldots, m\} \) is the HF indicator (daily) with \( m \) denoting the number of HF periods within the LF time. The HF variables are \( \chi_H(\tau_L, \kappa) \in \mathbb{R}^{KH \times 1}, \quad K_H \geq 1 \) observations, whereas \( \chi_L(\tau_L, \kappa) \in \mathbb{R}^{KL \times 1}, \quad K_L \geq 1 \) are latent LF variables since they are not observed at HF (daily).

The MF process is given by all the HF variables \( \{\chi_H(\tau_L, j)\}_{j=1}^m \tau_L \) and only aggregate LF variables \( \{\chi_L(\tau_L)\}_{\tau_L} \) and the MF-V AR model contains all the observable variables in a mixed frequency vector:

\[ \chi(\tau_L) = [\chi_H(\tau_L, 1)', \ldots, \chi_H(\tau_L, m)', \chi_L(\tau_L)']'. \] (15)

where the dimension of the MF-vector is \( K = K_L + mK_H \) and the MF block, \( \chi_H(\tau_L) \), is conventionally observed after the HF block of variables, \( \chi_H(\tau_L, m) \). \( X(\tau_L) \) follows a VAR(\( p \)) model for some \( p \geq 1 \) as follows:

\[ \chi(\tau_L) = \sum_{k=1}^{p} A_k \chi(\tau_L - k) + \epsilon(\tau_L) \] (16)

where \( A_k \) are the \( K \times K \) matrices for \( k = 1, \ldots, p \) and the error vector \( \epsilon(\tau_L) \) is a strictly stationary martingale difference. The condition for stationarity applies here as in the LF-V AR case, and variables log-differences and first differences are taken. After having estimated the MF-V AR model illustrated in Formula 16, we test for Granger causality in mixed frequency case defined as in Ghysels et al. (2016) who relies, in turn, on Dufour and Renault (1998) definition of:

**Definition A.1.** Granger (Non)-Causality at Different Horizons. \( y \) does not cause \( x \) at horizon \( h \) given \( l \) (we denote it as \( yNGC_l x \mid l \)) if: \( P[x(\tau_L + h) \mid x(-\infty, \tau_L) + z(-\infty, \tau_L)] = P[x(\tau_L + h) \mid l(\tau_L)]\forall \tau_L \in \mathbb{Z} \) and \( y \) does not cause \( x \) up to horizon \( h \) given \( l \) (\( yNGC_k x \mid l \)) if \( yNGC_k x \mid l \) for all \( k = 1, \ldots, h \).

Considering \( W(\tau_L) = [x(\tau_L)', y(\tau_L)', z(\tau_L)']' \) as a vector of random variables, \( l(\tau_L) = W(-\infty, \tau_L) \) is the Hilbert space spanned by the vector \( W(\tau) \mid \tau \leq \tau_L \). In other words \( l(\tau_L) = x(-\infty, \tau_L) + y(-\infty, \tau_L) + z(-\infty, \tau_L) \). \( P[x(\tau_L + h) \mid l(\tau_L)] \) is the best linear prediction of \( x(\tau_L + h) \) based on \( l(\tau_L) \) which, according to definition A.1 is unchanged whether the past and present values of \( y \) are available or not. Further details on the notation and specification of the Granger causality at different horizon in Dufour and Renault (1998).


CBOE (2009). The CBOE volatility index-VIX, white paper.


