

# The impact of potential crowd behaviours on emergency evacuation: an evolutionary game theoretic approach

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**Abstract**—Crowd dynamics have important applications in evacuation management systems relevant to organizing safer large scale gatherings. For crowd safety, it is very important to study the evolution of potential crowd behaviours by simulating the crowd evacuation process. Planning crowd control tasks via studying the impact of crowd behavioural evolution towards evacuation simulation could mitigate the possibility of crowd disasters that may happen. During a typical emergency evacuation scenario, conflict among agents occurs when agents intend to move to the same location as a result of the interaction of agents within their nearest neighbours. The effect of the agent response towards their neighbourhood is vital in order to understand the effect of variation of crowd behaviours towards the whole environment. In this work, we model crowd motion subject to exit congestion under uncertainty conditions in a continuous space via computer simulations. We model best-response, risk-seeking, risk-averse and risk-neutral behaviours of agents via certain game theory notions. We perform computer simulations with heterogeneous populations in order to study the effect of the evolution of agent behaviours towards egress flow under threat conditions. Our simulation results show the relation between the local crowd pressure and the number of injured agents. We observe that when the proportion of agents in a population of risk-seeking agents is increased, the average crowd pressure, average local density and the number of injured agents get increased. Besides that, based on our simulation results, we can infer that crowd disaster could be prevented if the agent population are full of risk-averse and risk-neutral agents despite circumstances that lead to threat consequences.

**Index Terms**—Simulation of dynamic systems; Multi-agent systems; Agent-based models; Dynamic games.

## I. INTRODUCTION

No doubt, there are many positive effects when people congregate together. However, there are also several negative outcomes when the density of people grows too high, such as crowd disasters, severe traffic delays, and pollution. Further, densely populated areas could also lead to emergency evacuation where people attempt to move away immediately from the threat place due to the proximity of people and their frequent interactions. Emergency evacuation could also happen due to natural disasters, fire, traffic accidents, building structural failure and so on.

The prevailing evacuation management systems depend mainly on human power to assist the evacuees during an

emergency evacuation scenario. Although the organisers of large gatherings might have done the necessary preparation, it is difficult to anticipate the behaviours of a crowd during an event that may lead to a possible crowd disaster. Uncertainty issues such as the lack of information of other agents' actions, location, the severity of the emergency evacuation, and the safer evacuation exits during evacuation scenarios could add complexities to the evacuation management tasks [26]. Evacuation simulation of crowd dynamics [4], [38] have important applications in evacuation management system relevant to organizing safer large scale gatherings. Crowd dynamic models has been classified into different types depending upon how the scheme treats the pedestrians and the level of detail of the models, viz. macroscopic models, mesoscopic models and microscopic models. Macroscopic models [5], [23], [24] consolidate the whole crowd as a single entity, while mesoscopic models [12], [19], [21], [29] obtain general view of the crowd movement by separating the crowd into different small groups.

Macroscopic and mesoscopic models shrug off the importance of the behaviours and characteristics of individual agents where individual agents are considered as irrelevant to the movement of the whole crowd. Microscopic model is considered as a complex system [15] that involves both physical laws and each agent's characteristic in a crowd [33]. Through microscopic models, each agent's behaviours can be simulated and the consequences of emergent behaviours of the whole crowd can be observed [2]. Microscopic model is intended to model and simulate the actions and interactions of autonomous agents in order to examine their effects on the whole evacuation process.

In this regard, game theoretic models prove to be efficient for assessing the outcome of the dynamic behaviours of whole crowd. This is because by utilizing game theory oriented evacuation simulations, the agents will be able to examine all of the available options and choose the best strategy based on their principle. However, each agents's final payoffs will depend on the strategies chosen by other agents. Although social force evacuation model [10], [14] is based on behavioural aspects as well, the underlying assumption that all agents have homogeneous properties seems to be unrealistic. In the

evacuation scenario, crowd is typically composed of different types of individuals [51]. Studies on behavioral evolution of crowd takes into account the behaviours associated with both cooperators as well as defectors [22], [39]. In this regard, evolutionary game theory can be used to elucidate this. Basically, evolutionary game theory offers efficient computational models to make meaningful and robust decisions among interacting agents. Besides that, evolutionary game theoretic evacuation model have also been proven to be an effective model to study the crowd dynamics in terms of individual interactions that are entailed in microscopic models [9]. Consequently, in recent years a number of evolutionary game theory oriented research contributions have been proposed to model crowd behaviours during evacuation scenarios [1], [3], [9], [18], [25], [35], [41], [46]–[48], [53].

Besides that, in a  $n \times 2$  ( $n$  agents, two strategies) symmetric evacuation game as proposed in [1], [9], [18], [25], [35], [41], [48], there are only two different strategies that can be adopted by agents which are cooperator ( $C$ ) and defector ( $D$ ). However, in reality, there can be more strategies adapted by agents during evacuation scenarios such as evaluator and retaliator. By convention, cooperators usually don't fight for attaining the desired position. In contrast, defectors tend to be very aggressive in attaining desired position. An Evaluator will assess the opponent in terms of size. The evaluators will act as a cooperator if the opponent is large in size and act as a defector if the opponent is at most equal in size. While retaliator escalates only when the adversary escalates. When two retaliator meets, both would act as cooperator.

Up to now, evacuation model under certainty [18], [35], [48] and uncertainty [25] has been proposed where both scenarios are separately considered, which is inconsistent with reality. In contrast to previous work, we intend to study the effect of heterogeneous population [20], [30] where agents of uncertainty behaviours (risk seeking, risk averse and risk neutral behaviours of agents) and certainty behaviours (best-response agents) are combined. Evacuation simulation in the area of uncertainty and certainty is important in order to study the crowd behaviours during emergency scenarios.

Furthermore, Wirz et al. [49] suggested that it is crucial to understand the behaviours and situation of the crowd. The connection of crowd behaviours and evolution of different types of behaviours towards escape flow remain unexplored in previous literatures. Therefore, in this work we intend to investigate the effect of various crowd behaviours towards egress flow and the relationship between evolution of crowd behaviours and the occurrences of the crowd turbulence which is believed as potential indicators to alarm crowd disasters [6], [13], [16], [27], [28].

## II. METHODS

### A. Proposed spatial evacuation model

Regarding agents' movement in continuous space, we utilized the social force model as described in [25]. Next, we

present spatial evacuation as defined in [25]. Each agent has its own estimated evacuation time,  $T_i$  as defined by

$$T_i = \frac{d_i}{\|v(\mathbf{r}, t)\|}, \quad (1)$$

which depends on the distance between an agent and the exit,  $d_i$  and local speed of agent,  $\|v(\mathbf{r}, t)\|$ :

$$\|v(\mathbf{r}, t)\| = \frac{\sum_j v_j}{n}, \quad (2)$$

which is the mean speed of the agents around the central location  $\mathbf{r}$  of the  $i^{\text{th}}$  agent at time  $t$ . Agents within a skin to skin distance of less than  $80\text{cm}$  to agent  $i$  are considered in Equation (2), thus,  $v_j$  refers to the speed of an agent at time  $t$ , while  $n$  refers to total number of agents around the considered area at time  $t$ .

We perceive the crowd evacuation process as an evacuation game that is played with the objective to reduce the evacuation time. At each time step, the agent interact with its nearest neighbors. All the conflicting neighboring agents are identified and solved according to certain rules. Thus, the winners of the conflicts and the agents who are not involved in conflicts with their neighboring agents move to their desired positions. The simulation ends when all the agents have finally evacuated the room.

In order to solve the conflicting agents, at first we need to find the conflicting neighbors. The proposed neighborhood rules in [25] have been utilized in this work. Then, taking into account the interaction of neighboring agents between  $i$  and  $i_c$  ( $i_c$  are the agents other than  $i$  in the scenario (analogous to the complement of  $i$ )), the mean estimated evacuation time of these neighboring agents is defined as  $T_{i(i_c)} = \frac{T_i + \sum_{i_c} T_{i_c}}{1 + n_{i_c}}$ , where  $n_{i_c}$  refers to the number of neighboring agents for  $i$ . In cases where the neighboring agents tend to interact with each other, we need to solve the conflicts so that only one winner will be able to move. The winner can overtake other agents and reach the desired position and gain the utility by reducing his estimated evacuation time by  $\Delta t$ , while the loser(s) will remain in the current location and lose the utility where the loser's estimated evacuation time will increase by the same quantity  $\Delta t$ . As a result, the cost of each winner agent will get reduced to an utility that amounts to  $\Delta u(T_{i(i_c)})$  and the cost of each loser agent(s) increases by the same amount.

For each step taken by an agent, the distance  $d_i$  between the agent and the exit eventually gets reduced by  $\Delta d$  which is defined as  $\Delta d = \|v(\mathbf{r}, t)\| \times \Delta t$  where  $\|v(\mathbf{r}, t)\|$  is the local speed of an agent  $i$  as defined in Equation (2).  $\Delta t$  is assumed to be a constant value of  $0.8s$  as proposed in [25]. Then, we define the difference in estimated evacuation time of conflicting agents for each step as  $\Delta u(T_{i(i_c)}) = \frac{\Delta d}{|v_i^0|}$  where  $|v_i^0|$  refers to the preferred speed of an agent. When there is an empty space available, the winner of the conflicts will try his best to utilize his preferred speed in order to move to that empty space. This justifies the fact that we have deployed the preferred speed instead of the local speed of an agent in calculating the cost function  $\Delta u(T_{i(i_c)})$ .

Compared to work in [25], in this work, a new  $n \times 4$  ( $n$  agents, four strategies) symmetric evacuation game is proposed

which consists of proposed strategies viz., cooperator (C), defector (D), evaluator (E) and retaliator (R). Previous works have assumed that payoff of an agent are not influenced by the size of the opponent(s). However, in reality, size of the opponent(s) indeed influences the payoffs that have been received by the conflicting agents. For example, when two defectors are competing to move to a desired position, the larger and mightier agent will usually be able to dominate move towards the target, while the smaller agent usually will not be able to do so. For simplicity the larger and mightier agent(s) will be simply referred to as larger agents in the rest of the discussions. Here, large opponent(s) for current agent  $i$  is set if the center of mass distance between agents  $i$  and  $i_c$  is less than or equal to  $2cm$ ,  $d_{i(i_c)} \leq 2cm$ . While equal opponent(s) for current agent  $i$  is set if  $-2cm < d_{i(i_c)} < 2cm$ . Thus, large opponents(s) for current agent  $i$  is about  $10kg$  and more, while, equal opponent(s) for current agent  $i$  is between  $-10kg$  and  $10kg$  (these values obtained by using equation mass of an agent as described in [25]). If no large opponent and equal opponent for current agent  $i$ , then it indicates that agent  $i$  is the largest from among the conflicting agents.

Besides that, the conflicting neighbours will also face a conflict cost which could be attributed due to some energy loss, the possibility of getting injuries, time delay in movements, loosing some favourable positions in the pedestrian space and so on. When the conflicting neighbours (especially in the case where the conflicting neighbours consist of at least two defectors) attempt to push with each other in order to move, there will be a little delay in time. In this proposed work, conflict cost is denoted by the time delay  $t_d$  where  $t_d > 0$ . Then, we can define the rules that will enable us to decide the winner of the conflicts as follows:

- (1) For the case of a conflict with  $n_{def}$  defectors and  $n_{coop}$  cooperator(s) where  $n_{def} > 1$ ,  $n_{coop} \geq 0$ :
  - (1.1) When large defector,  $L_{n_{def}}$  is single, the large defector will be able to move while all the other defectors and cooperators will remain at the same location. The payoff for the large defector is to gain the utility by reducing the cost of  $\Delta u(T_{i(i_c)})$ . Whereas, the payoff for the other defectors and cooperator(s) is to loose the utility by penalizing the cost to  $-\Delta u(T_{i(i_c)})$ .
  - (1.2) When the number of large defectors are more than one, the large defectors will try and deliberately rush in order to move towards the target. As a result of this conflict one of the large defectors will be able to move while the rest of the defector(s) and all the cooperators would remain at the same location. Due to the equiprobable chance available in getting to the next move by the large defectors, the payoff for the large defectors is to gain the utility by reducing the cost of  $\frac{\Delta u(T_{i(i_c)})}{L_{n_{def}}}$ . Besides that, the large defectors will face a conflict cost which is denoted by the time delay  $t_d$ . When the large defectors try and push with each other in order to move, there will be a little delay in time. Thus, the payoff for the large defectors is to gain

the utility by reducing the cost of  $\frac{\Delta u(T_{i(i_c)})}{L_{n_{def}}} - t_d$ . While the payoff for the other defectors and cooperator(s) is to lose the utility by increasing the cost of  $-\Delta u(T_{i(i_c)})$ .

- (1.3) When there is no large defector, the defectors will try and push in order to move. As a result of this conflict one of the defectors will be able to move while the rest of the defector(s) and all the cooperators would remain at the same location. The payoff for the defectors is to gain the utility by reducing the cost of  $\frac{\Delta u(T_{i(i_c)})}{n_{def}} - t_d$ . While the payoff for the cooperator(s) is to loose the utility by increasing the cost of  $-\Delta u(T_{i(i_c)})$ .
- (2) For the case of a conflict with  $n_{coop}$  cooperators,  $n_{coop} \geq 1$  and one defector, the defector will be able to move while all the cooperators will remain at the same location. The payoff for the defector is to gain the utility by reducing the cost of  $\Delta u(T_{i(i_c)})$ . Whereas, the payoff for the cooperator(s) is to loose the utility by penalizing the cost to  $-\Delta u(T_{i(i_c)})$ .
- (3) For the case of a conflict with  $n_{coop}$  cooperators,  $n_{coop} > 1$  and no defector, no winner is selected. Even though there are no winner and loser, the payoff is set equal to all cooperators as the conflicting agents will move together with the crowd based on the social force model. Therefore, the payoff for the cooperators is set to gain the utility by reducing the cost of  $\frac{\Delta u(T_{i(i_c)})}{n_{coop}}$ .

The overall rules in order to decide the winner of the conflicts as described above can be summarized as furnished in Algorithm 1.

Based on the aforementioned assertions in Algorithm 1, a new  $n \times 4$  game matrix is built as shown in Table I. The payoff shown in Table I accounts only for the row wise agents since all the other agents will get an identical payoff for similar type of interactions. When a strategy is chosen by the agents in the row, the payoff received for the concerned agent is given in the corresponding cell of the matrix.

### B. Proposed updating strategies

Each agent intends to play the aforementioned game with its nearest neighbors except the agents who are behind. Parallel update scheme [42] is utilized where strategies of all agents are updated simultaneously. However, here we assume that agents update their strategies for each taken step which is  $0.8s$ . We study four types of agent behaviours viz., best-response, risk-seeking, risk-averse and risk-neutral. Under uncertainty conditions, risk-seeking, risk-averse and risk-neutral agents are considered where these type of agents has been discussed in [25]. While, under certainty conditions, best-response agent is considered. In the context of an evacuation scenario, *certainty condition* refers to the ability of agents to respond by observing other agents' strategies in their neighbourhood during previous instances. Hence, best-response agents are myopic since they do not consider the future or far back strategies of other agents in their neighbourhood [18], [48].

Here, a strategy of an agent  $i$  is considered as best response if the strategy of this agent on period  $t$  is at least as good as

TABLE I  
PAYOFF TABLE FOR A TYPICAL  $n \times 4$  EVACUATION GAME ( $L$ : NUMBER OF LARGE DEFECTOR OPPONENT(S) TO CURRENT AGENT  $i$ ,  $EQ$ : NUMBER OF EQUAL DEFECTOR OPPONENT(S) TO CURRENT AGENT  $i$ ,  $x$ : NUMBER OF NEIGHBORING DEFECTOR(S) TO CURRENT AGENT  $i$ ,  $y$ : NUMBER OF NEIGHBORING COOPERATOR(S) TO CURRENT AGENT  $i$ ,  $s_m$  REFERS TO PAYOFF IN ROW  $m$  WHILE  $o_n$  REFERS TO PAYOFF IN COLUMN  $n$ ,  $E$  REFERS TO EVALUATOR, WHILE  $R$  REFERS TO RETALIATOR)

	$L \geq 1$		$L = 0 \text{ and } EQ \geq 1$		$L = 0 \text{ and } EQ = 0$	
	$o_1$	$o_2$	$o_1$	$o_2$	$o_1$	$o_2$
	$0D, yC(y \geq 1)$	$xD, yC(x \geq 1, y \geq 0)$	$0D, yC(y \geq 1)$	$xD, yC(x \geq 1, y \geq 0)$	$0D, yC(x \geq 1, y \geq 1)$	$xD, yC(x \geq 1, y \geq 0)$
$s_1 (C)$	$\frac{\Delta u(T_{i(i_c)})}{y+1}$	$-\Delta u(T_{i(i_c)})$	$\frac{\Delta u(T_{i(i_c)})}{y+1}$	$-\Delta u(T_{i(i_c)})$	$\frac{\Delta u(T_{i(i_c)})}{y+1}$	$-\Delta u(T_{i(i_c)})$
$s_2 (D)$	$\Delta u(T_{i(i_c)})$	$-\Delta u(T_{i(i_c)}) - t_d$	$\Delta u(T_{i(i_c)})$	$\frac{\Delta u(T_{i(i_c)})}{EQ+1} - t_d$	$\Delta u(T_{i(i_c)})$	$\Delta u(T_{i(i_c)}) - t_d$
$s_3 (E)$	$\frac{\Delta u(T_{i(i_c)})}{y+1}$	$-\Delta u(T_{i(i_c)})$	$\Delta u(T_{i(i_c)})$	$\frac{\Delta u(T_{i(i_c)})}{EQ+1} - t_d$	$\Delta u(T_{i(i_c)})$	$\Delta u(T_{i(i_c)}) - t_d$
$s_4 (R)$	$\frac{\Delta u(T_{i(i_c)})}{y+1}$	$-\Delta u(T_{i(i_c)}) - t_d$	$\frac{\Delta u(T_{i(i_c)})}{y+1}$	$\frac{\Delta u(T_{i(i_c)})}{EQ+1} - t_d$	$\frac{\Delta u(T_{i(i_c)})}{y+1}$	$\Delta u(T_{i(i_c)}) - t_d$

**Algorithm 1** Algorithm to decide the winner of the conflicts for the proposed automated spatial evacuation model

Identify strategy of current agent as defector or cooperater  
Identify total number of defectors, number of large and equal defectors, and cooperaters available in the neighbourhood of the current agent

Case based on number of defectors, large and equal defectors and cooperaters

**if** number of defectors  $>$  one and number of cooperaters  $\geq$  zero **then**

**if** number of large defector = one **then**

The large defector will be able to move while the rest of the defector(s) and all the cooperaters would remain at the same location

Payoff for the large defector  $\leftarrow \Delta u(T_{i(i_c)}) - t_d$

Payoff for defectors  $\leftarrow -\Delta u(T_{i(i_c)}) - t_d$

Payoff for cooperaters  $\leftarrow -\Delta u(T_{i(i_c)})$

**else if** number of large defectors  $>$  one **then**

One of the large defectors will be able to move while the rest of the defector(s) and all the cooperaters would remain at the same location

Payoff for large defectors  $\leftarrow \frac{\Delta u(T_{i(i_c)})}{L^{n_{def}}} - t_d$

Payoff for defectors  $\leftarrow -\Delta u(T_{i(i_c)}) - t_d$

Payoff for cooperaters  $\leftarrow -\Delta u(T_{i(i_c)})$

**else if** number of large defector = zero **then**

One of the defectors will be able to move while the rest of the defector(s) and all the cooperaters would remain at the same location

Payoff for defectors  $\leftarrow \frac{\Delta u(T_{i(i_c)})}{n_{def}} - t_d$

Payoff for cooperaters  $\leftarrow -\Delta u(T_{i(i_c)})$

**end if**

**else if** number of defector = one and number of cooperaters  $\geq$  one **then**

The single defector will be able to move while all the cooperaters will remain at the same location

Payoff for defectors  $\leftarrow \Delta u(T_{i(i_c)})$

Payoff for cooperaters  $\leftarrow -\Delta u(T_{i(i_c)})$

**else if** number of defector = zero and number of cooperaters  $>$  one **then**

No winner and loser, the payoff is set equal to all cooperaters as the conflicting agents will move together with the crowd based on the social force model

Payoff for cooperaters  $\leftarrow \frac{\Delta u(T_{i(i_c)})}{n_{coop}}$

**end if**

every other strategies of this agent against any other action of other agents  $i_c$  on previous period  $(t - 1)$  as defined in Equation 3. In Equation 3,  $s_i^t$  refers to strategy of agent  $i$  on period  $t$ ,  $s_{i_c}^{(t-1)}$  refers to strategy of other conflicting agents,  $i_c$  on period  $t - 1$ ,

$$s_i^t \in BR(s_{i_c}^{(t-1)}) \text{ iff } \forall s_{-i}^t \in S_i^t, u_i(s_i^t, s_{i_c}^{t-1}) \geq u_i(s_{-i}^t, s_{i_c}^{t-1}) \quad (3)$$

In this proposed automated evacuation model, agents update their preferred speed based on the Available Safe Egress Time ( $T_{ASET}$ ) and the Required Safe Egress Time ( $T_{RSET}$ ).

The  $T_{ASET}$  is the amount of time that elapses between the beginning of an emergency evacuation and the development of untenable conditions. While, the  $T_{RSET}$  is the amount of time (also measured from the beginning of emergency evacuation) required for agents to safely evacuate. Here,  $T_{ASET}$  is modelled using Equation 4. In Equation 4,  $T_{ASET_o}$  refers to the total available safe egress time at the beginning of the emergency evacuation, while,  $T_{elapsed}$  refers to the total time elapsed since the beginning of the emergency evacuation.  $T_{RSET}$  has been modelled in a new way as furnished in Equation 5. In Equation 5,  $T_i$  refers to the agent's estimated evacuation time and  $T_{iREQ}$  refers to agent's estimated evacuation time due to the total agents in front of the current agent  $i$ .  $T_{iREQ}$  is defined as in Equation 6, where  $n_i$  refers to total number of agents in front of the current agent  $i$  up to the exit, while  $\beta$  refers to the current flow at the exit door which means number of agents evacuated in a second. We assume that if no exit flow at the current period of a second, then the agents' estimated evacuation time depends on their speed and distance only as defined in Equation 1. Thus,  $T_{iREQ}$  is assumed to be 0 if there is no exit flow at that particular second as modelled in Equation 6.

$$T_{ASET} = T_{ASET_o} - T_{elapsed} \quad (4)$$

$$T_{RSET} = T_i + T_{iREQ} \quad (5)$$

$$T_{iREQ} = \begin{cases} n_i/\beta, & \text{if } \beta > 0 \\ 0, & \text{if } \beta = 0 \end{cases} \quad (6)$$

Previous works [18], [31], [34], [40], [48], [52] in crowd evacuation simulation lack to relate between available safe egress time  $T_{ASET}$  and agents' preferred speed, which is not fully realistic. In reality, agents will increase their preferred speed when they are in risky conditions where the required safe egress time  $T_{RSET}$  is quite low compared to the available safe egress time  $T_{ASET}$ . In order to achieve realistic crowd evacuation simulation, we propose a new speed parameter,  $r$  which will be multiplied with preferred speed whenever  $T_{RSET}$  is more than  $0.8 \times T_{ASET}$  which intends to indicate high risk for the agents as modelled in Equation 7. In this Equation 7, preferred speed of agents is assumed to be  $1.34m/s$  because this value is considered as the approximate mean value of the comfort walking speed for agents as specified in [11], [52]. Agents will try their best in order to reach a safe place prior to facing a worst condition, thus agents will exert themselves to reach the safer place by updating their preferred speed. Here, we assume that the agents could be evacuated safely if  $T_{RSET}$  is not greater than  $0.8 \times T_{ASET}$ .

$$|v_i^0| = \begin{cases} 1.34 \times r, & \text{if } T_{RSET} > 0.8 \times T_{ASET} \\ 1.34, & \text{otherwise} \end{cases} \quad (7)$$

### C. Simulations

In this section, we present our computer simulations with respect to the proposed spatial evacuation model. Here, we examine evacuation under potential threat conditions. Thus,

we set  $T_{ASET_o}$  which refers to the total available safe egress time at the beginning of the evacuation to  $60s$  to indicate evacuation under threat condition. For our simulations, we consider a rectangular room of size  $18m \times 17m$  which consists of a single door of length  $1m$  located at the center of one of the walls. The pedestrian room space at the range of locations  $x = 18$  and  $y = 0$  to  $y = 17$  belong to the walls and cannot be occupied by agents except at  $x = 18$  and  $y = 8$  to  $y = 9$  where the door is symmetrically located. Initially, 200 agents are placed at random positions in the range  $0 < x < 17$ ,  $1 < y < 16$  at time,  $t = 0$ .

Critical conditions of the crowd can be characterised using three main attributes, namely, density, speed and flow of the crowd [28]. [43] has investigated the relationship between these three characteristics in the context of large crowds and concluded that higher density will reduce the walking speed of the crowd and vice versa. Meanwhile, flow rate is a product of density and speed. Johansson [28] has proposed a measure called crowd pressure which can be seen as an early warning sign for critical crowd situations. Crowd pressure is computed as a product of local velocity variance and local crowd density as furnished below:

$$P(\vec{r}, t) = \rho(\vec{r}, t) \times Var_{\vec{r},t}(\vec{V}), \quad (8)$$

where  $P(\vec{r}, t)$  is the local density measured at place  $\vec{r}(x, y)$  and time  $t$  and  $Var_{\vec{r},t}(\vec{V})$  is the local velocity variance. Johansson [28] concluded from his study of analyzing crowd disaster in Mina at the 12th of January 2006 that only crowd pressure and density were useful to indicate the critical crowd conditions. Thus, in this work, we intend to study the effect of crowd behaviours towards crowd local density and crowd pressure.

Usually a typical crowd will be excited more to leave the room very fast near the exit door and hence clogging will happen near the exit door [11], [17], [31], [50]. This clogging will affect the evacuation time, flow and pressure of the crowd [36], [40]. Thus, we investigate the effect of crowd behaviours towards density and crowd pressure near the exit door at the location of  $x = 17$  and  $y = 8.5$  and at  $x = 15$  and  $y = 8.5$  which are at distance of  $1m$  and  $3m$  respectively from the center of the exit door.

We perform simulations with respect to a heterogeneous population where agents of uncertainty behaviours viz., risk-seeking, risk-averse and risk-neutral behaviours of agents and certainty behaviours pertaining to best-response agents are combined in order to study the effect towards egress. For the better comprehension of crowd dynamics during egress, we study the density and the local crowd pressure for various time delays caused by conflicts by repeating the simulations for 10 runs with different random frequencies of cooperators, defectors, evaluators and retaliators placed at random initial locations. We observe that for the aforementioned average values for the time frame of  $15s$  to  $(0.7 \times TotalEscapeTime)s$  a typical crowd could form an arch-like blocking near the exit. For each type of simulation, the number of agents with regard to one of the behaviours has been fixed, while the number of agents with the other three behaviours were randomly selected.

For instance, if the number of risk-averse agents is fixed to 50 agents, the remaining 150 agents will be chosen randomly from remaining three types of agents, viz. risk-seeking, risk-neutral and best-response agents. Then, the average values from 10 simulation runs will be studied as will be discussed in the following section. Here, we have included only two sources of randomness in the simulation model, viz. the random initial locations, and the number of other three behaviours in each of the simulation runs.

### III. RESULTS AND DISCUSSION

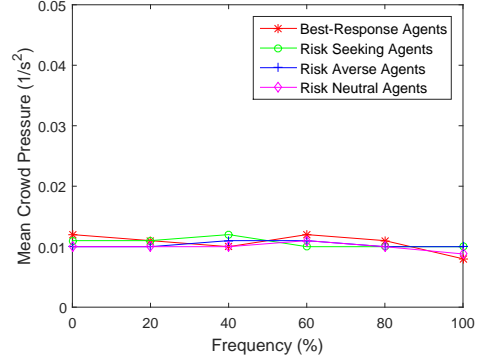
First of all, we set the speed parameter,  $r = 2$  and then followed by  $r = 3$ . The results for average crowd pressure for the case  $1m$  distance and that of the  $3m$  distance from the exit door are shown in Figures 1 and 2 respectively, while average local density at  $1m$  distance and at  $3m$  distance from the exit door are shown in Figures 3 and 4 respectively.

Johansson [28] has analysed video recordings of the crowd disaster that was encountered on January 12, 2006 at Mina during the last day of the Hajj, where 363 pilgrims lost their lives. The results in [28] showed that crowd turbulence started when the local crowd pressure is more than  $0.02s^{-2}$ , while crowd disaster happens when the crowd pressure is between  $0.03$  to  $0.05s^{-2}$ . Besides that, it is reported in [10], [44] that agents are injured if the pressure acting to the agents which is the total value of radial forces directed to them divided by their circumference is more than  $1600Nm^{-1}$ .

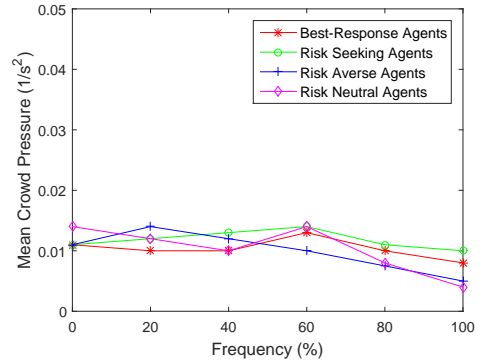
Our simulation results shows the relation between the local crowd pressure and number of injured agents as shown in Figures 2b and 5a, 2c and 5b and also 2d and 5c. Based on these simulation results, we observe that agents are getting injured when the value of local crowd pressure is more than  $0.02s^{-2}$ . In terms of the behaviours of the crowd, we observe that when the proportion of agents in a population of risk seeking agents get increased, the average crowd pressure, average local density and the number of agents injured increases as well except for the crowd pressure at  $1m$  distance from the exit door as shown in Figure 1. Although the local density is quite high at the  $1m$  distance from the exit door as displayed in Figure 3, the value of the average crowd pressure at that specific location is less than  $0.02s^{-2}$ . This indicates that high density alone which means overcrowding alone cannot be utilized as a critical crowd condition and this is in agreement with the previous work of [28].

We also found that when the proportion of agents in a population of risk averse and risk neutral agents increased to 100 percent, the average crowd pressure is quite low and very little number of injured agents even though the speed parameter  $r$  and the conflict time delay  $t_d$  are set to 3 and  $1.2s$  respectively. Therefore, we perceive that crowd disaster could be prevented if the agents' population are full of risk averse and risk neutral agents even under potential threat conditions.

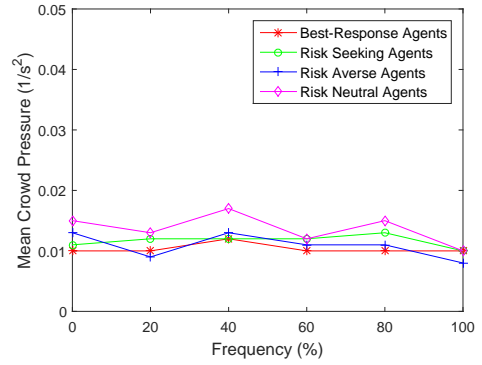
Besides that, we observed that crowd pressure is high when the speed parameter  $r$  is set to three and the conflict time delay  $t_d$  gets increased to  $1.2s$ . Even when the speed parameter  $r$  is set to two, our results in Figure 2b indicates that when  $t_d$  is increased to  $1.2s$ , the value of average crowd pressure is



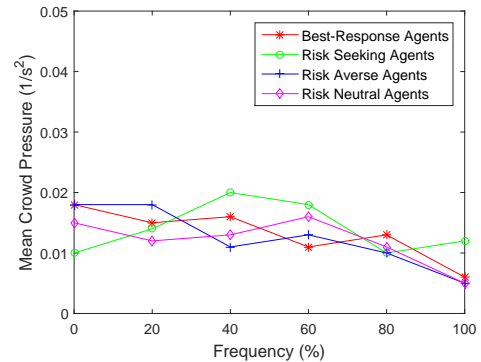
(a)  $r = 2$  and  $t_d = 0.5s$



(b)  $r = 2$  and  $t_d = 1.2s$



(c)  $r = 3$  and  $t_d = 0.5s$



(d)  $r = 3$  and  $t_d = 1.2s$

Fig. 1. The effect of different proportions of crowd behaviours towards mean of crowd pressure about  $1m$  from the center of the exit door.

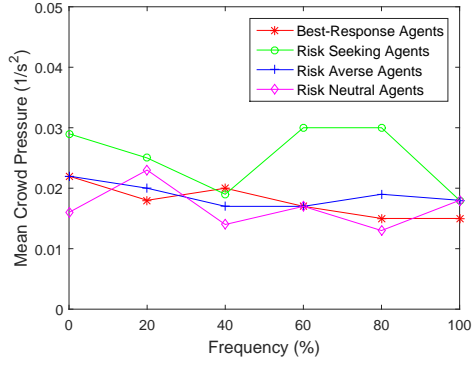
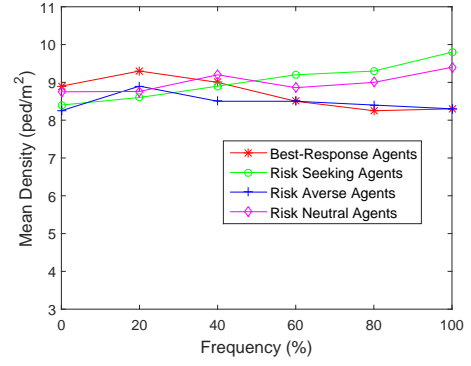
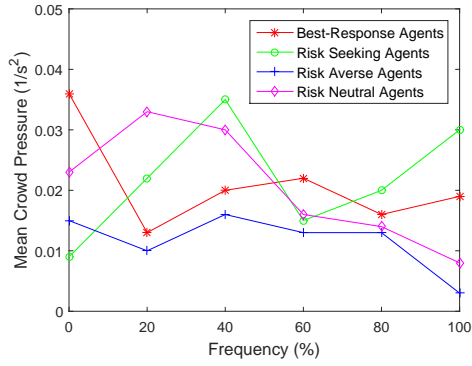
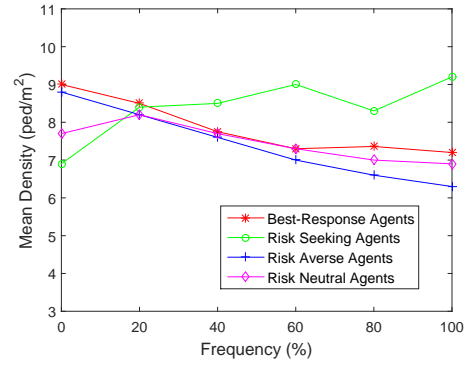
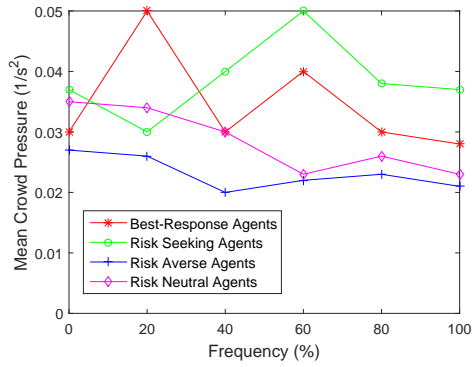
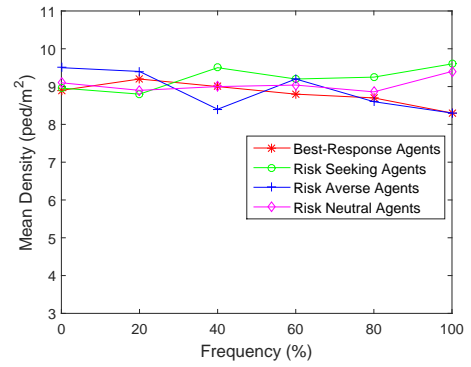
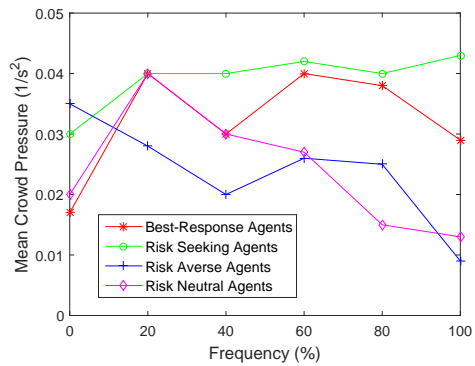
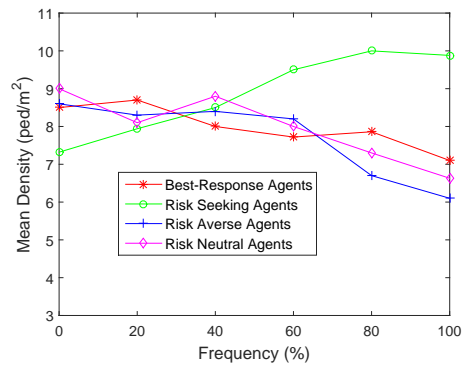
(a)  $r = 2$  and  $t_d = 0.5s$ (a)  $r = 2$  and  $t_d = 0.5s$ (b)  $r = 2$  and  $t_d = 1.2s$ (b)  $r = 2$  and  $t_d = 1.2s$ (c)  $r = 3$  and  $t_d = 0.5s$ (c)  $r = 3$  and  $t_d = 0.5s$ (d)  $r = 3$  and  $t_d = 1.2s$ (d)  $r = 3$  and  $t_d = 1.2s$ 

Fig. 2. The effect of different proportions of crowd behaviours towards mean of crowd pressure about  $3m$  from the center of the exit door.

Fig. 3. The effect of different proportions of crowd behaviours towards mean of crowd local density about  $1m$  from the center of the exit door.

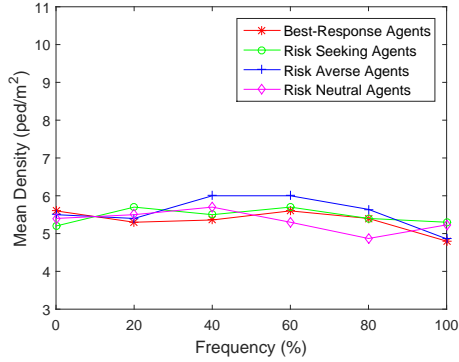
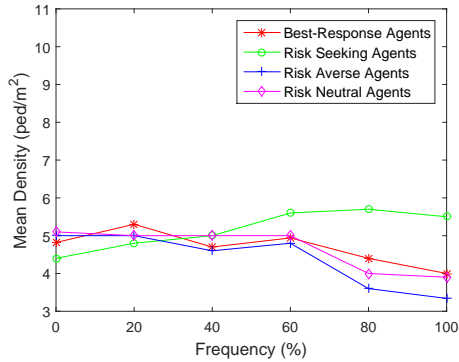
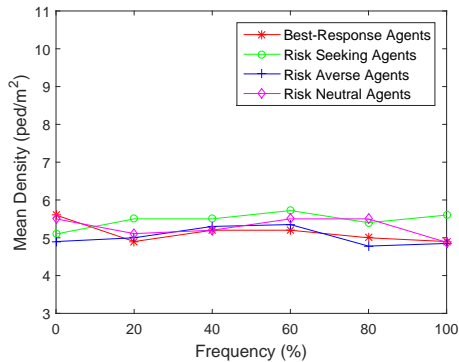
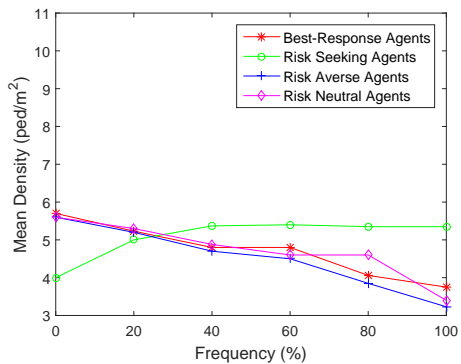
(a)  $r = 2$  and  $t_d = 0.5s$ (b)  $r = 2$  and  $t_d = 1.2s$ (c)  $r = 3$  and  $t_d = 0.5s$ (d)  $r = 3$  and  $t_d = 1.2s$ 

Fig. 4. The effect of different proportions of crowd behaviours towards mean of crowd local density about  $3m$  from the center of the exit door.

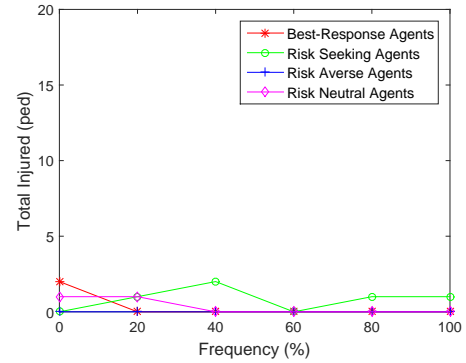
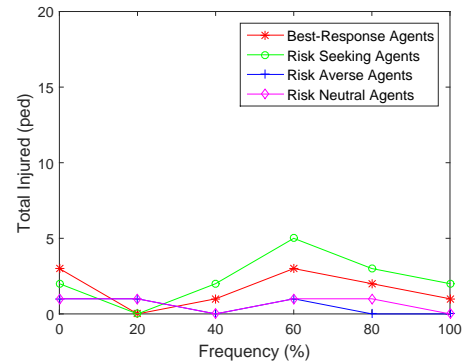
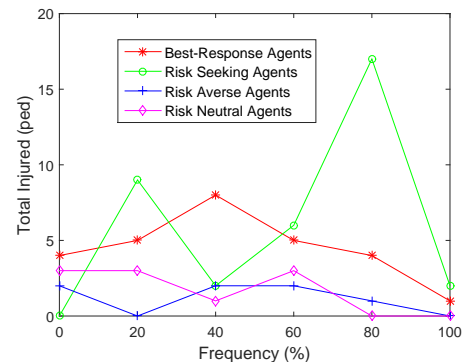
(a)  $r = 2$  and  $t_d = 1.2s$ (b)  $r = 3$  and  $t_d = 0.5s$ (c)  $r = 3$  and  $t_d = 1.2s$ 

Fig. 5. The effect of different proportions of crowd behaviours towards total injured agents.

getting more than  $0.03s^{-2}$  while Figure 5a shows that there are few agents who are prone to injuries. Thus, our results indicate that conflict time delay can be a means also for the occurrences of crowd accident when the available safe egress time  $T_{ASET}$  is quite low than the required safe egress time  $T_{RSET}$ .

#### A. Simulation for the case of a Homogeneous Population

Based on the simulation results discussed above, we further hypothesize that, increasing risk-seeking agents' population could lead to crowd disasters, while increasing risk-averse and risk-neutral agents could prevent the crowd disaster. Thus, how these behaviours of agents get evolved and how they update



their strategies during evacuation scenarios need to be addressed as well. To empirically justify these potential issues we have experimented computer simulations in a typical homogeneous population where we assume that the agents' behaviours are unchanged throughout the simulations. This aids us to better study on how these agents update their strategies based on a particular mode of behaviour. Table II shows the average equilibrium achieved by different types of behaviours for different conflict time delays. In Table II, we assume a conflict time delay,  $t_d = 0.5s$  for a low density condition, while conflict time delay,  $t_d = 1.0s$  for a high density condition. The justification is that more time delay is required for the winning agents amidst conflicts as the density grows higher.

Based on Table II, when the density of the crowd is less, we observe that risk-averse agents will act as cooperators half of the time and act as defectors or evaluators during remaining half. While, risk-neutral agents will act as a cooperators approximately one out of five times and as defectors or evaluators during the rest of the time. When the density grows higher, both risk-averse agents and risk-neutral agents will act mostly as cooperators. Thus, we perceive that in order to prevent crowd disaster, agents should exhibit risk-averse or risk-neutral behaviour as aforementioned.

#### B. Comparison with an experimental evacuation

1) *The Stapelfeldt Experiments:* The Stapelfeldt experiments were conducted in 1986, where 100 police cadets were grouped and evacuated [45] from a room in a chosen school gymnasium. The experimental evacuations were conducted through a single exit of variable width, utilising exit widths of 0.75, 0.80, 1.50 and 1.60m. The exit widths are changed by the opening and closing of a set of double doors. The experimental information revealed here concerns the evacuation of the room under ordinary conditions, without the effect of threat conditions. The dimensions of the experimental room are not revealed, but it has been stated that the room was rectangular in shape [8], [37], [45].

The data generated from this experiment is presented and compared against the model of the classical social force model [10], the social force model in [32] and our proposed spatial evacuation model as furnished in Table III. For our proposed spatial evacuation model, in order to indicate normal evacuation scenario we have set the  $T_{ASET_0}$  which refers to the total available safe egress time at the beginning of evacuation to 250s. Besides that, we have utilized dynamic conflict time delay  $t_d$  in which  $t_d$  is updated for each agent depending on the local crowd density (Equation 8). When the density is about  $4person/m^2$  walking contact among the agents started to occur [43]; we have set  $t_d = 0.5s$  if the local crowd densities less than  $4person/m^2$ . When the local crowd density is between  $4person/m^2$  and  $5.55person/m^2$ , we have set the conflict time delay  $t_d = 1.0s$ . This is because it is reported in [7] that possible crowd forces begin to occur when density reaches  $5.55person/m^2$ . Thus, we chose more conflict time delay. When local crowd density is more than  $5.55person/m^2$ , we have set  $t_d = 1.5s$  indicating that more time delay is needed for the agents to be a winner of the conflict as the density grows higher.

As can be seen in Table III, the average escape times by using the social force model of Helbing et al. [10] and Li et al. [32] generate significantly longer escape times than our proposed model. Compared to experimental results, the social force model proposed in [32] produced an average escape time quite similar to our proposed model when the exit widths are 1.5m and 1.6m. However, when the exit widths are 0.75m and 0.8m, Li et al.'s [32] model produced an average escape time longer than our proposed model (Table III). These results show that our proposed spatial evacuation model produced robust and relevant results since the average escape times produced by our proposed model are quite similar to the experimental results in [45] for all exit widths.

## IV. CONCLUSION

We have systematically investigated the effect on egress under uncertainty and certainty scenarios that could possibly arise during emergency evacuations. In particular, we examine the best-response, risk-seeking, risk-averse and risk-neutral behaviours of agents (pedestrians) using the norms of a typical evolutionary game theory approach. We have simulated evacuation scenarios in a continuous space using the classical social force model, where the impatient and patient agents have been modeled with different individual parameter settings. In summary, the main contributions of our research are as follows:

- (1) Systematic investigation on the effect of evolution of crowd behaviours that are prevalent in the heterogeneous population pertaining to critical conditions of the crowd via evolutionary game theory oriented simulations under potential threat conditions.
- (2) Formulation of the dynamical cost function for each of the agents with the incorporation of the size of the agents and conflict time delay.
- (3) Development of an automated spatial evacuation model.
- (4) Important findings on how a typical crowd should evolve and behave in order to prevent crowd disaster.

We have set out a framework that can be used by designers of crowd control and evacuation systems. They will have to re-run our model with their specific values for parameters such as room size, repulsive force and its range, angle under which agents are in conflict, etc. Our simulations transparently show what kind of crowd behaviours can be expected during various evacuation scenarios. One of the limitations of the proposed work is we have assumed that the agents interact only with their neighbors. It would also be interesting in future to consider also groups of agents that act as a cluster (e.g. families) that would surely show another behavior with respect to each other. Another limitation of the current proposed work is we have included only two sources of randomness in the simulation model, viz. the random initial locations, and the number of other three behaviours in the simulation. For future avenues or research, we would examine the effect of random behaviour of the crowd towards evacuation. In the near future, we will also consider a detailed investigation of evacuation scenarios in rooms of different sizes subject to the presence of obstacles.

TABLE II  
AVERAGE STRATEGIES DURING THE EQUILIBRIUM STATE ACHIEVED BY DIFFERENT TYPES OF AGENTS' BEHAVIOURS.

	$r = 2$ $t_d = 0.5s$	$r = 2$ $t_d = 1.0s$	$r = 3$ $t_d = 0.5s$	$r = 3$ $t_d = 1.0s$
<b>Risk-seeking</b>	$D = 37\%$ $E = 62\%$ $C = 0\%$ $R = 1\%$	$D = 46\%$ $E = 53\%$ $C = 0\%$ $R = 1\%$	$D = 47\%$ $E = 52\%$ $C = 0\%$ $R = 1\%$	$D = 48\%$ $E = 51\%$ $C = 0\%$ $R = 1\%$
<b>Risk-averse</b>	$D = 16\%$ $E = 16\%$ $C = 51\%$ $R = 17\%$	$D = 0\%$ $E = 2\%$ $C = 94\%$ $R = 4\%$	$D = 19\%$ $E = 21\%$ $C = 42\%$ $R = 18\%$	$D = 2\%$ $E = 2\%$ $C = 93\%$ $R = 3\%$
<b>Risk-neutral</b>	$D = 34\%$ $E = 46\%$ $C = 19\%$ $R = 1\%$	$D = 13\%$ $E = 15\%$ $C = 70\%$ $R = 2\%$	$D = 38\%$ $E = 42\%$ $C = 19\%$ $R = 1\%$	$D = 11\%$ $E = 13\%$ $C = 72\%$ $R = 4\%$
<b>Best-response</b>	$D = 26\%$ $E = 28\%$ $C = 37\%$ $R = 9\%$	$D = 18\%$ $E = 17\%$ $C = 62\%$ $R = 3\%$	$D = 28\%$ $E = 28\%$ $C = 34\%$ $R = 10\%$	$D = 19\%$ $E = 22\%$ $C = 56\%$ $R = 3\%$

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TABLE III  
 AVERAGE ESCAPE TIME OF OUR PROPOSED MODEL COMPARED WITH OTHER MODELS. DATA FOR SOCIAL FORCE MODEL [10], SOCIAL FORCE MODEL IN [32] AND OUR PROPOSED MODEL ARE RESULTS OF THE AVERAGE ESCAPE TIME OVER 10 SIMULATIONS. EW REFERS TO EXIT WIDTH.

	$EW = 0.75m$	$EW = 0.80m$	$EW = 1.50m$	$EW = 1.60m$
<b>Experimental Results (s)</b>	55	50	30	26
<b>Social Force Model in [10] (s)</b>	130	115	38	33.8
<b>Social Force Model in [32](s)</b>	94.37	87.64	31.3	29.24
<b>Proposed Model (s)</b>	$66.15 \pm 1.43(SD)$	$60.75 \pm 2.5(SD)$	$31.36 \pm 1.5(SD)$	$30 \pm 1.5(SD)$

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