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Incipient Fault Detection for Traction Motors of High-Speed Railways Using an Interval Sliding Mode Observer

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Abstract—This paper proposes a stator-winding incipient shorted-turn fault detection method for the traction motors used in China high-speed railways. Firstly, a mathematical description for incipient shorted-turn faults is given from the quantitative point of view to preset the fault detectability requirement. Then, an interval sliding mode observer is proposed to deal with uncertainties caused by measuring errors from motor speed sensors. The active robust residual generator and the corresponding passive robust threshold generator are proposed based on this particularly designed observer. Furthermore, design parameters are optimized to satisfy the fault detectability requirement. This developed technique is applied to an electrical traction motor to verify its effectiveness and practicability.

Index Terms—Incipient fault detection; interval sliding mode observer; traction motors.

I. INTRODUCTION

A demand for rail transportation rapidly increasing, safety and customer satisfaction have become two of the most important concerns for China Railway High-speed (CRH). To deal with these issues, intelligent vehicle fault diagnosis, fault-tolerant and monitoring techniques [1]-[4] have been developed to find out and tolerate faulty components. Traction motors are core power equipments to convert electricity into mechanical energy in electrical traction systems of high-speed railways. The sixteen traction motors in each CRH are all three-phase squirrel-cage asynchronous motors, which are the most important components to determine the riding quality. However, as claimed in [5], this kind of motors has limitations that it will result in premature incipient faults occurring on stators. The actual fault modes of stators in [5] are broken down into the following five groups: turn-to-turn, coil-to-coil, open circuit, phase-to-phase and coil-to-ground. It is turn-to-turn faults that are the initial stages and quite difficult to detect due to their incipient nature. However, the initial turn-to-turn faults may generate increasing heat, and a direct phase-to-phase or phase-to-ground faults. Thus, the motor is quickly drooped off the line. Therefore, incipient shorted-turn fault detection is essential to avoid serious failures and improve safety of high-speed railways.

During the past decades, rare model-based fault detection (FD) results are available for stator-winding incipient shorted-turn faults. One reason is that it is quite difficult to obtain the accurate motor faulty mathematical model and expressions of external and internal electromagnetic interferences. On the other hand, most of the traditional model-based FD strategies such as [6], [7] and [8] mainly focus on abrupt faults rather than incipient faults. Comparing with abrupt faults, incipient faults evolve more slowly and are smaller in amplitude [9], which needs FD schemes with strong detectability. In [10], the exponential function is used to characterize the small evolution rate feature for incipient faults and FD schemes with adaptive thresholds are proposed to detect the incipient faults. The methods developed in both of the two published papers [9] and [11] are aimed to deal with small evolution rate issue for incipient faults as well. However, the small amplitude feature is rarely considered in the existing FD works, even no proper mathematical description is available for incipient faults from the quantitative point of view to characterize the small amplitude feature, which motivates this paper to propose a mathematical description for incipient shorted-turn faults. Due to the small amplitude feature, stator-winding incipient shorted-turn faults are easily submerged by disturbances and uncertainties caused by measuring errors from speed sensors. Therefore, to detect incipient shorted-turn faults, particular incipient fault detection (IFD) technique should be developed to possess not only strong robustness to disturbances and uncertainties, but also strong sensitiveness to incipient shorted-turn faults.

In traditional robust FD systems such as [12] and [13], the residual generator is firstly designed and optimized to get a good trade-off between sensitivity to faults and robustness against disturbances, which is called as active robust FD in [14] where the design freedom locating only on the dynamics of residual generators can not satisfy the detectability requirements for incipient faults. Using interval observer technique proposed in [15], an alternative approach is applied to design dynamical threshold generator to produce more proper thresholds, known as passive robust FD proposed in [14]. Interval observers are proposed in [15] for the first time to estimate the set of admissible values of states, and then developed in [16], [17], [18] and [19] etc., which have been summarized in the review paper [20]. In [16], a quasi-LPV approximation for nonlinearities is built based on interval analysis and then, interval observer is designed for the quasi-LPV system using the cooperativity theory. For the planar systems with complex...
poles, the time-varying interval observer is designed in [17]. In [18], the $L_1/L_2$ performance is introduced to design optimal interval observers for nonnegative LPV systems and for more general ones. Unobservable nonlinear systems are considered and corresponding interval observers are designed in [19]. An algorithm that propagates the uncertainties is proposed in [14] based on zonotopes and an interval linear-parameter-varying (LPV) observer is implemented to design the passive fault diagnosis method. In the passive interval observer based fault diagnosis methods, observer gains plays an important role because they determine residual sensitivities to faults and the associated adaptive thresholds derived from the uncertainties, which are analyzed in [21] detailedly. On the other hand, sliding mode techniques are not only used for control [23], [24] and [25], but also used for fault diagnosis extensively [9], [11], [22], [26], [27] and [28] because of inherent robustness to matched uncertainties and disturbances. Recently, sliding mode techniques are used for interval observer design such as in [22], [29] and [30] to improve the inherent robustness to matched uncertainties. High-order sliding mode techniques are used to design interval observers for LPV systems in [30], an interval sliding mode observer is constructed via a convex sum of an upper estimator and a lower estimator in [29]. Therefore, to combine interval observers and sliding mode observer techniques together to design active robust residual generators and passive robust threshold generators will be pertinent way.

Recently, an interval sliding mode observer is proposed in [22] to detect incipient sensor faults for linear time-invariant systems. Built on the author’s previous work in [22], IFD schemes with detailed analysis and solid results are developed for the traction motors used in CRH in this paper. A faulty dynamical model with parameter uncertainties for the traction motors with stator-winding shorted turns is introduced from [31]. A novel quantitative mathematical description for incipient shorted-turn faults is presented via a proposed scale variable. An interval sliding mode diagnostic observer is proposed particularly for the faulty dynamical model which can compensate for observer unmatched uncertainties caused by measuring errors from the motor speed sensors. Then, IFD schemes, including residual generator and threshold generator, are proposed based on this diagnostic observer. Furthermore, parameters in these IFD schemes are optimized such that the fault detectability is satisfied. The contribution of this paper is summarized as follows:

1) A mathematical description for stator-winding incipient shorted-turn faults is given from the quantitative point of view.
2) A novel interval sliding mode diagnostic observer is proposed for faulty dynamical model of traction motors with uncertainties.
3) IFD schemes using active and passive robust FD techniques are proposed to satisfy the preset fault detectability requirements.

Notation: In this paper, without special illustrate, $\| \cdot \|$ represents the 2–norm of a matrix or a vector. For a real matrix or a vector $M$, $M > 0$ ($M \geq 0$) means that all its entries are positive (nonnegative). For any vector $x \in \mathbb{R}^n$, $|x| = \text{col}(|x_1|, \ldots, |x_n|)$ where $x_1, \ldots, x_n$ are elements of $x$.

II. Preliminaries

One lemma usually used for interval observer design is shown as follows.

**Lemma 1:** ( [30] ) Let $x \in \mathbb{R}^n$ be a vector variable satisfying $x \in [\bar{x}, \bar{x}]$ for some $\bar{x}, \bar{x} \in \mathbb{R}^n$. If $A \in \mathbb{R}^{n \times n}$ is a constant matrix, then $Ax \in [\phi, \phi]$ where $\phi = A^+ \bar{x} - A^- \bar{x}$.

Then the following lemma is introduced based on Lemma 1.

**Lemma 2:** Let $x \in \mathbb{R}^n$ be a vector variable satisfying $x \in [\bar{x}, \bar{x}]$ for some $\bar{x}, \bar{x} \in \mathbb{R}^n$, and $\omega \in \mathbb{R}$ be a scalar variable satisfying $\omega \in [\omega, \omega]$ for some $\omega, \omega \in R$. Then $\omega x \in [\phi, \phi]$ where

$$\phi = -\omega x + x^T \omega - \bar{x} \omega + \omega^T \bar{x} - \omega \bar{x} = -\omega \bar{x} + x^T \omega - \bar{x} \omega + \omega^T \bar{x} - \bar{x} \omega + (\omega \bar{x} - \omega \bar{x})(\bar{x} - \bar{x}).$$

Furthermore, if $\omega - \omega \leq 2 \Delta \omega$, then $\omega x - \phi \in [0, \bar{x}]$ and $\phi - \omega x \in [0, \bar{x}]$ where

$$\bar{x} = (2 \Delta \omega + \omega) \varepsilon + 2 \Delta \omega (x^+ + \bar{x})$$

with $\varepsilon = \bar{x} - x$ and $\bar{\omega} = x - \bar{x}$.

**Proof:** See Appendix A.

A. Stator-Winding Shorted-Turn Faults

Stator windings $a$, $b$ and $c$ with shorted-turn faults on phase $a$ is shown in Fig. 1 where $as_2$ represents the shorted turns. Denote $\mu$ as the fraction of shorted turns. Then the leakage inductance of the shorted turns is $\mu Ls$ where $Ls$ is the per-phase leakage inductance, and the fault impedance is resistance ($R_f$). Using the reference frame transformation theory presented in [32], the machine equations can then be obtained in complex $dq$ variables as presented in [31]. Let $\lambda_{dr}$ and $\lambda_{qf}$ represent...
stator magnetic flux linkage in dq coordinates respectively, and $i_d$ and $i_q$ represent stator currents in dq coordinates respectively. Considering the electromagnetic interferences on positive-sequence and negative-sequence currents, a fourth-order state-space presentation with single phase stator-winding shorted-turn faults is obtained by

$$
\begin{align}
\lambda_{q_r} &= a_{11}\lambda_{q_r} - n_p \omega_r \lambda_{d_r} + a_{13}i_{q_3} + f_{11}i_f, \\
\lambda_{d_r} &= n_p \omega_r \lambda_{q_r} + a_{22}\lambda_{d_r} + a_{24}i_{q_4}, \\
i_{q_3} &= a_{31}\lambda_{q_r} + a_{32}n_p \omega_r \lambda_{d_r} + a_{33}i_{q_5} + b_1v_{q_3} + f_{31}i_f \\
&+ f_{32}v_{q_5} + d_{1}(t), \\
i_{d_5} &= -a_{41}n_p \omega_r \lambda_{q_r} + a_{42}\lambda_{d_r} + a_{44}i_{q_5} + b_2v_{d_5} + d_{2}(t), \\
y &= \text{col}(i_{q_3}, i_{d_5})
\end{align}
$$

where $\omega_r$ is the time-varying rotate speed, both $d_1(t)$ and $d_2(t)$ represent the electromagnetic interferences, $a_{11} = -\frac{1}{T}, a_{13} = \frac{T}{\sigma L_T}, a_{13} = a_{33} = -\frac{\omega_q}{\sigma L_T} - \frac{\omega_q}{\sigma L_T}, a_{41} = a_{32}, a_{42} = a_{31}, a_{44} = a_{33}, b_1 = \frac{\omega_q}{\sigma L_T} - \frac{\omega_q}{\sigma L_T}, b_2 = b_1, f_{31} = \frac{\omega_q}{\sigma L_T} - \frac{\omega_q}{\sigma L_T}$ and $f_{32} = \frac{\omega_q}{\sigma L_T} - \frac{\omega_q}{\sigma L_T}$.

Let $z_1 = \text{col}(\lambda_{q_r}, \lambda_{d_r})$ and $z_2 = \text{col}(i_{q_3}, i_{d_5})$. Then the system (1)-(4) can be written in a compact form

$$
\begin{align}
z_1 &= A_{11}z_1 + A_{12}z_2 + F_{12} + I_f, \\
z_2 &= A_{21}z_1 + A_{22}z_2 + B_{34}v + F_{34} + d, \\
y &= \text{Col}(z_1, z_2)
\end{align}
$$

where $f = \text{col}(i_f, v_{oa})$ represents fault caused by shorted turns, $v = \text{col}(v_{q_3}, v_{d_5}), d = \text{col}(d_1, d_2)$ and

$$
\begin{align}
A_{11} &= \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}, & \Delta A_{11} &= \begin{bmatrix} 0 & -n_p \omega_r \\ n_p \omega_r & 0 \end{bmatrix}, \\
A_{21} &= \begin{bmatrix} a_{31} & 0 \\ 0 & a_{42} \end{bmatrix}, & \Delta A_{21} &= \begin{bmatrix} 0 & -a_{32}n_p \omega_r \\ a_{32}n_p \omega_r & 0 \end{bmatrix}, \\
A_{12} &= \begin{bmatrix} a_{13} & 0 \\ 0 & a_{24} \end{bmatrix}, & A_{22} &= \begin{bmatrix} a_{33} & 0 \\ 0 & a_{44} \end{bmatrix}, \\
F_{12} &= \begin{bmatrix} f_{11} & 0 \\ 0 & 0 \end{bmatrix}, & F_{34} &= \begin{bmatrix} f_{31} & f_{32} \\ 0 & 0 \end{bmatrix}, \\
B_{34} &= \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}, & C &= \begin{bmatrix} 0 & I_2 \end{bmatrix}.
\end{align}
$$

It can be seen that $A_{11}$ and $A_{21}$ are independent of $\omega_r$, but $A_{11}$ and $A_{21}$ rely on $\omega_r$. It is assumed throughout this paper that the measured speed signal $\hat{\omega}_r \in \Omega_{\omega_r}$ where

$$
\Omega_{\omega_r} = \{\hat{\omega}_r \in R | |\hat{\omega}_r - \omega_r| \leq \Delta \omega_r, \Delta \omega_r \in R\}.
$$

Thus, the upper bound and the lower bound of $\omega_r$ can be obtained by $\omega_r = \hat{\omega}_r + \Delta \omega_r$ and $\omega_r = \hat{\omega}_r - \Delta \omega_r$ respectively, and further, $\omega_r - \omega_r \leq 2\Delta \omega_r, 0 \leq \omega_r - \omega_r \leq 2\Delta \omega_r, 0 \leq \omega_r - \omega_r \leq 2\Delta \omega_r$.

Define

$$
\varphi_1 := \begin{bmatrix} 0 & -n_p \\ 0 & 0 \end{bmatrix}, \varphi_2 := \begin{bmatrix} 0 & 0 \\ n_p & 0 \end{bmatrix}, \\
\varphi_3 := \begin{bmatrix} 0 & a_{32}n_p \\ 0 & 0 \end{bmatrix}, \varphi_4 := \begin{bmatrix} 0 & 0 \\ -a_{41}n_p & 0 \end{bmatrix}.
$$

Then $\varphi_1 < 0, \varphi_2 > 0, \varphi_3 > 0$ and $\varphi_4 < 0$. Moreover, since $a_{32} = a_{41}, \varphi_3 = -a_{32}\varphi_1$ and $\varphi_4 = -a_{32}\varphi_2$. Let $\varphi_1 := \omega_r \omega_r$. Then

$$
\Delta A_{11}z_1 = \varphi_1 f_1 + \varphi_2 f_2, \Delta A_{21}z_1 = \varphi_3 f_1 + \varphi_4 f_2.
$$

Based on Lemma 2, for $z_1 \in [z, \xi]$ and $\omega_r \in [\omega_r, \bar{\omega}_r]$, there exist $\varphi_1$ and $\varphi_1$ such that $\varphi_1 \in [\varphi_1, \bar{\varphi}_1]$. Then

$$
\varphi_1 z_1 \in \begin{bmatrix} \varphi_1 \varphi_1 \\ \varphi_3 \varphi_3 \end{bmatrix}, \varphi_2 z_1 \in \begin{bmatrix} \varphi_2 \varphi_2 \\ \varphi_2 \varphi_2 \end{bmatrix}, \varphi_3 z_1 \in \begin{bmatrix} \varphi_3 \varphi_3 \\ \varphi_4 \varphi_4 \end{bmatrix}.
$$

A reasonable assumption in this study on $f, d_1(t)$ and $d_2(t)$ are presented as follows.

**Assumption 1:** There exist constants $d, \bar{d}$ and $\bar{f}$ such that $d \leq d(t) \leq \bar{d}$ and $||f|| \leq \bar{f}$.

**Remark 1:** Because the fault $f$ in system (6)-(7) caused by shorted turns is low-frequency, the electromagnetic interferences with low frequencies are the most significant factor to influence fault detectability, which are also mainly considered in this paper. Therefore, it is also reasonable for low-frequency electromagnetic interferences $d$ to make this assumption. In addition, the assumption for $d$ and $f$ in Assumption 1 is popular in interval observers and sliding mode observers (see [26], [30] and [20]).

### B. Incipient Shorted-Turn Fault Description

From (6)-(8), the transfer functions from $f$ to $y$ and from $d$ to $y$ are obtained respectively by $G_f(s) = C(sI - A(\omega_r))^{-1} \text{Col}(F_{12}, F_{34})$ and $G_d(s) = C(sI - A(\omega_r))^{-1}$ where

$$
A(\omega_r) = \begin{bmatrix} A_{11} + \Delta A_{11} & A_{12} \\ A_{21} + \Delta A_{21} & A_{22} \end{bmatrix}.
$$

Then, two incremental quantities, $\Delta y_f$ and $\Delta y_d$ caused by $f$ and $d$ respectively, can be described by $\Delta y_f = G_f(s)f$ and $\Delta y_d = G_d(s)d$. Thus,

$$
\inf_{\Delta y_d \neq 0} \frac{||\Delta y_f||_2}{||\Delta y_d||_2} = \inf_{\omega} \frac{\inf_{\omega} g(G_f(j\omega))}{\sup_{\omega} \tilde{g}(G_d(j\omega))} \times \inf_{\omega} \frac{||f||_2}{||d||_2},
$$

where $g(\cdot)$ and $\tilde{g}(\cdot)$ represent the minimum and maximum singular values respectively, and $\omega$ is operating frequency of the induction motors.

Now, a scale variable to describe the developing process of incipient shorted-turn faults $f$ is ready to be defined by

$$
\Gamma = \inf_{\Delta y_d \neq 0} \frac{||\Delta y_f||_2}{||\Delta y_d||_2} = \inf_{\omega} \frac{||f||_2}{||d||_2},
$$

where $\alpha = \inf_{\omega} g(G_f(j\omega))/\sup_{\omega} \tilde{g}(G_d(j\omega))$.

**Remark 2:** It should be pointed out that the scale variable $\Gamma$ defined in (13) provides a quantity relationship between fault $f$ and disturbance $d$ to some extent for traction motors. It can also be used to distinguish the incipient faults from other abrupt faults.

For practical induction motors, there exist preset constants $\Gamma$ and $\bar{\Gamma}$ such that the developing process of stator-winding shorted-turn faults is divided into three levels. The first level
is $D.1: 0 \leq \Gamma < \Gamma$. In this case, it is unnecessary to detect the shorted-turn faults because $f$ is small sufficiently in amplitude and the induction motor operates safely. The second level is $D.2: \Gamma < \Gamma < \Gamma$. The shorted-turn faults begin to affect the normal operation and degrade the performances of motors. However, $f$ is not large enough in amplitude so that it is challenging to detect. The third level is $D.3: \Gamma < \Gamma < +\infty$. In this case, the turns have become shorted seriously which even stop the running of the motors.

In this study, the shorted-turn faults belonging to $D.2$ are mainly considered, which are the so-called “incipient shorted-turn faults”. A set includes all the incipient shorted-turn faults can be defined by

\[
\Omega_{f,\Gamma} = \left\{ f \mid \Gamma \in [\bar{\Gamma}, \bar{\Gamma}] \right\} .
\]

(14)

It should be pointed out that if all $f \in \Omega_{f,\Gamma}$ are detectable, then the fault detectability of the proposed FD schemes is characterized by $\Omega_{f,\Gamma}$.

The objective of this paper is to design IFD schemes for the motor system (6)-(8) such that the detectability is able to be characterized by $\Omega_{f,\Gamma}$, i.e. all incipient faults $f \in \Omega_{f,\Gamma}$ are detectable, by

1) proposing an interval sliding mode diagnostic observer,

2) proposing a novel residual generator and an interval threshold generator.

III. INCIPIENT FAULT DETECTION SCHEMES

A. INTERVAL SLIDING MODE DIAGNOSTIC OBSERVER DESIGN

A fault diagnostic observer will be developed for system (6)-(8) in this section using interval estimation and sliding mode techniques, which will provide interval estimate for $z_1$ in fault-free scenario in the presence of uncertainty $\Delta A_{11} z_1$ and reconstruction for $z_2$ with uncertainty $\Delta A_{22} z_2$ and disturbance $d$ in both fault and fault-free scenarios.

1) Observer Structure Design: Firstly, denote $\bar{z}_1$, $\bar{z}_2$, $\bar{z}_1$ and $\bar{z}_2$ as the estimates of upper bound and lower bound of $z_1$ and $z_2$, respectively. Based on the structure of dynamical equations in (6) and (7), the following observer structure is proposed:

\[
\begin{align*}
\dot{\bar{z}}_1 &= A_{11} \bar{z}_1 + \varphi_1 \phi_1 + \varphi_2 \phi_1 + A_{12} y + L_1 (\bar{z}_1 - \bar{z}_1) + A_2 D[t_1] \{ \phi_1(\bar{z}_1 - \bar{z}_1) \} ; \\
\dot{\bar{z}}_2 &= A_{12} \bar{z}_1 + \varphi_1 \phi_1 + \varphi_2 \phi_1 + A_{12} y + L_1 (\bar{z}_1 - \bar{z}_1) - A_2 D[t_1] \{ \phi_1(\bar{z}_1 - \bar{z}_1) \} + K_1 D[t_1] y ; \\
\dot{\bar{z}}_2 &= A_{22} \bar{z}_2 + \varphi_1 \phi_1 + \varphi_2 \phi_1 + A_{22} \bar{z}_2 + B_{12} \bar{z}_2 ; \\
\dot{\bar{z}}_2 &= A_{22} \bar{z}_2 + \varphi_1 \phi_1 + \varphi_2 \phi_1 + A_{22} \bar{z}_2 + B_{12} \bar{z}_2 + K_2 y.
\end{align*}
\]

(15)-(18)

where the initial values satisfy $\bar{z}_1(0) \leq z_1(0) \leq \bar{z}_1(0)$ and $\bar{z}_2(0) \leq z_2(0) \leq \bar{z}_2(0)$, the gain matrices $A_2$ and $A_2$, $K_1$ and $K_2$ are particularly added here to compensate for observer unmatched uncertainty, and the nonnegative matrix $L_1$ is used to ensure interval estimation. They will be specified later. The matrices $L_2$, $K_2$ and $K_2$ are to be designed to guarantee the occurrence of sliding mode. The nonlinear function $\nu$ is designed as $\nu = \text{col}(\text{sign}(\bar{z}_2 - \bar{z}_2), \text{sign}(\bar{z}_2 - \bar{z}_2)))$. The dead-zone operator $D[-]$ is defined by

\[
D[t_1] = \begin{cases} 1, & t > t_1, \\ 0, & t \leq t_1. \end{cases}
\]

The time instant $t_1$ is the time when sliding mode occurs which will be specified later.

**Remark 3:** The dead-zone operator $D[-]$ is used here to guarantee that before sliding mode occurs, the observer (15)-(16) can provide an interval estimate for $z_1$ by guaranteeing that $\varphi_1 \phi_1 + \varphi_2 \phi_1 + A_2 D[t_1] \{ \phi_1(\bar{z}_1 - \bar{z}_1) \} - K_1 D[t_1] y \geq \Delta A_{11} z_1 \geq \varphi_1 \phi_1 + \varphi_2 \phi_1 - A_2 D[t_1] \{ \phi_1(\bar{z}_1 - \bar{z}_1) \} + K_1 D[t_1] y.

\[
\nabla
\]

**Remark 4:** It should be pointed out that the observer unmatched uncertainty caused by $\Delta A_{11} z_1$ is quite challenging to compensate. Most of related works, for example, [28], [20], [33] and [18], use robust methods such as $L_1$ and $L_2$ gains to address this issue. Different from the developed interval observer by [20] and [18] for LPV systems, a special observer structure (15)-(18) using the sliding mode technique is proposed where $K_1 D[t_1] y$ and $K_2 D[t_1] y$ in (15) and (16) are particularly designed to compensate for observer unmatched uncertainty caused by $\Delta A_{11} z_1$, which facilitates to improve fault detectability.

\[
\nabla
\]

2) Observer Parameters Design: Now, it is ready to design observer parameters. Firstly, throughout this paper, it is assumed that the incipient fault occurrence time instant $t_0 > t_1$, which is reasonable because $t_1$ is adjustable. Due to the dead-zone operator $D[-]$, the stability analysis and interval estimation will be divided into two phases: $t \leq t_1$ phase and $t > t_1$ phase. It should be noted that the $t \leq t_1$ phase is fault-free. The parameter design objective is given as follows:

1) for $t \leq t_1$, $z_1 \in \{\bar{z}_1, \bar{z}_1\}$, $z_1 - \bar{z}_1$ and $\bar{z}_1 - \bar{z}_1$ are ultimately bounded and $\bar{z}_1$ and $\bar{z}_1$ are driven to a sliding surface.

2) for $t > t_1$, uncertainty caused by $\Delta A_{11} z_1$ is compensated for, $z_1 - \bar{z}_1$ and $\bar{z}_1 - \bar{z}_1$ are still ultimately bounded, $z_1 \in \{\bar{z}_1, \bar{z}_1\}$ in fault-free scenario, and $\bar{z}_2$ and $\bar{z}_2$ remain on the same sliding surface in both fault and fault-free scenarios.

Define the estimate errors as follows:

\[
\begin{align*}
\hat{e}_1 &:= \bar{z}_1 - z_1, \quad \hat{e}_1 := z_1 - \bar{z}_1, \\
\hat{e}_2 &:= \bar{z}_2 - z_2, \quad \hat{e}_2 := z_2 - \bar{z}_2.
\end{align*}
\]

By comparing (15)-(18) with (6)-(7), the estimate error dynamics are obtained by

\[
\begin{align*}
\dot{\hat{e}}_1 &= A_{11} \hat{e}_1 + \varphi_1 (\phi_1 - \phi_1) + \varphi_2 (\phi_1 - \phi_1) + L_1 (\hat{e}_1 + \bar{e}_1) + A_2 D[t_1] \{ \phi_1(\bar{z}_1 - \bar{z}_1) \} ; \\
\dot{\hat{e}}_2 &= A_{12} \hat{e}_1 + \varphi_1 (\phi_1 - \phi_1) + \varphi_2 (\phi_1 - \phi_1) + \bar{e}_1 + \hat{e}_1 + A_2 D[t_1] \{ \phi_1(\bar{z}_1 - \bar{z}_1) \} + K_1 D[t_1] y ; \\
\dot{\hat{e}}_2 &= A_{22} \hat{e}_1 + \varphi_1 (\phi_1 - \phi_1) + \varphi_2 (\phi_1 - \phi_1) + A_{22} \hat{e}_2 + B_{12} \hat{e}_2 + \hat{e}_2, \\
\dot{\hat{e}}_2 &= A_{22} \hat{e}_1 + \varphi_1 (\phi_1 - \phi_1) + \varphi_2 (\phi_1 - \phi_1) + A_{22} \hat{e}_2 + B_{12} \hat{e}_2 + K_2 y.
\end{align*}
\]

(19)-(22)
Let $e_1 = \text{col}(\tilde{e}_1, e_1)$ and $e_2 = \text{col}(\tilde{e}_2, e_2)$. Then the error system (19)-(22) can be written in a compact form

\begin{align}
\dot{e}_1 &= A_1 e_1 + A_2 D(t_e) e_2 + \Phi_1 + F_1 C_{f} f \\
&\quad - K_1 D(t_e) v, \\
\dot{e}_2 &= A_2 e_1 + A_3 e_2 + \Phi_2 + \bar{d} + F_2 C_{f} f - K_2 v,
\end{align}

where $\bar{d} = \text{col}(d - \tilde{d}, d - \tilde{d}) < 0$,

\begin{align}
A_1 &= \begin{bmatrix} A_{11} + L_1 & L_1 \\ L_1 & A_{11} + L_1 \end{bmatrix}, & A_2 &= \begin{bmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{bmatrix}, \\
A_3 &= \text{diag}(A_{21}, A_{22}), & A_4 &= \begin{bmatrix} A_{22} + L_2 \\ L_2 \end{bmatrix}, \\
\Phi_1 &= \begin{bmatrix} \varphi_1(\bar{\phi}_1 - \phi_1) + \varphi_2(\bar{\phi}_1 - \phi_1) \\ \varphi_1(\bar{\phi}_1 - \phi_1) + \varphi_2(\bar{\phi}_1 - \phi_1) \end{bmatrix}, & K_1 &= \begin{bmatrix} \tilde{K}_1 \\ \tilde{K}_1 \end{bmatrix}, \\
\Phi_2 &= \begin{bmatrix} \varphi_3(\bar{\phi}_1 - \phi_1) + \varphi_4(\bar{\phi}_1 - \phi_1) \\ \varphi_3(\bar{\phi}_1 - \phi_1) + \varphi_4(\bar{\phi}_1 - \phi_1) \end{bmatrix}, & K_2 &= \begin{bmatrix} \tilde{K}_2 \\ \tilde{K}_2 \end{bmatrix},
\end{align}

$\Phi_1 = \begin{bmatrix} \varphi_1(\bar{\phi}_1 - \phi_1) + \varphi_2(\bar{\phi}_1 - \phi_1) \\ \varphi_1(\bar{\phi}_1 - \phi_1) + \varphi_2(\bar{\phi}_1 - \phi_1) \end{bmatrix}$, $K_1 = \begin{bmatrix} \tilde{K}_1 \\ \tilde{K}_1 \end{bmatrix}$, $\Phi_2 = \begin{bmatrix} \varphi_3(\bar{\phi}_1 - \phi_1) + \varphi_4(\bar{\phi}_1 - \phi_1) \\ \varphi_3(\bar{\phi}_1 - \phi_1) + \varphi_4(\bar{\phi}_1 - \phi_1) \end{bmatrix}$, $K_2 = \begin{bmatrix} \tilde{K}_2 \\ \tilde{K}_2 \end{bmatrix}$.

$F_1 = \text{diag}(F_{12}, F_{13})$, $F_2 = \text{diag}(F_{34}, F_{34})$ and $C_{f} = \text{col}(-I_2, I_2)$.

Recalling $\varphi_3 = -\alpha_3 \varphi_1$ and $\varphi_4 = -\alpha_3 \varphi_2$, there exists a nonsingular nonnegative matrix $T_0$ such that $\Phi_1 = T_0 \Phi_2$ where

$$T_0 = \frac{1}{\alpha_3} \begin{bmatrix} 0 & I_2 \\ I_2 & 0 \end{bmatrix}.$$ 

A new coordinate transformation $(e_1, e_2) \rightarrow (e_s, e_2)$ where $e_s = e_1 + T e_2$ with $T = -T_0 D(t_e)$ is introduced. Then

\begin{align}
\dot{e}_s &= (A_1 + TA_3) e_s + (A_2 D(t_e) + T A_4 - (A_1 + TA_3) T) e_2 \\
&\quad + T \bar{d} + (F_1 + TF_2) C_{f} f - (K_1 D(t_e) + TK_2) v, \\
\dot{e}_2 &= A_3 e_s + (A_4 - A_3 T) e_2 + \Phi_2 + \bar{d} + F_2 C_{f} f - K_2 v.
\end{align}

**Remark 5:** Generally speaking, it is not necessary to require $T_0$ to be nonnegative. However, for the considered traction motor system (6)-(8), $T_0$ is nonnegative, which is useful in the following mathematical derivation. \(\nabla\)

The t < t phase. For $t \leq t_s$, $F_1 C_{f} f = 0$ and it follows from (23) that $\dot{e}_1 = A_1 e_1 + F_1 C_{f} f$. Based on Lemma 2, there exist $\chi_1$ and $\bar{\chi}_1$ such that $e_s, z_1 - \bar{\chi}_1, \bar{\chi}_1$ and $z_1 - \bar{\chi}_1$ in $[0, \chi_1], \bar{\chi}_1$ and $z_1 - \bar{\chi}_1$ in $[0, \bar{\chi}_1]$. Then

$$\Phi_1 \leq \Delta A_1 e_1 + \Phi_1 \tag{27}$$

where $\Delta A_1 = \begin{bmatrix} \tilde{A}_{11} \\ \tilde{A}_{13} \\ \tilde{A}_{14} \end{bmatrix}$

Then the following proposition is ready to be presented.

**Proposition 1:** If there exists a nonnegative matrix $L_1$ such that $\Phi_1 \leq \Delta A_1 e_1 + \Phi_1$, then

(i) for $t \leq t_s$, $e_s > e_1 > 0$.

(ii) for $t \leq t_s$, $\|e_s\| < \|w_1(t)\|$ where $w_1(t)$ is ultimately bounded.

\textbf{Proof:} Firstly, it should be noted that for $t \leq t_s$, $T = 0$ and $e_s = e_1$. From (11) that $\Phi_1 > 0$. Therefore, based on the positive system theory [34], with the Metzler matrix $A_1$, $e_1 > 0$ for $0 \leq t \leq t_s$.

Furthermore, for $t \leq t_s$, if $A_1 > A_1 + \Delta A_1$, then it yields from (9) that $A_1 e_1 > (A_1 + \Delta A_1) e_1$. Thus, it follows from (29) that $\dot{e}_1 = A_1 e_1 + \Phi_1$. By Comparison Principle provided by [35], if $0 < e_1(0) < w_1(0)$, then $0 < e_1 \leq w_1$ where $w_1$ is the state of system $w_1 = A_1 w_1 + \Phi_1$. Since $A_1$ is the Hurwitz and Metzler matrix, based on positive system theory, $w_1 > 0$ and is ultimately bounded associated with $A_1$ and $\Phi_1$.

Hence, the result follows. \(\blacksquare\)

**Remark 6:** Since $\omega$ is time varying, $A_1 + \Delta A_1$ is a time-varying system matrix. The stability condition for LPV systems with constant uncertain parameters developed in [18] (Theorem 7) does not work any more. In Proposition 1, the Metzler matrix $A_1$ is introduced to deal with this problem. \(\nabla\)

The $t > t_s$ phase. For $t > t_s$, with the parameter selection $K_1 = -TK_2$, $A_2 = -TA_4 + (A_1 + TA_3) T$, it yields from (25) that

\begin{align}
\dot{e}_s &= (A_1 + TA_3) e_s + \bar{d} + (F_1 + TF_2) C_{f} f.
\end{align}

Then the following Proposition is ready to be presented.

**Proposition 2:** If $K_1 = -TK_2$, $A_2 = TA_4 + (A_1 + TA_3) T$ and there exists a nonnegative matrix $L_1$ such that $A_1 + TA_3$ is the Metzler and Hurwitz matrix, then

(i) in fault-free scenario, for $t > t_s$, $e_s > 0$, $z_1 \in [z_1^-, z_1^+]$ and $e_s$ is ultimately bounded,

(ii) in both fault and fault-free scenarios, for $t > t_s$, $e_s$ is ultimately bounded and $\|e_s\| < w_2(t)$ where $w_2(t)$ is a positive scalar function determined later. \(\nabla\)

\textbf{Proof:} It can be seen from (30) that with the selected $K_1$ and $A_2$, the observer unmatched uncertainty $\Phi_1$ disappears, which means that it is compensated for. In fault-free scenario, based on the positive system theory [34], with the Hurwitz matrix $A_1 + TA_3$, $\bar{d} > 0$, $(F_1 + TF_2) C_{f} f = 0$ and condition $e_s(t_s) = 0$, $e_s(t) > 0$ for $t > t_s$. Since during the sliding, $e_2 = 0$, $e_1 = e_s > 0$, that is $z_1 \in [z_1^-, z_1^+]$. Furthermore, since $A_1 + TA_3$ is the Hurwitz matrix, $e_s$ is ultimately bounded in both fault and fault-free scenarios.

In addition, using Comparison Principle, the reference [36] provides the method to obtain the positive scalar function $w_2(t)$ via constructing Lyapunov functions. So the construction of $w_2(t)$ is omitted here. Hence, the result follows. \(\blacksquare\)

Since $\Phi_1 = T_0 \Phi_2$ and $T_0 \geq 0$, it follows from (27) that $\Phi_2 \leq \Delta A_1 e_1 + \Phi_2$ where $\Delta A_1 = T_0^{-1} \Delta A_1$ and $\Phi_2 = T_0^{-1} \Phi_1$ with

$$T_0^{-1} = \begin{bmatrix} 0 & I_2 \\ I_2 & 0 \end{bmatrix}.$$
being nonnegative matrix. Then it is obtained from (26) that
\[ \|\Phi_2\| \leq \|\Delta A_3\| \cdot \|e_r - T e_2\| + \|\Phi_2\| \]
\[ \leq \|\Delta A_3\| \cdot \|e_r\| + \|\Delta A_3 T\| \cdot \|e_2\| + \|\Phi_2\|. \quad (31) \]

For error system (25)-(26), consider the sliding surface
\[ S = [\text{col}(e_r, e_s)] e_2 = 0. \quad (32) \]

Next, it is focused on the design of parameters \( L_2 \) and \( K_2 \) to guarantee the reachability condition with respect to sliding surface \( S \) for both \( t \leq t_s \), phase and \( t > t_s \), phase in both fault and fault-free scenarios. The following proposition is ready to be presented.

**Proposition 3:** The error system (26) is driven to sliding surface \( S \) in (32) before \( t_s \) and maintains on it thereafter if there exists a matrix \( L_2 \) such that

1. There exists the Hurwitz and Metzler matrix \( \bar{A}_4 \) such that \( (\bar{A}_4)_{ii} \geq (A_4 - A_1 T + ||\Delta A_4 T||E_4)_{ii} \) and \( (\bar{A}_4)_{ij} \geq (A_4 - A_1 T + ||\Delta A_4 T||E_4)_{ij} \) for \( i, j = 1, \ldots, 4, i \neq j \) where \( (\cdot)_{ij} \) represent the element of \( \text{ith} \) row and \( \text{jth} \) column of the matrix.

2. The gain matrix \( K_2 = \lambda_{\max}(P_2)P_2^{-1}c \) where \( P_2 \) satisfies
\[ \bar{A}_4^T P_2 + P_2 \bar{A}_4 < 0 \quad (33) \]

and \( c \) satisfies
\[ c \geq [||A_1|| + ||\Delta A_1||\max(||w_1(t)||, ||w_2(t)||)] \]
\[ + 4\|T_0^{-1}\| \cdot \|\varphi_2 - \varphi_1\| \Delta \omega_s \sqrt{\frac{1}{z_1^2} + \frac{z_2}{z_1^2}} \]
\[ + 4\max(||\tilde{P}_2||, ||\tilde{d}||) \cdot ||F_2 C_f|| \bar{f} + \eta \quad (34) \]

with \( \eta \) being any positive constant.

**Proof:** It follows from Propositions 1 and 2 that \( ||e_r|| \leq ||w_1(t)|| \) for \( t \leq t_s \) and \( ||e_r|| \leq w_2(t) \) for \( t > t_s \). Then \( ||e_r|| \leq \max(||w_1(t)||, ||w_2(t)||) \) for \( t \geq 0 \). Since \( \bar{A}_4 = T_0^{-1} \Phi_1, \|
\Phi_2\| \leq 4\|T_0^{-1}\| \cdot ||\varphi_2 - \varphi_1\| \Delta \omega_s \sqrt{\frac{1}{z_1^2} + \frac{z_2}{z_1^2}} \). In addition, from Assumption 1, \( \bar{d} \leq 4\max(||\tilde{P}_2||, ||\tilde{d}||) \). Let \( \nu(t) = \frac{1}{2} \epsilon_2^T P_2 \nu_2 \). It is worth mentioning that based on positive system theory in [34], \( P_2 \) is a diagonal positive matrix. The time derivative of \( V \) along (26) is
\[ V = \frac{1}{2} \epsilon_2^T((A_4 - A_1 T)^T P_2 + P_2(A_4 - A_1 T)) \epsilon_2 \]
\[ + e_2^T P_2 A_3 e_s + e_2^T P_2 \Phi_2 + e_2^T P_2 d + e_2^T P_2 F_2 C_f f \]
\[ - \epsilon_2^T P_2 K_2 \nu \]
\[ \leq \frac{1}{2} \epsilon_2^T((A_4 - A_1 T + ||\Delta A_4 T||E_4)^T P_2 \]
\[ + P_2(A_4 - A_1 T + ||\Delta A_4 T||E_4)) \epsilon_2 + \lambda_{\max}(P_2) ||e_2|| \]
\[ \cdot (||A_3|| + ||\Delta A_3|| ||e_r|| + ||\tilde{P}_2|| + ||\tilde{d}|| + ||F_2 C_f|| \bar{f} + c) \]
\[ \leq \frac{1}{2} \epsilon_2^T(\bar{A}_4^T P_2 + P_2 \bar{A}_4) ||e_2|| - \lambda_{\max}(P_2) ||e_2|| \eta \]
\[ \leq - \sqrt{2} \eta \lambda_{\min}(P_2) \bar{f}. \]

where the first inequality is obtained based on (31) and the second inequality is obtained based on \( e_2^T((A_4 - A_1 T + ||\Delta A_4 T||E_4)^T P_2 + P_2(A_4 - A_1 T + ||\Delta A_4 T||E_4)) e_2 \leq ||e_2|| (\bar{A}_4^T P_2 + P_2 \bar{A}_4) ||e_2|| \].

Therefore, the reachability condition is satisfied. Furthermore, from [37], \( e_2 \) is driven to the sliding surface \( S \) in (32) before \( t_s \) and maintains on it thereafter where
\[ t_s = \frac{||e_2(0)|| \lambda_{\min}(P_2)}{\eta \lambda_{\max}(P_2)}. \quad (35) \]

Hence, the result follows.

**Remark 7:** It can be seen from (35) that the sliding mode occurrence time \( t_s \) can be reduced by decreasing \( e_2(0) \) and increasing \( \eta \). The value \( e_2(0) \) can be adjusted by choosing appropriate initial value for the observer dynamics. The value \( \eta \) is the reachability which can be chosen freely. This confirms that it is reasonable to assume fault occurrence time \( t_0 > t_s \).

**B. Residual and Interval Threshold Generation**

For \( t > t_s \), the sliding mode has occurred, and thus \( \dot{e}_2 = e_2 = 0, e_1 = e_s \). Then it follows from (30) that
\[ \dot{e}_1 = (A_1 + T A_3) e_1 + (F_1 + T F_2) C_f f + T \tilde{d}. \quad (36) \]

**Remark 8:** The equation (23) for \( e_1 \) can not be computed by ordinary differential equation (ODE) theory because of the existence of the discontinuous \( v \). It is necessary to introduce (36) such that traditional fault diagnosis methods for continuous systems can be applied.

To generate residuals, an estimator \( \hat{z}_2 \) for \( z_2 \) should be firstly constructed. Referring the structures of \( \hat{z}_2 \) and \( \hat{z}_1 \) in (17) and (18) respectively, the estimator \( \hat{z}_2 \) is constructed as
\[ \hat{z}_2 = \frac{1}{2}(A_2 (\hat{z}_1 + \hat{z}_2) + (\varphi_3 + \varphi_4)(\bar{\Phi}_1 + \bar{\Phi}_1)) \]
\[ + A_{22} \hat{z}_2 + B_{34} v. \quad (37) \]

Define \( r := z_2 - \hat{z}_2 \). Then in fault scenario, the residual generator is obtained, which, according to (26), can be expressed as
\[ \dot{r} = C_r A_3 e_1 + A_{22} r + C_r (\hat{\Phi}_2 + \hat{\Phi}_1) + F_{34} f. \quad (38) \]

where \( C_r = \left[ -\frac{1}{2} I_2, \frac{1}{2} I_2 \right] \). To simplify the symbols, system (36) and (38) are written in a compact form
\[ \dot{H} = A_H H + D_H d_H + F_H f, \quad (39) \]
\[ r = C_H H \quad (40) \]

where \( H = \text{col}(e_1, r), d_H = \text{col}(\Phi_2, d) \) and
\[ A_H = \left[ \begin{array}{cc} A_1 + T A_3 & 0 \\ C_r A_3 & A_{32} \end{array} \right], \quad D_H = \left[ \begin{array}{cc} 0 & T \\ C_r & C_r \end{array} \right], \quad F_H = \left[ \begin{array}{cc} (F_1 + T F_2) C_f f \\ F_{34} \end{array} \right], \quad C_H = \left[ \begin{array}{cc} 0 & I_2 \end{array} \right]. \]

It can be seen from (39) and (40) that the gain matrix \( L_1 \) affects both the robustness from \( d_H \) to \( r \) and sensitiveness to \( f \) to \( r \). Reference [8] has provided a number of approaches such as \( H_2 \) to \( H_2 \) trade-off approach and \( H_\infty \) to \( H_\infty \) trade-off approach etc. to optimize \( L_1 \). In this paper, the optimization for \( L_1 \) is omitted and it is supposed that \( L_1 \) has been determined to satisfy the requirements in Propositions 1 and 2.

The determination of a threshold is to find out the tolerant limit for disturbances and model uncertainties under fault-free scenario [8]. Accordingly, the interval threshold should
be generated to include residual \( r \) in fault-free scenario. Two estimators \( \hat{Z} \) and \( \bar{Z} \) are firstly constructed as

\[
\hat{Z} = A_2 Z_1 + \phi_1 \bar{Z} + B_3 v \quad \bar{Z} = A_2 Z_1 + \phi_4 \bar{Z} + B_3 v
\]

where \( L_r \) is the design gain matrix to determine later. Let

\[
A_Z := \begin{bmatrix} A_2 + L_r & L_r \\ L_r & A_2 + L_r \end{bmatrix}
\]

Then it is easy to obtain that if \( A_Z \) is the Metzler matrix, then \( z_2 = [\hat{Z} \bar{Z}] \) in fault-free scenario.

Define \( \bar{r} := \bar{Z} - Z_2 \) and \( r := Z - Z_2 \). Then \( r \in [\bar{r}, \bar{r}] \) in fault-free scenario. Furthermore, the threshold generator is obtained by

\[
\begin{align*}
\bar{r} &= C_r A_3 e_1 + C_r (\bar{F} d + \bar{d}) + (A_2 + 2 L_r) \bar{r} \\
\bar{r} &= C_r A_3 e_1 + C_r (\bar{F} d + \bar{d}) + (A_2 + 2 L_r) r
\end{align*}
\]

where \( e_1 \) is determined by (36) in fault-free scenario, \( C_r = [1/2, 1/2] \) and \( C_\bar{r} = -C_r \). Similar with (39)-(40), the equations (36), (44) and (45) can be written in a compact form

\[
\begin{align*}
\bar{r} &= \bar{A} \bar{r} + D \bar{d} \\
\bar{r} &= \bar{A} \bar{r} + D \bar{d}
\end{align*}
\]

where \( \bar{A} = \begin{bmatrix} 1 & T A_3 & 0 & 0 \\
A_2 + 2 L_r & A_2 & 0 & 0 \\
A_2 & 0 & 0 & 0 \\
\end{bmatrix} \) and \( \bar{D} = \begin{bmatrix} C_r A_3 & 0 \\
C_r & 0 \\
C_\bar{r} & 0 \\
C_\bar{r} & 0 \\
\end{bmatrix} \), \( \bar{A} \) is the evaluated values of \( \bar{r} \) and \( \bar{D} \bar{r} \) from Assumption 1.

Next, we introduce the calculation approach for \( L_r \) as follows: Firstly, split the \( r \) in system (39)-(40) into two components \( r_f \) and \( r_d \) caused by \( f \) and \( d \) respectively, i.e.,

\[
\begin{align*}
r(s) &= r_f(s) + r_d(s) \\
r_f(s) &= G_f(s) f(s) \\
r_d(s) &= G_d(s) d(s)
\end{align*}
\]

where \( r(s), r_f(s), r_d(s), f(s) \) and \( d(s) \) are the Laplace transforms for \( r(t), r_f(t), r_d(t), f(t) \) and \( d(t) \) respectively, \( G_f(s) \) and \( G_d(s) \) being transfer functions from \( f \) to \( r_f \) and from \( d \) to \( r_d \) respectively. Then \( |r(s)| ≥ |[|r_f(s)| - |r_d(s)|]| = |r_f(s)| - |r_d(s)| \) as long as \( |r_f(s)| > |r_d(s)| \). Suppose that \( |r_f(s)| > |r_d(s)| \). Then

\[
\inf_{r_f} |r_f(s)| > \inf_{r_d} |r_d(s)| - \sup_{r_d} |r_d(s)|.
\]

As stated in [38], to detect the incipient faults \( f \in \Omega_{f,T} \), it requires that

\[
\inf_{\epsilon, \gamma} |r_f(s)| > \inf_{\epsilon, \gamma} |r_d(s)| \quad \text{if and only if there exists a symmetric positive definite (SPD) matrix } P_k \text{ such that}
\]

\[
\begin{bmatrix} A^T_k P_k + P_k A_k + C^T R C_R - \gamma^2(\epsilon, \Gamma) \\
\end{bmatrix} > 0
\]

where \( \gamma(\epsilon, \Gamma) = \inf_{\epsilon, \gamma} G_f(j \omega) - \sup_{\gamma} G_d(j \omega) \).

It should be pointed out that since \( z_1 \) is inherent bounded, \( \hat{F}_1 \) in (28) is also bounded, which results in that \( \hat{F}_2 \) is bounded due to \( \hat{F}_2 = T_0^{-1} \hat{F}_1 \). From Propositions 1, 2 and 3, both \( e_1 \) and \( e_2 \) are both bounded. Thus, \( \hat{F}_2 \) in \( dH \) is bounded, and then there exists a constant \( dH \) such that \( |dH| > 0 \) such that \( |dH| ≤ |dH| \). Also, from Assumption 1, \( |d| \in [\min|d|, |d|] \), \( \max|d|, |d| \). Thus, there exists a constant \( \epsilon \) such that \( \epsilon ≥ \epsilon \) where \( \epsilon = \max \frac{|d|}{dH} \).

Therefore, based on the well-known bounded real lemma, the calculation approach for \( L_r \) is obtained as follows: the inequality (52) holds if and only if there exists a symmetric positive definite (SPD) matrix \( P_k \) gain matrix \( L_r \) such that

\[
\begin{bmatrix} A^T_k P_k + P_k A_k + C^T R C_R - \gamma^2(\epsilon, \Gamma) \\
\end{bmatrix} > 0
\]

holds for any \( \Gamma \in [\Gamma, \Gamma] \) and \( \epsilon ≥ \epsilon \). Hence, the following proposition is ready to be presented.

**Proposition 4:** All the incipient faults \( f \in \Omega_{f,T} \) are detected if there exist a nonnegative matrix \( L_r \) and a SPD. matrix \( P_k \) such that \( A_Z \) defined in (43) is the Metzler matrix and (53) holds for any \( \Gamma \in [\Gamma, \Gamma] \) and \( \epsilon ≥ \epsilon \).
D. Incipient Fault Detection Decision

As traditional FD in [12] and [6], the following logical relationship is used to determine the occurrence of incipient shorted-turn faults

\( J \leq J_{th}, \rho = 0, \)

\( J > J_{th}, \rho = 1. \)

Therefore, the decision on occurrence for incipient shorted-turn faults is made if \( J(T_d) > J_{th}(T_d) \) for \( t > T_d \) where the detection time instant \( T_d \) satisfies \( T_d > t_0 \) to \( t_s \). To simplify the expression, let \( \rho = 0 \) represent the case (1) and \( \rho = 1 \) case (2).

Then, the following algorithm is ready to be presented.

**Algorithm 1:** The procedure to detect incipient shorted-turn faults for traction motors based on interval sliding mode diagnostic observer (15)-(18)

**Step 1:** Determine \( \bar{\Delta}, \Delta \) and \( \bar{\Delta}_f \) in Assumption 1, and \( \Gamma \) and \( \bar{\Gamma} \) to describe incipient shorted-turn faults

**Step 2:** Select \( L_1 \) and \( L_2 \) to satisfy the conditions of Propositions 1, 2 and 3

**Step 3:** Select \( K_1 \), \( K_{11} \), \( K_2 \), \( K_3 \) and \( A_2 \) to satisfy the conditions in Propositions 2 and 3

**Step 4:** Select \( L_s \) to satisfy the conditions of Proposition 4

**Step 5:** Construct residual generator (38) and interval threshold generator (41)-(42), and then determine \( J \) and \( J_{th} \) by (48).

**Remark 9:** Built on the authors’ previous work [22], the incipient fault detection schemes with detailed analysis and solid results are developed for the traction motors used in CRH in this paper. Comparing with [22], the differences are shown as follows. The considered systems are different. A class of linear time-invariant systems is considered in [22], while in this paper, a specific traction motor system with uncertainties is considered, which is more practical. The designed fault diagnostic observers are different. Due to the uncertainties in the paper, Lemma 2 is introduced to obtain interval bounds for \( \Delta A_{11} z_1 \) and \( \Delta A_{21} z_1 \), and an interval sliding mode observer structure (15)-(18) with dead-zone operator \( \bar{\Delta}[] \) and corresponding observer parameters \( L_1 \), \( L_2 \), \( \bar{K}_1 \), \( K_{11} \), \( \bar{K}_2 \), \( K_2 \) and \( A_2 \) are particularly proposed and designed.

IV. Verification

Following the procedure given in Algorithm 1, the incipient shorted-turn fault detection schemes are constructed as follows:

**Step 1:** The reference stator currents are set as \( i_{qs} = 100 \text{A} \) and \( i_{ds} = 0 \text{A} \). For the traction induction motor used in CRH, electromagnetic interferences in amplitude are approximate 10% of the reference stator currents. Therefore, \( d_1 \) and \( d_2 \) in this simulation are set as

\( d_1 = 10 \sin(250t) \text{A}, \quad d_2 = 10 \cos(250t) \text{A} \)

and then \( \bar{d} \) and \( d \) in Assumption 1 are selected as \( \bar{d} = \text{col}(10, 10) \) and \( d = \text{col}(-10, -10) \).

To determine \( \Gamma \) and \( \bar{\Gamma} \), the gains of \( \bar{g}(G_d(j \omega)) \) and \( g(G_f(j \omega)) \) used by (13) should be given firstly through the amplitude-frequency bode diagrams of \( G_d(j \omega) \) and \( G_f(j \omega) \). The nominal physical parameters of the traction induction motor used in CRH are given in Table I obtained from the traction and driving control system-fault injection benchmark (TDCS-FIB) (see [39] and [11]). In this simulation, the fraction of shorted turns is set as \( \mu = 5\% \). Then the transfer functions \( G_d(s) \) and \( G_f(s) \) which rely on the time-varying motor speed \( \omega_r \) can be obtained. However, since \( \omega_r \) varies with respect to time between zero and the maximum speed 3000rpm, it is impossible to plot the amplitude-frequency bode diagrams of \( G_d(j \omega) \) and \( G_f(j \omega) \) for every \( \omega_r \in [0, 3000 \text{rpm}] \). So amplitude-frequency bode diagrams for only \( \omega_r = 200 \text{rpm}, 1500 \text{rpm} \) and \( 3000 \text{rpm} \) are plotted in Fig. 2, which can describe the rough ranges for \( \bar{g}(G_d(j \omega)) \) and \( g(G_f(j \omega)) \). In addition, disturbances with low frequencies affect incipient fault detection significantly because the fault-related signals caused by shorted turns are low-frequency. Therefore, we mainly focus on the frequencies belongs to \([0, 10^4 \text{Hz}]\). It can be seen from Fig. 2 that

\[
\sup_{\omega \in [10^2, 3000 \text{rpm}]} g(G_f(j \omega)) = 12.4100,
\inf_{\omega \in [10^2, 3000 \text{rpm}]} g(G_f(j \omega)) = 0.0059.
\]

Thus, based on (13), \( a = 4.7542 \times 10^{-4} \).

Table I: Nominal parameters of the traction motor.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>Rated power</td>
<td>300 Kw</td>
</tr>
<tr>
<td>( V )</td>
<td>Rated voltage</td>
<td>2500 V</td>
</tr>
<tr>
<td>( I )</td>
<td>Rated current</td>
<td>106 A</td>
</tr>
<tr>
<td>( RPM )</td>
<td>Rated rotating speed</td>
<td>3000 rpm</td>
</tr>
<tr>
<td>( L_s )</td>
<td>stator inductance</td>
<td>0.0343 H</td>
</tr>
<tr>
<td>( L_r )</td>
<td>rotor inductance</td>
<td>0.0343 H</td>
</tr>
<tr>
<td>( L_m )</td>
<td>mutual inductance</td>
<td>0.0328 H</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>leakage factor</td>
<td>10.0856</td>
</tr>
<tr>
<td>( R_s )</td>
<td>stator resistance</td>
<td>0.1142</td>
</tr>
<tr>
<td>( R_r )</td>
<td>rotor resistance</td>
<td>0.1460</td>
</tr>
<tr>
<td>( n_p )</td>
<td>number of pole pairs</td>
<td>4</td>
</tr>
</tbody>
</table>

Fig. 2. Amplitude-frequency bode diagrams of \( G_f(j \omega) \) and \( G_d(j \omega) \).
given in Table I. In addition, from [31], there is a differential relationship between \( v_{st} \) and \( i_f \) that

\[
  i_f = -\frac{R_f}{L_f} i_f - \frac{3v_{st}}{\mu L_s}.
\]

Then the steady-state value of \( i_f \) is induced and a rough range for the incipient fault \( f \) can also be obtained by \( \|f\| \in [1.31 \times 10^4, 2.63 \times 10^5] \). Thus, based on the definition of \( \Gamma \) given in (13), \( \Gamma = 4.4045 \) and \( \Gamma = 8.8427 \). Therefore, \( \Omega_{f,r} \) for this induction motor is specified by \( \Omega_{f,r} = \{f \mid \Gamma \in [2.2023, 4.5558]\} \).

**Step 2:** Considering the measuring accuracy of the speed sensors and the electromagnetic interferences imposing on the speed sensors, the measuring error in CRH is assumed to be approximate \( \pm 3.5\% \) of the rated speed \( RPM \). So the radius \( \Delta \omega_r \) is chosen as \( \Delta \omega_r = 105 \) rpm. A closed-loop tracking control structure for the traction motor is constructed using the PID control technique to regulate the stator currents, which ensures the existences of \( L_1, L_2 \) and \( L_s \) through regulate eigenvalues of the system matrices \( A_{11} \) and \( A_{22} \). Based on Propositions 1, 2 and 3, it can be calculated that

\[
  L_1 = \begin{bmatrix} 1.2940 & 2.9364 \\ 3.3982 & 1.2870 \end{bmatrix},
\]

\[
  L_2 = \begin{bmatrix} 11.8978 & 11.3190 \\ 11.3190 & 11.8978 \end{bmatrix}.
\]

**Step 3:** Based on Propositions 2 and 3, \( R_1, K_1, R_2 \) and \( K_2 \) are selected where \( \eta = 10 \) and \( P_2 = 0.0081L_s \). Also, the matrix \( A_2 \) is obtained by

\[
  A_2 = \begin{bmatrix} -1.0684 & -1.6439 & 84.7134 & -1.6562 \\ -1.7420 & -1.0681 & -1.7298 & 84.7136 \\ 84.7303 & -1.2917 & -1.0514 & -1.2794 \\ -1.2447 & 84.7263 & -1.2570 & -1.0555 \end{bmatrix}.
\]

Then the interval sliding mode diagnostic observer (15)-(18) is constructed. For simulation purpose, the zero-sequence component of stator voltage is set as \( v_{st} = -250v \) for \( t > 2s \), which is about 10\% of the rated voltage \( V \). Thus, \( f \in \Omega_{f,r} \). To verify both acceleration and uniform motions in this simulation, \( \omega_r \) is set as

\[
  \omega_r = \begin{cases} 
    1000 \text{ rpm}, & t < 3, \\
    1000 + 250(t - 3) \text{ rpm}, & 3 < t < 5, \\
    1500 \text{ rpm}, & t \geq 5.
  \end{cases}
\]

Time responses of the interval sliding mode diagnostic observer are presented in Figs. 3 - 6. It is shown in Figs. 3 and 4 that before incipient faults occur, i.e., for \( t < 2s \), \( \tilde{z}_1 - z_1 \geq 0 \), \( z_1 - \tilde{z}_1 \geq 0 \) and \( \tilde{z}_1 \leq z_1 \leq \tilde{z}_1 \). From Figs. 5 and 6, it can be seen that \( z_2 \) is driven to zero and the sliding mode occurs before 0.5s. Thus, \( t_s = 0.5s \). Furthermore, after sliding mode occurs, i.e., \( t > t_s \), the estimate intervals become obviously tighter in Fig. 3 than the ones for \( t \leq t_s \), which verifies the effectiveness of our proposed technique to compensate for observer unbalanced uncertainties \( \Delta A_{11}z_1 \) for the traction motors.

**Step 4:** With the determined gain matrix \( L_1 \), the transfer functions of \( G_{f_f}(j\omega) \) and \( G_{r_f}(j\omega) \) can be obtained and their
amplitude-frequency bode diagrams are shown in Fig. 7. Then it can obtain that
\[
\sup_{\omega \in [0, 10^3]} \omega \left| G_{rf}(j\omega) \right| = 1.117 \times 10^{-2},
\]
\[
\inf_{\omega \in [0, 10^3]} \omega \left| G_{rd}(j\omega) \right| = 2.709 \times 10^{-5}.
\]

From Fig. 3, the steady value of \( e_1 \) approximates zero, which implies that \( \Phi_2 \) in \( d_\infty \) approximates zero for small sufficiently \( z_1 \). Therefore, the parameter \( \xi \approx 1 \). Hence, based on (13),
\[
\gamma^2 (\Gamma) = 0.575.
\]

By solving LMI formed by (53), a feasible solution can be obtained by
\[
L_r = \begin{bmatrix} 1.2256 & 3.1989 \\ 3.1975 & 1.2229 \end{bmatrix}.
\]

**Step 5:** The residual generator based on (37) and interval threshold generator based on (41)-(42) are constructed using above calculated design parameters, and then \( J \) and \( J_{th} \) given by (48) are determined.

Time responses of \( r, \bar{r}, \underline{r}, J, J_{th} \) and \( \rho \) are presented in Figs. 8 and 9. It can be seen from Fig. 8 that \( r \) escapes from the interval \([\underline{r}, \bar{r}]\) after 2.0s due to the fault occurrence. The residual \( J \) in Fig. 9 exceeds \( J_{th} \) at about 2.6s, and the incipient fault indicate variable \( \rho \) becomes 1 and maintains it for \( t > 2.65s \). Therefore, based on the decision principle, this incipient turn fault is detected at time instant \( T_d = 2.65s \).

**V. Conclusion**

This paper has presented a stator-winding incipient shorted-turn fault detection method for traction motors. The mathematical description of incipient shorted-turn faults has been given from the quantitative point of view. A novel interval sliding mode observer has been particularly designed as diagnostic observer to compensate for observer uncertainties caused by measuring errors from the motor speed sensors. Then, an active robust residual generator and a passive robust threshold generator have been proposed and the design parameters have also been optimized such that the considered incipient shorted-turn faults can be detected. Simulations based on a traction motor used in CRH have been presented in the paper to demonstrate the effectiveness and practicability.
**Appendix A**

PROOF OF LEMMA 2

**Proof:** Note that $\omega x$ satisfies the following inequalities

$$\omega x = \omega x - \frac{1}{\omega} (\omega - \omega) x - \omega (x - \bar{x})$$

$$= \omega (x - \bar{x}) - \omega (x - \bar{x})$$

$$= \frac{1}{\omega} (\omega - \omega) (x - \bar{x}) = 0,$$

$$\ddot{\omega} \dot{x} = \omega (x - \bar{x}) + \dot{\omega} (\dot{x} - \bar{x}) + (\ddot{\omega} - \omega) (\bar{x} - \bar{x})$$

$$= - (\ddot{x} - \bar{x}) + \dot{\omega} (\dot{x} - \bar{x}) + (\ddot{\omega} - \omega) (\bar{x} - \bar{x}) = 0.$$  

Then $-\omega x + \omega x \leq \omega x - \bar{\omega} x + \bar{\omega} x + (\ddot{\omega} - \omega) (\bar{x} - \bar{x}).$ Using Lemma 1 for $\omega x, \dot{\omega} x, \ddot{\omega} x$ and $\bar{\omega} x$, $\omega x \in [\bar{\omega}, \bar{\omega}]$ follows. Furthermore,

$$\omega x = \frac{1}{\omega} (\omega - \omega) \dot{x} + \ddot{x} + \dot{\omega} (\dot{x} - \bar{x}) + \ddot{\omega} (\dot{x} - \bar{x})$$

$$\leq 2 (\omega + \ddot{\omega}) \dot{x} + \dot{\omega} (\dot{x} - \bar{x}) + 2 (\ddot{\omega} (\dot{x} - \bar{x})$$

and

$$\ddot{\omega} = \omega x$$

$$\leq - (\ddot{\omega} - \omega) \dot{x} + \ddot{\omega} (\dot{x} - \bar{x}) + \ddot{x} + \omega (\ddot{\omega} - \omega)$$

$$+ \ddot{\omega} (\dot{x} - \bar{x}) + \ddot{\omega} (\dot{x} - \bar{x})$$

$$\leq 2 (\omega + \ddot{\omega}) \dot{x} + \ddot{\omega} (\dot{x} - \bar{x}) + 2 (\ddot{\omega} (\dot{x} - \bar{x}).$$

Hence, the result follows.

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