# An Econometric Investigation of Hedging Performance of Stock Index Futures in Korea: Dynamic versus Static Hedging

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## Abstract

Employing daily data of stock index and stock index futures, this paper empirically investigates the hedging effectiveness of time-varying hedge ratios of emerging futures markets using South Korea as a case. This paper employs eight variants of GARCH models to estimate the hedge ratios along with the conventional methods, and compares the hedging effectiveness of these estimated hedge ratios across model specifications using both within-sample and out-of-sample forecasting performances. In contrast to recent research findings, hedging performance based on a conventional OLS method outperforms the GARCH class models.

Key words: Stock index futures; time-varying hedge ratio; GARCH model; hedging effectiveness.

JEL classification: G1; G13; G15

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## 1. Introduction

The last five decades have seen tremendous interest in modelling and forecasting of the optimal hedge ratios (OHR) and alternative hedging strategies applied to the commodity and financial futures.[1] Stock index futures contracts, in particular, offer opportunities for unbundling the market and non-market components of risk and return to investment banks, security houses, fund managers and individual investors. Fund managers use stock index futures to alter, temporarily, the systematic risk of a portfolio without having to buy or sell its constituent stock. They are routinely used in program trading and index arbitrage to achieve portfolio insurance. Hedgers use the markets as a means to avoid the risk associated with price changes in the related cash markets. The determination of optimal hedge ratio helps the investor to choose the optimal portfolios with suitable futures and a reasonable number of futures contracts. Consequently, a large body of empirical literature has accumulated in recent years examining the issues of relative effectiveness of sophisticated hedging methods over much simpler and intuitively appealing traditional hedging methods using currencies, commodities, stock indices, and interest rate products (Sultan and Hasan 2008).

Given the plethora of literature, there is a gap in the current research strand. Most previous studies confined their attention to more developed and mature financial markets and exchanges. Quite surprisingly, there has been little research to examine the behaviour of time-varying hedge ratios for emerging markets.[2] This article, therefore, investigates the behaviour of dynamic hedge ratios in the stock and futures markets of South Korea, using alternative variants of GARCH models, and compares the hedging effectiveness of optimal hedge ratios across those models. More specifically, using daily data of the stock spot and futures markets of South Korea – within the framework of bivariate standard GARCH, GARCH-BEKK, GARCH-ECM, GARCH-X, and asymmetric GARCH-GJR, GARCH-DCC models – this paper estimates the time-varying hedge ratios over the period January 2000 to August 2017 and compares the hedging performances of those hedge ratios. In addition, we have employed two customised variants of GARCH models – namely the Markov switching volatility ARCH (MSVARCH) model (Hamilton and Susmel 1994, Turner et al. 1989), and the asymmetric non-linear smooth-transition generalised autoregressive conditional heteroscedasticity (ANST-GARCH) model (Anderson et al. 1999, Nam et al. 2001) – to capture regime-switching and asymmetric behaviour. None of the previous studies has employed these two variants of GARCH model.

Our analysis contributes to the existing literature in the following ways: First, we have estimated the time-varying hedge ratios using longer time span and frequency of data. A number of important events, regimes and episodes in the financial markets have characterised this long period. These include the post-Asian financial crisis, the dot-com boom (until March 2000), aftermath of the bubble burst (after March 2000), the bankruptcy of WorldCom (July 2000-October 2000), a period of the stable and upward trending markets (April 2002-July 2007), the recent global financial crisis (August 2007-September/October 2008)[3], and aftermath of the financial crisis followed by a period of great recession and euro-zone crisis. In a recent study on the forecasting performance of 125 variants of GARCH models, Laurent et al. (2012) note that during unstable periods such as the dot-com bubble, the superior models consist of sophisticated GARCH specifications such as orthogonal and dynamic conditional correlation (DCC) embedded with leverage effect. During tranquil periods, GARCH with specifications such as conditional correlation and symmetry in the variance perform well. Finally, during the 2007-2008 financial crisis, GARCH specification with non-stationarity in the conditional variance process generates superior forecast. The selection and use of eight alternative variants of GARCH model would successfully capture these varying features of asymmetry, regime shifts, and unstable and calm market conditions which are embedded in our data series. Bivariate GARCH, GARCH-BEKK, GARCH-ECM, GARCH-X, and GARCH-DCC are the principal variants employed in previous research; the use of ANST-GARCH and MRVARCH would accommodate the issues of regimes shift, asymmetry and non-stationary variances, respectively.

Second, we have evaluated the hedging performance using two non-overlapping out of sample forecast to ameliorate sampling effect and to obtain more robust results. Third, we have investigated the hedging effectiveness using two distinct frameworks of utility evaluations – i.e. (a) the mean-variance and (b) exponential utility approaches. Fourth, we have computed the minimum capital risk requirement (MCRR) using those hedge ratios to ascertain the superiority of an alternative hedging strategy that holds capital adequacy requirement of the fund at a minimum level. Given that hedge ratios of various portfolios are predictable, an investor always prefers a portfolio with a lower financial capital to reach the maximum of risk reduction.

The stock and futures exchanges of South Korea represent a major exchange in the Asia Pacific region and within the global exchanges in terms of both market capitalisation and trading volume. Furthermore, previous research on the Korean markets aimed at investigating the relative effectiveness of dynamic hedging yields mixed results with a number of studies found no evidence of outperformance of complex econometric models over a much simpler hedging method (see Alexander and Barbosa, 2007, and Moon et al. 2009 and Copeland and Zhu 2010), and others found the comparative efficacy of more sophisticated econometric models (see Lai et al. 2009). Sim and Zurbruegg (2001a) noted that the comparative performance of a constant hedge ratio vis-a-vis the time-varying hedge ratio improved in the South Korean market after the Asian financial crisis. Given the significance of the Korean markets and the conflicting evidence, we attempt to re-assess empirically the comparative efficacy of dynamic hedging as an interesting case study.

The paper is organised as follows: Section 2 describes and discusses the optimal hedge ratio and the eight GARCH models. Section 3 presents a brief review of literature on the Korean market. The data and preliminary diagnostics are described in Section 4. Sections 5-7 offer the empirical results based upon estimating conventional and dynamic hedging models, and the final section offers a summary and conclusion.

## 2. Estimation of Optimal Hedge Ratios and the GARCH Models

### 2.1 The Hedge Ratio

Johnson's (1960) risk minimising hedge ratio $h^*$ is defined as

$$h^* = -\frac{\sigma_{c,f}}{\sigma_f^2} = -\frac{\mathrm{cov}(R_c, R_f)}{\mathrm{var}(R_f)} \quad , \tag{1}$$

where $R_c$ and $R_f$ denote returns on spot and future indices. The optimal hedge ratio (OHR) then is computed as the slope coefficient of the following regression,

$$R_{ct} = \alpha + \beta R_{ft} + \varepsilon_t \, , \tag{2}$$

where $\varepsilon_t$ is an error term.[4] A $\beta = 0$ implies unhedged position; $\beta = 1$ signifies a fully hedged position; and $\beta < 1$ implies a partial hedge.

It is now well-known in the literature that the conventional hedging model has shortcomings. As the distribution of futures and spot prices are changing through time, $h^*$ which is expressed as the ratio of covariance between futures returns and cash returns and variance of futures returns, moves randomly through time (see Cecchetti et al. 1988, Baillie and Myers 1991, and Kroner and Sultan 1993). Therefore eq. (2) should be modified as:

$$h_T^* = \frac{\mathrm{cov}(R_{T+1}^c, R_{T+1t}^f I\Omega_T)}{\mathrm{var}(R_{T+1}^f I\Omega_T)} \quad (3)$$

In eq. (3), conditional moments are changing as the information set, $\Omega_T$, is updated; consequently, the number of futures contracts held and the optimal hedge ratio will also change through time – hence the t subscripts of $h_T^*$. Under the condition of time-varying distribution, the bivariate GARCH model is utilised to estimate the time-varying hedge ratios to approximate the dynamic hedging strategies.

### 2.2. Bivariate GARCH Model

The time-varying hedge ratios are estimated from eight variants of GARCH models: standard GARCH, GARCH-ECM, GARCH-BEKK, GARCH-GJR and GARCH-X, GARCH-DCC, ANST-GARCH and MSVARCH. The following bivariate GARCH (p, q) model is applied to returns from the stock cash and futures markets[5],

$$y_t = \mu + \varepsilon_t \quad (4)$$

$$\varepsilon_t / \Omega_{t-1} \sim N(0, H_t) \quad (5)$$

$$\mathrm{vech}(H_t) = C + \sum_{i=1}^{p} A_i \mathrm{vech}(\varepsilon_{t-i})^2 + \sum_{j=1}^{q} B_j \mathrm{vech}(H_{t-j}) \quad (6)$$

where $y_t = (r_t^c, r_t^f)$ is a (2x1) vector containing stock returns from the cash and futures markets. $H_t$ is a (2x2) conditional covariance matrix, C is (3x1) parameter vector of constants, $A_i$ and $B_j$ are (3x3) parameter matrices, and vech is the column stacking operator that stacks the lower triangular portion of a symmetric matrix.

To make the estimation amenable, Engle and Kroner (1995) have suggested various restrictions to be imposed on the parameters of $A_i$ and $B_j$ matrices. A parsimonious representation may be achieved by imposing a diagonal restriction on the parameter matrices so that each variance and covariance element depends only on its own past values and prediction errors. The following equations represent a diagonal vech bivariate GARCH (1, 1) conditional variance equation(s):

$$H_{11,t} = C_1 + A_{11}(\varepsilon_{1,t-1})^2 + B_{11}(H_{11,t-1}) \tag{7a}$$

$$H_{12,t} = C_2 + A_{22}(\varepsilon_{1,t-1}, \varepsilon_{2,t-1}) + B_{22}(H_{12,t-1}) \tag{7b}$$

$$H_{22} = C_3 + A_{33}(\varepsilon_{2,t-1})^2 + B_{33}(H_{22,t-1}). \tag{7c}$$

In the bivariate GARCH (1, 1) model, the diagonal vech parameterisation involves nine conditional variance parameters.

Using the bivariate GARCH model, the time-varying hedge ratio can be computed as

$$h_t^* = \hat{H}_{12,t} / \hat{H}_{22,t}, \tag{8}$$

where $\hat{H}_{12,t}$ is the estimated conditional covariance between the cash and futures returns, and $\hat{H}_{22,t}$ is the estimated conditional variance of futures returns. Since the conditional covariance is time-varying, the optimal hedge would be time-varying too.

**2.3. GARCH-ECM Model**

When the bivariate GARCH model incorporates the error correction term in the mean equation, it becomes the GARCH-ECM model which is presented as

$$y_t = \mu + \delta(u_{t-1}) + \varepsilon_t, \qquad (9)$$

where $u_{t-1}$ denotes the lagged error-correction term, retrieved from the cointegration regression. Therefore, a bivariate GARCH-ECM model will be employed to account for the long-run relationship and basis risk (see Kroner and Sultan 1993). Equation 8 still represents the hedge ratio.

**2.4. Bivariate GARCH-BEKK Model**

In the BEKK model as suggested by Engle and Kroner (1995), the conditional covariance matrix is parameterised to

$$\mathrm{vech}(H_t) = C'C + \sum_{k=1}^{k}\sum_{i=1}^{p} A'_{ki}\varepsilon_{t-i}\varepsilon'_{t-1}A_{ki} + \sum_{k=1}^{k}\sum_{j=1}^{q} B'_{kj}H_{t-j}B_{kj} \quad . \qquad (10)$$

Eqs. (4) and (5) also apply to the BEKK model and are defined as before. In eq. (10) $A_{ki}$, i = 1,...q, k = 1,...k, and $B_{kj}$ j = 1,...q, k = 1,..k are NxN matrices. The GARCH-BEKK model is sufficiently general that it guarantees the conditional covariance matrix, $H_t$ to be positive definite, and renders significant parameter reduction in the estimation. For example, a bivariate BEKK GARCH (1, 1) parametrisation needs to estimate only 11 parameters in the conditional variance-covariance structure. The time-varying hedge ratio from the BEKK model is again represented by eq. (8).

**2.5. Bivariate GARCH-GJR Model**

Along with the leptokurtic distribution of stock returns data, empirical research has shown a negative correlation between current returns and future volatility (Black 1976, Christie 1982). This negative effect of current returns on future variance is sometimes called the leverage effect (Bollerslev et al. 1992). Glosten et al. (1993) provide a modification to the GARCH model that allows positive and negative innovations to returns which have a different impact on conditional variance.[6] Glosten et al. (1993) suggest that the asymmetry effect can also be captured simply by incorporating a dummy variable in the original GARCH.

$$\sigma_t^2 = \alpha_0 + \alpha u_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} + \beta \sigma_{t-1}^2, \qquad (11)$$

where $I_{t-1} = 1$ if $u_{t-1} > 0$; otherwise $I_{t-1} = 0$. Thus, the ARCH coefficient in a GARCH-GJR model switches between $\alpha + \gamma$ and $\alpha$, depending on whether the lagged error term is positive or negative. The time-varying hedge ratio based on the GARCH-GJR model is also expressed as eq. (8).

**2.6. Bivariate GARCH-X Model**

The GARCH-X model is an extension of the GARCH-ECM model as it incorporates the square of error correction term in the conditional covariance matrix. In the GARCH-X model, conditional heteroscedasticity may be modelled as a function of lagged squared error correction term, in addition to the ARMA terms in the variance/covariance equations:

$$\mathrm{vech}(H_t) = C + \sum_{i=1}^{p} A_i \mathrm{vech}(\varepsilon_{t-i})^2 + \sum_{j=1}^{q} B_j \mathrm{vech}(H_{t-j}) + \sum_{k=1}^{k} D_k \mathrm{vech}(u_{t-1})^2 \quad .(12)$$

A significant positive effect may imply that the further the series deviate from each other in the short run, the harder they are to predict. The hedge ratio again is presented by eq. (8).

**2.7. Bivariate GARCH-DCC**

The preceding variants of the GARCH model assume constant correlation in the conditional covariance matrix. Tse and Tusi (2002) developed the dynamic conditional correlational GARCH (GARCH-DCC) model by allowing the conditional correlation to vary over time. The DCC model is often the most accurate in terms of forecasting depending on the criteria (Engle 2002). The bivariate covariance matrix of DCC can be expressed as

$$H_t = \begin{bmatrix} h_{c,t}^2 & h_{cf,t} \\ h_{cf,t} & h_{f,t}^2 \end{bmatrix} = \begin{bmatrix} h_{c,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix} \begin{bmatrix} h_{c,f} & 0 \\ 0 & h_{f,t} \end{bmatrix}, \qquad (12)$$

where $\rho_t$ is the time-varying conditional correlation coefficient of spot and futures returns at time t. The conditional correlation is specified as an autoregressive moving average process,

$$\rho_t = (1-\theta_1-\theta_2)\rho + \theta_1\rho_{t-1} + \theta_2\phi_{t-1} \,. \qquad (13)$$

Eq. (8) is again used to compute the hedge ratio.

**2.8. ANST-GARCH**

The asymmetric non-linear smooth-transition generalised autoregressive conditional heteroscedasticity (ANST-GARCH) model was proposed by Nam et al. (2001). Following Anderson et al. (1999) and Nam et al. (2001), we apply the ANST-GARCH model to capture the asymmetric effect on mean and variance equations. The ANST-GARCH model has the following specification,

$$R_t = \mu + [\phi_1 + \phi_2 F(\varepsilon_{t-1})]R_{t-1} + \varepsilon_t$$

$$h_t = a_0 + a_1\varepsilon_{t-1}^2 + a_2 h_{t-1} + [b_0 + b_1\varepsilon_{t-1}^2 + b_2 h_{t-1}]F(\varepsilon_{t-1}) \,,$$

where $\varepsilon_t \mid I_{t-1} = v_t\sqrt{h_t}$, $v_t \xrightarrow{iid} N(0,1)$

$F(\varepsilon_{t-1}) = \{1+\exp[-\gamma(\varepsilon_{t-1})]\}^{-1}$, the parameter, $\gamma$ governs the speed of transition between volatility regimes and $I_{t-1}$ is known information set at time $t$.

The main difference between ANST-GARCH and GARCH models is that the former one measures regime-switching behaviour of volatility in the variance using the $F$ function. A significant non-zero $\phi_2$ indicates the existence of asymmetric mean reversion; $b_1 + b_2$ represents persistence of a shock to the conditional variance. When $\gamma$ is nearly 0, $F \cong 1$ and the ANST-GARCH turns out to a GARCH (1, 1) model.

### 3.9. MSVARCH

The regime-switching ARCH (RSVARCH) model combines regime-switching volatility with ARCH effects within each regime. This model extends the switching ARCH model of Hamilton and Susmel (1994) by allowing all volatility parameters to switch across regimes. The specification for the conditional variance for the RSARCH model is specified (see Hamilton and Susmel 1994 and Turner et al. 1989) as

$$r_t = \mu_t + \varepsilon_t = \eta_t \sqrt{h_{t,S_t}}$$

$$h_{t,S_t} = \alpha_{0,S_t} + \alpha_{1,S_t} \varepsilon_{t-1}^2 + \beta_{1,S_t} h_{t-1}.$$

This study assumes that $S_t = 2$. $\alpha_{0,S_t}, \alpha_{1,S_t}, \beta_{1,S_t}$ are assumed to be non-negative to ensure positive conditional variance, and $\alpha_{1,S_t} + \beta_{1,S_t}$ measures the persistence of shocks on conditional variance. The corresponding Markov Chain transition probability is given by

$$P = \begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix} \text{ where } \Pr\left(S_t = i \mid S_{t-1} = j\right) = p_{ij,} \text{ for } i, j = 1, 2 ,$$

and we have $\pi_1 = \dfrac{1-p}{2-p-q}$, $\pi_2 = \dfrac{1-q}{2-p-q}$, where $\pi_j$ is the unconditional probability of being in regime $j$.

We estimate cash and futures markets' returns, respectively, applying ANST-GARCH and MSVARCH models in the first step, and then we compute the time-varying hedge ratio as follows:

$$hr_t = \frac{\text{cov}(R_s, R_f)}{\text{var}(R_f)} = \rho_{R_s, R_f} \cdot \frac{\sigma_{R_s}}{\sigma_{R_f}}.$$

It is hypothesised that time-varying hedge ratios would be different across different variants of GARCH model. Therefore, the next question that arises is: which one is more effective? As stated earlier in this paper we apply all the above methods to estimate the hedge ratio, and compare their effectiveness. We also compare the hedging performance of dynamic hedging strategies with traditional hedging methods.

## 3. Literature Review

The search for alternative hedging strategies and modelling the optimal hedge ratio has generated considerable research at both theoretical and empirical levels for almost four decades. This section has drawn only from the experience of South Korea and recent research to furnish readers with an overview of the state-of-the-art research in this area.

Zanotti et al. (2010) investigate comparative efficacy of hedging performance of futures hedge ratios using daily data from electricity markets: Nord Pool, EEX and Power Next. The study employed five alternative econometric models – static OLS, dynamic OLS, a constant correlation (CCC) GARCH, and two dynamic time-varying correlations models, namely GARCH-DCC and exponential DCC. Their results suggest that the GARCH models attain maximum hedging effectiveness when volatility is relatively high. In the case of Powernext which is the most recent and less liquid market, hedging does not lead to variance reduction. In two other markets, future trading reduces the risk of electricity portfolios.

Hatemi and Roca (2014) investigate the movements of optimal hedge ratio using weekly US and UK equity markets data spanning the period January 1999 to

September 2009, accounting for two potential structural breaks. The empirical finding shows that there is one negative shift and one positive shift in the optimal hedge ratio in the US; while there is only one significant and positive shift in the optimal hedge ratio in the UK. Hatemi and Roca (2014) contend that accounting for the structural changes in the hedge ratios tend to avoids frequent rebalancing and higher costs, which are associated with time-varying hedge ratios.

Kenourgios et al. (2008) investigate the hedging effectiveness of S&P 500 stock futures contract using weekly data spanning the period July 1992 to June 2002. The minimum variance hedge ratios (MVRs) are estimated using alternative methods – namely OLS, ECM, bivariate GARCH, EGARCH and GARCH-ECM models. Their results indicate that the optimal hedge ratio that incorporates nonstationarity, long-run equilibrium relationship and short-run dynamics is reliable and useful for hedgers. Furthermore, the error correction model outperforms the conventional OLS, the ECM with GARCH errors, and the GARCH and EGARCH (1, 1) models in terms of risk reduction. Their in-sample analysis also suggests that the ECM provides better forecast with about 12% reduction in RMSEs.

Juhl et al. (2012) examine the effect of the hedge horizon on optimal hedge size and effectiveness in a cointegrated system using a simple regression method and an error correction model (ECM). The study demonstrated that both specifications yield similar results in the case of hedge horizon. That is, the estimated hedge ratio and regression $R^2$ both tend to be small when price changes are measured over short intervals but, as the hedge horizon lengthened, both measures will converge toward one.

Kawaller and Koch (2013) provide two recommendations for hedging practitioners attempting to qualify for special hedge accounting treatment. First, they propose an alternative measure to the traditional dollar offset ratio. In this form, they suggest division by the starting value of the hedge item rather than division by the change in the value of hedged items. This measure is less likely to exceed acceptable boundary conditions during periods of calm markets. Second, they propose an alternative metric – the $R^2$ analogue – which measures the proportion of total risk that would be mitigated if the hedger used the regression slope coefficient on the hedge ratio.

Alexander and Barbosa (2007) conducted an out-of-sample performance test using daily observations of seven exchange indices from six equity markets (Brazil, France, Hong Kong, Korea, the UK and the USA): CAC 40, FTSE 100, Hang Seng Composite, IBOVESPA, KOSPI 200, NASDAQ 100 and S&P 500. They found no evidence that complex econometric models, including GARCH, EWMA and ECM can improve the simple ordinary least squares hedge ratio.

Moon et al. (2009) investigated the relative effectiveness of hedging performance based on alternative modelling techniques such as the conventional OLS, GARCH, and rolling OLS using the daily data of Korea Securities Dealers Automated Quotation (KOSDAQ) markets. The result shows that the simple rolling OLS is superior to all the popular multivariate GARCH models.[7]

Copeland and Zhu (2010) compared various dynamic hedge ratios with the standard OLS hedge ratios for six markets: Australia, Germany, Japan, Korea, the UK and the USA. They found no clear benefits of using more sophisticated hedging models. Copeland and Zhu (2010) contend that complex econometric models including GARCH introduce too much noise to provide a cost-effective hedge.

Lai et al. (2009) have estimated optimal hedge ratios using daily data over the period January 1998 to June 2005 for five East Asian markets: Hong Kong, Japan, Korea, Singapore and Taiwan.[8] Their results show that hedge ratios constructed from a bivariate Copula-threshold-GARCH model are the best performing in variance reduction for all markets except Japan and Singapore.

Sim and Zurbruegg (2001a) investigated the impact of the Asian financial market crisis on the hedging effectiveness of the South Korean index futures using daily data over the period May 1996 to March 1999 within the framework of a bivariate error-correction GARCH model. Their results indicate a decline in the persistence of conditional volatility within the market prices after the crisis. As a result, the comparative performance of a constant hedge ratio vis-a-vis the time-varying hedge ratio improved after the Asian financial crisis.

The general impression from the foregoing discussion is that the choice of optimal hedge ratio and the effectiveness of dynamic hedging is an issue of ongoing research to the financial practitioners and researchers. Given the conflicting evidence of the relative effectiveness of dynamic hedging, we have re-examined the issue using eight variants of GARCH model to offer a more parsimonious time-series approach using a longer time span and more recent data from the Korean exchange.

## 4. Data and Diagnostics

The models are estimated using daily data spanning January 2000 to August 2017 on stock indices and their counterpart futures contracts from South Korea. Empirical evaluation of hedging performance using daily data has tremendous value for money managers, who adjust their portfolios as often as daily (Figlewski 1986). The KOSPI 200 index consists of 200 big companies of the stock market division of the Korea Exchange. The KOSPI is calculated as current capitalisation (at the time of comparison) divided by the base market capitalisation. KOSPI 200 is important because it is listed on futures and option markets and is one of the most actively traded indices in the world.[9,10] All futures price indices are continuous series.[11] The data are collected from DATASTREAM International. To avoid the sample effect and overlapping issue, two out-of-sample periods are considered, including a one-year period (2015) and a two-year period (2016 to 2017). All models are estimated for the periods 2000-2014 and 2000-2015, and the estimated parameters are applied recursively for forecasting hedge ratios over the horizons of 2015 and 2016-2017.

Descriptive statistics relating to the distribution of return indices are presented in Table 1. These statistics are mean, standard deviation, variance, a measure of skewness, a measure of excess kurtosis (normal value is 3), the Jarque-Bera statistics, and unit root test results of cash and future price indices. The table also presents higher order autocorrelation Q, and ARCH effects in the returns indices series. The values of the skewness statistics indicate that the density function is negatively skewed for future return indices and positively skewed for the cash return indices. The values of the excess kurtosis statistic are greater than 2, which suggests that the density function has a fat tail. The values of the Jarque-Bera statistic are high, suggesting that the return indices are not normally distributed. Judged by the skewness, excess kurtosis and Jarque-Bera statistics, it can be inferred that the return indices exhibit 'fat tails' in both markets. The data series have also been checked for stationarity using the Elliott-Rothenberg-Stock Dickey-Fuller generalised least squares (DF-GLS) unit root test. The DF-GLS test results indicate that each of the return indices series has no unit root. Tests for autocorrelation in the first moments using the Q(20) statistic indicate that none is present in any of the indices. Finally, tests for ARCH using Engle's LM statistic generally support the hypothesis of time-varying variances.

## 5. Empirical Results

In this section we formally evaluate the effectiveness of conventional and time-varying regression results of cash returns and future returns (eq. (2) by using the Cochrane-Orcutt method. Here, daily spot changes in the index are regressed on daily changes in the nearby index futures contract. Table 2 presents the results. Parameter estimates of the future returns in eq. (2) represent the constant minimum variance hedge ratio (t-stats in parentheses). The coefficient attached to the future returns variable is positive and highly significant. The hedge ratio is found to be .9039. This statistic indicates that a substantial portion of variability in the cash market is hedged using the futures instruments.

The standard GARCH, GARCH-BEKK, GARCH-DCC, ANST-GARCH and MSVARCH models are estimated without the error correction term in the mean equation. GARCH-ECM and GARCH-GJR incorporate the error correction term in the mean equation whereas the GARCH-X model incorporates the error correction term both in mean and variance equations. The results are reported in Table 3. For reasons of economy and brevity, we only report and discuss parameters that are of interest to us.[12] The ARCH coefficients ($A_{11}$ and $A_{22}$) are significant. These parameters indicate the amount of influence that past residuals have on current residuals. The GARCH coefficients ($B_{11}$ and $B_{22}$) represent the influence of past volatility on future volatility. The coefficients are positive and significant in all cases except for GARCH-GJR which produces negative effect on future volatility. The parameters representing the error correction term ($\delta_1$) in the GARCH-GJR and GARCH-X models for cash market are negative, large and economically significant while they are small and positive for the GARCH-ECM model, at the conventional level of significance. The size of coefficients ranges from -0.1584 (GARCH-X) to -0.1545 (GARCH-GJR). The absolute sizes of the parameters suggest that day-to-day deviations really do have a significant impact on the absolute levels of the cash indices. The error correction coefficient in the mean equation of futures return is positive and statistically significant in the cases of all three GARCH models of South Korea.[13] Alternatively, the result may be interpreted as when an increase in short-run deviation lowers the cash returns but increases the future returns. This is a distinguishing feature of the emerging markets as opposed to developed and mature markets where day-to-day deviations do not have much of an impact on the absolute levels of the cash and futures returns as such deviations are arbitraged anyway. The error correction coefficients in the conditional variance equations are positive and significant in cases of both cash and futures returns. This suggests that the further the series deviate from each other in the short run, the harder they are to predict.

In Table 4, we present all parameter estimates of cash and futures markets' returns from ANST-GARCH and MSVARCH models. For the ANST-GARCH model all parameter estimates are significant. The parameter, $\phi_2 \neq 0$ and is significant which indicates that mean reversion is asymmetric for both markets. The return is negatively correlated ($\phi_1 < 0$) under a shock for cash return, and return serial correlation is 0.03589 ($\phi_1 + \phi_2 > 0$) under a positive shock. Cash returns are positively correlated under a prior positive return shock, and the positive return shock is persistent. In other words, the markets over-react to bad news while under-reacting to good news. Compared to the futures market, the speed of mean reversion is slower in the cash market. Turning to the MSVARCH model, based on the two-state assumption, all parameter estimates are significant at the 1% level, which indicates that ARCH effects are significant for both cash and futures markets. The unconditional probability ranges from 0.0393 to 0.9607. The MSVARCH model displays strong persistence; the expected durations are 123 and 38 trading days for the cash market, while those for the futures markets are even higher.

We further test for stationarity of the estimated hedge ratios. Results are not reported here to save space. The unit test results indicate that the dynamic hedge ratios are mean-reverting, signifying that the effect of a shock to the hedge ratios would eventually die out. We also find that the hedge ratios follow the AR(1) process and the result shows that hedge ratios associated with all variants of GARCH models are positively serially correlated, which suggests that if a hedge ratio is large this week, it is expected to remain large next week in the absence of a new shock (Kroner and Sultan 1993).

**In-sample Variance Reduction**

There are four hedging strategies for within-sample and out-of-sample periods hedge ratio comparison – no hedge, naive hedge, the conventional minimum variance hedge, and conditional hedge. Within the conditional hedge the paper applies eight different GARCH models to estimate eight different hedge ratios for each market. In the case of no hedge, the investor takes no position in the futures markets to offset market risk in the cash market. In the case of naive hedge, the hedger takes a position in both markets by the same amount but in the opposite direction. The conventional minimum variance hedge ratio is estimated using the OLS. Finally, the conditional hedge is based on the time-varying hedge ratios obtained using the different GARCH modelling techniques.

Comparison between the effectiveness of different hedge ratios is drawn by constructing portfolios implied by the computed ratios, and the change in the variance of these portfolios indicates the hedging effectiveness of the hedge ratios. The portfolios are constructed as ($R^c_t$ - $h_t^* R^f_t$), where $R^c_t$ is the log difference of the cash (spot) prices, $R^f_t$ is the log difference of the futures prices, and $h_t^*$ is the estimated optimal hedge ratio.[14] The variance of these constructed portfolios is estimated and compared to represent portfolio risk. We define variance reduction as

$$\frac{Var(U) - Var(H)}{Var(U)},$$

where Var (U) is the variance of a benchmark portfolio and Var (H) signifies variance of the minimum variance hedged portfolio.

First, we examine the within-sample risk reduction performance of these models. The results are reported in the second and fifth columns of Table 5. The GARCH-GJR variant exhibits the lowest risk reduction among dynamic hedging models. In contrast, all variants of dynamic hedging models fail to outperform the traditional OLS method. An investor would actually increase the risk of their portfolio using the conditional hedging model in the case of Korea. This result is consistent with Alexander and Barbosa (2007), Moon et al. (2009), and Copeland and Zhu (2010).

We compared a percentage in-sample variance reduction of the conventional OLS model with a given benchmark model. There is a modest improvement in the OLS hedge compared with the naive hedge and different variants of GARCH models except the ANST-GARCH. The worst performance is in the case of ANST-GARCH where the conditional models fail quite miserably. The potential risk reduction of 764.3% of traditional OLS hedging method compared with no hedge strategies is substantial in Korea. Given the absolute lack of dominance of the conditional hedging model over the traditional method, an investor needs to carefully evaluate all possible hedging methods to identify the most appropriate hedging strategies that fit the data and the investor's utility preference schedule.

**Out-of-sample Variance Reduction**

Baillie and Myers (1991) contend that the more reliable measure of hedging effectiveness is indicated by a comparison of hedged portfolio variance performance using hedge ratios in the out-of-sample periods estimated by different methods. Therefore, we compare the hedging effectiveness of the different methods during two different non-overlapping out-of-sample time periods: from January 2015 to December 2015 (one year), and from January 2016 to August 2017 (nearly two years). Two different lengths of out-of-sample periods are applied to check whether changing the length has any significant effect on the hedging effectiveness of the hedge ratios. Two different lengths are also applied to avoid the sampling effect and overlapping effect. All versions of the GARCH are estimated for the period 2000 to 2014 first, and then the estimated parameters are applied to recursively forecast hedge ratios over the one-year out-of-sample time period. Similarly the GARCH models are estimated over the period 2000 to 2015 and the estimated parameters are used to forecast hedge ratios over the longer out-of-sample time period.

The third and sixth columns of Table 4 show the variance of the shorter out-of-sample and percentage change in variance, respectively. Among the models, OLS performs best.

The fourth and seventh columns of Table 5 demonstrate the variance of the longer out-of-sample and percentage change in variance, respectively. The results show that the out-of-sample portfolio based on OLS hedge outperforms all principal variants of GARCH models. The standard GARCH outperforms other variants of GARCH models and ANST-GARCH does worst within the GARCH. Changing the length of the out-of-sample period does not affect the performance of the hedge ratios much, over the different time horizons.

## 6. Evaluation of Hedging Performance using Utility Functions

The reductions in the variance are quite small in the large majority of in-sample tests, but this is expected given that daily data have been applied. As Kroner and Sultan (1993) contend, small size improvements in portfolio risk do not imply that the economic viability of the proposed strategy is questionable. The GARCH-based portfolio should be applied if it makes the investor's utility greater than the reduction in the return caused by the transaction cost incurred. Therefore, we have investigated the economic significance of the time-varying hedge ratio within the utilitarian framework using two distinct approaches – i.e. (i) the mean-variance utility function, and (ii) the exponential utility function.

The mean-variance utility function is augmented by the transaction cost

$$\mathrm{EU}(R_{ct} - h_t^* R_{ft}) = \mathrm{E}(R_{ct} - h^* R_{ft}) - Q - \psi \,\mathrm{var}(R_{ct} - h_t^* R_{ft}), \tag{9}$$

where Q signifies the transaction cost to attenuate the utility level. Following Kroner and Sultan (1993), we assume the expected return to the hedged portfolio to be zero and the value of the coefficient of risk tolerance ($\psi$) to be 4. Therefore, the average utility from hedging in a given trading day is $-Q - 4\,\mathrm{var}(R_{ct} - h_t^* R_{ft})$. In this paper we assume a typical round-trip cost of 0.00072% based on the KOSTAR future contract value.[15] The results are reported in Table 6. Evaluation of the mean-variance utility function (MV) shows that the OLS-based hedging strategy yields maximum expected utility. The entries underneath the column ΔMV demonstrate the utility gains in the GARCH class models with respect to the OLS-based hedging strategy. For example, the utility loss for the GARCH-BEKK model with respect to OLS in South Korea is 2.03%.

An alternative measure of hedging performance in recent research has underscored the role of skewness and kurtosis of portfolio returns.[16] As Alexander and Barbosa (2008) contend, the hedging performance evaluation based on the proportional variance reduction does not incorporate the effect of variance reduction on skewness and kurtosis. The minimum variance hedged portfolio is designed to have very low return volatility, but a high kurtosis indicates that the hedge can be spectacularly wrong on just a few days and a negative skewness indicates that it would be losing rather than making money.

Therefore, the second measure of hedging effectiveness which accounts for both skewness and kurtosis is derived from the following certainty equivalent (CE) exponential utility function[17]

$$\mathrm{U}(\mathrm{w}) = -\psi \exp(-\mathrm{w}/\psi), \qquad (10)$$

where w signifies wealth. The exponential function has the property, U(w) = E[U(w)]. Using Taylor expansion of U(w) around the mean value and taking expectation operator up to the fourth term, and after suitable transformation, the certainty equivalent utility function may be approximated as

$$\mathrm{CE} = \mu - \frac{\sigma^2}{2\psi} + \frac{\varphi}{6\psi^2} - \frac{\kappa}{24\psi^3}, \qquad (11)$$

where the third and fourth moments $\varphi = \mathrm{E}[(\mathrm{w}-\mu)^3]$ and $\kappa = \mathrm{E}[(\mathrm{w}-\mu)^4]$ signify skewness and kurtosis, respectively. Eq. (11) indicates that when $\psi > 0$, there is an aversion to risk associated with increasing variance, negative skewness and increasing kurtosis.

The results are reported in Table 6. The entries underneath column CE show certainty equivalent utility associated with different hedging strategies. The bold numbers indicate the maximum (minimum) (dis)utility. Results show that the ANST-

GARCH hedge yields best results for Korea, followed by the GARCH-DCC hedge. However, this method does not consider the issues of transaction cost and portfolio rebalancing.

## 7. Hedging Effectiveness Minimum Capital Risk Requirement

Given that hedge ratios of various portfolios obtained from GARCH models are predictable, fund managers always prefer a portfolio with a lower financial capital. One popular approach is the calculation of Value at Risk (VaR) using the in-house economic model to estimate the Minimum Capital Risk Requirements (MCRR). In this section, we are evaluate hedging effectiveness by estimating and comparing Minimum Capital Risk Requirement (MCRR) for portfolio returns obtained under alternative hedging models.

To obtain reliable VaR estimates and for computational ease, we have shortened our sample period using 10 years of observations. For example, VaR estimates based on historical simulation calculate 5% worst-case scenarios using 500 observations; the Monte Carlo simulation requires 10,000 real and synthetic observations for the calculation of down-side risks. Increasing the number of observations by going back further in time is not desirable to obtain precise estimates of MCRR. Therefore, we truncate the sample period from January 2000 to December 2009. To reduce the bulkiness of the results, we calculate the MCRR for the portfolio returns based on the estimated hedge ratios using standard GARCH, GARCH-BEKK, GARCH-ECM, GARCH-GJR and GARCH-X models.

We calculate the MCRR for 1-day, 10-day, 20-day, 30-day and 60-day investment horizons, by simulating densities of portfolio returns using Efron’s (1982) bootstrapping methodology which is based on a multivariate GARCH (1, 1) model.[18] The Monte Carlo simulation procedure used 10,000 simulated paths of portfolio returns based on a GARCH (1, 1) model to generate an empirical distribution of the maximum loss.

Table 7 presents the estimates of the MCRR obtained from the condition of alternative hedging models. The top panel of Table 7A presents MCRR for a short hedge (long cash, short futures) and the lower panel of the Table A shows the results for a long hedge (long futures, short cash). For comparison purposes, we have also calculated the MCRR estimates from the comparable S&P 500 indices' data. Table 6B presents the estimated MCRR of S&P 500. The results are reported in terms of percentage returns. The results show that for short hedge, there is a modest gain in using MCRR estimates based on the GARCH hedging models in the KOSPI 200 compared to the unhedged position. For the short hedge, BARCH-BEKK performs better among the GARCH class of models in most cases. GARCH-X performs well at the one-day investment horizon. For the long hedge, the standard GARCH outperforms other variants of GARCH models at the short investment horizon, while GARCH-ECM and GARCH-GJR outperform their competing models at 30-day and 60-day horizons, respectively. The short hedge position appears to require more MCRR than comparable the long hedge position does over out-of-sample investment horizons. When we compare these results with those in Table 7B it is evident that, compared to the position in S&P 500, the position in KOSPI 200 requires far more capital under both short hedge and long hedge conditions. These comparative results suggest that the position in a KOSPI 200 contract is more risky than the S&P 500 position when compared with higher VaR results. Therefore, the benefit of using GARCH-based conditional models to calculate the MCRR is more marginal in the case of KOSPI 200 than in the case of S&P 500.

## 8. Summary, Implications and Conclusion

In this paper, we evaluate the effectiveness of the conditional hedging model in reducing the portfolio risk of an investor holding both cash stock and futures market positions using daily data and eight variants of the GARCH model – i.e. standard bivariate GARCH, GARCH-BEKK, GARCH-ECM, GARCH-X, GARCH-GJR, GARCH-DCC, ANST-GARCH and MSVARCH. The effectiveness of the hedge ratio is investigated by comparing the within-sample period (January 2000 to August 2017) and out-of-sample period performance of the different hedge ratios for two periods, January 2016-August 2017 and January 2015-December 2015 (one year). The two different lengths of out-of-sample periods are applied to investigate the effect of changing the length on the hedging effectiveness of the hedge ratios. The two different periods are also applied to avoid sample effect and overlapping issues.

Results from within-sample show that, overall, all variants of dynamic hedging models fail to outperform the traditional OLS method. Both shorter and longer out-of-sample period results show that the OLS hedge outperforms all principal variants of GARCH models. The paper further investigates the economic significance of the time-varying hedge ratio within the utilitarian framework using two distinct approaches – i.e. (i) the mean-variance utility function, and (ii) the exponential utility function. The OLS-based hedging strategy yields maximum expected utility compared to dynamic hedging models in the mean-variance utility framework. Results based on exponential utility function suggest that the ANST-GARCH-based hedging model yields maximum utility for portfolio investors. Finally, we have estimated the Value at Risk of portfolio positions based on various hedging models using Bootstrapping techniques. The result shows that the benefit of using GARCH-based conditional models to calculate the MCRR is very modest in the case of KOSPI 200 compared to the case of S&P 500.

In the post-GARCH era, the issue of dynamic hedging received much popularity and acceptance due to the ability of the GARCH models to account for the co(variance) of portfolio and futures returns. However, the modelling technique requires computational elegance and a high level of quantitative sophistication for an informed investor, despite the fact that there are critics of GARCH-based dynamic hedging models. As Fink et al. (2005) contend, GARCH models can provide significant numerical optimisation challenges, most notably due to the joint difficulty of estimating a large number of parameters and a likelihood function which is not globally concave. To alleviate the maximisation problem in the nonlinear estimation routine, the model requires a wide range of starting values. Furthermore, hedging strategies based on GARCH models require frequent rebalancing of the portfolio positions. The benefit of too frequent rebalancing tends to be offset by transaction costs.[19] Copeland and Zhu (2010) contend that complex econometric models including GARCH introduce too much noise to provide a cost-effective hedge. The testable implications of our results and previous research based on the stock futures data of Korea, together with the critical evaluation, suggest that investors would be better served by simply using the OLS hedge ratios and periodically updating the resulting hedge ratios using some simple and intuitively reasonable updating scheme, such as rolling-window hedge.[20] Our results also imply that the comparative hedging performance based on the hedge ratios obtained from different econometric models applied depends upon the market under study and the length of forecasting horizon.

## Notes

1. For example, see Working (1953), .Johnson (1960), Silber (1985) and Fortune (1989).
2. However, studies of Sim and Zurbruegg (2001, Alexander and Barbosa (2007), Lai et al. (2009), Moon et al. (2009) and Hasan and Choudhry (2013) ) are exceptions.
3. For a discussion, see Laurent et al. (2012).

4. The OLS estimation of the hedge ratio from eq. (2) is based on the assumption of time-invariant asset distributions suggested by Ederington (1979) and Anderson and Danthine (1980).
5. This section has drawn extensively from Hasan and Choudhry (2013).
6. There is more than one GARCH model available that is able to capture the asymmetric effect in volatility. According to Engle and Ng (1993), the Glosten et al. (1993) model is the best at parsimoniously capturing this asymmetric effect.
7. Moon et al. (2009) estimated diagonal VEC GARCH, matrix diagonal GARCH, constant conditional correlation GARCH, BEKK GARCH, and principal component GARCH model.
8. Lai et al. (2009) used the daily data of the Korea Stock Exchange Composite Price Index (KORCOMP) and the future price indices of KOSPI 200.
9. On the Korea Stock Exchange, the Korea Stock Price Index 200 future was launched in May 1996 and its trading contracts reached a volume of nearly 34 million by 2005 with a trading value of nearly 18 billion USD. Alexander and Barbosa (2007) noted that the Korean Stock Exchange was the fifth-largest exchange for trading of index futures contracts in 2005 after the CME, Eurex, Euronext and the National Stock Exchange of India.
10. The study of Alexander and Barbosa (2007) used KOSPI 200 data; Moon et al. (2009) utilised Korea Securities Automated Quotation (KOSDAQ) data while Lai et al. (2009) used the daily data of Korea Stock Exchange Composite Price Index (KORCOMP) and the future price indices of KOSPI 200 in their study.
11. The continuous series is a perpetual series of futures prices. It starts at the nearest contract month, which forms the first values for the continuous series, either until the contract reaches its expiry date or until the first business day of the actual contract month. At this point, the next trading contract month is taken.
12. Many diagnostic tests are not reported or discussed to conserve space. However, they are available upon request.
13. The result is quite plausible, pointing to the notion that if the error correction term is statistically negative and significant in one equation, then the term would be positive in another equation in a bivariate model.
14. In the case of the constant ratio the time subscript does not exist.
15. Moon et al. (2009) reported that a typical round-trip cost is around 0.00072% of KOSTAR future contract value in the Korean market. Yang and Lai (2009) noted that the transaction cost ranges between 0.005% and 0.01% in the major global exchanges which are trading financial contracts of DJIA, S&P500,, NASDAQ100, FTSE100, CAC40, DAX30 and Nikkei225. Rossi and Zucca (2002) noted a transaction cost of 0.0015% in the Italian bond market.
16. For example, see Cremers et al. (2004), Harvey et al. (2004) and Alexander and Barbosa (2008).
17. This part is drawn from Alexander and Barbosa (2008).
18. Interested readers are referred to Brooks et al. (2002), and Jorion (2007) for a detailed discussion.
19. For example, see Sim and Zurbruegg (2001b) and Kofman and McGlenchy (2005) for a discussion.
20. The study of Moon et al. (2009) shows that the simple rolling OLS is superior to all the popular multivariate GARCH models.

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**Table 1. Descriptive statistics of stock spot and futures indices returns**

| Statistics | **Korea** | |
|---|---|---|
| | Cash Return | Future Return |
| Mean | .000192 | .000186 |
| Variance | .000224 | .000270 |
| Std. Dev. | .015623 | .016436 |
| Skewness | -.48056 | -.370422 |
| Kurtosis | 6.42395 | 5.48021 |
| Jarque-Bera | 8023.29 | 5815.59 |
| Stationarity: $t_\mu$ | -13.210[a] | -20.509[a] |
| $t_\tau$ $t_\tau$ | -26.461[a] | -29.631[a] |
| ARCH(1) | 131.55 | 113.68 |
| Q(20) | 45.390 | 52.870 |

Note: $t_\mu$ and $t_\tau$ are the Elliot-Rothenberg-Stock Dickey-Fuller generalised least squares (DF-GLS) unit root test statistics with allowance for a constant and a trend, respectively. 5% critical values of $t_\mu$ and $t_\tau$ are -1.948 and -3.190 (see Elliot-Rothenberg-Stock 1996, Table 1).

**Table 2. Bivariate regression results of the constant minimum hedge ratio model**

| Country | Constant | Futures returns | Diagnostic | F-test |
|---|---|---|---|---|
| Korea | -.0000239 (0.3337) | 0.9039* (207.53) | $R^2$ = .904 DW= 2.758 | 43071.0 |

**Table 3. Parameter estimates of conditional hedging model of South Korea**

| Dependent Variable | GARCH | GARCH-ECM | GARCH-X | GARCH-BEKK | GARCH-GJR | GARCH-DCC |
|---|---|---|---|---|---|---|
| $\mu_1$ | 8.52e-04[a] | 8.80e-04[a] | 7.19e-04[a] | 0.00072[a] | -0.00077[a] | 0.00088[a] |
| | (3.40110) | (4.9373) | (3.7318) | (4.0239) | (-0.9215) | (4.7178) |
| $\delta_1$ | | 8.58e-04[a] | -0.15841[a] | | -0.1545[a] | |
| | | (4.7028) | (-2.4553) | | (1.0204) | |
| $\mu_2$ | 8.56e-04[a] | -0.18815[a] | 7.07e-04[a] | 0.00071[a] | 0.00072[a] | 0.00088 [a] |
| | (3.3830) | (-3.7958) | (3.5750) | (3.8913) | (3.4219) | (4.6024) |
| $\delta_2$ | | 0.17158[a] | 0.23200[a] | | 0.18586[a] | |
| | | (3.4120) | (3.5343) | | (22.776) | |
| $C_{11}$ | 6.08e-06[a] | 7.66e-06[a] | 4.71e-06[a] | 0.00114[a] | 2.67e-04[a] | 2.36e-06[a] |
| | (5.9458) | (9.0341) | (9.2329) | (4.6162) | (43.994) | (6.1163) |
| $A_{11}$ | 0.08251[a] | 0.07255[a] | 0.07319[a] | 0.16501[a] | 0.00643[a] | 0.08152[a] |
| | (7.7332) | (13.167) | (18.376) | (3.5503) | (7.7089) | (15.406) |
| $B_{11}$ | 0.89684[a] | 0.8981[a] | 0.87924[a] | 1.16134[a] | -0.02611[a] | 0.91552[a] |
| | (133.52) | (144.75) | (153.35) | (50.951) | (-29.218) | (177.28) |
| $D_{11}$ | | | 0.60072[a] | | 0.00556[a] | 0.03532[a] |
| | | | (9.5028) | | (2.8157) | (7.2070) |
| $C_{21}$ | 5.76e-06[a] | 7.41e-06[a] | 4.49e-06[a] | 0.00056[b] | 3.41e-04[a] | |
| | (5.9076) | (9.0893) | (9.1054) | (2.1282) | (51.080) | |
| $A_{12}$ | 0.07952[a] | 0.07056[a] | 0.07008[a] | -0.12104 [a] | 0.00325[a] | |
| | (7.4838) | (13.312) | (18.124) | (-2.6017) | (7.9915) | |
| $A_{21}$ | | | | 0.10537[b] | | |
| | | | | (2.2897) | | |
| $B_{12}$ | 0.90045[a] | 0.90092[a] | 0.88227[a] | 0.22749[a] | -0.29921[a] | |
| | (7.4838) | (150.41) | (156.47) | (9.3570) | (-14.765) | |
| $B_{21}$ | | | | -0.1990[a] | | |
| | | | | (-8.6133) | | |
| $D_{21}$ | | | 0.60773[a] | | | |
| | | | (9.5723) | | | |
| $C_{22}$ | 5.73e-06[a] | 7.45e-06[a] | 4.48e-06[a] | 1.70e-08 | 3.32e-05 [a] | 2.42e-06[a] |
| | (5.6039) | (8.8712) | (9.0766) | (4.8e-05) | (12.521) | (6.6834) |
| $A_{22}$ | 0.08002[a] | 0.07137[a] | 0.06983[a] | 0.37806[a] | 0.00290[a] | 0.08067[a] |
| | (7.5430) | (13.214) | (18.063) | (8.2707) | (7.4675) | (16.617) |
| $B_{22}$ | 0.90186[a] | 0.90191[a] | 0.88480[a] | 0.75262[a] | 0.8768 | 0.91650[a] |
| | (41.955) | (147.85) | (163.37) | (31.997) | (101.41) | (197.70) |
| $D_{22}$ | | | 0.60988[a] | 0.00114[a] | 0.00242[a] | 0.94694[a] |
| | | | (9.3475) | (4.6162) | (2.7084) | (128.23) |
| $P$ | 0.95091 | | | | | |
| L | 31080.7 | 31934.2 | 31553.4 | 31201.436 | 31620.2 | 31248.0 |
| $\Phi$ | 0.86967 | 0.89632 | 0.87033 | 0.92595 | 0.84582 | 0.90452 |

Note: a, b and c imply significance at 1%, 5% and 10% levels, respectively; figures in parentheses underneath the coefficients are t-statistics. $\rho$ is the within-sample correlation coefficient between cash and futures returns. Log-L is the log-likelihood and $\varphi$ signifies the first-order serial correlation coefficient in the hedge ratio derived from an AR(1) model.

**Table 4. Parameter estimates from ANST-GARCH and MSVARCH models of conditional hedging model of South Korea**

| Dependent Variable | ANST-GARCH | ANST-GARCH | MSVARCH | MSVARCH |
|---|---|---|---|---|
| | $R_c$ | $R_f$ | $R_c$ | $R_f$ |
| $\mu$ | 0.03927[a] | 0.04497[a] | 0.05511[a] | 0.04598[b] |
| | (0.0000) | (0.0000) | (2.8193) | (2.0813) |
| $\phi_1$ | -0.02359[a] | 0.06340[a] | | |
| | (0.0008) | (0.0000) | | |
| $\phi_2$ | 0.05948[a] | 0.02647[a] | | |
| | (0.0002) | (0.0000) | | |
| $a_0(\alpha_0)$ | 0.00003[a] | 0.00009[a] | 0.98411[a] | 1.38643[a] |
| | (243.93) | (1320.18) | (59.799) | (64.239) |
| $a_1(\alpha_1)$ | 0.02013[a] | -.000003[a] | 0.32133[a] | 0.29975[a] |
| | (11.207) | (-20.1956) | (11.152) | (10.952) |
| $a_2(\beta_1)$ | 0.06049[a] | 0.17392[a] | 0.78928[a] | 0.43621[a] |
| | (9.44036) | (0.17392) | (59.799) | (64.239) |
| $b_0$ | -0.00005[a] | -0.00008[a] | | |
| | (-1185.3) | (100.564) | | |
| $b_1$ | 0.06076[a] | 0.000003[a] | | |
| | (12.597) | (16.0172) | | |
| $b_2$ | 0.04983[a] | -0.21947[a] | | |
| | (3.7776) | (-1122.60) | | |
| $\gamma$ | 12.7208[a] | 14.1895[a] | | |
| | (13.052) | (169.517) | | |
| p | | | 0.97712[a] | 0.96655[a] |
| | | | (22.181) | (16.022) |
| q | | | 0.94382[a] | 0.18226[a] |
| | | | (17.590) | (12.286) |
| $\rho$ | 0.95091 | | | |
| L | 9604.2475 | 5474.4020 | -8332.0422 | -7783.0395 |
| $\varphi$ | -0.01194 | | 0.833523 | |
| $\pi_1$ | | | 0.2894 | 0.0393 |
| $\pi_2$ | | | 0.7106 | 0.9607 |
| $d_1$ | | | 123.14 | 353.99 |
| $d_2$ | | | 38.82 | 275.37 |

Note: a, b and c imply significance at 1%, 5% and 10% levels, respectively; figures in parentheses underneath the coefficients are t-statistics. $\rho$ is the within-sample correlation coefficient between cash and futures returns. Log-L is the log-likelihood and $\varphi$ signifies the first-order serial correlation coefficient in the hedge ratio derived from an AR(1) model. $\pi_j$ is the unconditional probability of being in regime $j$ and $d_j$ is the half-life or expected duration of the j-th state.

**Table 5. Portfolio variance reduction**

| | Variance | | | Variance reduction | | |
|---|---|---|---|---|---|---|
| | In-sample | Out-of-sample 2015 | Out-of-sample 2016-2017 | In-sample(%) | Out-of-sample 2015 (%) | Out-of-sample 2016-2017 (%) |
| No Hedge | .006102 | .004500 | .006087 | 764.3 | 667.9 | 767.1 |
| Naïve Hedge | .000747 | .000610 | .000725 | 5.807 | 4.095 | 3.276 |
| OLS | .000706 | .000586 | .000702 | - | - | - |
| GARCH Hedge | .000718 | .000588 | .000712 | 1.699 | 0.341 | 1.424 |
| GARCH-BEKK | .000724 | .000598 | .000723 | 2.549 | 2.047 | 2.991 |
| GARCH-GJR | .000708 | .000588 | .000774 | 0.283 | 0.341 | 10.25 |
| GARCH-ECM | .000718 | .000592 | .000706 | 1.699 | 1.023 | 0.569 |
| GARCH-X | .000715 | .000593 | .000714 | 1.274 | 1.194 | 1.709 |
| GARCH-DCC | .000721 | .000607 | .000720 | 2.124 | 3.583 | 2.564 |
| ANST-GARCH | .005097 | .002441 | .004475 | 621.9 | 316.5 | 537.4 |
| MSVARCH | .000766 | .000599 | .000753 | 8.498 | 2.2184 | 7.264 |

Note: The fifth, sixth and seventh columns show percentage in-sample variance reductions of OLS hedge compared to other hedging models.

**Table 6.In-sample hedging performance using utility function**

| **South Korea** | Mean Return | Volatility | Skewness | Kurtosis | MV | ΔMV | CE |
|---|---|---|---|---|---|---|---|
| Unhedged | .000958 | .006102 | -.480556[a] | 6.423949[a] | -0.02512 | | -0.008992 |
| Naive | .000030 | .000747 | -.126914[a] | 11.39052[a] | -0.00370 | | -0.015411 |
| OLS | .000024 | .000706 | -.160821[a] | 12.05113[a] | **-0.00354** | | -0.009585 |
| BGARCH | -.000068 | .000718 | -.169256 | 1.089971[a] | -0.00359 | 1.35% | **-0.002630** |
| GARCHBEKK | -.000106 | .000724 | -.118941 | 6.103481[a] | -0.00361 | 2.03% | -0.005409 |
| GARCH-GJR | .000121 | .000708 | -.199432[c] | 6.309633[a] | -0.00355 | 0.02% | -0061527 |
| GARCH-ECM | -.000060 | .000718 | -.129846 | 6.230429[a] | -0.00359 | 1.35% | -0.006334 |
| GARCH-X | .000002 | .000715 | -.135433 | 6.299222[a] | -0.00358 | 1.01% | -0055916 |
| GARCH-DCC | -.000065 | .000721 | -.117880 | 6.164275[a] | -0.00360 | 1.69% | -0.005396 |
| ANST-GARCH | .004535 | .005097 | -.170801 | 2.102226[a] | -0.02110 | 475.93 | 0.0007512 |
| MSVARCH | .000031 | .000766 | -.224045[b] | 5.399300[a] | -0.00378 | 6.89 | -0.003812 |

**Table-7A. MCRR Estimates-GARCH Hedging Models for Korea**

| Days | Unhedged | Naïve Hedge | GARCH | GARH-ECM | GARCH-X | GARCH-BEKK | GARCH-GJR |
|---|---|---|---|---|---|---|---|
| A. Long cash and short futures: | | | | | | | |
| 1 | 0.075635 | 0.056136 | 0.048729 | 0.058819 | 0.048169 | 0.056367 | 0.055876 |
| 10 | 0.239415 | 0.208204 | 0.224966 | 0.226590 | 0.232154 | 0.214631 | 0.220946 |
| 20 | 0.351449 | 0.307227 | 0.334677 | 0.328243 | 0.317796 | 0.300063 | 0.320870 |
| 30 | 0.432825 | 0.400145 | 0.402204 | 0.404677 | 0.393287 | 0.389864 | 0.413343 |
| 60 | 0.628453 | 0.564234 | 0.584424 | 0.585854 | 0.615720 | 0.553359 | 0.586102 |
| B. Short cash and long futures: | | | | | | | |
| 1 | 0.027731 | 0.070933 | 0.024168 | 0.024614 | 0.025477 | 0.024262 | 0.025129 |
| 10 | 0.081347 | 0.221984 | 0.071695 | 0.075153 | 0.073658 | 0.073614 | 0.072380 |
| 20 | 0.114920 | 0.311698 | 0.102648 | 0.106101 | 0.106098 | 0.105683 | 0.103278 |
| 30 | 0.147357 | 0.387505 | 0.126995 | 0.123839 | 0.130989 | 0.132868 | 0.126748 |
| 60 | 0.211694 | 0.570341 | 0.185841 | 0.187921 | 0.191762 | 0.191656 | 0.179441 |

**Table-7B. MCRR Estimates-GARCH Hedging Models for S&P500**

| Days | Unhedged | Naïve Hedge | GARCH | GARH-ECM | GARCH-X | GARCH-BEKK | GARCH-GJR |
|---|---|---|---|---|---|---|---|
| A. Long cash and short futures: | | | | | | | |
| 1 | 0.016121 | 0.005997 | 0.00375 | .003094 | .003813 | 0.00305 | .003485 |
| 10 | 0.049140 | 0.012559 | 0.01267 | .010474 | .012311 | 0.01002 | .011635 |
| 20 | 0.069428 | 0.017497 | 0.01761 | .015568 | .016808 | 0.01445 | .016137 |
| 30 | 0.087110 | 0.020916 | 0.02106 | .018259 | .020674 | 0.01827 | .020039 |
| 60 | 0.128711 | 0.030758 | 0.03051 | .027979 | .029369 | 0.02758 | .028397 |
| B. Short cash and long futures: | | | | | | | |
| 1 | 0.019131 | 0.004082 | 0.00416 | .00402 | .00409 | 0.003691 | .00404 |
| 10 | 0.057588 | 0.012341 | 0.21231 | .01209 | .01245 | 0.011617 | .01297 |
| 20 | 0.083857 | 0.017751 | 0.0179 | .01643 | .01766 | 0.016770 | .018207 |
| 30 | 0.101431 | 0.022043 | 0.02181 | .02189 | .02206 | 0.020577 | .02206 |
| 60 | 0.146370 | 0.030871 | .03112 | .03108 | .03065 | 0.029314 | .03141 |