Decentralized Sliding Mode LFC for Nonlinear Interconnected Power System with Time Delay*

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Abstract— **This paper considers a decentralised sliding mode load frequency control (LFC) for multi-area power system with uncertain time-varying parameters and delay. Since delays can exert a destabilizing effect on the overall system, it is necessary to maximize the delay bound in order to regularize the deviation in frequency and tie-line power. Robustness is improved by taking advantage of the system structure and uncertainty bounds. A sliding surface is designed, which guarantees the stability of the sliding motion and the stability of the sliding motion is analyzed based on Lyapunov-Razumikhin function which has a fast changing rate. A delay dependent decentralized sliding mode control is synthesized to drive the system to the sliding surface in finite time, and maintain a sliding motion afterward. The effectiveness of the proposed method is tested via a two-area interconnected power system.**

I. INTRODUCTION

With the advancement of scientific technology, power systems are becoming larger and more complex. The problem of decentralised control and stabilization of interconnected systems have become a major design goal [1], [2]. Since communication technology constitutes the main backbone for power system data transmission, it is characterized by open communication infrastructure which are unreliable due to time delay, packet loss, and communication failure. As a direct consequence, constant and time varying delay must be considered in order to enhance the performance of LFC systems [3], [4], [5].

It is widely known that time delay is usually a source of instability and performance degradation in control systems. Huge number of results have been dedicated to the development and stability of time delay system (see, e.g. [6], [7], [8]). Two main techniques based on Lyapunov-Krasovskii functional and Lyapunov-Razumikhin function have been largely used to deal with time delay. In most of the existing works, delay independent and delay dependent stability problems have been analyzed specifically. While there have been various arguments as to which method is more suitable for a specific problem, for lack of ambiguity, each of these strategies has its advantages. The Lyapunov-Krasovskii method has proved very effective in dealing with time-varying delays [9], [6], [4]. The Lyapunov-Razumikhin method, when compared with the Lyapunov-Krasovskii method, can be used to deal with large constant delay and time varying delay with fast change rate [10], [11].

Sliding mode control (SMC) technique due to its strong robustness properties, fast and good transient response has proved very effective in dealing with system uncertainties even if delays are involved ([12], [13], [14]). Recent work carried out in LFC with time delay has recorded remarkable improvement amidst differences in techniques. Within the context of sliding mode, some did not consider time delay in the design procedure see [15], [5], [16]. Whereas in some others, the interconnection (tie-line) was dealt with as part of the model uncertainty see [4], [17], [18].
This work proposes a decentralised sliding mode LFC scheme for power system with time-varying delays and non-linear delayed disturbances. The assumptions for nonlinear terms are imposed on the transformed systems to avoid unnecessary conservatism. Lyapunov- Razumikhin theorem is used as a main analysis tool to deal with the delay, thereby deriving a set of conditions which guarantee that the derived sliding motion is stable. Then under assumption that each subsystem states are accessible, a delay dependent decentralized sliding mode control is synthesized such that the power system remains uniformly asymptotically stable in the presence of uncertainties and time delays. Compared with [4], the tie-line is dealt with as a separate entity and sensor to controller delay is considered. In addition, the system not only allows the bounds on the uncertainty have more general non-linear form but robustness is enhanced by fully using the bounds on the uncertainties in this paper. A two area LFC model is simulated to show the feasibility of the developed results and the effectiveness of the proposed method.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

The dynamics of the multiarea LFC power system are described by the following equation [4].

$$\begin{aligned}\dot{x}_i &= A_i x_i + E_i x_{id_i} + B_i(u_i + G_i(t, x_i, x_{id_i})) \\ &\quad + F_i(t, x_i, x_{id_i}) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \Upsilon_{ij}(t, x_j, x_{jd_j}) \qquad (1)\end{aligned}$$

where $x_i = [\Delta f_i \ \Delta P_{mi} \ \Delta P_{vi} \ \Delta E_i \ \Delta P_{tie}]^T \in \Omega_i \subset \Re^5$, is the state vector, $u_i \in \Re$ is the control input at the governor terminal of the i-th subsystem. The system matrices $A_i \in \Re^{5\times 5}$, $E_i \in$

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$\Re^{5\times5}$, $B_i \in \Re^{5\times1}$ are given by

$$A_i = \begin{bmatrix} -\frac{D_i}{M_i} & \frac{1}{M_i} & 0 & 0 & -\frac{1}{M_i} \\ 0 & -\frac{1}{T_{chi}} & \frac{1}{T_{chi}} & 0 & 0 \\ \frac{-1}{RT_{gi}} & 0 & \frac{-1}{T_{gi}} & 0 & 0 \\ \beta_i & 0 & 0 & 0 & 1 \\ 2\pi\sum_{\substack{j=1\\j\neq i}}^{N} T_{ij} & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

$$E_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{K_{Pi}}{T_{Pi}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{T_{gi}} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_{g_i}} \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

$$\Upsilon_{ij} = \begin{bmatrix} 0_{4\times1} \\ -2\pi\sum_{\substack{j=1\\j\neq i}} T_{ij} \end{bmatrix}, \; F_i = \begin{bmatrix} \frac{-1}{M_i} \\ 0_{4\times1} \end{bmatrix} \Delta P_{di} \tag{4}$$

where all the parameters above have the same physical meanings as in [4]. The uncertainties $G_i(\cdot)$ and $F_i(\cdot)$ are used to model matched and mismatched nonlinear perturbation respectively, with appropriate dimensions; where the terms $\sum_{\substack{j=1\\j\neq i}}^{N} \Upsilon_{ij}(\cdot)$ describe the unknown interconnection of the i-th subsystem; $x_{id_i} := x_i(t-d_i)$ represent the delayed state where $d_i := d_i(t)$ is the time varying delay which is assumed to be known, non-negative and bounded in $\Re^+ := \{t \mid t \geq 0\}$, that is

$$\bar{d}_i := \sup_{t\in\Re^+} \{d_i(t)\} < \infty$$

The initial condition related to the delay is given by

$$x_i(t) = \phi_i(t), \qquad t \in [-\bar{d}_i, 0] \tag{5}$$

where $\phi_i(\cdot)$ is continuous in $[-\bar{d}_i, 0]$. It is assumed that all the nonlinear functions are smooth enough such that the unforced interconnected system has a unique continuous solution.

The objective of this paper is to design a decentralized sliding mode controller

$$u_i = \varphi_i(x_i, x_{id_i})$$

such that the closed loop system under the assumption that all the states are available is asymptotically stable in the presence of disturbances and time varying delay.

III. Basic Assumptions

Assumption 1. The matrix pair (A_i, B_i) is controllable for $i = 1, 2, .., N$. It should be noted that for a LFC controlled system, $T_{gi} \neq 0$. Matrices B_i are full rank for $i = 1, 2, .., N$.

It follows from [19] that there exists a coordinate transformation $\hat{x}_i = \hat{T}_i x_i$ such that the matrix triple (A_i, E_i, B_i) in system (1) in the new coordinate, has the structure

$$\hat{A}_i = \begin{bmatrix} \hat{A}_{i1} & \hat{A}_{i2} \\ \hat{A}_{i3} & \hat{A}_{i4} \end{bmatrix}, \hat{E}_i = \begin{bmatrix} \hat{E}_{i1} & \hat{E}_{i2} \\ \hat{E}_{i3} & \hat{E}_{i4} \end{bmatrix}, \hat{B}_i = \begin{bmatrix} 0 \\ \hat{B}_{i2} \end{bmatrix} \tag{6}$$

where $\hat{A}_{i1} \in \Re^{4\times4}$, $\hat{E}_{i1} \in \Re^{4\times4}$, and specifically $\hat{B}_{i2} \in \Re$ is a positive scalar for $i = 1, 2, ..., N$. Further, the fact that (A_i, B_i) is controllable implies that $(\hat{A}_{i1}, \hat{A}_{i2})$ is controllable (see, [20]), and thus there exists a matrix $K_i \in \Re^{1\times4}$ such that $\hat{A}_{i1} - \hat{A}_{i2}K_i$ is Hurwitz stable. Further, consider the transformation matrix $z_i = \tilde{T}_i \hat{x}_i$, with $\tilde{T}_i$ defined by

$$\tilde{T}_i = \begin{bmatrix} I_4 & 0 \\ K_i & 1 \end{bmatrix} \tag{7}$$

Let

$$T_i = \tilde{T}_i \hat{T}_i \tag{8}$$

It follows from the analysis above that in the new coordinate $z_i = T_i x_i$, system (1) has the following form

$$\begin{aligned} \dot{z}_i \;=\; & \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} z_i + \begin{bmatrix} E_{i1} & E_{i2} \\ E_{i3} & E_{i4} \end{bmatrix} z_{id_i} + \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} \\ & \times \left(u_i + G_i(t, T_i^{-1}z_i, T_i^{-1}z_{id_i})\right) + F_i(t, T_i^{-1}z_i, T_i^{-1}z_{id_i}) \\ & + \sum_{\substack{j=1\\j\neq i}}^{N} \Upsilon_{ij}(t, T_i^{-1}z_j, T_i^{-1}z_{jd_j}) \end{aligned} \tag{9}$$

where

$$\begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} = T_i A_i T_i^{-1} \tag{10}$$

$$\begin{bmatrix} E_{i1} & E_{i2} \\ E_{i3} & E_{i4} \end{bmatrix} = T_i E_i T_i^{-1}, \quad T_i B_i = \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} \tag{11}$$

Assumption 2. There exist known continuous non-negative functions $\zeta_i(\cdot)$, $\kappa_i(\cdot)$, and $\alpha_i(\cdot)$ such that

$$\|G_i(\cdot)\| \leq \zeta_i(t, z_i(t), \|z_{id_i}\|) \tag{12}$$

$$\|F_i^b(\cdot)\| \leq \alpha_i^a(t, z_i(t))\|z_i\| + \alpha_i^b(t, \|z_{id_i}\|)\|z_{id_i}\| \tag{13}$$

$$\|\Upsilon_{ij}^b(\cdot)\| \leq \kappa_{ij}^a(t, z_j)\|z_j\| + \kappa_{ij}^b(t, \|z_{jd_j}\|)\|z_{jd_j}\| \tag{14}$$

IV. Sliding mode control analysis and design

Section III shows that there exists new coordinates $z_i = T_i x_i$ such that in the new coordinate $z = \mathrm{col}(z_1, z_2 ... z_N)$, the system (1) can be described in (9). System (9) can be rewritten by

$$\begin{aligned} \dot{z}_i^a &= A_{i1}z_i^a + A_{i2}z_i^b + E_{i1}z_{id_i}^a + E_{i2}z_{id_i}^b \\ &\quad + F_i^a(t, z_i, z_{id_i}) + \sum_{\substack{j=1\\j\neq i}}^{N} \Upsilon_{ij}^a(t, z_j, z_{jdj}) \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{z}_i^b &= A_{i3}z_i^a + A_{i4}z_i^b + E_{i3}z_{id_i}^a + E_{i4}z_{id_i}^b \\ &\quad + B_{i2}(u_i + G_i(t, z_i, z_{id_i})) + F_i^b(t, z_i, z_{id_i}) \\ &\quad + \sum_{\substack{j=1\\j\neq i}}^{N} \Upsilon_{ij}^b(t, z_j, z_{jdj}) \end{aligned} \tag{16}$$

where $z_i := \mathrm{col}(z_i^a, z_i^b)$ with $z_i^a \in \Re^4$ and $z_i^b \in \Re$, A_{i1} is Hurwitz stable, and B_{i2} is a positive scalar. Thus

$$G_i(t, z_i, z_{id_i}) := G_i(t, T_i^{-1}z_i, T_i^{-1}z_{id_i}) \tag{17}$$

$$\begin{bmatrix} F_i^a(t, z_i, z_{id_i}) \\ F_i^b(t, z_i, z_{id_i}) \end{bmatrix} := T_i F_i(t, T_i^{-1}z_i, T_i^{-1}z_{id_i}) \tag{18}$$

$$\begin{bmatrix} \Upsilon_{ij}^a(t, z_j, z_{jdj}) \\ \Upsilon_{ij}^b(t, z_j, z_{jdj}) \end{bmatrix} := T_i \Upsilon_{ij}(t, T_i^{-1}z_j, T^{-1}z_{jdj}) \tag{19}$$

where $\Upsilon_{ij}^a(\cdot) \in \Re^{4\times 1}$, $F_i^a(\cdot) \in \Re^{4\times 1}$; $\Upsilon_{ij}^b(\cdot) \in \Re$, $F_i^b(\cdot) \in \Re$ for $i, j = 1, 2, ..., N$. It should be noted that all the matrices A_{ij}, E_{ij} and B_{i2} can be obtained from (10) and (11). Thus the transformed system (15)-(16) is well defined.

A. Stability of sliding motion

From the structure of system (15) and (16), define a switching function of the form

$$\sigma_i(z_i) = S_{i2} z_i^b, \qquad i = 1, 2, ..., N \tag{20}$$

Therefore the local sliding surface for the interconnected system (9) is described as

$$\sigma_i(z_i) = S_{i2} z_i^b = 0, \qquad i = 1, 2, ..., N \tag{21}$$

Since A_{i1} in (15) is stable, for any $Q_i > 0$, the following Lyapunov equation has a unique solution $P_i > 0$ such that

$$A_{i1}^T P_i + P_i A_{i1} = -Q_i, \qquad i = 1, 2, ..., N. \tag{22}$$

From the structure of system (15)-(16), the sliding motion of system (1) associated with the sliding surface (21) is dominated by system (15). When dynamic system (15) is limited to the sliding surface (21), from the fact that S_{i2} is nonsingular, the reduced order dynamics can be described as

$$\dot{z}_i^a = A_{i1} z_i^a + E_{i1} z_{id_i}^a + \Psi_i(t, z_i^a, z_{id_i}^a) + \sum_{\substack{j=1\\ j\neq i}}^{N} \tilde{\Upsilon}_{ij}^a(t, z_j^a, z_{jdj}^a) \tag{23}$$

where $z_i^a = \text{col}\left(z_{i1}^a, z_{i2}^a, ..., z_i^a\right)$, $z_i^b = \text{col}\left(z_{i1}^b, z_{i2}^b, .., z_i^b\right)$, and

$$\Psi_i(t, z_i^a, z_{id_i}^a) \;:=\; F_i^a(t, z_i, z_{id_i})|_{z_i^b = 0} \tag{24}$$

$$\tilde{\Upsilon}_{ij}(t, z_j^a, z_{jd_j}^a) \;:=\; \Upsilon_{ij}^a(t, z_j, z_{jd_j})|_{z_j^b = 0} \tag{25}$$

where $F_i^a(\cdot)$ and $\Upsilon_{ij}^a(\cdot)$ are defined in (18)-(19) respectively. System (23) is obtained by applying (21) to the system (15).

From Assumption 2, it is clear to see that $F_i^a(\cdot)$ and $\Upsilon_{ij}^a(\cdot)$ are bounded by a known continuous function. Therefore, on the sliding surface, the bounds on (13) and (14) can be described as

$$\begin{aligned} \|P_i^{1/2}\Psi_i(t, z_i^a, z_{idi}^a)\| \;&\leq \beta_i^a(t, z_i^a)\|P_i^{1/2} z_i^a\| \\ &\quad + \beta_i^b\left(t, \|z_{id_i}^a\|\right)\|P_i^{1/2} z_{id_i}^a\| \end{aligned} \tag{26}$$

$$\begin{aligned} \|P_j^{1/2}\tilde{\Upsilon}_{ij}(t, z_j^a, z_{jdj}^a)\| \;&\leq \chi_{ij}^a(t, z_j^a)\|P_j^{1/2} z_j^a\| \\ &\quad + \chi_{ij}^b(t, \|z_{jd_j}^a\|)\|P_j^{1/2} z_{jd_j}^a\| \end{aligned} \tag{27}$$

where the functions $\beta_i^a(\cdot)$, $\beta_i^b(\cdot)$, $\chi_{ij}^a(\cdot)$, and $\chi_{ij}^b(\cdot)$ are nondecreasing.

Theorem 1. Under Assumptions 1-2, the sliding motion of system (15)-(16) associated with the sliding surface (21), governed by (23) is uniformly asymptotically stable if $W^T + W > 0$, where the matrix

$$W = \begin{bmatrix} w^a & w^b \\ w^c & w^d \end{bmatrix}_{2N\times 2N} \tag{28}$$

Here $w^a = (w_{ij}^a)_{N\times N}$, $w^b = (w_{ij}^b)_{N\times N}$, $w^c = (w_{ij}^c)_{N\times N}$, $w^d = (w_{ij}^d)_{N\times N}$ and

$$w_{ij}^a = \begin{cases} \lambda_{min}(P_i^{-1/2} Q_i P_i^{-1/2}) - 2\beta_i^a(\cdot) - q_i, & \text{if } i = j \\ -\lambda_{max}(P_i^{1/2} P_j^{-1/2})\chi_{ij}^a(\cdot), & \text{if } i \neq j \end{cases}$$

$$w_{ij}^b = \begin{cases} \|P_i^{1/2} E_{i1} P_i^{-1/2}\| + \beta_i^b(\cdot), & \text{if } i = j \\ -\lambda_{max}(P_i^{1/2} P_j^{-1/2})\chi_{ij}^b(\cdot), & \text{if } i \neq j \end{cases}$$

$$w_{ij}^d = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

where $w_{ij}^b = w_{ij}^c$ for $i, j = 1, 2, ..., N$.

Proof. For system (23), consider a Lyapunov function candidate

$$V(z_1^a, z_2^a, ..., z_n^a) = \sum_{i=1}^{N} (z_i^a)^T P_i z_i^a \tag{29}$$

It follows from (22) that the time derivative of V along the trajectories of system (23) is given as

$$\begin{aligned} \dot{V}(z_i^a)|_{(23)} &= -\sum_{i=1}^{N} (z_i^a)^T Q_i z_i^a + 2\sum_{i=1}^{N} (z_i^a)^T P_i E_{i1} z_{id_i}^a \\ &\quad + 2\sum_{i=1}^{N} (z_i^a)^T P_i \Psi_i(\cdot) + 2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} (z_i^a)^T P_i \tilde{\Upsilon}_{ij}^a(\cdot) \end{aligned} \tag{30}$$

It is straight forward to see that

$$\begin{aligned} (z_i^a)^T Q_i z_i^a &= (z_i^a)^T P_i^{1/2} P_i^{-1/2} Q_i P_i^{-1/2} P_i^{1/2} z_i^a \\ &\leq \lambda_{min}(P_i^{-1/2} Q_i P_i^{-1/2})\|P_i^{1/2} z_i^a\|^2 \end{aligned} \tag{31}$$

and

$$\begin{aligned} 2\sum_{i=1}^{N} (z_i^a)^T P_i E_{i1} z_{id_i}^a &= 2\sum_{i=1}^{N} (z_i^a)^T P_i^{1/2} P_i^{1/2} E_{i1} P_i^{-1/2} P_i^{1/2} z_{id_i}^a \\ &\leq 2\sum_{i=1}^{N} \|P_i^{1/2} z_i^a\|\|P_i^{1/2} E_{i1} P_i^{-1/2}\|\|P_i^{1/2} z_{id_i}^a\| \end{aligned} \tag{32}$$

From (26), it follows that

$$\begin{aligned} 2\sum_{i=1}^{N} (z_i^a)^T P_i \Psi_i(\cdot) &= 2\sum_{i=1}^{N} (z_i^a)^T P_i^{1/2} P_i^{1/2} \Psi_i(\cdot) \\ &\leq 2\sum_{i=1}^{N} \|(P_i^{1/2} z_i^a)^T\|\|(P_i^{1/2}\Psi_i(\cdot))\| \\ &\leq 2\sum_{i=1}^{N} \|P_i^{1/2} z_i^a\|\left(\beta_i^a(\cdot)\|P_i^{1/2} z_i^a\| + \beta_i^b(\cdot)\|P_i^{1/2} z_{id_i}^a\|\right) \\ &= 2\sum_{i=1}^{N} \left(\beta_i^a(\cdot)\|P_i^{1/2} z_i^a\|^2 + \beta_i^b(\cdot)\|P_i^{1/2} z_i^a\|\|P_i^{1/2} z_{id_i}^a\|\right) \end{aligned} \tag{33}$$

It is clear that all the subsystems in (15) has the same dimension 4. From (27)

$$\begin{aligned} &2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} (z_i^a)^T P_i \tilde{\Upsilon}_{ij}^a(\cdot) \\ &= 2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} (z_i^a)^T P_i^{1/2} P_i^{1/2} P_j^{-1/2} P_j^{1/2} \tilde{\Upsilon}_{ij}^a(\cdot) \\ &\leq 2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} \lambda_{max}(P_i^{1/2} P_j^{-1/2})\|P_i^{1/2} z_i^a\|\Bigg(\chi_{ij}^a(\cdot)\|P_j^{1/2} z_j^a\| \\ &\quad + \chi_{ij}^b(\cdot)\|P_j^{1/2} z_{jd_j}^a\|\Bigg) \end{aligned} \tag{34}$$

Substituting (31), (32), (33) and (34) into (30) yields

$$\dot{V}(z_i^a(t))|_{(23)} \leq -\sum_{i=1}^{N} \lambda_{min}(P_i^{-1/2}Q_iP_i^{-1/2})\|P_i^{1/2}z_i^a\|^2$$
$$+2\sum_{i=1}^{n}\|P_i^{1/2}z_i^a(t)\|\|P_i^{1/2}E_{i1}P_i^{-1/2}\|\|P_i^{1/2}z_{id_i}^a\|$$
$$+2\sum_{i=1}^{N}\Big(\beta_i^a(t,z_i^a)\|P_i^{1/2}z_i^a\|^2+\beta_i^b(t,z_{id}^a)\times$$
$$\|P_i^{1/2}z_i^a\|\|P_i^{1/2}z_{id_i}^a\|\Big)+2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\lambda_{max}(P_i^{1/2}P_j^{-1/2}) \tag{35}$$
$$\Big(\chi_{ij}^a(\cdot)\|P_i^{1/2}z_i^a\|\|P_j^{1/2}z_j^a\|+\chi_{ij}^b(\cdot)\|P_i^{1/2}z_i^a\|\|P_j^{1/2}z_{jd_j}^a\|\Big)$$

For any constant $q_i > 1$, assume that

$$V(z_{1d_1}^a, z_{2d_i}^a, ..., z_{nd_n}^a) \leq q_iV(z_1^a, z_2^a, ..., z_n^a) \tag{36}$$

which from (29) implies that

$$q_i\sum_{i=1}^{N}\|P_i^{1/2}z_i^a\|^2-\sum_{i=1}^{N}\|P_i^{1/2}z_{id_i}^a\|^2 \geq 0 \tag{37}$$

Further from (35) and (37), it follows that

$$\dot{V}(z_i^a(t))|_{(23)} \leq$$
$$-\Bigg\{\sum_{i=1}^{N}\Big(\lambda_{min}(P_i^{-1/2}Q_iP_i^{-1/2})-2\beta_i^a(\cdot)-q_i\Big)\|P_i^{1/2}z_i^a\|^2$$
$$+\sum_{i=1}^{N}\|P_i^{1/2}z_{id_i}^a\|^2-2\sum_{i=1}^{N}\Big(\|P_i^{1/2}E_{i1}P_i^{-1/2}\|+\beta_i^b(\cdot)\Big)$$
$$\times\|P_i^{1/2}z_i^a(t)\|\|P_i^{1/2}z_{id_i}^a\|-2\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\lambda_{max}(P_i^{1/2}P_j^{-1/2})\chi_{ij}^a(\cdot)$$
$$\times\|P_i^{1/2}z_i^a\|\|P_j^{1/2}z_j^a\|-\lambda_{max}(P_i^{1/2}P_j^{-1/2})\chi_{ij}^b(\cdot)$$
$$\times\|P_i^{1/2}z_i^a\|\|P_j^{1/2}z_{jd_j}^a\|\Bigg\}$$
$$\leq -Z^T(W+W^T)Z \tag{38}$$

where

$$Z =: \Big(\|P_1^{1/2}z_1^a\|, ..., \|P_N^{1/2}z_N^a\|, \|P_1^{1/2}z_{1d_1}^a\|, \cdots, \|P_N^{1/2}z_{Nd_N}^a\|\Big)^T$$

Hence from Theorem 1.4 in [7], the conclusion follows based on the condition that $W^T+W>0$. ■

B. Decentralized Control design

Next, it is necessary to design a decentralized sliding mode control law for the interconnected systems such that the systems state is driven to the sliding surface (21). A well known reachability condition for the interconnected systems (15)-(16) is described by [12]

$$\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|} < 0 \tag{39}$$

Consider the following decentralized control.

$$u_i = -(S_{i2}B_i)^{-1}\Big\{S_{i2}H_i+\Big(\Xi_i+\eta_i\Big)\text{sgn}(\sigma_i(z_i))\Big\} \tag{40}$$

where

$$H_i = A_{i3}z_i^a + A_{i4}z_i^b + E_{i3}z_{id_i}^a + E_{i4}z_{id_i}^b \tag{41}$$

is the linear component.

$$\begin{aligned}\Xi_i \quad = \quad & \|S_{i2}B_i\|\zeta_i(t,z_i,\|z_{id_i}\|)+\|S_{i2}\|\Big(\kappa_{ij}^a(t,z_j)\|z_j\| \\ & +\kappa_{ij}^b(t,\|z_{jd_j}\|)\|z_{jd_j}\|\Big)+\|S_{i2}\|\Big(\alpha_i^a(t,z_i)\|z_i\| \\ & +\alpha_i^b(t,\|z_{id_i}\|)\|z_{id_i}\|\Big)\end{aligned} \tag{42}$$

where η_i is a positive constant. The functions $\zeta_i(\cdot)$, $\kappa_i^a(\cdot)$, $\kappa_i^b(\cdot)$, $\alpha_i^a(\cdot)$, and $\alpha_i^b(\cdot)$ are given in Assumption 2.

Theorem 2. Consider the system (15)-(16). Under Assumption 2, a delay dependent, decentralized control law in (40) with Ξ_i defined by (42) drives the system (15)-(16) to the sliding surface (21) and maintain a sliding motion on it thereafter.

Proof. From (20) and (16), it is easy to see that

$$\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|}=\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)}{\|\sigma_i(z_i)\|}\Big\{S_{i2}H_i+S_{i2}B_iu_i$$
$$+S_{i2}B_iG_i(\cdot)+S_{i2}F_i^b(\cdot)+S_{i2}\sum_{\substack{j=1\\ j\neq i}}^{N}\Upsilon_{ij}^b(\cdot)\Big\} \tag{43}$$

It follows that substituting u_i in (40) into (43),

$$\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|}=\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)}{\|\sigma_i(z_i)\|}\Bigg\{S_{i2}H_i+S_{i2}B_i$$
$$\Bigg(-(S_{i2}B_i)^{-1}\Big\{S_{i2}H_i+\Big(\Xi_i+\eta_i\Big)\text{sgn}(\sigma_i(z_i))\Big\}\Bigg)$$
$$+S_{i2}B_iG_i(\cdot)+S_{i2}F_i^b(\cdot)+S_{i2}\sum_{\substack{j=1\\ j\neq i}}^{N}\Upsilon_{ij}^b(\cdot)\Bigg\} \tag{44}$$

Then substituting (42) into (44)

$$\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|}=\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)}{\|\sigma_i(z_i)\|}\Bigg\{S_{i2}H_i+S_{i2}B_i$$
$$\Bigg(-(S_{i2}B_i)^{-1}\Big\{S_{i2}H_i+\Big(\|S_{i2}B_i\|\zeta_i(t,z_i,\|z_{id_i}\|)$$
$$+\|S_{i2}\|\Big(\alpha_i^a(t,z_i)\|z_i\|+\alpha_i^b(t,\|z_{id_i}\|)\|z_{id_i}\|\Big)$$
$$+\|S_{i2}\|\Big(\kappa_{ij}^a(t,z_j)\|z_j\|+\kappa_{ij}^b(t,\|z_{jd_j}\|)\|z_{jd_j}\|\Big)$$
$$+\eta_i\Big)\text{sgn}(\sigma_i(z_i))\Big\}\Bigg)+S_{i2}B_iG_i(\cdot)+S_{i2}F_i^b(\cdot)$$
$$+S_{i2}\sum_{\substack{j=1\\ j\neq i}}^{N}\Upsilon_{ij}^b(\cdot)\Bigg\} \tag{45}$$

From Assumption 2, it follows that

$$\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|} \le \Bigg(-\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)\mathrm{sgn}(\sigma_i(z_i))}{\|\sigma_i(z_i)\|}\|S_{i2}B_i\|\zeta_i(\cdot)$$
$$+\|S_{i2}B_i\|\|G_i(\cdot)\|\Bigg) - \Bigg(\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)\mathrm{sgn}(\sigma_i(z_i))}{\|\sigma_i(z_i)\|}\|S_{i2}\|$$
$$\Big(\alpha_i^a(t,z_i)\|z_i\| + \alpha_i^b(t,\|z_{id_i}\|)\|z_{id_i}\|\Big) - \|S_{i2}\|\|F_i(\cdot)\|\Bigg)$$
$$-\Bigg(\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)\mathrm{sgn}(\sigma_i(z_i))}{\|\sigma_i(z_i)\|}\|S_{i2}\|\Big(\kappa_{ij}^a(t,z_j)\|z_j\|$$
$$+\kappa_{ij}^b(t,\|z_{jd_j}\|)\|z_{jd_j}\|\Big) - \sum_{\substack{j=1\\ j\neq i}}^{N}\|S_{i2}\|\|\Upsilon_{ij}^b(\cdot)\|\Bigg)$$
$$-\sum_{i=1}^{N}\frac{\sigma_i^T(z_i)\mathrm{sgn}(\sigma_i(z_i))}{\|\sigma_i(z_i)\|}\eta_i \le -\sum_{i=1}^{N}\eta_i < 0 \quad (46)$$

where the fact that $\|\sigma_i(z_i)\| \le \sigma_i^T(z_i)\mathrm{sgn}(\sigma_i(z_i))$ [14] was used above. This shows that the reachability condition holds. Hence the conclusion follows. ■

From Sliding mode control theory, Theorems 1 and 2 together guarantee that the closed-loop systems formed by applying controllers (40)–(42) to the system (15)-(16) are asymptotically stable.

V. Simulation Example

In order to illustrate the obtained results, consider a two-area LFC power system with time varying network delays. The parameters and data used in obtaining the simulation results are from [4], [5]. The areas are interconnected via a tie-line. Thus, as the system load varies, turbine speed also varies which in turn changes the frequency $\Delta f(t)$ and the tie-line power $\Delta P_{tie}(t)$ of the generating unit.

By using the algorithm in [20], the coordinate transformation $z_i = T_i x_i$ for $i = 1.2$ can be obtained with T_i defined by

$$T_1 = T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 30 & 62 & 42.5 & 11.50 & 1 \end{bmatrix}$$

Hence it follows from (9) that

Area 1:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{13} & A_{14} \end{bmatrix} = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -30 & -62 & -42.5 & -11.50 & 1 \\ \hline -89 & -156 & -43 & -8 & 3 \end{array}\right] \quad (47)$$

$$\begin{bmatrix} E_{11} & E_{12} \\ E_{13} & E_{14} \end{bmatrix} = \left[\begin{array}{cccc|c} 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} \\ 0 & 0 & 0.01 & 0.08 & 0 \\ \hline 0.38 & 50.36 & 0.06 & 0.45 & 0 \end{array}\right]$$

$$B_1 = \begin{bmatrix} 0_{4\times1} \\ 1 \end{bmatrix}, F_1 = \begin{bmatrix} -0.08(z_{1d_1}^a)^2 \\ 0_{3\times1} \\ \hline -2.48(z_1^b)^2 \end{bmatrix}, \Upsilon_{12} = \begin{bmatrix} 0_{4\times1} \\ \hline -0.31\sin(z_2^b) \end{bmatrix}$$

Area 2:

$$\begin{bmatrix} A_{21} & A_{22} \\ A_{23} & A_{24} \end{bmatrix} = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -30 & -62 & -42.5 & -11.5 & 1 \\ \hline -105 & -188 & -69 & -13 & 4 \end{array}\right] \quad (48)$$

$$\begin{bmatrix} E_{21} & E_{22} \\ E_{23} & E_{24} \end{bmatrix} = \left[\begin{array}{cccc|c} 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} & 0_{3\times1} \\ 0 & 0 & 0.009 & 0.069 & 0 \\ \hline 0.001 & 40.76 & 0.055 & 0.438 & 0 \end{array}\right]$$

$$B_2(\cdot) = \begin{bmatrix} 0_{4\times1} \\ 1 \end{bmatrix}, \; F_2(\cdot) = \begin{bmatrix} -0.07(z_{1d_1}^a)^2 \\ 0_{3\times1} \\ \hline -2.06(z_1^b)^2 \end{bmatrix} \quad (49)$$

Based on Assumption 2. The unknown matched uncertainty $G_i(\cdot)$ and $F_i^b(\cdot)$ is assumed to satisfy

$$\|G_1(\cdot)\| \le \underbrace{3|z_{1d_1}^b|\sin^2(z_1)}_{\zeta_1(\cdot)}$$
$$\|G_2(\cdot)\| \le \underbrace{2\sin^2 z_2^a\|z_2\|}_{\zeta_2(\cdot)}$$

$$\|F_1^b(\cdot)\| \le \underbrace{\sin^2(z_1)\|z_1\| + 0.8\|z_{1d_1}\|}_{\alpha_1(\cdot)}$$
$$\|F_2^b(\cdot)\| \le \underbrace{2\cos^2(z_2)\|z_2\| + 0.5\|z_{2d_2}\|}_{\alpha_2(\cdot)}$$
$$\|\Upsilon_{12}^b(\cdot)\| \le \underbrace{0.5\cos^2(z_2)\|z_2\|}_{\kappa_{12}(\cdot)}$$

Therefore, on the sliding surface (21), when the sliding motion takes place. Since A_{11} and A_{21} in (47)-(48) are stable, it follows that choosing $Q_1 = Q_2 = I_2$ and solving the lyapunov equation (22) has a unique solution

$$P_1 = P_2 \begin{bmatrix} 2.0559 & -0.5 & -0.8979 & 0.5 \\ -0.5 & 0.8979 & -0.5 & -1.7320 \\ -0.8979 & -0.5 & 1.7320 & -0.5 \\ 0.5 & -1.7320 & -0.5 & 9.9248 \end{bmatrix}$$

and thus

$$P_1^{\frac{1}{2}} = P_2^{\frac{1}{2}} \begin{bmatrix} 1.3419 & -0.3066 & -0.3959 & 0.0667 \\ -0.3066 & 0.6680 & -0.3726 & -0.4678 \\ -0.3959 & -0.3726 & 1.1890 & -0.1507 \\ 0.0667 & -0.4678 & -0.1507 & 3.1111 \end{bmatrix}$$

Further, from (26)-(27),

$$\beta_1^a(\cdot) = 3\sin^2(z_1^a), \quad \beta_1^b(\cdot) = 2\sin^2(t)$$
$$\beta_2^a(\cdot) = 2|z_2^a|^4, \quad \beta_2^b(\cdot) = |z_{2d_2}^a|$$
$$\chi_{12}^a(\cdot) = 3\sin^2(z_2^a), \quad \chi_{12}^b(\cdot) = 0$$

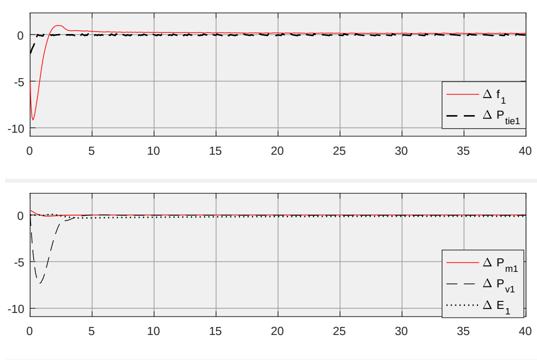

Fig. 1: Evolution of system responses for area 1

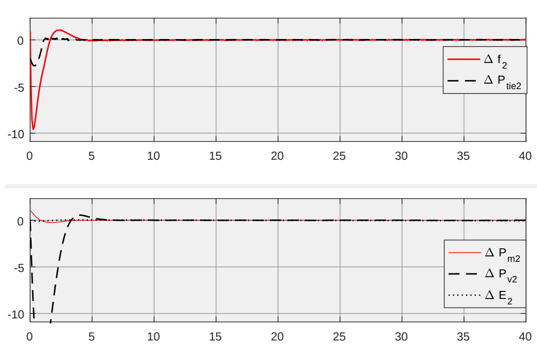

Fig. 2: Evolution of system responses for area 2

By direct calculation, W defined in (28) is given by

$$W = \begin{bmatrix} 10.3761 & -3.5 & 2.9826 & 0 \\ -3.5 & 12.3761 & 0 & 1.7195 \\ 2.9826 & 0 & 1 & 0 \\ 0 & 1.7195 & 0 & 1 \end{bmatrix}$$

It is easy to verify that the conditions in Theorem 1 are satisfied. Thus from Theorem 1, the sliding motion associated with the sliding surface is uniformly asymptotically stable. From theorem 2, the decentralised control law is given by (40)-(42).

For simulation purposes, the time delay $d_1(t) = d_2(t) = 6 + 2\sin^2(t)$ is chosen for the two area LFC system. Based on Theorem 1 and 2, the LFC system is uniformly asymptotically stable for $t \geq 0$. Figs 1 & 2 shows the systems response in areas 1 & 2 from their initial values, under load perturbation and time varying delay. However, based on Theorem 2, the delay and disturbances in the system can be compensated by proper designed controller, which converges rapidly. Thereby driving all error deviations to zero, which verifies the effectiveness of the designed controller.

VI. CONCLUSION

A decentralized state feedback sliding mode control based on the Lyapunov-Razumikhin theorem has been proposed for nonlinear interconnected power systems with time-varying delays and uncertainties. By employing the delay and exploiting the bounds on the uncertainties in both the sliding motion analysis and the control design, conservatism is reduced and the robustness is enhanced. The simulation results on a 2-area interconnected power systems has demonstrated the effectiveness of the obtained results and further illustrates the feasibility of the proposed methodology. In the future, it will be interesting to explore the delay independent control for LFC power systems.

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