Comments on the paper: The most energy efficient way to charge the capacitor in a RC circuit D. Wang 2017 *Phys. Educ.* **52** 065019

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Abstract

In a recent paper (Wang. D. (2017) Phys. Educ. 52 065019) a comparison was made between the efficiency in charging a capacitor ($C$) in series with a resistor ($R$) using either a voltage source or a constant current source. The paper concluded that using a current source was more efficient. We show that this is not correct when the energy loss within the current source is considered. It is also shown that the energy loss is not dependent on the charging rate. A formal proof using calculus and simpler graphical arguments are presented.

In a recent paper a comparison was made between the efficiency in charging a capacitor ($C$) in series with a resistor ($R$) using either a voltage source or a constant current source [1]. The main conclusion of the analysis was that it is more efficient to charge $C$ in a series $R$-$C$ circuit to a given voltage using a current source rather than a constant voltage source. The author also suggests that the inefficiency in charging $C$ in the constant voltage source case is somehow associated with the rapid changes in the current. Although the mathematics in the paper is correct, we wish to point out that this analysis has a flaw associated with it and thus the main conclusion of the paper and its physical reason are incorrect.

The defect in the analysis is that it ignores the internal construction of a current source. We show below that when the energy dissipated in the current source is also taken into account the efficiencies of using a current and voltage source are in fact the same.

In general a current source circuit will consist of a constant voltage source and some control element in series with it. The voltage source is depicted in fig. 1 by a battery of $V_B$ volts whilst the control element is depicted by a bipolar transistor with a control input at its base. The control input is adjusted automatically by other circuitry (not shown) so the current through the transistor is of a particular value. We assume the base current is negligible so that the current supplied by the battery flows into $C$. Control elements other than a bipolar transistor can of course be used and the discussion below is general and does not depend on this selection. We include in the circuit in fig. 1 the fixed series resistance $R$ as in reference [1]. However, it should be noted that when the current is determined by a current generator, a series resistance is not needed to charge a capacitor. Inspection of fig. 1 shows that if $R$=0 Ω we have $C$ charged directly from the current source whilst if the control element is replaced by a short circuit, then the circuit reduces to the classic case of a $C$ being charged by a resistor and voltage source. Below we show that these two extremes and any intermediate case where both $R$ and control element are in the circuit, the energy loss in charging $C$ is the same and independent of the charging rate.

We denote by $I(t)$ the current which for generality and for further discussions is assumed to be a function of time. The total energy supplied by the battery, $U_B$, is given by

$$U_B = V_B \int_0^T I(t)dt = V_B Q(T)$$

(1)

where $Q(T)$ is the total charge supplied in time $T$. This energy is depicted graphically by the area shown in fig. 2(a). From $Q=CV_C$ the voltage $V_C$ across any capacitor $C$ will rise linearly with the charge supplied to it as shown in fig 2(b). Formally, if $V_C(t)$ is the capacitor voltage at time $t$ then the energy supplied to the capacitor is

$$U_C = \int_0^T V_c(t)I(t)dt = \int_0^{Q(T)} V_c(Q)dQ = \frac{1}{C}\int_0^{Q(T)} QdQ = \frac{Q(T)^2}{2C} = \frac{1}{2}V_C(T)Q(T)$$

(2)

which is the usual expression for the energy stored in $C$. Graphically, this is given by the area under the triangle shown in fig. 2(b). Applying Kirchhoff's voltage law (KVL), the voltage $V_E(t)$ across the control element and resistance is given by

$$V_E(t) = V_B - V_C(t)$$

(3)

Hence formally the energy dissipated in the control element and resistance combined, $U_E$, is

$$U_E = \int_0^T V_E(t)I(t)dt = \int_0^{Q(T)} (V_B - V_c(Q))dQ = V_B Q(T) - \frac{Q^2(T)}{2C}$$
$$= V_B Q(T) - \frac{1}{2}V_C(T)Q(T)$$

(4)

Alternatively, a simpler proof of eqn. (4) not involving calculus is to argue that if $V_C$ rises linearly with the charge supplied, then due to KVL $V_E$ falls linearly with the charge supplied. The energy $U_E$, is thus given by the area under the trapezium shown in fig. 2(c). Hence

$$U_E = \frac{V_B + (V_B - V_C(T))}{2}Q(T) = V_B Q(T) - \frac{1}{2}V_C(T)Q(T)$$

(5)

which is the same as eqn. (4). Thus the energy dissipated in the control element plus series resistance will always be equal to the energy supplied by the source minus the energy stored in the capacitor, which is simply the conservation of energy.

Inspection of eqns. (1), (2) and (4) or (5) show that they are applicable no matter how the current $I(t)$ varies with $t$ , or indeed if it is a constant, provided its time integral is equal to $Q(T)$. Further, the equations are independent of the relative proportions of $U_E$ lost in the control element and $R$. Thus if $R$=0 Ω then all the energy $U_E$ is dissipated in the control element of the current source. At the other extreme if the control element is replaced by a short circuit and $R$≠0 Ω, which is the classic circuit where $C$ is charged by constant voltage source and $R$, then $U_E$ is the energy dissipated in $R$. For these two extremes and any intermediate case, $U_E$ has the same value.

Further, the above equations show that the energy loss, $U_E$, is independent of how rapidly the charging current changes, *i.e*. on the function $I(t)$. This is counter to the assertion previously made that the cause of the apparent loss in efficiency was due to current surges [1].

In summary, it does not matter if the $C$ in a series $R$-C circuit is charged by a voltage source or a current source (with either a constant current or a time dependent current); when the energy loss in the control element of the current source is taken into account, the energy loss is the same.

**References**

[1] Wang. D. The most energy efficient way to charge the capacitor in a RC circuit. (2017) Phys. Educ. 52 065019

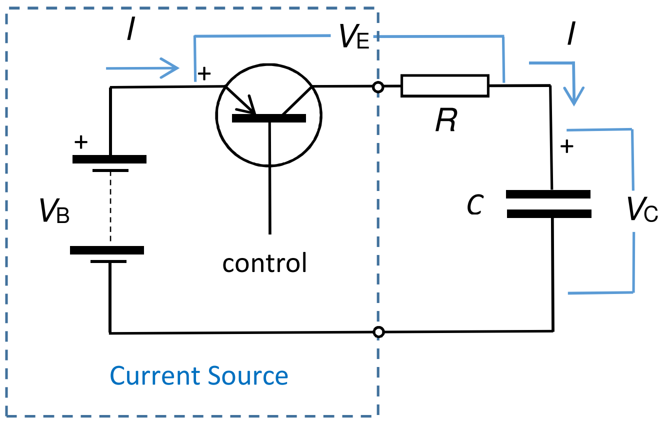


Figure 1. Showing basic elements of a current source charging a $C$ in a series $R$-$C$ circuit.

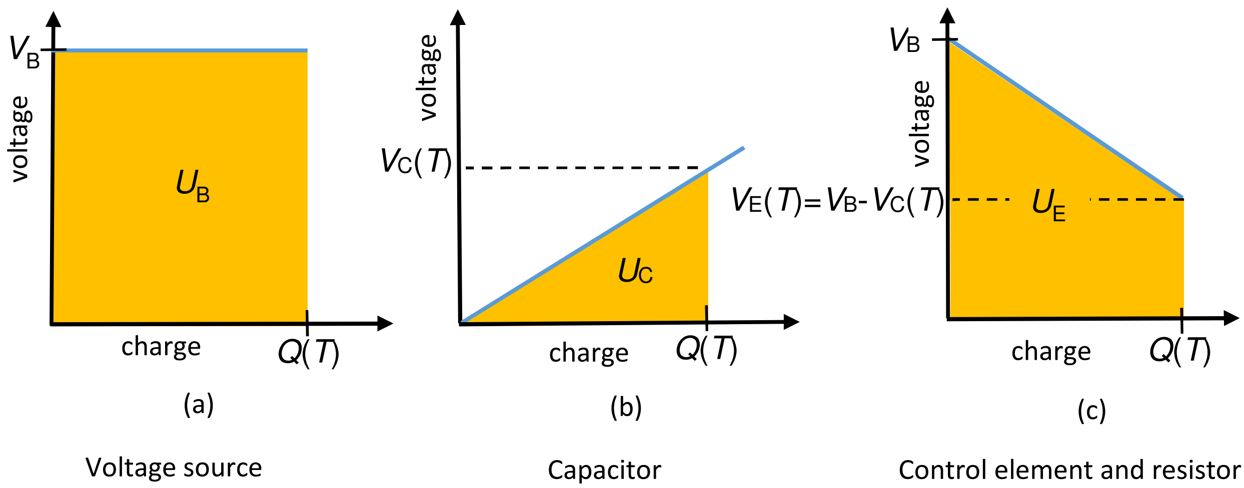


Figure 2. Showing voltage against charge for each element (a) voltage source, (b) capacitor, (c) control element and resistor.

**The Author:**

Robert Oven is a Senior Lecture and Director of Education in the School of Engineering and Digital Arts at the University of Kent. He has taught numerous physical electronics, analogue electronics and electromagnetism courses at degree level since 1986. He was also Chair of the Foundation Year Programme in the School for over 20 years. His research interests are in the areas of ion implantation and more recently ion diffusion in glass and optical measurements. He has published 47 peer reviewed papers mainly in these areas.