LOSS COVERAGE IN INSURANCE MARKETS: WHY ADVERSE SELECTION IS NOT ALWAYS A BAD THING

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IAA COLLOQUIUM IN OSLO
Acknowledgement

Bursary from
- IAA-International Actuarial Association
- The Colloquium
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- Background
- How does insurance work?
- Risk classification Scheme
- Adverse Selection
- Loss Coverage
- Demand function
  - Iso-elastic demand function
- Equilibrium Premium
- Results on adverse selection and loss coverage
- Summary and Further research
- References
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Background

How insurance works and risk classification scheme

Regulators

Restrict risk classification
E.g. European Gender Directive

π₁ = π₂ = ⋯ = πₙ = πₑ
Pooled Premium

Risk Classification

Risk-group 1
- Risk: μ₁

Risk-group 2
- Risk: μ₂

Risk-group n
- Risk: μₙ

π₁ → Insurers

π₂ → Insurers

πₙ → Insurers

Insurers

Fair Premium
πᵢ = μᵢ

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Insurance Risk
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Adverse Selection

0, \pi_1, \pi_2, \pi_3, \pi_e, \ldots, \pi_7, \pi_8, \ldots, \pi_n, 1.
Adverse Selection

0, π₁, π₂, π₃, πₑ, ..., π₇, π₈, ..., πₙ, 1.

Typical definition

Purchasing decision is positively correlated with losses
-Chiappori and Salanie (2000) “Positive Correlation Test”
Adverse Selection

$0, \pi_1, \pi_2, \pi_3, \pi_e, \ldots, \pi_7, \pi_8, \ldots, \pi_n, 1.$

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Empirical results are mixed and vary by market.
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Empirical results are mixed and vary by market.

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<thead>
<tr>
<th>Insurance Type</th>
<th>Reference(s)</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Insurance</td>
<td>Cawley and Philipson (1999)</td>
<td>X</td>
</tr>
<tr>
<td>Auto Insurance</td>
<td>Chiappori and Salanie (2000)</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Cohen (2005)</td>
<td>O</td>
</tr>
<tr>
<td>Health Insurance</td>
<td>Cardon and Hendel (2001)</td>
<td>X</td>
</tr>
</tbody>
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Adverse Selection

- Restricting risk classification \(\Rightarrow\) Policy is over-subscribed by high risks \textbf{BAD}?
Adverse Selection

- Restricting risk classification $\Rightarrow$ Policy is over-subscribed by high risks **BAD**?
- **Good measure?**

Definition

Adverse Selection (AS) = expected claim per policy \[ E[QL] \]
\[ \text{expected loss per risk} = E[Q]E[L], \] (1)

Adverse Selection Ratio:

\[ S = \frac{AS \text{ at pooled premium}}{\pi} \]  
\[ = \frac{AS \text{ at risk-differentiated premiums}}{1} \]  
> 1 $\Rightarrow$ **Adverse Selection**.

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Adverse Selection

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(1)

where \( Q \): quantity of insurance; \( L \): risk experience.
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> 1 ⇒ **Adverse Selection**.
Example

- A population of 1000
- Two risk groups
  - 200 high risks with risk 0.04
  - 800 low risks with risk 0.01
- No moral hazard
Example
Full risk classification
## Example

Full risk classification

<table>
<thead>
<tr>
<th></th>
<th>Low risks</th>
<th>High risks</th>
<th>Aggregate</th>
</tr>
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<tbody>
<tr>
<td>Risk</td>
<td>0.01</td>
<td>0.04</td>
<td>0.016</td>
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No adverse selection.
Example

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# Example

Restriction on risk classification - Case 1

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Moderate adverse selection
### Example

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Adverse Selection Ratio (S) 1.25 > 1

**Moderate adverse selection**
Example
Restriction on risk classification-Case 2
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Heavier adverse selection suggests pooling is always bad. But is it?

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Heavier adverse selection
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Loss Coverage

Aim of insurance: provide protection for those who suffer losses.

▶ High risks most need insurance.

▶ Restriction on risk classification seems reasonable.

Thomas (2008, 2009) "Loss Coverage":

**Definition**

\[
\text{Loss Coverage (LC)} = \frac{\text{insured expected losses}}{\text{population expected losses}} \tag{3}
\]

**Loss Coverage Ratio:**

\[
C = \frac{\text{LC at a pooled premium } \pi_e}{\text{LC at a risk-differentiated premium } \pi_i} \tag{4}
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No restriction on risk classification
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No adverse selection.
Example
Restriction on risk classification-Case 1
### Example

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**Moderate adverse selection** \(S = 1.25\) but **favorable loss coverage.**
Example
Restriction on risk classification-Case 2
## Example

### Restriction on risk classification - Case 2

<table>
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<tr>
<th></th>
<th>Low risks</th>
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<tr>
<td>Risk</td>
<td>0.01</td>
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*Heavier adverse selection \((S = 1.3462)\) and worse loss coverage. Loss coverage might be a better measure!*

M Hao (SMSAS-University of Kent)
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**Loss coverage ratio \((C)\)\:** 0.875<1

**Heavier adverse selection \((S = 1.3462)\) and worse loss coverage. Loss coverage might be a better measure!**
Table of contents

- Background
  - How does insurance work?
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- Adverse Selection
- Loss Coverage
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  - Iso-elastic demand function
- Equilibrium Premium
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- Summary and Further research
- References
Demand Function

**Definition**

\[ d(\mu, \pi) : \text{the proportional demand for insurance for risk} \ \mu \ \text{at premium} \ \pi. \]
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- \frac{\partial}{\partial \pi} d(\mu, \pi) < 0 : a decreasing function of premium.
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- \( \frac{\partial}{\partial \pi} d(\mu, \pi) < 0 : \) a decreasing function of premium.
- \( d(\mu_1, \pi) < d(\mu_2, \pi) : \) the proportional demand is greater for the higher risk-group.

Demand elasticity:

\[ \epsilon(\mu, \pi) = -\frac{\partial}{\partial \pi} \frac{d(\mu, \pi)}{d(\mu, \pi)} \]

i.e. sensitivity of demand to premium changes.
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Demand elasticity: \( \epsilon(\mu, \pi) = - \frac{\partial d(\mu, \pi)}{d(\mu, \pi)} / \frac{\partial \pi}{\pi} \) i.e. sensitivity of demand to premium changes.
Demand Function

Iso-elastic demand function

\[ \varepsilon(\mu, \pi) = \lambda, \text{ i.e. constant} \]  

(5)
Iso-elastic demand function

\[ \epsilon(\mu, \pi) = \lambda, \text{ i.e. constant} \]  \hspace{1cm} (5)

\[ d(\mu, \pi) = \tau \left[ \frac{\pi}{\mu} \right]^{-\lambda}. \]  \hspace{1cm} (6)
Iso-elastic demand function

$\tau = 1, \mu = 0.01, \lambda = 0.4, 0.8$ and $1.2$
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$$d(\mu_1, \pi_e)(\pi_e - \mu_1)p_1 + d(\mu_2, \pi_e)(\pi_e - \mu_2)p_2 = 0. \quad (7)$$
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“Profit” from low risk-group = “Loss” from high risk-group

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If $\lambda_1 = \lambda_2 = \lambda$, 

\[ \text{(Fair-premium demand-share)} \]
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If $\lambda_1 = \lambda_2 = \lambda$,

$$\pi_e = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^\lambda + \alpha_2 \mu_2^\lambda},$$ \hspace{1cm} (8)

where

$$\alpha_i = \frac{\tau_i \rho_i}{\tau_1 \rho_1 + \tau_2 \rho_2}, \; i = 1, 2$$ \hspace{1cm} (9)

(Fair-premium demand-share)
Unique equilibrium premium

\[ p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04, \lambda_1 = \lambda_2 = 1 \]
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Results on adverse selection and loss coverage

Results on adverse selection

\[ S = \pi e^{\alpha_1 \mu_1} + \alpha_2 \mu_2. \]  

(10)
Results on adverse selection

Adverse Selection Ratio

\[ S = \frac{\pi e}{\alpha_1 \mu_1 + \alpha_2 \mu_2}. \]  \hspace{1cm} (10)

\[ \alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}, \quad i = 1, 2 \]

(Fair-premium demand-share)
Results: Adverse Selection Ratio (S)

\[ \rho_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; \rho_2 = 1000, \tau_2 = 1, \mu_2 = 0.04 \]
Results on loss coverage

Loss Coverage Ratio

\[ C = \frac{1}{\pi e^\lambda} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}. \] (11)
Results: Loss Coverage Ratio (C)

\[ p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.04 \]
Results: Loss Coverage Ratio (C)

\[ p_1 = 9000, \tau_1 = 1, \mu_1 = 0.01; p_2 = 1000, \tau_2 = 1, \mu_2 = 0.03, 0.04, 0.05, 0.08 \]
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Summary

When there is restriction on risk classification, a pooled premium $\pi$ is charged across all risk-groups. Adverse selection may not be a good measure. Loss coverage is an alternative metric. Adverse selection is not always a bad thing! A moderate level of adverse selection can increase loss coverage.
Summary

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Loss coverage is an alternative metric.

Adverse selection is not always a bad thing! A moderate level of adverse selection can increase loss coverage.
Further Research

- Other/more general demand e.g. $d(\mu, \pi) = \tau e^{1-(\frac{\pi}{\mu})^\lambda}$.
- Loose restriction on demand elasticities.
- Partial restriction on risk classification.


References


Questions?
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Thank you!