Content Offloading via D2D Communications Based on User Interests and Sharing Willingness

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***Abstract*—As a promising solution to offload cellular traffic, device-to-device (D2D) communication has been adopted to help disseminate contents. In this paper, the D2D offloading utility is maximized by proposing an optimal content pushing strategy based on the user interests and sharing willingness. Specifically, users are classified into groups by their interest probabilities and carry out D2D communications according to their sharing willingness. Although the formulated optimization problem is nonconvex, the optimal solution is obtained in closed-form by applying Karush-Kuhn-Tucker conditions. The theoretical and simulation results show that more contents should be pushed to the user group that is most willing to share, instead of the group that has the largest number of interested users.**

I. Introduction

According to the mobile traffic forecast report published by Cisco [1], the current cellular network infrastructure is facing an explosive traffic growth. An interesting observation has revealed that a large portion of the mobile traffic is composed of duplicate requests for the commonly interested contents [2]. Therefore, the device-to-device (D2D) communication is proposed to assist content dissemination [3]. Specifically, instead of serving the duplicate requested content to each interested user individually, the base station (BS) pushes it to a properly selected subset of users (seed users). For other non-seed users having interests, D2D links can be exploited to acquire the contents from the nearby seed users. If there are no seed users in proximity, they will acquire it from BS. By employing D2D communications in disseminating the content of common interest, the traffic load of the non-seed users can be offloaded from the cellular network [4].

The performance of this D2D assisted offloading scheme is highly dependent on the designed content pushing strategy [5]–[7], which determines the selection of seed users for pushing. A number of approaches have been proposed to address this pushing strategy design problem [8]–[10]. Since the content to be offloaded is based on the common interests of users, the current pushing strategies were investigated according to the distribution of heterogeneous user interests to achieve the maximum offloading utility [9]–[11]. In previous work, it was assumed that the users will always accept the pushing from BS unconditionally [9]–[12]. However, in fact, only when the users are interested in the pushed content they will accept the pushing. Otherwise, the pushing request will be ignored or refused. In addition, it is worth to mention that, in [9], [10], the seed users were assumed to be altruistic to share with others. The results in [11] showed that this assumption was unpractical. Moreover, if all the seed users do not want to share with others, there will be no content offloaded via D2D links. As a result, it becomes important to also consider the sharing willingness of users in pushing strategy design.

The successful D2D sharing was greatly influenced by the sharing willingness of the seed users, but the probability of willing to share was assumed to be a fixed value for all users in [12]. In fact, there are always some users who are more willing to share than others [13], [14], which leads to the different levels of sharing willingness. Since tracking each user's sharing willingness costs high consumption of resources such as memory and power, the sharing willingness of users was estimated in group manner in [15]. The difference in the sharing willingness of user groups adds another dimension to the offloading problem, and it further complicates the pushing strategy design. Therefore, based on the user interests and sharing willingness, the optimal pushing strategy is investigated in this paper to maximize the offloading utility.

In this paper, according to different user interests, users are classified into groups and have different sharing probabilities due to different levels of sharing willingness. BS selects the seed users from each group under a pushing probability, but only the interested users will accept pushing. Furthermore, the content sharing via D2D links for non-seed users is affected by the share probability of the seed users that have the content. The optimization problem is formulated to optimize the pushing probability of each group for maximizing the offloading utility, which is defined as the average number of users that can get the interested content via D2D links per unit area. Though the problem is nonconvex, the global optimal solution is derived in closed-form by applying the Karush-Kuhn-Tucker (K.K.T) conditions. Finally, the offloading performance obtained by the optimal pushing strategy is illustrated by the simulation results. It is shown that more content should be pushed to the users with high sharing willingness for them to carry out D2D communications.

II. System Model

Consider a cellular network where users can share their cached contents via D2D links with others. As shown in Fig. 1, the D2D transmission distance is denoted by the radius r. The reference content for dissemination is first pushed by the BS to the selected seed users, which are represented by the shaded

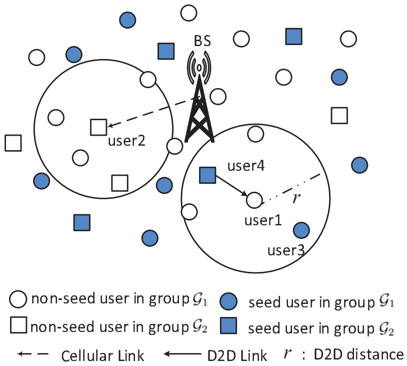

Fig. 1: An example of D2D assisted offloading with M=2

circles and squares. If there are seed users in proximity, the non-seed users then download the reference content via D2D links. Otherwise, they will turn to BS for content downloading.

According to their interests to the reference content, users are classified into M disjoint groups. Let $\mathcal{M} = \{1, 2, \cdots, M\}$ represents the set of groups, and the group m is expressed by $\mathcal{G}_m$. For the reference content, w_m is defined as the interest probability of group $\mathcal{G}_m$. w_m stands for the probability that a user in group $\mathcal{G}_m$ want the content, where $0 \le w_m \le 1, m \in \mathcal{M}$. That is to say, w_m denotes that how much the users in group $\mathcal{G}_m$ is interested in the reference content. Another crucial feature is the users' sharing willingness, which is evaluated by the share probability of a group [14], [15]. Let ρ_m denote the share probability of group $\mathcal{G}_m$. ρ_m shows the probability that a user in group $\mathcal{G}_m$ is willing to share content with others. Suppose that the distribution of users' locations in each group is independent of the other group, and it is modeled as a Poisson Point Process (P.P.P) [16]. The density of user in group $\mathcal{G}_m$ is denoted by $\lambda_m, m \in \mathcal{M}$. Besides, it is defined that $t_m = \lambda_m w_m$ is the interested density of group $\mathcal{G}_m$, which means the average number of users from group $\mathcal{G}_m$ in a unit area that is interested in the reference content.

As shown in Fig. 1, users are divided into $\mathcal{G}_1$ and $\mathcal{G}_2$ according to their interests, which are represented by the circles and squares. The non-seed users can get the reference content from the nearby seed users belonging to same or another group. For example, user 1 can get the content from user 3 with probability ρ_1, and from user 4 with probability ρ_2 via D2D links. Since user 2 do not have seed users in proximity, therefore, it requests the content from BS.

It is assumed that the selection of seed users is done randomly by the BS. The probability that a user in group $\mathcal{G}_m$ that will be selected for pushing is denoted by c_m. Let l_m denotes the density of seed users in group $\mathcal{G}_m$ that accepts the content pushing. Under the P.P.P model, l_m is given as

$$l_m = \lambda_m w_m c_m = t_m c_m. \tag{1}$$

The w_m in (1) shows that only the interested users will accept the pushing. Similarly, the density of non-seed users in group $\mathcal{G}_m$ who are also interested in the reference content is denoted as n_m, which is obtained as

$$n_m = \lambda_m w_m (1 - c_m) = t_m (1 - c_m). \tag{2}$$

Since these non-seed users are also interested in the reference content, they will request the content from BS or the nearby seed-users having the content. Let $\mathbb{P}$ denote the D2D probability. It means that, for a non-seed user, there is at least one seed-users in proximity that have the content and will share it via D2D transmission. According to the P.P.P model, in area A, the probability that there are n users is calculated as

$$P(n, A) = \frac{(\lambda A)^n}{n!} \exp(-\lambda A), \tag{3}$$

where λ is the user density in the bounded area A. Let L denote the density of the seed-users that have the content and will share it via D2D links. It is obtained as

$$L = \sum_{m=1}^{M} \rho_m l_m = \sum_{m=1}^{M} t_m \rho_m c_m. \tag{4}$$

Based on (3), the D2D probability $\mathbb{P}$ is obtained as

$$\mathbb{P} = 1 - P(0, \pi r^2) = 1 - \exp(-\pi r^2 L). \tag{5}$$

where $P(0, \pi r^2)$ is the probability that no users will share content to a non-seed user within D2D range r.

III. Problem Formulation

To characterize the offloading performance in this system, we first define the system offloading utility denoted by $\mathbb{U}$, which is

$$\mathbb{U} = \sum_{m=1}^{M} n_m \mathbb{P}. \tag{6}$$

From (6), $\mathbb{U}$ can be regarded as the average number of interested users per unit area that can get the reference content via D2D links, which is similar with the offloading performance measurement in [17]. Substituting (2) and (5) in (6), we have

$$\mathbb{U} = \left(\sum_{m=1}^{M} t_m (1 - c_m) \right) \left(1 - \exp\left(-B \sum_{k=1}^{M} t_k \rho_k c_k \right) \right), \tag{7}$$

where B represents the D2D area, i.e. $B = \pi r^2$.

Given t_m and ρ_m in each group, $\mathbb{U}$ reflects the offloading ability achieved by the pushing probability c_m in each group. For instance, If the content is pushed to every user, i.e. $c_m = 1$ for all m, there is no D2D offloading. Moreover, if $c_m = 0$ for all m, there are no seed user in cell, and every interested user will request the BS for downloading. Consequently, $\mathbb{U} = 0$ in both cases. Therefore, the optimal pushing probability c_m for each group $\mathcal{G}_m$ need to be investigated. The optimization problem is formulated as

$$\mathcal{P}1 : \max_{\boldsymbol{c}} \quad \mathbb{U}(\boldsymbol{c}), \tag{8a}$$

$$\text{s.t.} \quad 0 \le c_m \le 1, m \in \mathcal{M}. \tag{8b}$$

In (8a), vector $\boldsymbol{c} = [c_1, c_2, \cdots, c_M]$ represents the pushing strategy of the system. The constraint (8b) ensures that $c_m, \forall m \in \mathcal{M}$ is a valid probability. Since the pushing will be refused when there is no one interested in the reference content, it is assumed that $t_m \neq 0, \forall m \in \mathcal{M}$ in the following analysis.

IV. Solution Analysis

In this section, the solution for problem $\mathcal{P}1$ will be analyzed in two cases; the *different* sharing and *partial-same* sharing case. In *different* sharing case, different groups has different sharing probabilities, i.e $\rho_k \neq \rho_m, \forall m \neq k$. In *partial-same* case, part of groups have the same share probability. The case that each group has the same share probability is included in the *partial-same* case, and thus it is not discussed separately.

A. Different Sharing

By checking the Hessian matrix, it is easy to know that problem $\mathcal{P}1$ is nonconvex. Therefore it is very hard to directly get the optimal pushing strategy, which is denoted by the vector $\boldsymbol{c}^* = [c_1^*, c_2^*, \cdots, c_M^*]$.

However, we can still derive the optimal solution $\boldsymbol{c}^*$ in closed-form by the following proof line. First, a special structure of the optimal solution $\boldsymbol{c}^*$ is revealed by Proposition 4.1. Second, the special structure of $\boldsymbol{c}^*$ is associated with the order of sharing probabilities in the Proposition 4.2. Then, by applying the special structure of $\boldsymbol{c}^*$ in K.K.T conditions, one case of the optimal solutions is given in Theorem 4.1. Finally, the general closed-form expression of $\boldsymbol{c}^*$ is summarized in Theorem 4.2.

***Proposition** 4.1:* There is at most one group in $\boldsymbol{c}^*$ that are in the range $0 < c_i^* < 1$, and for all the other groups, i.e. $\forall j \neq i$, $c_j^* = 0$ or $c_j^* = 1$.

Proof: The proof is provided in Appendix A. ■

Furthermore, the relationship between c_m and ρ_m is investigated in the following proposition.

***Proposition** 4.2:* When $0 < c_m^* < 1$ holds for group $\mathcal{G}_m$, if group $\mathcal{G}_i$ has the share probability that $\rho_i < \rho_m$, then $c_i^* = 0$; If group $\mathcal{G}_j$ has the share probability that $\rho_j > \rho_m$, then $c_j^* = 1$.

Proof: The proof is provided in Appendix B. ■

Besides, the following three corollaries can be inferred from the proof of Proposition 4.2.

***Corollary** 4.1:* If $c_m^* = 0$ holds for group $\mathcal{G}_m$, group $\mathcal{G}_i$ with $\rho_i < \rho_m$ has the optimal pushing probability that $c_i^* = 0$.

***Corollary** 4.2:* If $c_m^* = 1$ holds for group $\mathcal{G}_m$, group $\mathcal{G}_j$ with $\rho_j > \rho_m$ has the optimal pushing probability that $c_j^* = 1$.

The proof of Corollary 4.1 and Corollary 4.2 is similar with Proposition 4.2, so that they are omitted.

***Corollary** 4.3:* It is assumed that the M groups are sorted in the rising order of sharing probabilities, i.e. $\rho_1 < \cdots < \rho_M$. For group $\mathcal{G}_m$, if $0 < c_m^* < 1$, then

$$c_m^* = \frac{1}{B\rho_m t_m}(B\Theta_m + 1 - \mathcal{W}(\exp(B\Psi_m + B\rho_m\Theta_m + 1))) \quad (9)$$

where $\Theta_m = \sum_{i=1}^{m} t_i$, $\Psi_m = \sum_{j=m+1}^{M} \rho_j t_j$. $\mathcal{W}$ is the Lambert-W function [18].

Proof: The proof is provided in Appendix C. ■

However, a key problem still remaining is to find the special group $\mathcal{G}_m$ with $0 < c_m^* < 1$. To solve this problem, Proposition 4.3 is introduced to show the uniqueness of the sufficient and necessary conditions in Theorem 4.1.

***Proposition** 4.3:* It is assumed that groups are sorted in the order that $\rho_1 < \cdots < \rho_M$. There is at most one group that satisfies the following conditions simultaneously.

$$\begin{cases} 1 + B\rho_m \sum_{i=1}^{m} t_i > \exp\left(B \sum_{j=1+m}^{M} t_j\rho_j\right), & (10) \\ 1 + B\rho_m \sum_{i=1}^{m-1} t_i < \exp\left(B \sum_{j=m}^{M} t_j\rho_j\right). & (11) \end{cases}$$

Proof: The proof is provided in Appendix D. ■

***Theorem** 4.1:* When M groups are sorted in the rising order of ρ_m, i.e. $\rho_1 < \cdots < \rho_M$, the optimal solution to problem $\mathcal{P}1$ is $\boldsymbol{c}^* = [\underbrace{0, \cdots, 0}_{m-1}, c_m^*, \underbrace{1, \cdots, 1}_{M-m}]$, where c_m^* is given by (9), *if and only if* (10) and (11) hold simultaneously.

Proof: The proof is provided in Appendix E. ■

Theorem 4.1 shows the sufficient and necessary conditions for the optimal pushing strategy. Moreover, the conditions in Theorem 4.1 also ensure that (9) is feasible. Finally, the uniqueness proved by the Proposition 4.3 is matched with Proposition 4.1, which shows that the optimal pushing strategy in Theorem 4.1 is exclusive.

For simplicity, we define two functions as follows,

$$f^1(m) = 1 + B\rho_m \sum_{i=1}^{m} t_i - \exp\left(B \sum_{j=1+m}^{M} t_j\rho_j\right), \quad (12)$$

$$f^0(m) = \exp\left(B \sum_{j=m}^{M} t_j\rho_j\right) - B\rho_m \sum_{i=1}^{m-1} t_i - 1. \quad (13)$$

From Theorem 4.1, we can infer the following corollary.

***Corollary** 4.4:* For group $\mathcal{G}_m$, if $f^1(m) \leq 0$, then $c_m^* = 0$; For group $\mathcal{G}_m$, if $f^0(m) \leq 0$, then $c_m^* = 1$.

The proofs of Corollary 4.4 is similar to the "*if*" part of Theorem 4.1, and thus omitted for brevity.

Based on the foregoing analysis, at the *different* sharing case, a closed-form optimal solution of the nonconvex problem $\mathcal{P}1$ is summarized in the following theorem.

***Theorem** 4.2:* Assuming that M groups are sorted in the order $\rho_1 < \cdots < \rho_M$, the optimal solution of problem $\mathcal{P}1$ is

$$\boldsymbol{c}^* = \begin{cases} [\underbrace{0, \cdots, 0}_{m}, \underbrace{1, \cdots, 1}_{M-m}], & \begin{cases} f^1(m) \leq 0, \\ f^0(m+1) \leq 0. \end{cases} \\ [\underbrace{0, \cdots, 0}_{m-1}, c_m, \underbrace{1, \cdots, 1}_{M-m}], & \begin{cases} f^1(m) > 0, \\ f^0(m) > 0. \end{cases} \end{cases} \quad (14)$$

where c_m is given by (9).

Proof: The first case is obtained from Corollary 4.4, and second case is obtained from Theorem 4.1. ■

B. Partial-same Sharing

In the *Partial-same* sharing case, the optimal pushing strategy is proved to be not unique by the following proposition. However, a special case of the alternative optimal pushing strategies is given in the proof of Proposition 4.4.

***Proposition** 4.4:* If n groups have the same share probability, where $2 \leq n \leq M$, the optimal pushing probabilities of these n groups are not unique.

Proof: The proof is provided in Appendix F. ■

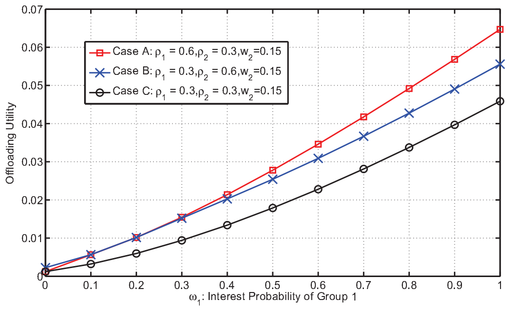

Fig. 2: Offloading performance versus w_1

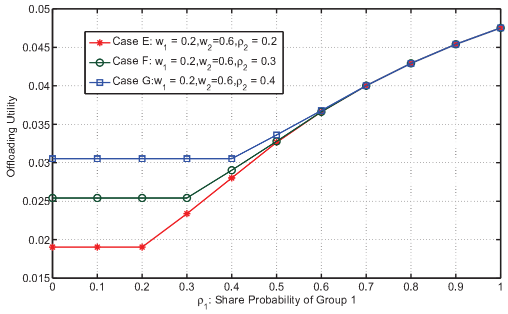

Fig. 3: Offloading performance versus ρ_1

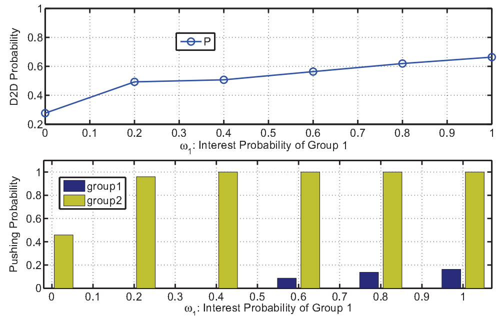

Fig. 4: D2D probability and pushing strategy versus w_1 in Case B

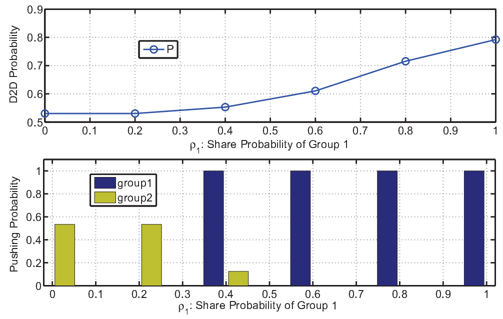

Fig. 5: D2D probability and pushing strategy versus ρ_1 in Case F

V. Simulation Results

In this section, the offloading utility achieved by the optimal pushing strategy is shown in simulation results, and the impacts of interests and sharing willingness on the optimal pushing strategy are also investigated. It is observed that the proposed pushing strategy can be easily extended to multiple contents, so only a reference content is adopted in the simulation. The D2D communication range is set to be $r = 5m$. The user density λ_m of each group is set to be 0.1 users per m^2. The total number of the user group in the simulation is $M = 2$, and the two groups are named as group 1 and group 2. The interest probability and share probability of group 1 are denoted by w_1 and ρ_1, respectively. Similarly, w_2 and ρ_2 represent the interest and share probability of group 2.

Fig. 2 shows the system offloading utility versus the interest probability of group 1 in 3 different cases. It is shown that the offloading utility in all the considered cases increases with w_1. The reason is that the number of the interested users in group 1 increases by increasing w_1. Therefore, more non-seed users will get the reference content via D2D links, and the offloading utility increases. In Fig. 2, Case C has the lowest offloading utility because the ρ_1 and ρ_2 in this case are smallest. Case B has a larger start point compared with others, because the ρ_2 in this case are largest. However, the offloading utility in Case B is gradually less than Case A especially when w_1 is much larger than w_2. As w_1 increases, most of the interested users in Case B are from group 1, which has a low share probability, i.e. $\rho_1 = 0.3$. While in Case A, most of the interested users are from the group with high share probability, i.e. $\rho_1 = 0.6$. Therefore, the seed users in Case A are more willing to carry out D2D, and thus the offloading utility of Case A is larger than Case B.

Fig. 3 shows the offloading utility versus the share probability of group 1 in 3 different cases. It is observed that the offloading utility increases only when $\rho_1 > \rho_2$ for all cases. The reason is that the pushing is only made to group 2 when $\rho_1 < \rho_2$. Therefore, increasing ρ_1 in this interval will not increase the offloading utility. When $\rho_1 > \rho_2$, the offloading utility increases with ρ_1 in all cases, since the users in group 1 start receiving pushing from BS. Therefore, it makes the non-seed users easier to find a seed user who is willing to share. In Fig. 3, when $w_1 < 0.5$, the ordering of the offloading utilities for Case E, Case F and Case G is determined by the ρ_2 in each case. However, the offloading utilities in all cases approach to the same with the growth of ρ_1. This is because the offloading performance is dominated by the group with high sharing willingness, i.e. group 1.

Fig. 4 shows the D2D probability $\mathbb{P}$ and the associated optimal pushing strategy versus w_1 in Case C. D2D share probability increases with w_1 due to that more pushing efforts are made. The slope of $\mathbb{P}$ becomes slow when $w_1 > 0.2$ because the interested users in the high sharing group, i.e. group 2, have already been pushed with content. The increased pushing efforts are made to users with low sharing willingness, i.e. group 1. When w_1 increases, the pushing probability in group 2 increases to 1 due to its higher share probability. When $w_1 = 0.4$, the number of seed users in group 2 is not large enough to cope with the increased number of interested non-seed users in group 1. Consequently, this leads to the increase of the pushing probability in group 1.

Fig. 5 shows the D2D probability $\mathbb{P}$ and the optimal pushing strategy versus ρ_1 in Case F. $\mathbb{P}$ stays the same when $\rho_1 < \rho_2$ due to the same pushing strategy in this interval. When $\rho_1 > \rho_2$, $\mathbb{P}$ increases with ρ_1. The reason is explained by the changes in the optimal pushing strategy. When $\rho_1 < \rho_2$, the content is only pushed to group 2 due to its high sharing willingness. Only when $\rho_1 > \rho_2$, the content is pushed to group 1. However, it is interesting to see that all users in group 1 are pushed with content, while the pushing probability in group 2 decreases with the growth of ρ_1. This is because the seed users from group 1 are more willing to share so that the pushing effort in group 2 can be saved. Although group 2 has the largest number of interested users, it does not have the largest pushing probability.

VI. Conclusion

In this paper, based on the heterogeneous user interests and different levels of sharing willingness, the content pushing strategy has been investigated to maximize the D2D offloading utility. Fortunately, the optimal solution to the nonconvex problem has been obtained in closed-form by applying K.K.T conditions. It is observed that the pushing probability for the group with the largest number of interested users depends on other groups' sharing behaviors. In other words, if there are plenty of seed users from other groups willing to share, no content should be pushed to this group. Furthermore, it is more crucial to push contents to the users who are more willing to share for them carrying out D2D communications.

Appendix A
Proof of Proposition 4.1

It is assumed that c_i^* and c_j^* are both larger than zero and less than one, i.e. $0 < c_i^* < 1$ and $0 < c_j^* < 1$, for two different groups $\mathcal{G}_i$ and $\mathcal{G}_j$. Since the $\mathbb{U}(\boldsymbol{c})$ achieves the maximum at $\boldsymbol{c}^*$, $\boldsymbol{c}^*$ is the solution to the following equation sets,

$$\frac{\partial \mathbb{U}(\boldsymbol{c})}{\partial c_i} = 0, \frac{\partial \mathbb{U}(\boldsymbol{c})}{\partial c_j} = 0. \tag{A.1}$$

Substituting (7) into (A.1), we have

$$1 + B\rho_i \sum_{m=1}^{M} t_m(1 - c_m) = \exp\left(B \sum_{k=1}^{M} t_k \rho_k c_k\right), \tag{A.2}$$

$$1 + B\rho_j \sum_{m=1}^{M} t_m(1 - c_m) = \exp\left(B \sum_{k=1}^{M} t_k \rho_k c_k\right). \tag{A.3}$$

For the *different* sharing case, we have $\rho_i \neq \rho_j, \forall i \neq j$. Consequently, there is no solution to (A.1). Therefore, at most one group $\mathcal{G}_i$ in the optimal pushing strategy has the pushing probability $c_i^* \in (0, 1)$. ■

Appendix B
Proof of Proposition 4.2

For all the feasible solutions to problem $\mathcal{P}1$, the linear independence constraint qualification (LICQ) [19] is satisfied. Therefore, the LICQ constrain qualification also applies at the global optimum, which means that the Karush-Kuhn-Tucker (K.K.T) conditions are necessary conditions for the global optimum. The following proof is based on this conclusion. The Lagrangian associated with $\mathcal{P}1$ is

$$\mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{c}) = \left(\sum_{m=1}^{M} t_m(1 - c_m)\right)\left(1 - \exp\left(-B \sum_{k=1}^{M} t_k \rho_k c_k\right)\right) + \sum_{m=1}^{M} \alpha_m c_m - \sum_{m=1}^{M} \beta_m (c_m - 1). \tag{B.1}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_M]$, $\boldsymbol{\beta} = [\beta_1, \beta_2, \cdots, \beta_M]$. α_m and β_m are the non-negative dual variables which are associated with the constraints $c_m \geq 0$ and $c_m - 1 \leq 0$. Thus, the following K.K.T conditions are obtained.

$$\frac{\partial \mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{c})}{\partial c_m} = 0, \forall m \in \mathcal{M}, \tag{B.2}$$

$$\alpha_m c_m = 0, \forall m \in \mathcal{M}, \tag{B.3}$$

$$\beta_m (c_m - 1) = 0, \forall m \in \mathcal{M}, \tag{B.4}$$

$$\alpha_m \geq 0, \beta_m \geq 0, \forall m \in \mathcal{M}, \tag{B.5}$$

$$0 \leq c_m \leq 1, \forall m \in \mathcal{M}. \tag{B.6}$$

From (B.2), we have

$$\exp\left(-B \sum_{k=1}^{M} t_k \rho_k c_k\right)\left(B\rho_m \sum_{m=1}^{M} t_m(1 - c_m) + 1\right) - 1 = \frac{1}{t_m}(\beta_m - \alpha_m), \forall m \in \mathcal{M}. \tag{B.7}$$

At the global optimum, if $0 < c_m^* < 1$, then $\alpha_m = 0$, and $\beta_m = 0$. Therefore, we have the following equation,

$$B\rho_m \sum_{m=1}^{M} t_m(1 - c_m) + 1 = \exp\left(B \sum_{k=1}^{M} t_k \rho_k c_k\right). \tag{B.8}$$

For the group $\mathcal{G}_i$, the associated K.K.T condition is $\frac{\partial \mathcal{L}}{\partial c_i} = 0$, which is written as,

$$\exp\left(-B \sum_{k=1}^{M} t_k \rho_k c_k\right)\left(B\rho_i \sum_{m=1}^{M} t_m(1 - c_m) + 1\right) = \frac{1}{t_i}(\beta_i - \alpha_i) + 1. \tag{B.9}$$

If $\rho_i < \rho_m$, combing (B.9) with (B.8), it can be inferred that

$$\beta_i - \alpha_i < 0. \tag{B.10}$$

Given the condition (B.5) , we have $\alpha_i > 0$ and $c_i^* = 0$ for group $\mathcal{G}_i$. Similarly, for group $\mathcal{G}_j$, if $\rho_j > \rho_m$, from the K.K.T condition that $\frac{\partial \mathcal{L}}{\partial c_j} = 0$, it is inferred that

$$\beta_j - \alpha_j > 0. \tag{B.11}$$

Therefore, $\beta_j > 0$ and $c_j^* = 1$. ■

Appendix C
Proof of Corollary 4.3

Suppose that the M groups are sorted in the order that $\rho_1 < \cdots < \rho_M$. If the groups $\mathcal{G}_i$ has the index that $1 \leq i \leq m - 1$, then $c_i^* = 0$. If the groups $\mathcal{G}_j$ has the index that $m + 1 \leq j \leq M$, then $c_j^* = 1$. In this case, (B.8) is rewritten as

$$B\rho_m \left(\sum_{i=1}^{m} t_i - t_m c_m^*\right) + 1 = \exp\left(\sum_{j=m+1}^{M} B t_j \rho_j + B t_m \rho_m c_m^*\right). \tag{C.1}$$

The following equation can be employed to solve the equation (C.1)

$$e^{ax+b} = cx + d \rightarrow x = -\frac{d}{c} - \frac{1}{a}\mathcal{W}\left(-\frac{a}{c}e^{b - \frac{ad}{c}}\right), \tag{C.2}$$

where $\mathcal{W}$ is the Lambert-W function. Then (9) is obtained. ■

Appendix D
Proof of Proposition 4.3

The uniqueness is proved by contradiction. Suppose that two groups $\mathcal{G}_m$ and $\mathcal{G}_k, m \neq k$ both satisfy (11) and (10) at the same time. Without loss of generality, we assume that $\rho_k > \rho_m$. In this case, $k - 1 \geq m$ due to the fact that M

groups are sorted in the order $\rho_1 < \cdots < \rho_M$. For group $\mathcal{G}_k$, the condition (11) is

$$1 + B\rho_k \sum_{i=1}^{k-1} t_i < \exp\left(B \sum_{j=k}^{M} t_j \rho_j\right). \tag{D.1}$$

However, for the left hand side (LHS) of (D.1), the following inequality holds,

$$1 + B\rho_m \sum_{i=1}^{m} t_i < 1 + B\rho_k \sum_{i=1}^{k-1} t_i. \tag{D.2}$$

For the right hand side (RHS) of (D.1), it is obtained that

$$\exp\left(B \sum_{j=k}^{M} t_j \rho_j\right) \le \exp\left(B \sum_{j=m+1}^{M} t_j \rho_j\right). \tag{D.3}$$

Substituting (D.2) and (D.3) into (D.1), it is inferred that

$$1 + B\rho_m \sum_{i=1}^{m} t_i < \exp\left(B \sum_{j=m+1}^{M} t_j \rho_j\right). \tag{D.4}$$

However, it is easy to know that (D.4) is contradictory to (10) for group $\mathcal{G}_m$.

Similarly, we can prove the contradiction when $\rho_k < \rho_m$. Therefore, at most one group can satisfy (10) and (11) at the same time in the *different* sharing case. ■

APPENDIX E
PROOF OF THEOREM 4.1

The proofs for both the "*if*" part and the "*only if*" part are based on contradiction.

We first consider the proof for the "*if*" part. If (10) and (11) hold for group $\mathcal{G}_m$ at the same time, it is assumed that the optimal pushing probability of group $\mathcal{G}_m$ is either $c_m^* = 1$ or $c_m^* = 0$. In the following part, it is shown that the assumption $c_m^* = 1$ contradicts (11) and $c_m^* = 0$ contradicts (10).

If we assume that $c_m^* = 1$, then $c_j^* = 1$ for groups $\mathcal{G}_j$ with $m+1 \le j \le M$ according to corollary 4.2. Therefore, the optimal pushing strategy can be written as $\boldsymbol{c}^* = [c_1^*, \cdots, c_{m-1}^*, \underbrace{1, \cdots, 1}_{M-m+1}]$. The K.K.T condition (B.7) is reduced to

$$\exp\left(-B\sum_{i=1}^{m-1} t_i\rho_i c_i^* - B\sum_{j=m}^{M} t_j\rho_j\right)\left(B\rho_m\sum_{i=1}^{m-1} t_i(1-c_i^*)+1\right) = \frac{1}{t_m}(\beta_m - \alpha_m) + 1. \tag{E.1}$$

Moreover, since $c_m^* = 1$, then $\alpha_m = 0$ and $\beta_m \ge 0$. The following inequality can be obtained from (E.1).

$$B\rho_m\sum_{i=1}^{m-1} t_i(1-c_i^*) + 1 \ge \exp\left(B\sum_{i=1}^{m-1} t_i\rho_i c_i^* + B\sum_{j=m}^{M} t_j\rho_j\right). \tag{E.2}$$

Since $0 \le c_i \le 1$, from the RHS of (E.2), it is obtained that

$$\exp\left(B\sum_{i=1}^{m-1} t_i\rho_i c_i^* + B\sum_{j=m}^{M} t_j\rho_j\right) \ge \exp\left(B\sum_{j=m}^{M} t_j\rho_j\right). \tag{E.3}$$

From the LHS of (E.2), the following inequality is inferred.

$$B\rho_m \sum_{i=1}^{m-1} t_i + 1 \ge B\rho_m \sum_{i=1}^{m-1} t_i(1-c_i^*) + 1. \tag{E.4}$$

By combining (E.4) and (E.3) with (E.2), the following inequality is obtained

$$B\rho_m \sum_{i=1}^{m-1} t_i + 1 \ge \exp\left(B \sum_{j=m}^{M} t_j\rho_j\right), \tag{E.5}$$

, which contradicts condition (11).

If we assume that $c_m^* = 0$, then $c_i^* = 0$ for groups $\mathcal{G}_i$ with $1 \le i \le m-1$ according to corollary 4.1. Therefore, the optimal pushing strategy can be written as $\boldsymbol{c}^* = [\underbrace{0, \cdots, 0}_{m}, c_{m+1}^*, \cdots, c_M^*]$. In addition, the dual variables associated with groups $\mathcal{G}_i$ are obtained as $\alpha_m \ge 0$ and $\beta_m = 0$, which results from the fact that $c_m^* = 0$.

Consequently, the following inequality is inferred from the associated K.K.T condition $\frac{\partial \mathcal{L}}{\partial c_m} = 0$,

$$B\rho_m\sum_{j=m+1}^{M} t_j(1-c_j^*) + B\rho_m\sum_{i=1}^{m} t_i + 1 \le \exp\left(B\sum_{j=m}^{M} t_j\rho_j c_j^*\right). \tag{E.6}$$

Due to the fact that $0 \le c_j \le 1$, the following inequalities are inferred from the RHS and LHS of (E.6), respectively.

$$\exp\left(B \sum_{j=m}^{M} t_j\rho_j c_j^*\right) \le \exp\left(B \sum_{j=m}^{M} t_j\rho_j\right), \tag{E.7}$$

$$B\rho_m \sum_{j=m+1}^{M} t_j(1-c_j^*) + B\rho_m\sum_{i=1}^{m} t_i + 1 \ge B\rho_m\sum_{i=1}^{m} t_i + 1. \tag{E.8}$$

Combine (E.7) and (E.8) with (E.6), the following inequality is readily obtained.

$$B\rho_m \sum_{i=1}^{m} t_i + 1 \le \exp\left(B \sum_{j=m}^{M} t_j\rho_j\right). \tag{E.9}$$

Obviously, (E.9) contradicts (10).

Overall, if (10) and (11) hold for group $\mathcal{G}_m$ simultaneously, its optimal pushing probability is larger than 0 and less than 1. Meanwhile, the optimal solution to $\mathcal{P}1$ is $\boldsymbol{c}^* = [\underbrace{0, \cdots, 0}_{m-1}, c_m^*, \underbrace{1, \cdots, 1}_{M-m}]$, where c_m^* is given by (9).

Next, consider the "*only if*" part.

Suppose that $\boldsymbol{c}^* = [\underbrace{0, \cdots, 0}_{m-1}, c_m^*, \underbrace{1, \cdots, 1}_{M-m}]$ is the optimal pushing strategy, but (10) or (11) is not satisfied for group $\mathcal{G}_m$. Nevertheless, it is shown that there exists a different pushing strategy which achieves a larger offloading utility than $\boldsymbol{c}^*$.

The offloading utility achieved by $\boldsymbol{c}^*$ is

$$\mathbb{U}(\boldsymbol{c}^*) = \left(\sum_{i=1}^{m} t_i - t_m c_m^*\right)\left(1 - \exp\left(B\sum_{j=m+1}^{M} t_j\rho_j - Bt_m\rho_m c_m^*\right)\right). \tag{E.10}$$

We first assume that the condition (10) is not satisfied. Since (C.1) holds for c_m^*, we substitute the LHS of (C.1) into (E.10), the following result is obtained.

$$\mathbb{U}(\boldsymbol{c}^*) = \frac{B\rho_m L_m^2}{B\rho_m L_m + 1}. \tag{E.11}$$

where $L_m = \sum_{i=1}^{m} t_i - t_m c_m^*$.

When the condition (10) is not satisfied, it means

$$1 + B\rho_m \sum_{i=1}^{m} t_i \le \exp\left(B \sum_{j=1+m}^{M} t_j \rho_j\right). \tag{E.12}$$

A feasible pushing strategy is denoted as $\boldsymbol{c}^0 = [\underbrace{0, \cdots, 0}_{m}, \underbrace{1, \cdots, 1}_{M-m}]$. The objective value achieved by $\boldsymbol{c}^0$ is

$$\mathbb{U}(\boldsymbol{c}^0) = \left(\sum_{i=1}^{m} t_i\right)\left(1 - \exp\left(-B \sum_{j=m+1}^{M} t_j \rho_j\right)\right). \tag{E.13}$$

According to (E.12), it is inferred that

$$\mathbb{U}(\boldsymbol{c}^0) \ge \frac{B\rho_m \left(\sum_{i=1}^{m} t_i\right)^2}{B\rho_m \sum_{i=1}^{m} t_i + 1}. \tag{E.14}$$

For function $u(x) = \frac{ax^2}{ax+1}$, we have $u'(x) = \frac{a^2x^2+2ax}{(ax+1)^2} > 0$, $\forall x > 0$. Hence $u(x)$ is an increasing function with respect to x. Therefore, we have $\mathbb{U}(\boldsymbol{c}^0) > \mathbb{U}(\boldsymbol{c}^*)$ due to the fact that $\sum_{i=1}^{m} t_i > L_m$. This contradicts with the presumption that $\boldsymbol{c}^*$ is the optimal pushing strategy.

If the condition (11) is not satisfied, we can find another feasible solution denoted by $\boldsymbol{c}^1 = [\underbrace{0, \cdots, 0}_{m-1}, \underbrace{1, \cdots, 1}_{M-m+1}]$. It is easy to verify that $\mathbb{U}(\boldsymbol{c}^1) > \mathbb{U}(\boldsymbol{c}^*)$, which also contradicts that $\boldsymbol{c}^*$ is global optimum. The proof is similar with the procedure from (E.11) to (E.14), and it is omitted for brevity.

Overall, $\boldsymbol{c}^* = [\underbrace{0, \cdots, 0}_{m-1}, c_m^*, \underbrace{1, \cdots, 1}_{M-m}]$ is the optimal pushing strategy to problem $\mathcal{P}1$, where c_m^* is given by (9), *only if* the (10) and (11) hold simultaneously.

By combining the "*if*" part and the "*only if*" part, Theorem 4.1 is proved. ■

APPENDIX F
PROOF OF PROPOSITION 4.4

Sort the M groups in the order that $\rho_1 < \cdots < \rho_{k1} = \cdots = \rho_{kn} < \cdots < \rho_M$. Let $\mathcal{K} = \{k1, \cdots, kn\}$ stands for the set of the n groups with the same share probability. It is defined a new group 0 with share probability $\rho_0 = \rho_{k1} = \cdots = \rho_{kn}$, and its request density is represented by t_0. We have the following equations hold for the new group 0.

$$t_0 = \sum_{k=k1}^{kn} t_k, t_0 c_0 = \sum_{k=k1}^{kn} t_k c_k. \tag{F.1}$$

where c_0 is the pushing probability of group 0.

After replacing the groups in set $\mathcal{K}$ by group 0, problem $\mathcal{P}1$ is reduced to the *different* sharing case with $M - n + 1$ groups. The optimal solution can then be obtained directly from Theorem 4.2.

We denote the optimal pushing strategy of group 0 as c_0^*. However, for a given c_0^*, there exists multiple $(c_{k1}^*, \cdots, c_{kn}^*)$ that satisfy the following condition.

$$t_0 c_0^* = \sum_{k=k1}^{kn} t_k c_k^*. \tag{F.2}$$

For example, a special case is $c_{k1}^* = \cdots = c_{kn}^* = c_0^*$. ■

REFERENCES

[1] C. V. N. Index, "Global mobile data traffic forecast update 2014–2019 white paper, feb 2015," *See: http://www. cisco. com/c/en/us/solutions/collateral/service-provider/visual-networking-index-vni/white_paper_c11-520862. html*, 2015.

[2] M. Cha, H. Kwak, P. Rodriguez, Y.-Y. Ahn, and S. Moon, "I tube, you tube, everybody tubes: analyzing the world's largest user generated content video system," in *Proceedings of the 7th ACM SIGCOMM conference on Internet measurement*. ACM, 2007, pp. 1–14.

[3] X. Wang, Z. Sheng, S. Yang, and V. C. M. Leung, "Tag-assisted social-aware opportunistic device-to-device sharing for traffic offloading in mobile social networks," *IEEE Wireless Communications*, vol. 23, no. 4, pp. 60–67, August 2016.

[4] V. Sciancalepore, D. Giustiniano, A. Banchs, and A. Hossmann-Picu, "Offloading cellular traffic through opportunistic communications: Analysis and optimization," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 1, pp. 122–137, 2016.

[5] S. Ioannidis, A. Chaintreau, and L. Massoulie, "Optimal and scalable distribution of content updates over a mobile social network," in *INFOCOM 2009*. IEEE, 2009, pp. 1422–1430.

[6] J. Whitbeck, Y. Lopez, J. Leguay, V. Conan, and M. D. De Amorim, "Push-and-track: Saving infrastructure bandwidth through opportunistic forwarding," *Pervasive and Mobile Computing*, vol. 8, no. 5, pp. 682–697, 2012.

[7] R. Wang, X. Peng, J. Zhang, and K. B. Letaief, "Mobility-aware caching for content-centric wireless networks: modeling and methodology," *IEEE Communications Magazine*, vol. 54, no. 8, pp. 77–83, August 2016.

[8] F. Mezghani, R. Dhaou, M. Nogueira, and A. L. Beylot, "Offloading cellular networks through v2v communications: How to select the seed-vehicles?" in *2016 IEEE International Conference on Communications (ICC)*, May 2016, pp. 1–6.

[9] H. H. Cheng and K. C. J. Lin, "Source selection and content dissemination for preference-aware traffic offloading," *IEEE Transactions on Parallel and Distributed Systems*, vol. 26, no. 11, pp. 3160–3174, Nov 2015.

[10] Y. Li, M. Qian, D. Jin, P. Hui, Z. Wang, and S. Chen, "Multiple mobile data offloading through disruption tolerant networks," *IEEE Transactions on Mobile Computing*, vol. 13, no. 7, pp. 1579–1596, July 2014.

[11] Y. Guo, L. Duan, and R. Zhang, "Cooperative local caching and file sharing under heterogeneous file preferences," *arXiv preprint arXiv:1510.04516*, 2015.

[12] J. Hu, L. L. Yang, and L. Hanzo, "Delay analysis of social group multicast-aided content dissemination in cellular system," *IEEE Transactions on Communications*, vol. 64, no. 4, pp. 1660–1673, April 2016.

[13] J. Li, Q. Yang, P. Gong, and K. S. Kwak, "End-to-end multiservice delivery in selfish wireless networks under distributed node-selfishness management," *IEEE Transactions on Communications*, vol. 64, no. 3, pp. 1132–1142, 2016.

[14] Y. Li, G. Su, D. O. Wu, D. Jin, L. Su, and L. Zeng, "The impact of node selfishness on multicasting in delay tolerant networks," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 5, pp. 2224–2238, 2011.

[15] R. A. Shaikh, H. Jameel, B. J. d'Auriol, H. Lee, S. Lee, and Y.-J. Song, "Group-based trust management scheme for clustered wireless sensor networks," *IEEE Transactions on Parallel and Distributed Systems*, vol. 20, no. 11, pp. 1698–1712, 2009.

[16] S. N. Chiu, D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic geometry and its applications*. John Wiley & Sons, 2013.

[17] B. Chen, C. Yang, and A. F. Molisch, "Cache-enabled device-to-device communications: Offloading gain and energy cost," *arXiv preprint arXiv:1606.02866*, 2016.

[18] E. W. Weisstein, "Lambert w-function," 2002.

[19] R. G. Eustaquio, E. W. Karas, and A. A. Ribeiro, "Constraint qualifications for nonlinear programming," *Federal University of Parana*, 2008.