# Mobility Management in Small Cell Networks

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***Abstract*—The cell sojourn time and the handoff rate are considered as the main parameters in the mobility management of the cellular systems. In this paper, we address the mobility management in a two-tier heterogeneous network (HetNet) and propose a framework to study the impact of different system parameters on the handoff rate and the small cell sojourn time. In the proposed framework, the overlapping coverage among the small cells and the number of overlaps on the path of a reference user equipment (UE) are derived to obtain the actual time that the reference UE spends in each small cell during its movement from the starting point to the destination point. The results show the accuracy of the analysis in this paper in comparison to the analysis when ignoring the impact of the overlaps. The results also show the importance of considering the overlaps among the small cells in dense HetNets.**

## I. INTRODUCTION

The mobility management is important and essential in the cellular systems [1]. It is anticipated that the small cells with different frequency (e.g. high frequency) will be deployed densely in the cellular systems, and the mobility management in these systems will be very challenging and complex. Therefore, there is real need for developing an accurate model to evaluate the system performance and also to design a new system that is more suitable for the mobility management in the future. In the cellular networks, both the handoff rate and the cell sojourn time are considered as the main parameters in the mobility management. These are also used to estimate the UEs speed [2]–[4]. In the dense heterogeneous networks (HetNets), taking the overlaps among the small cells into account when modelling the small cells coverage is essential for accurate speed estimation and estimating the required resources at the cells from different tiers.

Recently, the handoff rate and the cell sojourn time in the cellular systems have received attention [2], [3], [5]–[7]. In [6], the handoff rate and the cell sojourn time in a one-tier network were investigated. However, the future cellular networks will include small cells with different frequency bands (e.g. high frequency small cells). When studying the mobility management in HetNets, modelling the cells has taken two main directions, Voronoi Tessellations cells (VTCs) assumption and regular shapes assumption (e.g. circle and hexagonal). Regarding the first direction, the VTCs assumption has been considered in the HetNet [2], [3]. In [2], the number of handoffs made during a time window was used to estimate the UE's speed in small cell networks. Stochastic geometry was used to derive approximations to the Cramer-Rao lower bound (CRLB) for the speed estimate of a UE. In [3], the UE's speed was estimated by using the cell sojourn time, where CRLB for the sojourn time-based speed estimation was analysed. Both [2], [3] assumed that the small cells in the network form VTCs which means that the whole network is covered by the small cells. However, a huge infrastructure will be required for the high frequency small cells to cover the whole network as the high frequency suffers from very large propagation loss [8]. Also this assumption restricts the analysis to a one-tier cellular system similar to [6]. Considering the second direction, both [5] and [7] assumed that the small cells in two-tier HetNets have regular shapes. [5] addressed the cell sojourn time in a two-tier HetNet where the small cells were assumed to have fixed hexagonal shapes in the network and the overlap coverage among the small cells was not taken into consideration. [7] investigated the mobility in a two-tier HetNet and also derived the sojourn time and cross-tier handoff rate. The overlaps among small cells of ellipse shape on a reference UE's path was also neglected in this work. Therefore, some of the intra-tier handoffs (handoffs among small cells due to overlaps) will be counted as cross-tier handoffs. Ignoring the overlaps will not only affect the accuracy of the handoff rate analysis but also affect the accuracy of the cell sojourn time as shown later in this paper.

The contribution of this paper is to propose a mobility framework with taking into consideration the overlaps among the small cells. The locations of the small cells base stations (SCBSs), the macro cells base station (MCBSs) and the way-points of a reference UE during its movement in the system are randomly distributed on the plane and form independent Poisson point processes (PPPs). The distribution of the SCSBs around a reference UE's path is studied and the small cells crossed by the reference UE during one movement is mapped into marked point process (MPP) on $\mathbb{R}^+$. This assumption is validated through simulations. Based on the above mapping, a novel framework is proposed to model the coverage of the small cells and the overlap coverage among these small cells on the reference UE's path in a dense HetNet. The small cell sojourn time and the handoff rate are derived by using the proposed framework.

The rest of this paper is structured as follows: Section II describes the system model and the mobility model. The small cells distribution is investigated in Section III. In Section IV, the small cell sojourn time in a two-tier HetNet is derived. In Section V, the total handoff rate is studied and derived. In Section VI, the system performance is shown by numerical and simulation results. Conclusions are drawn in Section VII.

## II. System Model

Consider a two-tier HetNet in Fig. 1. Each tier is characterized by the tuple $\{\lambda_k, \alpha_k\}$, where $k$ takes a value of 1 or 2, $\lambda_k$ is the base stations density and $\alpha_k$ is the path-loss exponent of the $k$th tier. The first-tier (macro cells) uses low frequency and the second tier (small cells) uses high frequency. It is assumed that the MCBSs and the SCBSs in the network are randomly distributed as independent PPP $\Phi_k$ with density $\lambda_k$ [9]. Due to a big difference in transmit powers and propagation losses between the first and the second tiers, load imbalance and minimization of the small cells coverage take place if the tier association is based on the maximum received power as it was assumed in [11]. Since there is no interference between the first tier and the second tier, it is assumed that the association to the small cells is based on the minimum received power from any small cell [12]. Therefore any UE will be associated to the $j$th small cell when the received power satisfies the condition below:

$$\rho_{min} \leq \rho_j \geq \max_{i,j \in \Phi_2} \rho_i \tag{1}$$

where $\rho_{min}$ is the minimum received power to consider the UEs in the small cells coverage, $\rho_j$ and $\rho_i$ are the received power from the $j$th small cell and the $i$th small cell respectively.

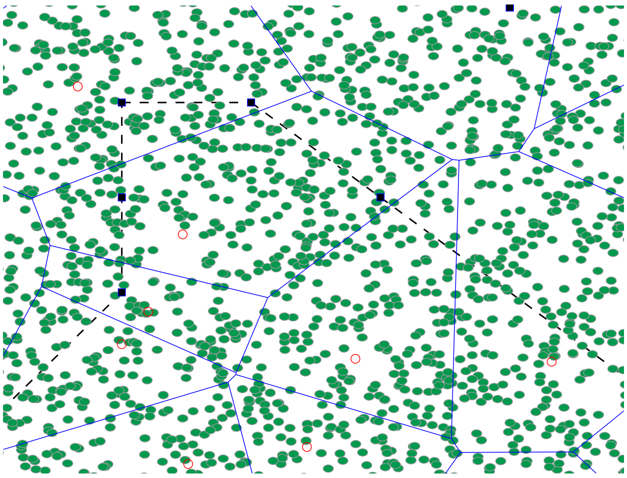

Fig. 1. System Model. Red circles represent the MCBSs, green dots represent the small cells coverage, black squares represent the waypoints, and the black dashed lines represent the reference UE's path between any two waypoints.

Based on the above condition, the probability that a reference UE ($\mathcal{U}_0$) connected to any small cell in the system can be obtained similar to [10]. Without loss of generality, when $\mathcal{U}_0$ is located at the origin and all the small cells transmit with the same power $p_2$, the probability of $\mathcal{U}_0$ being in the second tier coverage is obtained as:

$$\begin{aligned} \mathcal{A}_2 &= 1 - \mathbb{P}\big[\rho_0 < \rho_{min}\big] \\ &= 1 - \exp\Big( - \pi\lambda_2 \big(\frac{\rho_{min}}{\mathscr{L}_2 p_2}\big)^{\frac{-2}{\alpha_2}}\Big) \end{aligned} \tag{2}$$

where $\mathbb{P}[.]$ indicates the probability, $\rho_0$ is the received power form the nearest small cell and $\mathscr{L}_2$ is the path-loss of the high frequency at 1 meter.

The RWP proposed in [6] is considered in this paper. The movement trace of any UE is modelled by the quadruples $\{W_{l-1}, W_l, V_l, T_l\}_{l\in\mathbb{L}}$ where $l$ denotes the $l$th movement. During the $l$th movement, $W_{l-1}$ and $W_l$ denote the starting waypoint and destination waypoint respectively, and $V_l$ and $T_l$ denote the velocity of the UE and pause time respectively. The velocities $V_l$ are independent and identically distributed (i.i.d.) with distribution $P_V(.)$ and pause times $T_l$ are i.i.d with distribution $P_T(.)$. The waypoints $\{W_0, W_1, \cdots W_{l-1}, W_l, \cdots W_L\}$ are a homogeneous PPP $\Phi_w(l)$ with density $\lambda_w$, and the nearest point in $\Phi_w(l)$ is selected as the destination waypoint:

$$W_l = \arg\min_{w \in \Phi_w(l)} \parallel w - W_{l-1} \parallel \tag{3}$$

where $\parallel . \parallel$ indicates the Euclidean distance. Since the transition lengths are i.i.d [6], the expected value of the transition length during the $l$th period can be obtained as:

$$\mathbb{E}[\parallel W_l - W_{l-1} \parallel] = \frac{1}{2\sqrt{\lambda_w}} \tag{4}$$

## III. Small Cells

In this section, we investigate the coverage of small cells on $\mathcal{U}_0$'s path by taking into consideration that some overlaps may take place on the path. Since all transition lengths are i.i.d, for brevity we consider the $\mathcal{U}_0$'s path from $W_0$ to $W_1$ ($\mathcal{P}_0$). Consider $D_j$ as the vertical distance between $\mathcal{P}_0$ and the $j$th SCBS with radius $r_j$. The number of small cells crossed by $\mathcal{U}_0$ is given by:

$$N_0 = \sum_{j \in \Phi_2} \mathbf{1}(D_j \leq r_j) \tag{5}$$

where $\mathbf{1}(.)$ is the indicator function. It is assumed that $\mathscr{A}_2$ is the area surrounding $\mathcal{P}_0$, and any small cell will be crossed by $\mathcal{U}_0$ if its SCBS is located in this area. Since SCBSs are distributed as PPP, the number of SCBSs in $\mathscr{A}_2$ has a Poisson distribution. Therefore, the expected number of small cells crossed by $\mathcal{U}_0$ is obtained as:

$$\mathbb{E}[N_0] = \lambda_2 \mathscr{A}_2 \tag{6}$$

To enhance the tractability, the total coverage of cells have been assumed to have a regular shape (e.g. circle) for estimating the handoff and the sojourn time in the cellular systems [2], [3], [6]. This assumption holds in estimating the small cells coverage in the inter-frequency deployment [12], if the overlap coverage among the small cells is taken into consideration. Since the association to the second tier is based on $\rho_{min}$, the coverage of small cells is independent of the distance to the MCBSs. Therefore, it is assumed that the footage of any small cell forms a circle (including some overlaps). Given that the $i$th small cell is crossed by $\mathcal{U}_0$, the covered segment of $\mathcal{P}_0$ by the $i$th small cell with radius $r_i$ and at distance $\tau_i$ from $\mathcal{P}_0$ can be obtained as:

$$C_i = \sqrt{4r_i^2 - 4\tau_i^2}, \qquad \tau_i \leq r_i \tag{7}$$

The coverage of each small cell on $\mathcal{P}_0$ is a random variable depending on the small cell's radius and the distance from its SCBS to the path. The probability density function (PDF) of any small cell coverage on $\mathcal{P}_0$ is derived by using the transforming density function as shown below:

$$\begin{aligned} f_{C_i}(c) &= f_{\tau_i}(\tau(c_i))\,|\frac{d\tau}{dc}| \\ &\overset{(a)}{=} \frac{1}{r_i}\frac{d}{dc}(\sqrt{r_i^2-\frac{c_i^2}{4}}) \\ &= \frac{c}{4r_i^2\sqrt{1-\frac{c^2}{4r_i^2}}} \end{aligned} \tag{8}$$

where $0 \leq c \leq 2r_i$, (a) follows from Eq. (7) and $f_{\tau_i}(\tau)$ is the PDF of $\tau_i$ which is uniformly distributed in $[0, r_i]$. The expected value of any small cell coverage crossed by $\mathcal{U}_0$ is:

$$\mathbb{E}[C_i] = \int_0^{2r_i} c f_{C_i}(c)dc \tag{9}$$

The integral limits are from the fact that the maximum and the minimum coverage of any small cell with radius $r_i$ on $\mathcal{P}_0$ are $2r_i$ and 0 respectively. The result in Eq. (9) includes some overlap coverage on $\mathcal{P}_0$. The overlaps can be ignored when the density of small cells is very low. However, it is anticipated that the small cell density in the future cellular networks is very high and overlap coverage needs to be taken into consideration. The overlap coverage on $\mathcal{P}_0$ depends on various parameters such as the density of small cells and the coverage of each small cell. Finding the number of overlaps and the overlap areas will help to estimate the small cell sojourn time and the handoff rate precisely. Before, the overlap coverage on $\mathcal{P}_0$ is investigated, we make an assumption based on the following definition.

**Definition** When $\mathbb{R}^\delta$ is a $\delta$-dimensional Euclidean space, a uniform PPP on $\mathbb{R}^\delta \times [0, \eta]$ of intensity $\lambda$ can be interpreted as a MPP on $\mathbb{R}^\delta$ with marks from $[0, \eta]$ and intensity $\eta\lambda$ [13].

**Assumption**: Without loss of generality, if $r_j = r, \forall j$ and the point $W_0$ is at the origin, the SCBSs at distance of $r$ or less from the line that starts from the origin and passes through $W_1$, can be interpreted as a MPP on $\mathbb{R}^+ \times [0, r]$, $\bar{\Phi}_2 = \{(\bar{y}_i, \tau_i)\}$ of intensity $\bar{\lambda}_2$, where $\bar{y}$ represent the nearest points (NPs) on the line to the SCBSs of the crossed small cells as shown in Fig. 2. The NPs are assumed to be distributed on the line as PPP, the accuracy of this assumption is validated through simulations in Fig. 3. $\tau_i$ represents the vertical distance from the location of the $i$th SCBS to the NP $\bar{y}_i$. Since the locations of the SCBSs are uniformly distributed and can be at any distance from $\mathcal{P}_0$, it is also assumed that $\tau$ is uniformly distributed in the range $[0, r]$. The density of the new process $\bar{\Phi}_2$ can be obtained as:

$$\begin{aligned} \mathbb{E}[\bar{N_0}] &= \mathbb{E}[N_0] \\ \bar{\lambda}_2 \parallel W_1 - W_0 \parallel &= \mathscr{A}_2\lambda_2 \\ \bar{\lambda}_2 &= \frac{\mathscr{A}_2\lambda_2}{\parallel W_1 - W_0 \parallel} \\ \bar{\lambda}_2 &= 2r\lambda_2 \end{aligned} \tag{10}$$

where $\mathbb{E}[N_0]$ is obtained in Eq. (6), $\bar{N}_0$ represents the number of NPs on $\mathcal{P}_0$ and $\mathscr{A}_2 = 2r \parallel W_1 - W_0 \parallel$. When the NPs are set in order according to the distance from $W_0$ as $(\bar{y}_1, \bar{y}_2, \cdots, \bar{y}_i, \cdots, \bar{y}_{\bar{N}_0})$, the NP inter-distance (e.g. the first NP inter-distance represents the distance between the points $\bar{y}_1$ and $\bar{y}_2$) has an exponential distribution:

$$\mathbb{P}(\parallel \bar{y}_{i+1} - \bar{y}_i \parallel \leq \mathrm{d}) = 1 - \exp(-\bar{\lambda}_2\mathrm{d}) \qquad \mathrm{d} > 0 \tag{11}$$

The expected value of the distance from $W_0$ to $\bar{y}_i$ can be

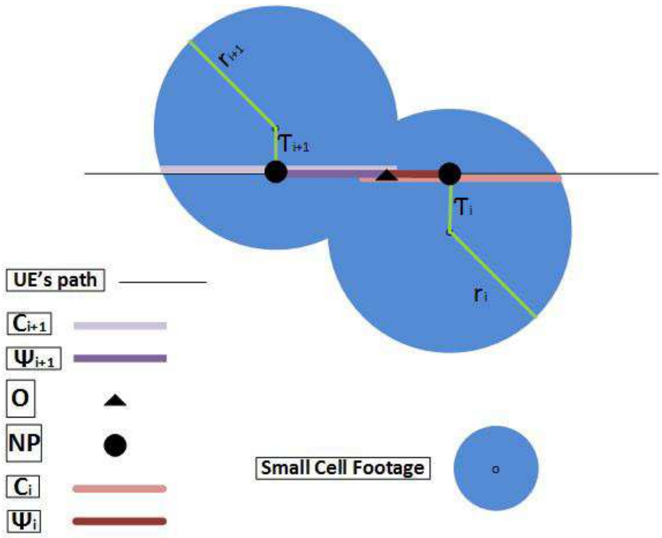


Fig. 2. The coverage of small cells on $\mathcal{P}_0$.

obtained as:

$$\begin{aligned} \mathbb{E}[\parallel \bar{y}_i - W_0 \parallel] &= \int_{-\infty}^{\infty} \mathrm{d}\, f_{(\parallel \bar{y}_i - W_0 \parallel)}(\mathrm{d})d\mathrm{d} \\ &= \int_0^{\infty} \mathrm{d}\frac{\bar{\lambda}_2^i}{\Gamma(i)}\mathrm{d}^{i-1}e^{-\bar{\lambda}_2\mathrm{d}}d\mathrm{d} \\ &= \frac{i}{\bar{\lambda}_2} \end{aligned} \tag{12}$$

where $f_{(\parallel \bar{y}_i - W_0 \parallel)}$ represents the PDF of the distance $\parallel \bar{y}_i - W_0 \parallel$ [9], $\Gamma(.)$ represents the gamma function. Next, the coverage of one overlap on $\mathcal{P}_0$ is investigated.

**Lemma 1** The expected value of one overlap coverage on $\mathcal{P}_0$ can be obtained as:

$$\mathbb{E}[\mathfrak{C}_i] = \frac{\mathbb{E}[\bar{C}_i]}{2} \tag{13}$$

where $\bar{C}_i = \frac{C_i}{2} + \frac{C_{i+1}}{2}$ represents the maximum distance between the $i$th NP and $(i+1)$th NP for the $i$th overlap to occur, as shown in Fig. 2.

*Proof*: See Appendix A.

After finding the expected value of any overlap coverage on $\mathcal{P}_0$, the expected number of overlaps taking place on $\mathcal{P}_0$ is obtained as follows.

**Lemma 2** The expected number of overlaps on $\mathcal{P}_0$ can be expressed as:

$$\mathbb{E}[N_{OL}] = \mathbb{E}[N_{OL,max}]\Big(1 - e^{-\bar{\lambda}_2 \int_0^{4r} \bar{c}\, f_{\bar{C}_i}(\bar{c})\, d\bar{c}}\Big) \tag{14}$$

where $\mathbb{E}[N_{OL,max}] = \mathcal{A}_1\Big(\frac{\bar{\lambda}_2}{2\sqrt{\lambda_w}} - 1\Big) + \frac{\mathcal{A}_2}{2}\Big(\frac{\bar{\lambda}_2}{\sqrt{\lambda_w}} - 1\Big)$

*Proof*: See Appendix B.

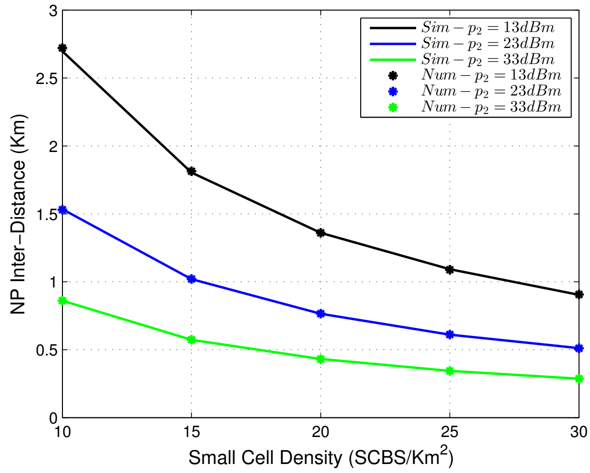


Fig. 3. Assumption 1 is validated in this figure, where Num. is the numerical result and obtained by using Eq. (12) and Sim. is the results obtained from simulations.

## IV. Sojourn Time

The cell sojourn time is defined as the expected time that $\mathcal{U}_0$ stays in a coverage of small cell of interest and it directly affects the efficiency of system resources utilization. In this section we consider two scenarios, when the pause time equals zero ($T = 0$). We also consider a scenario when the pause time is not zero. Since all transition lengths are i.i.d, the expected sojourn time will be derived during one transition time (e.g. $\| W_1 - W_0 \|$). The small cells crossed by $\mathcal{U}_0$ have different coverages on $\mathcal{P}_0$ as they have different transmit powers and they are located at different distances from $\mathcal{P}_0$. Since the cell association among the small cells tiers is based on the maximum received power, the overlap coverages will be served by different small cells depending on the transmit power and the locations of the SCBSs around the path. The expected value of the $i$th small cell's footage on $\mathcal{P}_0$, served by the $(i+1)$th small cell is obtained in the following Lemma.

**Lemma 3** Given that an overlap occurs on the path between the $i$th small cell and the $(i+1)$th small cell, the expected value of the $i$th small cell's footage served by the $(i+1)$th small cell due to overlapping is expressed as:

$$\mathbb{E}[\chi_{(i+1)\mapsto i}] = \frac{\mathbb{E}[\mathfrak{C}_i](\frac{p_{2,i}}{p_{2,i+1}})^{\frac{-1}{\alpha_2}}}{1 + (\frac{p_{2,i}}{p_{2,i+1}})^{\frac{-1}{\alpha_2}}} \tag{15}$$

*Proof*: See Appendix C.

### A. *Pause Time* $= 0$

The sojourn time in a small cell of interest can be expressed in the next Theorem.

**Theorem 1** The expected sojourn time during one movement when $T = 0$ can be expressed:

$$\mathbb{E}[\mathcal{S}^0] = \frac{1}{v}\Big(\mathbb{E}[C_i] - \mathbb{P}_{OL}\big(\mathbb{E}[\chi_{(i+1)\mapsto i}] + \mathbb{E}[\chi_{(i-1)\mapsto i}]\big)\Big) \tag{16}$$

where $\mathbb{P}_{OL}$ is the probability that the reference small cell overlaps with either the $(i+1)$th or the $(i-1)$th small cells on $\mathcal{P}_0$ and is obtained in Eq. (B4).

*Proof*: Given that the $i$th small cell crossed by $\mathcal{U}_0$, the sojourn time that $\mathcal{U}_0$ stays in the $i$th small cell coverage when $V = v$ can be expressed as:

$$\mathcal{S}^0 = \frac{C_i - \Xi}{v} \tag{17}$$

where $\Xi$ represents the $i$th small cell's footage on the path served by other small cells due to overlapping. $\Xi$ can take a value between 0 when no overlap occurs, and $C_i$ when one overlap or more occur with other small cells on the path. Given that the $i$th small cell has $C_i$ coverage on the path and overlaps with other small cells, the expectation of $\Xi$ can be expressed as:

$$\mathbb{E}[\Xi] = \mathbb{P}_{OL}\big(\mathbb{E}[\chi_{(i+1)\mapsto i}] + \mathbb{E}[\chi_{(i-1)\mapsto i}]\big) \tag{18}$$

The result in Eq. (16) is reached.

### B. *Pause Time* $\neq 0$

Since the waypoints are distributed randomly as a PPP on the plane with density $\lambda_w$, the probability that $\mathcal{U}_0$ spends the pause time in the small cell of interest can be found as follows.

**Lemma 4** The probability that $\mathcal{U}_0$ spends the pause time in the reference small cell is the probability of the destination point ($W_1$) served by that small cell:

$$\mathbb{P}_{\mathcal{S}} = \frac{\pi\Big(\frac{\rho_{min}}{\mathscr{L}_2 p_{2,0}}\Big)^{-\frac{2}{\alpha_2}}}{\mathscr{A}_2\Big(\big(\frac{p_{2,j}}{p_{2,0}}\big)^{\frac{2}{\alpha_2}} + 1\Big)} \tag{19}$$

where $p_{2,0}$ and $p_{2,j}$ are the transmit powers of the small cell of interest and the $j$th small cell respectively.

*Proof*: See Appendix D.

The expected sojourn time that $\mathcal{U}_0$ spends in any small cell when $T \neq 0$ and $V = v$ can be expressed as:

$$\mathbb{E}[\mathcal{S}] = \mathcal{A}_2\mathbb{E}[\mathcal{S}^s] + \mathcal{A}_1\mathbb{E}[\mathcal{S}^0] \tag{20}$$

where $\mathbb{E}[\mathcal{S}^s] = \mathbb{P}_{\mathcal{S}}(T + \mathbb{E}[\mathcal{S}^0]) + (1 - \mathbb{P}_{\mathcal{S}})\mathbb{E}[\mathcal{S}^0]$ represents the expected time that $\mathcal{U}_0$ spends in each small cell when $W_1$ is located in the small cells coverage.

## V. Handoff Rate

The handoff rate is defined as the expected number of handoffs taking place per unit time. It is considered as one of the important parameters in the cellular systems as it affects the amount of signalling. The total handoff rate can be expressed as:

$$\mathbb{E}[\mathcal{H}_T] = \frac{\mathbb{E}[N_{HF}]}{\mathbb{E}[\mathcal{T}_0]} \tag{21}$$

where $\mathbb{E}[\mathcal{T}_0] = \frac{1}{2v\sqrt{\lambda_w}}$ represents the total time that $\mathcal{U}_0$ needs to travel along $\mathcal{P}_0$ and $N_{HF}$ represents the number of handoffs that $\mathcal{U}_0$ experiences during the same movement. The maximum number of handoffs that $\mathcal{U}_0$ can experience on

$\mathcal{P}_0$, is $2N_0$ when no overlap takes place on the path. Since some of the handoffs will be among the small cells due to overlapping, the total number of handoffs can be expressed as $2N_0 - N_{OL}$. Therefore the expectation of the total handoff rate becomes:

$$\mathbb{E}[\mathcal{H}_T] = \frac{2\mathbb{E}[N_0] - \mathbb{E}[N_{OL}]}{\mathbb{E}[\mathcal{T}_0]} \tag{22}$$

## VI. Numerical Results

In this section, simulation and numerical results are presented to validate the analysis and to show the impact of different parameters such as the waypoint density $\lambda_w$, the transmit power of the small cells and the density of the small cells in the system on the handoff rate and the cell sojourn time. Some figures in this section include two cases. The first case (A) represents the analysis in this paper which considers the overlaps among the small cells. The second case (B) represents the analysis when the overlaps are ignored. It is assumed that the minimum power $\rho_{min} = -90$ dBm.

Fig. 4 shows the total handoff rate for different values of the small cell density when the transmit power of the small cells are either 33 dBm or 36 dBm. As expected, it is shown that the total handoff rate increases when the density of the small cells increases. The total handoff rate also increases when the transmit power of the small cells increases. This is because the UEs cross more small cells when they have larger footage. Fig. 4 also shows a comparison between the case when the overlaps are taken into account (A), and the case of ignoring the overlaps (B) on the reference UE's path. The total handoff rate in (B) is always greater, as the overlaps are ignored and two handoffs are assumed to take place for each small cell.

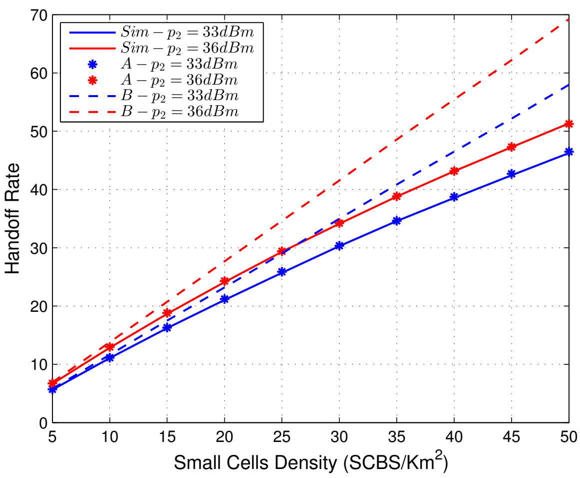


Fig. 4. The handoff rate, where $(A)$ considers the cell overlap, $(B)$ does not consider the cell overlap, $Sim$ are the results from simulations, $\alpha_2 = 4$ and $\lambda_w = 0.01$.

The expectation of the cell sojourn time is shown in Fig. 5 in the two cases (A) and (B) for different small cells density and different transmit powers. It is seen that the case (A) is very accurate in different densities of HetNets. Fig. 5 also shows that the cell sojourn is minimized in the dense HetNet due to the small cells overlapping. Furthermore, it is also seen that when the overlaps are ignored, e.g. case (B), the cell sojourn time becomes independent of the small cell density. However, our analysis (A) and the simulations show that the cell sojourn time is not only affected by the footages of the small cells (the transmit powers), but also affected by the small cell density. The gap between the two cases (A) and (B) increases when the small cell density increases which implies that the analysis becomes very inaccurate when ignoring the overlaps.

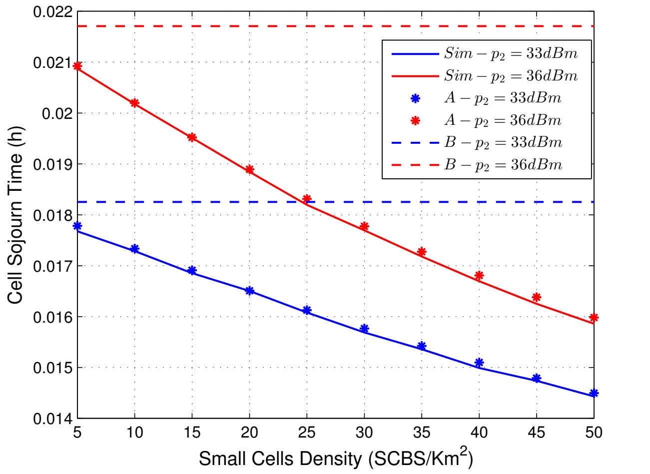


Fig. 5. The Cell sojourn time, where $(A)$ considers the cell overlap, $(B)$ does not consider the cell overlap, $Sim$ are the results from simulations, $\alpha_2 = 4$, $T = 0$ and $\lambda_w = 0.01$.

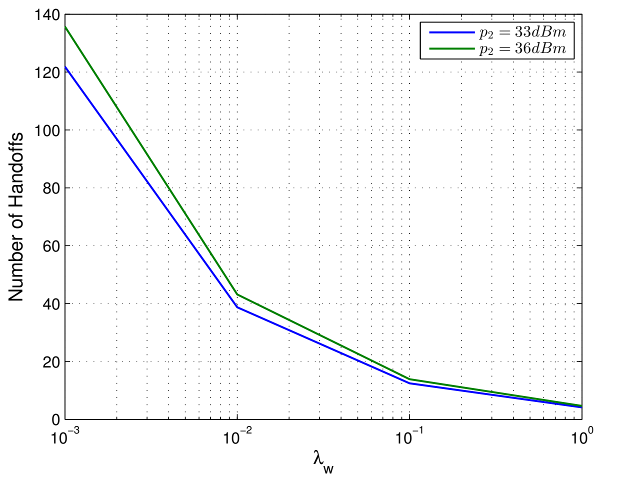


Fig. 6. The number of handoffs, where $\lambda_2 = 40$ and $\alpha_2 = 4$.

The number of handoffs for different values of the mobility parameter $\lambda_w$ is shown in Fig. 6. It is shown that the total number of handoffs during one movement increases when the mobility parameter decreases. This is because the expected distance that the reference UE needs to travel from the starting point to the destination point decreases when $\lambda_w$ increases as shown in Eq. (4). Once again, the impact of the small cell transmit power on the number of handoffs is shown in this figure, where the number of handoffs are always greater when the transmit power is higher e.g. 36 dBm.

## VII. Conclusion

In this paper, the stochastic geometry tool was used to propose a novel mobility framework to model and analyse the main mobility parameters such as the handoff rate and the small cell sojourn time. In the proposed framework, the overlaps among the small cells on a reference UE's path was taken into consideration. The simulation results showed that ignoring the overlaps can affect the accuracy of the small cell sojourn time and the handoff rate significantly. It was also shown that the small cell sojourn time becomes independent of the small cell density when the overlaps were not considered. However, the simulation and the numerical results showed that increasing the small cell density can reduce the small cell sojourn time due to overlapping.

## Appendix A

Given that the $i$th and the $(i+1)$th small cells with coverage $C_i$ and $C_{i+1}$ respectively are overlapped on $\mathcal{P}_0$. From Fig 2, the $i$th small cell overlaps with the $(i+1)$th small cell on $\mathcal{P}_0$ if $\bar{y}_i$ and $\bar{y}_{i+1}$ are at distance $\bar{C}_i$ or less. Thus, any overlap coverage can be expressed as:

$$\mathfrak{C}_i = \begin{cases} \bar{C}_i - \parallel \bar{y}_{i+1} - \bar{y}_i \parallel, & \parallel \bar{y}_{i+1} - \bar{y}_i \parallel < \bar{C}_i \\ 0, & \text{otherwise} \end{cases} \tag{A1}$$

When the $i$th small cell and the $(i+1)$th small cell are overlapped, the distance between $\bar{y}_i$ and $\bar{y}_{i+1}$ is uniformly distributed in the range $[0, \frac{C_i+C_{i+1}}{2}]$. Therefore, the expectation of $\mathfrak{C}_i$ becomes:

$$\mathbb{E}[\mathfrak{C}_i] = \begin{cases} \frac{\mathbb{E}[\bar{C}_i]}{2}, & \parallel \bar{y}_{i+1} - \bar{y}_i \parallel < \bar{C}_i \\ 0, & \text{otherwise} \end{cases} \tag{A2}$$

The PDF of $\bar{C}_i$ is the convolution of the PDFs of $C_i$ and $C_{i+1}$. Since $C_i$ and $C_{i+1}$ are independent random variables, the joint PDF of both $C_i$ and $C_{i+1}$ can be expressed as $f_{C_i C_{i+1}}(c_i, c_{i+1}) = f_{C_i}(c_i) f_{C_{i+1}}(c_{i+1})$. Therefore the PDF of $\bar{C}_i$ is obtained as:

$$\begin{aligned} f_{\bar{C}_i}(\bar{c}) &= \int_{-\infty}^{\infty} \Big( \frac{d}{d\bar{c}} \int_0^{\bar{c}-c_i} f_{C_i C_{i+1}}(c_i, c_{i+1}) dc_{i+1} \Big) dc_i \\ &= \int_{-\infty}^{\infty} f_{C_i C_{i+1}}(c_i, \bar{c} - c_i) dc_i \\ &\overset{(a)}{=} \int_0^{\bar{c}} \frac{\bar{c} c_i (\bar{c} - c_i)\, dc_i}{32 r_i^2 r_{i+1}^2 \sqrt{1 - \frac{c_i^2}{4r_i^2}} \sqrt{1 - \frac{(\bar{c}-c_i)^2}{4r_{i+1}^2}}} \\ &\overset{(b)}{=} \int_0^{\bar{c}} \frac{\bar{c} c_i (\bar{c} - c_i)\, dc_i}{16 r^3 \sqrt{8r^2 - c_i^2 - (\bar{c} - c_i)^2}} \end{aligned} \tag{A3}$$

where (a) follows from Eq. (8) and from the fact that all small cells have the same distribution around $\mathcal{P}_0$, and (b) follows from that $r_i = r, \forall i$. The expected value of $\bar{C}_i$ is obtained as:

$$\mathbb{E}[\bar{C}_i] = \int_0^{4r} \bar{c}\, f_{\bar{C}_i}(\bar{c})\, d\bar{c} \tag{A3}$$

where the integral limits follow from $C_i$ and $C_{i+1}$ being independent and from that the maximum summation of both small cells coverage can be $2r_i + 2r_{i+1} = 4r$ when both are maximum $C_i = 2r_i$ and $C_{i+1} = 2r_{i+1}$, and the minimum summation of both small cells coverage can be 0 when both are minimum $C_i = C_{i+1} = 0$. The desired results in Eq. (13) is reached after solving Eq. (A3).

## Appendix B

Since the overlap between the $i$th and the $(i+1)$th small cells occurs when the distance between $\bar{y}_i$ and $\bar{y}_{i+1}$ is equal or less than $\bar{C}_i$, the number of overlaps can be expressed as:

$$N_{OL} = \sum_{i=2}^{N_0} \mathbf{1}(\parallel \bar{y}_i - \bar{y}_{i-1} \parallel \leq \bar{C}_{i-1}) \tag{B1}$$

the expected number of overlaps can be expressed as:

$$\mathbb{E}[N_{OL}] = \mathbb{E}[N_{OL,max}]\mathbb{P}_{OL} \tag{B2}$$

where $\mathbb{P}_{OL}$ is defined as the probability of two consecutive small cells with coverage $C_i$ and $C_{i+1}$ overlapping on $\mathcal{P}_0$ and $\mathbb{E}[N_{OL,max}]$ represents the maximum number of overlaps that can occur on $\mathcal{P}_0$. Given that the number of small cells crossed by $\mathcal{U}_0$ is $N_0$, the maximum overlaps can take different values, for instance when $W_1$ is not located in the small cells coverage, the maximum number of overlaps occurring on the path will be $N_0 - 1$. However, when $W_1$ is located in the small cells coverage the maximum number of overlaps that can take place on $\mathcal{P}_0$ is either $N_0$ when $W_1$ is located in coverage of small cell that its SCBS is not located in $\mathscr{A}_2$, or $N_0 - 1$ when $W_0$ is located in a small cell that its SCBS belongs to $\mathscr{A}_2$.Therefore the expected maximum number of overlaps that can occur on the path can be expressed as

$$\mathbb{E}[N_{OL,max}] = \mathcal{A}_1 \Big( \frac{\bar{\lambda}_2}{2\sqrt{\lambda_w}} - 1 \Big) + \frac{\mathcal{A}_2}{2} \Big( \frac{\bar{\lambda}_2}{\sqrt{\lambda_w}} - 1 \Big) \tag{B3}$$

Since the NPs follow a PPP, the probability of the overlap occurring is obtained from the null probability as:

$$\begin{aligned} \mathbb{P}_{OL} &= 1 - \mathbb{P}\big[\text{No Overlap}\big] \\ &= 1 - \mathbb{P}\big[ \parallel \bar{y}_{i+1} - \bar{y}_i \parallel > \mathbb{E}[\bar{C}_i] \big] \\ &= 1 - \exp\big( - \bar{\lambda}_2 \int_0^{4r} \bar{c}\, f_{\bar{C}_i}(\bar{c})\, d\bar{c} \big) \end{aligned} \tag{B4}$$

where $\exp\big( - \bar{\lambda}_2 \int_0^{4r} \bar{c}\, f_{\bar{C}_i}(\bar{c})\, d\bar{c} \big)$ represents the probability of no overlap occurs or the probability that $\bar{y}_{i+1}$ is at distance greater than $\bar{C}_i$ from $\bar{y}_i$. The desired result in Eq. (14) is reached after substituting Eq. (B4) and Eq. (B3) in Eq. (B2).

## Appendix C

Assume that $O$ point on $\mathcal{P}_0$ where the received power from both the $i$th and the $(i+1)$th small cells are equal as shown in Fig. 2. The average received power at $O$ is:

$$\frac{p_{2,i}\mathscr{L}_2}{\Big(\sqrt{\tau_i^2 + \Psi_i^2}\Big)^{\alpha_2}} = \frac{p_{2,i+1}\mathscr{L}_2}{\Big(\sqrt{\tau_{i+1}^2 + \Psi_{i+1}^2}\Big)^{\alpha_2}} \tag{D1}$$

where $\Psi_i = \parallel \bar{y}_i - O \parallel$ and $\Psi_{i+1} = \parallel \bar{y}_{i+1} - O \parallel$ represent the actual one side coverage served by the $i$th small cell and the $(i+1)$th small cell on $\mathcal{P}_0$ respectively. Since the small cells can be located at any distance from $\mathcal{P}_0$ and they have different transmit powers, the point $O$ can be located either between $\bar{y}_i$ and $\bar{y}_{i+1}$, before $\bar{y}_i$ or after $\bar{y}_{i+1}$. Assuming that $\tau_i = \tau_{i+1} = 0$, the one side coverage of the $i$th small cell can be obtained as:

$$\begin{aligned}\Psi_i &= \Big(\frac{p_{2,i+1}}{p_{2,i}}\Big)^{\frac{-1}{\alpha_2}}\Psi_{i+1}\\ &\overset{(a)}{=} \frac{\big(\frac{p_{2,i+1}}{p_{2,i}}\big)^{\frac{-1}{\alpha_2}}}{1+\big(\frac{p_{2,i+1}}{p_{2,i}}\big)^{\frac{-1}{\alpha_2}}} \parallel \bar{y}_i - \bar{y}_{i+1} \parallel\end{aligned} \tag{D2}$$

Note that $\Psi_i = r_i$ and $\Psi_{i+1} = r_{i+1}$ when both the $i$th and the $(i+1)$th do not overlap on $\mathcal{P}_0$. (a) follows from $\parallel \bar{y}_i - \bar{y}_{i+1} \parallel = \Psi_i + \Psi_{i+1}$. The footage of the $i$th small cell served by the $(i+1)$th small cell can be obtained as:

$$\begin{aligned}\chi_{(i+1)\mapsto i} &= r_i - \Psi_i\\ &\overset{(b)}{=} r_i - \frac{\big(\frac{p_{2,i+1}}{p_{2,i}}\big)^{\frac{-1}{\alpha_2}}(r_i + r_{i+1} - \mathfrak{C}_i)}{1+\big(\frac{p_{2,i+1}}{p_{2,i}}\big)^{\frac{-1}{\alpha_2}}}\\ &\overset{(c)}{=} \frac{\mathfrak{C}_i\big(\frac{p_{2,i+1}}{p_{2,i}}\big)^{\frac{-1}{\alpha_2}}}{1+\big(\frac{p_{2,i+1}}{p_{2,i}}\big)^{\frac{-1}{\alpha_2}}}\end{aligned} \tag{D3}$$

where (b) follows from Eq. (D2) and $\parallel \bar{y}_i - \bar{y}_{i+1} \parallel = r_i + r_{i+1} - \mathfrak{C}_i$, and (c) follows from $r_i = r_{i+1}\big(\frac{p_{2,i+1}}{p_{2,i}}\big)^{\frac{-1}{\alpha_2}}$. Given that the $i$th small cell and $(i+1)$th small cell overlaps on $\mathcal{P}_0$, the expectation of $\mathfrak{C}_i$ is obtained in Lemma 1. Therefore the expected value of $\chi_{(i+1)\mapsto i}$ can be expressed as in Eq. (15).

## Appendix D

It is assumed that the small cell of interest is crossed by $\mathcal{U}_0$ and the destination waypoint is located at the origin. If the distance between the destination waypoint and the small cell of interest is donated by $\mathfrak{r}_0$, the probability of the destination point served by the small cell of interest is a conditional probability (it is assumed that the association among the small cells is based on the maximum received power) and it is expressed as:

$$\begin{aligned}\mathbb{P}_{\mathcal{S}} &= \mathbb{P}\bigg[\mathfrak{r}_0 < r_0 \mid \rho_0 > \max_{j\in\Phi_2, j\neq 0} \rho_j\bigg]\\ &= \mathbb{P}\bigg[\mathfrak{r}_0 < \Big(\frac{p_{min}}{\mathscr{L}_2 p_{2,0}}\Big)^{-\frac{1}{\alpha_2}}\bigg]\mathbb{P}\bigg[\frac{p_{2,0}\mathscr{L}_2}{\mathfrak{r}_0^{\alpha_2}} > \frac{p_{2,j}\mathscr{L}_2}{\mathfrak{r}_j^{\alpha_2}}\bigg]\\ &= \Bigg(\frac{\pi\Big(\frac{p_{min}}{\mathscr{L}_2 p_{2,0}}\Big)^{-\frac{2}{\alpha_2}}}{\mathscr{A}_2}\Bigg)\Bigg(\mathbb{P}\bigg[\mathfrak{r}_0 < \Big(\frac{p_{2,0}\mathfrak{r}_j^{\alpha_2}}{p_{2,j}}\Big)^{\frac{1}{\alpha_2}}\bigg]\Bigg)\\ &= \Bigg(\frac{\pi\Big(\frac{p_{min}}{\mathscr{L}_2 p_{2,0}}\Big)^{-\frac{2}{\alpha_2}}}{\mathscr{A}_2}\Bigg)\Bigg(\int_0^\infty \exp\Big(-\pi\lambda_2\Big(\frac{p_{2,j}}{p_{2,0}}\Big)^{\frac{2}{\alpha_2}}\mathfrak{r}_j^2\Big)\\ &\qquad f_{\mathfrak{r}_j}(\mathfrak{r})\Bigg)\end{aligned} \tag{E1}$$

where $\mathbb{P}\big[\mathfrak{r}_0 < r_0\big]$ is the probability that the destination waypoint is at distance less than $r_0$ from the small cell of interest, $\rho_0$ and $\rho_j$ are the received powers from the small cell of interest and the received power from the $j$th small cell respectively, and $\mathbb{P}\big[\rho_0 > \max_{j\in\Phi_2, j\neq 0}\rho_j\big]$ is the probability that $\mathcal{U}_0$ at the destination waypoint receives the maximum received power from the small cell of interest and $\mathfrak{r}_j$ is the distance between the $j$th small cell and the destination waypoint. Since the locations of both waypoints and SCBSs are uncorrelated and randomly distributed in the network, the variable $\mathfrak{r}_j$ is assumed to have a Rayleigh distribution with PDF $2\pi\lambda_2\mathfrak{r}_j e^{-\pi\lambda_2\mathfrak{r}_j^2}$. $\mathbb{P}_{\mathcal{S}}$ is obtained after solving Eq. (E1).

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