Adaptive Observer Design for a Class of Nonlinear Interconnected Systems with Uncertain Time Varying Parameters

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Abstract: In this paper, a class of nonlinear interconnected systems with uncertain time varying parameters is considered, in which both the interconnections and the isolated subsystems are nonlinear. The difference between the unknown time varying parameter and its corresponding nominal value is assumed to be bounded where the nominal value is not required to be known. A dynamical system is proposed and then, the error systems between the original interconnected system and the designed dynamical systems are analysed based on the Lyapunov direct method. A set of conditions is developed such that the augmented systems formed by the error dynamical systems and the designed adaptive laws, are globally uniformly bounded. Specifically, the estimation errors are asymptotically convergent to zero using LaSalle-Yoshizawa Theorem. Case study on a coupled inverted pendulum system is presented to demonstrate the developed methodology, and simulation shows that the proposed approach is effective and practicable.

Keywords: Nonlinear Interconnected Systems, Adaptive Observers, Time Varying Parameters, Lyapunov Direct Method.

1. INTRODUCTION

The development of advanced technologies has produced corresponding growth in the scale of engineering systems, and thus the scale of many practical systems becomes large in order to satisfy the increasing requirement for system performance. Such systems are called large scale systems and usually can be modelled by sets of lower-order ordinary differential equations which are linked through interconnections (Yan & Xie (2003), Bakule (2008), Mahmoud (2011) and Yan *et al.* (2013). Interconnected systems widely exists in the real world, for example, coupled inverted pendulums, energy systems and biological systems etc (see e.g. Bakule (2008) and Mahmoud (2011)). Study on interconnected systems has received great attention and many results have been obtained (see e.g. Bakule (2008) and Mahmoud (2011), Yan *et al* (2017)). Much of the existing work assumes that all system states are available in control design. However, for a practical system, only a subset of system states is usually available. In order to implement state feedback control schemes, one of possible choices is to design an observer to estimate system states, and then use the estimated states to form the feedback control loop.

Observer design has been studied for many years, and the early work can be dated back to the well known Luenberger observer. The majority of the early work about observer design is mainly for linear systems and the robust problem against various uncertainties was not considered. However, due to the mechanical wearing and modelling errors, many practical control systems involve unknown parameters. Recently, much literature has devoted to design an adaptive observer for nonlinear systems and many different methods have been developed in order to obtain high estimation performance in the presence of parametric uncertainty and/or unstructural uncertaity. Boizot *et al* (2010) developed an adaptive observer by using extended Kalman filter to reduce the effect of perturbations. However, in terms of the parameter estimation for nonlinear systems, it is usually very difficult to analysis the stability of the extended Kalman filter. Sliding mode techniques have been applied in Efimov *et al* (2016) to enhance the performance of the adaptive observer proposed by Yan & Edwards (2008a). It should be noted that unknown parameters considered in these papers are constant. An adaptive redesign of reduced order nonlinear observers are presented in Stamnes *et al* (2011) where the solution of a partial differential equation is required, which may not be possible in most of cases. An adaptive observer is designed for a class of MIMO uniformly observable nonlinear systems with linear and nonlinear parametrizations in Farza *et al* (2009) and the exponential converges of the error dynamics for both types of parametrization is guaranteed under the persistent excitation condition. Tyukin *et al* (2013) considered the problem of asymptotic reconstruction of the state and parameter. However, in both Farza *et al* (2009) and Tyukin *et al* (2013), it is required that the unknown parameters are constant. The literature in Yang & Liu (2016) proposed an adaptive state estimator for a class of multi-input and multi-output non-linear systems with uncertainties in the state and the output equations, in which the systems considered are not interconnected systems. The work in Pu *et al* (2015) proposed an adaptive observer which expands the extended state observer to nonlinear disturbed systems. However, the adaptive extended state observer is linear and requires that the error dynamics can be transformed into a canonical form.

The observer design for interconnected systems has been widely studied. A sliding mode observer has been presented in Yan & Edwards (2008b) for decentralised fault detection but the parameter uncertainty is not considered. Chen *et al* (2016) tried to overcome the limitation of the strict-feedback form and designed an observer to estimate the system state variables which is used to implement an adaptive neural network output-feedback control scheme. An adaptive interconnected observer is proposed for sensorless control of a synchronous motor in Hamida *et al* (2013), where the system considered includes only two subsystems and the parameters may vary but with known bounds. It should be noted that all the observers mentioned above are mainly used to implement a special control task. Therefore, strong limitation is unavoidably imposed on the considered interconnected systems such that the designed observer can perform an exact function to complete the task. Moreover, in most of the existing work, it is required that either the unknown parameters are constant (see e.g Efimov *et al* (2016)) or the nominal values of the unknown parameters are known (Pourgholi & Majd (2012)). The corresponding observation results for large scale nonlinear interconnected systems are very limited, particularly when uncertain parameters are involved.

In this paper, a class of nonlinear interconnected systems with uncertain time varying parameters are considered, in which both the isolated subsystems and the interconnections are nonlinear. Under the condition that the difference between the unknown time varying parameters and the corresponding uncertain nominal values are bounded by constants, adaptive updating laws are proposed to estimate the parameters. The persistent of excitation condition is not required. A set of sufficient conditions are proposed such that the error dynamics formed by the system states and the designed observers are asymptotically stable while the adaptive laws are uniformly bounded using LaSalle-Yoshizawa Theorem. The results obtained are applied to a coupled inverted pendulum systems, and simulation results are presented to demonstrate the effectiveness and feasibility of the developed results. The main contribution includes: (i) Both the interconnections and isolated subsystems take nonlinear forms. (ii) The unknown parameters considered in the system are time varying and the corresponding nominal values are not required to be known. (iii) The asymptotic convergence of the observation error between the considered systems states and the designed observers is guaranteed while the estimate errors of the time varying parameters are uniformly bounded.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider a nonlinear interconnected system composed of N subsystems as follows

$$\dot{x}_i = A_i x_i + f_i(x_i, u_i) + B_i\theta_i(t)\xi_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(x_j) \quad (1)$$

$$y_i = C_i x_i \quad (2)$$

where $x_i \in R^{n_i}$, $u_i \in U \in R^{m_i}$ (U is the admissible control set) and $y_i \in R$ are the state variables, inputs and outputs of the i-th subsystem respectively. The functions $f_i(\cdot)$ are known continuous, the scalars $\theta_i(t) \in R$ are unknown time varying parameter and $\xi_i(t) \in R$ are known regressor signals. The matrices $A_i \in R^{n_i\times n_i}, B_i \in R^{n_i\times m_i}$ and $C_i \in R^{1\times n_i}$ are constants, and C_i are full column rank. The terms $\sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}(x_j)$ are the known interconnections for $i = 1, \cdots, N$.

Assumption 1. The pairs (A_i, C_i) are observable for $i = 1, \cdots, N$.

From Assumption 1, there exist matrices L_i such that $A_i - L_iC_i$ are Hurwitz stable. This implies that, for any positive-definite matrices $Q_i \in R^{n_i\times n_i}$, the Lyapunov equations

$$(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i) = -Q_i \quad (3)$$

have unique positive-definite solutions $P_i \in R^{n_i\times n_i}$.

Assumption 2. There exist matrices $F_i \in R^{m_i\times 1}$ such that solutions P_i to the Lyapunov equations (3) satisfy the constraints

$$B_i^T P_i = F_i C_i, \qquad i = 1, \cdots, N \quad (4)$$

Assumption 3. The uncertain time varying parameters $\theta_i(t)$ satisfy

$$|\theta_i(t) - \theta_{0_i}| \le \epsilon_{0_i} \quad (5)$$

where θ_{0_i} are unknown constant, and ϵ_{0_i} are known constant for $i = 1, \cdots, N$.

Remark 1. Assumption 3 is to specify a class of uncertainties tolerated in the observer design. The unknown constants θ_{0_i} given in (5) are called the nominal value of the uncertain time varying parameters $\theta_i(t)$ throughout this paper. Different from the existing work, it is not required that θ_{0_i} in (5) are known but the bounds on the difference between the unknown time varying parameters $\theta_i(t)$ and their nominal values θ_{0_i} are assumed to be known.

For further analysis, the terms $B_i\theta_i(t)\xi_i(t)$ in system (1) are rewritten as

$$B_i\theta_i(t)\xi_i(t) = B_i[\theta_{0_i} + \epsilon_i(t)]\xi_i(t) \quad (6)$$

where the scalers $\epsilon_i(t) = \theta_i(t) - \theta_{0_i}$.

Assumption 4. The nonlinear terms $f_i(x_i, u_i)$ and $H_{ij}(x_j)$ satisfy the Lipschitz condition.

Assumption 4 implies that there exist nonnegative constants ℓ_{f_i} and $\ell_{H_{ij}}$ such that

$$\|f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)\| \le \ell_{f_i}(u_i)\|\hat{x}_i - x_i\| \quad (7)$$

$$\|H_{ij}(\hat{x}_j) - H_{ij}(x_j)\| \le \ell_{H_{ij}}\|\hat{x}_j - x_j\| \quad (8)$$

for $i = 1, 2, \cdots, N$ and $i \neq j$.

The Assumption 4 is used to guarantee the asymptotic convergence of the observation error.

3. MAIN RESULTS

From (6), system (1) can be rewritten as

$$\begin{aligned} \dot{x}_i &= A_i x_i + f_i(x_i, u_i) + B_i[\theta_{0_i} + \epsilon_i(t)]\xi_i(t) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} H_{ij}(x_j) && (9) \\ y_i &= C_i x_i && (10) \end{aligned}$$

For system (9)-(10), construct dynamical systems

$$\begin{aligned} \dot{\hat{x}}_i &= A_i\hat{x}_i + f_i(\hat{x}_i, u_i) + L_i(y_i - \hat{y}_i) \\ &\quad + B_i[\hat{\theta}_i(t) - \hat{\epsilon}_i(t)]\xi_i(t) - 2P_i^{-1}(F_iC_i)^T|\xi_i(t)|\ \epsilon_{0_i} \\ &\quad \times\psi_i(\hat{y}_i, y_i) + \sum_{\substack{j=1 \\ j\neq i}}^{N} H_{ij}(\hat{x}_j) && (11) \\ \hat{y}_i &= C_i\hat{x}_i && (12) \end{aligned}$$

where

$$\psi_i(\hat{y}_i, y_i) = \begin{cases} \dfrac{F_i(\hat{y}_i - y_i)}{\|F_i(\hat{y}_i - y_i)\|}, & F_i(\hat{y}_i - y_i) \neq 0 \\ 0, & F_i(\hat{y}_i - y_i) = 0 \end{cases} \quad (13)$$

for $i = 1, 2, \cdots, N$, and $\hat{\theta}_i(t)$ and $\hat{\epsilon}_i(t)$ are given by the adaptive laws as follows

$$\dot{\hat{\theta}}_i(t) = -2\delta_i(F_i(\hat{y}_i - y_i))^T\xi_i(t) \quad (14)$$

$$\dot{\hat{\epsilon}}_i(t) = 2(F_i(\hat{y}_i - y_i))^T\xi_i(t) \quad (15)$$

where δ_i are positive constants for $i = 1, 2, \cdots, N$. Let

$$\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_{0_i} \quad (16)$$

$$\tilde{\epsilon}_i(t) = \hat{\epsilon}_i(t) - \epsilon_{0_i} \quad (17)$$

where the unknown constants θ_{0_i} and the known constants ϵ_{0_i} satisfy the inequality in Assumption 3.

Let $e_{x_i} = \hat{x}_i - x_i$. Then, from systems (9)-(10) and (11)-(12), the error dynamical systems can be described by

$$\begin{aligned} \dot{e}_{x_i} &= (A_i - L_iC_i)e_{x_i} + [f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] + B_i\tilde{\theta}_i(t)\xi_i(t) \\ &\quad - B_i\hat{\epsilon}_i(t)\xi_i(t) - B_i\epsilon_i(t)\xi_i(t) \\ &\quad - 2P_i^{-1}(F_iC_i)^T|\xi_i(t)|\epsilon_{0_i}\psi_i(\hat{y}_i, y_i) && (18) \end{aligned}$$

where $\tilde{\theta}_i(t)$ is defined in (16).

Theorem 1. Under Assumptions $1 - 4$, the error dynamical systems (18) with adaptive laws (14) - (15) are globally uniformly bounded if the matrix $W^T + W$ is positive definite, where the matrix $W = [w_{ij}]_{N\times N}$ and its entries w_{ij} are defined by

$$w_{ij} = \begin{cases} \lambda_{\min}(Q_i) - 2\ell_{f_i}\|P_i\|, & i = j \\ -2\|P_i\|\ell_{H_{ij}}, & i \neq j \end{cases} \quad (19)$$

where P_i, and Q_i satisfy Lyapunov equation in (3). Further, the errors e_{x_i} given in (18) satisfy

$$\lim_{t\to\infty}\|e_{x_i}(t)\| = 0, \qquad i = 1, 2, \ldots, N \quad (20)$$

Proof. For system (14)-(15) and (18), consider the candidate Lyapunov function

$$V = \sum_{i=1}^{N} e_{x_i}^T P_i e_{x_i} + \frac{1}{2}\sum_{i=1}^{N}(\frac{1}{\delta_i}\tilde{\theta}_i^2(t) + \tilde{\epsilon}_i^2(t)) \quad (21)$$

where δ_i is a positive constants for $i = 1, 2, \cdots, N$. Then, from (18)

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N}(\dot{e}_{x_i}^T P_i e_{x_i} + e_{x_i}^T P_i \dot{e}_{x_i}) + \sum_{i=1}^{N}(\frac{1}{\delta_i}\tilde{\theta}_i(t)\dot{\hat{\theta}}_i(t) \\ &\quad + \tilde{\epsilon}_i(t))\dot{\hat{\epsilon}}_i(t)) \\ &= \sum_{i=1}^{N}\Big\{e_{x_i}^T[(A_i - L_iC_i)^TP_i + P_i(A_i - L_iC_i)]e_{x_i} \\ &\quad + 2e_{x_i}^TP_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\ &\quad + 2e_{x_i}^TP_i\sum_{\substack{j=1 \\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] \\ &\quad + 2e_{x_i}^TP_iB_i\tilde{\theta}_i(t)\xi_i(t) - 2e_{x_i}^TP_iB_i\epsilon_i(t)\xi_i(t) \\ &\quad - 2e_{x_i}^TP_iB_i\hat{\epsilon}_i(t)\xi_i(t) + \frac{1}{\delta_i}\tilde{\theta}_i(t)\dot{\hat{\theta}}_i(t) \\ &\quad + \tilde{\epsilon}_i(t)\dot{\hat{\epsilon}}_i(t) - 4e_{x_i}^TP_iP_i^{-1}(F_iC_i)^T|\xi_i(t)| \\ &\quad \times\epsilon_{0_i}\psi_i(\hat{y}_i, y_i)\Big\} && (22) \end{aligned}$$

By using condition (4) and $C_ie_{x_i} = \hat{y}_i - y_i$,

$$\begin{aligned} e_{x_i}^TP_iB_i &= ((P_iB_i)^Te_{x_i})^T = (B_i^TP_ie_{x_i})^T \\ &= (F_iC_ie_{x_i})^T = (F_i(\hat{y}_i - y_i))^T && (23) \end{aligned}$$

From (22) and (23)

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N}\Big\{e_{x_i}^T[(A_i - L_iC_i)^TP_i + P_i(A_i - L_iC_i)]e_{x_i} \\ &\quad + 2e_{x_i}^TP_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\ &\quad + 2e_{x_i}^TP_i\sum_{\substack{j=1 \\ j\neq i}}^{N}[H_{ij}(\hat{x}_j) - H_{ij}(x_j)] \\ &\quad + [2(F_i(\hat{y}_i - y_i))^T\xi_i(t) + \frac{1}{\delta_i}\dot{\hat{\theta}}_i(t)]\tilde{\theta}_i(t) \\ &\quad - 2(F_i(\hat{y}_i - y_i))^T\epsilon_i(t)\xi_i(t) \\ &\quad - 2(F_i(\hat{y}_i - y_i))^T\hat{\epsilon}_i(t)\xi_i(t) \\ &\quad + \tilde{\epsilon}_i(t)\dot{\hat{\epsilon}}_i(t) - 4(F_i(\hat{y}_i - y_i))^T|\xi_i(t)|\epsilon_{0_i}\psi_i(\hat{y}_i, y_i)\Big\} && (24) \end{aligned}$$

From (16), it can be seen that $\dot{\tilde{\theta}}_i(t) = \dot{\hat{\theta}}_i(t)$ and by substituting (13) and (14) into (24) gives

$$\begin{aligned}\dot{V} = \sum_{i=1}^{N} \Big\{ & e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} \\ & +2e_{x_i}^T P_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\ & +2e_{x_i}^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N} [H_{ij}(\hat{x}_j) - H_{ij}(x_j)] \\ & -2(F_i(\hat{y}_i - y_i))^T \epsilon_i(t)\xi_i(t) \\ & -2(F_i(\hat{y}_i - y_i))^T \hat{\epsilon}_i(t)\xi_i(t) \\ & +\tilde{\epsilon}_i(t))\dot{\tilde{\epsilon}}_i(t) - 4\|F_i(\hat{y}_i - y_i)\| \; |\xi_i(t)|\epsilon_{0_i} \Big\}\end{aligned}$$

From (17), it can be seen that $\dot{\tilde{\epsilon}}_i(t) = \dot{\hat{\epsilon}}_i(t)$.

$$\begin{aligned}\dot{V} = \sum_{i=1}^{N} \Big\{ & e_{x_i}^T[(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i)]e_{x_i} \\ & +2e_{x_i}^T P_i[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\ & +2e_{x_i}^T P_i \sum_{\substack{j=1 \\ j\neq i}}^{N} [H_{ij}(\hat{x}_j) - H_{ij}(x_j)] \\ & -2(F_i(\hat{y}_i - y_i))^T \epsilon_i(t)\xi_i(t) \\ & -[2(F_i(\hat{y}_i - y_i))^T \xi_i(t) - \dot{\tilde{\epsilon}}_i(t)]\hat{\epsilon}_i(t) - \epsilon_{0_i}\dot{\tilde{\epsilon}}_i(t) \\ & -4\|F_i(\hat{y}_i - y_i)\| \; |\xi_i(t)|\epsilon_{0_i} \Big\} \end{aligned} \tag{25}$$

Substituting (15) into (25) gives

$$\begin{aligned}\dot{V} \le & \sum_{i=1}^{N} \Big\{ - e_{x_i}^T Q_i e_{x_i} + 2\|e_{x_i}\|\|P_i\|[f_i(\hat{x}_i, u_i) - f_i(x_i, u_i)] \\ & +2\|e_{x_i}\|\|P_i\| \sum_{\substack{j=1 \\ j\neq i}}^{N} [H_{ij}(\hat{x}_j) - H_{ij}(x_j)] \\ & -2(F_i(\hat{y}_i - y_i))^T \xi_i(t)[\epsilon_i(t) + \epsilon_{0_i}] \\ & -4\|F_i(\hat{y}_i - y_i)\| \; |\xi_i(t)|\epsilon_{0_i} \Big\} \\ \le & \sum_{i=1}^{N} \{-e_{x_i}^T Q_i e_{x_i} + 2\|e_{x_i}\|\|P_i\|[\ell_{f_i}\|\hat{x}_i - x_i\|] \\ & +2\|e_{x_i}\|\|P_i\| \sum_{\substack{j=1 \\ j\neq i}}^{N} [\ell_{H_{ij}}\|\hat{x}_j - x_j\|]\} \\ \le & -\sum_{i=1}^{N} \{\lambda_{\min}(Q_i) - 2\|P_i\|\ell_{f_i}\|e_{x_i}\|^2 \\ & -\sum_{\substack{j=1 \\ j\neq i}}^{N} [2\|P_i\|\ell_{H_{ij}}]\|e_{x_i}\|\|e_{x_j}\| \Big\} \end{aligned} \tag{26}$$

Then, from the definition of the matrix W in (19) and the inequality above, it follows that

$$\dot{V} \le -\frac{1}{2} X^T[W^T + W]X$$

where $X = [\|\tilde{e}_{x_1}\|, \|\tilde{e}_{x_2}\|, \cdots, \|\tilde{e}_{x_N}\|]^T$.

From LaSalle-Yoshizawa Theorem in (Krstic *et al* (1995)) all the solutions of (18) are globally uniformly bounded and satisfy

$$\lim_{t\to\infty} X^T[W^T + W]X = 0 \tag{27}$$

Hence, the conclusion follows from $W^T + W > 0$. $\triangle$.

Remark 2. Theorem 1 shows that the estimated states $\hat{x}_i$ given in the observer (11) converge to the system states x_i in (1) asymptotically. In addition, it shows that the augmented systems formed by (18) and the adaptive laws (14)-(15) are uniformly bounded.

4. CASE STUDY: A COUPLED INVERTED PENDULUM

Consider the system formed by two inverted pendulums connected by a spring as given in Figure 1. Let $\varphi_1 = x_{11}, \varphi_2 = x_{21}, \dot{\varphi}_1 = x_{12}$, and $\dot{\varphi}_2 = x_{22}$ (see e.g. Chen & Li (2008)). The coupled inverted pendulums can be modelled as

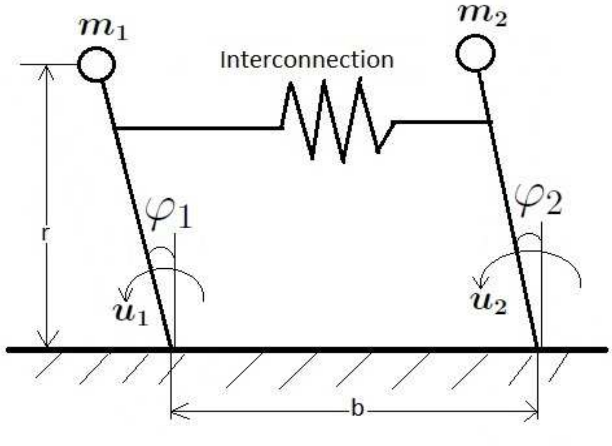

Fig. 1. Coupled inverted pendulums

$$\begin{aligned}\dot{x}_1 = & \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ (\frac{m_1 gr}{J_1} - \frac{kr^2}{4J_1})\sin(x_{11}) + \frac{1}{J_1}u_1 \end{bmatrix} \\ & + \begin{bmatrix} 0 \\ \frac{kr}{2J_1} \end{bmatrix}(l - b) + \begin{bmatrix} 0 \\ \frac{kr^2}{4J_1}\sin(x_{21}) \end{bmatrix}\end{aligned} \tag{28}$$

$$y_1 = [\,1 \;\; 1\,] \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \tag{29}$$

$$\begin{aligned}\dot{x}_2 = & \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ (\frac{m_2 gr}{J_2} - \frac{kr^2}{4J_2})\sin(x_{21}) + \frac{1}{J_2}u_2 \end{bmatrix} \\ & + \begin{bmatrix} 0 \\ \frac{kr}{2J_2} \end{bmatrix}(l - b) + \underbrace{\begin{bmatrix} 0 \\ \frac{kr^2}{4J_2}\sin(x_{11}) \end{bmatrix}}_{H_{21}(x_1)} +\end{aligned} \tag{30}$$

$$y_2 = [\,1 \;\; 1\,] \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \tag{31}$$

The end masses of pendulums are $m_1 = 1.5$ kg and $m_2 = 1$ kg, the moments of inertia are $J_1 = 5$ kg and $J_2 = 4$ kg, the constant of connecting spring is $k = 100$ N/m, the pendulum height is $r = 0.3$ m, and the gravitational acceleration is $g = 9.81$ m/s^2. In order to illustrate the developed theoretical results, it is assumed that $(l - b) = \theta_i(t)$ is an unknown time varying parameter for $i = 1, 2$ where l is the natural length of spring and b is the distance between the two pendulum hinges.

In order to avoid system states going to infinity, and for simulation purposes, the following feedback transformation is introduced

$$u_i = -k_i x_i + v_i, \qquad i = 1, 2 \tag{32}$$
$$k_1 = [\,10 \;\; 15\,] \qquad \text{and} \qquad k_2 = [\,8 \;\; 12\,]$$

Then, the system (28)-(31) can be rewritten as

$$\dot{x}_1 = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A_1} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0.4329\sin(x_{11}) + \frac{1}{5}v_1 \end{bmatrix}}_{f_1(x_1,u_1)}$$
$$+ \underbrace{\begin{bmatrix} 0 \\ 3 \end{bmatrix}}_{B_1}(l - b) + \underbrace{\begin{bmatrix} 0 \\ 0.45\sin(x_{21}) \end{bmatrix}}_{H_{12}(x_2)} \tag{33}$$

$$y_1 = \underbrace{[\,1 \;\; 1\,]}_{C_1} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \tag{34}$$

$$\dot{x}_2 = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A_2} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0.17325\sin(x_{21}) + \frac{1}{4}v_2 \end{bmatrix}}_{f_2(x_2,u_2)}$$
$$+ \underbrace{\begin{bmatrix} 0 \\ 3.75 \end{bmatrix}}_{B_2}(l - b) + \underbrace{\begin{bmatrix} 0 \\ 0.5625\sin(x_{11}) \end{bmatrix}}_{H_{21}(x_1)} \tag{35}$$

$$y_2 = \underbrace{[\,1 \;\; 1\,]}_{C_2} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \tag{36}$$

Choose $L_i = [0 \quad 0]$, and $Q_i = 4I$ for $i = 1, 2$. It follows that the Lyapunov equations (3) have unique solutions:

$$P_i = \begin{bmatrix} 5 & 1 \\ 1 & 1 \end{bmatrix}, \qquad i = 1, 2 \tag{37}$$

satisfying the condition (4) with $F_1 = 3$ and $F_2 = 3.75$. For simplicity, it is assumed that $\xi_i(t) = 1$, $\epsilon_{0_i} = 1$ and $\delta_i = 100$ for $i = 1, 2$.

By direct computation, it follows that the matrix $W^T + W$ is positive definite. Thus, all the conditions of Theorem 1 are satisfied. This implies that the dynamical system

$$\dot{\hat{x}}_1 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.4329\sin(\hat{x}_{11}) + \frac{1}{5}v_1 \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ 3 \end{bmatrix} (\hat{\theta}_1(t) - \hat{\epsilon}_1(t))$$
$$- \begin{bmatrix} 0 \\ 0.4 \end{bmatrix} \frac{(\hat{y}_1 - y_1)}{\|\hat{y}_1 - y_1\|} + \begin{bmatrix} 0 \\ 0.45\sin(\hat{x}_{21}) \end{bmatrix} \tag{38}$$

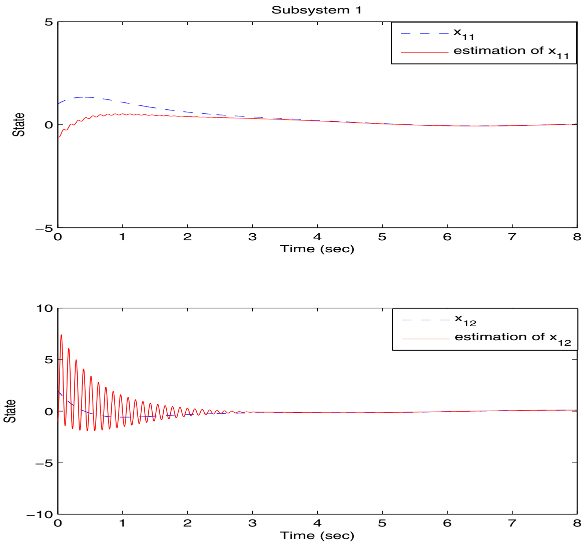

Fig. 2. The time response of the 1st subsystem states $x_1 = \text{col}\,(x_{11}, x_{12})$ and their estimation $\hat{x}_1 = \text{col}\,(\hat{x}_{11}, \hat{x}_{12})$.

$$\dot{\hat{\theta}}_1(t) = -200(3(\hat{y}_1 - y_1))^T \tag{39}$$
$$\dot{\hat{\epsilon}}_1(t) = 2(3(\hat{y}_1 - y_1))^T \tag{40}$$
$$\hat{y}_1 = [\,1 \;\; 1\,] \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{12} \end{bmatrix} \tag{41}$$

$$\dot{\hat{x}}_2 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.17325\sin(\hat{x}_{21}) + \frac{1}{4}v_2 \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ 3.75 \end{bmatrix} (\hat{\theta}_2(t) - \hat{\epsilon}_2(t))$$
$$- \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \frac{(\hat{y}_2 - y_2)}{\|\hat{y}_2 - y_2\|} + \begin{bmatrix} 0 \\ 0.5625\sin(\hat{x}_{11}) \end{bmatrix} \tag{42}$$

$$\dot{\hat{\theta}}_2(t) = -200(3.75(\hat{y}_2 - y_2))^T \tag{43}$$
$$\dot{\hat{\epsilon}}_2(t) = 2(3.75(\hat{y}_2 - y_2))^T \tag{44}$$
$$\hat{y}_2 = [\,1 \;\; 1\,] \begin{bmatrix} \hat{x}_{21} \\ \hat{x}_{22} \end{bmatrix} \tag{45}$$

is a robust observer of the system (33)-(36).

For simulation purpose, θ_{0_i} and $\theta_i(t)$ are chosen as 0 and $0.1\sin t$ respectively for $i = 1, 2$. Simulation in Figures 2 and 3 shows that the developed results are effective.

5. CONCLUSION

In this paper, an adaptive observer design for a class of nonlinear large scale interconnected systems with unknown time varying parameters has been proposed. It is assumed that the unknown parameters vary within a specific range. A set of sufficient conditions has been developed such that the error system with adaptive laws is globally uniformly bounded. Case study on a coupled inverted pendulum system shows the practicability of the developed observer scheme for nonlinear interconnected systems.

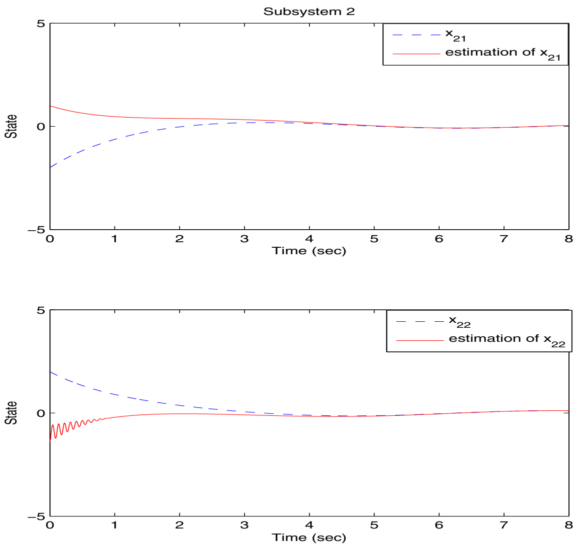

Fig. 3. The time response of the 2nd subsystem states $x_2 = \text{col}\ (x_{21}, x_{22})$ and their estimation $\hat{x}_2 = \text{col}\ (\hat{x}_{21}, \hat{x}_{22})$.

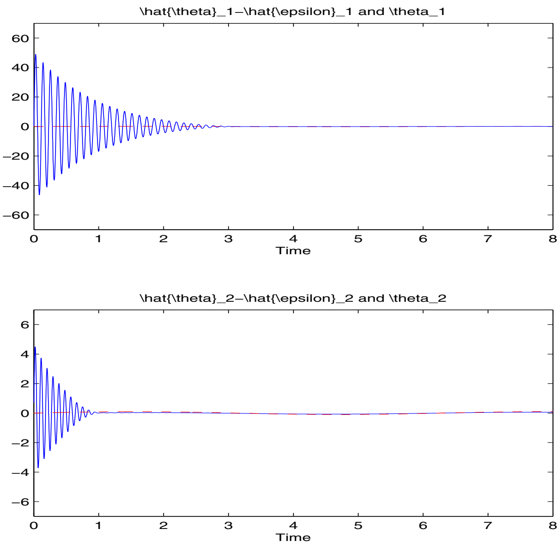

Fig. 4. Upper figure: the time response of $\hat{\theta}_1(t) - \hat{\epsilon}_1(t)$ (solid line) and $\theta_1(t)$ (dashed line); Bottom figure: the time response of $\hat{\theta}_2(t) - \hat{\epsilon}_2(t)$ (solid line) and $\theta_2(t)$ (dashed line).

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