The continuous single source location problem with capacity and zone-dependent fixed cost: Models and solution approaches*

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Abstract

The continuous capacitated single-source multi-facility Weber problem with the presence of facility fixed cost is investigated. A new mathematical model which incorporates multi-level type capacity (or design) and facility fixed cost that is capacity-based and zone-dependent is introduced. As no data set exists for this new location problem, a new data set based on convex polygons using triangular shape is constructed. A generalised two stage heuristic scheme that combines the concept of aggregation, an exact method, and an enhanced Cooper's alternate location-allocation method is put forward. A framework that embeds Variable Neighbourhood Search is also proposed. Computational experiments show that these matheuristics produce encouraging results for this class of location problems. The proposed approaches are also easily adapted to cater for a recently studied variant namely the single-source capacitated multi-facility Weber problem where they outperform those recently published solution methods.

Keywords: location, continuous space, capacity and fixed cost, single-source, matheuristics.

1. Introduction

The Multi-facility Weber problem (MFWP) deals with finding the location of m facilities in the continuous space and the allocation of each customer to the m chosen facilities so that the sum of the total transportation costs is minimised. This problem, also known as the planar location-allocation problem, is classified as the Multi-facility Weber problem if the demand

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or weight of all customers is unity and as the generalised MFWP otherwise. Cooper (1963 and 1972) shows that the objective function of MFWP is neither concave nor convex and may contain multiple local minima which makes the problem difficult to solve using exact methods. Hence, the MFWP falls in the realm of global optimisation problems. In addition, the MFWP is shown to be NP-hard, see Megiddo and Supowit (1984) and Sherali and Nordai (1988).

The single source location problem arises in situations where customers must be served by one facility only which is referred to as the single source capacitated multi-facility Weber problem (SCCMFWP). For instance, in a telecommunication network design, a user is assigned to a single base transceiver station, while when locating oil drill platforms, each oil well has to be allocated to one platform (Devine and Lesso, 1972 and Rosing, 1992). In real life applications, it is also worth taking into account a set up or opening cost of a facility which may be dependent on geographical areas (zones) and/or a throughput rate (capacity) of the facility. For example, for political, environmental or economic reasons, there are some governments that implement different tax policies for urban, suburban, and remote regions or regional restrictions as some areas are under government protections such as forests, lakes, rivers, etc. As a result, in some areas there may be cheaper opening costs of locating a facility whereas in others extortionate costs could be imposed. This paper proposes a mathematical model and solution methods to deal with such a strategic decision problem which, in many cases, require a massive investment.

In this study we aim to

(i) propose a new mathematical model for the SCCMFWP considering the fixed costs that are capacity-based and zone-dependent (SSCMFWP-FC)

(ii) develop an effective two stage heuristic approach and introduce an enhanced Variable Neighbourhood Search

(iii) construct new data sets for the SSCMFWP-FC and record promising results, and

(iv) produce several new best solutions for the recently studied location problem variant namely the SSCMFWP.

The paper is organised as follows. In the next section, a review of the relevant literature is provided. The section thereafter presents the mathematical models for both the SSCMFWP and the SSCMFWP-FC. In Section 4, the proposed solution frameworks are described followed by the computational results in Section 5. The adaptation and the implementation of our approach for the SSCMFWP problem are presented in Section 6. Finally, our conclusions and some highlights of future research are given in the last section.

2. Literature review

In this section we first briefly look at works that treat problems similar to ours. This is followed by a more detailed description of the few papers on the single-source capacitated multi-facility Weber Problem itself. For a comprehensive review on the MFWP which has attracted much research attention in the literature, the reader can refer to the works of Brimberg et al. (2008) and Brimberg et al. (2014). The discrete case is not reviewed here, but for completeness see Correia and Captivo (2003) for the capacitated case, and Correira and Captivo (2006) for the single source case.

(i) A brief on the capacitated MFWP (CMFWP)

Cooper (1972) is among the first who puts forward exact and heuristic methods for the CMFWP. The latter is the well-known alternating transportation-location (ATL for short) heuristic. Basically, ATL is a modification of the heuristic (ALA) originally developed by Cooper (1964) for the MFWP. This technique is based on alternately solving the location-allocation problem and the Transportation Problem (TP) until there is less than epsilon (ε) improvement found in the total cost. It is worth noting that once the facilities are located, the CMFWP reduces to the classical TP. Sherali and Shetty (1977) propose a convergent cutting plane algorithm, which is originally derived from a bilinear programming problem, to solve the rectilinear distance CMFWP.

Sherali and Tuncbilek (1992) revisit the problem using a distance proportional to the square of the Euclidean distance. They design a branch and bound algorithm to compute strong upper bounds. Sherali et al. (1994) formulate the rectilinear distance CMFWP as a mixed integer nonlinear programming model, and develop a reformulation-linearization technique to transform the problem into a linear mixed-integer program. Sherali et al. (2002) also design a branch and bound technique based on partitioning the allocation space to construct global optimisation procedures for the Euclidean and ℓ_p distance CMFWP.

Zainuddin and Salhi (2007) propose a perturbation-based heuristic to tackle the CMFWP. This scheme considers borderline customers whose locations lie approximately half-way between their nearest and their second nearest facilities. Aras et al. (2007a) develop three heuristic techniques which include Lagrangean heuristic, the discrete p-capacitated facility location heuristic which is similar to the p-median method of Hansen et al. (1998), and the cellular heuristic of Gamal and Salhi (2003) to solve the CMFWP with Euclidean, squared Euclidean, and ℓ_p distances. Aras et al. (2007b) adopt simulated annealing, threshold accepting, and genetic algorithms to deal with the CMFWP with rectilinear, Euclidean, squared Euclidean, and ℓ_p distances. In a following study, Aras et al. (2008) adapt their earlier approaches to tackle the CMFWP with rectilinear distance.

Luis et al. (2009) study the CMFWP by designing restricted regions within their constructive heuristic which forbid new locations to be sited too close to the previously found locations. A discretisation method which divides a continuous space into a discrete number of cells while embedding the use of restricted regions within the search is also put forward. Mohammadi et al. (2010) design two genetic algorithms (GAs); one for the location problem and the other for the allocation of customers to those open facilities. Luis et al. (2011) present a novel guided reactive greedy randomised adaptive search procedure by designing a framework that combines adaptive learning with the concept of restricted regions. Akyüz et al. (2014) study the CMFWP by developing two branch and bound algorithms with the first designed for the allocation space whereas the second for the partition of the location space.

(ii) A brief on the SSCMFWP

The literature on the SSCMFWP is very scarce. Gong et al. (1997) propose a hybrid evolutionary method based on GA to search the locatable area and hence find the global or near global solutions. In the allocation stage, a Lagrangean relaxation approach is applied. Experiments are carried out on randomly generated data with the number of facilities (m) varying from 2 to 6.

Manzour al-Ajdad et al. (2012) develop an iterative two phase heuristic algorithm to tackle the problem. In the first phase or location phase, the ALA method of Cooper (1964) is modified by introducing two assignment rules namely the simplified and parallel assignments respectively. In the second phase or the allocation phase, customers are allocated to facilities by solving optimally the generalised assignment problem. A simulated annealing algorithm is also used as an alternative solution in the first phase. Data sets taken from the literature are adapted accordingly to cater for the SSCMFWP. The results are compared with a general MINLP solver BARON that is run for a limited time. The authors claim that their proposed methods provide better results than BARON. Manzour et al. (2013) produce a simpler version to the one proposed by Manzour al-Ajdad et al. (2012) but with slightly inferior results.

Öncan (2013) investigates the SSCMFWP with Euclidean and Rectilinear distances by proposing three solution methods. The first one is the Single-Source ALA method which is an improved version of Cooper's ALA method (Cooper, 1964) when the allocation phase is solved optimally. In the second one, a very large neighbourhood search procedure is employed within the first method to solve the allocation problem efficiently. In the third method, a discrete approximation technique that uses Lagrangean Relaxation is put forward to find lower and upper bounding procedures for the SSCMFWP. Experiments are performed using three classes of instances from the literature as well as newly randomly generated data sets. Competitive empirical results are produced when compared to the published work though these are found to be relatively inferior in some instances to those given by Manzour al-Ajdad et al. (2012).

(iii) A brief on the Weber problem in the presence of fixed cost

Most of the literature on the facility location problems with fixed costs focus on the discrete space, see for instance the recent papers by Rahmani and MirHassani (2014), Guastaroba and Speranza (2014), Farahani et al. (2014), and Ho (2015). However, in the continuous location problem there is a shortage of references which investigate the presence of fixed costs. Brimberg et al. (2004) study the multi-source Weber problem with constant fixed cost and design a multiphase heuristic to deal with the problem. The discrete version of the problem is first solved to find an approximate number of facilities and then the facility configuration is improved by applying Cooper's ALA scheme. Brimberg and Salhi (2005) propose fixed costs which are zone-dependent for locating a single facility in the continuous space. The zones are defined as non-overlapping convex polygons. An efficient approach is presented to optimally solve the problem. A discretization approach to deal with multi facility problem is also designed. Both studies are focused on the uncapacitated case.

Luis et al. (2015) deal with CMFWP by introducing three types of fixed costs which are constant, zone-based, and continuous fixed cost functions. Heuristic methods that adopt the concept of restricted regions and a GRASP metaheuristic are used to tackle the problem. Competitive results are obtained when the methods are implemented using the four well-known data sets from the literature (see Brimberg et al. (2000). Hosseininezhad et al. (2015) propose a cross entropy heuristic to solve CMSWP with a zone-based fixed cost which includes production and installation costs. Numerical examples were generated as a platform to evaluate the methods. The results perform well when compared to the optimizer results performed by GAMS.

3. Mathematical formulation

In the single-source capacitated problem, we are given a set of customers, located at n fixed points, with their respective demands. The aim is to (i) locate m facilities in a continuous space, (ii) determine the capacity of each facility and (iii) allocate customers to exactly one facility without violating the capacity of the facility while minimising the sum of the transportation costs. We first present the mathematical model for the SSCMFWP followed by the one for the SSCMFWP-FC. For completeness we also provide at the end of this section, the discrete counterpart variants to SSCMFWP and SSCMFWP-FC as we shall use these for benchmarking purposes. We refer to these variants as the Discrete Multi Facility Location Problem (DMFLP) and the Discrete Multi Facility Location Problem with Fixed Cost (DMFLP-FC) respectively.

3.1. Mathematical model of the SSCMFWP

The following notations are used to describe the sets and parameters of the SSCMFWP model.

Notations

Sets and Parameters

I : set of customers with i as its index

m : the number of facilities

n : the number of customers

$a_i = \left(x_i^a, y_i^a\right)$: location of customer i where $a_i \in \Re^2, i \in I$;

w_i : demand or weight of customer i, $i \in I$;

Q_j : capacity of facility j, $j = 1,\ldots,m$;

Decision Variables

$$Y_{ij} = \begin{cases} 1, \text{ if customer i is assigned to facility } j, \forall i \in I ; j = 1,\ldots m; \\ 0, \text{otherwise} \end{cases}$$

$X_j = (x_j, y_j)$: coordinates of facility j where $X_j \in \Re^2$, $\forall j = 1,...,m$;

Let $d(X_j, a_i)$ be the Euclidean distance between facility j and customer i.

The mathematical model of the SSCMFWP can be formulated as follows:

$$\text{Minimise} \quad \sum_{j=1}^{m}\sum_{i\in I}\left(Y_{ij} \cdot d(X_j, a_i) \cdot w_i\right) \tag{1}$$

Subject to

$$\sum_{j=1}^{m} Y_{ij} = 1, \qquad \forall i \in I \tag{2}$$

$$\sum_{i\in I}\left(w_i \cdot Y_{ij}\right) \le Q_j, \quad \forall j = 1,...,m \tag{3}$$

$$Y_{ij} \in \{0,1\} \qquad \forall i \in I,\ j = 1,...,m \tag{4}$$

$$X_j \in \Re^2 \quad \forall j = 1,...,m \tag{5}$$

The objective function (1) is to minimise the sum of the transportation costs. Constraints (2) ensure that each customer's demand has to be satisfied by exactly one facility. Constraints (3) guarantee that capacity constraints of the facilities are not violated. Constraints (4) and (5) refer to the binary nature of the variables and the continuous location variables, respectively. It is worth noting that once the allocations are known, the problem turns into m pure single facility location problems where each one can be solved optimally by the well-known iterative equations given by Weiszfeld (1937).

We also note that once the m facilities are located, the problem reduces to the generalised assignment problem (GAP) which can theoretically be solved optimally by any suitable ILP solver such as CPLEX, Lingo, GuRobi, or Xpress-MP. The mathematical formulation of the GAP is relatively similar to SSCMFWP except that (5), relating to the location of facilities (X_j), is now fixed turning $d(X_j, a_i)$ to be known which reduces the problem to an integer (binary) linear programming problem (ILP). Equations (1) – (4) are then used to deal with the GAP. This is still relatively more difficult to solve due to the binary nature of the decision variables (Y_{ij}) compared to its counterpart the Transportation Problem (TP) which is usually applied at the allocation phase when solving the multi-source capacitated multi-facility Weber problem (see Luis et al., 2011).

3.2. Mathematical model of the SSCMFWP-FC

This subsection presents the mathematical model of the new problem SSCMFWP-FC where the fixed cost is taken into account and the capacity of the facilities is also considered as a decision variable. The fixed cost may not always be based on the chosen capacity but, in many situations, it is linked to the region/zone where the facility is located. Here, as the location of each facility is unknown, the region/zone is also treated as a decision variable. In this study, for simplicity, we consider the shape of each zone to be a convex polygon. If it is not the case, we simply decompose any non-convex polygon into a number of smaller convex areas as commonly used in the literature (see Fernandez et al., 2000).

The D-function (Chernov et al., 2009), which is used in our formulation, is utilised to determine whether a point is inside a convex polygon or not. For instance, given two points P1(x_1, y_1) and P2(x_2, y_2), the corresponding edge vector of the polygon $\overrightarrow{P_1P_2}$ has four parameters defined as follows:

$$\Delta = d(P1, P2) = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}\ ; \Omega = \frac{(y_1 - y_2)}{\Delta}; \Phi = \frac{(x_2 - x_1)}{\Delta}$$

$$\Psi = \frac{(x_1 \cdot y_2 - x_2 \cdot y_1)}{\Delta} \tag{6}$$

A point, say point P3 (x_3, y_3), is inside the polygon if it is on the right hand side of all edges. In other words, we compute $\Lambda = \Omega \cdot x_3 + \Phi \cdot y_3 + \Psi$ and check whether point P3 lies on the edge (i.e., $\Delta = 0$), left hand side (i.e., $\Delta > 0$) or right hand side(i.e., $\Delta < 0$).

The notations used for sets and parameters in this model are similar to the ones given earlier with the following minor additions:

Notations

Set and Parameters

R : set of regions/zones.

D_r : set of capacity designs for facilities located in zone/area r ($r \in R$).

F_{rd} : fixed cost of a facility located in zone r using design d ($r \in R, d \in D_r$)

b_{rd} : the capacity of a facility located in area r using design d ($r \in R, d \in D_r$)

E_r : set of edges of zone r ($r \in R$) with e as its index. The edges make up a convex polygon (zone) where one or more facilities can be located

Ω_{re}, Φ_{re}, and Ψ_{re} : the parameters for edge e of zone r ($r \in R$, $e \in E_r$) as defined in (6)

$U = \underset{i \in I}{Max}(Max(x_i^a, y_i^a, 0))$: a large number used in later equation to check whether a facility is inside a certain region/zone.

Decision Variables

$$Y_{ij} = \begin{cases} 1, \text{ if customer i is assigned to facility } j, \forall i \in I;\ j = 1,...m; \\ 0, \text{otherwise} \end{cases}$$

$$S_{jrd} = \begin{cases} 1, \text{ if facility } j \text{ is located in area r and using design } d, \forall r \in R; d \in D;\ j = 1,...,m; \\ 0, \text{otherwise} \end{cases}$$

$X_j = (x_j, y_j)$: coordinates of facility j where $X_j \in \Re^2$; $j = 1,...,m$.

Note that the design of a facility can be defined by the type of machinery, the capacity, etc. The problem SSCMFWP-FC can be modelled as a binary nonlinear problem as follows.

Objective function:

$$\text{Minimise} \quad \sum_{j=1}^{m}\sum_{i \in I}\left(Y_{ij} \cdot w_i \cdot d(X_j, a_i)\right) + \sum_{j=1}^{m}\sum_{r \in R}\sum_{d \in D_r}\left(F_{rd} \cdot S_{jrd}\right) \tag{7}$$

Subject to

$$\sum_{j=1}^{m} Y_{ij} = 1, \quad \forall i \in I \tag{8}$$

$$\sum_{i \in I}\left(Y_{ij} \cdot w_i\right) \le \sum_{r \in R}\sum_{d \in D_r}\left(b_{rd} \cdot S_{jrd}\right), \quad \forall j = 1,...,m \tag{9}$$

$$\sum_{r \in R}\sum_{d \in D_r} S_{jrd} = 1, \quad \forall j = 1,...,m \tag{10}$$

$$\Omega_{re} \cdot x_j + \Phi_{re} \cdot y_j + \Psi_{re} \le U \cdot (1 - S_{jrd}), \forall r \in R; e \in E_r; d \in D_r; j = 1,...,m; \tag{11}$$

$$Y_{ij} \in \{0,1\}, \quad \forall i \in I; j = 1,...,m \tag{12}$$

$$S_{jrd} \in \{0,1\}, \quad \forall r \in R; d \in D_r; j = 1,...,m; \tag{13}$$

$$X_j \in \Re^2, \quad \forall j = 1,...,m \tag{14}$$

The objective function (7) is to minimise the sum of the total costs including the transportation and the opening facilities fixed costs. Constraints (8) guarantee that each demand point is served by one facility. Constraints (9) ensure that capacity constraints of the facilities are met. Constraints (10) make sure that a facility located in an area with one capacity assigned. Constraints (11) indicate the region/zone of the open facilities. Constraints (12) and (13) refer to the binary nature of the variables whereas Constraints (14) specify the continuous location variables.

In case the location of the m facilities are fixed or known, the problem can be treated as the assignment problem. However, the decision is not only to assign each customer to which facility but also to determine the capacity required by each facility. We refer to this assignment problem as the generalised assignment problem with fixed cost (GAP-FC). As the location of each facility is known, its corresponding zone (area) is also known. Therefore, the fixed cost is now only related to the location and the capacity of the facility $(\hat{F}_{jd})$ considering the location (region) cost. The mathematical model for the GAP-FC is as follows.

<u>Decision Variables</u>

$$Y_{ij} = \begin{cases} 1, \text{ if customer } i \text{ is assigned to facility } j, \forall i \in I\,;\; j = 1,...m; \\ 0, \text{otherwise} \end{cases}$$

$$S_{jd} = \begin{cases} 1, \text{ if facility } j \text{ uses design } d, \; \forall d \in D;\; j = 1,..., m; \\ 0, \text{otherwise} \end{cases}$$

The GAP-FC model

Objective Function

$$\text{Minimise} \quad \sum_{j=1}^{m} \sum_{i \in I} \left(Y_{ij} \cdot w_i \cdot d(X_j, a_i) \right) + \sum_{j=1}^{m} \sum_{d \in D} \left(\hat{F}_{jd} \cdot S_{jd} \right) \tag{15}$$

Subject to

$$\sum_{j=1}^{m} Y_{ij} = 1, \qquad \forall i \in I \tag{16}$$

$$\sum_{i \in I} \left(Y_{ij} \cdot w_i \right) \le \sum_{d \in D} \left(b_d \cdot S_{jd} \right), \qquad \forall j = 1,..., m \tag{17}$$

$$\sum_{d \in D} S_{jd} = 1, \qquad \forall j = 1,..., m \tag{18}$$

$$Y_{ij} \in \{0,1\}, \qquad \forall i \in I;\; j = 1,..., m \tag{19}$$

$$S_{jd} \in \{0,1\}, \qquad \forall d \in D;\; j = 1,..., m; \tag{20}$$

3.3. Mathematical model for the DMFLP-FC

As the SSCMFWP-FC model is nonlinear and non-convex, it cannot be solved optimally by an exact method using commercial software optimizer such as CPLEX. For benchmarking purposes, we also propose its linear discrete counterpart model, the DMFLP-FC. Similar to the previous model, let J be a set of potential facilities. The zone (area) for each potential facility is also known as its location is known. Therefore, the zone is not treated as a decision variable as each potential site is a zone on its own right. In the model, the fixed cost $(\hat{F}_{jd})$ is now only related to the location and the capacity of the potential facility $(\hat{b}_{jd})$ considering the location (zone) cost. Each potential facility j has a set of capacity designs (D_j) based on its corresponding area. The notations used for sets and parameters in the DMFLP-FC model are relatively similar to those in previous models with J being an additional set indexed by j.

Decision Variables

$$Y_{ij} = \begin{cases} 1, \text{ if customer } i \text{ is assigned to facility } j, \forall i \in I;\ j \in J; \\ 0, \text{otherwise} \end{cases}$$

$$S_{jd} = \begin{cases} 1, \text{ if facility } j \text{ is open and uses design } d, \forall j \in J; d \in D_j; \\ 0, \text{otherwise} \end{cases}$$

Objective function:

$$\text{Minimise} \sum_{j \in J} \sum_{i \in I} \left(Y_{ij} \cdot w_i \cdot d(X_j, a_i) \right) + \sum_{j \in J} \sum_{d \in D_j} \left(\hat{F}_{jd} \cdot S_{jd} \right) \tag{21}$$

Subject to

$$\sum_{j \in J} Y_{ij} = 1, \quad \forall i \in I \tag{22}$$

$$\sum_{i \in I} \left(Y_{ij} \cdot w_i \right) \le \sum_{d \in D_j} \left(\hat{b}_{jd} \cdot S_{jd} \right), \quad \forall j \in J \tag{23}$$

$$\sum_{d \in D_j} S_{jd} \le 1, \quad \forall j \in J \tag{24}$$

$$\sum_{j \in J} \sum_{d \in D_j} S_{jd} = m \tag{25}$$

$$Y_{ij} - \sum_{d \in D_j} S_{jd} \le 0, \quad \forall i \in I, j \in J \tag{26}$$

$$Y_{ij} \in \{0,1\}, \quad \forall i \in I; j \in J \tag{27}$$

$$S_{jd} \in \{0,1\}, \quad \forall j \in J, d \in D_j \tag{28}$$

Constraints (25) ensure that the number of open facilities is now fixed to m.

Note that DMFLP can be obtained from the above model by setting in the objective function $F_{jd} = 0 \ \forall d \in D_j ; j \in J$, and replacing constraints (25) by $\sum_{j \in J} S_{jd} = \rho_d \ \forall d \in D$, with ρ_d referring to the capacity design at facility d while D_j becomes constant and set to D.

4. The proposed solution methods for the SSCMFWP-FC

The above mathematical model is interesting and appropriate for small size instances only and hence powerful heuristic methods are the best way forward to tackle such a difficult and challenging location problem. For an overview of heuristic search in general, the reader will find the recent book by Salhi (2017) to be informative and easy to read. In this paper, we propose two heuristic based-methods for solving the SSCMFWP-FC. The first one is a generalised two-stage heuristic while the second is a VNS-based metaheuristic.

4.1. A generalised two-stage heuristic method

This approach can be categorised as a multi-start method consisting of two stages. We refer to this as the generalised two-stage heuristic method (GTSHM) whose main steps are depicted in Figure 1. In the initialisation stage, these zones/areas with non-convex shapes are first decomposed into convex polygons. This is achieved by applying classical methods such as the ones proposed by Fernández et al. (2000) which are known to be efficient and easy to implement.

Stage 1 aims to find a relatively good initial solution by solving the discrete counterpart of DMFLP-FC. When n and |J| are large, the DMFLP-FC is not easy to solve optimally. One way to overcome this shortcoming is to adopt an aggregation approach. There are several schemes that could be adopted such as a simple but guided randomised approach, a hierarchical agglomerative clustering, a $p-$median-based approach, customer aggregation method as proposed by Sankaran (2000), among others. A comprehensive review on aggregation techniques for large facility location problems is given in Irawan and Salhi (2015b). In our study, the number of potential facility sites is reduced to μ sites (μ << n) while all customers are fully served. To determine the μ potential facility sites, we first build μ clusters by solving the uncapacitated p-median problem based on the customers' locations with $\mu = p$ using the well-known local search proposed by Resende and Werneck (2007).

Initialisation

a. Find the zones/areas whose shape are non-convex and decompose those into convex polygons.

b. Define the number of iterations (T) and the reduced number of potential sites (μ). Set $z_{best} = \infty$.

For t = 1 to T do the following stages:

Stage 1

a. Reduce the number of potential facilities from n to μ by heuristically solving the p-median problem to generate μ clusters where $\mu = p$. Determine the centroid of each cluster using Equation (29) and treat these centroids as potential facility sites.

b. Solve the reduced discrete problem (DMFLP-FC) which consists of n customers and μ potential facilities using an exact method (CPLEX). Let z be its objective function value and for all j = 1 to m let $X_j(x_j, y_j)$ be the coordinates of facility j, N_j the set of customers to be served by facility j, A_j the area of facility j, and K_j the capacity design of facility j.

c. Set $\mu = \mu$-1.

Stage 2

a. Apply the proposed local search given in Figure 5 using z, X_j, N_j, A_j, K_j $\forall j = 1,\ldots,m$ obtained from Stage 1. If a better solution is found update all the above information

b. If $z < z_{best}$ update $z_{best} = z$ along with $X_j^{best} \leftarrow X_j$, $N_j^{best} \leftarrow N_j$, $A_j^{best} \leftarrow A_j$, and $K_j^{best} \leftarrow K_j$ $\forall j = 1,\ldots,m$

End for

Figure 1. The two stage heuristic method (GTSHM)

Let $\tilde{N}_c$ be the set of customers that belong to cluster c, c = 1,…, μ. The centroid of each cluster is determined using the following equations:

$$\tilde{x}_c = \frac{\sum_{i \in \tilde{N}_c} w_i \cdot x_i^a}{\sum_{i \in \tilde{N}_c} w_i} \; ; \; \tilde{y}_c = \frac{\sum_{i \in \tilde{N}_c} w_i \cdot y_i^a}{\sum_{i \in \tilde{N}_c} w_i}, \quad \forall c = 1,\ldots,\mu \tag{29}$$

The centroids of clusters are then treated as a set of potential facility sites. To diversify the search, the value of μ is adjusted systematically where μ value is reduced by one for the next iteration. A similar methodology has shown to be promising when solving large p-median problems (Irawan et al., 2014; Irawan and Salhi, 2015a) and p-centre problems (Irawan et al., 2016). Here, the reduced DMFLP-FC is solved by CPLEX.

Speeding-up mechanism- To speed up the search process, we relax the DMFLP-FC by ignoring the integrality requirements on the allocation variables Y_{ij}. The resulting MIP is relatively much easier to solve without a significant loss in solution quality. We also consider near optimal solutions by terminating CPLEX when a duality gap (%Gap) reached α which we set to 2.5%. In addition to the objective function value (z), by solving the reduced DMFLP-FC the location of the m facilities along with their capacity is obtained. These solutions are then fed into the next stage and treated as the initial solution.

In Stage 2, we propose a local search based on the ALA heuristic introduced by Cooper (1964). Here, we introduce some enhancements to cater for the characteristics of the SSCMFWP-FC where a multi-level type capacity is considered and the fixed cost is capacity-based and zone-dependent. The proposed local search also incorporates the Weiszfeld's formula to find a new location for an open facility on the plane. The main steps of the proposed local search are given in Figure 2. The process of using Stages 1 and 2 is repeated until a prescribed maximum number of iterations (T) is reached. Note that in case the location of a facility happens to lie on the boundary of two adjacent zones (areas), we break the tie by selecting the zone with the cheapest fixed cost.

The idea behind the use of such a multi-start process is that there is a lack of correlation between a good initial solution and a good final solution. For instance, Manzour al-Ajdad et al. (2012) refer to the best solution only which can, in our views, be restrictive. In our preliminary study, it reveals that the best solution is obtained not necessarily from the best initial solution. A similar lack of correlation is also well known in location-routing problems where the best location at the location stage does not necessarily lead to the best overall cost when routing is involved, see Salhi and Rand (1989), and Salhi and Fraser (1996) for the case of homogeneous and heterogeneous vehicle fleet respectively.

Procedure LocalSearch (z, X_j, N_j, A_j, $K_j, \forall j = 1,\dots,m$**)**

1. Define ε and $\hat{T}$
2. Set $\hat{X}_j \leftarrow X_j$, $\hat{N}_j \leftarrow N_j$, $\hat{A}_j \leftarrow A_j$, and $\hat{K}_j \leftarrow K_j$ for all $\forall j = 1,\dots,m$
3. Do the following steps $\hat{T}$ times:
   a. Update $\hat{X}_j$ and $\hat{A}_j$ $(j = 1,\dots,m)$ using the following:
      (i). Set j = 1.
      (ii). Calculate the total cost of facility j with $\hat{f} = F_{rd} + \sum_{i \in \hat{N}_j} d(\hat{X}_j, a_i)$ where $d = \hat{K}_j$ and $r = \hat{A}_j$.
      (iii). Set $\bar{X}_j \leftarrow \hat{X}_j$
      (iv). Repeat the following steps:
      - Calculate $\tilde{X}_j$ using Weiszfeld's equations as follows:

$$\tilde{x}_j = \frac{\sum_{i \in \hat{N}_j} \frac{w_i \cdot x_i^a}{d(\bar{X}_j, a_i)}}{\sum_{i \in \hat{N}_j} \frac{w_i}{d(\bar{X}_j, a_i)}} \; ; \quad \tilde{y}_j = \frac{\sum_{i \in \hat{N}_j} \frac{w_i \cdot y_i^a}{d(\bar{X}_j, a_i)}}{\sum_{i \in \hat{N}_j} \frac{w_i}{d(\bar{X}_j, a_i)}} \tag{30}$$

      - If d($\bar{X}_j$, $\tilde{X}_j$) $\le \varepsilon$ go to Step 3a(v).
      - Determine the area ($\tilde{A}_j$) of the new location ($\tilde{X}_j$) of facility j and use tie breaker if necessary
      - Calculate $\tilde{f} = F_{rd} + \sum_{i \in \hat{N}_j} d(\tilde{X}_j, a_i)$ where $d = \hat{K}_j$ and $r = \tilde{A}_j$.
      - If $\tilde{f} < \hat{f}$ update $\hat{f} = \tilde{f}$ along with $\hat{X}_j \leftarrow \tilde{X}_j$ and $\hat{A} \leftarrow \tilde{A}_j$.
      - Update $\bar{X}_j \leftarrow \tilde{X}_j$
      (v). If j = m go to Step 3b, otherwise set j = j + 1 and go to Step 3a(ii).
   b. If d($\hat{X}_j$, X_j) $\le \varepsilon$ $\forall j = 1,\dots,m$ go to Step 4.
   c. Solve the GAP-FC using $\hat{X}_j$ and $\hat{A}_j$. Let $\hat{z}$ be its objective function value and record $\hat{K}_j$ and $\hat{N}_j$ $\forall j = 1,\dots,m$.
   d. If $\hat{z} < z$ update $z = \hat{z}$ along with $X_j \leftarrow \hat{X}_j$, $N_j \leftarrow \hat{N}_j$, $A_j \leftarrow \hat{A}_j$, and $K_j \leftarrow \hat{K}_j$ for all $\forall j = 1,\dots,m$
4. Return z along with X_j, N_j, A_j, and K_j for all $\forall j = 1,\dots,m$.

Figure 2. The proposed local search

4.2. Enhanced VNS-based methods

VNS is a metaheuristic technique that comprises a local search and a neighbourhood search. The former aims to intensify the search while the latter aims to escape from the local optima through diversification by systematically changing the neighbourhood. For various variants and successful applications of VNS, the reader can refer to Hansen et al. (2010) and Brimberg et al. (2014). In this section, we propose a VNS-based method which we refer to as the Enhanced VNS (EVNS). This metaheuristic which consists of two stages is presented in Figure 3.

Initialisation

a. Identify the zones/areas whose shape are non-convex and decompose them into convex polygons.

b. Define T, k_{max} and μ. Set $z_{best} = \infty$.

Stage 1

Apply Stage 1 of GTSHM (Figure 1) T times and take the solution that yields the smallest objective function value (z) along with X_j, N_j, A_j, and K_j $\forall j = 1,...,m$

Stage 2

a. Update $z = z_{best}$ along with $X_j \leftarrow X_j^{best}$, $N_j \leftarrow N_j^{best}$, $A_j \leftarrow A_j^{best}$, and $K_j \leftarrow K_j^{best}$, $\forall j = 1,...,m$

b. Set k = 1

c. Shaking
Update X_j, $\forall j = 1,...,m$, using Procedure Shaking given in Figure 7 with k as an input.

d. Local search
Apply the proposed local search, given in Figure 5, using z, X_j, N_j, A_j and K_j $\forall j = 1,...,m$.

e. Move or Not
If $z < z_{best}$ then
- Set k = 1 and $z_{best} = z$ along with $X_j^{best} \leftarrow X_j$, $N_j^{best} \leftarrow N_j$, $A_j^{best} \leftarrow A_j$ and $K_j^{best} \leftarrow K_j$, $\forall j = 1,...,m$.

Else
- Set k = k+1 and $z = z_{best}$ along with $X_j \leftarrow X_j^{best}$, $N_j \leftarrow N_j^{best}$, $A_j \leftarrow A_j^{best}$, and $K_j \leftarrow K_j^{best}$, $\forall j = 1,...,m$.

f. If $k \le k_{max}$ go back to Stage 2(c), otherwise stop.

Figure 3. The Enhanced VNS (EVNS)

In Stage 1, a relatively good initial solution is first obtained by solving T reduced discrete problems (DMFLP-FC) using an exact method. The solution that yields the smallest objective function value is chosen as the one to be fed into the VNS-based algorithm. The shaking process is conducted by removing a randomly chosen facility from the current solution configuration and introducing a facility located at a customer site that is randomly selected (i.e., customer i) outside the forbidden regions. In this study, we define a forbidden region by a circle centred at the existing facility site with a radius ($\hat{r}$).

The steps of the shaking process are provided in Figure 4. In the local search, the algorithm presented in Figure 2 is also used here to find the local optima. In the move or not move step, a larger neighbourhood will be systematically used if an improvement is not found (i.e., k=k+1), otherwise the search reverts back to the first (i.e., the smallest) one (i.e., k=1). The search terminates when $k > k_{max}$.

Procedure Shaking (k, $(X_j)_{j=1,\dots,m}$)

1. Define β and γ.
2. Calculate $\hat{r}$ using the following Equation:

$$\hat{r} = \frac{\sum_{j=1}^{m}\left(\underset{i \in N_j}{Max}\left(d\left(X_j, a_i\right)\right)\right)}{m} \tag{31}$$

3. Do the following step k times
   - (i) Set $v = 0$ and $\breve{r} = \hat{r}$.
   - (ii) Select randomly a facility, say facility $\hat{j}$, from the set of open facilities $(X_j)_{j=1,\dots,m}$
   - (iii) Choose randomly customer i.
   - (iv) For j = 1 to m ($j \neq \hat{j}$) do the following mini steps:
     - If $d(X_j, a_i) < \breve{r}$ then
       - Set $v = v + 1$.
       - If $v = \beta$ then set $\breve{r} = \gamma \cdot \breve{r}$ and $v = 0$.
       - Go back to Step 3(iii).
   - (v) Update $X_{\hat{j}} \leftarrow a_i$.

Figure 4. The main steps of Procedure Shaking

5. Computational results of the SSCMFWP-FC

We carried out extensive experiments to examine the performance of the proposed heuristic approaches. These were coded in C++ .Net 2012 where we also used the IBM ILOG CPLEX version 12.6 Concert Library. The tests were run on a PC with an Intel Core i5 CPU @ 3.20GHz processor, 8.00 GB of RAM and under Windows 7. As there is no data available in the literature for the SSCMFWP-FC, we constructed a newly generated dataset with $n = 100$ to 1000 with an increment of 100 where we randomly generated the demand of each customer between 1 and 10 and fixed the number of areas/zones $|R|$ to $0.1n$. Figure 5 illustrates an example when n = 1000 and |R| = 100. As the shape of each region/zone is a convex polygon, we propose for convenience a triangular shape. We classify these regions into three categories, namely category 1, 2 and 3 which represent cheap, average, and expensive regions respectively. We vary the value of m from 5 to 25 with an increment of 5.

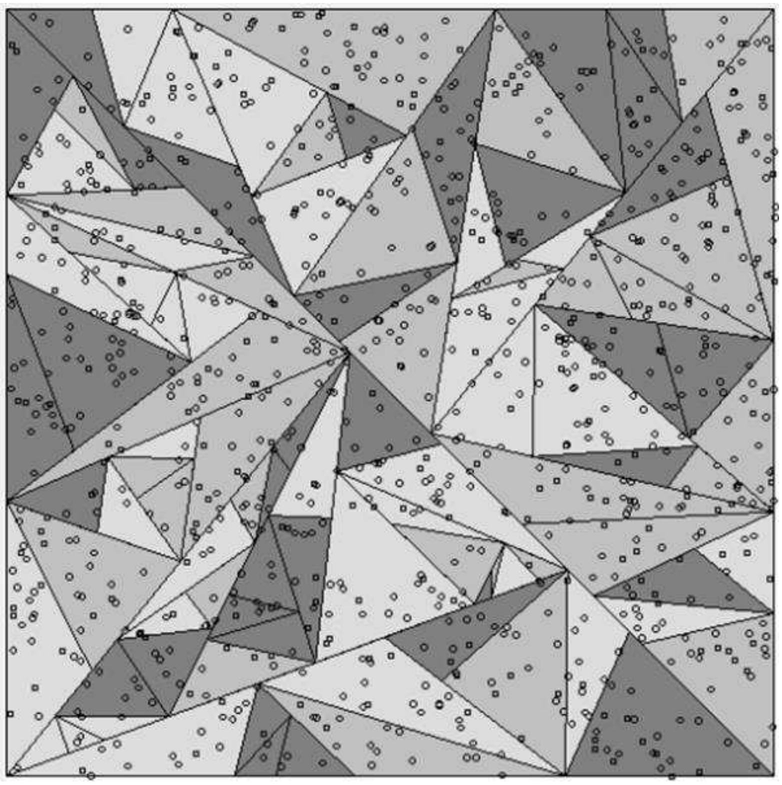

Figure 5. Illustration of a dataset with n = 1000 and |R| = 100 using generator of Figure 6

The set of possible capacities (D_r) and the fixed cost (F_{rd}) for opening a facility located in an area are also randomly generated based on the total customers demand, the average distance from one customer to others, and the number of open facilities (m). The main steps of the data set generator are given in Figure 6. Here, we set $|D_r|$ to 3 for all $r \in R$. The dataset can also be collected from the authors or downloaded from the CLHO (2016) website (http://www.kent.ac.uk/kbs/research/research-centres/clho/datasets.html).

Procedure Generating Capacities and Fixed Costs

1. Let denote $\hat{\delta}$ be the average distance among customers and calculate $\hat{\sigma} = \sum_{i \in I} w_i / m$
2. For each region r in R do the following steps:
   a. Let unit cost $\hat{c} = \hat{\delta}$, $\hat{c} = 1.25 \cdot \hat{\delta}$ and $\hat{c} = 1.5 \cdot \hat{\delta}$ for cheap, average and expensive region respectively.
   b. Set $\sigma^- = 0.7 \cdot \hat{\sigma}$ and $\sigma^+ = 0.9 \cdot \hat{\sigma}$. Set $b_{r1} = \sigma^- + \text{rand}[0, (\sigma^+ + 1 - \sigma^-)]$ and $F_{r1} = b_{r1} \cdot \hat{c}$. Set $\varphi = \sigma^+ - \sigma^-$
   c. Set $\sigma^- = \sigma^+ + 1$ and $\sigma^+ = \sigma^- + \varphi$
   d. Set $b_{r2} = \sigma^- + \text{rand}[0, (\sigma^+ + 1 - \sigma^-)]$ and $F_{r2} = b_{r2} \cdot \hat{c} \cdot 0.99$.
   e. Set $\sigma^- = \sigma^+ + 1$ and $\sigma^+ = \sigma^- + \varphi$
   f. Set $b_{r3} = \sigma^- + \text{rand}[0, (\sigma^+ + 1 - \sigma^-)]$ and $F_{r3} = b_{r3} \cdot \hat{c} \cdot 0.97$.

Figure 6. The Procedure for generating the capacities (D_r) and the fixed costs (F_{rd})

To assess our proposed approaches, we compare the obtained solutions with those found by the exact method using CPLEX for the DMFLP-FC problem (i.e., the discrete problem) where we limit the computing time of CPLEX to 3 hours. This strategy provides lower bound (LB), upper bound (UB), and duality Gap (%). The performance measure using the Dev (%) between the Z value obtained by the metaheuristic approach (Z_m) and the best known (Z^{bk}) is adopted where Dev (%) is calculated as follows:

$$\text{Dev}(\%) = \frac{Z_h - Z^{bk}}{Z^{bk}} \times 100 \tag{32}$$

with Z_h referring to the feasible solution cost obtained by either the exact method (UB) on the discrete problem or the metaheuristic method (h).

In our experimental study, we set parameters $\varepsilon = 0.0001$, $\mu = \min(10m, 0.75n)$, $\hat{T} = 5$, $\alpha = 2.5\%$. In addition, the parameter T is set to 10 for GTSHM whereas for EVNS we use T = 5, $k_{max} = 10$, β = 10 and γ = 0.5. Those parameters were chosen based on our preliminary experiments. The summary results are shown in Tables 1a and 1b where the results of the exact method on the DMFLP-FC (i.e., Gap (%) and CPU time (in seconds)) are given. It is interesting to note that CPLEX obtains the optimal solutions for the DMFLP-FC for all instances for $m = 5$ and n = 100, 200 and 400. However, when $n \geq 900$, CPLEX fails to obtain neither UB nor LB values within the 3 hours.

The bold numbers in Tables 1a and 1b refer to the best deviation found including ties. It can be observed that GTSHM produces better results when compared to EVNS and the exact method on the DMFLP-FC. The Average* in the tables refers to the average results based on the 39 instances that can be solved by the exact method on the DMFLP-FC. Based on the Average* indicator, GTSHM yields the smallest deviation of 0.0743% whereas EVNS and the exact method on the DMFLP-FC produce a deviation of 0.3391% and 2.5608% respectively. The Average+ in the tables indicates the average performance from all 50 instances. This measure can be calculated only for GTSHM and EVNS where GTSHM still performs better than EVNS as GTSHM produces a smaller deviation of 0.0739%. The GTSHM also obtains 41 best solutions whereas EVNS and the exact method attain 9 and 4 best solutions, respectively. However, GTSHM consumes slightly more computational time than EVNS. In general, the GTSHM is found to be the most suitable method for solving the SSCMFWP-FC as it provides good quality solutions within an acceptable computing time.

Table 1a. Computational Results for the SSCMFWP-FC

N	M	Best Known (Z^{bk})	Exact Method for the DLPFC			Proposed Methods			
						GTSHM		VNS	
			Dev (%)	Gap (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)
100	5	39,547.60	0.4745	0.01	6	**0.0000**	13	0.2742	9
	10	37,418.01	0.8014	0.01	78	**0.0000**	17	**0.0000**	27
	15	36,541.88	1.9348	0.01	193	**0.0000**	55	0.4594	79
	20	36,354.43	1.3569	0.01	7,902	**0.0000**	100	0.7362	110
	25	36,127.23	1.1703	0.01	265	**0.0000**	140	0.6475	97
200	5	162,465.44	0.2431	0.01	7,369	**0.0000**	30	0.0671	26
	10	153,293.61	0.3550	0.44	10,800	**0.0000**	192	0.0290	156
	15	150,452.26	0.3406	0.18	10,825	**0.0000**	229	0.0261	200
	20	147,763.09	0.6739	0.25	10,868	**0.0000**	344	**0.0000**	242
	25	146,315.41	0.7356	0.02	10,803	**0.0000**	332	0.2036	382
300	5	324,632.28	0.1328	0.27	10,800	0.3832	36	**0.0000**	28
	10	297,007.76	**0.0000**	0.18	10,807	0.1503	164	0.1149	297
	15	287,375.31	0.5080	0.24	10,807	**0.0000**	381	0.1998	317
	20	283,139.76	0.7441	0.38	10,838	**0.0000**	456	0.5593	453
	25	279,194.45	0.9059	0.32	10,804	**0.0000**	2,952	0.4542	1,476

Table 1b. Computational Results for the SSCMFWP-FC (continued)

N	m	Best Known (Z^{bk})	Exact Method for the DLPFC			Proposed Methods			
						GTSHM		VNS	
			Dev (%)	Gap (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)
400	5	590,426.48	**0.0000**	0.01	3,023	1.0868	55	2.1698	41
	10	548,815.92	**0.0000**	0.03	10,801	0.8253	151	0.7585	185
	15	530,597.49	0.1669	0.42	10,801	**0.0000**	556	0.1694	501
	20	522,212.16	0.3310	0.44	10,898	**0.0000**	677	0.1668	643
	25	516,077.45	0.8627	0.94	10,897	**0.0000**	1,112	0.0679	926
500	5	971,248.29	0.0588	0.32	10,801	**0.0000**	76	0.9664	77
	10	906,169.89	0.6894	0.31	10,801	**0.0000**	507	0.1752	338
	15	882,259.77	0.5752	0.46	10,801	**0.0000**	692	0.7751	616
	20	871,717.11	0.7856	0.77	10,801	**0.0000**	999	0.0672	899
	25	861,997.10	0.1707	0.26	10,801	**0.0000**	2,936	0.1717	1,476
600	5	1,352,323.29	0.3255	0.79	10,801	0.0846	144	**0.0000**	110
	10	1,264,461.05	0.0571	0.66	10,805	**0.0000**	1,059	0.2098	689
	15	1,214,664.57	0.7665	0.59	10,801	**0.0000**	1,338	0.7480	1,032
	20	1,187,100.93	1.4779	1.22	10,801	**0.0000**	1,107	0.2240	784
	25	1,172,741.02	1.3737	1.07	10,801	**0.0000**	2,427	**0.0000**	2,295
700	5	1,868,881.42	0.4377	1.05	10,802	**0.0000**	172	0.3459	99
	10	1,716,374.36	**0.0000**	0.14	10,802	0.0931	771	0.4542	847
	15	1,668,837.17	0.7989	0.71	10,803	**0.0000**	1,230	0.1175	965
	20	1,640,359.95	2.1913	1.95	10,802	**0.0000**	2,788	0.2657	1,979
	25	1,624,202.25	2.1404	2.12	10,803	**0.0000**	3,196	0.0381	3,214
800	5	2,467,417.90	NF	NF	NF	**0.0000**	267	0.0223	202
	10	2,215,644.68	71.6548	41.80	10,809	0.2743	873	**0.0000**	637
	15	2,160,250.08	2.1546	2.35	10,804	**0.0000**	1,257	0.6039	1,072
	20	2,132,376.26	0.6516	1.26	10,806	**0.0000**	2,055	0.9582	1,228
	25	2,115,741.63	1.8243	2.55	10,803	**0.0000**	2,153	**0.0000**	2,140
900	5	3,067,730.33	NF	NF	NF	0.1750	419	**0.0000**	361
	10	2,844,267.01	NF	NF	NF	**0.0000**	1,949	0.3012	1,447
	15	2,729,273.85	NF	NF	NF	**0.0000**	2,872	0.2761	2,029
	20	2,676,813.17	NF	NF	NF	**0.0000**	3,424	0.2703	2,258
	25	2,637,616.26	NF	NF	NF	**0.0000**	4,619	0.1491	4,249
1000	5	3,852,783.93	NF	NF	NF	0.6216	716	**0.0000**	444
	10	3,596,016.11	NF	NF	NF	**0.0000**	2,353	0.1215	1,464
	15	3,458,598.86	NF	NF	NF	**0.0000**	5,390	0.2384	4,065
	20	3,379,402.71	NF	NF	NF	**0.0000**	2,071	0.0797	1,714
	25	3,329,234.96	NF	NF	NF	**0.0000**	4,645	0.7525	3,712
Average*			2.5608		9,355	**0.0743**	866	0.3391	684
Average^{+}						**0.0739**	1,250	0.3087	973
#Best			4			**41**		9	

6. The adaptation of the proposed heuristic methods for the SSCMFWP

Our two proposed methods (GTSHM and EVNS), which are originally designed to solve the SSCMFWP-FC, can easily be adapted to tackle the simpler version namely SSCMFWP. We perform two scenarios, one with variable capacity of the facility where no available results are reproetd in the litertature and the other using constant capacity where published results do exist. Our revised approach contains the following minor modifications.

a) The exact method for solving the reduced discrete problem

The implementation of the exact method with CPLEX is to solve the reduced discrete problem for the DMFLP instead of DMFLP-FC. As a result, decision variables A_j and K_j, $j = 1,...,m$, are no longer required. Similar to the previous method, the DMFLP is also relaxed here by transforming variables Y_{ij} from integer to continuous.

b) The proposed local search

As the SSCMFWP does not consider the fixed cost (F_{rd}), the total cost of each open facility ($\hat{f}$ and $\tilde{f}$), as shown in Figure 2, is now based on the distance between the facility and its customers only. The parameter $\hat{T}$ is set to ∞ meaning that the stopping criteria of the search process of the new location for a facility is based on the distance between the new location and the previous one only (compared to ε). Besides, the process to determine the area of the new location $(\tilde{A}_j)$ is also not needed here. In Step 3c of Figure 2, instead of solving the GAP-FC to allocate customers to the new facilities, the classical GAP, which is relatively easier to solve, is considered instead.

6.1. Case of variable capacity- Experiments using newly generated dataset

This section presents the computational results of our adapted method where the capacity of the open facilities is not constant. As there is no available data in the literature relating to this problem, we use the newly generated dataset utilised in Section 5 where n = 100 to 1000 with an increment of 100 and the customer demand is generated randomly between 1 and 10. We also vary m from 5 to 25 with an increment of 5. There are three capacity designs for the

m open facilities with $K = \{1,2,3\}$ reflecting small, medium, and large capacity. The capacity of design k, β_k, is defined as follow:

$$\beta_k = \frac{\tau_k \cdot \sum_{i \in I} w_i}{m}, \ k \in K \tag{33}$$

τ_k is set to 0.8, 1, and 1.4 for $k = 1$, 2, and 3 respectively. The number of open facilities that use capacity design k, ρ_k, is calculated as $\rho_k = \sigma_k \cdot m$ where σ_k is set to 0.2, 0.4, and 0.4 for $k = 1$, 2, and 3 respectively.

Table 2a. Computational Results for the SSCMFWP using newly generated dataset

n	m	Best Known (Z^{bk})	Exact Method for the DLPFC			Proposed Methods			
						GTSHM		VNS	
			Dev (%)	Gap (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)
100	5	9,378.48	0.9905	0.00	1	**0.0000**	2.36	**0.0000**	1.92
	10	5,553.85	0.9141	0.00	1	**0.0000**	5.75	0.9259	6.02
	15	4,102.37	1.2179	0.01	1	0.4031	6.22	**0.0000**	5.84
	20	3,318.27	0.6810	0.00	4	**0.0000**	10.32	1.9220	9.55
	25	2,624.86	**0.0000**	0.01	3	0.8423	7.32	0.9799	7.60
200	5	36,556.81	0.3047	0.00	3	**0.0000**	5.82	0.0038	5.34
	10	24,858.34	1.0989	0.00	5	0.0007	18.21	**0.0000**	14.05
	15	19,322.80	0.3279	0.00	9	**0.0000**	38.47	0.1712	24.73
	20	15,537.97	0.6039	0.01	6	**0.0000**	38.45	**0.0000**	26.32
	25	12,924.38	0.7378	0.01	5	**0.0000**	35.55	**0.0000**	27.30
300	5	76,214.07	0.3655	0.00	14	**0.0000**	10.81	**0.0000**	10.24
	10	52,916.81	0.1247	0.00	20	**0.0000**	44.53	0.0227	33.63
	15	41,730.30	0.3091	0.01	10	**0.0000**	69.46	**0.0000**	46.53
	20	34,906.12	0.6832	0.01	12	**0.0000**	79.69	0.0044	53.24
	25	30,395.59	0.6995	0.01	19	**0.0000**	74.61	0.0426	52.97
400	5	146,200.26	0.2385	0.01	58	**0.0000**	21.44	0.0025	16.51
	10	98,533.76	0.7563	0.01	124	**0.0000**	121.59	0.1921	61.30
	15	74,791.00	0.1665	0.00	32	**0.0000**	118.01	0.1467	118.72
	20	62,655.34	0.6835	0.01	42	**0.0000**	106.10	0.1296	78.50
	25	54,845.44	0.5573	0.01	61	**0.0000**	106.82	0.0081	83.07
500	5	240,569.47	0.2214	0.01	138	0.1132	32.27	**0.0000**	32.53
	10	161,110.97	0.9518	0.00	239	**0.0000**	109.52	0.0232	72.20
	15	129,497.44	0.4632	0.01	391	**0.0000**	196.43	0.0198	137.12
	20	106,965.35	0.8537	0.01	168	**0.0000**	175.57	0.0040	172.18
	25	93,650.37	0.2481	0.01	344	**0.0000**	170.64	**0.0000**	118.90

Table 2b. Computational Results for the SSCMFWP using newly generated dataset (continued)

N	m	Best Known (Z^{bk})	Exact Method for the DLPFC			Proposed Methods			
						GTSHM		VNS	
			Dev (%)	Gap (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)
600	5	332,883.79	0.2295	0.01	602	0.0343	41.88	**0.0000**	77.17
	10	232,284.39	0.3074	0.01	794	**0.0000**	311.68	0.2846	178.57
	15	185,998.75	0.4736	0.00	600	0.0592	374.69	**0.0000**	225.79
	20	156,352.37	0.2357	0.01	525	**0.0000**	624.83	0.0998	346.61
	25	134,870.61	0.4174	0.01	188	**0.0000**	215.12	0.0079	150.72
700	5	462,057.44	0.1852	0.00	605	**0.0000**	65.84	**0.0000**	79.91
	10	314,415.42	0.5362	0.01	884	0.0325	213.99	**0.0000**	148.76
	15	252,399.38	0.4608	0.01	624	**0.0000**	464.16	0.0854	272.01
	20	215,202.11	0.3177	0.01	555	**0.0000**	449.24	0.3155	280.30
	25	187,164.60	0.4165	0.01	285	**0.0000**	299.70	0.0479	199.60
800	5	601,705.39	0.2120	0.00	1,385	0.0038	83.75	**0.0000**	59.89
	10	406,401.58	0.4342	0.01	1,555	0.0892	331.43	**0.0000**	274.05
	15	328,582.61	0.5350	0.01	4,178	**0.0000**	1,679.28	0.2165	872.92
	20	278,812.68	0.5261	0.01	2,998	**0.0000**	712.46	0.2502	506.59
	25	243,614.41	0.3784	0.01	5,850	0.0120	365.40	**0.0000**	395.33
900	5	759,185.58	0.2910	0.00	1,133	**0.0000**	99.61	**0.0000**	112.50
	10	527,455.13	0.2877	0.01	1,346	0.0027	383.69	**0.0000**	275.43
	15	424,515.93	0.2939	0.01	6,623	**0.0000**	869.02	0.0222	503.77
	20	364,407.43	0.1910	0.01	4,395	**0.0000**	777.95	0.6135	503.89
	25	322,183.39	0.2286	0.01	2,487	**0.0000**	726.70	0.0384	402.15
1000	5	966,903.15	0.0536	0.00	2,838	0.0116	124.69	**0.0000**	100.61
	10	663,115.73	72.1917	100.00	10,907	**0.0000**	656.66	0.1685	381.66
	15	527,104.49	0.1373	0.00	7,121	**0.0000**	1,487.82	0.5967	1,036.86
	20	446,532.01	**0.0000**	0.01	2,854	0.2864	743.82	0.2950	505.10
	25	388,398.57	0.3524	0.01	1,627	**0.0000**	725.15	0.0776	646.67
Average			1.8778		1,293.32	**0.0378**	288.69	0.1544	195.06
#Best			2			**37**		19	

In our experiments, the parameters setting for the proposed methods is similar to the one in Section 5 except $\mu = \min(10m, \min(0.75n, 150))$. The summary results of the proposed methods are shown in Tables 2a and 2b where the results of the exact method on the DMFLP (Gap (%) and CPU time in seconds) are also given. Here, almost all instances of the DMFLP can be solved optimally using CPLEX within 3 hours except when n = 1000 and m = 10. It can be noted that GTSHM produces better results when compared to EVNS and the exact method on the DMFLP. This claim is similar to the one gathered in our previous experiments on the SSCMFWP-FC. According to average results, GTSHM yields the smallest deviation of 0.0378% while EVNS and the exact method on the DMFLP produce deviations of 0.1544% and 1.8778% respectively. The GTSHM also obtains 37 best solutions whereas EVNS and the exact method on the DMFLP attain 19 and 2 best solutions, respectively. However, EVNS is found to run relatively faster than the other methods.

6.2. Case of constant capacity- Experiments using existing dataset

The performance of our adapted methods is also tested for the SSCMFWP when the capacity of a facility is constant. The three well known data sets from Brimberg et al. (2000) originally used for the multi-facility Weber problem are used here. These data sets are 50, 654, and 1060 customers and the demand of all data sets is set to one unit.

Here, the capacity of a facility is defined as the average total demand of all customers per facility (i.e., $Q_j = q = \left\lceil \sum_{i=1}^{n} w_i \middle/ m \right\rceil$, with $\lceil x \rceil$ being the smallest integer greater than or equal to x). This setting was initially proposed by Zainuddin and Salhi (2007) and also adopted by Luis et al. (2009 and 2011) for the case of CMFWP.

The experiments are performed by varying the number of facilities (m) from 2 to 25 for the 50 customers and 5 to 50 with an increment of 5 for the other two data sets. The solutions of the MFWP given by Brimberg et al. (2000) are set as 'lower bounds' for the SSCMFWP. These solutions are optimal for the 50 dataset and the best known solutions for the others. Though these 'lower bounds' may not be valid for the larger problems where the optimal solutions are unknown, these values can still be considered as good reference points as noted by Luis et al. (2009). To the best of our knowledge, the only empirical results that adopted such setting for these data sets are those by Manzour al-Ajdad et al. (2012) using the heuristics MA-TPP and MA-TPS for short, Manzour et al. (2013) using the procedure HM-PALAS for short, and finally Öncan (2013) using the heuristic TO-SSALA for short. For comparison purposes, the best methods proposed by these three studies are reported only.

In this experiment, the parameters setting for the proposed methods is similar to the one in Section 5 except that $\mu = \min(10m, 0.75n)$ for n = 50 and $\mu = m + T$ for other datasets. Tables 3, 4 and 5 present the summary results for n = 50, 654 and 1060 respectively. In the tables, Dev (%) is calculated using Equation (32) with Z^{bk} replaced by LB and Z_h refers to the objective function value produced by heuristic (h). In the ‘Dev (%)’ column of our proposed methods, ‘*’ refers to a new best solution. In the case of n = 50 customers, GTSHM produces better results when compared to EVNS and the other published results such as MA-TPP and HM-PALAS. In this case, GTSHM yields the smallest average deviation of 8.12%. Both GTSHM and EVNS produce 14 new best solutions out of 24.

Table 3. Computational results on the dataset with n = 50 for the SSCMFWP

m	LB	HM-PALAS		MA-TPP		Proposed Methods			
						GTSHM		VNS	
		Dev (%)	CPU (s)	Dev (%)	CPU(s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)
2	135.52	**0.85**	0.90	0.93	1.27	0.86	0.25	1.34	0.21
3	105.21	**1.06**	1.20	1.07	1.66	1.26	0.44	1.09	0.45
4	84.15	3.88	1.60	**2.76**	2.55	2.78	0.77	2.78	0.61
5	72.24	8.38	1.87	8.98	2.63	**5.96***	1.19	6.10	0.95
6	60.97	**0.91**	2.37	0.92	3.84	0.93	0.44	0.93	0.70
7	54.5	5.44	2.54	6.20	6.07	**3.42***	0.77	**3.42***	1.14
8	49.94	**2.45**	3.50	2.80	5.27	2.46	0.87	4.74	0.66
9	45.69	**3.51**	3.72	3.52	7.22	3.52	0.94	3.54	0.77
10	41.69	21.46	3.71	18.21	8.66	**15.72***	7.85	17.90	4.92
11	38.02	10.53	4.42	10.93	8.11	**6.14***	1.01	**6.14***	1.22
12	35.06	2.57	6.37	2.79	10.24	**2.07***	0.66	5.83	0.63
13	32.31	12.85	5.34	16.76	12.14	**8.87***	2.17	9.61	2.24
14	29.66	9.52	5.78	**4.70**	11.72	4.76	1.17	10.58	0.91
15	27.63	4.01	7.26	**2.39**	13.29	6.98	1.12	7.42	0.91
16	25.74	9.08	8.24	**3.98**	15.56	4.09	0.95	4.09	0.77
17	23.99	11.09	8.13	11.57	10.43	8.64	1.53	**8.23***	2.17
18	22.29	8.86	7.10	**7.20**	14.40	8.14	1.23	7.21	1.30
19	20.64	9.36	7.92	11.35	12.68	**9.04***	1.29	**9.04***	1.08
20	19.36	7.04	8.38	7.80	13.30	**6.56***	1.34	6.77	0.98
21	18.08	5.99	8.67	13.00	14.51	**5.11***	1.15	**5.11***	1.01
22	16.82	17.37	9.80	9.74	14.93	**5.09***	1.19	**5.09***	0.93
23	15.61	17.69	11.14	8.68	14.88	**4.67***	1.17	**4.67***	1.05
24	14.44	20.91	10.61	7.32	17.65	**4.88***	1.25	**4.88***	0.95
25	13.3	87.50	10.03	82.46	13.54	**73.02***	2.87	**73.02***	2.26
Average		11.76	5.86	10.25	9.86	**8.12**	1.40	8.73	1.20
#Best		5		4		**13**		9	

HM-PALAS : Priority-based ALA with simplified assignment proposed by Manzour et al. (2013)

MA-TPP : Two-phase with parallel assignment proposed by Manzour-al-Ajdad et al. (2012)

Table 4. Computational results on the dataset with n = 654 for the SSCMFWP

m	LB	TO-SSALA		HM-PALAP		MA-TPP		Proposed Methods			
								GTSHM		VNS	
		Dev (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)
5	209,068.80	**54.00**	300.20	60.47	5.63	**54.00**	8.20	**54.00**	7.73	**54.00**	5.94
10	115,339.03	**42.81**	1,104.50	54.67	11.45	**42.81**	17.96	**42.81**	14.88	**42.81**	10.58
15	80,177.04	**67.69**	2,586.40	81.42	19.18	**67.69**	29.82	**67.69**	21.33	**67.69**	22.67
20	63,389.02	**69.36**	4,278.80	74.90	35.50	71.36	49.91	**69.36**	26.26	69.42	31.71
25	52,209.51	**47.52**	6,560.80	67.72	41.73	**47.52**	64.35	**47.52**	32.46	**47.52**	41.99
30	44,705.19	76.34	9,362.30	109.58	60.74	76.40	72.96	77.63	37.18	**76.33***	34.09
35	39,257.27	**78.65**	13,080.40	89.48	75.86	82.80	104.76	78.76	39.96	79.70	73.05
40	35,704.41	47.63	17,414.10	62.30	104.92	44.71	139.41	44.71	40.62	**43.86***	56.82
45	32,306.97	55.77	21,321.30	75.72	98.25	59.35	237.81	**55.57***	50.29	55.58	91.60
50	29,338.01	31.27	25,870.60	42.68	2.00	31.31	184.77	**30.13***	52.87	38.45	31.74
Average		57.10	10,187.90	71.89	58.63	57.80	91.00	**56.82**	32.36	57.54	40.02
#Best		6		0		4		**7**		6	

TO-SSALA : Single Source Alternate Location-Assignment (SSALA) proposed by Oncan (2013)

HM-PALAP : Priority-based ALA with parralel assignment proposed by Manzour et al. (2013)

MA-TPP : Two-phase with parallel assignment proposed by Manzour-al-Ajdad et al. (2012)

Table 5. Computational results on the dataset with n = 1060 for the SSCMFWP

m	LB	TO-SSALA		HM-PALAS		MA-TPS		Proposed Methods			
								GTSHM		VNS	
		Dev (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)	Dev (%)	CPU (s)
5	1,851,879.9	**0.98**	801.90	1.57	10.92	1.06	15.82	1.06	16.27	1.06	11.38
10	1,249,564.8	**2.66**	3,487.50	6.94	23.80	3.20	35.86	3.11	25.87	3.13	23.49
15	980,132.13	**1.65**	7,589.70	4.19	45.42	1.68	90.20	**1.65**	33.69	**1.65**	44.81
20	828,802.00	**2.33**	12,526.30	9.60	60.24	3.46	141.15	3.47	42.87	3.47	61.24
25	722,061.19	**3.95**	20,756.60	8.64	84.61	5.17	190.12	4.42	58.84	4.15	56.38
30	638,263.00	4.07	27,303.40	9.80	133.35	5.32	273.63	**4.00***	61.64	4.45	92.22
35	577,526.63	3.56	40,235.30	6.31	164.81	4.26	422.27	**3.44***	75.50	3.46	101.00
40	529,866.19	6.89	56,384.40	10.37	244.25	7.72	526.11	**6.62***	78.71	6.77	95.75
45	489,650.00	8.92	67,218.30	13.80	354.98	9.02	842.86	9.20	88.60	**8.36***	97.34
50	453,164.00	**6.61**	74,328.60	10.48	479.87	7.80	816.89	6.99	90.51	7.49	193.27
Average		**4.16**	31,063.20	8.17	160.23	4.87	335.49	4.40	57.25	4.40	77.69
#Best		**6**		0		0		4		2	

TO-SSALA : Single Source Alternate Location-Assignment (SSALA) proposed by Oncan (2013)

HM-PALAS : Priority-based ALA with simplified assignment proposed by Manzour et al. (2013)

MA-TPS : Two-phase with simplified assignment proposed by Manzour-al-ajdad et al. (2012)

For the case of n = 654 customers, GTSHM also achieves better results when compared to the other heuristics (EVNS, TO-SSALA, HM-PALAP, and MA-TPP). Besides, GTSHM yields the smallest average deviation of 56.82% whereas EVNS, TO-SSALA, HM-PALAP, and MA-TPP produce 57.54%, 57.10%, 71.89%, and 57.80% respectively. Our methods produce 4 new best solutions out of 10. For the case of n = 1060 customers, both GTSHM and EVNS have relatively small deviations of 4.40% though marginally larger than the one obtained by TO-SSALA (4.16%). However, the average computational time required by TO-SSALA to solve one instance is recorded to be over 31,000 seconds (more than 8.6 hours) whereas GTSHM and EVNS only require approximately 57 and 78 seconds respectively. Note that the computer used to execute TO-SSALA is faster than the one for GTSHM and EVNS. In addition, 4 new best solutions out of 10 were also obtained here.

It is worth noting that though GTSHM requires 57.25 seconds on average and MA-TPS needs 335.49 seconds, the latter was coded in Matlab and run on Intel Dual Core@2.4GHz. Moreover, MA-TPS used an older version of CPLEX (version 10) to solve the assignment problem. According to http://cpuboss.com/compare-cpus, Intel Dual Core@2.4GHz is approximately 1.5 slower than Intel i5 3.2GHz. Based on Aruoba and Fernández-Villaverde (2014), on average, Matlab is 10 times slower than C++. A report from Tramontani (2014) reveals that CPLEX version 10 is approximately 8.5 times slower than CPLEX version 12.6 when solving MIP. In the GTSHM, CPLEX approximately makes up on average 96% of the total computing time. As an example, if GTSHM was to be coded in basic Matlab and executed on Intel Dual Core@2.4GHz, and assuming the coding structures used are similar in both approaches, GTSHM would on average approximately require 57.25*(0.96*8.5 +0.04*10)*1.5 =735.09 seconds, making it slightly more than twice slower. This observation shows that it is that simple to compare fairly the computing time required by different algorithms given the various ingredients under which the algorithms are written, implemented and run..

In brtief, based on our findings, we can report that the proposed methods (GTSHM and EVNS) perform rather well in all datasets as demonstrated by a total of 22 best solutions out of 44 which is an impressive result.

7. Conclusion and suggestions

A variant of the multi-facility Weber problem known as the single-source capacitated multi-facility Weber problem with opening facility fixed costs is studied. A new model (SSCMFWP–FC) that considers the presence of the fixed cost based on capacity and zone-dependent is also proposed. A framework that integrates the aggregation technique, the implementation of an exact method using CPLEX, and the enhanced well-known alternate location-allocation method of Cooper is put forward. A Variable Neighbourhood Search is then adapted to address the problem. A set of new instances which we generated for the new model is used to evaluate the performance of our heuristics. Very competitive results are also obtained when compared against the exact method on the discrete location problem (DMFLP-FC). The proposed methods are also adapted to solve the single-source capacitated multi-facility Weber problem (SSCMFWP) and are assessed on two types of datasets. The first one is the newly generated dataset used for the SSCMFWP–FC where our proposed methods perform very well when compared against DMFLP and the second one is a dataset available in the literature. The empirical results show that our proposed heuristics provide superior results on almost all instances when compared against the recently published ones.

The following research directions could be worth exploring in the future. In this study, the fixed cost of the zones is generated irrespective of the effect of continuity between adjacent zones. For example in our experiment if a location happens to be on the boundary we opt for the cheapest fixed cost. However, in practice the change in the fixed cost between two adjacent zones would not be drastically different. A new construction of the fixed cost that takes into account such a smoothness of the fixed cost is worth examining. The problem could also be extended to other classes of location problems such as the location of casualty collection points that arises in the case of catastrophic events (Drezner et al., 2006), the location routing problem on the plane (Salhi and Nagy, 2009), and the continuous competitive facility location problem (Redondo et al. 2013). From a technical view point, the current VNS approach can be modified to enhance its efficiency further by incorporating an adaptive memory mechanism to govern selection moves in a neighbourhood.

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