# Adaptive Position Tracking Control of High-speed Trains with Piecewise Dynamics*

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*Abstract*— **This paper addresses the adaptive position tracking control problem for high-speed trains with time-varying resistances and mass in the motion dynamics. To handel these time-varying parameters with piecewise constant characteristics, a piecewise constant model with unknown parameters is introduced for different train operation conditions. An integrated adaptive controller structure is constructed to have the capacity to achieve plant-model matching with known parameters and complete system parametrization with unknown parameters, which is desirable for adaptive tracking control. For the train position tracking requirement, the reference model system is specifically chosen. Stable adaptive laws are designed to update the adaptive controller parameters in the presence of the unknown piecewise constant system parameters. Closed-loop stability and asymptotic state tracking are proved. Simulation results on a high-speed train model are presented to illustrate the desired adaptive position tracking control performance.**

## I. INTRODUCTION

As high-speed trains is one of the most comfortable and rapidly transit systems, the control problem of trains have attracted a considerable number of studies to maintain safe and reliable operation. Due to the characteristics (high-speed) of the train, the automatic train operation plays an important role in driving the train, when the operation conditions change.

During the past years, some results on controller design for high-speed trains have been obtained, see, for example [1]-[4]. Among the existing results, the model-based controller design methods are always used, in which the constant or the variable model parameters with known upper bounds are employed. In practice, the dynamic motion model of the train is a time-varying nonlinear model dependent on the operating conditions. Especially, for the high-speed train, the aerodynamic resistance will change largely, when the train moves at a high-speed or passes a tunnel. Thus, the constants or bounded variable parameters cannot represent the characteristics of the system dynamics well, which motivates us to propose a new model to describe the high-speed train for the control design. In this paper, considered the piecewise characterizes of the train operating conditions, a new piecewise constant model with unknown parameters is presented to represent the train dynamics.

According to requirements on trains and lines, such as timetable, predefined platforms or emergency shutdown, the position trajectory calculations for high-speed trains is important for safe operations. Speed regulation is one of the main way to achieve the position trajectory tracking. To deal with the unknown parameters in the proposed piecewise constant model and to achieve good tracking performance (see [5], [6]), the adaptive techniques is suit for the controller design problem.

This paper is focused on the position tracking problem for the high-speed trains with the piecewise dynamic characteristics. A piecewise constant model is used to describe the train motion dynamics with its variable parameters. The controller structure, design conditions, and adaptive laws are derived to construct the automated train control scheme. The main contributions of this paper can be summarized as follows: (i) A piecewise constant model is introduced to describe the train motion dynamics with its piecewise dynamics. (ii) The adaptive controller with reference model, structure and adaptive laws is developed to achieve the train position tracking, in the present of the unknown piecewise constant parameters.

The rest of this paper is organized as follows: In Section II, the dynamical model of high-speed trains are introduced. In Section III, an adaptive controller is developed for the train with unknown parameters to position tracking. In Sections IV, a simulation study is presented to show the performance of the proposed method. Finally, some conclusions are given in Section V.

## II. PROBLEM FORMULATION

In this section, we will introduce the dynamic model of high-speed trains, which can be modelled as a piecewise constants model.

**High-speed Train Motion Model.** From [4], the longitudinal motion dynamics of a train can be described as:

$$M(t)\ddot{x}(t) = F(t) - F_r(t) - F_g(t) - F_c(t), \qquad (1)$$

where $x(t)$ is the position of the train, $M(t)$ is the mass of the train, $F(t)$ is the traction force, $F_r(t)$ is the general resistance, $F_g(t)$ is the force caused by motion on the grade, $F_c(t)$ is the force caused by motion on the curve. The force $F(t)$ acting on the train, is generated by the traction system to achieve the tractive effort or dynamic braking.

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According to [7], the Davis equation is usually used to express the the general resistance $F_r(t)$:

$$F_r(t)=a_r(t)+b_r(t)v(t)+c_r(t)v^2(t), \tag{2}$$

where $a_r(t)$ defines the train's rolling resistance component, $b_r(t)$ defines the train's linear resistance, $c_r(t)$ defines the train's nonlinear resistance; $v(t)$ is the speed of the train.

From [8], the grade resistance force $F_g$ and the curvature force are modeled as:

$$F_g(t)=M(t)g\sin\theta(t), \tag{3}$$
$$F_c(t)=0.004D(t)M(t), \tag{4}$$

where $\theta(t)$ is the slope angle of the current track. $D(t)$ is the degree of curvature and can be calculated by $D(t) = 0.5d_w/R(t)$, with $d_w$ being the distance between the front and rare wheels of the train, and $R(t)$ being the curve radius.

**Piecewise Dynamics Analysis.** The train's mass is a constant, when it is running between two stations. The mass parameter $M(t)$ can be modeled as a piecewise constant function depending on the displacement $x$ of the train.

The accurate models of coefficients $a_r$, $b_r$ and $c_r$ are complex, time-varying and dependent on many factors, which causes difficulty for the control design in practice. According to [9], the changes of the coefficients $a_r$, $b_r$ and $c_r$ mainly depend on the current conditions of the train (mass, speed, tunnel passing, etc.). These coefficients $a_r$, $b_r$ and $c_r$ can be considered as constants under the certain operating condition. The train operating conditions determines by the train displacement $x$ and velocity $\dot{x}$. Thus, the coefficients $a_r$, $b_r$ and $c_r$ can be modeled as the piecewise constants depending on the displacement $x$ and velocity $\dot{x}$ of the train.

According to the characteristics of the track line, the slope angle $\theta(t)$ and the degree of curvature $D(t)$ can be considered as a piecewise constant depending on the position $x$ of the train. For high-speed trains, the variables $x$ and $\dot{x}$ representing the displacement and velocity of the train, respectively, can be measured online by the speed sensors and track circuits.

From the above analysis, it can be seen that the motion model of the high-speed train is in general described by a time-varying dynamic equation, which can be approximated by certain piecewise constant functions, as the train operating conditions usually follow certain piecewise properties. In this paper, the controller design problem for the high-speed train modelled by a piecewise constant model will be focused, especially for the case that the parameters of the train are unknown.

**Piecewise Dynamic Model.** With the expressions (2)-(4) of the resistance forces, the train dynamic model (1) can be rewritten as

$$\begin{aligned} M(t)\ddot{x}(t)=&F(t)-\big(a_r(t)+b_r(t)\dot{x}(t)+c_r(t)\dot{x}^2(t)\big)\\ &-M(t)g\sin\theta(t)-0.004D(t)M(t). \end{aligned} \tag{5}$$

Define $m(t) = \dfrac{1}{M(t)}$, $a(t) = \dfrac{a_r(t)}{M(t)}$, $b(t) = \dfrac{b_r(t)}{M(t)}$, $c(t) = \dfrac{c_r(t)}{M(t)}$ and $\vartheta(t) = \sin\theta(t)$. Then, equation (5) can be rewritten as

$$\begin{aligned} \ddot{x}(t)=&m(t)F(t)-\big(a(t)+b(t)\dot{x}(t)+c(t)\dot{x}^2(t)\big)\big)\\ &-g\vartheta(t)-0.004D(t). \end{aligned} \tag{6}$$

During the train operation, we define $\Omega$ as the region for all possible system states $x(t)$ and $\dot{x}(t)$, with its $l$ subregions $\Omega_i$, $i = 1,\dots,l$. Because $m(t)$, $a(t)$, $b(t)$, $c(t)$, $\vartheta(t)$, and $D(t)$ are dependent on the displacement $x$ and velocity $\dot{x}$ of the train, the values of $(m(t),a(t),b(t),c(t),\vartheta(t),D(t))$ are determined as $(m(t),a(t),b(t),c(t),\vartheta(t),D(t)) = (m_i,a_i,b_i,c_i,\vartheta_i,D_i)$. If $(x(t),\dot{x}(t)) \in \Omega_i$, where $i = 1,\dots,l$, $m_i$, $a_i$, $b_i$, $c_i$, $\vartheta_i$, and $D_i$ are unknown constants. The time instants when $(x(t),\dot{x}(t))$ jumps from one region to another are known, due to the available $x(t)$ and $\dot{x}(t)$.

The indicator functions $\chi_i(t)$ are introduced to describe the piecewise constants of the parameters in equation (6), as follows:

$$\chi_i(t)=\begin{cases} 1, & \text{if } \ (x(t),\dot{x}(t))\in\Omega_i,\\ 0, & \text{otherwise,}\end{cases} \tag{7}$$

$$\sum_{i=1}^{l}\chi_i(t)=1,\quad \chi_p(t)\chi_q(t)=0,\quad \text{for}\ \ p\neq q. \tag{8}$$

We assume that the common boundaries do not exist, which imply that $(x(t),\dot{x}(t))$ only belongs to one region. Since $(x(t),\dot{x}(t)) \in \Omega_i$ can be available in real-time, the functions $\chi_i(t)$ defined in (7) can be known.

With $x_1 = x$ and $x_2 = \dot{x}$, the motion dynamics (5) can be expressed as

$$\dot{x}_1(t)=x_2(t), \tag{9}$$
$$\begin{aligned}\dot{x}_2(t)=&m(t)F(t)-a(t)-b(t)x_2(t)-c(t)x_2^2(t))\\ &-g\vartheta(t)-0.004D(t),\end{aligned} \tag{10}$$

where

$$m(t)=\sum_{i=1}^{l}m_i\chi_i(t),\quad a(t)=\sum_{i=1}^{l}a_i\chi_i(t), \tag{11}$$
$$b(t)=\sum_{i=1}^{l}b_i\chi_i(t),\quad c(t)=\sum_{i=1}^{l}c_i\chi_i(t), \tag{12}$$
$$\vartheta(t)=\sum_{i=1}^{l}\vartheta_i\chi_i(t),\quad D(t)=\sum_{i=1}^{l}D_i\chi_i(t), \tag{13}$$

with $m_i$, $a_i$, $b_i$, $c_i$, $\vartheta_i$, and $D_i$ being unknown constants, and $\chi_i(t)$ being the indicator functions defined in (7).

**Comparison.** For the existing works, the dynamic motion models for high-speed trains are considered with the constant parameters, or the unknown constant parameters and bounded uncertainties. In [10], the model (5) without the grade resistance and the curvature resistance terms is obtained, and the experiment is done to validate the acceptable accuracy of the obtained model with constant coefficients $a_r$, $b_r$ and $c_r$. Also, in [2], [11] and [12], without the grade resistance and the curvature resistance terms, the model (5) with the constant coefficients $a_r$, $b_r$ and $c_r$, is used to study the control problems. On the other hand, in [3], [4] and [13], to make the motion dynamic model of the train in a better accuracy and be more closed to the practical conditions, the parameters in the general resistance $F_r(t)$ are considered as unknown constants, and a bounded uncertain term is introduced to model the disturbances from rail conditions (ramp, tunnel, curvature, etc.).

In this work, we employ the ramp and curvature resistance model from [8], and describe the longitudinal motion of the high-speed train as equation (5). According to the characteristics of these parameters in resistances (see analysis in [7]-[8]), we assume that train conditions are piecewise changes and invariable in a certain rail. Further, considering the models studied in literatures [2]-[4], [10], [12] and [13], the piecewise constant model is proposed, which can be more closed to the real train, and its accuracy is enough to investigate the control problem. There exists some more complex train operating conditions that are not considered in this paper and need future study.

**Objective.** The objective of this paper is to develop an adaptive position tracking control for high-speed trains described by (9) and (10), with unknown friction parameters modeled in (11), (13), to guarantee the system stability and asymptotic tracking properties.

## III. Adaptive Controller Design

In Section II, the dynamic motion of high-speed trains is modeled as a piecewise constant nonlinear system with unknown parameters. For this class of system, the controller design has not been available. In this section, a new adaptive state feedback controller is proposed to achieve the closed-loop stability (signal boundedness) and state tracking for the high-speed train.

### *A. Reference Model System*

This paper is focused on the position tracking problem for high-speed trains. The reference model should be designed to make the reference trajectory $x_{d1}(t)$ satisfy the requirements based on timetable, platforms or some speed-reduction induced by the emergent conditions.

The following linear reference model system is used to produce the reference trajectory $x_d(t)$:

$$\dot{x}_d(t) = A_d x_d(t) + B_d r(t), \quad x_d(t) = [x_{d1}(t), x_{d2}(t)]^T \tag{14}$$

where $x_{d1}(t)$ is the desired position trajectory, $r(t) \in R$ is the reference input signal, which is continuous and bounded, $A_d$ is a stable matrix.

According to the structure of the train system (9)-(10), the reference model system should be chosen as the following structure:

$$\begin{bmatrix} \dot{x}_{d1}(t) \\ \dot{x}_{d2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_{d1} & -a_{d2} \end{bmatrix} \begin{bmatrix} x_{d1}(t) \\ x_{d2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b_d \end{bmatrix} r(t), \tag{15}$$

where $a_{d1} > 0$, $a_{d2} > 0$, and $b_d > 0$.

To design the reference input for the Distance-To-Go (DTG) curve under the reference model, the relationship between $x_{d1}(t)$ and $r(t)$ should be obtained firstly. The transfer function can be calculated as

$$x_{d1}(t) = \frac{b_d}{s^2 + a_{d2}s + a_{d1}}[r](t). \tag{16}$$

For the second order transfer function, the damping should not be less than 1, i.e., the reference model should work on the overdamping or critical damping condition to ensure there do not exist the overshoot in the output. Then, we can obtain $a_{d2}/2\sqrt{a_{d1}} \geq 1$. Further, according to the Distance-To-Go (DTG) curve and considered the speed operation conditions including acceleration, reacceleration, constant speed, deceleration, constant speed, redeceleration, and slowing down, the position trajectory is a piecewise continues function, which results in the input signal a piecewise continues function. The desired position trajectory can be transferred into the $s$ dominant. Then, through the transfer function (16), the input signal can be obtained. Further, via the inverse Laplace transform, we can obtain the input signal, i.e., the reference input $r(t)$.

### *B. Controller Structure*

As the reference model be chosen, we propose the following adaptive state feedback controller structure:

$$\begin{aligned} F(t) = {} & k_{x_1}(t)x_1(t) + k_{x_2}(t)x_2(t) + k_r(t)r(t) \\ & + \hat{a}(t) + \hat{c}(t)x_2^2(t) + g\hat{\vartheta}(t) + 0.004\hat{D}(t), \end{aligned} \tag{17}$$

where $r(t)$ is the reference input signal, $k_{x_1}(t)$, $k_{x_2}(t)$, $k_r(t)$, $\hat{a}(t)$, $\hat{c}(t)$, $\hat{\vartheta}(t)$, and $\hat{D}(t)$ are time-varying parameters defined as $k_{x_1}(t) = \sum_{i=1}^{l} k_{x_{1i}}(t)\chi_i(t)$, $k_{x_2}(t) = \sum_{i=1}^{l} k_{x_{2i}}(t)\chi_i(t)$, $k_r(t) = \sum_{i=1}^{l} k_{ri}(t)\chi_i(t)$, $\hat{a}(t) = \sum_{i=1}^{l} \hat{a}_i(t)\chi_i(t)$, $\hat{c}(t) = \sum_{i=1}^{l} \hat{c}_i(t)\chi_i(t)$, $\hat{\vartheta}(t) = \sum_{i=1}^{l} \hat{\vartheta}_i(t)\chi_i(t)$, $\hat{D}(t) = \sum_{i=1}^{l} \hat{D}_i(t)\chi_i(t)$, with $k_{x_{1i}}(t)$, $k_{x_{i2}}(t)$, $k_{ri}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, and $\hat{D}_i(t)$ being adaptive parameters (to be obtained from some stable adaptive laws, as the estimates of some nominal parameters to be defined), and $\chi_i(t)$ being defined in (7).

### *C. Plant-Model Matching*

To design an adaptive control law for the system (9)-(10) with unknown piecewise constant parameters, it is needed to derive the nominal control law which can satisfy the matching condition and achieve the tracking of $x_d(t)$ by $x(t)$, when implemented by true plant parameters. The nominal control law is designed as

$$\begin{aligned} F(t) = {} & k_{x_1}^*(t)x_1(t) + k_{x_2}^*(t)x_2(t) + k_r^*(t)r(t) \\ & + a^*(t) + c^*(t)x_2^2(t) + g\vartheta^*(t) + 0.004D^*(t), \end{aligned} \tag{18}$$

where the parameters $k_{x_1}^*(t)$, $k_{x_2}^*(t)$, $k_r^*(t)$, $a^*(t)$, $c^*(t)$, $\vartheta^*(t)$, $D^*(t)$ are defined as $k_{x_1}^*(t) = \sum_{i=1}^{l} k_{x_{1i}}^*\chi_i(t)$, $k_{x_2}^*(t) = \sum_{i=1}^{l} k_{x_{2i}}^*\chi_i(t)$, $k_r^*(t) = \sum_{i=1}^{l} k_{ri}^*\chi_i(t)$, $a^*(t) = \sum_{i=1}^{l} a_i^*\chi_i(t)$, $c^*(t) = \sum_{i=1}^{l} c_i^*\chi_i(t)$, $\vartheta^*(t) = \sum_{i=1}^{l} \vartheta_i^*\chi_i(t)$, $D^*(t) = \sum_{i=1}^{l} D_i^*\chi_i(t)$, with $k_{x_{1i}}^*$, $k_{x_{2i}}^*$, $k_{ri}^*$, $a_i^*$, $c_i^*$, $\vartheta_i^*$, $D_i^*$, being constants and satisfying:

$$a_{d1} = -m_i k_{x_{1i}}^*, \quad a_{d2} = b_i - m_i k_{x_{2i}}^*, \quad b_d = m_i k_{ri}^*, \tag{19}$$

$$a_i = m_i a_i^*, c_i = m_i c_i^*, \vartheta_i = m_i \vartheta_i^*, D_i = m_i \hat{D}_i^*. \tag{20}$$

The equations in (19)-(20) describe the plant-model matching conditions, that is, if the piecewise constant parameters $m_i$, $a_i$, $b_i$, $c_i$, $\vartheta_i$, and $D_i$ are known, then nominal parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, $k^*_{ri}$, $a^*_i$, $c^*_i$, $\vartheta^*_i$, $D^*_i$ exist to satisfy (19)-(20) and the nominal control law (18) results in the closed-loop system

$$\dot{x}_1(t) = x_2(t), \tag{21}$$

$$\dot{x}_2(t) = -a_{d1}x_1(t) - a_{d2}x_2(t) + b_d r(t), \tag{22}$$

which has a bounded solution $x_1(t)$, $x_2(t)$. The tracking errors $e_1(t) = x_1(t) - x_{d1}(t)$ and $e_2(t) = x_2(t) - x_{d2}(t)$ under the nominal control law satisfy:

$$\dot{e}_1(t) = e_2(t), \tag{23}$$

$$\dot{e}_2(t) = -a_{d1}e_1(t) - a_{d2}e_2(t), \tag{24}$$

which implies that $e_1(t)$ and $e_2(t)$ approach zero exponentially as $t \to \infty$, due to the choice of $a_{d1} > 0$ and $a_{d2} > 0$ to make $A_d$ stable.

### D. Tracking Error Equation

When the piecewise constant parameters $m_i$, $a_i$, $b_i$, $c_i$, $\vartheta_i$, and $D_i$ are unknown, it is required to use the adaptive control law (17) to ensure the stability and tracking of the closed-loop system. To design the adaptive update law for $k_{x_{1i}}(t)$, $k_{x_{2i}}(t)$, $k_{ri}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, and $\hat{D}_i(t)$, which are the estimates of the unknown constant parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, $k^*_{ri}$, $\xi^*$, $a^*_i$, $c^*_i$, $\vartheta^*_i$ and $D^*_i$, we define the parameter errors as $\tilde{k}_{x_{1i}}(t) = k^*_{x_{1i}} - k_{x_{1i}}(t)$, $\tilde{k}_{x_{2i}}(t) = k^*_{x_{2i}} - k_{x_{2i}}(t)$, $\tilde{k}_{ri}(t) = k^*_{ri} - k_{ri}(t)$, $\tilde{a}_i(t) = a^*_i - \hat{a}_i(t)$, $\tilde{c}_i(t) = c^*_i - \hat{c}_i(t)$, $\tilde{\vartheta}_i(t) = \vartheta^*_i - \hat{\vartheta}_i(t)$, $\tilde{D}_i(t) = D^*_i - \hat{D}_i(t)$, and use the control law (17) and the system (9)-(10) under the matching condition (19)-(20), to obtain the tracking error equations

$$\dot{e}_1(t) = e_2(t), \tag{25}$$

$$\begin{aligned}\dot{e}_2(t) = &-a_{d1}e_1(t) - a_{d2}e_2(t)\\ &+\sum_{i=1}^{l}\frac{1}{k^*_{ri}}b_d\Big(\tilde{k}_{x_{1i}}(t)\chi_i(t)x_1(t) + \tilde{k}_{x_{2i}}(t)\chi_i(t)x_2(t)\\ &+\tilde{k}_{ri}(t)\chi_i(t)r(t) + \tilde{a}_i(t)\chi_i(t) + \tilde{c}_i(t)\chi_i(t)x_2^2(t)\\ &+g\tilde{\vartheta}_i(t)\chi_i(t) + 0.004\tilde{D}_i(t)\chi_i(t)\Big),\end{aligned} \tag{26}$$

based on which the adaptive update laws for $k_{x_{1i}}(t)$, $k_{x_{2i}}(t)$, $k_{ri}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, and $\hat{D}_i(t)$, will be proposed.

### E. Adaptive Laws

With $e(t) = [e_1(t), \quad e_2(t)]^T$, the following parameter adaptive laws are used to update the controller parameters in (17):

$$\dot{k}_{x_{1i}}(t) = -\Gamma_{x_{1i}}x_1(t)e^T(t)P_dB_d\chi_i(t), \tag{27}$$

$$\dot{k}_{x_{2i}}(t) = -\Gamma_{x_{2i}}x_2(t)e^T(t)P_dB_d\chi_i(t), \tag{28}$$

$$\dot{k}_{ri}(t) = -\Gamma_{ri}r(t)e^T(t)P_dB_d\chi_i(t), \tag{29}$$

$$\dot{\hat{a}}_i(t) = -\Gamma_{ai}e^T(t)P_dB_d\chi_i(t), \tag{30}$$

$$\dot{\hat{c}}_i(t) = -\Gamma_{ci}x_2^2(t)e^T(t)P_dB_d\chi_i(t), \tag{31}$$

$$\dot{\hat{\vartheta}}_i(t) = -\Gamma_{\vartheta i}ge^T(t)P_dB_d\chi_i(t), \tag{32}$$

$$\dot{\hat{D}}_i(t) = -\Gamma_{Di}0.004e^T(t)P_dB_d\chi_i(t), \tag{33}$$

where $\Gamma_{x_{1i}}$, $\Gamma_{x_{2i}}$, $\Gamma_{ri}$, $\Gamma_{ci}$, $\Gamma_{ai}$, $\Gamma_{\vartheta i}$, and $\Gamma_{Di}$ are positive constants and $P_d > 0$, satisfying $A_d^TP_d + P_dA_d = -Q_d$, for some $Q_d > 0$.

### F. Stability Analysis

Based on the adaptive laws (27)-(33), the following stability and tracking properties can be obtained:

***Theorem 1****: For the piecewise constant system (9)-(10) and the reference model system (15), the controller (17) with its parameters updated by the adaptive laws (27)-(33) ensures the boundedness of all closed-loop signals, and the asymptotic state tracking:* $\lim_{t\to\infty} e(t) = 0$.

**Proof**: The values of the parameters $a_{d1}$, $a_{d2}$ and $b_d$ ensure the stability of (15), i.e., $x_d(t) \in L_\infty$.

The following continuous Lyapunov function is chosen:

$$\begin{aligned}V = e^TP_de + \sum_{i=1}^{l}\frac{1}{k^*_{ri}}\Big(&\Gamma^{-1}_{x_{1i}}\tilde{k}^2_{x_{1i}} + \Gamma^{-1}_{x_{2i}}\tilde{k}^2_{x_{2i}} + \Gamma^{-1}_{ri}\tilde{k}^2_{ri}\\ &+\Gamma^{-1}_{ai}\tilde{a}^2_i + \Gamma^{-1}_{ci}\tilde{c}^2_i + \Gamma^{-1}_{\vartheta i}\tilde{\vartheta}^2_i + \Gamma^{-1}_{Di}\tilde{D}^2_i\Big).\end{aligned} \tag{34}$$

Using the estimation error equations (25)-(26) and the adaptive laws (27)-(33), the time derivative of $V$ becomes

$$\dot{V} = -e^T(t)Q_de(t) \leq 0, \tag{35}$$

which indicates that the closed-loop system consisting of (25)-(26) and (27)-(33) is uniformly stable and its solutions is uniformly bounded, that is, $e(t)$, $x_1(t)$, $x_2(t)$, $k_{x_{1i}}(t)$, $k_{x_{2i}}(t)$, $k_{ri}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, $\hat{D}_i(t)$, and $\dot{e}(t)$ are all bounded. Then, with the structure of the controller (17), the boundedness of $\nu_0(t)$ is ensured. Further, (35) implies $e(t) \in L_2$ and so $\lim_{t\to\infty} e(t) = 0$. $\nabla$

### G. Multi-trains Tracking

It should be note that on the track line, there are always several trains operation simultaneously. For the following trains, the safe distances to the preceding train should be guaranteed by the controller. Right now, most of the existing controller design are based on the pre-specified speed or position trajectories, which are designed off-line considering the allocated running time and the characteristics of automatic train operation.

Consider the situation when there are two trains on the same track: The preceding train A with position $z(t)$ and following train B with position $x(t)$. Our task is to design a control law to guarantee the safe distance between these two trains to avoid possible collisions.

If we treat the position $z(t)$ of Train A as a reference point to design a control law for Train B, we can choose $x_{d1}(t) = z(t) - L_d$ (or less) with $L_d$ being the safe distance to keep between tao trains, as the desired reference signal for the position $x(t)$ of Train B to track.

If we treat the position $x(t)$ of Train B as a reference point to design a control law for Train A, we can choose $z_{d1}(t) = x(t) + L_d$ (or larger) with $L_d$ being the safe distance to keep between tao trains, as the desired reference signal for the position $z(t)$ of Train A to track.

For the model reference adaptive control, the controller is designed to track the desired trajectories resulted in the reference model with reference input. Thus, the reference models for the following trains should be considered the safe distance problem.

We give the multiple reference models as follows:

$$\dot{x}_{d1}^{j} = x_{d2}^{j}, \tag{36}$$
$$\dot{x}_{d2}^{j} = -a_{d1}^{j} x_{d1}^{j} - a_{d2}^{j} x_{d2}^{j} + b_{d}^{j} r^{j}(t), \tag{37}$$

where $j = 1, 2, \ldots, T_n$, $T_n$ is the number of the trains that work in one line simultaneously. When $j = 1$, equations (36)-(37) represent the headmost train. The controller proposed in (17) can be used directly to ensure the real-time position values determined by Distance-To-Go curve.

For the following trains, i.e., $j = 2, \ldots, T_n$, the safe distance should be considered as follows:

$$x_{d1}^{j-1}(t) - x_{d1}^{j}(t) = L_d, \tag{38}$$

where $L_d > 0$ is a constant representing the desired position of train $j$ behind the preceding one.

As the following train starts off after the preceding one and follows the timetable, the reference model (15) with the adaptive controller (17) can be used to achieve the targets. When there is any fault occurred in the preceding train and resulting in the speed reduction, the reference model should be modified on-line, especially the reference input. Thanks to the structure of the reference model chosen referring the plant, only the amplitudes of the input signals (parameters of the exponential function or jump function) should be modified to reduce the speed to guarantee the safe distance.

### *H. Summary*

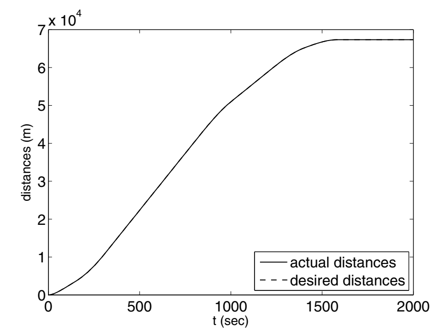
**Problem and Results.** In this paper, we focus on the position tracking control design problem for the high-speed trains. The motion dynamic model is used to the controller design, which brings in the time-varying frictions and results in a time-varying nonlinear model. To deal with the time-varying model, a piecewise model with unknown parameters is introduced to describe the train longitudinal motion dynamic model. Then, an integrated adaptive controller is proposed to achieve the state tracking (position and related speed tracking), in the presence of the unknown system piecewise constant parameters.

**Comparison.** Compared with some existed adaptive controller, an integrated adaptive controller is proposed in this paper to deal with the piecewise constants model in the presence of the unknown parameters. If a single adaptive control in [14] is used for the piecewise constants model, the transient responses will be appeared during the model switchings, that is when the piecewise model changes its sub-models, the single adaptive controller should adjust its control parameters to make the tracking error to zero, which leads to the transient responses (over shoots, oscillation, etc.,). If there is not enough dwelling time for each sub-model, the system would be unstable. The integrated adaptive controller can overcome the transient responses problem, because all the sub-models are considered and the controller can cover the all possible controller. If a set of controller is used, there should exist a decision maker to choose which controller is suitable for the working model. The proposed integrated adaptive controller does not need a special decision maker, because its matching conditions can achieve the decision automatically.

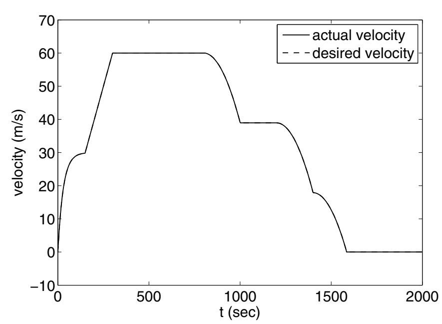
## IV. Simulation Study

A simulation study on a high-speed train is presented to demonstrate the effectiveness of the proposed adaptive controller. The system parameters are borrowed from a CRH type train ([15], [16]), in which 4 motors are considered.

**Tracking performance and reference model.** To verify the control scheme well and according to [15], [16], several operation conditions including acceleration, reacceleration, constant speed, deceleration, constant speed, redeceleration, and slowing down until fully stop are considered during the train operation. Choose the parameters of the reference model as $a_{d1} = 0.12$, $a_{d2} = 1.9$ and $b_d = 1/(500 \times 10^3)$.




(a) $x_1$ and $x_{d1}$




(b) $x_2$ and $x_{d2}$

Fig. 1. Distance and velocity trajectories.

**Simulation conditions.** In this section, we consider the train does not stop during the travel. The mass of the train does not change and is chosen as $M_i = M = 500$ ton. Considered the tunnel, slope and curvature in the rail, 4 modes will be considered. The parameters ($m_i$, $a_i$, $b_i$, $c_i$, $\vartheta_i$, and $D_i$ defined in (11)-(13)) are set as:

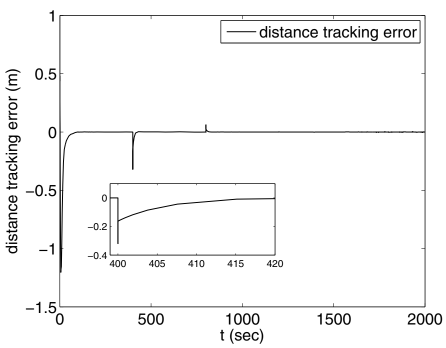
(i) For $t < 400$ s, the train bakes up. In this case, the coefficients are chosen as $a_1 = 2.25$, $b_1 = 1.9 \times 10^{-3}$, $c_1 = 3.2 \times 10^{-4}$, $\theta_1 = 0$, $D_1 = 0$.

(ii) During $400 \leq t < 800$ s, the train entrances the tunnel. Then only $c_2$ is replaced by $c_2 = 9.2 \times 10^{-4}$, with $a_2 = a_1$, $b_2 = b_1$, $\theta_2 = \theta_1$ and $D_2 = D_1$.

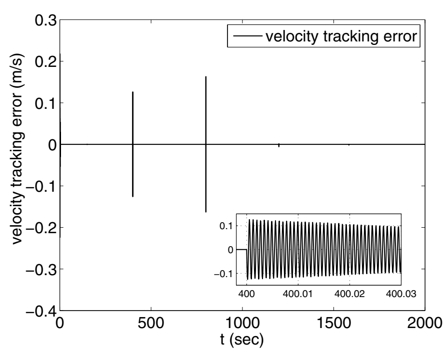
(iii) At 800 s, the train exits the tunnel and travels in the slope and curvature track. For $800 \leq t < 1200$ s, the coefficients are $c_3 = c_1 = 3.2 \times 10^{-4}$, $\theta_3 = 0.015$, $D_3 = 0.34$, with $a_3 = a_1$, $b_3 = b_1$.

(iv) After 1200 s, the train moves in the open air and horizontal track to slow down until fully stop. For $1200 \leq t < 2000$ s, the coefficients are the same as that of the baking up, i.e., $a_4 = a_1$, $b_4 = b_1$, $c_4 = c_1$, $\theta_4 = \theta_1$ and $D_4 = D_1$.

For simulation purpose, the initial sates are chosen as $x_d(0) = x(0) = [0 \;\; 0]^T$, and the initial parameter estimates are set as $95\%$ of their nominal values. The related gains of the adaptive laws in (27)-(33) are chosen as 2.




(a) $e_1$




(b) $e_2$

Fig. 2. Tracking errors.

**Simulation results.** Figs. 1-2 show the simulation results of the high-speed train dynamic motion modelled by a piecewise constant model. Fig. 1 shows the distances (a) and velocities (b) of the train and the reference model. Fig. 2 shows the state tracking errors including the distance (a) and velocity (b). From the simulation results, we can obtain that the proposed adaptive controller can achieve the close-loop stability and asymptotic tracking properties of the train even in the presence of parameters changes.

## V. CONCLUSIONS

In this paper, the adaptive position tracking problem is addressed for high-speed trains with the piecewise dynamics. A piecewise constant model is used to represent the train dynamics with piecewise dynamics. An adaptive controller are developed to deal with the unknown parameters of the piecewise model. Simulation results further confirm the effectiveness of the proposed controller.

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