Adaptive Compensation of Traction System Actuator Failures for High-Speed Trains

Zehui Mao, Gang Tao, *Fellow, IEEE*, Bin Jiang, *Senior Member, IEEE*, Xing-Gang Yan

***Abstract*—In this paper, an adaptive failure compensation problem is addressed for high-speed trains with longitudinal dynamics and traction system actuator failures. Considered the time-varying parameters of the train motion dynamics caused by time-varying friction characteristics, a new piecewise constant model is introduced to describe the longitudinal dynamics with variable parameters. For both the healthy piecewise constant system and the system with actuator failures, the adaptive controller structure and conditions are derived to achieve the plant-model matching. The adaptive laws are designed to update the adaptive controller parameters, in the presence of the system piecewise constant parameters and actuator failure parameters which are unknown. Based on Lyapunov functions, the closed-loop stability and asymptotic state tracking are proved. Simulation results on a high-speed train model are presented to illustrate the performance of the developed adaptive actuator failure compensation control scheme.**

***Index Terms*—Actuator failures, adaptive control, failure compensation, high-speed train.**

I. Introduction

High-speed trains with their fast and high loading capacities, have become more popular. In the recent years, a considerable number of studies have been focused on control design for the train systems (see, for example, [1]-[5]). To achieve high speed and loading, the increasing of the automatic train operating control capabilities of high-speed train is required, which may increase the possibility of traction system failures. The traction system generating the traction/breaking force consists of rectifiers, inverters, PWMs (pulse width modulations), traction motors, and mechanical drives, etc., among which PWMs, traction motors, and mechanical drives are considered as actuators. Actuator failures are often uncertain in patterns, amplitudes, and time instances. These failed actuators may deteriorate the train performance severely, resulting in time delay or cancellation of the other trains. Therefore, it is crucial for the traction system of high-speed trains to study the effective failure compensation technologies.

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Z. Mao and B. Jiang are with College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, 211106, China, Email: zehuimao@nuaa.edu.cn (Z. Mao), binjiang@nuaa.edu.cn (B. Jiang, corresponding author). The authors are also with Jiangsu Key Laboratory of Internet of Things and Control Technologies (Nanjing Univ. of Aeronautics and Astronautics).

G. Tao is with Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, VA 22904, U.S.A, and he is a visiting professor in Nanjing University of Aeronautics and Astronautics, Email: gt9s@virginia.edu.

X. Yan is with School of Engineering and Digital Arts, University of Kent, Canterbury, Kent CT2 7NT, United Kingdom, Email: x.yan@kent.ac.uk.

During the past years, some results on fault diagnosis and fault-tolerant control for high-speed trains have been obtained, see, for example [6]-[9]. It should be noted that the longitudinal dynamics of high-speed trains are usually used to study the automatic train control system or fault-tolerant control problem, since the control design is focused on the train handling, tracking and braking. Much of the existing work uses the longitudinal dynamic model with constants parameters, or the variable parameters with known upper bounds. However in practice, these parameters are time-varying and dependent on the track conditions. These constants or bounded variable parameters cannot represent the characteristics of the system dynamics well, which motivates the research to derive a new suitable model to describe the longitudinal dynamics of high-speed train for the control design. Specifically, a new piecewise constant model with unknown parameters is presented in this paper to solve the modeling problem.

On the other hand, when failures occur, it is necessary to utilize the failure compensation to guarantee the system stable and even asymptotic tracking. Until now, many results about the fault-tolerant control are available, see [10]-[17]. It should be noted that in these results, the parameters of the plants are assumed either known or unknown but are modeled as unknown inputs with bounds. Adaptive techniques can be used to the control problem, in which the parameters are unknown, to achieve good tracking performance (see [18], [19]), which is suitable for high-speed trains. But, the unknown failure problem with the unknown piecewise constant parameters in the high-speed train has not been studied.

This paper is focused on the actuator failure compensation problem for the longitudinal dynamics of high-speed trains with traction system actuator failures. A piecewise constant model is used to describe the longitudinal dynamic system with its variable parameters. The design conditions, controller structure, and adaptive laws are derived for both healthy and faulty cases to construct the automated train control scheme. The main contributions of this paper can be summarized as follows:

(i) For the variable high-speed train dynamics, a piecewise constant model is introduced to describe the longitudinal motion dynamics with traction system actuator failures.
(ii) The adaptive controller with design conditions, structure and adaptive laws is developed for the healthy case when the piecewise constant parameters are unknown.
(iii) For both the constant parameter model and piecewise constant parameter model, adaptive failure compensation schemes are designed for the high-speed train longitudinal motion system with unknown plant parameters and

traction system actuator failures with unknown failure time and parameters.

The rest of this paper is organized as follows: In Section II, the dynamical model of high-speed trains are introduced, and the actuator failure compensation problem is formulated. In Section III, an adaptive control scheme is developed for the healthy system with unknown parameters, as the baseline adaptive system. A simulation study is presented to show the performance of the proposed method. In Sections IV and V, the failure compensation schemes for both constant parameter and piecewise constant parameter systems, are developed, respectively. Simulations for these two cases are also presented to verify the effectiveness of the failure compensation schemes. Finally, some conclusions are given in Section VI.

II. Problem Formulation

In this section, we will introduce the dynamic model of high-speed trains and the model of traction system actuator failures. Further, the objective of this work and the design issues for adaptive control and failure compensation are formulated.

A. Longitudinal Motion Equation

The longitudinal train dynamical model is usually used to study train handling, traction and braking system design. The train can be considered as a mass point, whose forces are varying with the operating track conditions. By Newton's law, the longitudinal motion dynamics of a train can be described as [6]:

$$M(t)\ddot{x}(t) = F(t) - F_r(t) - F_g(t) - F_c(t), \tag{1}$$

where $x(t)$ is the displacement of the train, $M(t)$ is the mass of the train, $F(t)$ is the traction force, $F_r(t)$ is the general resistance, $F_g(t)$ is the force caused by motion on the grade, $F_c(t)$ is the force caused by motion on the curve. The force $F(t)$ acting on the train, is generated by the traction system to achieve the tractive effort or dynamic braking, which can represent the action on the train to reduce motion during the application of the brakes.

Since there are the load-unload cargoes on freight lines and on-off passengers on passenger lines [2], the mass of a train is varying between stations. When the train is operated on line, its mass is constant. Thus, the variable mass $M(t)$ can be modeled as a piecewise constant function depending on the displacement x of the train. For high-speed trains, the variables x and $\dot{x}$ representing the displacement and velocity of the train, respectively, can be measured online by the speed sensors and track circuits .

Remark 1: For modeling the train, there exist two types of train models, one of which is to treat a train as a cascade of point masses connected with couplers. The other one is to consider the whole train as a point mass [20]. Since the problem of in-train forces is not so important in short trains and the speed tracking is emphasized for high-speed trains (passenger trains), the in-train forces of the couplers are often not taken into account. Thus, in this study, we choose the later model to study the control and failure compensation problem.

B. Resistance Force Models

To obtain the dynamic motion system of the train, the resistance models should be studied firstly. Three types of resistances are usually considered to establish the motion equation.

General resistance. As modeled in [21], the general resistance $F_r(t)$ is approximated by a quadratic function, i.e., the Davis equation:

$$F_r(t) = a_r(t) + b_r(t)v(t) + c_r(t)v^2(t), \tag{2}$$

where $M(t)$ is the mass of the train, $v(t)$ is the speed of the train; $a_r(t)$ defines the train's rolling resistance component, (which contributes to the journey, rolling and track resistance); $b_r(t)$ defines the train's linear resistance, (which contributes to the flange friction, flange impact, rolling resistance between wheel and rail and wave action of the rail); $c_r(t)$ defines the train's nonlinear resistance, (which contributes to the rear drag, head-end wind pressure, turbulence between trains, yaw angle of wind tunnels and skin friction on the side of the train).

Coefficients a_r and b_r of the Davis equation refer to the mechanical resistances and are mass related. When a train moves at a high-speed, the mass independent term $c_r v^2$ becomes dominant [22]. It should be noted that the aerodynamic resistance force changes more significantly in term of coefficient c_r in tunnel than in an open air. The tunnel resistance is related to the train length, the ratio of the cross-sectional area of the train to the cross-sectional area of the tunnel, tunnel length and the tunnel roughness [23].

The changes of the coefficients a_r, b_r and c_r mainly depend on the current conditions of the train (mass, speed, tunnel passing, etc.). When the train is operating under certain conditions, these coefficients a_r, b_r and c_r can be considered as constants. The train operating conditions depend on the train displacement x and velocity $\dot{x}$. Thus, the coefficients a_r, b_r and c_r can be modeled as the piecewise constants depending on the displacement x and velocity $\dot{x}$ of the train.

Grade resistance. From [23], the grade resistance force F_g, which comes from the grade of track at the point of the train's location, is modeled as

$$F_g(t) = M(t) g \sin\theta(t), \tag{3}$$

where $\theta(t)$ is the slope angle of the current track. The track line is made up of several horizontal, slope and curvature tracks. For a certain slope track, the slope angle θ is a constant depending on the displacement x of the train.

Curvature force. From [23], the curvature force F_c, which comes from the curvature of track at the point of the train's location, is modeled as

$$F_c(t) = 0.004 D(t) M(t), \tag{4}$$

where $D(t)$ is the degree of curvature and can be calculated by $D(t) = 0.5 d_w / R(t)$, with d_w being the distance between the front and rare wheels of the train (the wheelbase length, a constant for a certain train), and $R(t)$ being the curve radius (a constant for a certain curvature track). According to the characteristics of the track line, $D(t)$ can be considered as a piecewise constant depending on the displacement x of the train.

From the above analysis, it is clear to see that the longitudinal motion model of the high-speed train is in general described by a time-varying dynamic equation. In practice, its time-variation can be approximated by certain piecewise constant functions, as the train operating conditions usually follow certain piecewise properties. In this paper, such a piecewise constant model and its control problems will be focused. Especially, the case when the parameters of the train are uncertain, will be considered.

C. Piecewise Dynamic Model

Using expressions (2)-(4) of the resistance forces, equation (1) can be rewritten as

$$M(t)\ddot{x}(t) = F(t) - \big(a_r(t) + b_r(t)\dot{x}(t) + c_r(t)\dot{x}^2(t)\big) - M(t)g\sin\theta(t) - 0.004D(t)M(t). \quad (5)$$

With $m(t) = \dfrac{1}{M(t)}$, $a(t) = \dfrac{a_r(t)}{M(t)}$, $b(t) = \dfrac{b_r(t)}{M(t)}$, $c(t) = \dfrac{c_r(t)}{M(t)}$ and $\vartheta(t) = \sin\theta(t)$, this equation can be rewritten as

$$\ddot{x}(t) = m(t)F(t) - \big(a(t) + b(t)\dot{x}(t) + c(t)\dot{x}^2(t)\big) - g\vartheta(t) - 0.004D(t). \quad (6)$$

According to the analysis above, $m(t)$, $a(t)$, $b(t)$, $c(t)$, $\vartheta(t)$, and $D(t)$ are piecewise constants and are dependent on the displacement x and velocity $\dot{x}$ of the train.

Define Ω as the region for all possible system states $x(t)$ and $\dot{x}(t)$ during the train operation, with its l subregions Ω_i, $i = 1, \ldots, l$. The values of $(m(t), a(t), b(t), c(t), \vartheta(t), D(t))$ are determined as $(m(t), a(t), b(t), c(t), \vartheta(t), D(t)) = (m_i, a_i, b_i, c_i, \vartheta_i, D_i)$, if $(x(t), \dot{x}(t)) \in \Omega_i$, where $i = 1, \ldots, l$, m_i, a_i, b_i, c_i, ϑ_i, and D_i are unknown constants. Due to the fact that $x(t)$ and $\dot{x}(t)$ are available, the time instants when $(x(t), \dot{x}(t))$ jumps from one region to another are known. Thus, the index "i" can represent the different operating conditions of the train.

To describe the piecewise constants of the parameters in equation (6), the indicator functions $\chi_i(t)$ are introduced as follows:

$$\chi_i(t) = \begin{cases} 1, & \text{if } (x(t), \dot{x}(t)) \in \Omega_i, \\ 0, & \text{otherwise}, \end{cases} \quad (7)$$

$$\sum_{i=1}^{l} \chi_i(t) = 1, \quad \chi_p(t)\chi_q(t) = 0, \quad \text{for } p \neq q. \quad (8)$$

It is assumed that there do not exist the common boundary, i.e., $(x(t), \dot{x}(t))$ only belongs to one region. Since the information about $(x(t), \dot{x}(t)) \in \Omega_i$ is available, the functions $\chi_i(t)$ defined in (7) are known.

Let $x_1 = x$ and $x_2 = \dot{x}$. The longitudinal motion dynamics (5) can be expressed as

$$\dot{x}_1(t) = x_2(t), \quad (9)$$

$$\dot{x}_2(t) = m(t)F(t) - a(t) - b(t)x_2(t) - c(t)x_2^2(t)) - g\vartheta(t) - 0.004D(t), \quad (10)$$

where

$$m(t) = \sum_{i=1}^{l} m_i\chi_i(t), \quad a(t) = \sum_{i=1}^{l} a_i\chi_i(t), \quad (11)$$

$$b(t) = \sum_{i=1}^{l} b_i\chi_i(t), \quad c(t) = \sum_{i=1}^{l} c_i\chi_i(t), \quad (12)$$

$$\vartheta(t) = \sum_{i=1}^{l} \vartheta_i\chi_i(t), \quad D(t) = \sum_{i=1}^{l} D_i\chi_i(t), \quad (13)$$

with m_i, a_i, b_i, c_i, ϑ_i, and D_i being unknown constants, and $\chi_i(t)$ being the indicator functions defined in (7).

Remark 2: Due to the time-varying parameters of the resistances, the longitudinal dynamic motion model of a train is time-varying. For the heavy haul trains, these parameters are always set as known constants, see [4], [5] and [23]. Recently, some work about the unknown constant parameters has been reported (see [2] and [8]). For high-speed trains, these parameters are varying, especially for c. In this paper, the piecewise constant model is introduced to describe the varying parameters coordinating with the displacement and velocity of the train. The proposed model is not only more convenient for analysis of a time-varying model, but also has the higher accuracy than that of the constant model, which is suitable for a practical control design. □

D. Actuator Failure Model

This paper is focused on dealing with the failures of actuators such as PWMs, traction motors and mechanical drives, which can lead to the traction force $F(t)$ abnormal. The general failures of the actuators, such as, IGBT (Insulated Gate Bipolar Transistor) failures (from PWMs), and failures caused by motor overheating, turn-to-turn short circuit of motor, slipping of mechanical drives, etc. These failures may result in motor stop or loss of effectiveness of motor torque. We consider there are n motors in a train. So, the resultant traction force $F(t)$ is the sum of the forces F_j, $j = 1, \ldots, n$, generated from the jth motor:

$$F(t) = \sum_{j=1}^{n} F_j(t). \quad (14)$$

The actuator failures can be modeled by

$$F_j(t) = \bar{F}_j(t) = \bar{F}_{j0} + \sum_{\rho=1}^{s_j} \bar{F}_{j\rho} f_{j\rho}(t), \quad t \geq t_j, \quad (15)$$

for some $j \in \{1, 2, \ldots, n\}$. Here, the failure occurring time instant t_j, failure index j, constants $\bar{F}_{j0}$ and $\bar{F}_{j\rho}$, are unknown, while the basis signals $f_{j\rho}(t)$ are known, and s_j are the number of the basis signals of the jth actuator failure.

Remark 3: It should be pointed out that the actuator failure model (15) is a completely parameterized form which represents many different types of actuator failures. For instances, when the motor is overheating or the rotor of the motor is locked, it will stop by the protecting system. In this case, $\bar{F}_{j0} = 0$ and $\bar{F}_{j\rho} = 0$, so $\bar{F}_j = 0$, for some $j \in \{1, 2, \ldots, n\}$

and $\rho = 1, \ldots, s_j$. When the mechanical drives slip, the force F_j will become constant, i.e., for some $j \in \{1, 2, \ldots, n\}$ and $\rho = 1, \ldots, s_j$, $\bar{F}_{j0}$ equals to a constant and $\bar{F}_{j\rho} = 0$, so $\bar{F}_j = \bar{F}_{j0}$ is a constant. When the failures (broken, aging) of the IGBTs in PWMs occur, the force may be time-varying, for which the control signal cannot be applied. These actuator failures may occur, but which types of the failure occur are unknown. □

From (15), the input of system (9)-(10) can be expressed as

$$F(t) = \sum_{j=1}^{n} \left(\sigma_j \nu_j(t) + (1 - \sigma_j)\bar{F}_j(t)\right), \tag{16}$$

where $\nu_j(t)$ is the applied control signal to be designed, and σ_j is the actuator failure pattern parameter with

$$\sigma_j = \sigma_j(t) = \begin{cases} 0, & \text{if the } j\text{th actuator fails, i.e.,} F_j(t) = \bar{F}_j(t), \\ 1, & \text{otherwise.} \end{cases} \tag{17}$$

There are n actuators in a train and up to $\bar{n}$ unknown actuator failures ($\bar{n} < n$), that is, any $\bar{n}$ of the n actuators may fail during the train operation. When an actuator fails, the failure time and failure parameters are unknown. For the adaptive actuator failure compensation problem of the high-speed train, the basic assumption is given as: (A1) for any up to $\bar{n}$ actuators fail, the remaining healthy actuators can still achieve the desired control objective.

Since the actuators in the power units use the same control signal, it follows from (15) and (16) that the system input can be expressed by

$$F(t) = k_\nu \nu_0(t) + \xi^T \varpi(t), \tag{18}$$

$$\xi = [\xi_1^T, \xi_2^T, \ldots, \xi_n^T]^T, \tag{19}$$

$$\xi_j = [\xi_{j0}, \quad \xi_{j1}, \ldots, \xi_{js_j}]^T \in R^{s_j+1}, \tag{20}$$

$$\varpi(t) = [1, f_{11}(t), \ldots, f_{1s_1}(t), \ldots, 1, f_{j1}(t), \ldots, f_{js_j}(t), \ldots, 1, f_{n1}(t), \ldots, f_{ns_n}(t)]^T, \quad \text{for} \quad j = 1, \ldots, n, \tag{21}$$

where $\nu_0(t)$ is a designed control signal, and k_ν is the actuator failure pattern parameter with ξ and $\varpi(t)$ to determine which actuators and what kind of failures occur. The parameter k_ν only takes one integer in the interval $[n - \bar{n}, n]$ to respect the different failures. The cases of different k_ν with parameters ξ and $\varpi(t)$ are listed as follows:

1) $k_\nu = n$: there is no failure, and $\xi = 0$.
2) $k_\nu = n - 1$: one actuator failure occurs. If the pth actuator fails, then $\xi_p = [\bar{F}_{p0}, \quad \bar{F}_{p1}, \ldots, \bar{F}_{ps_p}]^T$, and $\xi_q = 0$, for $q = 1, \ldots, n$ and $q \neq p$.
3) $k_\nu = n - \bar{n}$: $\bar{n}$ actuators fail. For the healthy actuators q, the terms $\xi_q = 0$, and for the others p failed actuators, $\xi_p = [\bar{F}_{p0}, \quad \bar{F}_{p1}, \ldots, \bar{F}_{ps_p}]^T$, with $p \neq q$.

Remark 4: Equations (18)-(21) have been used to describe the system input, which contain the healthy and faulty cases of the actuators. The different values of the coordinating parameters k_ν and ξ represent the corresponding different actuator failures. It should be noted that the parameters k_ν and ξ can change their values with the failure evolution. But for a time interval, the actuator failure patten is fixed, that is, actuators only fail at some time points. Considering the unknown failure time points, k_ν and ξ are piecewise constants with the unknown jump time. □

E. Objective and Design Issues

Objective. The objective of this paper is to develop an adaptive failure compensation scheme for high-speed trains described by (2), (9), and (10), with unknown friction parameters modeled in (11)-(13), and unknown actuator failures modeled in (18)-(21), to guarantee the system stability and asymptotic tracking properties even in the presence of actuator failures.

Design issues. To achieve the objective above, the following technical issues need to be solved:

1) Develop an adaptive controller for the healthy high-speed train modeled by piecewise constant model with unknown parameters, to guarantee system stability and asymptotic tracking.
2) Analyze matching conditions for healthy and failure cases to design the adaptive controller.
3) Design an adaptive failure compensation scheme for the piecewise constant model with unknown parameters and actuator failures.

In the subsequent sections, a failure compensation framework will be proposed, under which both healthy and faulty cases are studied. The failure compensation controller proposed for the actuator failures with unknown failure time and failure parameters can deal with the piecewise constant model with the unknown parameters, simultaneously.

III. Adaptive Controller Design for Healthy System

The analysis in Section III shows that the longitudinal motion of high-speed trains is modeled as a piecewise constant nonlinear system with unknown parameters. The controller design for this class of system has not been available in the existing results. A new adaptive state feedback controller is proposed for the first time to achieve the closed-loop stability (signal boundedness) and state tracking for the high-speed train motion control.

A. Reference Model System

The system is expected to track the reference trajectory $x_d(t)$, which is produced by a linear reference model system

$$\dot{x}_d(t) = A_d x_d(t) + B_d r(t), \quad x_d(t) = [x_{d1}(t), x_{d2}(t)]^T \tag{22}$$

where A_d is a stable matrix.

From the structure of the train system (9)-(10), the reference model system is chosen as

$$\begin{bmatrix} \dot{x}_{d1}(t) \\ \dot{x}_{d2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_{d1} & -a_{d2} \end{bmatrix} \begin{bmatrix} x_{d1}(t) \\ x_{d2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b_d \end{bmatrix} r(t), \tag{23}$$

where $a_{d1} > 0$, $a_{d2} > 0$, and $b_d > 0$, $r(t) \in R$ is the reference input signal, which is continuous and bounded.

Although the plant (9)-(10) is a piecewise constant model, the structure of the model is determined. The reference model can be chosen a common one, that is, the parameters in reference model (23) are constants. This choice can satisfy the design requirements and can simplify the controller design, which is more suitable for practical engineering.

B. Controller Structure

The following adaptive state feedback controller structure is proposed:

$$F(t) = k_{x_1}(t)x_1(t) + k_{x_2}(t)x_2(t) + k_r(t)r(t) + \hat{a}(t) + \hat{c}(t)x_2^2(t) + g\hat{\vartheta}(t) + 0.004\hat{D}(t), \quad (24)$$

where $r(t)$ is a reference input signal, $k_{x_1}(t)$, $k_{x_2}(t)$, $k_r(t)$, $\hat{a}(t)$, $\hat{c}(t)$, $\hat{\vartheta}(t)$, and $\hat{D}(t)$ are time-varying parameters defined as

$$k_{x_1}(t) = \sum_{i=1}^{l} k_{x_{1i}}(t)\chi_i(t), \quad k_{x_2}(t) = \sum_{i=1}^{l} k_{x_{2i}}(t)\chi_i(t), \quad (25)$$

$$k_r(t) = \sum_{i=1}^{l} k_{ri}(t)\chi_i(t), \quad \hat{a}(t) = \sum_{i=1}^{l} \hat{a}_i(t)\chi_i(t), \quad (26)$$

$$\hat{c}(t) = \sum_{i=1}^{l} \hat{c}_i(t)\chi_i(t), \quad \hat{\vartheta}(t) = \sum_{i=1}^{l} \hat{\vartheta}_i(t)\chi_i(t), \quad (27)$$

$$\hat{D}(t) = \sum_{i=1}^{l} \hat{D}_i(t)\chi_i(t), \quad (28)$$

with $\chi_i(t)$ being defined in (7), $k_{x_{1i}}(t)$, $k_{x_{i2}}(t)$, $k_{ri}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, and $\hat{D}_i(t)$ being the time-varying estimates of the nominal controller parameters:

$$k^*_{x_1}(t) = \sum_{i=1}^{l} k^*_{x_{1i}}\chi_i(t), \quad k^*_{x_2}(t) = \sum_{i=1}^{l} k^*_{x_{2i}}\chi_i(t), \quad (29)$$

$$k^*_r(t) = \sum_{i=1}^{l} k^*_{ri}\chi_i(t), \quad a^*(t) = \sum_{i=1}^{l} a^*_i\chi_i(t), \quad (30)$$

$$c^*(t) = \sum_{i=1}^{l} c^*_i\chi_i(t), \quad \vartheta^*(t) = \sum_{i=1}^{l} \vartheta^*_i\chi_i(t), \quad (31)$$

$$D^*(t) = \sum_{i=1}^{l} D^*_i\chi_i(t), \quad (32)$$

with $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, a^*_i, c^*_i, and ϑ^*_i, D^*_i being constants and satisfying:

$$a_{d1} = -m_i k^*_{x_{1i}}, \quad a_{d2} = b_i - m_i k^*_{x_{2i}}, \quad b_d = m_i k^*_{ri}, \quad (33)$$

$$a_i = m_i a^*_i, \quad c_i = m_i c^*_i, \quad \vartheta_i = m_i \vartheta^*_i, \quad D_i = m_i \hat{D}^*_i. \quad (34)$$

With (24) in (9)-(10) under (33)-(34), we have the closed-loop system

$$\dot{x}_1(t) = x_2(t), \quad (35)$$

$$\begin{aligned}\dot{x}_2(t) = & -a_{d1}x_1(t) - a_{d2}x_2(t) + b_d r(t) \\ & + \sum_{i=1}^{l} m_i\tilde{k}_{x_{1i}}(t)\chi_i(t)x_1(t) \\ & + \sum_{i=1}^{l} m_i\tilde{k}_{x_{2i}}(t)\chi_i(t)x_2(t) + \sum_{i=1}^{l} m_i\tilde{k}_{ri}(t)\chi_i(t)r(t) \\ & + \sum_{i=1}^{l} m_i\tilde{a}_i(t)\chi_i(t) + \sum_{i=1}^{l} m_i\tilde{c}_i(t)\chi_i(t)x_2^2(t) \\ & + g\sum_{i=1}^{l} m_i\tilde{\vartheta}_i(t)\chi_i(t) + 0.004\sum_{i=1}^{l} m_i\tilde{D}_i(t)\chi_i(t). \quad (36)\end{aligned}$$

where $\tilde{k}_{x_{1i}}(t) = k^*_{x_{1i}} - k_{x_{1i}}(t)$, $\tilde{k}_{x_{2i}}(t) = k^*_{x_{2i}} - k_{x_{2i}}(t)$, $\tilde{k}_{ri}(t) = k^*_{ri} - k_{ri}(t)$, $\tilde{a}_i(t) = a^*_i - \hat{a}_i(t)$, $\tilde{c}_i(t) = c^*_i - \hat{c}_i(t)$, $\tilde{\vartheta}_i(t) = \vartheta^*_i - \hat{\vartheta}_i(t)$.

The equations in (33)-(34) are the plant-model matching conditions; that is, if the plant parameters m_i, a_i, b_i, c_i, ϑ_i, and D_i are known, the nominal control law

$$F(t) = k^*_{x_1}(t)x_1(t) + k^*_{x_2}(t)x_2(t) + k^*_r(t)r(t) + a^*(t) + c^*(t)x_2^2(t) + g\vartheta^*(t) + 0.004D^*(t), \quad (37)$$

leads to the tracking errors $e_1(t) = x_1(t) - x_{d1}(t)$ and $e_2(t) = x_2(t) - x_{d2}(t)$ satisfy

$$\dot{e}_1(t) = e_2(t), \quad (38)$$

$$\dot{e}_2(t) = -a_{d1}e_1(t) - a_{d2}e_2(t), \quad (39)$$

which implies that $e_1(t)$ and $e_2(t)$ approach zero exponentially as $t \to \infty$, due to the choice of $a_{d1} > 0$ and $a_{d2} > 0$ to make A_d stable.

C. Adaptive Laws

To develop adaptive laws for updating the parameter estimates $k_{x_{1i}}(t)$, $k_{x_{i2}}(t)$, $k_{ri}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, and $\hat{D}_i(t)$, an error equation in terms of $e_1(t)$, $e_2(t)$, $\tilde{k}_{x_{1i}}(t)$, $\tilde{k}_{x_{2i}}(t)$, $\tilde{k}_{ri}(t)$, $\tilde{a}_i(t)$, $\tilde{c}_i(t)$, $\tilde{\vartheta}_i(t)$, $\tilde{D}_i(t)$ is needed. In view of (23) and (35)-(36), we have

$$\dot{e}_1(t) = e_2(t), \quad (40)$$

$$\begin{aligned}\dot{e}_2(t) = & -a_{d1}e_1(t) - a_{d2}e_2(t) \\ & + \sum_{i=1}^{l} \frac{1}{k^*_{ri}} b_d \Big(\tilde{k}_{x_{1i}}(t)\chi_i(t)x_1(t) + \tilde{k}_{x_{2i}}(t)\chi_i(t)x_2(t) \\ & + \tilde{k}_{ri}(t)\chi_i(t)r(t) + \tilde{a}_i(t)\chi_i(t) + \tilde{c}_i(t)\chi_i(t)x_2^2(t) \\ & + g\tilde{\vartheta}_i(t)\chi_i(t) + 0.004\tilde{D}_i(t)\chi_i(t)\Big). \quad (41)\end{aligned}$$

With $e(t) = [e_1(t), \; e_2(t)]^T$, the following parameter adaptive laws are used to update the controller parameters in (24):

$$\dot{k}_{x_{1i}}(t) = -\Gamma_{x_{1i}} x_1(t) e^T(t) P_d B_d \chi_i(t), \quad (42)$$

$$\dot{k}_{x_{2i}}(t) = -\Gamma_{x_{2i}} x_2(t) e^T(t) P_d B_d \chi_i(t), \quad (43)$$

$$\dot{k}_{ri}(t) = -\Gamma_{ri} r(t) e^T(t) P_d B_d \chi_i(t), \quad (44)$$

$$\dot{\hat{a}}_i(t) = -\Gamma_{ai} e^T(t) P_d B_d \chi_i(t), \quad (45)$$

$$\dot{\hat{c}}_i(t) = -\Gamma_{ci} x_2^2(t) e^T(t) P_d B_d \chi_i(t), \quad (46)$$

$$\dot{\hat{\vartheta}}_i(t) = -\Gamma_{\vartheta i} g e^T(t) P_d B_d \chi_i(t), \quad (47)$$

$$\dot{\hat{D}}_i(t) = -\Gamma_{Di} 0.004 e^T(t) P_d B_d \chi_i(t), \quad (48)$$

where $\Gamma_{x_{1i}}$, $\Gamma_{x_{2i}}$, Γ_{ri}, Γ_{ci}, Γ_{ai}, $\Gamma_{\vartheta i}$, and Γ_{Di} are positive constants and $P_d > 0$, satisfying $A_d^T P_d + P_d A_d = -Q_d$, for some $Q_d > 0$.

Remark 5: It should be noted that there is a sign function sign[·], for examples [18] and [19], in adaptive laws. However, the adaptive laws (42)-(48) do not involve the sign functions, due to sign$[k^*_{ri}]$ being positive. This can be obtained directly from the equation $b_d = m_i k^*_{ri}$, as b_d is chosen as positive constant and m_i is always positive. □

D. Stability Analysis

Based on the adaptive laws (42)-(48), the following stability and tracking properties can be obtained:

Theorem 1: *For the piecewise constant system (9)-(10) and the reference model system (23), the controller (24) with its parameters updated by the adaptive laws (42)-(48) ensures the boundedness of all closed-loop signals, and the asymptotic state tracking:* $\lim_{t\to\infty} e(t) = 0$.

Proof: The values of the parameters a_{d1}, a_{d2} and b_d ensure the stability of (23), i.e., $x_d(t) \in L_\infty$.

Consider the following continuous Lyapunov function

$$V = e^T P_d e + \sum_{i=1}^{l} \frac{1}{k^*_{ri}} \Big(\Gamma^{-1}_{x_{1i}} \tilde{k}^2_{x_{1i}} + \Gamma^{-1}_{x_{2i}} \tilde{k}^2_{x_{2i}} + \Gamma^{-1}_{ri} \tilde{k}^2_{ri} + \Gamma^{-1}_{ai} \tilde{a}^2_i + \Gamma^{-1}_{ci} \tilde{c}^2_i + \Gamma^{-1}_{\vartheta i} \tilde{\vartheta}^2_i + \Gamma^{-1}_{Di} \tilde{D}^2_i \Big). \quad (49)$$

With the estimation error in (40)-(41) and the adaptive laws in (42)-(48), the time derivative of V becomes

$$\dot{V} = -e^T(t) Q_d e(t) \leq 0, \quad (50)$$

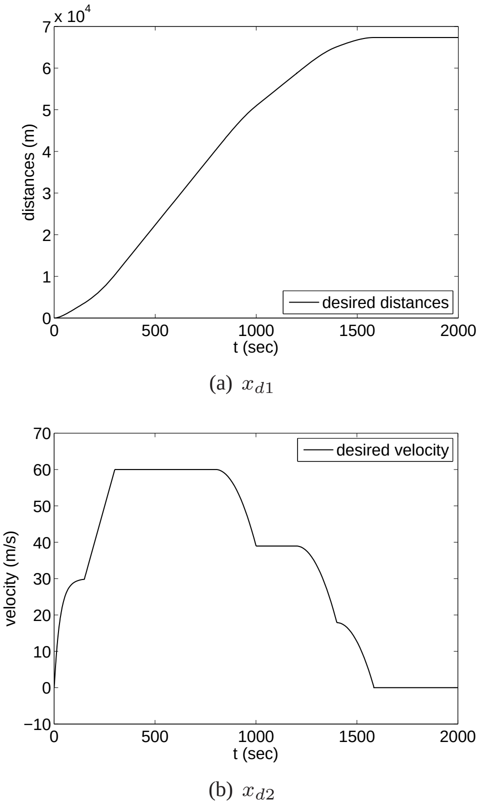
which indicates that the closed-loop system consisting of (40)-(41) and (42)-(48) is uniformly stable and its solutions is uniformly bounded, that is, $e(t)$, $x_1(t)$, $x_2(t)$, $k_{x_{1i}}(t)$, $k_{x_{2i}}(t)$, $k_{ri}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, $\hat{D}_i(t)$, and $\dot{e}(t)$ are all bounded. Then, with the structure of the failure compensation controller (24), the boundedness of $\nu_0(t)$ is ensured. Further, (50) implies $e(t) \in L_2$ and so $\lim_{t\to\infty} e(t) = 0$. ∇

The proposed adaptive control scheme can achieve the closed-loop stability and tracking performance of high-speed trains with its time-varying parameters modeled as piecewise constants. Compared with the existing results [2], [3], this method can relax the condition that the general resistance $F_r(t)$ of the unknown parameters $a(t)$, $b(t)$ and $c(t)$ should have a known bound.

E. Simulation Study

To demonstrate the effectiveness of the proposed adaptive controller, simulation study on a high-speed train is presented. The system parameters are borrowed from a CRH type train ([2], [24]), in which 4 motors are considered.

Tracking performance and reference model. To verify the control scheme well and according to [2], [25], [26], several operating conditions including acceleration, reacceleration, constant speed, deceleration, constant speed, redeceleration, and slowing down until fully stop, as shown in Fig. 1, are considered during the train operation. Choose the parameters of the reference model as $a_{d1} = 0.12$, $a_{d2} = 1.9$ and $b_d = 1/(500 \times 10^3)$. In the simulation, the reference input is calculated based on the distance and the velocity given in Fig. 1.



(a) x_{d1}

(b) x_{d2}

Fig. 1: Desired distance and velocity trajectories.

Simulation conditions. According to the tracking distance and velocity trajectories in Fig. 1, the train does not stop during the travel. So the mass of the train does not change, which is chosen as $M_i = M = 400$ ton. We consider the train consisting of 8 vehicles (4 locomotives and 4 carriages), in which the number of the actuators are 16. Due to the tunnel, slope and curvature which lead to the changes of the resistance coefficients in the travel, 4 modes will be considered for the healthy system (a_i, b_i, c_i, ϑ_i, and D_i are defined as equations (11)-(13), and a_i, b_i, c_i, ϑ_i are expressed in kN, kN s/m, kN s^2/m^2 and degree.)

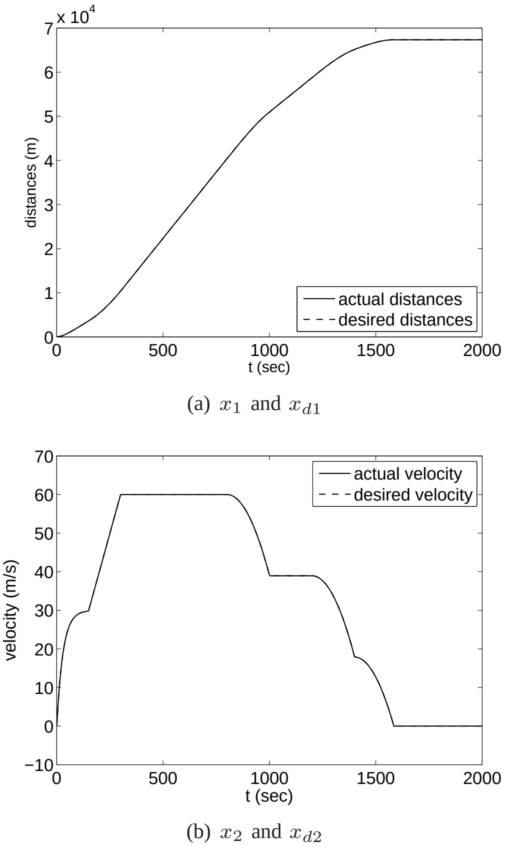
(i) For $t < 400$ s, the train bakes up. In this case, the coefficients are chosen as $a_1 = 3.25 \times 10^3$, $b_1 = 26.75$, $c_1 = 0.48$, $\theta_1 = 0$, and $D_1 = 0$.

(ii) During $400 \leq t < 800$ s, the train enters the tunnel. Then the coefficients are chosen as $a_2 = 3.25 \times 10^3$, $b_2 = 26.75$, $c_2 = 0.78$, $\theta_2 = 0$, and $D_2 = 0$.

(iii) At 800 s, the train leaves the tunnel and travels in the slope and curvature track. For $800 \leq t < 1200$ s, the coefficients are $a_1 = 3.25 \times 10^3$, $b_1 = 26.75$, $c_3 = 0.48$, $\theta_3 = 10$, and $D_3 = 0.34$.

(iv) After 1200 s, the train moves in the open air and horizontal track to slow down until fully stop. For $1200 \leq t < 2000$ s, the coefficients are chosen as $a_4 = 3.25 \times 10^3$, $b_4 = 26.75$, $c_4 = 0.48$, $\theta_4 = 0$, and $D_4 = 0$.

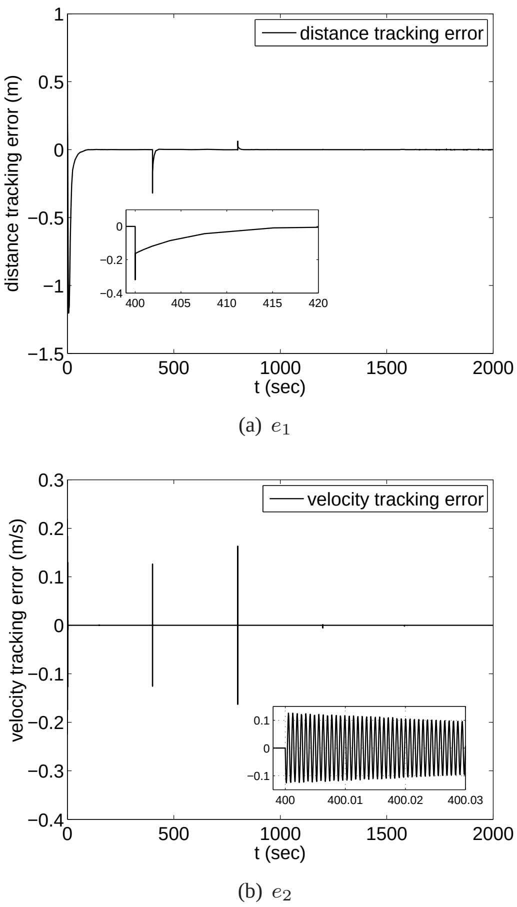
These 4 modes are used to construct the controller system, in which the parameters are unknown for the adaptive controller design.



(a) x_1 and x_{d1}

(b) x_2 and x_{d2}

Fig. 2: Distance and velocity trajectories for the piecewise constant healthy model.



(a) e_1

(b) e_2

Fig. 3: Tracking errors for the piecewise constant healthy model.

For simulation purpose, the initial sates are chosen as $x(0) = [-0.5 \quad 0]^T$, and the initial parameter estimates are chosen as 90% of their nominal values. The gains of the adaptive laws in (42)-(48) are chosen as 2.

Simulation results. Figs. 2-3 show the simulation results of the healthy traction system modeled by piecewise model. Fig. 2 shows the distances (a) and velocities (b) of the train and the reference model, in which the distance and velocity of the train are represented by solid lines, while the desired distance and velocity are used the dashed lines. Fig. 3 shows the state tracking errors including the distance (a) and velocity (b). From the simulation results, it can be seen that the proposed adaptive controller can achieve the close-loop stability and asymptotic tracking properties of the train even in the presence of parameters changes.

IV. Adaptive Failure Compensation Design for Single Operating Condition

In this section, an adaptive failure compensation scheme is proposed to guarantee the system stability and asymptotic tracking properties in the presence of uncertain actuator failures, for the single operating condition when train is operating under a certain rail condition, i.e., the parameters of the train dynamic model are unknown constants but uncertain actuator failures may occur. The key task is to design a failure compensation controller structure which can guarantee the plant-model matching and is capable of dealing with any possible failures: $F(t) = k_\nu \nu_0(t) + \xi^T \varpi(t)$, with k_ν and ξ being unknown piecewise constants.

A. System Description

Without loss of generality, assume that the train is operating under the ith dynamic model with the presence of actuator failures. From (18)-(21), the system (9)-(10) is given by

$$\dot{x}_1(t) = x_2(t), \tag{51}$$

$$\dot{x}_2(t) = m_i(k_\nu \nu_0(t) + \xi^T \varpi(t)) - a_i - b_i x_2(t) - c_i x_2^2(t) - g\vartheta_i - 0.004D_i, \tag{52}$$

where m_i, a_i, b_i, c_i, ϑ_i, and D_i are unknown constants; $\varpi(t)$ is the known function; $k_\nu = n$ and $\xi = 0$, before failures occur, and are unknown constants, after failures occur; ν_0 is the applied control signal to be designed with failure compensation to guarantee the closed-loop stability and asymptotic state tracking properties. Note that if an actuator of a train fails, its failure time and failure value are unknown. The index "i" is fixed to represent that the train is running in a certain operating condition, i.e., the train is on a certain rail.

B. Failure Compensation Controller Structure

We construct the adaptive failure compensation controller structure as:

$$\nu_0(t) = k_{x_{1i}}(t)x_1(t) + k_{x_{2i}}(t)x_2(t) + k_{ri}(t)r(t) + \hat{\xi}^T(t)\varpi(t) + \hat{a}_i(t) + \hat{c}_i(t)x_2^2(t) + g\hat{\vartheta}_i(t) + 0.004\hat{D}_i(t), \quad (53)$$

where $r(t)$ is a reference input signal, $k_{x_{1i}}(t)$, $k_{x_{2i}}(t)$, $k_{ri}(t)$, $\hat{\xi}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, and $\hat{D}_i(t)$ are are the time-varying estimates of the nominal controller parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i, defined to satisfy

$$a_{d1} = -m_i k_\nu k^*_{x_{1i}}, \quad a_{d2} = b_i - m_i k_\nu k^*_{x_{2i}}, \quad (54)$$
$$b_d = m_i k_\nu k^*_{ri}, \quad \xi = -k_\nu \xi^*, \quad a_i = m_i k_\nu a^*_i, \quad (55)$$
$$c_i = m_i k_\nu c^*_i, \quad \vartheta_i = m_i k_\nu \vartheta^*_i, \quad D_i = m_i k_\nu D^*_i. \quad (56)$$

Use the control law (53) and the system (51)-(52) under the conditions (54)-(56), to obtain

$$\dot{x}_1(t) = x_2(t), \quad (57)$$
$$\dot{x}_2(t) = -a_{d1}x_1(t) - a_{d2}x_2(t) + b_d r(t) + m_i k_\nu \tilde{k}_{x_{1i}}(t)x_1(t) + m_i k_\nu \tilde{k}_{x_{2i}}(t)x_2(t) + m_i k_\nu \tilde{k}_{ri}(t)r(t) + m_i k_\nu \tilde{\xi}^T(t)\varpi(t) + m_i k_\nu \tilde{a}_i(t) + m_i k_\nu \tilde{c}_i(t)x_2^2(t) + g m_i k_\nu \tilde{\vartheta}_i(t) + 0.004 m_i k_\nu \tilde{D}_i(t). \quad (58)$$

where $\tilde{k}_{x_{1i}}(t) = k^*_{x_{1i}} - k_{x_{1i}}(t)$, $\tilde{k}_{x_{2i}}(t) = k^*_{x_{2i}} - k_{x_{2i}}(t)$, $\tilde{k}_{ri}(t) = k^*_{ri} - k_{ri}(t)$, $\tilde{\xi}(t) = \xi^* - \hat{\xi}(t)$, $\tilde{a}_i(t) = a^*_i - \hat{a}_i(t)$, $\tilde{c}_i(t) = c^*_i - \hat{c}_i(t)$, $\tilde{\vartheta}_i(t) = \vartheta^*_i - \hat{\vartheta}_i(t)$, $\tilde{D}_i(t) = D^*_i - \hat{D}_i(t)$.

Similar to the healthy case, the equations in (54)-(56) are the plant-model matching conditions. The tracking errors $e_1(t) = x_1(t) - x_{d1}(t)$ and $e_2(t) = x_2(t) - x_{d2}(t)$ under the nominal failure compensation controller $\nu^*_0(t) = k^*_{x_{1i}}x_1(t) + k^*_{x_{2i}}x_2(t) + k^*_{ri}r(t) + \xi^{*T}\varpi(t) + a^*_i + c^*_i x_2^2(t) + g\vartheta^*_i + 0.004D^*_i(t)$, satisfy (38) and (39), which implies that $e_1(t)$ and $e_2(t)$ approach zero exponentially as $t \to \infty$.

As in [18], let (T_p, T_{p+1}), $p = 0, 1, \ldots, \mathcal{M}$, with $T_0 = 0$, be time intervals. During these time intervals, the actuator failure pattern is fixed, which means that the actuators only fail at time T_p, for $p = 0, 1, \ldots, \mathcal{M}$. Under Assumption (A1), we have $\mathcal{M} \le \bar{n}$ and $T_{\mathcal{M}+1} = \infty$. At time $T_{\bar{p}}$, $\bar{p} = 0, 1, \ldots, \mathcal{M}$, similar to that in [19], it is obtained that the unknown plant model matching parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i, change their values as:

$$k^*_{x_{1i}} = k^*_{x_{1i}(p)}, \quad k^*_{x_{2i}} = k^*_{x_{2i}(p)}, \quad k^*_{ri} = k^*_{ri(p)}, \quad (59)$$
$$\xi^* = \xi^*_{(p)}, \quad a^*_i = a^*_{i(p)}, \quad c^*_i = c^*_{i(p)}, \quad \vartheta^*_i = \vartheta^*_{i(p)}, \quad (60)$$
$$D^*_i = D^*_{i(p)}, \quad (61)$$

for $t \in (T_p, T_{p+1})$, $p = 0, 1, \ldots, \mathcal{M}$, that is, the plant-model matching parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i are piecewise constants, since under different failure conditions, the system has different characteristics.

C. Adaptive Laws

When the constant parameters m_i, k_ν, ξ, a_i, b_i, c_i, ϑ_i, and D_i are unknown, it is required to use the adaptive failure compensation controller (53) to ensure the stability of the closed-loop system.

Substituting (23) into (57)-(58) and with the matching condition $b_d = m_i k_\nu k^*_{ri}$, it follows that the tracking error dynamics are described by

$$\dot{e}_1(t) = e_2(t), \quad (62)$$
$$\dot{e}_2(t) = -a_{d1}e_1(t) - a_{d2}e_2(t) + \frac{1}{k^*_{ri}} b_d \Big(\tilde{k}_{x_{1i}}(t)x_1(t) + \tilde{k}_{x_{2i}}(t)x_2(t) + \tilde{k}_{ri}(t)r(t) + \tilde{\xi}^T(t)\varpi(t) + \tilde{a}_i(t) + \tilde{c}_i(t)x_2^2(t) + g\tilde{\vartheta}_i(t) + 0.004\tilde{D}_i(t) \Big). \quad (63)$$

Similar to the healthy case and from the marching conditions (54)-(56) , the sign of the parameter k^*_{ri} can be obtained as positive. With $e(t) = [e_1(t), \ \ e_2(t)]^T$ and sign$[k^*_{ri}]$ being positive, the following parameter adaptive laws are applied to update the controller parameters in (53):

$$\dot{k}_{x_{1i}}(t) = -\gamma_{x_{1i}} x_1(t) e^T(t) P_d B_d, \quad (64)$$
$$\dot{k}_{x_{2i}}(t) = -\gamma_{x_{2i}} x_2(t) e^T(t) P_d B_d, \quad (65)$$
$$\dot{k}_{ri}(t) = -\gamma_{ri} r(t) e^T(t) P_d B_d, \quad (66)$$
$$\dot{\hat{\xi}}(t) = -\gamma_\xi \varpi(t) e^T(t) P_d B_d, \quad (67)$$
$$\dot{\hat{a}}_i(t) = -\gamma_{ai} e^T(t) P_d B_d, \quad (68)$$
$$\dot{\hat{c}}_i(t) = -\gamma_{ci} x_2^2(t) e^T(t) P_d B_d, \quad (69)$$
$$\dot{\hat{\vartheta}}_i(t) = -\gamma_{\vartheta i} g e^T(t) P_d B_d, \quad (70)$$
$$\dot{\hat{D}}_i(t) = -\gamma_{Di} 0.004 e^T(t) P_d B_d, \quad (71)$$

where $\gamma_{x_{1i}}$, $\gamma_{x_{2i}}$, γ_{ri}, γ_ξ, γ_{ci}, γ_{ai}, $\gamma_{\vartheta i}$, and γ_{Di} are positive constants and $P_d > 0$, satisfying $A_d^T P_d + P_d A_d = -Q_d$, for some $Q_d > 0$.

D. Performance Analysis

We obtain the following stability and tracking properties:

Theorem 2: *The adaptive failure compensation controller (53) updated by the adaptive laws (64)-(71), applied to the faulty system (51)-(52), ensures that all closed-loop signals are bounded and the state tracking error $e(t)$ satisfies* $\lim_{t\to\infty} e(t) = 0$.

Proof: Consider the following candidate Lyapunov function

$$V = e^T P_d e + \frac{1}{k^*_{ri}} \Big(\gamma_\xi^{-1}\tilde{\xi}^T\tilde{\xi} + \gamma_{x_{1i}}^{-1}\tilde{k}^2_{x_{1i}} + \gamma_{x_{2i}}^{-1}\tilde{k}^2_{x_{2i}} + \gamma_{ri}^{-1}\tilde{k}^2_{ri} + \gamma_{ai}^{-1}\tilde{a}_i^2 + \gamma_{ci}^{-1}\tilde{c}_i^2 + \gamma_{\vartheta i}^{-1}\tilde{\vartheta}_i^2 + \gamma_{Di}^{-1}\tilde{D}_i^2 \Big). \quad (72)$$

For $t \in (T_p, T_{p+1})$ and $p = 0, 1, \ldots, \mathcal{M}$, $\dot{\tilde{k}}_{x_{1i}}(t) = -\dot{k}_{x_{1i}}(t)$, $\dot{\tilde{k}}_{x_{2i}}(t) = -\dot{k}_{x_{2i}}(t)$, $\dot{\tilde{k}}_{ri}(t) = -\dot{k}_{ri}(t)$, $\dot{\tilde{\xi}}(t) = -\dot{\hat{\xi}}(t)$, $\dot{\tilde{a}}_i(t) = -\dot{\hat{a}}_i(t)$, $\dot{\tilde{c}}_i(t) = -\dot{\hat{c}}_i(t)$, $\dot{\tilde{\vartheta}}_i(t) = -\dot{\hat{\vartheta}}_i(t)$, $\dot{\tilde{D}}_i(t) = -\dot{\hat{D}}_i(t)$, given that $k^*_{x_{1i}} = k^*_{x_{1i}(p)}$, $k^*_{x_{2i}} = k^*_{x_{2i}(p)}$, $k^*_{ri} = k^*_{ri(p)}$, $\xi^* = \xi^*_{(p)}$, $a^*_i = a^*_{i(p)}$, $c^*_i = c^*_{i(p)}$, $\vartheta^*_i = \vartheta^*_{i(p)}$, $D^*_i = D^*_{i(p)}$ are constant for $t \in (T_p, T_{p+1})$. It should be noted that $V(\cdot)$ as a function of t is not continuous because $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i, are piecewise constant parameters.

With the estimation errors in (62)-(63) and the adaptive laws in (64)-(71), the time derivative of V for $t \in (T_p, T_{p+1})$, $p = 0, 1, \ldots, \mathcal{M}$, becomes

$$\dot{V} = -e^T(t) Q_d e(t) \le 0, \quad (73)$$

Due to the finite number of failures in the system, $V(T_{\mathcal{M}})$ is finite. From

$$\dot{V} = -e^T(t)Q_d e(t) \leq 0, \quad t \in (T_{\mathcal{M}}, \infty), \tag{74}$$

the closed-loop system consisting of (62)-(63) and (64)-(71) is uniformly stable and its solutions is uniformly bounded. Therefore, all the variables $e(t)$, $x_1(t)$, $x_2(t)$, $k_{x_{1i}}(t)$, $k_{x_{2i}}(t)$, $k_{ri}(t)$, $\hat{\xi}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, $\hat{D}_i(t)$, and $\dot{e}(t)$ are bounded. Then, with the structure of the failure compensation controller (53), the boundedness of $\nu_0(t)$ is ensured. Further, (74) implies $e(t) \in L_2$ and so $\lim_{t\to\infty} e(t) = 0$. ∇

In this section, the adaptive failure compensation method is proposed for the constant case with unknown actuator failures, which can achieve the tracking errors convergent to zero, i.e., the distance and velocity of the train can track the desired trajectories.

E. Simulation Study

In this section, a simulation study result is given to demonstrate the effectiveness of the proposed failures compensation scheme. Consider the plant the same as that of the healthy case in Part E of Section III.

Simulation conditions. The desired tracking performance and reference model are taken the same as in Section III, which is shown in Fig. 1. In this section, only the constant parameter case is considered. The coefficients are chosen as $a_i = 3.25 \times 10^3$ (kN), $b_i = 26.75$ (kN s/m), $c_i = 0.48$ (kN s^2/m^2), $\theta_i = 10$ (degree), $D_i = 0.34$, with $M_i = M = 400$ ton, $g = 9.8$, $n = 16$.

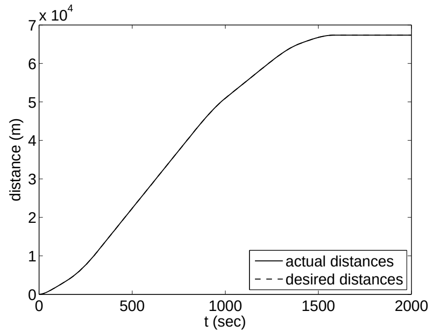
Due to the same type and control scheme applied for each motor, if there are more than one motor failed, the effectiveness of the failed motors can be considered as the failure from one motor. Then, the left healthy motors provide the traction force. Here, one motor failure is taken into consideration, which is expressed as: F_α fails for some $\alpha \in \{1, 2, 3, 4\}$,

$$F_\alpha(t) = \begin{cases} 2 \times 10^5, & \text{for } 400 \leq t < 600\text{s}; \\ 2 \times 10^5(1 + \sin(0.05t - 30)), & \text{for } 600 \leq t < 800\text{s}; \\ 0, & \text{for } 800 \leq t \leq 2000\text{s}; \end{cases}$$

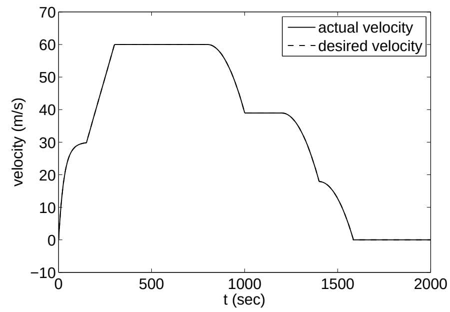
$F_\beta = \nu_\beta$, $\beta \neq \alpha$, $\beta \in \{1, 2, 3, 4\}$.

The initial conditions are chosen as $x_d(0) = x(0) = [0 \;\; 0]^T$, and the initial parameter estimates are 95% of their ideal values. The gains of the adaptive laws in (64)-(71) are chosen as 2.

Simulation results. Figs. 4-5 show the simulation results of the traction system with actuator failures. Fig. 4 shows the distances (a) and velocities (b) including the plant distance and velocity (solid) and the desired distance and velocity (dashed). Fig. 5 shows the tracking errors including the distance (a) and velocity (b). From the simulation results, it can be seen that the proposed adaptive controller can achieve the close-loop stability and asymptotic tracking properties of the train even in the presence of parameters changes.

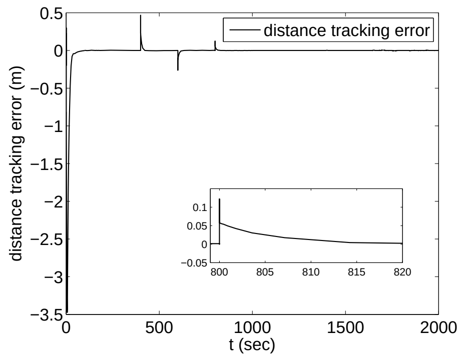


(a) x_1 and x_{d1}

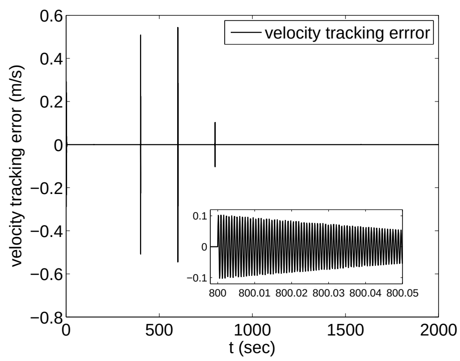


(b) x_2 and x_{d2}

Fig. 4: Distances and velocity for the constant case with actuator failures.



(a) e_1



(b) e_2

Fig. 5: Tracking errors for the constant case with actuator failures.

V. Adaptive Failure Compensation Design for Multiple Operating Conditions

In the above section, we have proposed an adaptive failure compensation scheme for the single operating condition, i.e., the parameters of the plant are constants. In this section, an adaptive failure compensation scheme will be developed for the multiple (whole) operating conditions, in the case where the parameters of the model are piecewise constants with known jump time and the actuator failures are expressed as piecewise constants with unknown jump time (occurrence time). The key task is to design a failure compensation controller with its adaptive laws, which is suitable for these two kinds of piecewise models, simultaneously.

A. Faulty System Description

With actuator failures modeled as in (18)-(21), the dynamic (9)-(10) becomes

$$\dot{x}_1(t) = x_2(t), \tag{75}$$

$$\begin{aligned}\dot{x}_2(t) =& \sum_{i=1}^{l} m_i\chi_i(t)(k_\nu\nu_0(t) + \xi^T\varpi(t)) \\ &- \sum_{i=1}^{l} a_i\chi_i(t) - \sum_{i=1}^{l} b_i\chi_i(t)x_2(t) - \sum_{i=1}^{l} c_i\chi_i(t)x_2^2(t) \\ &-g\sum_{i=1}^{l}\vartheta_i\chi_i(t) - 0.004\sum_{i=1}^{l} D_i\chi_i(t),\end{aligned} \tag{76}$$

where the piecewise constant parameters m_i, a_i, b_i, c_i, ϑ_i, and D_i are defined in (11)-(13); $\varpi(t)$ is the known function; $k_\nu = n$ and $\xi = 0$, before failures occur, and are unknown constants, after failures occur; ν_0 is the applied control signal to be designed with failure compensation to guarantee the system stability and asymptotic tracking properties. Note that if an actuator of a train fails, its failure time and failure value are unknown.

B. Failure Compensation Controller Structure

We propose the adaptive failure compensation controller structure as

$$\begin{aligned}\nu_0(t) =& k_{x_1}(t)x_1(t) + k_{x_2}(t)x_2(t) + k_r(t)r(t) + \hat{\xi}^T(t)\varpi(t) \\ &+\hat{a}(t) + \hat{c}(t)x_2^2(t) + g\hat{\vartheta}(t) + 0.004\hat{D}(t),\end{aligned} \tag{77}$$

where $r(t)$ is a reference input signal, $\hat{\xi}(t)$ is a time-varying parameters, $k_{x_1}(t)$, $k_{x_2}(t)$, $k_r(t)$, $\hat{a}(t)$, $\hat{c}(t)$, $\hat{\vartheta}(t)$, and $\hat{D}(t)$ are time-varying parameters defined as (25)-(28), with $k_{x_{1i}}(t)$, $k_{x_{2i}}(t)$, $k_{ri}(t)$, $\hat{\xi}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, and $\hat{D}_i(t)$ being the time-varying estimates of the nominal controller parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i, $i \in I$, satisfying

$$a_{d1} = -m_ik_\nu k^*_{x_{1i}}, \quad a_{d2} = b_i - m_ik_\nu k^*_{x_{2i}}, \tag{78}$$

$$b_d = m_ik_\nu k^*_{ri}, \quad \xi = -k_\nu\xi^*, \quad a_i = m_ik_\nu a^*_i, \tag{79}$$

$$c_i = m_ik_\nu c^*_i, \quad \vartheta_i = m_ik_\nu\vartheta^*_i, \quad D_i = m_ik_\nu D^*_i. \tag{80}$$

Use the controller (77) and the system (75)-(76) under (78)-(80), to obtain

$$\dot{x}_1(t) = x_2(t), \tag{81}$$

$$\begin{aligned}\dot{x}_2(t) =& -a_{d1}x_1(t) - a_{d2}x_2(t) + b_dr(t) \\ &+\sum_{i=1}^{l} m_ik_\nu\tilde{k}_{x_{1i}}(t)\chi_i(t)x_1(t) \\ &+\sum_{i=1}^{l} m_ik_\nu\tilde{k}_{x_{2i}}(t)\chi_i(t)x_2(t) + \sum_{i=1}^{l} m_ik_\nu\tilde{k}_{ri}(t)\chi_i(t)r(t) \\ &+\sum_{i=1}^{l} m_ik_\nu\chi_i(t)\tilde{\xi}(t)\varpi(t) + \sum_{i=1}^{l} m_ik_\nu\tilde{a}_i(t)\chi_i(t) \\ &+\sum_{i=1}^{l} m_ik_\nu\tilde{c}_i(t)\chi_i(t)x_2^2(t) + g\sum_{i=1}^{l} m_ik_\nu\tilde{\vartheta}_i(t)\chi_i(t) \\ &+0.004\sum_{i=1}^{l} m_ik_\nu\tilde{D}_i(t)\chi_i(t).\end{aligned} \tag{82}$$

where $\tilde{k}_{x_{1i}}(t) = k^*_{x_{1i}} - k_{x_{1i}}(t)$, $\tilde{k}_{x_{2i}}(t) = k^*_{x_{2i}} - k_{x_{2i}}(t)$, $\tilde{k}_{ri}(t) = k^*_{ri} - k_{ri}(t)$, $\tilde{\xi}(t) = \xi^* - \hat{\xi}(t)$, $\tilde{a}_i(t) = a^*_i - \hat{a}_i(t)$, $\tilde{c}_i(t) = c^*_i - \hat{c}_i(t)$, $\tilde{\vartheta}_i(t) = \vartheta^*_i - \hat{\vartheta}_i(t)$, $\tilde{D}_i(t) = D^*_i - \hat{D}_i(t)$.

The equations in (78)-(80) are the plant-model matching conditions. Similar to the healthy and single operating condition, the tracking errors $e_1(t) = x_1(t) - x_{d1}(t)$ and $e_2(t) = x_2(t) - x_{d2}(t)$ under the nominal failure compensation controller $\nu^*_0(t) = k^*_{x_1}(t)x_1(t) + k^*_{x_2}(t)x_2(t) + k^*_r(t)r(t) + \xi^*\varpi(t) + a^*(t) + c^*(t)x_2^2(t) + g\vartheta^*(t) + 0.004D^*(t)$, satisfy (38) and (39), which implies that $e_1(t)$ and $e_2(t)$ approach zero exponentially as $t \to \infty$.

Let (T_p, T_{p+1}) be the time intervals for $p = 0, 1, \ldots, \mathcal{M}$, with $T_0 = 0$, on which the actuator failure pattern is fixed, that is, the actuators only fail at time T_p, $p = 0, 1, \ldots, \mathcal{M}$. Moreover, $\mathcal{M} \leq \bar{n}$ and $T_{\mathcal{M}+1} = \infty$. For the piecewise constant model (75)-(76), let $\{T_q\}_{q=1}^{\infty}$ denote the known time instants at which (75)-(76) switches between modes. It should be noted that the actuator failure time T_p is unknown, but switching mode time T_q is known. Then, for the matching conditions, there are two possible cases depending on the actuator failure time T_p and T_{p+1}.

(i) $T_{q-1} < T_p < T_q$, $T_{q-1} < T_{p+1} < T_q$: The actuator failures occur before the system (75)-(76)switches mode. Assume at the time interval (T_{q-1}, T_q), the system (75)-(76) is under the ith dynamics model. Then, at time $T_{\bar{p}}$, $\bar{p} = 0, 1, \ldots, \mathcal{M}$, the unknown plant model matching parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i, change their values during the time intervals (T_{q-1}, T_q), such that

$$k^*_{x_{1i}} = k^*_{x_{1i(p)}}, \quad k^*_{x_{2i}} = k^*_{x_{2i(p)}}, \quad k^*_{ri} = k^*_{ri(p)}, \tag{83}$$

$$\xi^* = \xi^*_{(p)}, \quad a^*_i = a^*_{i(p)}, \quad c^*_i = c^*_{i(p)}, \tag{84}$$

$$\vartheta^*_i = \vartheta^*_{i(p)}, \quad D^*_i = D^*_{i(p)}, \tag{85}$$

for $t \in (T_p, T_{p+1})$, $p = 0, 1, \ldots, \mathcal{M}$. This means that the plant-model matching parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i are piecewise constants, during the time interval (T_{q-1}, T_q) (under the ith dynamics model).

(ii) $T_{q-1} < T_p < T_q$, $T_q < T_{p+1}$: The actuator failures occur after the system (75)-(76) switches mode. Assume at the time intervals (T_{q-1}, T_q) and (T_q, T_{q+1}), the system (75)-(76) are under the ith and $i+1$th dynamics models, separately. Then, at time $T_{\bar{q}}$, $\bar{q} = 0, 1, \ldots, \infty$, the unknown plant model matching parameters change their values during the time intervals (T_{q-1}, T_{p+1}), such that

$$k^*_{x_{1i}} = k^*_{x_{1i}(p)}, \quad k^*_{x_{2i}} = k^*_{x_{2i}(p)}, \quad k^*_{ri} = k^*_{ri(p)}, \tag{86}$$
$$\xi^* = \xi^*_{(p)}, \quad a^*_i = a^*_{i(p)}, \quad c^*_i = c^*_{i(p)}, \tag{87}$$
$$\vartheta^*_i = \vartheta^*_{i(p)}, \quad D^*_i = D^*_{i(p)}, \tag{88}$$

and

$$k^*_{x_{1i+1}} = k^*_{x_{1i+1}(p)}, \quad k^*_{x_{2i+1}} = k^*_{x_{2i+1}(p)}, \tag{89}$$
$$k^*_{ri+1} = k^*_{ri+1(p)}, \quad \xi^* = \xi^*_{(p)}, \tag{90}$$
$$a^*_{i+1} = a^*_{i+1(p)}, \quad c^*_{i+1} = c^*_{i+1(p)}, \tag{91}$$
$$\vartheta^*_{i+1} = \vartheta^*_{i+1(p)}, \quad D^*_{i+1} = D^*_{i+1(p)}, \tag{92}$$

that is, the plant-model matching parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i change their values according to the switching modes with known switching times.

Remark 6: For system operation in this case, there are two possible situations: (i) some different failures occur under a certain system mode and (ii) the system mode switches under a certain failure patten. The situation (i) is equivalent to the single operating condition addressed in Section IV. When k_ν and ξ change, for the certain ith dynamic system model, the parameters of the faulty system defined in (75) and (76) also change, and so do the matching parameters $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i. The treatment for the situation (ii) is similar to the case where the system mode switches, addressed in Section III, with a fixed actuator failure uncertainty. Then, it can be seen that the parameter of failures $\xi^*_{(p)}$ does not change, because this parameter is independent of the system parameters m_i, a_i, b_i, c_i, ϑ_i, and D_i. The whole system (75)-(76) switches between these two modes, in which the system mode switching time is known, and the parameter switchings caused by system mode changes are dealt via the plant-model matching and controller adaptation. □

C. Adaptive Laws

Using the controller (77) and the system (75)-(76) and from the matching condition $b_d = m_i k_\nu k^*_{ri}$, we have the tracking error equations

$$\dot{e}_1(t) = e_2(t), \tag{93}$$

$$\begin{aligned}\dot{e}_2(t) = & -a_{d1}e_1(t) - a_{d2}e_2(t) + \sum_{i=1}^{l} \frac{1}{k^*_{ri}} b_d \tilde{\xi}(t)\varpi(t)\chi_i(t) \\ & + \sum_{i=1}^{l} \frac{1}{k^*_{ri}} b_d \Big(\tilde{k}_{x_{1i}}(t)\chi_i(t)x_1(t) + \tilde{k}_{x_{2i}}(t)\chi_i(t)x_2(t) \\ & + \tilde{k}_{ri}(t)\chi_i(t)r(t) + \tilde{a}_i(t)\chi_i(t) + \tilde{c}_i(t)\chi_i(t)x_2^2 \\ & + g\tilde{\vartheta}_i(t)\chi_i(t) + 0.004\tilde{D}_i(t)\chi_i(t)\Big),\end{aligned} \tag{94}$$

Similar to the above two cases and according to the matching conditions (78)-(80), the sign of the parameter k^*_{ri} can be obtained to be positive. Since $e(t) = [e_1(t), \ \ e_2(t)]^T$ and sign$[k^*_{ri}]$ is positive, the following parameter adaptive laws are applied to update the controller parameters in (77):

$$\dot{k}_{x_{1i}}(t) = -\gamma_{x_{1i}} x(t) e^T(t) P_d B_d \chi_i(t), \tag{95}$$
$$\dot{k}_{x_{2i}}(t) = -\gamma_{x_{2i}} x(t) e^T(t) P_d B_d \chi_i(t), \tag{96}$$
$$\dot{k}_{ri}(t) = -\gamma_{ri} r(t) e^T(t) P_d B_d \chi_i(t), \tag{97}$$
$$\dot{\hat{\xi}}(t) = -\gamma_\xi \varpi(t) e^T(t) P_d B_d, \tag{98}$$
$$\dot{\hat{a}}_i(t) = -\gamma_{ai} e^T(t) P_d B_d \chi_i(t), \tag{99}$$
$$\dot{\hat{c}}_i(t) = -\gamma_{ci} x_2^2(t) e^T(t) P_d B_d \chi_i(t), \tag{100}$$
$$\dot{\hat{\vartheta}}_i(t) = -\gamma_{\vartheta i} g e^T(t) P_d B_d \chi_i(t), \tag{101}$$
$$\dot{\hat{D}}_i(t) = -\gamma_{Di} 0.004 e^T(t) P_d B_d \chi_i(t), \tag{102}$$

where $\gamma_{x_{1i}}$, $\gamma_{x_{2i}}$, γ_{ri}, γ_ξ, γ_{ci}, γ_{ai}, $\gamma_{\vartheta i}$, and γ_{Di} are positive constants and $P_d > 0$, satisfying $A_d^T P_d + P_d A_d = -Q_d$, for some $Q_d > 0$.

D. Performance Analysis

The performance of the adaptive controller is now analyzed to obtain the following stability and tracking properties:

Theorem 3: *For the faulty system (75)-(76) with actuator failures (18)-(21), the adaptive failure compensation controller (77) updated the adaptive scheme (95)-(102), ensures the closed-loop signals boundedness and the state tracking error* $e(t)$ *satisfying* $\lim_{t\to\infty} e(t) = 0$.

Proof: The values of the parameters a_{d1}, a_{d2} and b_d ensure the stability of (23), i.e., $x_d(t) \in L_\infty$.

For the tracking error dynamic equation (93)-(94), it should be noted that there are two kinds of the unknown switching parameters: one piecewise constant with the known switching times representing by $\chi_i(t)$, and the other from the actuator failures with unknown parameters in ξ, whose switching representing the failure changes is determined by failure pattern changes. A desirable Lyapunov function should be chosen to meet the characteristics of these two piecewise models. For either the healthy system case or the single operating condition with failures, only one of these characteristics is taken into account, and now for both cases, two kinds of characteristics are taken into account.

We choose the following candidate Lyapunov function:

$$\begin{aligned}V = & e^T P_d e + \sum_{i=1}^{l} \frac{1}{k^*_{ri}} \gamma_\xi^{-1} \tilde{\xi}^T \tilde{\xi} + \sum_{i=1}^{l} \frac{1}{k^*_{ri}} \Big(\gamma_{x_{1i}}^{-1} \tilde{k}^2_{x_{1i}} + \gamma_{x_{2i}}^{-1} \tilde{k}^2_{x_{2i}} \\ & + \gamma_{ri}^{-1} \tilde{k}^2_{ri} + \gamma_{ai}^{-1} \tilde{a}_i^2 + \gamma_{ci}^{-1} \tilde{c}_i^2 + \gamma_{\vartheta i}^{-1} \tilde{\vartheta}_i^2 + \gamma_{Di}^{-1} \tilde{D}_i^2\Big),\end{aligned} \tag{103}$$

where the term (containing $\tilde{\xi}$) about the failures is different from the term (last term, containing $\tilde{k}_{x_{1i}}$, $\tilde{k}_{x_{2i}}$, $\tilde{k}_{ri}$, $\tilde{a}_i$, $\tilde{c}_i$, $\tilde{\vartheta}_i$, and $\tilde{D}_i$,) about the model parameters, because the switches of the failures are achieved via the matching condition instead of the indicator functions $\chi_i(t)$ in (7). Also, $V(\cdot)$ as a function of t is not continuous, because $k^*_{x_{1i}}$, $k^*_{x_{2i}}$, k^*_{ri}, ξ^*, a^*_i, c^*_i, ϑ^*_i, D^*_i, are piecewise constant parameters. With the estimation errors in (93)-(94) and the adaptive laws in (95)-(102), the time derivative of V for $t \in (T_p, T_{p+1})$, $p = 0, 1, \dots, \mathcal{M}$, becomes

$$\dot{V} = -e^T(t) Q_d e(t) \le 0. \quad (104)$$

Since there are only a finite number of failures in the system, $V(T_{\mathcal{M}})$ is finite, and, from

$$\dot{V} = -e^T(t) Q_d e(t) \le 0, \quad t \in (T_{\mathcal{M}}, \infty), \quad (105)$$

the closed-loop system consisting of (81)-(82), (93)-(94) and (95)-(102) is uniformly stable and its solutions is uniformly ultimately bounded. That is, $e(t)$, $x_1(t)$, $x_2(t)$, $k_{x_{1i}}(t)$, $k_{x_{2i}}(t)$, $k_{ri}(t)$, $\hat{\xi}(t)$, $\hat{a}_i(t)$, $\hat{c}_i(t)$, $\hat{\vartheta}_i(t)$, $\hat{D}_i(t)$, and $\dot{e}(t)$ are all bounded. Then, with the structure of the failure compensation controller (77), the boundedness of $\nu_0(t)$ is ensured. Further, equation (105) implies $e(t) \in L_2$ and so $\lim_{t\to\infty} e(t) = 0$. ∇

Recall that for the case considered in this section, there are two kinds of parameter variations (see Remark 6), caused by either actuator failure changes or system mode changes. Our adaptive failure compensation controller (77) is so parametrized that both parameter variations can be handled, resulting in a complete system parametrization (94) and enabling the design of the stable adaptive laws (95)-(102). To handle unknown and switching parameters, those (from system modes) with know switching time instants are parametrized in the controller structure and those (from actuator failures) with unknown switching time instants are both parametrized in the controller and dealt with via the use of a piecewise Lyapunov function V.

We should also note that, compared with the existing results about the failure compensation for high-speed trains [7], [8], our proposed adaptive method does not require the known bounds of the failures, whose knowledge may not be obtained for some failures. Moreover, the proposed failure compensation scheme can guarantee the tracking errors (distance and velocity tracking errors) convergent to zero with the unknown actuator failures and is effective for the more general failures which can be expressed as the form (15).

E. Simulation Study

In this section, simulation study on the plant the same as that of the healthy plant will be presented.

Simulation conditions. Similar to Part E in Section III, 4 modes will be considered. Considering the failure modes, the switching times between modes are different from that of the healthy case in Section III. The system parameters a_i, b_i, c_i, ϑ_i, and D_i are defined as equations (11)-(13), and a_i, b_i, c_i, ϑ_i are expressed in kN, kN s/m, kN s^2/m^2 and degree.

(i) For $t < 400$ s, the train bakes up. In this case, the coefficients are chosen as $a_1 = 3.25 \times 10^3$, $b_1 = 26.75$, $c_1 = 0.48$, $\theta_1 = 0$, and $D_1 = 0$.
(ii) During $400 \le t < 800$ s, the train enters the tunnel. Then the coefficients are chosen as $a_2 = 3.25 \times 10^3$, $b_2 = 26.75$, $c_2 = 0.78$, $\theta_2 = 0$, and $D_2 = 0$.
(iii) At 800 s, the train leaves the tunnel and travels in the slope and curvature track. For $800 \le t < 1400$ s, the coefficients are $a_1 = 3.25 \times 10^3$, $b_1 = 26.75$, $c_3 = 0.48$, $\theta_3 = 10$, and $D_3 = 0.34$.
(iv) After 1400 s, the train moves in the open air and horizontal track to slow down until fully stop. For $1400 \le t < 2000$ s, the coefficients are chosen as $a_4 = 3.25 \times 10^3$, $b_4 = 26.75$, $c_4 = 0.48$, $\theta_4 = 0$, and $D_4 = 0$.

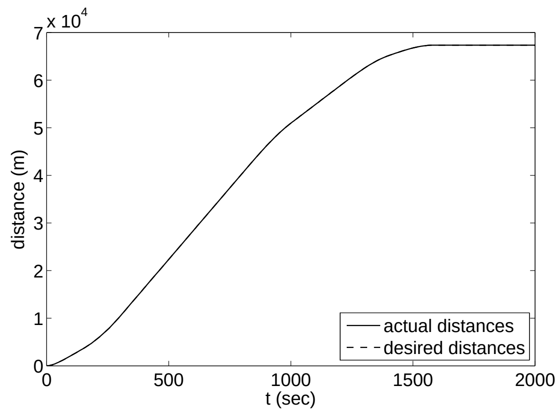
These 4 modes are used to construct the controller system, in which the parameters are unknown for the adaptive controller design.

Considering the failure modes, i.e., the failure occurs before or after the system mode switching, the following failures are chosen, with whose modes and patterns are the same as that of the signal-model case but the occurrence times are different. The failure is expressed as: F_α fails for some $\alpha \in \{1, 2, 3, 4\}$,

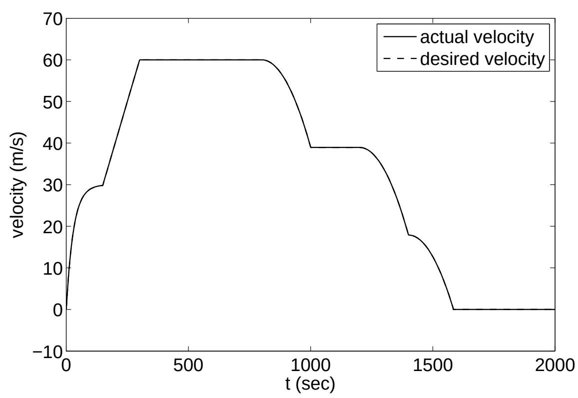
$$F_\alpha(t) = \begin{cases} 2 \times 10^5, & \text{for } 600 \le t < 1000\text{s}; \\ 2 \times 10^5(1 + \sin(0.05t - 30)), & \text{for } 1000 \le t < 1200\text{s}; \\ 0, & \text{for } 1200 \le t \le 2000\text{s}; \end{cases}$$

$F_\beta = \nu_\beta$, $\beta \ne \alpha$, $\beta \in \{1, 2, 3, 4\}$.

The initial conditions are chosen as $x_d(0) = x(0) = [0 \;\; 0]^T$, and the values of the initial parameter estimates are 95% of their ideal values. The gains of the adaptive laws in (95)-(102) are chosen as 2.



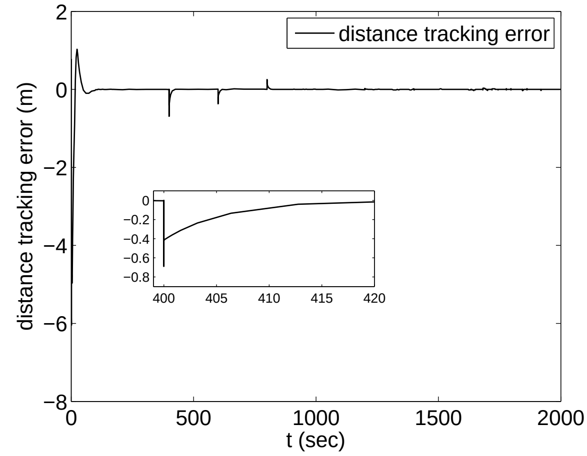
(a) x_1 and x_{d1}



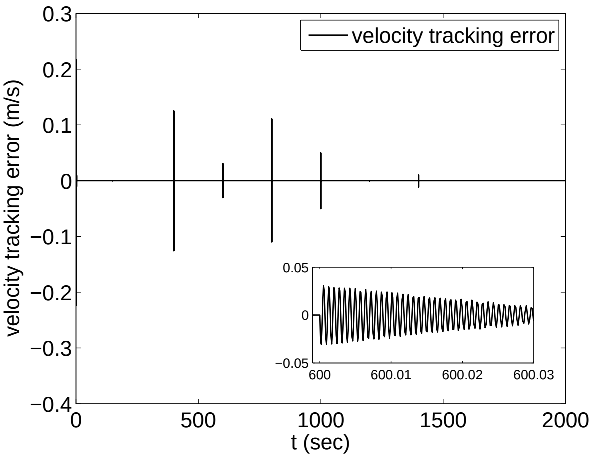
(b) x_2 and x_{d2}

Fig. 6: Distances and velocity trajectories for the piecewise constant model with actuator failures.

Simulation results. Figs. 6-7 show the simulation results of the traction system with actuator failures. Fig. 6 shows the distances (a) and velocities (b) including the plant distance and velocity (solid) and the desired distance and velocity (dashed). Fig. 7 shows the tracking errors including the distance (a) and velocity (b). From the simulation results, it can be seen that the proposed adaptive controller can achieve the close-loop stability and asymptotic tracking properties of the train even in the presence of parameters changes. practical situation.



(a) e_1



(b) e_2

Fig. 7: The tracking errors for the piecewise constant model with actuator failures

VI. Conclusions

In this paper, the adaptive failure compensation problem is addressed for high-speed trains with the longitudinal dynamics and traction system actuator failures, which are uncertain in time instants, values, and patterns. A new piecewise constant model with unknown parameters is introduced to represent the longitudinal dynamics with variable parameters. An adaptive failure compensation scheme, together with the adaptive controller for healthy system, are developed to deal with the unknown parameters in the plant and traction system actuator failures. Simulation results demonstrate the effectiveness of the obtained theoretical results.

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Zehui Mao received her Ph.D. degree in Control Theory and Control Engineering from Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2009. She is now an associate professor at the College of Automation Engineering in Nanjing University of Aeronautics and Astronautics, China. And she is now a visiting scholar in University of Virginia. She worked in the areas of fault diagnosis, with particular interests in nonlinear control systems, sampled-data systems and networked control systems. Her current research interests include fault diagnosis and fault-tolerant control of systems with disturbance and incipient faults, and high speed train and spacecraft flight control applications.

Gang Tao (S'84-M'89-SM'96-F'07) received the B.S. degree from the University of Science and Technology of China, Hefei, China, in 1982, and the M.S. and Ph.D. degrees from the University of Southern California, Los Angeles, CA, USA, during 1984-1989. He is currently a Professor at the University of Virginia, Charlottesville, VA, USA. He worked in the areas of adaptive control, with particular interests in adaptive control of systems with multiple inputs and multiple outputs and with nonsmooth nonlinearities and actuator failures, in stability and robustness of adaptive control systems, and in passivity characterizations of control systems. He has authored or coauthored six books, and more than 400 technical papers and book chapters. His current research interests include adaptive control of systems with uncertain actuator failures and nonlinearities, with structural damage and sensor uncertainties and failures, adaptive approximation-based control, and resilient aircraft and spacecraft flight control applications. He has served as an Associate Editor of Automatica, the International Journal of Adaptive Control and Signal Processing, and the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, a Guest Editor of the Journal of Systems Engineering and Electronics, and an Editorial Board Member of the International Journal of Control, Automation and Systems.

Bin Jiang obtained Ph.D. degree in Automatic Control from Northeastern University, Shenyang, China, in 1995. He had ever been postdoctoral fellow, research fellow and visiting professor in Singapore, France, USA and Canada, respectively. Now he is a Chair Professor of Cheung Kong Scholar Program in Ministry of Education and Dean of College of Automation Engineering in Nanjing University of Aeronautics and Astronautics, China. He currently serves as Associate Editor or Editorial Board Member for a number of journals such as IEEE Trans. On Control Systems Technology; Int. J. of Control, Automation and Systems; Nonlinear Analysis: Hybrid Systems; Acta Automatica Sinica; Journal of Astronautics; Control and Decision; Systems Engineering and Electronics Technologies. He is chair of Control Systems Chapter in IEEE Nanjing Section, a member of IFAC Technical Committee on Fault Detection, Supervision, and Safety of Technical Processes, a member of IFAC Technical Committee on Discrete Event and Hybrid System. His research interests include fault diagnosis and fault-tolerant control and their applications in aircraft, satellite and high-speed train.

Xing-Gang Yan received B.Sc. degree from Shaanxi Normal University, in 1985, M.Sc. degree from Qufu Normal University in 1991, and Ph.D. degree in Engineering from Northeastern University, China in 1997. Currently, he is a Senior Lecturer at the University of Kent, UK. He was a Lecturer at Qingdao University, China from 1991 to 1994. He worked as a Research Fellow/Associate in the University of Hong Kong, China, Nanyang Technological University, Singapore and the University of Leicester, UK. He is the Editor-in-Chief of the International Journal of Engineering Research and Science and Technology. His research interests include sliding mode control, decentralised control, fault detection and isolation, control and observation of nonlinear systems and time delay systems with applications.