Nonlinear Sliding Mode Control for Interconnected Systems with Application to Automated Highway Systems†‡

Jianqiu Mu[1], Xing-Gang Yan[1], Sarah K. Spurgeon[2,3] and Dongya Zhao[3]

***Abstract*—In this paper, a decentralised control strategy based on sliding mode techniques is proposed for a class of nonlinear interconnected systems. Both matched uncertainties in the isolated subsystems and mismatched uncertainties associated with the interconnections are considered. Under mild conditions, sliding mode controllers for each subsystem are designed in a decentralised manner by only employing local information. Conditions are determined which enable information on the interconnections to be employed within the decentralised controller design to reduce conservatism. The developed results are applied to an automated highway system. Simulation results pertaining to a high-speed following system are presented to demonstrate the effectiveness of the approach.**

***Index Terms*—Decentralised control, sliding mode techniques, nonlinear interconnected systems, automated highway systems.**

I. INTRODUCTION

A class of complex systems, including multi-machine power systems [1], automated highway systems [2] and multi-agent systems [3], [4], can be modelled as a collection of subsystems with appropriate interconnections [8], [5]. Such classes of systems are called large scale interconnected systems. The interconnections among subsystems together with the inherent nonlinearity of the coupled dynamics inevitably produce complex dynamics. Moreover, such classes of systems are frequently distributed in space. This may render a centralised control strategy difficult to implement as centralised controllers require that the controller in each subsystem can access all the state information relating to all the other subsystems. Problems such as network failure or blockage of communication channels may prevent information transfer among subsystems. This has motivated the development of decentralised control strategies in which each subsystem is controlled independently. The control is based only on local information, which not only enhances system reliability but reduces the overhead in information transfer. In view of this, decentralised strategies have been effectively applied in various areas such as fault diagnosis and discrete-event systems [6], [7].

It is well known that uncertainties or modelling errors may seriously affect control system performance. Specifically, for large scale interconnected systems, uncertainties experienced by one subsystem not only affect its own performance but usually affect the other subsystems' performance as well due to the interactions between the subsystems. Sliding mode control has been recognised as a powerful approach in dealing with uncertainties [9], [10]. The general process to design a sliding mode controller can be separated into two steps:
1). Design of a sliding surface such that the behaviour of the system in the sliding mode exhibits desired performance.
2). Design a control law to ensure that the system state can be driven to the previously designed sliding surface and then remains on it thereafter.
When the system is constrained to the sliding surface, a sliding motion which is governed by the corresponding sliding mode dynamics is said to occur. The closed loop system is completely insensitive to matched uncertainties in the sliding mode [9], [10]. The sliding mode approach can also be used to deal with systems in the presence of unmatched uncertainty [11] although the property of total insensitivity is frequently lost. However, in contrast to the case of centralised control, decentralised control can only use local information and thus the uncertainties within the interconnections may not be rejected, even if they are matched. Designing a decentralised control scheme to reject the effect of uncertainties in the interconnection terms is thus challenging.

The problem of robust decentralised controller design has received much attention and many results have been obtained. In [12], [13], [14], [15], only matched uncertainties are considered and the bounds on the matched uncertainties are assumed to be linear or polynomial. In terms of mismatched uncertainties, in order to achieve asymptotic stability, some limitations are unavoidable. Mismatched uncertainties have been considered in [11], [16] where centralised dynamical feedback controllers are designed which need more resources to exchange information between subsystems. A class of constraints called integral quadratic constraints is imposed on the considered systems to limit the structure of the original systems [16]. In some cases, adaptive techniques are applied to estimate an upper bound on the mismatched uncertainty which can then be used to counteract its effects [17]. This approach can be powerful when the uncertainty satisfies a

[1]Jianqiu Mu and Xing-Gang Yan are with Instrumentation, Control and Embedded Systems Research Group, School of Engineering & Digital Arts, University of Kent, CT2 7NT Canterbury, United Kingdom. (e-mail: jm838@kent.ac.uk; x.yan@kent.ac.uk).
[2]Sarah K. Spurgeon is with the Department of Electronic and Electrical Engineering, UCL, London, UK. (e-mail: s.spurgeon@ucl.ac.uk).
[3]Sarah K. Spurgeon and Dongya Zhao are with the College of Chemical Engineering, China University of Petroleum. (e-mail: dyzhao@upc.edu.cn).

† This paper was completed when Professor Sarah Spurgeon visited the China University of Petroleum as a Chang Jiang Scholar.
‡ This paper is partially supported by the China Chang Jiang Scholars Program, the National Natural Science Foundation of China (NSFC) (61473312) and the EPSRC via grant reference number EP/J017477/1.

linear growth condition. In [18], although the uncertainties are assumed to be functions, the system needs to be transformed into a special triangular structure. All the literature which considers mismatched uncertainties mentioned above inevitably requires extra resources and increases system complexity. This may be unattractive from the viewpoint of implementation. Specifically, output feedback control based results impose very strong limitations on the uncertainties and interconnections (see e.g. [12], [23], [20], [1]).

In this paper, a decentralised control strategy for a class of nonlinear interconnected systems is proposed based on a sliding mode control paradigm. In terms of the robustness, both matched uncertainties and mismatched unknown interconnections are considered. It is well known that to deal with interconnections is one of the main challenges for interconnected systems when decentralised control is considered. The main contribution of this work can be summarized as follows:

i). The uncertain interconnections are separated into two parts to reduce the conservatism.
ii). It is not required that the interconnections vanish at the origin.
iii). The bounds on the uncertainties have a more general form than those imposed within existing work.

Based on the approach proposed in [9], a sliding surface for each subsystem is designed. Together these constitute a composite sliding surface for the interconnected system. A set of sufficient conditions is developed such that the corresponding sliding motion is asymptotically stable when the system is restricted to the designed sliding surface. Then, a decentralised sliding mode control is designed to drive the large-scale interconnected system to the sliding surface in finite time. It is shown that if the uncertainties/interconnections possess a superposition property, a decentralised control scheme may be designed to counteract the effect of the uncertainty. Finally, the developed decentralised control scheme is applied to an automated highway system. Simulation results relating to a high-speed car following system show that the obtained results are effective. The study shows that limitations on the bounds assumed on the uncertainties and interconnections can be greatly reduced when compared with the output feedback case.

II. System Description and Preliminaries

Consider a nonlinear large-scale interconnected system composed of N subsystems where the i-th subsystem is described by

$$\begin{aligned}\dot{x}_i &= A_i x_i + B_i\left(u_i + \phi_i(t, x_i)\right) + \sum_{j=1}^{N} \Xi_{ij}(t, x_j) \\ &\quad + \psi_i(t, x) \qquad i = 1, 2, \ldots, N \end{aligned} \tag{1}$$

where $x_i \in \mathcal{D}_i \subset \mathcal{R}^{n_i}$ ($\mathcal{D}_i$ is the neighborhood of the origin $x_i = 0$), $u_i \in \mathcal{R}^{m_i}$ denote the state variables and inputs of the i-th subsystem, respectively. The matrix pairs (A_i, B_i) are constant with appropriate dimensions. The matched uncertainties are denoted by $\phi_i(t, x_i)$. The terms $\sum_{j=1}^{n} \Xi_{ij}(t, x_j)$ with $\Xi_{ij}(t, 0) = 0$ describe the known interconnection of the i-th subsystem. The nonlinear functions $\psi_i(t, x)$ represent the uncertain interconnections where $x = \mathrm{col}(x_1, x_2, \ldots, x_n)$ is the state of the whole interconnected system. It is assumed that all the nonlinear functions are sufficiently smooth such that the unforced system has a unique continuous solution.

It should be noted that

$$\begin{aligned}\sum_{j=1}^{N} \Xi_{ij}(t, x_j) &= \Xi_{i1}(t, x_1) + \ldots + \Xi_{i(i-1)}(t, x_{i-1}) \\ &\quad + \Xi_{ii}(t, x_i) + \Xi_{i(i+1)}(t, x_{i+1}) + \ldots + \Xi_{iN}(t, x_N) \\ &= \Xi_{ii}(t, x_i) + \sum_{\substack{j \neq i \\ j=1}}^{N} \Xi_{ij}(t, x_j)\end{aligned} \tag{2}$$

In this case, $\Xi_{ii}(t, x_i)$ can be considered as the known nonlinearity in the ith subsystem and the term $\sum_{\substack{j \neq i \\ j=1}}^{N} \Xi_{ij}(t, x_j)$ as the known interconnection within the ith subsystem. It will be shown that such a class of interconnections can be employed in decentralised controller design to reduce conservatism.

Definition 1 (see [8], [19]) The following systems:

$$\begin{aligned}\dot{x}_i &= A_i x_i + B_i\left(u_i + \phi_i(t, x_i)\right) + \psi_i(t, x) \\ &\qquad i = 1, 2, \ldots, N\end{aligned} \tag{3}$$

are called the isolated subsystems of the interconnected system (1).

Definition 2 (see [8], [19]) Consider the interconnected system (1). If the designed controller u_i for the i-th subsystem depends on the time t and states x_i of the i-th subsystem only, i.e.

$$u_i = \varrho_i(x_i, t), \quad (x_i, t) \in \mathcal{D}_i \times \mathcal{R}^+, \quad i = 1, 2, \ldots, N \tag{4}$$

then the control (4) is called a decentralised control.

Remark 1. From the Definitions 1 and 2 above, it is clear that the decentralized control paradigm for interconnected systems is different from the one adopted for multi-agent systems as the interconnected systems are interconnected through interconnection terms for the case of decentralised control. With a multi-agent system, the systems are interconnected through distributed controls [3], [8].

The objective of this paper is to design a decentralised control

$$u_i = \varrho_i(x_i, t), \qquad i = 1, 2, \ldots, N \tag{5}$$

for system (1) based on sliding mode techniques such that the corresponding closed-loop system formed by applying the controllers (5) to the system (1) is asymptotically stable. From (5), it can be seen that the local controller u_i in the i-th subsystem can only access the states of the i-th subsystem.

The following basic assumption is firstly imposed on the system (1).

Assumption 1. The matrix pairs (A_i, B_i) are controllable and $\mathrm{rank}(B_i) = m_i$ for $i = 1, 2, \ldots, N$.

Under the condition that $\mathrm{rank}(B_i) = m_i$ in Assumption 1, there exists an invertible matrix $\tilde{T}_i \in \mathcal{R}^{(n_i \times n_i)}$ such that after the coordinate transformation $\tilde{x}_i = \tilde{T}_i x_i$, the matrix pairs (A_i, B_i) with respect to the new coordinates $\tilde{x}_i$ have the following structure

$$\tilde{A}_i = \begin{bmatrix} \tilde{A}_{i1} & \tilde{A}_{i2} \\ \tilde{A}_{i3} & \tilde{A}_{i4} \end{bmatrix} = \tilde{T}_i A_i \tilde{T}_i^{-1} \quad (6)$$

$$\tilde{B}_i = \begin{bmatrix} 0 \\ \tilde{B}_{i2} \end{bmatrix} = \tilde{T}_i B_i \quad (7)$$

where $\tilde{A}_{i1} \in \mathcal{R}^{(n_i-m_i)\times(n_i-m_i)}$ and the matrix $\tilde{B}_{i2} \in \mathcal{R}^{m_i\times m_i}$ is nonsingular for $i = 1, 2, \ldots, N$. It should be noted that the matrix $\tilde{T}_i$ can be obtained using basic matrix theory.

Assume that (A_i, B_i) is controllable. From [9], it follows that the matrix pair $(\tilde{A}_{i1}, \tilde{A}_{i2})$ in (6) is controllable. Then, there exists a matrix $K_i \in \mathcal{R}^{(n_i-m_i)\times m_i}$ such that $\tilde{A}_{i1} - K_i\tilde{A}_{i2}$ is Hurwitz stable. Considering the system (1), introduce a new transformation matrix as follows:

$$T_i = \begin{bmatrix} I_{n_i-m_i} & 0 \\ K_i & I_{m_i} \end{bmatrix} \tilde{T}_i \quad (8)$$

It is clear that the matrix T_i is nonsingular. Define $z = \mathrm{col}(z_1, z_2, \ldots, z_N)$ where $z_i = T_i x_i$. Then in this new coordinate system, system (1) has the following form

$$\dot{z}_i = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} z_i + \begin{bmatrix} 0 \\ \tilde{B}_{i2} \end{bmatrix} (u_i + g_i(t, z_i)) + \sum_{j=1}^{N} \Gamma_{ij}(t, z_j) + \delta_i(t, z) \quad (9)$$

where $z_i \in T_i(D_i) := \Omega_i$, $A_{i1} = \tilde{A}_{i1} - \tilde{A}_{i2}K_i$ is stable, $T^{-1} \equiv: \mathrm{diag}\{T_1^{-1}, T_2^{-1}, \ldots, T_N^{-1}\}$, and

$$g_i(t, z_i) = \phi_i(t, T_i^{-1}z_i) \quad (10)$$

$$\Gamma_{ij}(t, z_j) \triangleq \begin{bmatrix} \Gamma_{ij}^a(t, z_j) \\ \Gamma_{ij}^b(t, z_j) \end{bmatrix} = T_i \Xi_{ij}(t, T_j^{-1}z_j) \quad (11)$$

$$\delta_i(t, z) \triangleq \begin{bmatrix} \delta_i^a(t, z) \\ \delta_i^b(t, z) \end{bmatrix} = T_i \psi_i(t, T^{-1}z) \quad (12)$$

where $\Gamma_{ij}^a(t, z_j) \in \mathcal{R}^{(n_i-m_i)}$, $\delta_i^a(t, z) \in \mathcal{R}^{(n_i-m_i)}$, $\Gamma_{ij}^b(t, z_j) \in \mathcal{R}^{m_i}$, and $\delta_i^b(t, z) \in \mathcal{R}^{m_i}$ for $i, j = 1, 2, \ldots, N$.

For further analysis, now partition $z_i = \mathrm{col}(z_i^a, z_i^b)$ where $z_i^a \in \mathcal{R}^{n_i-m_i}$ and $z_i^b \in \mathcal{R}^{m_i}$. Then the system (9) can be rewritten in the following form

$$\dot{z}_i^a = A_{i1}z_i^a + A_{i2}z_i^b + \sum_{j=1}^{N} \Gamma_{ij}^a(t, z_j) + \delta_i^a(t, z) \quad (13)$$

$$\dot{z}_i^b = A_{i3}z_i^a + A_{i4}z_i^b + \tilde{B}_{i2}(u_i + g_i(t, z_i)) + \sum_{j=1}^{N} \Gamma_{ij}^b(t, z_j) + \delta_i^b(t, z) \quad (14)$$

where the matrix A_{i1} in (13) is stable.

The following assumption is imposed on the uncertainty.

Assumption 2. There exist known continuous functions $\rho_i(t, z_i)$, $\eta_i^a(t, z)$ and $\eta_i^b(t, z)$ such that for $i = 1, 2, \ldots, N$,

(i) $\|g_i(t, z_i)\| \le \rho_i(t, z_i)$

(ii) $\|\delta_i^a(t, z)\| \le \eta_i^a(t, z)\|z\|$

(iii) $\|\delta_i^b(t, z)\| \le \eta_i^b(t, z)$

Remark 2. Assumption 2 is a limitation on all the uncertainties experienced by the interconnected system. It is required that bounds on the uncertainties are known. These bounds will be employed in the control design to reject the effects of the uncertainty. It should be emphasised that the bounds on the uncertainties in Assumption 2 have a more general form when compared with existing work [12], [20], [1], [23]. It should be noted that it is only required that $\delta_i^a(\cdot)$ vanish at the origin, and it is not required that $g_i(\cdot)$ and $\delta_i^b(\cdot)$ vanish at the origin.

III. Stability Analysis of the Sliding Motion

In this section, a sliding surface is designed for the system (9) and the stability of the corresponding sliding motion is analysed. A set of sufficient conditions is provided such that the sliding motion is asymptotically stable.

It is clear that system (13)-(14) has regular form. Choose the local sliding surface for the ith subsystem of the large-scale interconnected system (9) as follows:

$$\sigma_i(z_i) \equiv: z_i^b = 0, \qquad i = 1, 2, \ldots, N. \quad (15)$$

Then, the composite sliding surface for the interconnected system (13)-(14) is chosen as

$$\sigma(z) = 0 \quad (16)$$

where

$$\begin{aligned} \sigma(z) &\equiv: \mathrm{col}\left(\sigma_1(z_1), \sigma_2(z_2), \ldots, \sigma_N(z_N)\right) \\ &= \mathrm{col}\left(z_1^b, z_2^b, \ldots, z_N^b\right) \end{aligned}$$

Since A_{i1} in (13) is stable, for any $Q_i > 0$, the following Lyapunov equation has a unique solution $P_i > 0$ such that

$$A_{i1}^{\tau}P_i + P_iA_{i1} = -Q_i, \qquad i = 1, 2, \ldots, N. \quad (17)$$

During sliding motion, $z_i^b = 0$ for $i = 1, 2, \ldots, N$. Then, the sliding mode dynamics for the system (13)-(14) associated with the designed sliding surface (16) can be described by

$$\dot{z}_i^a = A_{i1}z_i^a + \sum_{j=1}^{n} \Gamma_{ij}^s(t, z_j^a) + \delta_i^s(t, z_1^a, z_2^a, \ldots, z_N^a) \quad (18)$$

where

$$\Gamma_{ij}^s(t, z_j^a) := \Gamma_{ij}^a(t, z_j)|_{z_j^b=0} \quad (19)$$

$$\delta_i^s(t, z_1^a, z_2^a, \ldots, z_N^a) := \delta_i^a(t, z)|_{(z_1^b, z_2^b, \ldots, z_N^b)=0} \quad (20)$$

Here $\Gamma_{ij}^a(t, z_j)$ and $\delta_i^a(t, z)$ are defined in (11) and (12) respectively.

Assumption 3. The functions $\Gamma_{ij}^s(\cdot)$ in (19) have the following decomposition:

$$\Gamma_{ij}^s(t, z_j^a) = \tilde{\Gamma}_{ij}^s(t, z_j^a)z_j^a \quad (21)$$

where $\tilde{\Gamma}_{ij}^s(t, z_j^a)$ is an appropriately-dimensioned matrix function for $i, j = 1, 2, \ldots, N$.

Remark 3. If the term $\Xi_{ij}(t, x_j)$ in system (1) is sufficiently smooth with $\Xi_{ij}(t, 0) = 0$, then $\Gamma_{ij}^s(t, z_j^a)$ will be smooth enough with $\Gamma_{ij}^s(t, 0) = 0$. From [20], it is straightforward to see that the decomposition (21) holds. It should be noted that in the system (13)-(14), the interconnection terms are $\Gamma_{ij}^a(t, z_j)$ and $\Gamma_{ij}^b(t, z_j)$, and only $\Gamma_{ij}^a(\cdot)$ are required to vanish at the origin. It is clear to see from (21) and (19) that the

Assumption 3 does not require that the interconnections $\Gamma_{ij}(\cdot)$ vanish at the origin. This is in comparison with all of the associated work [12], [16], [18], [20] where it is required that the interconnections vanish at the origin.

Under Assumptions 1-3, a reduced order interconnected system composed of N subsystems with dimension $n_i - m_i$ is obtained as follows:

$$\dot{z}_j^a = A_{i1} z_j^a + \sum_{j=1}^{n} \tilde{\Gamma}_{ij}^s(t, z_j^a) z_j^a + \delta_i^s(t, z_1^a, z_2^a, \ldots, z_N^a) \quad (22)$$

which represents the sliding mode dynamics relating to the sliding surface (16), where $z_i^a \in \mathcal{R}^{n_i - m_i}$ and $\tilde{\Gamma}_{ij}^s(t, z_j^a)$ is defined in (21).

Lemma 1: For the terms $\delta_i^s(t, z_1^a, z_2^a, \ldots, z_N^a)$ in system (22), if condition (ii) in Assumption 2 holds, then there exist continuous functions $\gamma_{ij}(\cdot)$ such that

$$\|\delta_i^s(t, z_1^a, z_2^a, \ldots, z_N^a)\| \leq \sum_{j=1}^{N} \gamma_i(t, z^a) \|z_j^a\| \quad (23)$$

where

$$\gamma_i(t, z^a) = \eta_i^a(t, z_1^a, 0, z_2^a, 0, \ldots, z_N^a, 0)$$

for $i = 1, 2, \ldots, N$, and $z^a = \mathrm{col}(z_1^a, z_2^a, \ldots, z_N^a)$.

Proof. From the definition of $\delta_i^s(\cdot)$ in (20), it follows that

$$\delta_i^s(t, z_1^a, z_2^a, \ldots, z_N^a) = \delta_i^a(t, z_1^a, 0, z_2^a, 0, \ldots, z_N^a, 0) \quad (24)$$

From condition (ii) in Assumption 2,

$$\|\delta_i^a(t, z)\| \leq \eta_i^a(t, z) \|z\| \quad (25)$$

From (24) and (25), it follows that

$$\begin{aligned}
&\|\delta_i^s(t, z_1^a, z_2^a, \ldots, z_N^a)\| \\
&\quad = \|\delta_i^a(t, z_1^a, 0, z_2^a, 0, \ldots, z_N^a, 0)\| \\
&\quad \leq \eta_i^a(t, z_1^a, 0, z_2^a, 0, \ldots, z_N^a, 0) \|z^a\| \\
&\quad \leq \sum_{j=1}^{N} \eta_i^a(t, z_1^a, 0, z_2^a, 0, \ldots, z_N^a, 0) \|z_j^a\| \\
&\quad \leq \sum_{j=1}^{N} \gamma_i(t, z^a) \|z_j^a\|
\end{aligned}$$

Hence the result follows. ■

The following result can now be presented.

Theorem 1: Consider the sliding mode dynamics given in equation (22). Under Assumptions 1-3, the sliding motion governed by (22) is asymptotically stable if there exists a domain Ω_{z^a} of the origin in $z^a \in \mathcal{R}^{\sum_{i=1}^N (n_i - m_i)}$ such that

$$M^\tau + M > 0$$

in $\Omega_{z^a} \backslash \{0\}$ where $M = (m_{ij})_{N \times N}$, and

$$m_{ij} = \begin{cases} \lambda_{min}(Q_i) - 2\|P_i\| \gamma_i(t, z^a) - \varsigma_{ii}(t, z_i^a), & i = j \\ -\varsigma_{ij}(t, z_j^a) - 2\|P_i\| \gamma_i(t, z^a), & i \neq j \end{cases}$$

where P_i and Q_i satisfy (17), and the functions $\varsigma_{ij}(\cdot)$ are defined by

$$\varsigma_{ij}(t, z_j^a) :\equiv \|P_i \tilde{\Gamma}_{ij}^s(t, z_j^a) + (\tilde{\Gamma}_{ij}^s)^\tau(t, z_j^a) P_i\|$$

with $\tilde{\Gamma}_{ij}^s(t, z_j^a)$ given by (21), and $\gamma_i(t, z^a)$ satisfy (23) for $i, j = 1, 2, \ldots, N$.

Proof. For system (22), consider the Lyapunov function candidate

$$V(t, z_1^a, z_2^a, \ldots, z_N^a) = \sum_{i=1}^{N} (z_i^a)^\tau P_i z_i^a \quad (26)$$

where P_i satisfies equation (17).

Then, from the lyapunov equation (17), the time derivative of $V(t, z_1^a, z_2^a, \ldots, z_N^a)$ along the trajectories of system (22) is given by

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{N} \Big\{ (\dot{z}_i^a)^\tau P_i z_i^a + (z_i^a)^\tau P_i \dot{z}_i^a \Big\} \\
&\leq \sum_{i=1}^{N} \Big\{ -\lambda_{min}(Q_i) \|z_i^a\|^2 \\
&\quad + 2\|z_i^a\| \|P_i\| \|\delta_i^s(t, z_1^a, z_2^a, \ldots, z_N^a)\| + \\
&\quad \sum_{j=1}^{N} \Big\| P_i \tilde{\Gamma}_{i1}^s(t, z_j^a) + (\tilde{\Gamma}_{ij}^s(t, z_j^a))^\tau z_j^a P_i \Big\| \|z_i^a\| \|z_j^a\| \Big\} \\
&\leq \sum_{i=1}^{N} \Big\{ -\lambda_{min}(Q_i) \|z_i^a\|^2 + \sum_{j=1}^{N} \varsigma_{ij}(t, z_j^a) \|z_i^a\| \|z_j^a\| \\
&\quad + 2\|z_i^a\| \|P_i\| \sum_{j=1}^{N} \gamma_i(t, z^a) \|z_j^a\| \Big\} \\
&= -\sum_{i=1}^{N} \Big\{ \lambda_{min}(Q_i) - 2\|P_i\| \gamma_i(t, z^a) \\
&\quad - \varsigma_{ii}(t, z_i^a) \Big\} \|z_i^a\|^2 + \\
&\quad \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \Big\{ \varsigma_{ij}(t, z_j^a) + 2\|P_i\| \gamma_i(t, z^a) \Big\} \|z_i^a\| \|z_j^a\| \\
&= -\frac{1}{2} Y^\tau (M^\tau + M) Y
\end{aligned} \quad (27)$$

where $Y \equiv: \mathrm{col}(\|z_1^a\|, \ldots, \|z_N^a\|)$.

Thus, the conclusion follows from $M^\tau + M > 0$. ■

Theorem 1 shows that the sliding motion corresponding to the designed sliding surface is asymptotically stable. Conditions to ensure this sliding motion is attained and maintained will be developed in the next section.

IV. Decentralised Sliding mode control design

A sliding mode control is designed to drive the system to the sliding surface. It is well known that an appropriate reachability condition is described by

$$\sigma^\tau(z) \dot{\sigma}(z) < 0$$

for a centralised system with switching surfaces $\sigma(z) = 0$. For the nonlinear interconnected system (1), the corresponding condition is described by

$$\sum_{i=1}^{N} \frac{\sigma_i^\tau(z_i) \dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|} < 0 \quad (28)$$

where $\sigma_i(z_i)$ is defined by (15). It should be noted that the condition (28) is proposed in [21] and has been widely used (see, *e.g.* [20]).

Consider system (13)-(14). In order to reduce the effects of the unknown interconnection $\delta_i^b(\cdot)$, consider the bounds $\eta_i^b(\cdot)$ in expression

$$\eta_i^b(t,z) = \sum_{j=1}^{N} \mu_{ij}(t,z_j) + \nu_i(t,z) \tag{29}$$

where $\nu_i(t,z)$ represents all the coupling terms which cannot be included in the term $\sum_{j=1}^{N} \mu_{ij}(t,z_j)$.

Remark 4. The interconnection decomposition in (29) is not unique and is introduced to reduce the conservatism caused by the interconnection terms within the control design. There is no general way to obtain the decomposition. The first interconnection term $\sum_{j=1}^{N} \mu_{ij}(t,z_j)$ has a superposition property. It will be shown that the term $\sum_{j=1}^{N} \mu_{ij}(\cdot)$ in (29) can be rejected by selection of an appropriate decentralised control and this will reduce conservatism. The second term, $\nu_i(t,z)$ in (29), cannot be rejected by the choice of decentralised control.

The objective is to design a decentralised sliding mode controller such that the reachability condition (28) is satisfied. For $i = 1, 2, \ldots, N$, the following control scheme is proposed:

$$\begin{aligned} u_i \;=\; & -\tilde{B}_{i2}^{-1}\Big\{A_{i3}z_i^a + A_{i4}z_i^b + \sum_{j=1}^{N} \Gamma_{ji}^b(t,z_i)\Big\} \\ & -\tilde{B}_{i2}^{-1}\mathrm{sgn}(z_i^b)\Big\{\|\tilde{B}_{i2}\|\rho_i(t,z_i) \\ & +\sum_{j=1}^{N} \mu_{ji}(t,z_i) + \zeta_i(t,z_i)\Big\} \end{aligned} \tag{30}$$

where $z_i = \mathrm{col}(z_i^a, z_i^b)$, $\rho_i(t,z_i)$ are defined in Assumption 2, $\mu_{ji}(t,z_i)$ satisfy (29) and $\zeta_i(t,z_i)$ is a reachability function which will be defined later.

Remark 5. From (30), it is clear to see that controller in the i-th subsystem can only access states z_i. The variables z_i, z_i^a and z_i^b in (30) can be obtained online as $z_i = T_i x_i$ and x_i is accessible online. The known interconnections $\Gamma_{ji}^b(\cdot)$, and the bounds on the matched uncertainties $\rho_i(\cdot)$ and unknown interconnections $\mu_{ji}(\cdot)$ are functions of z_i which are assumed to be known for the control design. In practical case, the bounds on uncertainties can be obtained by historical data or experiments.

Theorem 2: Consider the nonlinear interconnected system (9). Under Assumptions 1-3, the decentralised control (30) is able to drive system (9) to the composite sliding surface (16) and maintains a sliding motion on it thereafter if in the considered domain $\Omega = \Omega_1 \times \Omega_2 \cdots \times \Omega_N$, the functions $\zeta_i(t,z_i)$ in (30) satisfy

$$\sum_{i=1}^{N} \zeta_i(t,z_i) > \sum_{i=1}^{N} \nu_i(t,z) \tag{31}$$

in $\Omega\backslash\{0\}$ for all $t > 0$ with $\nu_i(t,z)$ defined in (29).

Proof. From the analysis above, all that needs to be proved is that the composite reachability condition (28) is satisfied. From (16), for $i = 1, 2, \ldots, N$,

$$\begin{aligned} \dot{\sigma}_i(z_i) = \dot{z}_i^b \;=\; & A_{i3}z_i^a + A_{i4}z_i^b \\ & +\tilde{B}_{i2}\big(u_i + \phi_i(t, T_i^{-1}z_i)\big) \\ & +\sum_{j=1}^{N} \Gamma_{ij}^b(t,z_j) + \delta_i^b(t,z) \end{aligned} \tag{32}$$

Substituting (30) into (32),

$$\begin{aligned} & \sum_{i=1}^{N} \frac{\sigma_i^\tau(z_i)\dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|} \\ & = \sum_{i=1}^{N} \Big\{ \frac{(z_i^b)^\tau}{\|z_i^b\|} \big\{ \delta_i^b(t,z) + \tilde{B}_{i2}\phi_i(t,T_i^{-1}z_i)\big\} \\ & \quad -\|\tilde{B}_{i2}\|\rho_i(t,z_i) - \sum_{j=1}^{N} \mu_{ji}(t,z_i) - \zeta_i(t,z_i)\Big\} \\ & \quad + \frac{(z_i^b)^\tau}{\|z_i^b\|} \Big\{ \sum_{i=1}^{N}\sum_{j=1}^{N} \Gamma_{ij}^b(t,z_j) - \sum_{i=1}^{N}\sum_{j=1}^{N} \Gamma_{ji}^b(t,z_i)\Big\} \\ & \leq \sum_{i=1}^{N} \|\tilde{B}_{i2}\phi_i(t,T_i^{-1}z_i)\| + \sum_{i=1}^{N} \|\delta_i^b(t,z)\| \\ & \quad -\sum_{i=1}^{N} \|\tilde{B}_{i2}\|\rho_i(t,z_i) - \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_{ji}(t,z_i) \\ & \quad -\sum_{i=1}^{N} \zeta_i(t,z_i) \end{aligned} \tag{33}$$

From Assumption 2 and the identity $\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} = \sum_{j=1}^{N}\sum_{i=1}^{N} a_{ji}$,

$$\begin{aligned} & \sum_{i=1}^{N} \|\delta_i^b(t,T^{-1}z)\| \\ & \leq \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_{ij}(t,z_j) + \sum_{i=1}^{N} \nu_i(t,z) \\ & = \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_{ji}(t,z_i) + \sum_{i=1}^{N} \nu_i(t,z) \end{aligned} \tag{34}$$

and

$$\begin{aligned} \|\tilde{B}_{i2}\phi_i(t,T_i^{-1}z_i)\| \;&\leq\; \|\tilde{B}_{i2}\|\|\phi_i(t,T_i^{-1}z_i)\| \\ &\leq\; \|\tilde{B}_{i2}\|\rho_i(t,z_i) \end{aligned} \tag{35}$$

Substituting inequalities (34) and (35) into (33),

$$\sum_{i=1}^{N} \frac{\sigma_i^\tau\dot{\sigma}_i}{\|\sigma_i\|} \leq -\sum_{i=1}^{N} \zeta_i(t,z_i) + \sum_{i=1}^{N} \nu_i(t,z) < 0 \tag{36}$$

Then the reachability condition (28) is satisfied. Hence, the result follows. ■

Remark 6. It should be noted that the functions $\zeta_i(\cdot)$ in (31) are design parameters. Theorem 2 shows that if $\zeta_i(\cdot)$ are designed to satisfy condition (31), then the well known reachability condition holds and a sliding mode will occur. Moreover, if all the interconnection functions $\nu_i(t,z)$ are bounded for $i=1,2,\ldots,N$ in the considered domain Ω, it is straightforward to see that (31) always holds by choosing appropriate $\zeta_i(\cdot)$.

From sliding mode control theory, Theorems 1 and 2 together guarantee that the closed-loop system formed by applying the decentralised controller (30) to the interconnected system (9) is asymptotically stable in the domain Ω.

It is clear to see that system (9) is an expression of system (1) in the new coordinates $z_i (z_i = T_i x_i)$. Partition T_i as follows

$$T_i = \begin{bmatrix} T_i^a \\ T_i^b \end{bmatrix} \tag{37}$$

where $T_i^a \in \mathcal{R}^{(n_i-m_i)\times n_i}$ and $T_i^b \in \mathcal{R}^{m_i\times n_i}$.

Then

$$\begin{bmatrix} z_i^a \\ z_i^b \end{bmatrix} := z_i = T_i x_i = \begin{bmatrix} T_i^a x_i \\ T_i^b x_i \end{bmatrix} \tag{38}$$

From the relationship between (1) and (9), it is straightforward to rewrite the control (30) in terms of the x coordinates to stabilize the system (1) using (38).

V. Case study - automated highway systems

In order to achieve high traffic flow rates and reduce congestion, an automated highway systems has been developed [22]. During the automated driving process, cars are driven automatically with both on-board lateral and longitudinal controllers. The lateral controller is used to steer the vehicle and the longitudinal controller is used to follow a lead vehicle at a safe distance. The stability and the robustness of the vehicle-following system will be considered as a case study to demonstrate the theoretical results. The dynamics of the vehicle-following system is described by [2]

$$\dot{\xi}_i = v_i - v_{(i-1)} \tag{39}$$

$$\dot{v}_i = \frac{1}{m_i}\left(-A_{ip}v_i^2 - d_i + f_i\right) \tag{40}$$

$$\dot{f}_i = \frac{1}{\kappa_i}\left(-f_i + u_i\right) \tag{41}$$

where ξ_i represents the distance between the ith and the $(i-1)$th vehicle, v_i is the velocity of the ith vehicle and f_i is the force applied to the longitudinal dynamics of the ith vehicle, where if $f_i > 0$ a forward driving force occurs and if $f_i < 0$, then a braking force takes place. m_i is the mass of the ith vehicle, d_i and κ_i are the constant frictional force and the engine brake time constant. The signal u_i is the control variable, where if $u_i > 0$, a throttle input results, and if $u_i < 0$ then a braking input occurs. Parameters are chosen as in [2]:

$$\begin{array}{lllllll} m_i & = & 1300\text{kg}, & A_{ip} & = & 0.3\text{Ns}^2/\text{m}^2, & d_i = 100\text{N} \\ \kappa_i & = & 0.2\text{s}, & v_0 & = & 20\text{m/s} & \end{array}$$

As in [24], a safety distance frequently used in automated highway systems based on the Time-Headway policy (CTH) is used in this design. The safety distance defined by the CTH policy is described by (e.g. see [24])

$$\xi_d(v_i) = \xi_{d0} + \beta v_i \tag{42}$$

where ξ_{d0} is the distance between stationary vehicles, and β is the so-called headway time. It is well known that the safety distance is closely related to the vehicle's velocity. Therefore, the safety distances in (42) are more practicable when compared with the work in [2] and [22] in which the safety distance is chosen as a constant.

Define $\xi_{d0} = 1$, $\beta = 0.5$ and $v_d = v_0$ as an ideal driving velocity, and let

$$x_{i1} = \xi_i - \xi_d(v_i) \tag{43}$$

$$x_{i2} = v_i - v_d \tag{44}$$

$$x_{i3} = \frac{f_i - A_{ip}v_0^2 - d_i}{1000} \tag{45}$$

for $i = 1,2,\ldots,6$. Then, a 6-vehicle following system can be described in the form of (1) as follows:

$$\begin{aligned} \dot{x}_i \;=\; & \underbrace{\begin{bmatrix} 0 & 1.0046 & -0.3846 \\ 0 & -0.0092 & 0.7692 \\ 0 & 0 & -5 \end{bmatrix}}_{A_i} x_i \\ & + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0.005 \end{bmatrix}}_{B} (u_i + 220 + \phi_i(x_i,t)) \\ & + \underbrace{\begin{bmatrix} -x_{(i-1)2} \\ 0 \\ 0 \end{bmatrix}}_{\Xi_{i(i-1)}} + \underbrace{\begin{bmatrix} 0.00046x_{i2}^2 \\ -0.00023x_{i2}^2 \\ 0 \end{bmatrix}}_{\Xi_{ii}} \\ & + \psi_i(t,x), \qquad i = 1,2,\ldots,6 \end{aligned} \tag{46}$$

where $\Xi_{ij} = 0$ if $i \neq j$ and $j \neq i-1$, and

$$\Xi_{i0} = \begin{bmatrix} -x_{02} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -v_0 + v_d \\ 0 \\ 0 \end{bmatrix} = 0$$

bounds of the unknown matched uncertainty $\phi_i(x_i,t)$ are assumed to satisfy

$$\|\phi_1(x_1,t)\| \leq 20|x_{11} + x_{12}| + 80|x_{13}| \tag{47}$$

$$\|\phi_2(x_2,t)\| \leq 25|x_{21} + x_{22}| + 75|x_{23}| \tag{48}$$

$$\|\phi_3(x_3,t)\| \leq 30|x_{31} + x_{32}| + 70|x_{33}| \tag{49}$$

$$\|\phi_4(x_4,t)\| \leq 35|x_{41} + x_{42}| + 65|x_{43}| \tag{50}$$

$$\|\phi_5(x_5,t)\| \leq 40|x_{51} + x_{52}| + 60|x_{53}| \tag{51}$$

$$\|\phi_6(x_6,t)\| \leq 45|x_{61} + x_{62}| + 55|x_{63}| \tag{52}$$

Remark 7. The high-speed following system is a physical system and the mass of each vehicle is relatively large and thus the corresponding driving/braking forces are large. It should be noted that the uncertainty added to the system in the current study is to illustrate the robustness of the designed control system to verify the results obtained in this paper. This element is not a feature of the system in [2].

Consider the system (46) in the domain

$$D_i = \{(x_{i1}, x_{i2}, x_{i3}) \quad |x_{i2}| < 20\} \tag{53}$$

which, from (44), implies that the maximus speed of all the cars is $40m/s$ (144 Km/h).

By using the algorithm in [9], the coordinate transformation $z_i = T_i x_i$ for $i = 1, 2, \ldots, 6$ can be obtained with T_i defined by

$$T_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 13 & 20.79 & 1 \end{bmatrix}$$

then the system (46) is transformed into the form (13)-(14) with

$$\begin{aligned} \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix} &= \left[\begin{array}{cc:c} 5 & 9 & -0.3846 \\ -10 & -16 & 0.7692 \\ \hdashline -77.88 & -111.668 & 5.9908 \end{array}\right] \\ \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} &= \left[\begin{array}{c} 0 \\ 0 \\ \hdashline 0.005 \end{array}\right] \end{aligned} \tag{54}$$

for $i = 1, 2, \ldots, 6$ and

$$\Gamma_{ii}(t, z_j) = \begin{bmatrix} \Gamma^a_{ii}(t, z_j) \\ \Gamma^b_{ii}(t, z_j) \end{bmatrix} = \left[\begin{array}{c} 0.000115x^2_{i2} \\ -0.00023x^2_{i2} \\ \hdashline -0.0033x^2_{i2} \end{array}\right]$$

for $i = 1, 2, \ldots, 6$ and

$$\begin{aligned} \Gamma_{i(i-1)} &= \begin{bmatrix} \Gamma^a_{i(i-1)}(t, z_j) \\ \Gamma^b_{i(i-1)}(t, z_j) \end{bmatrix} \\ &= \left[\begin{array}{c} -x_{(i-1)2} \\ 0 \\ \hdashline -13x_{(i-1)2} \end{array}\right], \qquad i = 2, \ldots, 6 \end{aligned}$$

The bounds on the unknown interconnections satisfy

$$\begin{aligned} \delta^a_1(\cdot) &\le 0.01\cos^2(z_{12})\|z_1\| + 0.008\sin^2(z_{21})\|z_2\| \\ \delta^a_2(\cdot) &\le 0.009\cos^2(z_{21})\|z_2\| + 0.016\sin^2(z_{13})\|z_1\| \\ &\quad + 0.0096\cos^2(z_{33})\|z_3\| \\ \delta^a_3(\cdot) &\le 0.008\sin^2(z_{32})\|z_3\| + 0.007\cos^2(z_{11})\|z_1\| \\ &\quad + 0.011\cos^2(z_{22})\|z_2\| + 0.0095\cos^2(z_{42})\|z_4\| \\ \delta^a_4(\cdot) &\le 0.011\cos^2(z_{41})\|z_4\| + 0.012\cos^2(z_{22})\|z_2\| \\ &\quad + 0.01\cos^2(z_{31})\|z_3\| + 0.0078\cos^2(z_{51})\|z_5\| \\ \delta^a_5(\cdot) &\le 0.012\sin^2(z_{51})\|z_5\| + 0.016\cos^2(z_{23})\|z_2\| \\ &\quad + 0.009\sin^2(z_{42})\|z_4\| + 0.0074\cos^2(z_{63})\|z_6\| \\ \delta^a_6(\cdot) &\le 0.02\sin^2(z_{63})\|z_6\| + 0.0075\sin^2(z_{13})\|z_1\| \\ &\quad + 0.012\sin^2(z_{51})\|z_5\| \\ \delta^b_1(\cdot) &\le \underbrace{0.24\cos^2(z_{12})\|z_1\|}_{\mu_{11}(t,z_1)} + \underbrace{0.192|z_{22}|}_{\mu_{12}(t,z_2)} \\ \delta^b_2(\cdot) &\le \underbrace{0.18\cos^2(z_{21})\|z_2\|}_{\mu_{22}(t,z_2)} + \underbrace{0.38\sin^2(z_{13})\|z_1\|}_{\mu_{21}(t,z_1)} + \\ &\quad \underbrace{0.32\sin^2(z_{22})\|z_1\| + 0.192\sin^2(z_{22}z_{33})\|z_3\|}_{\nu_2(t,z)} \\ \delta^b_3(\cdot) &\le \underbrace{0.2|z_{21}+z_{22}| + 0.1|z_{23}|}_{\mu_{32}(t,z_2)} \\ \delta^b_4(\cdot) &\le \underbrace{0.3|z_{11}+z_{13}| + 0.2|z_{12}|}_{\mu_{41}(t,z_1)} \\ &\quad + \underbrace{0.6|z_{51}+z_{52}| + 0.4|z_{53}|}_{\mu_{45}(t,z_5)} \\ \delta^b_6(\cdot) &\le \underbrace{0.6|z_{21}+z_{22}| + 0.4|z_{23}|}_{\mu_{62}(t,z_2)} \end{aligned}$$

It is clear that the known nonlinear interconnections $\Gamma_{ij}(t, z_j)$ in equation (21) can be expressed as

$$\begin{aligned} \Gamma^s_{ii} &= \begin{bmatrix} 0 & \frac{0.3}{1300}x_{i2} & 0 \\ 0 & -\frac{0.3}{1300}x_{i2} & 0 \\ 0 & -\frac{4.2864}{1300}x_{i2} & 0 \end{bmatrix}, \qquad i = 1, \ldots, 6 \\ \Gamma^s_{21} &= \Gamma^s_{32} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & -13 & 0 \end{bmatrix} \end{aligned}$$

which, by direct verification, satisfy (21). Now define the sliding surface as

$$\sigma(z_i) = z_{i3}, \qquad i = 1, \ldots, 6$$

Then, when the sliding motion takes place, from Lemma 1,

$$\begin{aligned} \delta^a_1(\cdot) &\le 0.01\cos^2(z_{12})\|z^a_1\| + 0.008\sin^2(z_{21})\|z^a_2\| \\ \delta^a_2(\cdot) &\le 0.009\cos^2(z_{21})\|z^a_2\| + 0.016\sin^2(z_{12})\|z^a_1\| \\ \delta^a_3(\cdot) &\le 0.008\sin^2(z_{32})\|z^a_3\| + 0.007\cos^2(z_{11})\|z^a_1\| \\ &\quad + 0.011\cos^2(z_{22})\|z^a_2\| \\ &\quad + 0.0095\cos^2(z_{42})\|z^a_4\| \\ \delta^a_4(\cdot) &\le 0.011\cos^2(z_{41})\|z^a_4\| + 0.012 \\ &\quad \cdot\cos^2(z_{22})\|z^a_2\| + 0.01\cos^2(z_{31})\|z^a_3\| \\ &\quad + 0.0078\cos^2(z_{51})\|z^a_5\| \\ \delta^a_5(\cdot) &\le 0.012\sin^2(z_{51})\|z^a_5\| + 0.009\sin^2(z_{42})\|z^a_4\| \\ \delta^a_6(\cdot) &\le 0.012\sin^2(z_{51})\|z^a_5\| \end{aligned}$$

Choose $Q_1 = 1000I_2, Q_2 = 234I_2, Q_3 = 23I_2, Q_4 = 1.3I_2, Q_5 = 0.05I_2$ and $Q_6 = 0.01I_2$, by solving the Lyapunov equation, (17) yields

$$\begin{aligned} P_1 &= \begin{bmatrix} 1577.27 & -931.82 \\ -931.82 & 613.64 \end{bmatrix} \\ P_2 &= \begin{bmatrix} 369.08 & -218.05 \\ -218.05 & 143.59 \end{bmatrix} \\ P_3 &= \begin{bmatrix} 36.28 & -21.43 \\ -21.43 & 14.11 \end{bmatrix} \\ P_4 &= \begin{bmatrix} 2.05 & -1.21 \\ -1.21 & 0.80 \end{bmatrix} \\ P_5 &= \begin{bmatrix} 0.079 & -0.047 \\ -0.047 & 0.031 \end{bmatrix} \\ P_6 &= \begin{bmatrix} 0.016 & -0.0093 \\ -0.0093 & 0.0061 \end{bmatrix} \end{aligned}$$

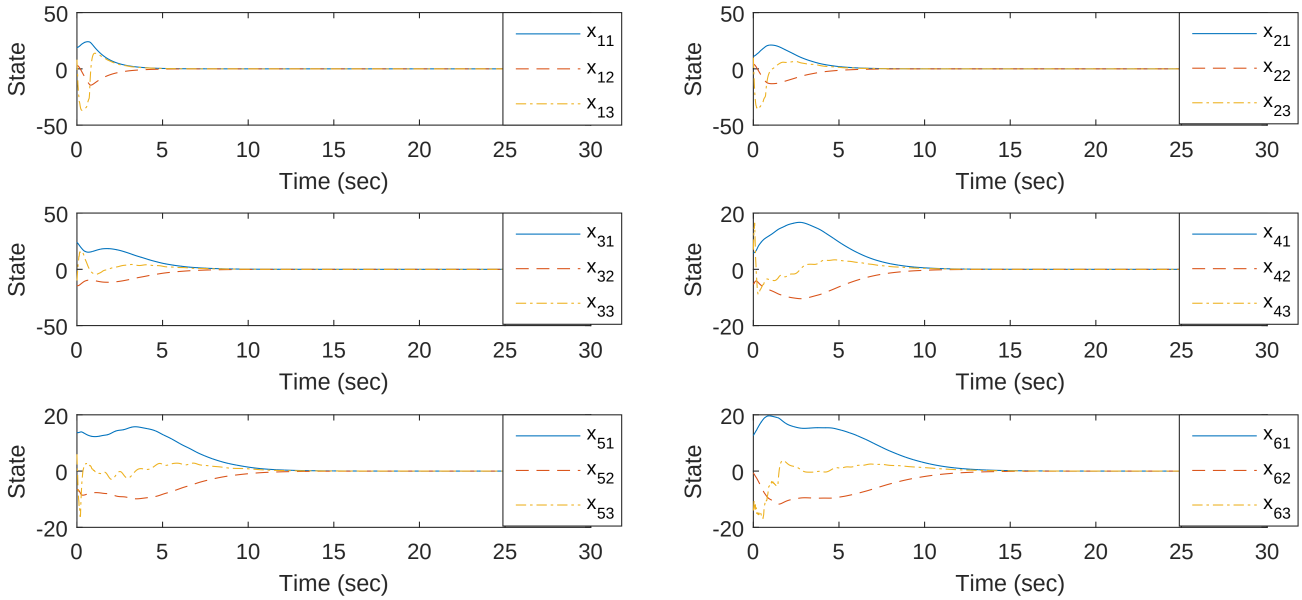

Fig. 1. Time responses of the state variables of the system (46)

Then, the matrix function M can be obtained. It is straightforward to verify that in the domain $\Omega = T(\mathcal{D}_1 \times \mathcal{D}_2 \times \cdots \times \mathcal{D}_6)$ where $\mathcal{D}_i$ are given in (53) for $i = 1, \ldots, 6$,

$$M^\tau + M > 0$$

It follows from Theorem 1 that the designed sliding mode is asymptotically stable.

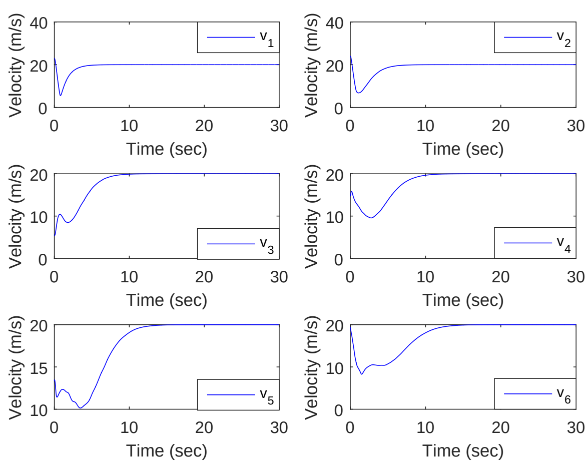

Fig. 2. Time responses of the velocities of the vehicles

Choose

$$\begin{array}{ll} \zeta_1 = 200 + 0.32\|z_1\| & \zeta_4 = 200 \\ \zeta_2 = 200 & \zeta_5 = 200 \\ \zeta_3 = 200 + 0.192\|z_3\| & \zeta_6 = 200 \end{array}$$

From (30), the controller u_i for $i = 1, \ldots, 3$ is well defined and the condition (31) in Theorem 2 is satisfied in the considered domain.

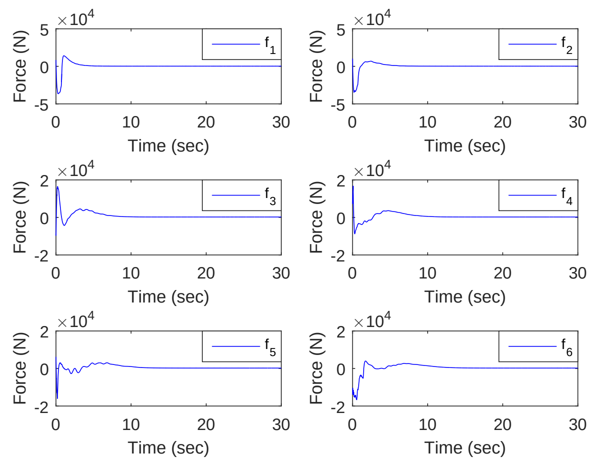

Fig. 3. Time responses of the driving/braking forces of the vehicles

Simulation results are obtained and shown in Fig.1-Fig.5. The time responses of all the system states are shown in Fig.1. From Fig.1, it is clear to see that all subsystems are stabilized even in the presence of uncertainties. The time response of velocities, driving/braking forces and distances with safe distances defined in (42) are shown in Fig.2-Fig.4 respectively. According to Fig.4, all cars are running within the prescribed safe distance to avoid collision. In Fig.3, it is clear to see that some subsystems, e.g. the 4th and 5th subsystems, experienced relatively large disturbances. However, owing to the robustness of the controller with respect to matched uncertainties when in the sliding mode, the closed-loop performance is robust. The control input signals applied to the system (46) are shown in Fig.5. It should be noted that a boundary layer approximation

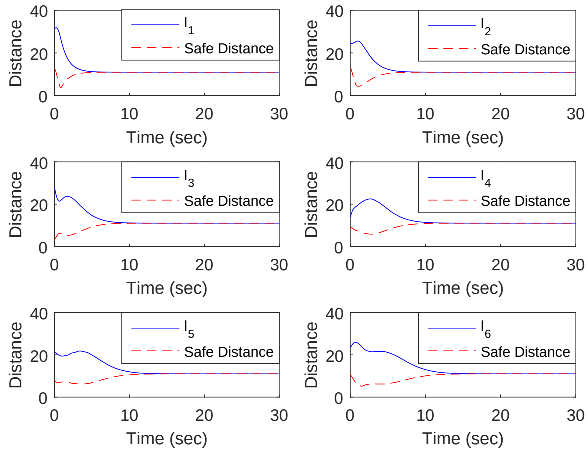

Fig. 4. Time responses of the actual distances between vehicles and the safe distance defined in (42)

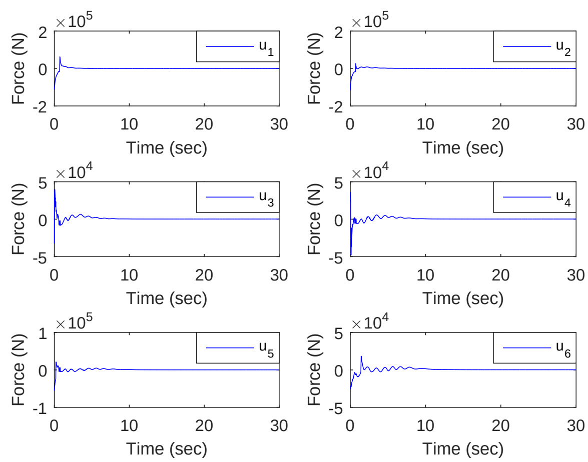

Fig. 5. Time responses of the system input

is used in the simulation, and thus there is no chattering. The simulation results show that the proposed approach is effective.

Remark 8. From the simulation example, it is clear to see that the bounds on the uncertainties have a more general form in this paper when compared with the existing work in [2] and [25]. In fact, in [2], the uncertainties are inevitably assumed to be a linear combination of known nonlinear functions in order to adaptively compensate parameter uncertainty. Furthermore, the bounds on the interconnections are assumed to satisfy a linear growth condition (i.e. $\|\delta_i\| \leq \sum_{j=0}^{N} c_j \|x_j\|$). In [25], an adaptive fuzzy control is applied on an automated highway system. In order to counteract the effect of the uncertainties, the bounds on the interconnection terms are assumed to have a special structure [25]. It should be noticed that in (54), B_{i2} is constant. However, the results developed in this paper can also be applied to the case that B_{i2} is nonsingular matrix.

VI. Conclusion

A decentralised state feedback sliding mode control law has been proposed to asymptotically stabilise a class of nonlinear interconnected systems with known and unknown interconnections in the considered domain. Both matched and mismatched uncertainties are considered. The bounds on the uncertainties can be functions instead of constants or polynomial bounds as considered in previous work. Both known interconnections and the bounds on the unknown interconnections have been fully considered in the control design to reduce conservatism. The developed results are applicable to a wide class of interconnected systems. Simulations based on a vehicle-following system have been presented to show that the results obtained are effective.

References

[1] X.-G. Yan, C. Edwards, S. K. Spurgeon and J.A.M. Bleijs, Decentralised sliding-mode control for multimachine power systems using only output information, IEEE Proc.-Control Theory Appl., vol.151, pp.627-635, Sep. 2004.

[2] J. T. Spooner, and K. M. Passino, Adaptive control of a class of decentralized nonlinear systems, IEEE Trans. on Automat. Control, vol. 41, no. 2, pp. 280-284, Feb. 1996.

[3] X. Wang, N. Xiao, L. Xie, E. Frazzoli and D. Rus, Decentralised dynamic games for large population stochastic multi-agent systems, IET Control Theory & Applications, vol. 9, no. 3, pp. 503-510, 2015.

[4] B. Mirkin, P. O. Gutman, Y. Shtessel, Asymptotic sliding mode control approach to adaptive distributed tracking problem for multi-agent nonlinear delayed systems, International Journal of Control, vol. 85, no. 11, pp. 503-510, 2015.

[5] J. Mu, X.-G. Yan and S. K. Spurgeon, Decentralised sliding mode control for a class of nonlinear interconnected systems. American Control Conference (ACC), Chicago, USA, pp. 5170 - 5175, 2015.

[6] Q. Liu, S.J. Qin, T. Chai, Decentralized fault diagnosis of continuous annealing processes based on multilevel PCA, IEEE Trans. on Automation Science and Engineering, vol. 10, no. 3, pp. 687 - 698, 2013.

[7] S. Jiang, R. Kumar, S. Takai, W. Qiu, Decentralized control of discrete-event systems with multiple local specifications, IEEE Trans. Automation Science and Engineering, vol. 7, no. 3, pp. 512 - 522, 2010.

[8] M. S. Mahmoud. Decentralized Systems with Design Constraints. Springer-Verlag London Limited, 2011.

[9] C. Edwards and S. K. Spurgeon, Sliding mode control: Theory and applications. London: Taylor & Francis, 1998.

[10] V. Utkin, J. Guldner and M. Shijun, Sliding Mode Control in Electro-mechanical Systems. London: Taylor & Francis, 1998.

[11] X.-G. Yan, S. K. Spurgeon and C. Edwards, Memoryless static output feedback sliding mode control for nonlinear systems with delayed disturbances, IEEE Trans. on Automatic Control, vol. 59, No.7, 1906-1912, 2014.

[12] M. Aldeen and H. Trinh, Decentralised feedback controllers for uncertain interconnected dynamic systems. IEE Proc. Part D: Control Theory Appl., vol. 140, no. 6, pp. 429-434, 1993.

[13] C.-F. Cheng, Disturbances attenuation for interconnected systems, Int. J. Control, vol. 66, no. 2, pp. 213-224, 1997.

[14] L. Jiang, Q. H. Wu and J. W. Wen, Decentralised nonlinear adaptive control for multimachine power systems via high-gain perturbation observer, IEEE Trans. on Circuits and Systems (I), vol. 51, no. 10, pp. 2052-2059, Oct. 2004.

[15] M. Yang, F. Yang, C.-S. Wang, and P. Wang, Decentralised sliding mode load frequency control for multi-area power systems, IEEE Trans. on Power Systems, vol. 28, no. 4, pp. 4301-4309, Aug. 2013.

[16] V. A. Ugrinovskii, I. R. Petersen, A. V. Savkin and E. Y. Ugrinovskaya, Decentralised state-feedback stabilization and robust control of uncertain large-scale systems with intergrally constrained interconnection, Systems and Control Letters, vol. 41, no. 2, pp. 107-119, Jun. 2000.

[17] C. C. Cheng and Y. Chang, Design of decentralised adaptive sliding mode controllers for large-scale systems with mismatched perturbations, Int. J. Control, vol. 81, no. 10, pp. 1507-1518, Oct. 2008.

[18] Z.-P. Jiang, Recent developments in decentralised nonlinear control, in 2004 8th Int. Conf. on Control, Automation, Robotics and Vision, Kunming, China, 2004.

[19] X.-G. Yan, J.-J. Wang, X.-Y. L$\ddot{u}$ and S.-Y. Zhang, Decentralised Output Feedback Robust Stabilization for a Class of Nonlinear Interconnected Systems with Similarity, IEEE Trans. Automatic Control, vol. 43, No. 2, pp. 294-299, Feb. 1998.

[20] X.-G. Yan, C. Edwards, S. K. Spurgeon, Decentralised robust sliding mode control for a class of nonlinear interconnceted systems by static output feedback, Automatica, vol. 40, pp. 613-620, Apr. 2004.

[21] K. C. Hsu, Decentralised variable-structure control design for uncertain large-scale systems with series nonlinearities, Int. J. control, vol. 68, no. 6, pp. 1231-1240, Jan. 1997.

[22] S. E. Shladover, C. A. Desoer, J. K. Hedrick, M. Tomizuka, J. Walrand, W.-B. Zhang, D. H. McMahon, H. Peng, S. Sheikholeslam, N. McKeown, Automatic vehicle control developments in the PATH program, IEEE Trans. Veh. Technol., vol. 40, no. 1, pp. 114-130, 1991.

[23] S. Tong, S. Sui, Y. Li, Adaptive fuzzy decentralised output stabilization for stochastic nonlinear large-scale systems with unknown control directions, IEEE Trans. on Fuzzy Systems, vol. 22, no. 5, 1365 - 1372, 2014.

[24] A. Ferrara, C. Vecchio, Second order sliding mode control of vehicles with distributed collision avoidance capabilities, Mechatronics, vol. 19, pp. 471-477, 2009.

[25] Y.-S. Huang, Z.Y- Wang, Decentralised adaptive fuzzy control for a class of large-scale MIMO nonlinear systems with strong interconnection and its application to automated highway systems, Information Sciences, vol. 274, pp. 210-224, 2014.

Jianqiu Mu received BEng in Electrical Engineering and Automation from Chongqing University, Chongqing, China, in 2012, MSc (distinction) in Advanced Electronic System Engineering from University of Kent, Canterbury, UK, in 2013. He is currently pursuing the Ph.D degree with Instrumentation, Control and Embedded Systems Research Group from University of Kent, Canterbury, UK. His current research interests include sliding mode control, decentralized control, nonlinear control systems and mobile robots.

Xing-Gang Yan received the B.Sc. degree in Applied Mathematics from Shaanxi Normal University, in 1985, the M.Sc. degree in Control and Optimisation from Qufu Normal University in 1991, and the Ph.D. degree in Control Engineering from Northeastern University, P. R. China in 1997. Currently, he is Senior Lecturer at the University of Kent, United Kingdom. He was a Lecturer in Qingdao University, P. R. China from 1991 to 1994. He worked as a Research Fellow or Research Associate in the Northwestern Polytechnical University, China, the University of Hong Kong, China, Nanyang Technological University, Singapore and the University of Leicester, United Kingdom. He is the Editor-In-Chief of the International Journal of Engineering Research and Science & Technology. His research interests include sliding mode control, decentralised control, fault detection and isolation, nonlinear control and time delay systems with applications. He has published three books and over 140 referred papers in the areas.

Sarah K. Spurgeon OBE, FREng, FInstMC, FIET, FIMA is Professor of Control Engineering and Head of Department of Electronic and Electrical Engineering at University College London in the UK. She is President of the Institute of Measurement and Control. Sarah Spurgeon's research interests are in the area of systems modelling and analysis, robust control and estimation in which areas she has published over 270 refereed research papers. She was awarded the Honeywell International Medal for 'distinguished contribution as a control and measurement technologist to developing the theory of control' in 2010 and an IEEE Millennium Medal in 2000. She is currently a member of the Council of the International Federation of Automatic Control (IFAC) and a member of the General Assembly of the European Control Association.

Dongya Zhao received BEng from Shandong University, Jinan, China, in 1998, MSc from Tianhua Institute of Chemical Machinery & Automation, Lanzhou, China, in 2002 and PhD from Shanghai Jiao Tong University, Shanghai, China, in 2009. He was a research fellow in Nanyang Technological University during July 2011 to July 2012. Since 2002, he has been with College of Chemical Engineering, China University of Petroleum, where he is currently a professor. His research interests include robot control, sliding mode control, process modelling and control, nonlinear system control and analysis. Professor Zhao is a committee member of Technical Committee on Process control, Chinese Association of Automation, senior member of Chinese Association of Automation and a member of IEEE.