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Robust Sliding Mode Observers for Large Scale Systems with Application to a Multimachine Power System

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Abstract: In this paper, a class of interconnected systems with structured and unstructured uncertainties is considered where the known interconnections and uncertain interconnections are nonlinear. The bounds on the uncertainties are employed in the observer design to enhance the robustness when the structure of the uncertainties is available for design. Under the condition that the structure distribution matrices of the uncertainties are known, a robust sliding mode observer is designed and a set of sufficient conditions is developed to guarantee that the error dynamics are asymptotically stable. In the case that the structure of uncertainties is unknown, an ultimately bounded approximate observer is developed to estimate the system states using sliding mode techniques. The results obtained are applied to a multimachine power system, and simulation for a two machine power system is presented to demonstrate the feasibility and effectiveness of the developed methods.

1. Introduction

The development of advanced technologies has produced corresponding growth in the scale of physical systems, and thus the scale of many practical systems becomes large in order to satisfy the increasing requirement for system performance. Such systems are called large scale systems and usually can be modeled by sets of lower-order ordinary differential equations which are linked through interconnections. These systems are typically called large scale interconnected systems (see, e.g.[1, 2, 3, 4]). Large scale interconnected systems widely exists in the real world, for example, energy systems and biological systems etc [1, 2]. One of the most important examples of interconnected systems is the interconnected power system or multimachine power system which consists of multi power generators connected via a power distribution network [5]. Naturally, the model of the power system is inherently nonlinear containing disturbances and uncertainties [6, 5]. As a consequence, the transient stability of power systems is a big challenge.

Transient stability is the ability of a power system to maintain the dynamic behaviour of the system at the steady state to meet different load demands or follow any significant unpredictable behaviour [7, 6]. Therefore, large numbers of researchers have developed control techniques to enhance the reliability of the power supply [8, 9]. Nonlinear optimal control of a multimachine power system is considered in [10] in which improved performance is achieved in terms of the transient stability and robustness under different fault conditions. The direct feedback linearization method is widely used to design controllers for interconnected power systems [11, 12]. However, the linearization technique may not be applicable for a complex network of interconnected systems and in this case it is necessary to consider multimachine power system with nonlinear interconnections. Recently, sliding mode controllers have been successfully applied for large scale power systems due to their performance and robustness against various disturbances [13]. A decentralized continuous higher-order sliding mode excitation control scheme is proposed in [14] to enhance transient stability and robustness of a multimachine power system. The authors in [15] used sliding mode techniques combined with a decentralized coordinated excitation and steam valve adaptive control to obtain high performance for the terminal voltage and the rotor speed simultaneously in the presence of a sudden fault and a wide range of operating conditions. In all the results mentioned above, it is assumed that all the system state variables are available for design. However, in practice, only a subset of state variables is accessible/measurable. In order to implement these control schemes, one of the choices is to design an observer to estimate system states, and then use the estimated states to form the feedback loop whenever possible. Therefore, a state estimation process is very important in both theoretical analysis and practical design.

It should be noted that observer-based controller design has been studied extensively for power systems. An observer-based controller proposed in [16] by combining a variable structure control with a reduced-order observer, which is then applied to a power system stabilizer. However, the observer-based controller is designed for a linear system and the system considered incorporates one power system. In addition, there are no unstructured uncertainties considered in the system. The research in [17] considers controller design for nonlinear systems and a nonlinear observer is used to estimate the unmeasurable states. This requires that the system can be represented in a Hamiltonian and triangular form. The authors in [18] designed a decentralized controller, i.e., for each subsystem a local controller is designed, using sliding mode techniques. This work does not involve observer design. The authors in [19] develop a functional observer approach for load frequency control of highly interconnected power networks. A quasi-decentralized functional observer is used to generate the control signal rather than estimate all the system states without considering any uncertainties. A load frequency control strategy based on sliding mode techniques and a disturbance observer is proposed in [20] . Although the authors consider uncertainties in the structure of the power system model, the observer designed is just for a power system instead of a multimachine power system. In [21] a controller which uses a nonlinear observer is developed for multimachine power systems to improve the transient stability. However, the authors did not consider the impact of disturbances. In [22] an unknown-input observer is deployed which can estimate the system states as well as perform fault detection and isolation. This is applied to a three-bus power system with one generator and two loads. It should be pointed out that the power system model considered in [22] is a differential algebraic model which is called a singular system (see [23]). Moreover, from the point of view of observation, observer based controller design imposes strong requirements on the considered system as the designed observer is for a specific task. In addition, observer design in the presence of unknown signals is challenging. An extended Kalman filter is used in [24, 25] to enhance frequency estimation of distorted power signals. However, in real time, it is difficult to implement this Kalman filter due to the poor flexibility in dealing with higher order systems. In addition, sliding mode techniques have advantages over Kalman filter approaches for electric power systems. One of these advantages is that robustness of the sliding-mode observer to parameter uncertainties and external noise can be guaranteed [26, 27]. State estimation and sliding mode control for a special class of stochastic dynamic systems, semi

Markovian jump systems, is presented in [28]. The authors designed a state observer to estimate unmeasured state components, and then synthesize a sliding mode control law based on the state estimates. The exact feedback linearization technique is used to design a nonlinear observer in [29] when the power system can be fully linearized. A sliding mode observer is presented in [30] to develop a robust observer-based nonlinear controller. This is used to construct the state variables of the system and estimate the perturbation. A sliding mode observer is designed in [31] where the observation error dynamics are ultimately stable instead of asymptotically stable as the structure of the uncertainties is not available. Moreover, results relating to sliding mode observer design for multimachine power systems are limited. It should be noted that when the structure of the interconnections is known and when the interconnections have certain properties, it is possible to design an asymptotic observer to obtain estimates with high accuracy. However, when the structure of the interconnections is not available, to design an asymptotic observer is challenging. In this case an approximate observer for large scale interconnected systems may satisfy the practical requirements.

In this paper, robust sliding mode observers are established for a class of interconnected systems in the presence of uncertainties. Both the known nonlinear interconnections and uncertain interconnections are considered. All the uncertainties are bounded by nonlinear functions. These nonlinear terms differentiate the contribution of this work from the state of the art. Coordinate transformations are introduced to simplify the system structure and transform the interconnected system to a new form with a particular structure which facilitates observer design. A set of sufficient conditions is developed such that the error dynamics are asymptotically stable when the structure of the uncertainties is known and satisfies the constrained Lyapunov equation. In the case where the structure of the uncertainties is not available but the bounds on the uncertainties are known, an ultimately bounded sliding mode observer is proposed to estimate the states of the interconnected systems, where the estimation error is dependent on the magnitude of the uncertainties. The results obtained are applied to multimachine power systems. Simulation results for a two machine power system are presented to demonstrate the effectiveness of the developed results. The main contribution includes: (i) The developed results can be applied to a wide class of interconnections due to the assumption on the limitations of the known interconnections and the wide class of bounds assumed on the unknown interconnections; (ii) Both a robust asymptotic observer and approximate observer are developed; (iii) the developed results are applicable to multimachine power systems of large order which shows their utility.

2. System description and Preliminaries

Consider a nonlinear interconnected system composed of N subsystems as follows

$$\dot{x}_i = A_i x_i + B_i u_i + \Delta\phi_i(x_i, u_i) + M_i(x) + \Delta M_i(x) \tag{1}$$

$$y_i = C_i x_i \tag{2}$$

where $x_i \in R^{n_i}$, $u_i \in U \in R^{m_i}$ (U is the admissible control set) and $y_i \in R^{p_i}$ with $m_i \leq p_i \leq n_i$ are the state variables, inputs and outputs of the i-th subsystem respectively. The matrix triples (A_i, B_i, C_i) are constant with appropriate dimensions and C_i are full row rank. The terms $\Delta\phi_i(x_i, u_i)$ and $\Delta M_i(x)$ are the uncertainties in the i-th isolated subsystems and interconnections respectively. The terms $M_i(x)$ are the known interconnections for $i = 1, \cdots, N$.

Assumption 1. The uncertainties $\Delta\phi_i(x_i, u_i)$ and $\Delta M_i(x)$ have the decomposition

$$\Delta\phi_i(x_i, u_i) = H_i^a \Delta\xi_i(x_i, u_i), \qquad \Delta M_i(x) = H_i^b \Delta E_i(x) \tag{3}$$

where $H_i^a \in R^{n_i \times k_i}$ and $H_i^b \in R^{n_i \times r_i}$ are the distribution matrices of the uncertainties, and

$$\|\Delta\xi_i(x_i, u_i)\| \le \rho_i(x_i, u_i) \qquad \text{and} \qquad \|\Delta E_i(x)\| \le \sigma_i(x) \tag{4}$$

where $\rho_i(x_i, u_i)$ is known and Lipshitz about x_i uniformly for $u_i \in U$, and $\sigma_i(x)$ is known and Lipshitz about x.

Since theC_i are full row rank, there exist nonsingular matrices T_{c_i} such that

$$\bar{A}_i = \begin{bmatrix} \bar{A}_{i1} & \bar{A}_{i2} \\ \bar{A}_{i3} & \bar{A}_{i4} \end{bmatrix} := T_{c_i} A_i T_{c_i}^{-1}, \tag{5}$$

$$\bar{B}_i = \begin{bmatrix} \bar{B}_{i1} \\ \bar{B}_{i2} \end{bmatrix} := T_{c_i} B_i, \qquad \bar{C}_i = \begin{bmatrix} 0 & I_{p_i} \end{bmatrix} := C_i T_{c_i}^{-1} \tag{6}$$

where $\bar{A}_{i1} \in R^{(n_i-p_i)\times(n_i-p_i)}$, $\bar{B}_{i1} \in R^{(n_i-p_i)\times m_i}$ and $\bar{B}_{i2} \in R^{p_i \times m_i}$ for $i = 1, \cdots, N$.

Then in the new coordinates

$$\bar{x}_i = T_{c_i} x_i \tag{7}$$

system (1)-(2) can be rewritten as

$$\dot{\bar{x}}_{i1} = \bar{A}_{i1}\bar{x}_{i1} + \bar{A}_{i2}\bar{x}_{i2} + \bar{B}_{i1}u_i + \bar{H}_{i1}^a \Delta\bar{\phi}_i(\bar{x}_i, u_i) + \bar{M}_{i1}(\bar{x}) + \bar{H}_{i1}^b \Delta\bar{M}_i(\bar{x}) \tag{8}$$

$$\dot{\bar{x}}_{i2} = \bar{A}_{i3}\bar{x}_{i1} + \bar{A}_{i4}\bar{x}_{i2} + \bar{B}_{i2}u_i + \bar{H}_{i2}^a \Delta\bar{\phi}_i(\bar{x}_i, u_i) + \bar{M}_{i2}(\bar{x}) + \bar{H}_{i2}^b \Delta\bar{M}_i(\bar{x}) \tag{9}$$

$$y_i = \bar{x}_{i2} \tag{10}$$

where $\bar{x} = \text{col}(\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_N)$, $\bar{x}_i = \text{col}(\bar{x}_{i1}, \bar{x}_{i2})$, $\bar{x}_{i1} \in R^{n_i - p_i}$, $\bar{x}_{i2} \in R^{p_i}$, $\bar{A}_{ij}$ and $\bar{B}_{il}$ are defined in (5)-(6) for $j = 1, 2, 3, 4$, $l = 1, 2$, $i = 1, 2, \cdots, N$, and

$$\begin{bmatrix} \bar{H}_{i1}^a \\ \bar{H}_{i2}^a \end{bmatrix} : = T_{c_i} H_i^a, \qquad \begin{bmatrix} \bar{H}_{i1}^b \\ \bar{H}_{i2}^b \end{bmatrix} := T_{c_i} H_i^b \tag{11}$$

$$\begin{bmatrix} \bar{M}_{i1}(x) \\ \bar{M}_{i2}(x) \end{bmatrix} : = T_{c_i} M_i(x) \tag{12}$$

$$\Delta\bar{\phi}_i(\bar{x}_i, u_i) = \Delta\xi_i(T_{c_i}^{-1}\bar{x}_i, u_i) \tag{13}$$

$$\Delta\bar{M}_i(\bar{x}) = \Delta E_i(T_{c_i}^{-1}\bar{x}) \tag{14}$$

where $\bar{H}_{i1}^a \in R^{(n_i-p_i)\times k_i}$, $\bar{H}_{i1}^b \in R^{(n_i-p_i)\times r_i}$, and $\bar{M}_{i1}(\cdot) \in R^{(n_i-p_i)}$ for $i = 1, 2, \cdots, N$.

Assumption 2. The matrix pair $(\bar{A}_i, \bar{C}_i)$ in (5)-(6) is observable for $i = 1, 2, \cdots, N$.

Under Assumption 2, there exists a matrix L_i such that $\bar{A}_i - L_i\bar{C}_i$ is stable, and thus for any $Q_i > 0$ the Lyapunov equation

$$(\bar{A}_i - L_i\bar{C}_i)^T P_i + P_i(\bar{A}_i - L_i\bar{C}_i) = -Q_i \tag{15}$$

has an unique solution $P_i > 0$ for $i = 1, 2, \cdots, N$.

Assumption 3. There exist matrices $F_i^a \in R^{k_i \times p_i}$ and $F_i^b \in R^{r_i \times p_i}$ such that the solution P_i to the Lyapunov equation (15) satisfies the constraint

$$\bar{H}_i^{aT} P_i = F_i^a \bar{C}_i \tag{16}$$

$$\bar{H}_i^{bT} P_i = F_i^b \bar{C}_i \tag{17}$$

For further analysis, introduce partitions of P_i and Q_i which are conformable with the decomposition in (8)-(10) as follows

$$P_i = \begin{bmatrix} P_{i1} & P_{i2} \\ P_{i2}^T & P_{i3} \end{bmatrix}, \qquad Q_i = \begin{bmatrix} Q_{i1} & Q_{i2} \\ Q_{i2}^T & Q_{i3} \end{bmatrix} \tag{18}$$

where $P_{i1} \in R^{(n_i-p_i)\times(n_i-p_i)}$ and $Q_{i1} \in R^{(n_i-p_i)\times(n_i-p_i)}$. Then, from $P_i > 0$ and $Q_i > 0$, it follows that $P_{i1} > 0$, $P_{i3} > 0$, $Q_{i1} > 0$ and $Q_{i3} > 0$.

The following results are required for further analysis.

Lemma 1. If P_i and Q_i have the partition in (18), then under Assumption 3, the following results hold

(i). $P_{i1}^{-1}P_{i2}\bar{H}_{i2}^a + \bar{H}_{i1}^a = 0$ if (16) is satisfied.
(ii). $P_{i1}^{-1}P_{i2}\bar{H}_{i2}^b + \bar{H}_{i1}^b = 0$ if (17) is satisfied.
(iii). The matrix $A_{i1} + P_{i1}^{-1}P_{i2}A_{i3}$ is Hurwitz stable if the Lyapunov equation (15) is satisfied.

Proof. See Lemma 2.1 in [32].

3. Sliding mode observer design

Consider the system in (8)-(10). Introduce a linear coordinate transformation

$$z_i = \underbrace{\begin{bmatrix} I_{n_i-p_i} & P_{i1}^{-1}P_{i2} \\ 0 & I_{p_i} \end{bmatrix}}_{T_i} \bar{x}_i \tag{19}$$

In the new coordinate system z_i, system (8)-(10) has the following form

$$\begin{aligned} \dot{z}_{i1} &= (\bar{A}_{i1} + P_{i1}^{-1}P_{i2}\bar{A}_{i3})z_{i1} + (\bar{A}_{i2} - \bar{A}_{i1}P_{i1}^{-1}P_{i2} + P_{i1}^{-1}P_{i2}(\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2}))z_{i2} \\ &\quad + \bar{B}_{i1}u_i + P_{i1}^{-1}P_{i2}\bar{B}_{i2}u_i + \bar{M}_{i1}(T^{-1}z) + P_{i1}^{-1}P_{i2}\bar{M}_{i2}(T^{-1}z) \qquad (20) \\ \dot{z}_{i2} &= \bar{A}_{i3}z_{i1} + (\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2})z_{i2} + \bar{B}_{i2}u_i + \bar{H}_{i2}^a\Delta\bar{\phi}_i(T_i^{-1}z_i, u_i) \\ &\quad + \bar{M}_{i2}(T^{-1}z) + \bar{H}_{i2}^b\Delta\bar{M}_i(T^{-1}z) \qquad (21) \\ y_i &= z_{i2} \qquad (22) \end{aligned}$$

where $z_i = \text{col}(z_{i1}, z_{i2})$ with $z_{i1} \in R^{n_i-p_i}$. From Assumption 1, (13) and (14)

$$\|\Delta\bar{\phi}_i(T_i^{-1}z_i, u_i)\| \le \rho_i((T_iT_{c_i})^{-1}z_i, u_i) := \bar{\rho}_i(z_i, u_i) \tag{23}$$

$$\|\Delta\bar{M}_i(T^{-1}z)\| \le \sigma_i((TT_c)^{-1}z) := \bar{\sigma}_i(z) \tag{24}$$

and $\bar{\rho}_i(z_i, u_i), \bar{\sigma}_i(z)$ satisfy the Lipschitz condition

$$\|\bar{\rho}_i(z_i, u_i) - \bar{\rho}_i(\hat{z}_i, u_i)\| \le \ell_{\bar{\rho}_i}\|z_i - \hat{z}_i\| \tag{25}$$

$$\|\bar{\sigma}_i(z) - \bar{\sigma}_i(\hat{z})\| \le \ell_{\bar{\sigma}_i}\|z - \hat{z}\| \tag{26}$$

Here $\ell_{\bar{\rho}_i}$ may be a function of u_i.

For system (20)-(22), consider a dynamical system

$$\begin{aligned}\dot{\hat{z}}_{i1} &= (\bar{A}_{i1} + P_{i1}^{-1}P_{i2}\bar{A}_{i3})\hat{z}_{i1} + (\bar{A}_{i2} - \bar{A}_{i1}P_{i1}^{-1}P_{i2} + P_{i1}^{-1}P_{i2}(\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2}))y_i \\ &\quad +\bar{B}_{i1}u_i + P_{i1}^{-1}P_{i2}\bar{B}_{i2}u_i + \bar{M}_{i1}(T^{-1}\hat{z}) + P_{i1}^{-1}P_{i2}\bar{M}_{i2}(T^{-1}\hat{z}) \qquad (27)\\ \dot{\hat{z}}_{i2} &= \bar{A}_{i3}\hat{z}_{i1} + (\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2})\hat{z}_{i2} + \bar{B}_{i2}u_i + \bar{M}_{i2}(T^{-1}\hat{z}) + d_i(\cdot) \qquad (28)\\ \hat{y}_i &= \hat{z}_{i2} \qquad (29)\end{aligned}$$

where $\hat{z} = \mathrm{col}(\hat{z}_1, y)$, and the injection term $d_i(\cdot)$ is defined by

$$d_i(\cdot) = (\|\bar{H}_{i2}^a\|\bar{\rho}_i(\hat{z}_i, u_i) + \|\bar{H}_{i2}^b\|\bar{\sigma}_i(\hat{z}) + \|\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2}\|\|y_i - \hat{y}_i\| + k_i)\mathrm{sgn}(y_i - \hat{y}_i) \qquad (30)$$

where $\bar{\rho}_i(\hat{z}_i, u_i) = \bar{\rho}_i(\hat{z}_{i1}, y_i, u_i)$ and $\bar{\sigma}_i(\hat{z}) = \bar{\sigma}_i(\hat{z}_{11}, y_1, \hat{z}_{21}, y_2, \cdots, \hat{z}_{N1}, y_N)$.

Let $e_{i1} = z_{i1} - \hat{z}_{i1}$ and $e_{y_i} = y_i - \hat{y}_i$. Then from (20)-(22) and (27)-(29), the error dynamics are described by

$$\begin{aligned}\dot{e}_{i1} &= (\bar{A}_{i1} + P_{i1}^{-1}P_{i2}\bar{A}_{i3})e_{i1} + [\bar{M}_{i1}(T^{-1}z) - \bar{M}_{i1}(T^{-1}\hat{z})]\\ &\quad +P_{i1}^{-1}P_{i2}[\bar{M}_{i2}(T^{-1}z) - \bar{M}_{i2}(T^{-1}\hat{z})] \qquad (31)\\ \dot{e}_{y_i} &= \bar{A}_{i3}e_{i1} + (\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2})e_{y_i} + [\bar{M}_{i2}(T^{-1}z) - \bar{M}_{i2}(T^{-1}\hat{z})]\\ &\quad +\bar{H}_{i2}^a\Delta\bar{\phi}_i(T_i^{-1}z_i, u_i) + \bar{H}_{i2}^b\Delta\bar{M}_i(T^{-1}z) - d_i(\cdot) \qquad (32)\end{aligned}$$

where $d_i(\cdot)$ is given in (30) for $i = 1, 2, \cdots, N$.

From the structure of the transformation matrix T_i in (19) and the fact that $\hat{z}_i = \mathrm{col}(\hat{z}_{i1}, y_i)$, it follows that

$$\|T_i^{-1}z_i - T_i^{-1}\hat{z}_i\| = \|T_i^{-1}(z_i - \hat{z}_i)\| = \left\|T_i^{-1}\begin{bmatrix} e_{i1} \\ 0 \end{bmatrix}\right\| = \|e_{i1}\|$$

From the analysis above, it is straightforward to see

$$\|T^{-1}z - T^{-1}\hat{z}\| = \|e_1\| \qquad (33)$$

where

$$e_1 := \mathrm{col}(e_{11}, e_{21}, \cdots, e_{N1}) \qquad (34)$$

Therefore,

$$\begin{aligned}\|\bar{M}_{i1}(T^{-1}z) - \bar{M}_{i1}(T^{-1}\hat{z})\| &\leq \ell_{\bar{M}_{i1}}\|e_1\| \qquad (35)\\ \|\bar{M}_{i2}(T^{-1}z) - M_{i2}(T^{-1}\hat{z})\| &\leq \ell_{\bar{M}_{i2}}\|e_1\| \qquad (36)\end{aligned}$$

The following conclusion is ready to be presented:

Theorem 1. Under Assumptions $1-3$, the error system (31) is asymptotically stable if the matrix $W^T + W$ is positive definite, where the matrix $W = [w_{ij}]_{N\times N}$, and its entries w_{ij} are defined by

$$w_{ij} = \begin{cases}\lambda_{min}(Q_{i1}) - 2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}} + \|P_{i2}\|\ell_{\bar{M}_{i2}}\big], & i = j\\ -2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}} + \|P_{i2}\|\ell_{\bar{M}_{i2}}\big], & i \neq j\end{cases} \qquad (37)$$

where P_{i1}, P_{i2} and Q_{i1} are given in (18).
Proof. For system (31), consider a Lyapunov function candidate

$$V = \sum_{i=1}^{N} e_{i1}^T P_{i1} e_{i1}$$

Then, the time derivative of V along the trajectories of system (31) is given by

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{N} \Big\{ e_{i1}^T [P_{i1}(\bar{A}_{i1} + P_{i1}^{-1} P_{i2} \bar{A}_{i3})^T + (\bar{A}_{i1} + P_{i1}^{-1} P_{i2} \bar{A}_{i3}) P_{i1}] e_{i1} \\
&\quad +2\|P_{i1}\| \|e_{i1}\| \big\{ \big[\ell_{\bar{M}_{i1}} + \|P_{i1}^{-1} P_{i2}\| \ell_{\bar{M}_{i2}} \big] \|e_1\| \big\} \Big\} \\
&\leq \sum_{i=1}^{N} \Big\{ - e_{i1}^T Q_{i1} e_{i1} + 2\|e_{i1}\| \big\{ \big[\|P_{i1}\| \ell_{\bar{M}_{i1}} + \|P_{i2}\| \ell_{\bar{M}_{i2}} \big] \|e_1\| \big\} \Big\} \qquad (38)
\end{aligned}$$

From the definition of e_1 in (34)

$$\|e_1\| \leq \sum_{j=1}^{N} \|e_{j1}\| = \|e_{i1}\| + \sum_{\substack{j=1 \\ j\neq i}}^{N} \|e_{j1}\| \qquad (39)$$

Then, from (38) and (39)

$$\begin{aligned}
\dot{V} &\leq \sum_{i=1}^{N} \Big\{ - e_{i1}^T Q_{i1} e_{i1} + 2\|e_{i1}\| \big\{ \big[\|P_{i1}\| \ell_{\bar{M}_{i1}} + \|P_{i2}\| \ell_{\bar{M}_{i2}} \big] \big[\|e_{i1}\| + \sum_{\substack{j=1 \\ j\neq i}}^{N} \|e_{j1}\| \big] \big\} \Big\} \\
&\leq \sum_{i=1}^{N} \Big\{ - e_{i1}^T Q_{i1} e_{i1} + 2\big[\|P_{i1}\| \ell_{\bar{M}_{i1}} + \|P_{i2}\| \ell_{M_{i2}} \big] \|e_{i1}\|^2 \\
&\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} 2\big[\|P_{i1}\| \ell_{\bar{M}_{i1}} + \|P_{i2}\| \ell_{\bar{M}_{i2}} \big] \|e_{i1}\| \|e_{j1}\| \Big\} \\
&\leq -\sum_{i=1}^{N} \Big\{ \big\{ \lambda_{min}(Q_{i1}) - 2\big[\|P_{i1}\| \ell_{\bar{M}_{i1}} + \|P_{i2}\| \ell_{\bar{M}_{i2}} \big] \big\} \|e_{i1}\|^2 \\
&\quad - \sum_{\substack{j=1 \\ j\neq i}}^{N} 2\big[\|P_{i1}\| \ell_{\bar{M}_{i1}} + \|P_{i2}\| \ell_{\bar{M}_{i2}} \big] \|e_{i1}\| \|e_{j1}\| \Big\} \qquad (40)
\end{aligned}$$

Then, from the definition of the matrix W in (37) and the inequality above, it follows that

$$\dot{V} \leq -\frac{1}{2} X^T [W^T + W] X$$

where $X = [\|e_{11}\|, \|e_{21}\|, \cdots, \|e_{N1}\|]^T$. Hence, the conclusion follows from $W^T + W > 0$. $\triangle$
Remark 1. From the error dynamics (31)-(32), it is clear to see that the e_{i1} dynamics interact with

the dynamics e_{y_i} through the interconnection terms $\bar{M}_{i1}(\cdot)$ and $\bar{M}_{i2}(\cdot)$. From the inequalities (35) and (36), it follows that the interconnections on the right-hand side of equation (31) are bounded by functions of e_1 only. The proof of Theorem 1 further shows that the stability of the error dynamics (31) are actually independent of e_{y_i}. This fact will be used to show the stability of the sliding motion later.

Remark 2. From the stability of Theorem 1, it follows that e_1 is bounded and thus there exists a constant $\beta > 0$ such that

$$\|e_1\| \le \beta, \tag{41}$$

where β can be estimated using the approach given in [32].

For system (31)-(32), consider a sliding surface

$$S \;=\; \{(e_{11}, e_{y_1}, e_{21}, e_{y_2}, \cdots, e_{N1}, e_{y_N}) \big| e_{y_1} = 0, e_{y_2} = 0, \cdots, e_{y_N} = 0\} \tag{42}$$

From the structure of the error dynamical system (31)-(32), it follows that the sliding mode of the error system (31)-(32) with respect to the sliding surface (42) is the system (31) when limited to the sliding surface (42). From Remark 1 and Theorem 1, the sliding mode associated with the sliding surface S given in (42) is asymptotically stable if the the conditions of Theorem 1 hold. All that remains is to determine the gains k_i in (30) such that the system (31)-(32) can be driven to the sliding surface S in finite time and a sliding motion maintained thereafter.

Theorem 2. Under Assumptions 1-3, system (31)-(32) is driven to the sliding surface (42) in finite time and remains on it if

$$k_i \ge (\|\bar{A}_{i3}\| + \ell_{\bar{M}_{i2}} + \|\bar{H}^a_{i2}\|\ell_{\bar{\rho}} + \|\bar{H}^b_{i2}\|\ell_{\bar{\sigma}})\beta + \eta \tag{43}$$

where β is determined by (41) and η is a positive constant.

Proof. From (32)

$$\begin{aligned}
\sum_{i=1}^{N} e_{y_i}^T \dot{e}_{y_i} \;&=\; \sum_{i=1}^{N} e_{y_i}^T \Big\{ \bar{A}_{i3} e_{i1} + (\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}) e_{y_i} + [\bar{M}_{i2} - \hat{\bar{M}}_{i2}] \\
&\quad + \bar{H}^a_{i2} \Delta\bar{\phi}_i(T_i^{-1} z_i, u_i) + \bar{H}^b_{i2} \Delta\bar{M}_i(T^{-1} z) - d_i(\cdot) \Big\} \\
&\le\; \sum_{i=1}^{N} \Big\{ \|\bar{A}_{i3}\| \|e_{i1}\| \|e_{y_i}\| + \ell_{\bar{M}_{i2}} \|e_{y_i}\| \|e_1\| + \|\bar{H}^a_{i2}\| \bar{\rho}_i(z_i, u_i) \|e_{y_i}\| \\
&\quad + \|\bar{H}^b_{i2}\| \bar{\sigma}_i(z) \|e_{y_i}\| + \|(\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}) \|e_{y_i}\|^2 - \|e_{y_i}\| \big\{ \|\bar{H}^a_{i2}\| \bar{\rho}_i(\hat{z}_{i1}, y_i, u_i) \\
&\quad + \|\bar{H}^b_{i2}\| \bar{\sigma}_i(\hat{z}) + \|\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}\| \|e_{y_i}\| + k_i) \mathrm{sgn}(e_{y_i}) \big\} \Big\}
\end{aligned} \tag{44}$$

From (41), $\|e_{i1}\| \le \beta$. Applying (41) to (44), it follows that

$$\sum_{i=1}^{N} e_{y_i}^T \dot{e}_{y_i} \;\le\; \sum_{i=1}^{N} \Big\{ \big\{ ((\|\bar{A}_{i3}\| + \ell_{\bar{M}_{i2}} + \|\bar{H}^a_{i2}\| \ell_{\bar{\rho}_i} + \|\bar{H}^b_{i2}\| \ell_{\bar{\sigma}_i})\beta - k_i \big\} \|e_{y_i}\| \Big\} \tag{45}$$

Applying (43) to (45)

$$\sum_{i=1}^{N} e_{y_i}^T \dot{e}_{y_i} \le -\eta \sum_{i=1}^{N} \|e_{y_i}\| \tag{46}$$

which implies that

$$e_y^T \dot{e}_y \leq -\eta \|e_y\|$$

where $e_y = \text{col}(e_{y_1}, e_{y_2}, \cdots, e_{y_N})$ and the inequality $\|e_y\| \leq \sum_{i=1}^N \|e_{y_i}\|$ is applied to obtain the inequality above. This shows that the reachability condition is satisfied. Hence the conclusion follows. $\triangle$

The study above shows that (27)-(29) is an asymptotic observer of the system (20)-(22).

If the structure of the uncertainties $\Delta\phi_i(x_i, u_i)$ and $\Delta M_i(x)$ in the system (1)-(2) are unknown, which implies that Assumption 1 does not hold, then an asymptotic observer usually is not available. In this case, an ultimately bounded observer will be designed. The following Assumption is required.

Assumption 4. The uncertainties $\Delta\phi_i(x_i, u_i)$ and $\Delta M_i(x)$ in system (1)-(2) satisfy

$$\begin{aligned} \|\Delta\phi_i(x_i, u_i)\| &\leq \varepsilon_i & (47)\\ \|\Delta\bar{M}_i(x)\| &\leq \Upsilon_i & (48) \end{aligned}$$

where ε_i and Υ_i are positive constants.

In this case, in the new coordinate z the system (1)-(2) is described by

$$\begin{aligned} \dot{z}_{i1} &= (\bar{A}_{i1} + P_{i1}^{-1}P_{i2}\bar{A}_{i3})z_{i1} + (\bar{A}_{i2} - \bar{A}_{i1}P_{i1}^{-1}P_{i2} + P_{i1}^{-1}P_{i2}(\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2}))z_{i2}\\ &\quad +\bar{B}_{i1}u_i + P_{i1}^{-1}P_{i2} \times \bar{B}_{i2}u_i + \bar{M}_{i1}(T^{-1}z) + P_{i1}^{-1}P_{i2}\bar{M}_{i2}(T^{-1}z)\\ &\quad +\Delta\tilde{\phi}_{i1}(T_i^{-1}z_i, u_i) + \Delta\tilde{M}_{i1}(T^{-1}z) & (49)\\ \dot{z}_{i2} &= \bar{A}_{i3}z_{i1} + (\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2})z_{i2} + \bar{B}_{i2}u_i + \bar{M}_{i2}(T^{-1}z)\\ &\quad +\Delta\tilde{\phi}_{i2}(T_i^{-1}z_i, u_i) + \Delta\tilde{M}_{i2}(T^{-1}z) & (50)\\ y_i &= z_{i2} & (51) \end{aligned}$$

where

$$\begin{bmatrix} \Delta\tilde{\phi}_{i1}(T_i^{-1}z_i, u_i) \\ \Delta\tilde{\phi}_{i2}(T_i^{-1}z_i, u_i) \end{bmatrix} = T_i^{-1} \begin{bmatrix} \Delta\phi_{i1}(T_i^{-1}z_i, u_i) \\ \Delta\phi_{i2}(T_i^{-1}z_i, u_i) \end{bmatrix} \quad (52)$$

$$\begin{bmatrix} \Delta\tilde{M}_{i1}(T^{-1}z) \\ \Delta\tilde{M}_{i2}(T^{-1}z) \end{bmatrix} = T_i^{-1} \begin{bmatrix} \Delta\bar{M}_{i1}(T^{-1}z) \\ \Delta\bar{M}_{i2}(T^{-1}z) \end{bmatrix} \quad (53)$$

and $z_i = \text{col}(z_{i1}, z_{i2})$ with $z_{i1} \in R^{n_i - p_i}$. From (47)-(48), there are constants ε_i^a, ε_i^b, Υ_i^a and Υ_i^b such that

$$\begin{aligned} \|\Delta\tilde{\phi}_{i1}(T_i^{-1}z_i, u_i)\| &\leq \varepsilon_i^a & (54)\\ \|\Delta\tilde{\phi}_{i2}(T_i^{-1}z_i, u_i)\| &\leq \varepsilon_i^b & (55)\\ \|\Delta\tilde{M}_{i1}(T^{-1}z)\| &\leq \Upsilon_i^a & (56)\\ \|\Delta\tilde{M}_{i2}(T^{-1}z)\| &\leq \Upsilon_i^b & (57) \end{aligned}$$

Now consider the dynamical systems

$$\begin{aligned} \dot{\hat{z}}_{i1} &= (\bar{A}_{i1} + P_{i1}^{-1}P_{i2}\bar{A}_{i3})\hat{z}_{i1} + (\bar{A}_{i2} - \bar{A}_{i1}P_{i1}^{-1}P_{i2} + P_{i1}^{-1}P_{i2}(\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2}))y_i\\ &\quad +\bar{B}_{i1}u_i + P_{i1}^{-1}P_{i2}\bar{B}_{i2}u_i + \bar{M}_{i1}(T^{-1}\hat{z}) + P_{i1}^{-1}P_{i2}\bar{M}_{i2}(T^{-1}\hat{z}) & (58)\\ \dot{\hat{z}}_{i2} &= \bar{A}_{i3}\hat{z}_{i1} + (\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2})\hat{z}_{i2} + \bar{B}_{i2}u_i + \bar{M}_{i2}(T^{-1}\hat{z}) + d_i(\cdot) & (59)\\ \hat{y}_i &= \hat{z}_{i2} & (60) \end{aligned}$$

where $\hat{z} = \text{col}(\hat{z}_1, y)$. The injection term $d_i(\cdot)$ is defined by

$$\begin{aligned} d_i(\cdot) &= (\|\Delta\tilde{\phi}_{i2}(T_i^{-1}\hat{z}_i, u_i)\| + \|\Delta\tilde{M}_{i2}(T^{-1}\hat{z})\| \\ &\quad + \|\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2}\|\|y_i - \hat{y}_i\| + k_i)\text{sgn}(y_i - \hat{y}_i) \end{aligned} \tag{61}$$

Let $e_{i1} = z_{i1} - \hat{z}_{i1}$ and $e_{y_i} = y_i - \hat{y}_i$. Then from (49)-(51) and (58)-(60), the error dynamical equation is described by

$$\begin{aligned} \dot{e}_{i1} &= (\bar{A}_{i1} + P_{i1}^{-1}P_{i2}\bar{A}_{i3})e_{i1} + [\bar{M}_{i1}(T^{-1}z) - \bar{M}_{i1}(T^{-1}\hat{z})] + P_{i1}^{-1}P_{i2}[\bar{M}_{i2}(T^{-1}z) \\ &\quad - \bar{M}_{i2}(T^{-1}\hat{z})] + \Delta\tilde{\phi}_{i1}(T_i^{-1}z_i, u_i) + \Delta\tilde{M}_{i1}(T^{-1}z) \end{aligned} \tag{62}$$

$$\begin{aligned} \dot{e}_{y_i} &= \bar{A}_{i3}e_1 + (\bar{A}_{i4} - \bar{A}_{i3}P_{i1}^{-1}P_{i2})e_{y_i} + [\bar{M}_{i2}(T^{-1}z) - \bar{M}_{i2}(T^{-1}\hat{z})] \\ &\quad + \Delta\tilde{\phi}_{i2}(T_i^{-1}z_i, u_i) + \Delta\tilde{M}_{i2}(T^{-1}z) - d_i(\cdot) \end{aligned} \tag{63}$$

The following result is ready to be presented:

Theorem 3. Under Assumptions 2 and 4, the system (62) is ultimately bounded stable if the function matrix $W^T + W$ is positive definite, where the matrix $W = [w_{ij}]_{N\times N}$, and its entries w_{ij} are defined by

$$w_{ij} = \begin{cases} \lambda_{min}(Q_{i1}) - 2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}} + \|P_{i2}\|\ell_{\bar{M}_{i2}}\big], & i = j \\ -2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}} + \|P_{i2}\|\ell_{\bar{M}_{i2}}\big], & i \neq j \end{cases} \tag{64}$$

where P_{i1}, P_{i2} and Q_{i1} are from (18).

Proof. Consider a Lyapunov function candidate for the system (62)

$$V = \sum_{i=1}^{N} e_{i1}^T P_{i1} e_{i1}$$

where P_{i1} is defined in (18).

Following a similar proof as in Theorem 1, it is obtained

$$\begin{aligned} \dot{V} \leq &- \sum_{i=1}^{N} \Big\{ \big\{\lambda_{min}(Q_{i1}) - 2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}} + \|P_{i2}\|\ell_{\bar{M}_{i2}}\big]\big\}\|e_{i1}\| \\ &- \sum_{\substack{j=1 \\ j\neq i}}^{N} 2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}} + \|P_{i2}\|\ell_{\bar{M}_{i2}}\big]\|e_{j1}\|\Big\}\|e_{i1}\| + 2\sum_{i=1}^{N}\|P_{i1}\|\big[\varepsilon_i^a + \Upsilon_i^a\big]\|e_{i1}\| \end{aligned} \tag{65}$$

Then, from the definition of the matrix W in Theorem 2 and the inequality above, it follows that

$$\begin{aligned} \dot{V} &\leq -\frac{1}{2}X^T[W^T + W]X + \mu X \\ &= -(\frac{1}{2}\lambda_{min}(W^T + W)\|X\| - \mu)\|X\| \end{aligned} \tag{66}$$

where $\mu = 2\sqrt{\sum_{i=1}^{N}(\|P_{i1}\|\big[\varepsilon_i^a + \Upsilon_i^a\big])^2}$ and $X = [\|e_{11}\|, \|e_{21}\|, \cdots, \|e_{N1}\|]^T$. It is clear to see that $\dot{V}$ is negative definite if $\mu < \frac{1}{2}\lambda_{min}(W^T + W)$. Therefore system (62) is ultimately bounded. Hence the result follows. △

For system (62)-(63), consider the same sliding surface S given in (42). It is straightforward to see that Theorem 3 implies that the sliding mode of the system (62)-(63) associated with the sliding surface S given in (42) is ultimately bounded.

The objective now is to determine the gains k_i in (61) such that the system can be driven to the sliding surface S in (42) in finite time and a sliding motion maintained thereafter.

Theorem 4. Under Assumptions 2 and 4, the system (62)-(63) is driven to the sliding surface (42) in finite time and remains on it if

$$k_i \geq (\|\bar{A}_{i3}\| + \ell_{\bar{M}_{i2}} + \ell_{\Delta\tilde{\phi}_{i2}} + \ell_{\Delta\tilde{M}_{i2}})\beta + \eta \tag{67}$$

where β is determined by (41) and η is a positive constant.

Proof. The proof of Theorem 4 can be obtained directly by following the proof of Theorem 2. It is omitted here.

Remark 3. The results above show that the sliding mode observers of the interconnected system (1)-(2) in z coordinates are given by (27)-(29) or (58)-(60). Let

$$\hat{x}_i = (T_i T_{c_i})^{-1}\hat{z}_i, \qquad\qquad i = 1, 2, \ldots, N \tag{68}$$

where T_{c_i} and T_i are given in (7) and (19) respectively and $\hat{z}_i$ are given in (27)-(29) or (58)-(60) for $i = 1, 2, \ldots, N$. Therefore the varibles $\hat{x}_i$ given in (68) are the estimate of the states x_i of the interconnected system (1)-(2) for $i = 1, 2, \ldots, N$.

4. Case study: multimachine power system

In this section, a case study on a multimachine power system is developed. In this case, the state variable of each machine is given by $x_i = [x_{i1}\, x_{i2}\, x_{i3}] = [\delta_i - \delta_{i0}\ \ \omega_i\ \ \Delta P_{ei}]$ with $\Delta P_{ei} \equiv: P_{ei} - P_{mi0}$ where δ_i is the generator power angle [rad], P_{ei} is electrical power [p.u.], and ω_i is relative speed [rad/s] for $i = 1, 2, \cdots, N$. It is assumed that $P_{mi} = P_{mi0} =$ constant. All the symbols and terms are the same as in [33]. Then by using direct feedback linearsation compensation for the power system as in [34], the multimachine power system can be described by the system (1)-(2) with

$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{D_i}{2H_i} & -\frac{\omega_0}{2H_i} \\ 0 & 0 & -\frac{1}{T'_{doi}} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ T'_{doi} \end{bmatrix}, \qquad C_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{69}$$

From the matrix C_i, it is clear to see that the measured states are the generator power angle δ_i [rad] and the electrical power P_{ei} [p.u.]. The objective is to mainly estimate the relative speed ω_i [rad/s] for $i = 1, 2, \cdots, N$.

The known and uncertain interconnections are given by

$$\bar{M}_i(x) = 0, \qquad \Delta\bar{M}_i(x) = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{H_i^b} \Phi_i(x)$$

where

$$|\Phi_i(x)| \leq \sum_{j=1}^{N} (\gamma_{ij}^{I} |\sin\delta_j| + \gamma_{ij}^{II} |\omega_j|)$$

with the constants γ_{ij}^{I} and γ_{ij}^{II} defined by

$$\begin{aligned}
\gamma_{ij}^{I} &= \frac{4}{|T'_{doj}|_{min}}|P_{ei}|_{max} \\
\gamma_{ij}^{II} &= |Q_{ei}|_{max}
\end{aligned}$$

for $i = 1, 2, \cdots, N$, and

$$\|\Delta \bar{M}_i(x)\| = |\Phi_i(x)| \leq \sum_{j=1}^{N} (\gamma_{ij}^{I} |\sin x_{j1}| + \gamma_{ij}^{II} |x_{j2}|) \tag{70}$$

The input control variables are

$$v_{fi} = I_{qi} K_{ci} u_{fi} - (x_{di} - x'_{di}) I_{qi} I_{di} - P_{mi0} - T'_{doi} Q_{ei} \omega_i$$

Choose

$$T_{c_i} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{for} \quad i =, 1, 2, \cdots, N.$$

Following the transformation $\bar{x}_i = T_{c_i} x_i$, the system matrices become

$$\begin{aligned}
\bar{A}_i &= T_c A_i T_c^{-1} = \begin{bmatrix} -\frac{D_i}{2H_i} & 0 & -\frac{\omega_0}{2H_i} \\ 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{T'_{doi}} \end{bmatrix} && (71) \\
\bar{B}_i &= T_c B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T'_{doi}} \end{bmatrix}, \qquad \bar{C}_i = \begin{bmatrix} 0 & I_{p_i} \end{bmatrix} && (72)
\end{aligned}$$

and

$$\bar{M}_i(\bar{x}) = 0,\ \Delta \bar{M}_i(\bar{x}) = T_c \Delta \bar{M}_i(x) = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{H_i^b} \Phi_i(x) \tag{73}$$

Comparing $(5) - (6)$, it follows that

$$\begin{aligned}
\bar{A}_{i1} &= -\frac{D_i}{2H_i}, \qquad \bar{A}_{i2} = \begin{bmatrix} 0 & -\frac{\omega_0}{2H_i} \end{bmatrix}, \qquad \bar{A}_{i3} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \bar{A}_{i4} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{T'_{doi}} \end{bmatrix} \\
\bar{B}_{i1} &= 0, \qquad \bar{B}_{i2} = \begin{bmatrix} 0 \\ \frac{1}{T'_{doi}} \end{bmatrix}, \qquad \Delta \bar{M}_{i1} = 0, \qquad \Delta \bar{M}_{i2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Phi_i(x)
\end{aligned}$$

For simulation purposes, consider a two machine power system where all the parameters are chosen as in [33]. In order to illustrate the obtained results, the following uncertainties are added to the isolated subsystems

$$\begin{aligned}
\Delta\phi_1(x_1,u_1) &= \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix}}_{H_1^a} \underbrace{x_{11}\sin u_1}_{\Delta\xi_1(x_1,u_1)} \\
\Delta\phi_2(x_2,u_2) &= \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix}}_{H_2^a} \underbrace{\sin^2(x_{21}+x_{23})}_{\Delta\xi_2(x_2,u_2)}
\end{aligned}$$

It is straightforward to see

$$\begin{aligned}
|\Delta\xi_1(x_1,u_1)| &\le |x_{11}|\,|\sin u_1| := \rho_1(x_1,u_1) \\
|\Delta\xi_2(x_2,u_2)| &\le |\sin^2(x_{21}+x_{23})| := \rho_2(x_2,u_2)
\end{aligned}$$

Then, let $Q_1 = Q_2 = I_3$. The solutions of Lyapunov equation (15) are given by

$$P_1 = \begin{bmatrix} 0.5841 & -0.135 & 0 \\ -0.135 & 0.2304 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 0.6799 & -0.3 & 0 \\ -0.3 & 0.3485 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

The transformation matrix T_i in the equation $z_i = T_i\bar{x}_i$ is given by

$$T_1 = \begin{bmatrix} 1 & -0.2311 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; T_2 = \begin{bmatrix} 1 & -0.4412 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{74}$$

Therefore, under the transformation $x_i = (T_iT_{c_i})^{-1}z_i$ with T_{c_i} and T_i defined in (71) and (74), the two machine power system can be described in z coordinates as follows

$$\dot{z}_{11} = -0.704z_{11} + \begin{bmatrix} -0.0555 & -39.27 \end{bmatrix} z_{12} \tag{75}$$

$$\begin{aligned}
\dot{z}_{12} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} z_{11} + \begin{bmatrix} 0.0788535 & 0 \\ 0 & -0.1449 \end{bmatrix} z_{12} + \begin{bmatrix} 0 \\ 0.1449 \end{bmatrix} u_1 \\
&\quad + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \Delta\bar{\phi}_1(T^{-1}z_1,u_1) + \bar{M}_{12}(T^{-1}z) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta\bar{M}_{12}(T^{-1}z)
\end{aligned} \tag{76}$$

$$y_1 = z_{12} \tag{77}$$

$$\dot{z}_{21} = -0.4941z_{21} + \begin{bmatrix} -0.1 & -30.8 \end{bmatrix} z_{22} \tag{78}$$

$$\begin{aligned}
\dot{z}_{22} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} z_{21} + \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1256 \end{bmatrix} z_{22} + \begin{bmatrix} 0 \\ 0.1256 \end{bmatrix} u_2 \\
&\quad + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \Delta\bar{\phi}_2(T^{-1}z_2,u_2) + \bar{M}_{22}(T^{-1}z) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta\bar{M}_{22}(T^{-1}z)
\end{aligned} \tag{79}$$

$$y_2 = z_{22} \tag{80}$$

where $z_{i1} \in R,\ \ z_{i2} := \mathrm{col}(z_{i21}, z_{i22}) \in R^2$. From $(74) - (74)$.

$$\begin{aligned} \|\bar{\rho}_1(z_1, u_1)\| &\leq |z_{121}||\sin u_1| \qquad (81)\\ \|\bar{\rho}_2(z_2, u_2)\| &\leq |\sin^2(z_{221} + z_{222})| \qquad (82) \end{aligned}$$

By direct calculation $\ell_{\bar{\rho}_1} = 1,\ \ \ell_{\bar{\rho}_2} = 2$. From (24) and (70).

$$\begin{aligned} |\bar{\sigma}_1(z)| &\leq (\gamma_{11}^I |\sin z_{121}| + \gamma_{11}^{II}|(z_{11} + 0.2311 z_{121}|)) \\ &\quad + (\gamma_{12}^I |\sin z_{221}| + \gamma_{12}^{II}|(z_{21} + 0.4412 z_{221}|)) \qquad (83)\\ |\bar{\sigma}_2(z)| &\leq (\gamma_{21}^I |\sin z_{121}| + \gamma_{21}^{II}|(z_{11} + 0.2311 z_{121}|)) \\ &\quad + (\gamma_{22}^I |\sin z_{221}| + \gamma_{22}^{II}|(z_{21} + 0.4412 z_{221}|)) \qquad (84) \end{aligned}$$

Therefore,

$$\begin{aligned} |\bar{\sigma}_1(z) - \bar{\sigma}_1(\hat{z})| &= \\ &\begin{bmatrix} \gamma_{11}^{II} & \gamma_{11}^I + 0.2311\gamma_{11}^{II} & 0 & \gamma_{12}^{II} & \gamma_{12}^I + 0.4412\gamma_{12}^{II} & 0 \end{bmatrix} \begin{bmatrix} |z_{11} - \hat{z}_{11}| \\ \|z_{12} - \hat{z}_{12}\| \\ |z_{21} - \hat{z}_{21}| \\ \|z_{22} - \hat{z}_{22}\| \end{bmatrix} \qquad (85)\\ |\bar{\sigma}_2(z) - \bar{\sigma}_2(\hat{z})| &= \\ &\begin{bmatrix} \gamma_{21}^{II} & \gamma_{21}^I + 0.2311\gamma_{21}^{II} & 0 & \gamma_{22}^{II} & \gamma_{22}^I + 0.4412\gamma_{22}^{II} & 0 \end{bmatrix} \begin{bmatrix} |z_{11} - \hat{z}_{11}| \\ \|z_{12} - \hat{z}_{12}\| \\ |z_{21} - \hat{z}_{21}| \\ \|z_{22} - \hat{z}_{22}\| \end{bmatrix} \qquad (86) \end{aligned}$$

where $\gamma_{11}^I = 0.9,\ \ \gamma_{12}^I = 0.7355,\ \ \gamma_{11}^{II} = \gamma_{12}^{II} = 1.4$ and $\gamma_{21}^I = 0.966,\ \ \gamma_{22}^I = 0.788,\ \ \gamma_{21}^{II} = \gamma_{22}^{II} = 1.5$. Thus $\ell_{\bar{\sigma}_1} = 2.69224$ and $\ell_{\bar{\sigma}_2} = 2.88532$.

By direct computation, it follows that the matrix $W^T + W$ is positive definite. Thus, all the conditions of Theorem 1 are satisfied. Therefore the following dynamical system is an asymptotic observer of the system (75)-(80)

$$\begin{aligned} \dot{\hat{z}}_{11} &= -0.704\hat{z}_{11} + \begin{bmatrix} -0.0555 & -39.27 \end{bmatrix} \hat{z}_{12} \qquad (87)\\ \dot{\hat{z}}_{12} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \hat{z}_{11} + \begin{bmatrix} 0.0788535 & 0 \\ 0 & -0.1449 \end{bmatrix} \hat{z}_{12} + \begin{bmatrix} 0 \\ 0.1449 \end{bmatrix} u_1 + d_1(\cdot) \qquad (88)\\ \hat{y}_1 &= \hat{z}_{12} \qquad (89)\\ \dot{\hat{z}}_{21} &= -0.4941\hat{z}_{21} + \begin{bmatrix} -0.1 & -30.8 \end{bmatrix} \hat{z}_{22} \qquad (90)\\ \dot{\hat{z}}_{22} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \hat{z}_{21} + \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1256 \end{bmatrix} \hat{z}_{22} + \begin{bmatrix} 0 \\ 0.1256 \end{bmatrix} u_2 + d_2(\cdot) \qquad (91)\\ \hat{y}_2 &= \hat{z}_{22} \qquad (92) \end{aligned}$$

where the terms $d_1(\cdot)$ and $d_1(\cdot)$ are defined by

$$\begin{aligned} d_1(\cdot) &= \Big(\left\| \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \right\| \bar{\rho}_1(T^{-1}\hat{z}_1, u_1) + \left\| \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\| \bar{\sigma}_1(T^{-1}\hat{z}) \\ &\quad + \left\| \begin{bmatrix} 0.0788535 & 0 \\ 0 & -0.1449 \end{bmatrix} \right\| \|y_1 - \hat{y}_1\| + k_1\Big)\mathrm{sgn}(y_1 - \hat{y}_1) \\ d_2(\cdot) &= \Big(\left\| \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \right\| \bar{\rho}_2(T^{-1}\hat{z}_2, u_2) + \left\| \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\| \bar{\sigma}_2(T^{-1}\hat{z}) \\ &\quad + \left\| \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1256 \end{bmatrix} \right\| \|y_2 - \hat{y}_2\| + k_2\Big)\mathrm{sgn}(y_2 - \hat{y}_2) \end{aligned}$$

where k_1 and k_2 are given by

$$\begin{aligned} k_1 &\geq \Big(\left\| \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\| + \ell_{\bar{M}_{12}} + \left\| \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \right\| \ell_{\bar{\rho}_1} + \left\| \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\| \ell_{\bar{\sigma}_1}\Big)\beta + \eta \\ k_2 &\geq \Big(\left\| \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\| + \ell_{\bar{M}_{22}} + \left\| \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \right\| \ell_{\bar{\rho}_2} + \left\| \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\| \ell_{\bar{\sigma}_2}\Big)\beta + \eta \end{aligned}$$

Therefore, $\hat{x}_i = (T_i T_{c_i})^{-1}\hat{z}_i$ is an estimate of $x_i = [x_{i1} \;\; x_{i2} \;\; x_{i3}] = [\delta_i - \delta_{i0} \;\; \omega_i \;\; \Delta P_{ei}]$ where T_{c_i} and T_i are defined in (71) and (74) respectively. The simulation results presented in Figs 1 and 2 show the effectiveness of the designed observer. It should be noted that the estimation process is implemented on-line.

Remark 4. As in existing work in [14, 15, 21, 33], both the multimachine power system considered and the interconnections are nonlinear. However, most work focuses on control design or observer-based control design. In this paper, the observer can be applied to the multimachine power system as shown in the example. Specifically the interconnections are nonlinear and all the uncertainties are bounded by nonlinear functions which encompasses a large class of disturbances.

5. Conclusion

In this paper, robust sliding mode observers have been designed for a class of nonlinear interconnected systems with uncertainties. The known nonlinear interconnections and uncertain nonlinear interconnections have been dealt with separately to reduce the effects of the interconnections without introducing unnecessary conservatism. A set of sufficient conditions has been provided such that the error dynamics are asymptotically stable if the structure of the uncertainties is known. All the bounds on the uncertainties involved are nonlinear and are employed in the observer design to reject/reduce the effect of uncertainties. An ultimately bounded sliding mode observer is proposed to estimate the states of the interconnected system if the structure of the uncertainties is not available. A case study relating to a multimachine power system has been used to demonstrate the proposed approach.

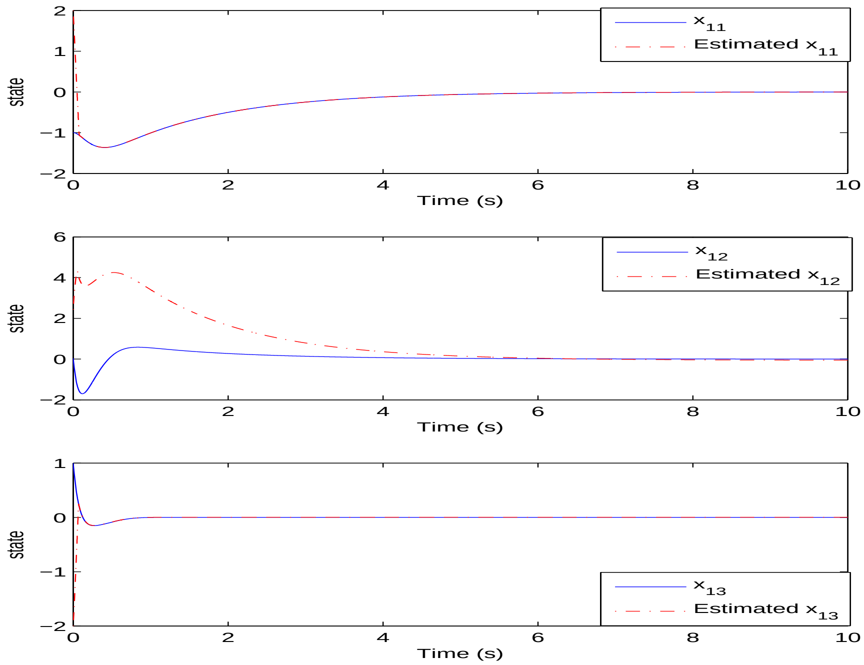

Fig. 1. *The time response of the 1st subsystem states* $x_1 = col\,(x_{11}, x_{12}, x_{13})$ *and their estimation* $\hat{x}_1 = col\,(\hat{x}_{11}, \hat{x}_{12}, \hat{x}_{13})$

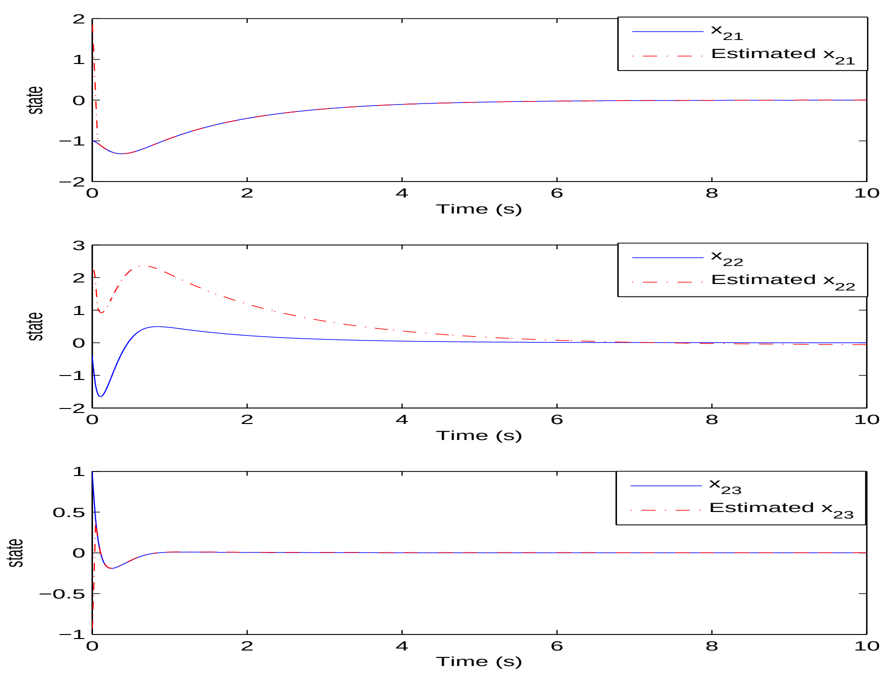

Fig. 2. *The time response of the 2nd subsystem states* $x_2 = col\,(x_{21}, x_{22}, x_{23})$ *and their estimation* $\hat{x}_2 = col\,(\hat{x}_{21}, \hat{x}_{22}, \hat{x}_{23})$

6. References

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