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Variable structure observer for a class of nonlinear large-scale interconnected systems with uncertainties*

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***Abstract*— In this paper, a variable structure observer design approach is proposed for a class of nonlinear, large-scale interconnected systems in the presence of unstructured uncertainty. The modern geometric approach is exploited to explore the system structure and a transformation is developed to facilitate observer design. Using the Lyapunov direct method, a robust asymptotic observer is presented which exploits the internal dynamic structure of the system as well as the structure of the uncertainties. The bounds on the uncertainties are nonlinear and are employed in the observer design to reject the effect of the uncertainties. A numerical example is presented to illustrate the approach and the simulation results show that the proposed approach is effective.**

I. Introduction

The development of advanced technologies has produced corresponding growth in physical systems. Such systems are frequently called system of systems or large-scale systems and can frequently be expressed by sets of lower-order ordinary differential equations which are linked through interconnections. Such models are typically called large scale interconnected systems (see, e.g. [2], [8], [17], [18],). Large-scale interconnected systems have been studied since the 1970s [11]. Early work focussed on linear systems. Subsequent results used decentralised control frameworks for nonlinear large scale interconnected systems. In much of these work, it is assumed that all the system state variables are available for use by the controller [2], [8]. However, this may be limiting in practice as only a subset of state variables may be available/measureable. It becomes of interest to establish observers to estimate the system states and then use the estimated states to replace the true system states in order to implement state feedback decentralised controllers. It is also the case that observer design has been heavily applied for fault detection and isolation [10], [15]. This further motivates the study of observer design for nonlinear large scale interconnected systems.

Sliding mode techniques have been used to design observers for nonlinear interconnected power systems in [1]. An adaptive observer is designed for a class of interconnected systems in [14] in which it is required that the isolated nominal subsystems are linear. Observer schemes for interconnected systems are proposed in [7], [10], [12], [15] where the obtained results are unavoidably conservative as it is required that the designed observer can be used for certain fault detection and isolation problems. Robust observer design is considered in [9] for a class of linear large scale dynamical systems where it is required that the interconnections satisfy quadratic constraints. In [13] a new decentralized control scheme which uses estimated states from a decentralised observer within a feedback controller is proposed. This uses a design framework based on linear matrix inequalities and is thus applicable for linear systems. A robust observer for nonlinear interconnected systems based on a constrained Lyapunov equation has been developed [16]. A PI observer is utilized for nonlinear interconnected systems for disturbance attenuation in [5] and interconnected nonlinear dynamical systems are considered in [3] where the authors combine the advantages of input-to-state dynamical stability and use reduced order observers to obtain quantitative information about the state estimation error. This work does not, however, consider uncertainties. It should be noted that in all the existing work relating to observer design for large scale interconnected systems, it is required that either the isolated subsystems are linear or the interconnections are linear. Moreover, most of the designed observers are used for special purposes such as fault detection and thus they impose specific requirements on the class of interconnected systems considered.

In this paper, a class of nonlinear interconnected systems with disturbances are considered. Fundamentally the work in [4] is extended for large scale systems. A robust variable structure observer is established based on a simplified system structure by using Lyapunov analysis. The structure of the internal dynamics and the uncertainty bounds are fully used in the observer design. These bounds are allowed to have a general nonlinear form. The difference between the output of the actual plant and the output of the observer is zero, and the observer states converge to the system states even if the system is not stable. A simulation example shows that the proposed approach is effective.

II. System Description and Problem Statement

Consider the nonlinear interconnected systems

$$\dot{x}_i(t) = f_i(x_i) + g_i(x_i)u_i + \Delta f_i(x_i) + \sum_{\substack{j=1\\ j\neq i}}^{N} D_{ij}(x_j) \quad (1)$$

$$y_i(t) = h_i(x_i), \quad i = 1, 2, \cdots, N \quad (2)$$

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where $x_i \in \Omega_i \subset R^{n_i}$ (Ω_i is a neighbourhood of the origin), $y_i \in R$ and $u_i \in U_i \subset R$ (U_i is an admissible control set) are the state, output and input of the i-th subsystem respectively, $f_i(x_i) \in R^{n_i}$ and $g_i(x_i) \in R^{n_i}$ are smooth vector fields defined in the domain Ω_i, and $h_i(x_i) \in R$ are smooth in the domain Ω_i for $i = 1, 2, \cdots, N$. The term $\Delta f_i(x_i)$ includes all the uncertainties experienced by the i-th subsystem. The term $\sum_{\substack{j=1 \\ j\neq i}}^{N} D_{ij}(x_j)$ is the nonlinear interconnection of the i-th subsystem.

Definition 1 The systems

$$\begin{aligned} \dot{x_i}(t) &= f_i(x_i) + g_i(x_i)u_i + \Delta f_i(x_i) && (3) \\ y_i(t) &= h_i(x_i) && (4) \end{aligned}$$

for $i = 1, 2, \cdots, N$ are called the isolated subsystems of the systems $(1) - (2)$.

$$\begin{aligned} \dot{x_i}(t) &= f_i(x_i) + g_i(x_i)u_i && (5) \\ y_i(t) &= h_i(x_i), \quad i = 1, 2, \cdots, N && (6) \end{aligned}$$

are called the nominal isolated subsytems of the systems $(1) - (2)$.

In this paper, under the assumption that the isolated subsystems $(5) - (6)$ have uniform relative degree r_i in the considered domain Ω_i, the interconnected systems $(1) - (2)$ are to be analysed. The objective is to explore the system structure based on a geometric transformation to design a robust asymptotic observer for the interconnected system $(1) - (2)$.

III. System Analysis and Assumptions

In this section, some assumptions are introduced to facilitate the observer design.

Assumption 1. The nominal isolated subsystem $(5) - (6)$ has uniform relative degree r_i in domain $x_i \in \Omega_i$ for $i = 1, 2, \cdots, N$.

Under Assumption 1, it follows from [6] that there exists a coordinate transformation

$$T_i : x_i \to \mathrm{col}(\zeta_i, \eta_i) \tag{7}$$

where

$$\zeta_i = \begin{bmatrix} \zeta_{i1} \\ \zeta_{i2} \\ \vdots \\ \zeta_{ir_i} \end{bmatrix} = \begin{bmatrix} h_i(x_i) \\ L_f h_i(x_i) \\ \vdots \\ L_f^{r_i-1} h_i(x_i) \end{bmatrix} \in R^{r_i} \tag{8}$$

for $i = 1, 2, \cdots, N$, and $\eta_i \in R^{n_i - r_i}$ is defined by

$$\eta_i = \begin{bmatrix} \eta_{i1} \\ \eta_{i2} \\ \vdots \\ \eta_{n_i-r_i} \end{bmatrix} = \begin{bmatrix} \phi_{i(r_i+1)}(x_i) \\ \phi_{i(r_i+2)}(x_i) \\ \vdots \\ \phi_{in_i}(x_i) \end{bmatrix}, \tag{9}$$

for $i = 1, 2, \cdots, N$. The functions $\phi_{i(r_i+1)}(x_i)$, $\phi_{i(r_i+2)}(x_i)$, $\cdots, \phi_{in_i}(x_i)$ can be obtained by solving the following partial differential equations:

$$L_{g_i}\phi_i(x_i) = 0, \quad x_i \in \Omega_i, \quad i = 1, 2, \cdots, N. \tag{10}$$

From [6], it follows that in the new coordinate system (ζ_i, η_i), the nominal isolated subsystem $(5) - (6)$ is equivalent to following form

$$\begin{aligned} \dot{\zeta}_i &= A_i\zeta_i + \beta_i(\zeta_i, \eta_i, u_i) && (11) \\ \dot{\eta}_i &= q_i(\zeta_i, \eta_i) && (12) \\ y_i &= C_i\zeta_i && (13) \end{aligned}$$

where

$$\begin{aligned} A_i &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{r_i \times r_i} && (14) \\ C_i &= \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \in R^{1\times r_i} && (15) \\ \beta_i(\zeta_i, \eta_i, u_i) &= \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \bar{\beta}_i(\zeta_i, \eta_i, u_i) \end{bmatrix} && (16) \end{aligned}$$

where

$$\bar{\beta}_i(\zeta_i, \eta_i, u_i) = L_{f_i}^{r_i} h_i(T_i^{-1}(\zeta_i, \eta_i)) + L_{g_i} L_{f_i}^{r_i-1} h_i(T_i^{-1}(\zeta_i, \eta_i))u_i$$

It is clear to see that the pair (A_i, C_i) is observable. Thus, there exists a matrix L_i such that $A_i - L_iC_i$ is Hurwitz stable. This implies that, for any positive-definite matrix $Q_i \in R^{r_i\times r_i}$, the Lyapunov equation

$$(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i) = -Q_i \tag{17}$$

has a unique positive-definite solution $P_i \in R^{r_i \times r_i}$ for $i = 1, 2, \cdots, N$.

Assumption 2. The uncertainty $\Delta f_i(x_i)$ in (1) satisfies

$$\frac{\partial T_i}{\partial x_i}\Delta f_i(x_i) = \begin{bmatrix} E_i\Delta\Psi(x_i) \\ 0 \end{bmatrix} \tag{18}$$

where $T_i(\cdot)$ is given in (7), $E_i \in R^{r_i\times r_i}$ is a constant matrix satisfying

$$E_i^T P_i = H_iC_i \tag{19}$$

with P_i satisfying (17), and $\|\Delta\Psi_i(x_i)\| \le \kappa_i(x_i)$, where $\kappa_i(x_i)$ is continuous and Lipschitz about x_i in the domain Ω_i for $i = 1, 2, \cdots, N$.

Remark 1. Denote the nonlinear uncertain term $\Delta\Psi_i(x_i)$ in (18) in the new coordinate frame (ζ_i, η_i) by $\Delta\Phi_i(\zeta_i, \eta_i)$ i.e.

$$\Delta\Phi_i(\zeta_i, \eta_i) = [\Delta\Psi_i(\zeta_i, \eta_i)]_{x_i = T_i^{-1}(\zeta_i,\eta_i)} \tag{20}$$

From Assumption 2, there exists a function $\rho_i(\zeta_i, \eta_i)$ such that

$$\|\Delta\Phi_i(\zeta_i, \eta_i)\| \le \rho_i(\zeta_i, \eta_i) \tag{21}$$

and $\rho_i(\zeta_i, \eta_i)$ satisfies the Lipschitz condition in $T_i(\Omega_i)$. Thus for any (ζ_i, η_i) and $(\hat{\zeta}_i, \hat{\eta}_i) \in T_i(\Omega_i)$,

$$\|\rho_i(\zeta_i, \eta_i) - \rho_i(\hat{\zeta}_i, \hat{\eta}_i)\| \le l_i^a\|\zeta_i - \hat{\zeta}_i\| + l_i^b\|\eta_i - \hat{\eta}_i\| \tag{22}$$

where both l_i^a and l_i^b are positive constants. Consider the interconnections $D_{ij}(x_j)$ in system (1). Then partition the term $\frac{\partial T_i}{\partial x_i}D_{ij}(x_j)$ as follows

$$\frac{\partial T_i}{\partial x_i}D_{ij}(x_j)\big|_{x_j=T_j^{-1}(\zeta_j,\eta_j)} = \begin{bmatrix} \Gamma_{ij}^a(\zeta_j,\eta_j) \\ \Gamma_{ij}^b(\zeta_j,\eta_j) \end{bmatrix} \tag{23}$$

where $\Gamma_{ij}^a(\zeta_j,\eta_j) \in R^{r_i}$, $\Gamma_{ij}^b(\zeta_j,\eta_j) \in R^{n_i-r_i}$ for $i = 1,2,\cdots,N$ and $i \neq j$.

Assumption 3. The nonlinear terms $\Gamma_{ij}^a(\zeta_j,\eta_j) \in R^{r_i}$, $\Gamma_{ij}^b(\zeta_j,\eta_j) \in R^{n_i-r_i}$ satisfy the Lipschitz condition in $T_i(\Omega_i)$.

Assumption 3 implies that there exist positive constants α_{ij}^a, α_{ij}^b, μ_{ij}^a and μ_{ij}^b such that

$$\begin{aligned} \|\Gamma_{ij}^a(\zeta_i,\eta_i) - \Gamma_{ij}^a(\hat\zeta_i,\hat\eta_i)\| &\le \alpha_{ij}^a\|\zeta_j - \hat\zeta_j\| \\ &\quad + \alpha_{ij}^b \parallel \eta_j - \hat\eta_j\| \end{aligned} \tag{24}$$

$$\begin{aligned} \|\Gamma_{ij}^b(\zeta_i,\eta_i) - \Gamma_{ij}^b(\hat\zeta_i,\hat\eta_i)\| &\le \mu_{ij}^a\|\zeta_j - \hat\zeta_j\| \\ &\quad + \mu_{ij}^b \parallel \eta_j - \hat\eta_j\| \end{aligned} \tag{25}$$

for $i = 1,2,\cdots,N$ and $i \neq j$. From (11) – (13) and the analysis above, it follows that under Assumption 2, in the new coordinate system (ζ_i,η_i) the system (1) – (2) can be described by

$$\begin{aligned} \dot\zeta_i &= A_i\zeta_i + \beta_i(\zeta_i,\eta_i,u_i) + E_i\Delta\Psi_i(\zeta_i,\eta_i) \\ &\quad + \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ij}^a(\zeta_j,\eta_j) \end{aligned} \tag{26}$$

$$\dot\eta_i = q_i(\zeta_i,\eta_i) + \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ij}^b(\zeta_j,\eta_j) \tag{27}$$

$$y_i = C_i\zeta_i \tag{28}$$

where A_i and C_i are given in (14) and (15) respectively, $\beta_i(\cdot)$ is defined in (16) and $\Gamma_{ij}^a(\cdot)$ and $\Gamma_{ij}^b(\cdot)$ are defined in (23).

Remark 2. Since $\beta_i(\cdot)$ is continuous in the domain $T_i(\Omega_i)$, it is straightforward to see that there exists a subset in a domain $T_i(\Omega_i)$ such that the function $\beta_i(\cdot)$ is Lipschitz in the subset

$$\begin{aligned} &\parallel \beta_i(\zeta_i,\eta_i,u_i) - \beta_i(\hat\zeta_i,\hat\eta_i,u_i) \parallel \le \\ &\qquad v_i^a(u_i) \parallel \zeta_i - \hat\zeta_i \parallel + v_i^b(u_i) \parallel \eta_i - \hat\eta_i \parallel \end{aligned} \tag{29}$$

where $v_i^a(u_i)$ and $v_i^b(u_i)$ are function of u_i for $i = 1,2,\cdots,N$.

Assumption 4. The function $q_i(\zeta_i,\eta_i)$ in equation (27) has the following decomposition

$$q_i(\zeta_i,\eta_i) = M_i\eta_i + \theta_i(\zeta_i,\eta_i) \tag{30}$$

where $M_i \in R^{(n_i-r_i)\times(n_i-r_i)}$ is a Hurwitz matrix and $\theta_i(\zeta_i,\eta_i)$ are Lipschitz in domain $T_i(\Omega_i)$.

Under Assumption 4, there exist constants τ_i^a and τ_i^b such that.

$$\begin{aligned} \parallel \theta_i(\zeta_i,\eta_i) - \theta_i(\hat\zeta_i,\hat\eta_i) \parallel &\le \tau_i^a \parallel \zeta_i - \hat\zeta_i \parallel \\ &\quad + \tau_i^b \parallel \eta_i - \hat\eta_i \parallel \end{aligned} \tag{31}$$

where $i = 1,2,\cdots,N$. Further, from the fact that M_i is Hurwitz stable for $\Lambda_i > 0$, the following Lyapunov equation has a unique solution $\Pi_i > 0$

$$M_i^T\Pi_i + \Pi_iM_i = -\Lambda_i \tag{32}$$

IV. Nonlinear Observer Synthesis

In this section an observer is designed for the transformed systems (26) – (28) and then an observer for the interconnected systems (26) – (28) is synthesised. For system (26) – (28), construct dynamical systems

$$\begin{aligned} \dot{\hat\zeta}_i &= A_i\hat\zeta_i + L_i(y_i - C_i\hat\zeta_i) + \beta_i(\hat\zeta_i,\hat\eta_i,u_i) \\ &\quad + K_i(y,\hat\zeta_i,\hat\eta_i) + \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ij}^a(\hat\zeta_j,\hat\eta_j) \end{aligned} \tag{33}$$

$$\dot{\hat\eta}_i = M_i\hat\eta_i + \theta_i(\hat\zeta_i,\hat\eta_i) + \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ij}^b(\hat\zeta_j,\hat\eta_j) \tag{34}$$

where the term $K_i(y_i,\hat\zeta_i,\hat\eta_i)$ is defined by

$$K_i(y_i,\hat\zeta_i,\hat\eta_i) = \begin{cases} \frac{P_i^{-1}C_i^T(y_i - C_i\hat\zeta_i)}{\|y_i - C_i\hat\zeta_i\|} \parallel H_i \parallel \rho_i(\hat\zeta_i,\hat\eta_i), & y_i - C_i\hat\zeta_i \neq 0 \\ 0, & y_i - C_i\hat\zeta_i = 0 \end{cases} \tag{35}$$

where P_i and H_i satisfy (17) and (19) respectively. It should be pointed out that the structure of the proposed observer in (33) – (34) is variable due to the term defined in (35). Therefore, it is called variable structure observer throughout this paper. The following results are ready to be presented.

Theorem 1. Suppose Assumptions 1 – 4 hold. Then, the dynamical system (33)–(34) is a robust asymptotic observer of system (26)–(28), if the function matrix $W^T(\cdot)+W(\cdot)$ is positive definite in the domain Ω, where the matrix $W(\cdot) = [w_{ij}(\cdot)]_{2N\times 2N}$, and its entries $w_{ij}(\cdot)$ are defined by

$$w_{ij} = \begin{cases} \lambda_{\min}(Q_i) - 2\lambda_{\max}(P_i)v_i^a - 2l_i^a\|C_i\|\|H_i\|, \\ \qquad i = j, 1 \le i \le N, \\ -2\lambda_{\max}(P_i)\alpha_{ij}^a, \\ \qquad i \neq j, 1 \le i \le N, 1 \le j \le N \\ \lambda_{\min}(\Lambda_{i-N}) - 2\lambda_{\max}(\Pi_{i-N})\tau_{i-N}^b, \\ \qquad i = j, N+1 \le i \le 2N, \\ -2\lambda_{\max}(\Pi_{(i-N)})\mu_{(i-N)(j-N)}^b, \\ \qquad i \neq j, N+1 \le i \le 2N, N+1 \le j \le 2N \\ -2[\lambda_{\max}(P_i)v_i^b + l_i^b\|C_i\|\|H_i\| + \lambda_{\max}(\Pi_i)\tau_i^a], \\ \qquad j - i = N, 1 \le i \le N, N+1 \le j \le 2N \\ -2\lambda_{\max}(P_i)\alpha_{i(j-N)}^b, \\ \qquad j - i \neq N, 1 \le i \le N, N+1 \le j \le 2N \\ 0, \; i - j = N, N+1 \le i \le 2N, 1 \le j \le N \\ -2\lambda_{\max}(\Pi_{i-N})\mu_{(i-N)j}^a, \\ \qquad i - j \neq N, N+1 \le i \le 2N, 1 \le j \le N \end{cases}$$

Proof. Let $e_{\zeta_i}=\zeta_i-\hat{\zeta}_i$ and $e_{\eta_i}=\eta_i-\hat{\eta}_i$ for $i=1,2,\cdots,N$. Compare systems $(26)-(28)$ and $(33)-(34)$. It follows that the error dynamical systems are described by

$$\begin{aligned}\dot{e}_{\zeta_i} &= (A_i-L_iC_i)e_{\zeta_i}+\beta_i(\zeta_i,\eta_i,u_i)-\beta_i(\hat{\zeta}_i,\hat{\eta}_i,u_i)\\ &\quad +E_i\Delta\Psi_i(\zeta_i,\eta_i)-K_i(y_i,\hat{\zeta}_i,\hat{\eta}_i)\\ &\quad +\sum_{\substack{j=1\\ j\neq i}}^{N}\Gamma_{ij}^{a}(\zeta_j,\eta_j)-\sum_{\substack{j=1\\ j\neq i}}^{N}\Gamma_{ij}^{a}(\hat{\zeta}_j,\hat{\eta}_j)\end{aligned}\tag{36}$$

$$\begin{aligned}\dot{e}_{\eta_i} &= M_ie_{\eta_i}+\theta_i(\zeta_i,\eta_i)-\theta_i(\hat{\zeta}_i,\hat{\eta}_i)\\ &\quad +\sum_{\substack{j=1\\ j\neq i}}^{N}\Gamma_{ij}^{b}(\zeta_j,\eta_j)-\sum_{\substack{j=1\\ j\neq i}}^{N}\Gamma_{ij}^{b}(\hat{\zeta}_j,\hat{\eta}_j)\end{aligned}\tag{37}$$

Now, for the system (36) and (37) consider the following candidate Lyapunov function

$$V=\sum_{i=1}^{N}e_{\zeta_i}^{T}P_ie_{\zeta_i}+\sum_{i=1}^{N}e_{\eta_i}^{T}\Pi_ie_{\eta_i}\tag{38}$$

Then, the time derivative of the candidate Lyapunov function can be described by

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}\Big\{(\dot{e}_{\zeta_i}^{T}P_ie_{\zeta_i}+e_{\zeta_i}^{T}P_i\dot{e_{\zeta_i}})\\ &\quad +(\dot{e}_{\eta_i}^{T}\Pi_ie_{\eta_i}+e_{\eta_i}^{T}\Pi_i\dot{e}_{\eta_i})\Big\}\end{aligned}\tag{39}$$

Substituting both $\dot{e}_{\zeta_i}$ in (36) and $\dot{e}_{\eta_i}$ in (37) into equation (39), it follows by direct computation that the time derivative of the function V in (38) can be described by

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}\Big\{e_{\zeta_i}^{T}[(A_i-L_iC_i)^{T}P_i+P_i(A_i-L_iC_i)]e_{\zeta_i}\\ &\quad +2e_{\zeta_i}^{T}P_i[\beta_i(\zeta_i,\eta_i,u_i)-\beta_i(\hat{\zeta}_i,\hat{\eta}_i,u_i)]\\ &\quad +2[e_{\zeta_i}^{T}P_iE_i\Delta\Psi_i(\zeta_i,\eta_i)-e_{\zeta_i}^{T}P_iK_i(y_i,\hat{\zeta}_i,\hat{\eta}_i)]\\ &\quad +2e_{\zeta_i}^{T}P_i\sum_{\substack{j=1\\ j\neq i}}^{N}[\Gamma_{ij}^{a}(\zeta_j,\eta_j)-\Gamma_{ij}^{a}(\hat{\zeta}_j,\hat{\eta}_j)]\\ &\quad +e_{\eta_i}^{T}(M_i^{T}\Pi_i+\Pi_iM_i)e_{\eta_i}+2e_{\eta_i}^{T}\Pi_i\\ &\quad \times[\theta_i(\zeta_i,\eta_i)-\theta_i(\hat{\zeta}_i,\hat{\eta}_i)]\\ &\quad +2e_{\eta_i}^{T}\Pi_i\sum_{\substack{j=1\\ j\neq i}}^{N}[\Gamma_{ij}^{b}(\zeta_j,\eta_j)-\Gamma_{ij}^{b}(\hat{\zeta}_j,\hat{\eta}_j)]\Big\}\end{aligned}\tag{40}$$

From (19),(21),(22) and (35)

(i) If $y_i-C_i\hat{\zeta}_i=0$, then

$$\begin{aligned}&e_{\zeta_i}^{T}P_iE_i\Delta\Phi_i(\zeta_i,\eta_i)-e_{\zeta_i}^{T}P_iK_i(y_i,\hat{\zeta}_i,\hat{\eta}_i)\\ &\qquad = e_{\zeta_i}^{T}C_i^{T}H_i^{T}\Delta\Phi_i(\zeta_i,\eta_i)\\ &\qquad = [H_i(y_i-C_i\hat{\zeta})\Delta\Phi_i(\zeta_i,\eta_i)]=0\end{aligned}$$

(ii) If $y_i-C_i\hat{\zeta}_i\neq 0$, then

$$\begin{aligned}&e_{\zeta_i}^{T}P_iE_i\Delta\Phi_i(\zeta_i,\eta_i)-e_{\zeta_i}^{T}P_iK_i(y_i,\hat{\zeta}_i,\hat{\eta}_i)\\ &= e_{\zeta_i}^{T}C_i^{T}H_i^{T}\Delta\Phi_i(\zeta_i,\eta_i)-e_{\zeta_i}^{T}P_i\frac{P_i^{-1}C_i^{T}(y_i-C_i\hat{\zeta}_i)}{\|y_i-C_i\hat{\zeta}_i\|}\\ &\quad\cdot\|\ H\ \|\ \rho_i(\hat{\zeta}_i,\hat{\eta}_i)\\ &= \|e_{\zeta_i}^{T}C_i^{T}\|\|H_i\|\rho_i(\zeta_i,\eta_i)-\frac{e_{\zeta_i}^{T}C_i^{T}C_ie_{\zeta_i}}{\|C_ie_{\zeta_i}\|}\|H_i\|\rho_i(\hat{\zeta}_i,\hat{\eta}_i)\\ &= \|e_{\zeta_i}^{T}C_i^{T}\|\|H_i\|\{\rho_i(\zeta_i,\eta_i)-\rho_i(\hat{\zeta}_i,\hat{\eta}_i)\}\\ &\leq \|e_{\zeta_i}^{T}C_i^{T}\|\|H_i\|\{l_i^{a}\|\zeta_i-\hat{\zeta}_i\|+l_i^{b}\|\eta_i-\hat{\eta}_i\|\}\end{aligned}$$

Then, from (i) and (ii), it follows that

$$\begin{aligned}&e_{\zeta_i}^{T}P_iE_i\Delta\Phi_i(\zeta_i,\eta_i)-e_{\zeta_i}^{T}P_iK_i(y_i,\hat{\zeta}_i,\hat{\eta}_i)\\ &\qquad\leq\|e_{\zeta_i}C_i\|\|H_i\|\{l_i^{a}\|e_{\zeta_i}\|+l_i^{b}\|e_{\eta_i}\|\}\end{aligned}\tag{41}$$

Substituting (24), (25), (29), (31), and (41) into (40) yields

$$\begin{aligned}\dot{V} &\leq -\sum_{i=1}^{N}\Big\{\big[\lambda_{\min}(Q_i)-2\lambda_{\max}(P_i)v_i^{a}-2l_i^{a}\|C_i\|\|H_i\|\big]\\ &\quad\cdot\|e_{\zeta_i}\|^{2}-\big[\sum_{\substack{j=1\\ j\neq i}}^{N}2\lambda_{\max}(P_i)\alpha_{ij}^{a}\big]\|e_{\zeta_i}\|\|e_{\zeta_j}\|\\ &\quad -\big[2\lambda_{\max}(P_i)v_i^{b}+2l_i^{b}\|C_i\|\|H_i\|\\ &\quad +2\lambda_{\max}(\Pi_i)\tau_i^{a}\big]\|e_{\zeta_i}\|\|e_{\eta_i}\|\big]-2\sum_{\substack{j=1\\ j\neq i}}^{N}\lambda_{\max}(P_i)\alpha_{ij}^{b}\\ &\quad\cdot\|e_{\zeta_i}\|\|e_{\eta_j}\|]-\sum_{\substack{j=1\\ j\neq i}}^{N}2\lambda_{\max}(\Pi_i)\mu_{ij}^{a}\|e_{\zeta_j}\|\|e_{\eta_i}\|\\ &\quad +\big[\lambda_{\min}(\Lambda_i)-2\lambda_{\max}(\Pi_i)\tau_i^{b}\big]\|e_{\eta_i}\|^{2}\\ &\quad -\big[\sum_{\substack{j=1\\ j\neq i}}^{N}2\lambda_{\max}(\Pi_i)\mu_{ij}^{b}\big]\|e_{\eta_i}\|\|e_{\eta_j}\|\Big\}\end{aligned}$$

Then, from the definition of the matrix $W(\cdot)$ and the inequality above, it follows that

$$\dot{V}\leq-\frac{1}{2}X^{T}[W^{T}(\cdot)+W(\cdot)]X$$

where $X=[\|e_{\zeta_1}\|,\|e_{\zeta_2}\|,\cdots,\|e_{\zeta_N}\|,\|e_{\eta_1}\|,\|e_{\eta_2}\|,\cdots,\|e_{\eta_N}\|]^{T}$. Since $W^{T}(\cdot)+W(\cdot)$ is positive definite in the domain $T(\Omega_1)\times U_1\times T(\Omega_2)\times U_2\times\cdots\times T(\Omega_N)\times U_N$, it is clear that $\dot{V}|_{(33)-(34)}$ is negative definite. Therefore

$$\lim_{t\to\infty}\|\zeta_i(t)-\hat{\zeta}_i(t)\|=0\ \text{ and }\ \lim_{t\to\infty}\|\eta_i(t)-\hat{\eta}_i(t)\|=0\tag{42}$$

Hence, the conclusion follows. △

Assume that $\frac{\partial T_i(\zeta_i,\eta_i)}{\partial(\zeta_i,\eta_i)}$ is bounded in $T_i(\Omega_i)$ for $i=1,2,\cdots,N$. There exists a positive constant γ_i such that

$$\left\|\frac{\partial T_i(\zeta_i,\eta_i)}{\partial(\zeta_i,\eta_i)}\right\|\leq\gamma_i,\ (\zeta_i,\eta_i)\in T_i(\Omega_i),\ i=1,2,\cdots,N$$

Define $\hat{x}_i=T_i^{-1}(\hat{\zeta}_i,\hat{\eta}_i),\ i=1,2,\cdots,N$ Then,

$$\begin{aligned}\|x_i-\hat{x}_i\| &= \|T_i^{-1}(\zeta_i,\eta_i)-T_i^{-1}(\hat{\zeta}_i,\hat{\eta}_i)\|\\ &\leq \gamma_i(\|\zeta_i-\hat{\zeta}_i\|+\|\eta_i-\hat{\eta}_i\|)\end{aligned}\tag{43}$$

Then, from (42) and (43), it follows that

$$\lim_{t\to\infty} \|x_i(t) - \hat{x}_i(t)\| = 0$$

This implies that $\hat{x}_i$ is an estimate of x_i for $i = 1, 2, \cdots, N$.

Remark 3 From the analysis above, it is clear to see that, in this paper, it is not required that either the nominal isolated subsystems or the interconnections are linearisable. The uncertainties are bounded by nonlinear functions and are fully used in the observer design in order to reject the effects of the uncertainties, and thus robustness is enhanced. The designed observer is an asymptotic observer and the developed results can be extended to the global case if the associated conditions hold globally.

V. Numerical example

Consider the nonlinear interconnected systems:

$$\begin{aligned}
\dot{x}_1 &= \underbrace{\begin{bmatrix} x_{12} \\ -0.1\sin x_{12} \\ -3x_{11}^2 - 3.25x_{13} - 2x_{12} \end{bmatrix}}_{f_1(x_1)} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{g_1(x_1)} u_1 \\
&\quad + \underbrace{\begin{bmatrix} \Delta\sigma_1 \\ 0.5\Delta\sigma_1 \\ -2\Delta\sigma_1 \end{bmatrix}}_{\Delta f_1(x_1)} + \underbrace{\begin{bmatrix} 0.2(x_{21}^2 + x_{22}) \\ 0 \\ 0.1\sin x_{21} \end{bmatrix}}_{D_{12}} \qquad (44)
\end{aligned}$$

$$y_1 = \underbrace{x_{11}}_{h_1(x_1)} \qquad (45)$$

$$\begin{aligned}
\dot{x}_2 &= \underbrace{\begin{bmatrix} -x_{21} \\ -x_{21}^2 - 3x_{22} + \cos(x_{21}^2 + x_{22}) - 1 \\ -2x_{23} + 0.2x_{21}^2 \end{bmatrix}}_{f_2(x_2)} \\
&\quad + \underbrace{\begin{bmatrix} 1 \\ -2x_{21} \\ 0 \end{bmatrix}}_{g_2(x_2)} u_2 + \underbrace{\begin{bmatrix} -\Delta\sigma_2 \\ 2x_{21}\Delta\sigma_2 \\ 0 \end{bmatrix}}_{\Delta f_2(x_2)} \\
&\quad + \underbrace{\begin{bmatrix} 0 \\ 0.1\sin(x_{13} + 2x_{11}) \\ 0 \end{bmatrix}}_{D_{21}} \qquad (46)
\end{aligned}$$

$$y_2 = \underbrace{x_{21}}_{h_2(x_2)} \qquad (47)$$

where $x_1 = \mathrm{col}(x_{11}, x_{12}, x_{13})$ and $x_2 = \mathrm{col}(x_{21}, x_{22}, x_{23})$, $h(x) = (h_1(x_1), h_2(x_2))^T$ and $u(t) = (u_1(t), u_2(t))^T$ are the system state, output and input respectively, D_{12} and D_{21} are interconnected systems, and $\Delta f_1(x_1), \Delta f_2(x_2)$ are the uncertainties experienced by the system which satisfy

$$\begin{aligned}
||\Delta f_1(x_1)|| &= 0.1|x_{13} + 2x_{11}|\sin^2 t \qquad (48) \\
||\Delta f_2(x_2)|| &= 0.1x_{21}^2|\cos t| \qquad (49)
\end{aligned}$$

The domain considered is

$$\Omega = \Big\{(x_{11},\ x_{12},\ x_{13},\ x_{21},\ x_{22},\ x_{23}), \ \Big|\ |x_{11}| < 3, \\ |x_{21}| \le 1.3,\ x_{11},\ x_{12},\ x_{13},\ x_{21},\ x_{22},\ x_{23} \in R\Big\} \quad (50)$$

By direct computation, it follows that the first subsystem has a uniform relative degree 2, and the second subsystem has a uniform relative degree 1. The corresponding transformations are obtained as follows:

$$T_1 : \begin{cases} \zeta_{11} = x_{11} \\ \zeta_{12} = x_{12} \\ \eta_1 = x_{13} + 2x_{11} \end{cases} \qquad T_2 : \begin{cases} \zeta_2 = x_{21} \\ \eta_{21} = x_{21}^2 + x_{22} \\ \eta_{22} = x_{23} \end{cases}$$

In the new coordinates, the system $(44) - (47)$ can be described by:

$$\begin{aligned}
\dot{\zeta}_1 &= \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A_1} \begin{bmatrix} \zeta_{11} \\ \zeta_{12} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -0.1\sin\zeta_{11} + u_1 \end{bmatrix}}_{\beta_1} \\
&\quad + \underbrace{\begin{bmatrix} \Delta\sigma_1(\zeta_1, \eta_1) \\ 0.5\Delta\sigma_1(\zeta_1, \eta_1) \end{bmatrix}}_{E_1\Delta\Psi(\zeta_1,\eta_1)} + \underbrace{\begin{bmatrix} 0.2\eta_{21} \\ 0 \end{bmatrix}}_{\Gamma_{12}^a} \qquad (51) \\
\dot{\eta}_1 &= \underbrace{-3.25\eta_1 + 0.25\zeta_{11}^2}_{q_1(\zeta_1,\eta_1)} + \underbrace{0.4\eta_{21} + 0.1\sin\zeta_2}_{\Gamma_{12}^b} \qquad (52) \\
y_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \zeta_{11} \\ \zeta_{12} \end{bmatrix} \qquad (53) \\
\dot{\zeta}_2 &= \underbrace{-}_{A_2}\zeta_2 + \underbrace{u_2}_{\beta_2} - \underbrace{\Delta\sigma_2(\zeta_2, \eta_2)}_{E_2\Delta\Psi(\zeta_2,\eta_2)} \qquad (54) \\
\dot{\eta}_2 &= \underbrace{\begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} \eta_{21} \\ \eta_{22} \end{bmatrix} + \begin{bmatrix} \cos\eta_{21} - 1 \\ 0.2\zeta_2^2 \end{bmatrix}}_{q_2(\zeta_2,\eta_2)} \\
&\quad + \underbrace{\begin{bmatrix} 0.1\sin\eta_1 \\ 0 \end{bmatrix}}_{\Gamma_{21}^b} \qquad (55) \\
y_2 &= \zeta_2
\end{aligned}$$

where $\zeta_1 = (\zeta_{11},\ \zeta_{12})^T$, $\eta_1 \in R$, $\zeta_2 \in R$, and $\eta_2 = (\eta_{21}, \eta_{22})^T$.

From $(48) - (49)$

$$\begin{aligned}
||\Delta\Psi_1(\zeta_1, \eta_1)|| &\le ||\Delta\sigma_1(\zeta_1, \eta_1)|| \\
||\Delta\Psi_2(\zeta_2, \eta_2)|| &\le ||\Delta\sigma_2(\zeta_2, \eta_2)|| \\
||\Delta\sigma_1(\zeta_1, \eta_1)|| &\le \underbrace{0.1|\eta_1|\sin^2 t}_{\rho_1(\cdot)} \\
||\Delta\sigma_2(\zeta_2, \eta_2)|| &\le \underbrace{0.1\zeta_2^2|\cos t|}_{\rho_2(\cdot)}
\end{aligned}$$

Then, for the first subsystem, choose

$$L_1 = \begin{bmatrix} 3 & 2 \end{bmatrix}^T, \qquad Q = I$$

It follows that the Lyapunov equation (17) has a unique solution:

$$P_1 = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

and the solution to equation (19) is $H_1 = 0.25$. As $M_1 = -3.25$, let $\Lambda_1 = 3.25$. Thus the solution of equation (32) is $\Pi_1 = 0.5$. Now, for the second subsystem, choose

$$L_2 = 0, \qquad Q_2 = 2$$

It follows that the Lyapunov equation (17) has a unique solution $P_2 = 1$ and the solution to equation(19) is $H_2 = -1$. As

$$M_2 = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$$

let

$$\Lambda_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{56}$$

Then,

$$\Pi_2 = \begin{bmatrix} 0.1667 & 0 \\ 0 & 0.25 \end{bmatrix}$$

By direct computation, it follows that the matrix $W^T + W$ is positive definite in the domain Ω defined in (50).
Thus, all the conditions of Theorem 1 are satisfied which implies that $(33) - (34)$ is an observer. Based on the parameters provided above, the observer $(33) - (34)$ has been well defined.
For simulation purposes, the controllers are chosen as:

$$u_1 = -\zeta_{11} - 2\zeta_{12} \quad \text{and} \quad u_2 = \cos\zeta_2 + 5$$

The simulation results in Figure 1 shows that the designed observer estimates the states of the interconnected system $x_1 = \text{col}(x_{11}, x_{12}, x_{13})$ and $x_2 = \text{col}\ (x_{21}, x_{22}, x_{23})$ in $(44) - (57)$ even though the system is not asymptotically stable.

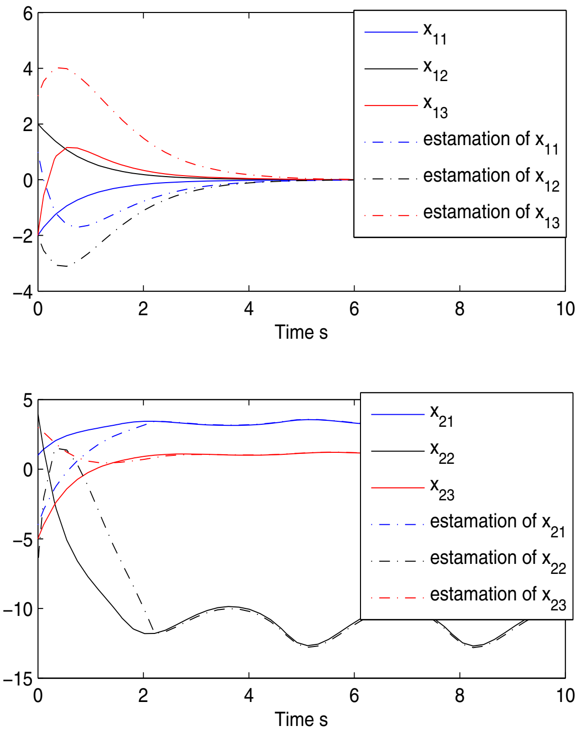

Fig. 1. The time response of the system states $x_1 = \text{col}\ (x_{11}, x_{12}, x_{13})$ and its estimation $\hat{x}_1 = \text{col}\ (\hat{x}_{11}, \hat{x}_{12}, \hat{x}_{13})$, the states $x_2 = \text{col}(x_{21}, x_{22}, x_{23})$ and its estimation $\hat{x}_2 = \text{col}\ (\hat{x}_{21}, \hat{x}_{22}, \hat{x}_{23})$

VI. Conclusions

In this paper, a class of nonlinear large scale interconnected systems with uniform relative degree have been considered. An asymptotic observer is developed for an uncertain system representation using a Lyapunov approach together with a geometric transformation which has been employed to exploit the system structure. It is not required that either the isolated nominal subsystems or the interconnections are linearizable. Robustness to uncertainties is enhanced by using the system structure and the structure of the uncertainties within the design framework.

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