Adverse Selection and Loss Coverage
in insurance markets

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2 Why do people buy insurance?

3 What drives demand for insurance?

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Background

Adverse selection:
If insurers cannot charge risk-differentiated premiums, then:
- higher risks buy more insurance, lower risks buy less insurance,
- raising the pooled price of insurance,
- lowering the demand for insurance,
usually portrayed as a bad outcome, both for insurers and for society.

In practice:
Policymakers often see merit in restricting insurance risk classification
- EU ban on using gender in insurance underwriting.
- Moratoria on the use of genetic test results in underwriting.

Question:
How can we reconcile theory with practice?
Motivating example

1. No adverse selection
   - Risk-differentiated premiums = 0.01
     - Weighted average premium = 0.016
     - Loss coverage = 50%
   - Risk-differentiated premiums = 0.04

2. Some adverse selection
   - Pooled premiums = 0.03
     - Weighted average premium = 0.03
     - Loss coverage = 56%
We ask:

- **Why** do people buy insurance?
- **What** drives demand for insurance?
- **How much** of population losses is compensated by insurance (with and without risk classification)?
- **Which** regime is most beneficial to society?

We find:

**Social welfare** is maximised by maximising **loss coverage**.

**Definition (Loss coverage)**

Expected population losses compensated by insurance.
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Why do people buy insurance?

Assumptions

Consider an individual with

- an initial wealth $W$,
- exposed to the risk of loss $L$,
- with probability $\mu$,
- utility of wealth $U(w)$, with $U'(w) > 0$ and $U''(w) < 0$,
- an opportunity to insure at premium rate $\pi$. 
Utility of wealth

Utility

$U(W)$

Utility of wealth loss coverage

$U(W-L)$

W-L

Wealth

W

Why do people buy insurance?
Expected utility: Without insurance

$U(W) - L \quad W$

$U(W - L)$

$W - L \quad W - \mu L \quad W$

$E[U(W)] = (1 - \mu)U(W) + \mu U(W - L)$
Expected utility: Insured at fair actuarial premium

Utility

\[ U(W) \]

\[ U(W - \mu L) \]

\[ U(W - L) \]

W - L

W - \mu L

W

Why do people buy insurance?

Expected utility: With insurance

Wealth

Fair premium

\[ \mu L \]
Maximum premium tolerated: $\pi_c$

Utility

$U(W)$
$U(W - \mu L)$
$U(W - \pi_c L)$
$U(W - L)$

Wealth

$W - L$
$W - \pi_c L$
$W - \mu L$
$W$

$(1 - \mu)U(W) + \mu U(W - L)$

Fair premium
$\mu L$

Maximum premium tolerated
$\pi_c L$

$U(W)$
$U(W - \pi_c L)$
$U(W - \mu L)$

Why do people buy insurance?

Maximum premium tolerated

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Loss Coverage

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Modelling demand for insurance

Simplest model:
If everybody has exactly the same $W, L, \mu$ and $U(\cdot)$, then:

- All will buy insurance if $\pi < \pi_c$.
- None will buy insurance if $\pi > \pi_c$.

**Reality:** Not all will buy insurance even at fair premium. *Why?*

**Heterogeneity:**
- Individuals are **homogeneous** in terms of underlying risk.
- But they can be **heterogeneous** in terms of risk-aversion.

**Source of Randomness:**
An individual’s utility function: $U_\gamma(w)$, where parameter $\gamma$ is drawn from random variable $\Gamma$ with distribution function $F_\Gamma(\gamma)$. 
Demand is a survival function

**Standardisation**

As certainty equivalent is invariant to positive affine transformations, we assume $U_\gamma(W) = 1$ and $U_\gamma(W - L) = 0$ for all $\gamma$.

**Condition for buying insurance:**

Given a premium $\pi$, an individual will buy insurance if:

$$
U_\gamma(W - \pi L) > (1 - \mu) U_\gamma(W) + \mu U_\gamma(W - L) = (1 - \mu).
$$

With insurance

Without insurance

**Demand as a survival function:**

Given a premium $\pi$, insurance demand, $d(\pi)$, is the survival function:

$$
d(\pi) = P[U_\Gamma(W - \pi L) > 1 - \mu].
$$
What drives demand for insurance?

Demand is a survival function

\[(1 - \mu)U(W) + \mu U(W - L)\]

\[U(W - L)\]

\[U(W)\]

\[W - L\]

\[W - \pi L\]

\[W\]

\[W\]

\[U(W - \pi L)\]

Density

\[d(\pi)\]

Utility

Wealth
Illustrative example: $W = L = 1$ and $U_\gamma(w) = w^\gamma$:

$$F_\Gamma(\gamma) = P[\Gamma \leq \gamma] = \begin{cases} 
0 & \text{if } \gamma < 0 \\
\tau \gamma^\lambda & \text{if } 0 \leq \gamma \leq (1/\tau)^{1/\lambda} \\
1 & \text{if } \gamma > (1/\tau)^{1/\lambda},
\end{cases}$$
Illustrative example: $W = L = 1$ and $U_\gamma(w) = w^\gamma$:

$$d(\pi) = P \left[ U_\Gamma (W - \pi L) > 1 - \mu \right] \approx \tau \left( \frac{\mu}{\pi} \right)^\lambda$$

$$\Rightarrow \epsilon(\pi) = \left| \frac{\partial d(\pi)}{d(\pi)} \right| = \lambda \quad \text{(constant elasticity} \Rightarrow \text{Iso-elastic demand}).$$

Iso-elastic demand for insurance

Fair-premium demand

$\lambda = 1$

$\lambda = 2$
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Risk classification

Assume all have same $W = L = 1$ and constant demand elasticity $\lambda$.

Risk-groups

Suppose the population can be divided into 2 risk-groups, with:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: $p_1$ and $p_2$;
- fair premium demand: $d_1(\mu_1) = \tau_1$ and $d_2(\mu_2) = \tau_2$, i.e.
  \[ d_i(\pi) = \tau_i \left( \frac{\pi}{\mu_i} \right)^{-\lambda}, \quad i = 1, 2; \]
- premiums offered: $\pi_1$ and $\pi_2$ (note that $\pi_1 = \pi_2$ is allowed).
Equilibrium

For a randomly chosen individual:

Define random variables:

\[ Q = I \text{ [Individual is insured]}; \]
\[ X = I \text{ [Individual incurs a loss]}; \]
\[ \Pi = \text{Premium offered to the individual}. \]

Equilibrium

Expected Premium:
\[ E[Q\Pi] = \sum_i p_i d_i(\pi_i) \pi_i. \]

Expected Claim:
\[ E[QX] = \sum_i p_i d_i(\pi_i) \mu_i. \]

Equilibrium:
\[ E[Q\Pi] = E[QX]. \]
Case 1: Risk-differentiated premium

Observations:
If risk-differentiated premiums are allowed,
- One possible equilibrium is achieved when \( \pi_i = \mu_i \).
- No losses for insurers.
- No (actuarial/economic) adverse selection.

Loss coverage (Population losses compensated by insurance):
Loss coverage: \( E[QX] = \sum_i p_i d_i(\mu_i) \mu_i = \sum_i p_i \tau_i \mu_i \).
**Case 2: Pooled premium**

**Equilibrium:**

If only a pooled premium $\pi_1 = \pi_2 = \pi_0$ is allowed,

\[
E[Q\Pi] = \sum_i p_i d_i(\pi_0) \pi_0;
\]

\[
E[QX] = \sum_i p_i d_i(\pi_0) \mu_i;
\]

\[
E[Q\Pi] = E[QX] \Rightarrow \pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^\lambda + \alpha_2 \mu_2^\lambda}, \text{ where } \alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}.
\]

**Observation:**

No losses for insurers! $\Rightarrow$ No (actuarial) adverse selection.
Case 2: Pooled premium

\[ \pi_0 = \alpha_1 \mu_1 + \alpha_2 \mu_2 \]

**Observation:**

Pooled equilibrium is greater than average premium charged under full risk classification: \( \pi_0 > \alpha_1 \mu_1 + \alpha_2 \mu_2 \) \( \Rightarrow \) (Economic) adverse selection.
Case 2: Pooled premium

Observation:
Aggregate demand (cover) is lower than under full risk classification ⇒ (Economic) adverse selection.
Loss coverage ratio

**Loss coverage under pooled premium:**

Loss coverage: \( E[QX] = \sum_i p_i d_i(\pi_0) \mu_i \).

**Loss coverage ratio:**

\[
C = \frac{\text{Loss coverage for pooled premium}}{\text{Loss coverage for risk-differentiated premium}},
\]

\[
= \frac{\sum_i p_i d_i(\pi_0) \mu_i}{\sum_i p_i d_i(\mu_i) \mu_i},
\]

\[
= \frac{1}{\pi_0^\lambda} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}.
\]
How much of population losses is compensated by insurance?

Loss coverage ratio

- $\lambda < 1 \Rightarrow$ Pooled premium $\succ$ Full risk classification.
- $\lambda > 1 \Rightarrow$ Pooled premium $\prec$ Full risk classification.
- Empirical evidence suggests $\lambda < 1$ in many insurance markets.
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Social welfare

Definition (Social welfare)

Social welfare, $S$, is the sum of all individuals’ expected (standardised) utilities:

$$S = E \left[ Q U_{i} (W - \Pi L) \right] + (1 - Q) \left[ (1 - X) U(W) + X U(W - L) \right],$$

$$= \sum_{i} \left[ d_{i}(\pi_{i}) U_{i}^{*} (W - \pi_{i} L) + (1 - d_{i}(\pi_{i})) \{(1 - \mu_{i}) U(W) + \mu_{i} U(W - L)\} \right] p_{i},$$

where $U_{i}^{*} (W - \pi_{i} L)$ is the expected utility of $i$-th risk-group’s insured population.

Linking social welfare to loss coverage

Assuming $L\pi_{i} \approx 0$, so that $U(W) = U_{i}^{*} (W - \pi_{i} L)$, gives:

$$S = \text{Positive multiplier} \times \sum_{i} p_{i} d_{i}(\pi_{i}) \mu_{i} + \text{Constant}.$$

Loss coverage provides a good proxy (which depends only on observable data) for social welfare (which depends on unobservable utilities).

**Result:** Maximising loss coverage maximises social welfare.
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Conclusions

Adverse selection need not always be adverse.

Restricting risk classification increases loss coverage if $\lambda < 1$.

Loss coverage is an observable proxy for social welfare.
References


