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Insurance Risk Classification

How much is socially optimal?

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Heriot-Watt University, March 2016
Contents

1. Introduction

2. Why do people buy insurance?

3. What drives demand for insurance?

4. How much of population losses is compensated by insurance?

5. Which regime is most beneficial to society?

6. Conclusions
Introduction

Background

Adverse selection:
If insurers cannot charge risk-differentiated premiums, then:
- higher risks buy more insurance, lower risks buy less insurance,
- raising the pooled price of insurance,
- lowering the demand for insurance,
usually portrayed as a bad outcome, both for insurers and for society.

In practice:
Policymakers often see merit in restricting insurance risk classification
- EU ban on using gender in insurance underwriting.
- Moratoria on the use of genetic test results in underwriting.

Question:
How can we reconcile theory with practice?
# Motivating example

## 1. No adverse selection

<table>
<thead>
<tr>
<th>Risk-differentiated premiums = 0.01</th>
<th>Weighted average premium</th>
<th>Loss coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Low-risk individuals" /></td>
<td>0.016</td>
<td>50%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Risk-differentiated premiums = 0.04</th>
<th><img src="image" alt="High-risk individuals" /></th>
</tr>
</thead>
</table>

## 2. Some adverse selection

<table>
<thead>
<tr>
<th>Pooled premiums = 0.03</th>
<th>Weighted average premium</th>
<th>Loss coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Low-risk pooled" /></td>
<td>0.03</td>
<td>56%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><img src="image" alt="High-risk pooled" /></th>
</tr>
</thead>
</table>
We ask:

- **Why** do people buy insurance?
- **What** drives demand for insurance?
- **How much** of population losses is compensated by insurance (with and without risk classification)?
- **Which** regime is most beneficial to society?

**Definition (Loss coverage)**

Expected population losses compensated by insurance.

**We find:**

Social welfare is maximised by maximising loss coverage.
## Contents

1. Introduction

2. Why do people buy insurance?

3. What drives demand for insurance?

4. How much of population losses is compensated by insurance?

5. Which regime is most beneficial to society?

6. Conclusions
Assumptions

Consider an individual with

- an initial wealth $W$,
- exposed to the risk of loss $L$,
- with probability $\mu$,
- utility of wealth $U(w)$, with $U'(w) > 0$ and $U''(w) < 0$,
- an opportunity to insure at premium rate $\pi$. 
Utility of wealth

Utility

$U(W)$

$U(W-L)$

Wealth

$W-L$

$W$
Expected utility: Without insurance

\[ U(W) - \mu L \]

\[ U(W - L) \]

Expected utility:

\[ (1 - \mu)U(W) + \mu U(W - L) \]
Expected utility: Insured at fair actuarial premium

\[ U(W) = \begin{cases} U(W - \mu L) & \text{for insured} \\ U(W) & \text{for uninsured} \end{cases} \]

Fair premium: \( \mu L \)
Why do people buy insurance?

Maximum premium tolerated: \( \pi_C \)

Utility

(1 - \( \mu \))\( U(W) \) + \( \mu U(W - L) \)

\( U(W - \pi_c L) \)

\( U(W - \mu L) \)

\( U(W - L) \)

Wealth

\( W - L \)

\( W - \pi_c L \)

\( W - \mu L \)

\( W \)

Fair premium

\( \mu L \)

\( \pi_c L \)

Maximum premium tolerated
Contents

1 Introduction

2 Why do people buy insurance?

3 What drives demand for insurance?

4 How much of population losses is compensated by insurance?

5 Which regime is most beneficial to society?

6 Conclusions
Modelling demand for insurance

Simplest model:

If everybody has exactly the same $W$, $L$, $\mu$ and $U(\cdot)$, then:

- All will buy insurance if $\pi < \pi_c$.
- None will buy insurance if $\pi > \pi_c$.

**Reality:** Not all will buy insurance even at fair premium. *Why?*

Heterogeneity:

- Individuals are **homogeneous** in terms of underlying risk.
- But they can be **heterogeneous** in terms of risk-aversion.

Source of Randomness:

An individual’s utility function: $U_\gamma(w)$, where parameter $\gamma$ is drawn from random variable $\Gamma$ with distribution function $F_\Gamma(\gamma)$. 
Demand is a survival function

**Standardisation**

As certainty equivalent is invariant to positive affine transformations, we assume $U_\gamma(W) = 1$ and $U_\gamma(W - L) = 0$ for all $\gamma$.

**Condition for buying insurance:**

Given a premium $\pi$, an individual will buy insurance if:

$$U_\gamma(W - \pi L) > (1 - \mu) U_\gamma(W) + \mu U_\gamma(W - L) = (1 - \mu).$$

With insurance

Without insurance

**Demand as a survival function:**

Given a premium $\pi$, insurance demand, $d(\pi)$, is the survival function:

$$d(\pi) = P[U_\Gamma(W - \pi L) > 1 - \mu].$$
What drives demand for insurance?

Demand is a survival function

\[
(1 - \mu)U(W) + \mu U(W - L) + \pi U(W - L)
\]

Density

\[ U(W - \pi L) \]

Utility

U(W)

U(W-L)

W-L

W - \pi L

W

Wealth

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Illustrative example: $W = L = 1$ and $U_\gamma(w) = w^\gamma$:

$$F_\Gamma(\gamma) = P[\Gamma \leq \gamma] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau \gamma^\lambda & \text{if } 0 \leq \gamma \leq (1/\tau)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau)^{1/\lambda}, \end{cases}$$
Illustrative example: $W = L = 1$ and $U_\gamma(w) = w^\gamma$:

\[ d(\pi) = P \left[ U_\Gamma (W - \pi L) > 1 - \mu \right] \approx \tau \left( \frac{\mu}{\pi} \right)^\lambda \]

\[ \Rightarrow \epsilon(\pi) = \left| \frac{\partial d(\pi)}{\partial \pi} \right| = \lambda \quad \text{(constant elasticity} \Rightarrow \text{Iso-elastic demand)} \]
Contents

1. Introduction
2. Why do people buy insurance?
3. What drives demand for insurance?
4. How much of population losses is compensated by insurance?
5. Which regime is most beneficial to society?
6. Conclusions
Risk classification

Assume all have same $W = L = 1$ and constant demand elasticity $\lambda$.

Risk-groups

Suppose the population can be divided into 2 risk-groups, with:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: $p_1$ and $p_2$;
- fair premium demand: $d_1(\mu_1) = \tau_1$ and $d_2(\mu_2) = \tau_2$, i.e.

$$d_i(\pi) = \tau_i \left( \frac{\pi}{\mu_i} \right)^{-\lambda}, \quad i = 1, 2;$$

- premiums offered: $\pi_1$ and $\pi_2$ (note that $\pi_1 = \pi_2$ is allowed).
Equilibrium

For a randomly chosen individual:

Define random variables:

\[ Q = I \ [ \text{Individual is insured} ] ; \]
\[ X = I \ [ \text{Individual incurs a loss} ] ; \]
\[ \Pi = \text{Premium offered to the individual}. \]

Equilibrium

Expected Premium:
\[ E[Q \Pi] = \sum p_i d_i(\pi_i) \pi_i. \]

Expected Claim:
\[ E[QX] = \sum p_i d_i(\pi_i) \mu_i. \]

Equilibrium:
\[ E[Q \Pi] = E[QX]. \]
Case 1: Risk-differentiated premium

Observations:

If risk-differentiated premiums are allowed,

- One possible equilibrium is achieved when $\pi_i = \mu_i$.
- No losses for insurers.
- No (actuarial/economic) adverse selection.

Loss coverage (Population losses compensated by insurance):

Loss coverage: $E[QX] = \sum_i p_i d_i(\mu_i) \mu_i = \sum_i p_i \tau_i \mu_i.$
Case 2: Pooled premium

Equilibrium:
If only a pooled premium $\pi_1 = \pi_2 = \pi_0$ is allowed,

\[
E[Q\Pi] = \sum_i p_i d_i(\pi_0) \pi_0; \\
E[QX] = \sum_i p_i d_i(\pi_0) \mu_i; \\
E[Q\Pi] = E[QX] \Rightarrow \pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^\lambda + \alpha_2 \mu_2^\lambda}, \text{ where } \alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}.
\]

Observation:
No losses for insurers! $\Rightarrow$ No (actuarial) adverse selection.
Case 2: Pooled premium

Observation:

Pooled equilibrium is greater than average premium charged under full risk classification: $\pi_0 > \alpha_1 \mu_1 + \alpha_2 \mu_2 \Rightarrow \text{(Economic) adverse selection.}$
**Case 2: Pooled premium**

![Graph showing aggregate demand (cover) vs. demand elasticity (λ). The graph indicates a downward trend as λ increases, suggesting reduced aggregate demand compared to full risk classification.](image)

**Observation:**
Aggregate demand (cover) is lower than under full risk classification \( \Rightarrow \) (Economic) adverse selection.
Loss coverage ratio

Loss coverage under pooled premium:
Loss coverage: $E[QX] = \sum_i p_i d_i(\pi_0) \mu_i$.

Loss coverage ratio:

$$C = \frac{\text{Loss coverage for pooled premium}}{\text{Loss coverage for risk-differentiated premium}},$$

$$= \frac{\sum_i p_i d_i(\pi_0) \mu_i}{\sum_i p_i d_i(\mu_i) \mu_i},$$

$$= \frac{1}{\pi_0^{\lambda}} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}.$$
How much of population losses is compensated by insurance?

**Loss coverage ratio**

- $\lambda < 1 \Rightarrow$ Pooled premium $\succ$ Full risk classification.
- $\lambda > 1 \Rightarrow$ Pooled premium $\prec$ Full risk classification.
- Empirical evidence suggests $\lambda < 1$ in many insurance markets.
1. Introduction

2. Why do people buy insurance?

3. What drives demand for insurance?

4. How much of population losses is compensated by insurance?

5. Which regime is most beneficial to society?

6. Conclusions
Social welfare

**Definition (Social welfare)**

Social welfare, $S$, is the sum of all individuals’ expected (standardised) utilities:

$$S = E \left[ Q U_{\Gamma}(W - \Pi L) \right] + (1 - Q) \left[ (1 - X) U(W) + X U(W - L) \right],$$

where

$$S = \sum_{i} \left[ d_i(\pi_i) U_i^*(W - \pi_i L) + (1 - d_i(\pi_i)) \left\{ (1 - \mu_i) U(W) + \mu_i U(W - L) \right\} \right] p_i,$$

where $U_i^*(W - \pi_i L)$ is the expected utility of $i$-th risk-group’s insured population.

**Linking social welfare to loss coverage**

Assuming $L\pi_i \approx 0$, so that $U(W) \approx U_i^*(W - \pi_i L)$, gives:

$$S \approx \text{Positive multiplier} \times \sum_{i} p_i d_i(\pi_i) \mu_i + \text{Constant}.$$  

Loss coverage provides a good proxy (which depends only on observable data) for social welfare (which depends on unobservable utilities).

**Result:** Maximising loss coverage maximises social welfare.
Contents

1 Introduction

2 Why do people buy insurance?

3 What drives demand for insurance?

4 How much of population losses is compensated by insurance?

5 Which regime is most beneficial to society?

6 Conclusions
Conclusions

Adverse selection need not always be adverse.

Restricting risk classification increases loss coverage if $\lambda < 1$.

Loss coverage is an observable proxy for social welfare.
References


