Insurance Risk Classification

How much is socially optimal?

Pradip Tapadar

University of Kent, Canterbury, CT2 7NF, UK
P.Tapadar@kent.ac.uk

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Background

Adverse selection:

If insurers cannot charge **risk-differentiated** premiums, then:

- higher risks buy more insurance, lower risks buy less insurance,
- raising the **pooled** price of insurance,
- lowering the demand for insurance,

usually portrayed as a bad outcome, both for **insurers** and for **society**.

In practice:

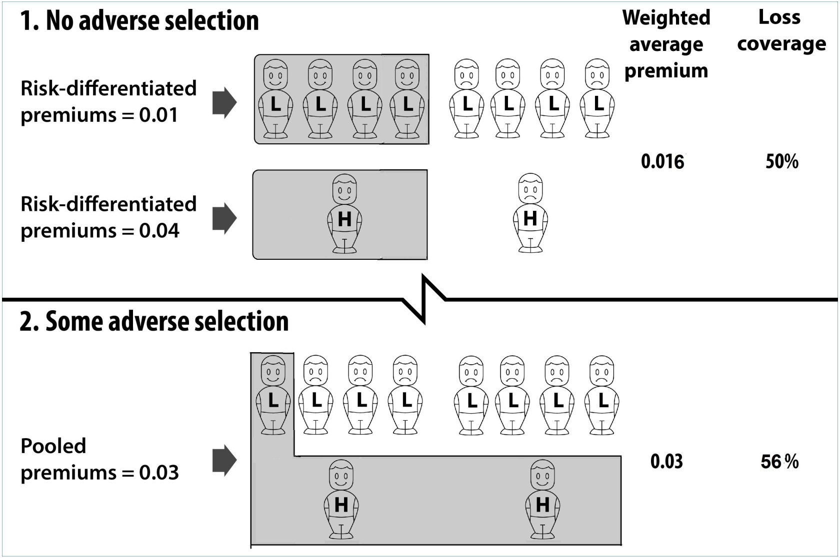
Policymakers often see merit in restricting insurance risk classification

- EU ban on using gender in insurance underwriting.
- Moratoria on the use of genetic test results in underwriting.

Question:

How can we reconcile theory with practice?

Motivating example



Agenda

We ask:

- **Why** do people buy insurance?
- **What** drives demand for insurance?
- **How much** of population losses is compensated by insurance (with and without risk classification)?
- **Which** regime is most beneficial to society?

Definition (Loss coverage)

Expected population losses compensated by insurance.

We find:

Social welfare is maximised by maximising **loss coverage**.

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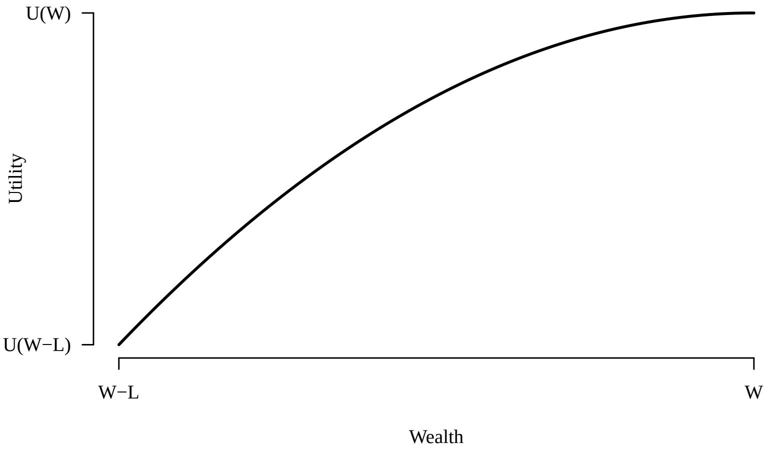
Why do people buy insurance?

Assumptions

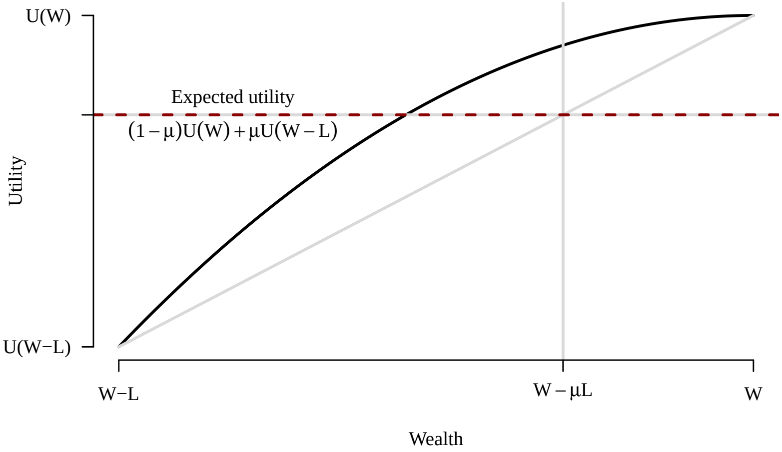
Consider an individual with

- an initial wealth W,
- exposed to the risk of loss L,
- with probability μ,
- utility of wealth $U(w)$, with $U'(w) > 0$ and $U''(w) < 0$,
- an opportunity to insure at premium rate π.

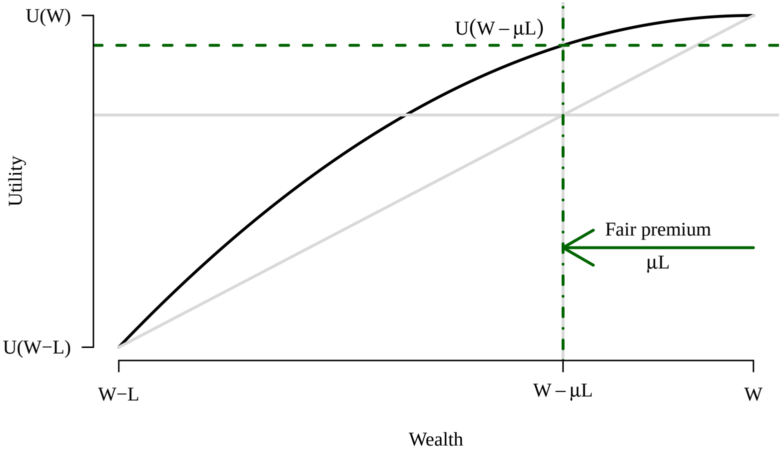
Utility of wealth



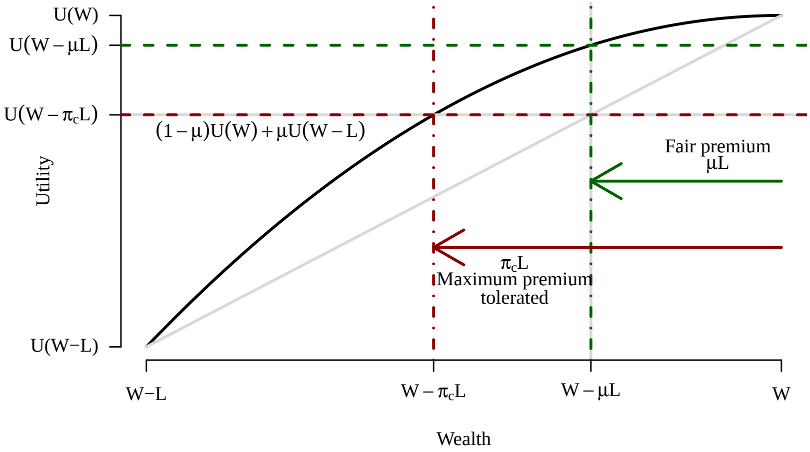
Expected utility: Without insurance



Expected utility: Insured at fair actuarial premium



Maximum premium tolerated: π_c



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Modelling demand for insurance

Simplest model:

If everybody has exactly the same W, L, μ and $U(\cdot)$, then:

- All will buy insurance if $\pi < \pi_c$.
- None will buy insurance if $\pi > \pi_c$.

Reality: Not all will buy insurance even at fair premium. **Why?**

Heterogeneity:

- Individuals are **homogeneous** in terms of underlying risk.
- But they can be **heterogeneous** in terms of **risk-aversion**.

Source of Randomness:

An individual's utility function: $U_\gamma(w)$, where parameter γ is drawn from random variable Γ with distribution function $F_\Gamma(\gamma)$.

Demand is a survival function

Standardisation

As certainty equivalent is invariant to positive affine transformations, we assume $U_\gamma(W) = 1$ and $U_\gamma(W - L) = 0$ for all γ.

Condition for buying insurance:

Given a premium π, an individual will buy insurance if:

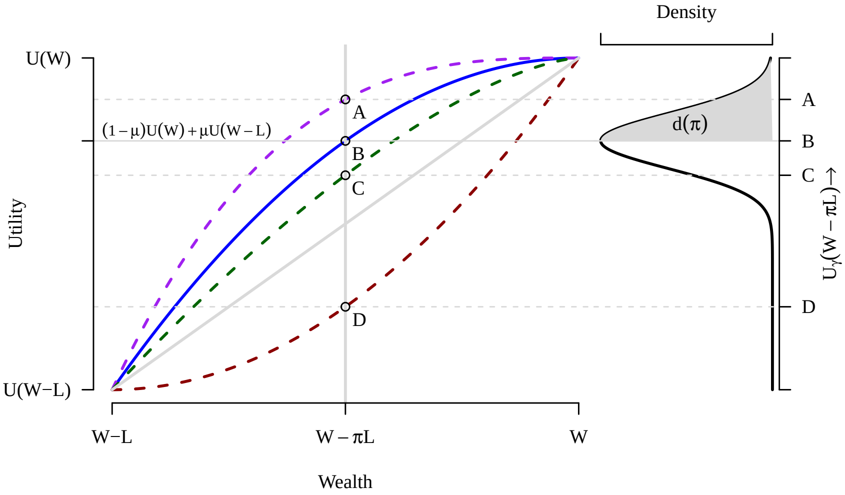
$$\underbrace{U_\gamma\left(W - \pi L\right)}_{\text{With insurance}} > \underbrace{\left(1-\mu\right)U_\gamma(W) + \mu U_\gamma(W - L) = \left(1-\mu\right)}_{\text{Without insurance}}.$$

Demand as a survival function:

Given a premium π, insurance demand, $d(\pi)$, is the survival function:

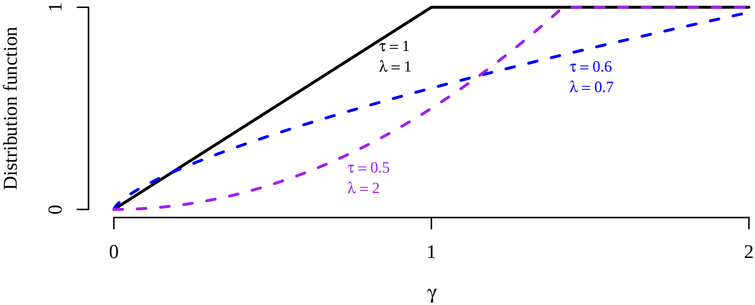
$$d(\pi) = \mathrm{P}\left[U_\Gamma\left(W - \pi L\right) > 1 - \mu\right].$$

Demand is a survival function



Illustrative example: $W = L = 1$ and $U_\gamma(w) = w^\gamma$:

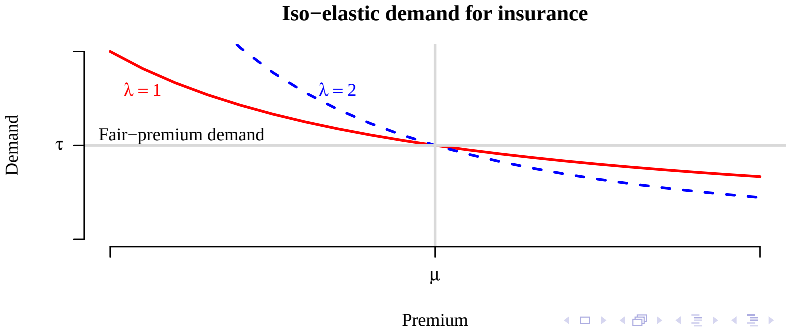
$$F_\Gamma(\gamma) = \mathrm{P}\left[\Gamma \le \gamma\right] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau\,\gamma^\lambda & \text{if } 0 \le \gamma \le (1/\tau)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau)^{1/\lambda}, \end{cases}$$



Illustrative example: $W = L = 1$ and $U_\gamma(w) = w^\gamma$:

$$d(\pi) = \mathrm{P}\left[U_\Gamma\left(W - \pi L\right) > 1 - \mu\right] \approx \tau\left(\frac{\mu}{\pi}\right)^\lambda$$

$$\Rightarrow \epsilon(\pi) = \left|\frac{\frac{\partial d(\pi)}{d(\pi)}}{\frac{\partial \pi}{\pi}}\right| = \lambda \quad \text{(constant elasticity} \Rightarrow \text{Iso-elastic demand).}$$



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Risk classification

Assume all have same $W = L = 1$ and constant demand elasticity λ.

Risk-groups

Suppose the population can be divided into 2 risk-groups, with:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: p_1 and p_2;
- fair premium demand: $d_1(\mu_1) = \tau_1$ and $d_2(\mu_2) = \tau_2$, i.e.

$$d_i(\pi) = \tau_i \left(\frac{\pi}{\mu_i} \right)^{-\lambda}, \quad i = 1, 2;$$

- premiums offered: π_1 and π_2 (note that $\pi_1 = \pi_2$ is allowed).

Equilibrium

For a randomly chosen individual:

Define random variables:

$$Q = I\;[\,\text{Individual is insured}\,]\,;$$
$$X = I\;[\,\text{Individual incurs a loss}\,]\,;$$
$$\Pi = \text{Premium offered to the individual.}$$

Equilibrium

Expected Premium: $$E[Q\Pi] = \sum_i p_i\, d_i(\pi_i)\, \pi_i.$$

Expected Claim: $$E[QX] = \sum_i p_i\, d_i(\pi_i)\, \mu_i.$$

Equilibrium: $$E[Q\Pi] = E[QX].$$

Case 1: Risk-differentiated premium

Observations:

If risk-differentiated premiums are allowed,

- One possible equilibrium is achieved when $\pi_i = \mu_i$.
- No losses for insurers.
- No (actuarial/economic) adverse selection.

Loss coverage (Population losses compensated by insurance):

Loss coverage: $E[QX] = \sum_i p_i \, d_i(\mu_i) \, \mu_i = \sum_i p_i \, \tau_i \, \mu_i$.

Case 2: Pooled premium

Equilibrium:

If only a pooled premium $\pi_1 = \pi_2 = \pi_0$ is allowed,

$$E[Q\Pi] = \sum_i p_i \, d_i(\pi_0) \, \pi_0;$$

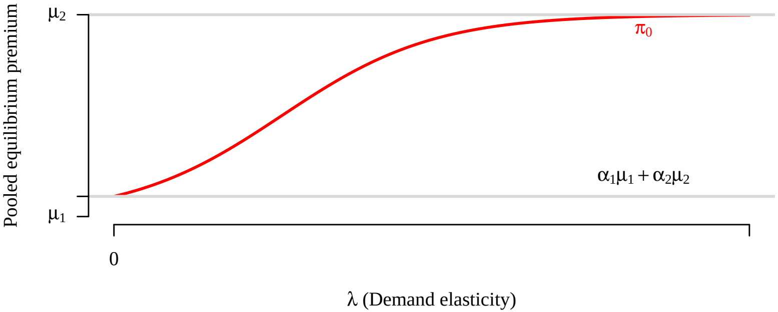
$$E[QX] = \sum_i p_i \, d_i(\pi_0) \, \mu_i;$$

$$E[Q\Pi] = E[QX] \Rightarrow \pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^{\lambda} + \alpha_2 \mu_2^{\lambda}}, \text{ where } \alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}.$$

Observation:

No losses for insurers! $\Rightarrow$ No (actuarial) adverse selection.

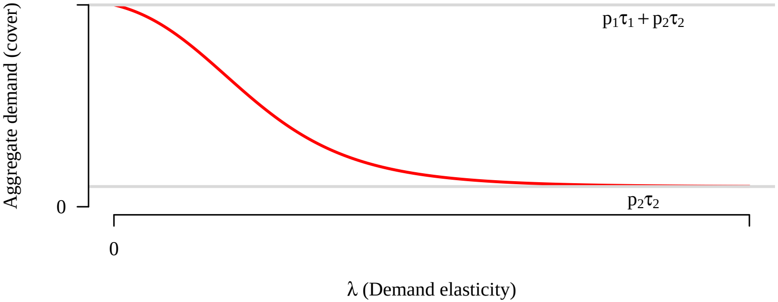
Case 2: Pooled premium



Observation:

Pooled equilibrium is greater than average premium charged under full risk classification: $\pi_0 > \alpha_1\mu_1 + \alpha_2\mu_2 \Rightarrow$ (Economic) adverse selection.

Case 2: Pooled premium



Observation:

Aggregate demand (cover) is lower than under full risk classification $\Rightarrow$ (Economic) adverse selection.

Loss coverage ratio

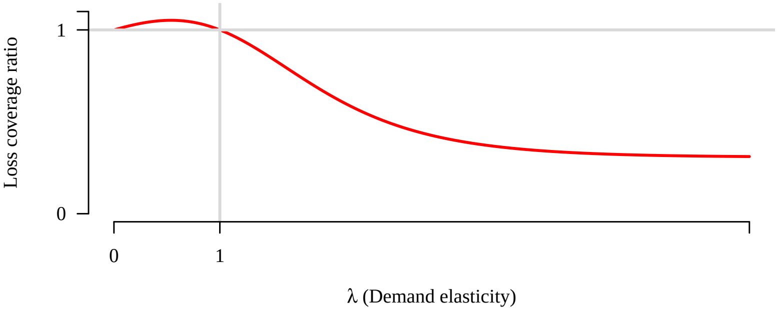
Loss coverage under pooled premium:

Loss coverage: $E[QX] = \sum_i p_i \, d_i(\pi_0) \, \mu_i$.

Loss coverage ratio:

$$
\begin{aligned}
C &= \frac{\text{Loss coverage for pooled premium}}{\text{Loss coverage for risk-differentiated premium}}, \\
&= \frac{\sum_i p_i \, d_i(\pi_0) \, \mu_i}{\sum_i p_i \, d_i(\mu_i) \, \mu_i}, \\
&= \frac{1}{\pi_0^\lambda} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}.
\end{aligned}
$$

Loss coverage ratio



- $\lambda < 1 \Rightarrow$ Pooled premium $\succ$ Full risk classification.
- $\lambda > 1 \Rightarrow$ Pooled premium $\prec$ Full risk classification.
- Empirical evidence suggests $\lambda < 1$ in many insurance markets.

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Social welfare

Definition (Social welfare)

Social welfare, S, is the sum of all individuals' expected (standardised) utilities:

$$\begin{aligned} S &= \mathrm{E}\left[Q\, U_\Gamma(W - \Pi L)\right] + (1-Q)\left[(1-X)\, U(W) + X\, U(W-L)\right], \\ &= \sum_i \Big[\underbrace{d_i(\pi_i) U_i^*(W - \pi_i L)}_{\text{Insured population}} + \underbrace{(1 - d_i(\pi_i))\left\{(1-\mu_i)U(W) + \mu_i U(W-L)\right\}}_{\text{Uninsured population}} \Big] p_i, \end{aligned}$$

where $U_i^*(W - \pi_i L)$ is the expected utility of i-th risk-group's insured population.

Linking social welfare to loss coverage

Assuming $L\pi_i \approx 0$, so that $U(W) \approx U_i^*(W - \pi_i L)$, gives:

$$S \approx \text{Positive multiplier} \times \underbrace{\sum_i p_i d_i(\pi_i)\mu_i}_{\textbf{Loss Coverage}} + \text{Constant}.$$

Loss coverage provides a good proxy (which depends only on observable data) for social welfare (which depends on unobservable utilities).

Result: Maximising loss coverage maximises social welfare.

Contents

Conclusions

Adverse selection need not always be adverse.

Restricting risk classification increases loss coverage if $\lambda < 1$.

Loss coverage is an observable proxy for social welfare.

References

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