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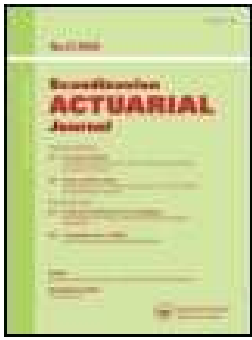
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# Insurance loss coverage and social welfare

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## ABSTRACT

Restrictions on insurance risk classification may induce adverse selection, which is usually perceived as a bad outcome, both for insurers and for society. However, a social benefit of modest adverse selection is that it can lead to an increase in 'loss coverage', defined as expected losses compensated by insurance for the whole population. We reconcile the concept of loss coverage to a utilitarian concept of social welfare commonly found in the economic literature on risk classification. For iso-elastic insurance demand, ranking risk classification schemes by (observable) loss coverage always give the same ordering as ranking by (unobservable) social welfare.

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## 1. Introduction

In personal insurance markets, regulators often impose some restrictions on risk classification. Such restrictions appear motivated by social concerns, such as distaste for discrimination, or a desire to redress perceived health or other disadvantages of high risks. But they can also induce adverse selection, which is usually seen as a bad outcome, both for insurers and for society.

The usual basic argument is as follows. If insurers can, they will charge risk-differentiated prices to reflect different risks. If instead insurers are banned from differentiating between lower and higher risks and have to pool all risks at one price, the equilibrium price will seem cheap to higher risks and expensive to lower risks. Higher risks will buy more insurance, and lower risks will buy less. Also, since the number of higher risks is typically smaller than the number of lower risks, the total number of risks insured will fall. The usual argument focuses on this reduction in coverage, e.g. 'This reduced pool of insured individuals reflects a decrease in the efficiency of the insurance market' (Dionne & Rothschild 2014, p. 185).

Thomas (2008) suggested that in some circumstances, there is a counter-argument to this perception that adverse selection always represents a bad outcome for society. The rise in equilibrium price under pooling reflects a shift in coverage towards higher risks. If the shift in coverage is large enough, it can more than outweigh the fall in numbers insured. In these circumstances, despite fewer risks being insured under pooling, expected losses compensated by insurance – 'loss coverage' – can be higher. Since more risk is being voluntarily traded and more losses are being compensated, this might be regarded as a better outcome from pooling.

The present paper makes a connection between the criterion of loss coverage and utilitarian criteria of social welfare commonly found in economic literature. Specifically, we show that if insurance demand is iso-elastic, the risk classification scheme which maximises loss coverage also maximises social welfare. From a utilitarian policymaker's perspective, this may be a useful result, because

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maximising loss coverage does not require knowledge of individuals' (generally unobservable) utility functions; loss coverage is based solely on observable quantities.

The rest of the paper is organised as follows. Section 2 illustrates with simple examples the possibility that loss coverage and social welfare can both be increased by moderate adverse selection. Section 3 reviews related literature. Section 4 sets up the model of insurance demand, equilibrium and loss coverage, and Section 5 extends this to include social welfare. Section 6 considers iso-elastic demand, including the main results of the paper. Section 7 gives conclusions.

## 2. Illustrative example

This section uses a simple example to illustrate the possibility that both loss coverage and utilitarian social welfare can be increased by adverse selection. The loss coverage part of this material is not new, but it may be helpful to review, because the argument that an aggregate benefit can sometimes arise from adverse selection seems counter-intuitive to many people.

Consider a population of nine risks (say lives). Assume that all losses and insurance cover are for unit amounts, and that the probability of loss is unaffected by the purchase of insurance (i.e. no moral hazard). Also assume that individuals can be divided into a high risk-group and a low risk-group, based on information which is fully observable by insurers. Then consider three alternative scenarios for risk classification: actuarially fair premiums, pooled premiums (with some adverse selection), and pooled premiums (with severe adverse selection), as shown in Figure 1.

In Scenario 1 in the upper panel of Figure 1, each  $H$  represents one higher risk and each  $L$  represents one lower risk. The population has the typical predominance of lower risks: a lower risk-group of six risks each with a probability of loss 0.01, and a higher risk-group of three risks each with a probability of loss 0.04. The demand response of each risk-group to an actuarially fair price is the same: exactly two-thirds of the members of each risk-group buy insurance.<sup>1</sup> The shading symbolises that a total of six risks are covered.

The weighted average of the premiums paid is  $(4 \times 0.01 + 2 \times 0.04)/6 = 0.02$ . Higher and lower risks are insured in the same proportions as they exist in the population, so there is no adverse selection. The expected losses compensated by insurance for the whole population can be indexed by:

$$\text{Loss coverage} = \frac{\text{Expected compensated losses}}{\text{Expected population losses}} = \frac{4 \times 0.01 + 2 \times 0.04}{6 \times 0.01 + 3 \times 0.04} = 66.7\%.$$

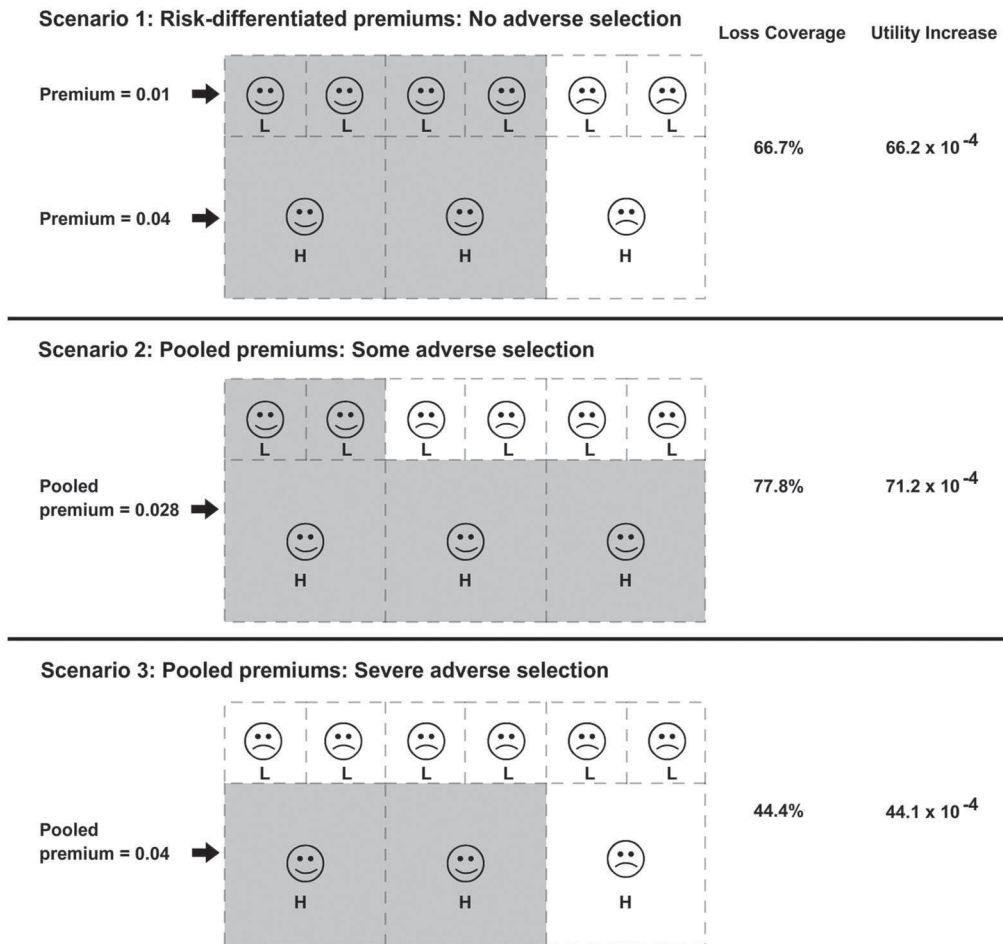
In Scenario 2 in the middle panel of Figure 1, risk classification has now been banned, and so insurers have to charge a common pooled premium to both higher and lower risks. Higher risks buy more insurance, and lower risks buy less (adverse selection). The pooled premium is set as the weighted average of the true risks, so that expected profits on low risks exactly offset expected losses on high risks. This weighted average premium is  $(2 \times 0.01 + 3 \times 0.04)/5 = 0.028$ . The shading symbolises that five risks (compared with six previously) are covered.

Note that the weighted average premium is higher in Scenario 2, and the number of risks insured is lower. But despite the adverse selection in Scenario 2, the expected losses compensated by insurance for the whole population are now higher. The loss coverage is:

$$\text{Loss coverage} = \frac{2 \times 0.01 + 3 \times 0.04}{6 \times 0.01 + 3 \times 0.04} = 77.8\%.$$

In our view Scenario 2, with a higher expected fraction of the population's losses compensated by insurance – higher loss coverage – is superior from a social viewpoint to Scenario 1. The perceived improvement in Scenario 2 arises not *despite* adverse selection, but *because* of adverse selection.

<sup>1</sup> The common two-thirds take-up under actuarially fair prices is not critical to our argument; all that is required is that take-up by higher risks is less than 100%, so that some increase in take-up by higher risks is possible when risk classification is banned.



**Figure 1.** Three scenarios for risk classification.

However, a ban on risk classification can reduce loss coverage, if the adverse selection which the ban induces becomes too severe. This possibility is illustrated in Scenario 3, the lower panel of Figure 1. Only two higher risks, and no lower risks, buy insurance. The loss coverage is:

$$\text{Loss coverage} = \frac{2 \times 0.04}{6 \times 0.01 + 3 \times 0.04} = 44.4\%.$$

Up to this point, the changes in insurance purchasing decisions as we moved through the three scenarios in Figure 1 were merely plausible intuitions; we have not explained why (for example) two of the four lower risks who purchased insurance in Scenario 1 dropped out of the market when we moved to Scenario 2, but the other two lower risks stayed in the market.

One way to rationalise different purchasing decisions by individuals with the same probabilities of loss is to postulate a distribution of risk preferences across individuals in each risk-group. Given an insurance price, each individual then makes their own decision, based on the certainty-equivalence principle (i.e. whether or not purchasing insurance increases their expected utility).

As a simple example, suppose all individuals have a power utility function  $U(w) = w^\gamma$  with different positive parameter  $\gamma$  for different individuals. Then if individuals have initial wealth of unit amount, the purchasing decisions represented in Figure 1 are consistent with the following distribution of risk preferences (the calculations are outlined in Appendix 1):

- one-third of the members of each risk-group have  $\gamma = 0.3$  (strong risk-aversers);
- one-third  $\gamma = 0.7$  (moderate risk-aversers); and
- one-third  $\gamma = 1.1$  (risk-lovers).

The postulated distribution of risk preferences also enables us to evaluate an alternative criterion for different risk classification schemes: the expected utility for an individual chosen at random from the population. This criterion, often referred to as ‘social welfare’, is common in economic literature on insurance risk classification (e.g. Hoy 2006). For the three scenarios in Figure 1, the *increase* in this expected utility, compared with a situation where no insurance is offered, is as follows (the calculations are outlined in Appendix 1):

Scenario 1:  $66.2 \times 10^{-4}$

Scenario 2:  $71.2 \times 10^{-4}$

Scenario 3:  $44.1 \times 10^{-4}$

Note that the ranking of the scenarios by either criterion, loss coverage or social welfare, is the same: Scenario 2 (highest), Scenario 1, Scenario 3 (lowest). The ‘same ranking’ property depends on the postulated distribution of risk preferences, and hence insurance demand functions. This paper shows that the ‘same ranking’ property holds not just for this example, but also for all iso-elastic insurance demand functions.

### 3. Literature review

The model of insurance markets implied by the illustrative example above differs from economic models derived from Rothschild & Stiglitz (1976) in two main ways.

First, in our model insurers compete only on price, and not on quantity. When different prices for different risks are banned (e.g. as in Scenarios 2 and 3), then for institutional or regulatory reasons, insurers do *not* attempt to separate risk-groups by menus of contracts offering different levels of cover priced at different rates. In this respect, our model is more in the spirit of Akerlof (1970). We justify this approach by noting that some important markets, such as life insurance, have the institutional feature of non-exclusive contracting, and so separation via contract menus is not feasible. Furthermore, we observe that the concept of separation via contract menus is often not salient to practitioners even in markets where it seems theoretically feasible.<sup>2</sup>

Second, in our model individuals with identical probabilities of loss can have different utility functions, and so unlike the representative agents from each risk-group in Rothschild–Stiglitz type models, they do not all make the same purchasing decision. This leads to an equilibrium where not all individuals are insured; this corresponds to the empirical reality of most voluntary insurance markets.<sup>3</sup>

Previous papers (Thomas 2008, 2009; Hao et al. 2016) and a book (Thomas 2017) formalised the insight that some adverse selection can increase loss coverage, as highlighted in the illustrative example. In all this earlier work, insurance demand from each risk-group was represented by a demand function with output a number between 0 and 1, to reflect the empirical observation that not all

<sup>2</sup> As regards life insurance, Rothschild–Stiglitz type models are inconsistent in important ways with empirical data (e.g. Cawley & Philipson 1999). As regards practice in other markets, most actuarial pricing textbooks make no reference to the concept of menus of contracts as screening devices (e.g. Gray & Pitts 2012, Friedland 2013, Parodi 2014). Some textbooks specifically counsel *against* any thought of using the level of deductible as a pricing factor (e.g. Ohlsson & Johansson 2010).

<sup>3</sup> For example, in life insurance, the Life Insurance Market Research Organisation (LIMRA) states that 44% of US households have some individual life insurance (LIMRA 2013). The American Council of Life Insurers states that 144 m individual policies were in force in 2013 (American Council of Life Insurers 2014, p. 72); the US adult population (aged 18 years and over) at 1 July 2013 as estimated by the US Census Bureau was 244 m. In health insurance, only 14.6% of the US population has individually purchased private cover (US Census Bureau, 2015), albeit substantially more have employer group cover or Medicare or Medicaid government cover.

individuals with the same probabilities of loss make the same purchasing decisions. The variation in purchasing decisions across persons with the same probabilities of loss was characterised as stochastic; no reference was made to individual utilities as a driver of individual decisions.

The loss coverage literature contrasts with economic literature on insurance risk classification, as summarised in surveys such as Hoy (2006), Einav & Finkelstein (2011) and Dionne & Rothschild (2014). Economic literature typically takes a utility-based approach: representative agents from each risk-group make purchasing decisions which maximise their expected utilities, and the outcomes of different risk classification schemes are then evaluated by a social welfare function which is a (possibly weighted) sum of expected utilities over the whole population.

The present paper reconciles, under certain assumptions, the different approaches described in the preceding two paragraphs. Specifically, we show that if insurance demand is iso-elastic, the risk classification scheme which maximises loss coverage also maximises utilitarian social welfare.

#### 4. The insurance market model

Section 4 formulates insurance demand, market equilibrium and loss coverage using the same framework as in Hao et al. (2018), which is then extended in Section 5 to include social welfare.

##### 4.1. Insurance demand with heterogeneous risk preferences

Consider an individual with an initial wealth  $W$ , who is exposed to the risk of losing an amount of  $L$  (with  $L \leq W$ ) with probability  $\mu$ . The individual's utility function  $U(w)$  is increasing in wealth  $w$  (i.e.  $U'(w) > 0$ ), but not necessarily risk-averse (i.e.  $U''(w)$  can have either sign).

The individual is offered insurance against the full amount of loss  $L$  at premium rate  $\pi$  per unit of loss, i.e. for premium  $\pi L$  (in this paper we do not consider partial cover). She will choose to buy insurance if  $\pi$  is low enough to satisfy:

$$U(W - \pi L) > (1 - \mu)U(W) + \mu U(W - L). \quad (1)$$

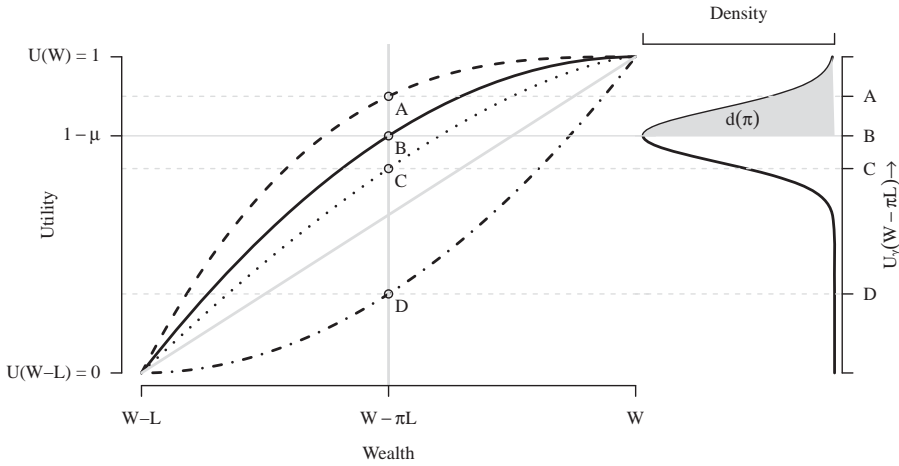
Now consider a group of individuals as described above, all with the same initial wealth  $W$ , potential loss  $L$  and risk  $\mu$ , but who may have different utility functions  $U_\gamma(w)$ . Each individual knows his own utility function, which is parameterised by the positive real number  $\gamma$ . A particular individual will choose to buy insurance if and only if the following condition is satisfied for the combination of the offered premium  $\pi$  and their particular utility function  $U_\gamma(w)$ :

$$U_\gamma(W - \pi L) > (1 - \mu)U_\gamma(W) + \mu U_\gamma(W - L), \quad (2)$$

Since decisions based on Equation (2) do not depend on the origin and scale of a utility function, we can standardise utilities at the 'end points' to  $U_\gamma(W) = 1$  and  $U_\gamma(W - L) = 0$ . Equation (2) then becomes:

$$U_\gamma(W - \pi L) > (1 - \mu). \quad (3)$$

Figure 2 provides a graphical representation of Equation (3), showing utility functions of four individuals with the same probability of loss  $\mu$ . The concave utility curves, with points  $A$ ,  $B$  and  $C$ , represent risk-averse individuals, where higher concavity represents higher risk aversion. We also show a convex utility curve, with point  $D$ , which represents a risk-loving (or risk-neglecting) individual. (As mentioned previously, the model does not require that all individuals are risk-averse.) For the individual at point  $A$ , the utility with insurance,  $U_{\gamma_A}(W - \pi L)$ , exceeds the critical value  $(1 - \mu)$ , where  $\gamma_A$  is the individual's utility function parameter. So the individual buys insurance. For the individuals at points  $C$  and  $D$ , the inverse applies, so they do not purchase insurance. The individual at point  $B$  is indifferent. Insurance purchase is denoted by the shaded area,  $d(\pi)$ , under the density graph for  $U_\Gamma(W - \pi L)$ .



**Figure 2.** Heterogeneous utility functions within a risk-group, leading to proportional insurance demand.

Now consider the perspective of an insurer, who cannot observe individuals’ utility functions. For given offered premium  $\pi$ , all the insurer can observe of insurance purchasing behaviour is the proportion of individuals who buy insurance. We call this a (proportional) demand function and denote it by  $d(\pi)$ . We have:

$$d(\pi) = P[U_{\Gamma}(W - \pi L) > (1 - \mu)], \tag{4}$$

where  $U_{\Gamma}(W - \pi L)$  is a random variable denoting the utility of the fixed wealth amount  $W - \pi L$  as observed by the insurer (the capital subscript- $\Gamma$  denoting that the insurer cannot observe any particular individual’s risk preference  $\gamma$ ).

A key feature of this model is that demand can be less than 1 when an actuarially fair premium is charged. The presence of some ‘risk-lovers’ generates a demand function consistent with the observation that not everyone buys insurance. Although ‘risk-loving’ or ‘risk-seeking’ are the usual stylised descriptions, the phenomenon might better be characterised as ‘risk-neglecting’. Whatever the description, heterogeneity of risk aversion provides a flexible model for the observed reality we wish to model, i.e. in extant insurance markets with risk-differentiated prices, many individuals do not purchase insurance.<sup>4</sup>

We note the following three properties of demand for insurance:

- (a)  $0 \leq d(\pi) \leq 1$ , so  $d(\pi)$  is a valid probability.
- (b)  $d(\pi)$  is non-increasing in  $\pi$ , i.e. demand for insurance cannot increase when premium increases. This can be shown as follows: For utility functions with  $U'(w) > 0$ , if  $\pi_1 < \pi_2$ , the random variable  $U_{\Gamma}(W - \pi_1 L)$  is statewise dominant<sup>5</sup> over the random variable  $U_{\Gamma}(W - \pi_2 L)$ . So,

$$\begin{aligned} \pi_1 < \pi_2 &\Rightarrow P[U_{\Gamma}(W - \pi_1 L) > 1 - \mu] \geq P[U_{\Gamma}(W - \pi_2 L) > 1 - \mu] \\ &\Rightarrow d(\pi_1) \geq d(\pi_2). \end{aligned} \tag{5}$$

- (c) Each individual’s decision is completely deterministic, given what they know. But to the insurer it appears stochastic, given what the insurer knows.

<sup>4</sup> Another possible explanation for some non-purchasing is expense and profit loadings. But this cannot explain why different individuals who are offered the same premiums for the same probabilities of loss often appear to make different decisions.

<sup>5</sup> One random variable is statewise dominant over a second if the first is at least as high as the second in all states of nature, with strict inequality for at least one state. It is an absolute form of dominance.



## 4.2. Equilibrium

Suppose the population can be sub-divided into  $n$  distinct risk-groups with probabilities of loss given by  $\mu_1, \mu_2, \dots, \mu_n$ . For convenience, we assume  $0 < \mu_1 < \mu_2 < \dots < \mu_n < 1$ . The proportion of the population belonging to risk-group  $i$  is  $p_i$ , and the insurance demand (based on Equation 4) is  $d_i(\pi_i)$ .

For an individual chosen at random from the population, we define

- (a) an indicator random variable  $Q$  representing the insurance purchasing decision (1 = purchase, 0 otherwise);
- (b) an indicator random variable  $X$  representing the loss event (1 = loss occurs, 0 otherwise);
- (c) a random variable  $\Pi$  representing the premium chargeable, if the individual purchases insurance.

Then conditional on membership of risk-group  $i$ :

- (a)  $Q$  is Bernoulli with parameter  $d_i(\pi_i)$ ;
- (b)  $X$  is a Bernoulli with parameter  $\mu_i$ ; and
- (c)  $\Pi$  takes the value  $\pi_i$ .

We assume that  $Q$  and  $X$  are independent, again conditional on membership of risk-group  $i$  (i.e. no moral hazard).

Then for the individual chosen at random from the population, the expected premium income is  $E[Q\Pi L]$ , and the expected insurance claim is  $E[QXL]$ .

We call any vector of premiums  $\underline{\pi} = (\pi_1, \pi_2, \dots, \pi_n)$  charged by insurers a risk classification regime. We assume that competition between insurers leads to expected profits  $\rho(\underline{\pi}) = 0$  in equilibrium. That is:

$$\rho(\underline{\pi}) = E[Q\Pi L] - E[QXL] = L \sum_{i=1}^n d_i(\pi_i) \pi_i p_i - L \sum_{i=1}^n d_i(\pi_i) \mu_i p_i = 0 \quad (6)$$

and dividing both sides by the fixed loss amount  $L$  we can write this as

$$\sum_{i=1}^n d_i(\pi_i) p_i (\pi_i - \mu_i) = 0, \quad (7)$$

which we refer to as the *equilibrium condition*.

We also assume that competition drives all insurers to classify risks to the maximum extent which regulation permits. Depending on applicable regulation, two polar cases of risk classification regime are as follows:

- Full risk classification, under which  $\pi_i = \mu_i$ . We denote this regime by  $\underline{\pi} = \underline{\mu} = (\mu_1, \mu_2, \dots, \mu_n)$ .
- No risk classification (risk pooling), under which all  $\pi_i = \pi_0$ , a constant.

## 4.3. Loss coverage

Loss coverage under a risk classification  $\underline{\pi}$  that leads to equilibrium is defined as the expected losses across the whole population that are compensated by insurance, that is:

$$\text{Loss coverage : } LC(\underline{\pi}) = E[QXL], \quad (8)$$

where  $\underline{\pi}$  satisfies the equilibrium condition in Equation (7).

Since the fixed loss amount  $L$  is the same for all individuals, we shall find it convenient to define standardised loss coverage (per unit of loss) as:

$$\text{Standardised loss coverage : } K(\underline{\pi}) = E[QX]. \quad (9)$$

For comparison purposes, we use loss coverage under full risk classification regime as a reference level, and hence define the *loss coverage ratio* as follows:

$$\text{Loss coverage ratio : } C = \frac{K(\underline{\pi})}{K(\underline{\mu})}. \quad (10)$$

## 5. Social welfare

Our approach to social welfare is in the same spirit as Hoy (2006): we assume cardinal and interpersonally comparable utilities, and assign equal weights to the utilities of all individuals. This equal-weights approach is based on the Harsanyi (1955) ‘veil of ignorance’ argument: that is, behind the (hypothetical) ‘veil of ignorance’, where one does not know what position in society (e.g. high risk or low risk) one occupies, the appropriate probability to assign to being any individual is  $1/n$ , where  $n$  is the number of individuals in society. Alternative risk classification regimes can then be compared by comparing expected utility in each regime for the (hypothetical) individual utility-maximiser behind the ‘veil of ignorance’.

In the model defined above, suppose an individual is selected at random from the whole population. The individual’s expected utility can be written as follows:

$$\text{Social welfare} = E[QU_{\Gamma}(W - \Pi L) + (1 - Q)[(1 - X)U_{\Gamma}(W) + XU_{\Gamma}(W - L)]], \quad (11)$$

where the first part represents random utility if insurance is purchased, and the second part the random utility if insurance is not purchased.

We assumed earlier that all individuals had the same utilities at the ‘end-points’,  $U(W) = 1$  and  $U(W - L) = 0$ . This relied on the property that certainty-equivalent decisions do not depend on the origins and scales of utility functions. But this argument cannot be directly extended to Equation (11), because individuals’ utilities can differ for identical levels of wealth, which has direct implications for social welfare.

However, without any standardisation, Equation (11) is susceptible to being dominated by a ‘utility monster’ who derives more utility from a given level of wealth than all other individuals combined (see Nozick 1974, Bailey 1997). This makes it unsuitable for policy purposes. So we propose to continue standardising the ‘end-points’,  $U(W) = 1$  and  $U(W - L) = 0$ , as before. This standardisation implies that the same ‘disutility of uninsured loss’ is assigned to all individuals, but utility if insurance is purchased  $U_{\Gamma}(W - \Pi L)$  differs between individuals. Under this standardisation, social welfare can be expressed as:

$$S(\underline{\pi}) = E[QU_{\Gamma}(W - \Pi L) + (1 - Q)(1 - X)]. \quad (12)$$

To derive an expression for  $S$ , we consider the constituent parts of Equation (12) separately. First:

$$\begin{aligned} & E[QU_{\Gamma}(W - \Pi L)] \\ &= \sum_{i=1}^n E[QU_{\Gamma}(W - \pi_i L) \mid \mu = \mu_i]P[\mu = \mu_i], \end{aligned} \quad (13)$$

$$\begin{aligned} &= \sum_{i=1}^n \{E[U_{\Gamma}(W - \pi_i L) \mid U_{\Gamma}(W - \pi_i L) > (1 - \mu_i)] \\ &\quad \times P[U_{\Gamma}(W - \pi_i L) > (1 - \mu_i)] p_i\} \end{aligned} \quad (14)$$

$$= \sum_{i=1}^n U_i^*(W - \pi_i L) d_i(\pi_i) p_i, \quad \text{using Equation (4),} \quad (15)$$

where  $U_i^*(W - \pi_i L) = E[U_\Gamma(W - \pi_i L) \mid U_\Gamma(W - \pi_i L) > (1 - \mu_i)]$  represents the expected utility of individuals purchasing insurance in risk-group  $i$ .

The second part of Equation (12) can be written as:

$$\begin{aligned} & E[(1 - Q)(1 - X)] \\ &= \sum_{i=1}^n E[(1 - Q)(1 - X) \mid \mu = \mu_i] P[\mu = \mu_i], \end{aligned} \quad (16)$$

$$= \sum_{i=1}^n [(1 - d_i(\pi_i)) (1 - \mu_i)] p_i. \quad (17)$$

Combining Equations (15) and (17), we get the following expression for social welfare:

$$S = \sum_{i=1}^n \left[ \underbrace{d_i(\pi_i) U_i^*(W - \pi_i L)}_{\text{Insured population}} + \underbrace{(1 - d_i(\pi_i)) (1 - \mu_i)}_{\text{Uninsured population}} \right] p_i, \quad (18)$$

$$= \underbrace{\left( \sum_{i=1}^n d_i(\pi_i) \mu_i p_i \right)}_{\text{Standardised loss coverage}} \quad (19)$$

$$- \underbrace{\sum_{i=1}^n d_i(\pi_i) [1 - U_i^*(W - \pi_i L)] p_i}_{\text{Adjustment to account for premiums}} + \underbrace{\sum_{i=1}^n [(1 - \mu_i)] p_i}_{\text{Constant as a function of } \pi_i}$$

$$= K(\underline{\pi}) - PA(\underline{\pi}) + \text{Constant}, \quad (20)$$

where  $PA(\underline{\pi})$  is the ‘premium adjustment’.

Equation (20) applies for any utility (and hence insurance demand) functions, but the ‘premium adjustment’ depends on the unobservable expected utility of insurance purchasers,  $U_i^*(W - \pi_i L)$ . The next section shows that for iso-elastic demand, a simpler relationship between loss coverage and social welfare applies.

## 6. The iso-elastic case

### 6.1. Iso-elastic demand

In the model in Section 4.1, assume a power utility function:

$$U_\gamma(w) = \left[ \frac{w - (W - L)}{L} \right]^\gamma, \quad (21)$$

so that  $U_\gamma(W - L) = 0$  and  $U_\gamma(W) = 1$  at the ‘end-points’.<sup>6</sup> This particular form of utility function leads to:

$$\text{relative risk aversion coefficient} : -w \frac{U_\gamma''(w)}{U_\gamma'(w)} = (1 - \gamma) \left[ \frac{w}{w - (W - L)} \right]. \quad (22)$$

<sup>6</sup> For  $W = L = 1$ , this reduces to the utility function  $U(w) = w^\gamma$  used in the illustrative example in Section 2.

Given that all individuals have the same  $W$  and same  $L$ , Equation (22) implies that heterogeneous risk preferences can be modelled as individual drawings  $\gamma$  from the distribution of a positive random variable  $\Gamma$ . In the special case  $W = L$ , Equation (22) reduces to  $(1 - \gamma)$ , the familiar case of constant relative risk aversion.

Demand for insurance at a given premium  $\pi$  is then:

$$d(\pi) = P[U_\Gamma(W - \pi L) > 1 - \mu], \tag{23}$$

$$= P[(1 - \pi)^\Gamma > 1 - \mu], \tag{24}$$

$$= P[\Gamma \log(1 - \pi) > \log(1 - \mu)], \text{ as } \log \text{ is monotonic,} \tag{25}$$

$$= P\left[\Gamma < \frac{\log(1 - \mu)}{\log(1 - \pi)}\right], \text{ as } \log(1 - \pi) < 0, \tag{26}$$

$$\approx P\left[\Gamma < \frac{\mu}{\pi}\right], \text{ as } \log(1 - x) \approx -x, \text{ for small } x, \tag{27}$$

where ‘small  $x$ ’ implies that both premium and probability of loss are small relative to total wealth. This assumption of ‘small  $x$  relative to wealth’ is necessary for us to make the link from an specific distribution of individual risk aversion to an iso-elastic formula for aggregate demand. It seems reasonable for many insurances.

Now suppose the risk aversion variable  $\Gamma$  has the following distribution across the population:

$$F_\Gamma(\gamma) = P[\Gamma \leq \gamma] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau\gamma^\lambda & \text{if } 0 \leq \gamma \leq (1/\tau)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau)^{1/\lambda}, \end{cases} \tag{28}$$

where  $\tau$  and  $\lambda$  are positive parameters. If  $\tau = \lambda = 1$ , this corresponds to the simplest possible distribution for  $\Gamma$ , the uniform distribution on  $(0,1)$ . By changing the parameters we can specify a wide variety of other distributions of  $\Gamma$ : the parameter  $\lambda$  controls the shape, and  $\lambda$  and  $\tau$  together determine the upper limit of the distribution of  $\Gamma$ .<sup>7</sup> Using this distribution for  $\Gamma$ , demand for insurance from the  $i$ -th risk-group based on Equation (27) takes the form:

$$d_i(\pi) = \tau_i \left(\frac{\mu_i}{\pi}\right)^\lambda, \tag{29}$$

subject to a cap of 1 (when all members of a risk-group purchase insurance, demand cannot increase further). Note that the parameter  $\tau_i$ , which was used in specifying the distribution of risk aversion variable  $\Gamma$ , turns out to be equivalent to the *fair-premium demand*,  $d_i(\mu_i)$ , that is the demand when an actuarially fair premium is charged. The formula (29) corresponds to iso-elastic demand, the constant demand elasticity being:

$$\epsilon(\pi) = -\frac{\partial \log(d_i(\pi))}{\partial \log \pi} = \lambda. \tag{30}$$

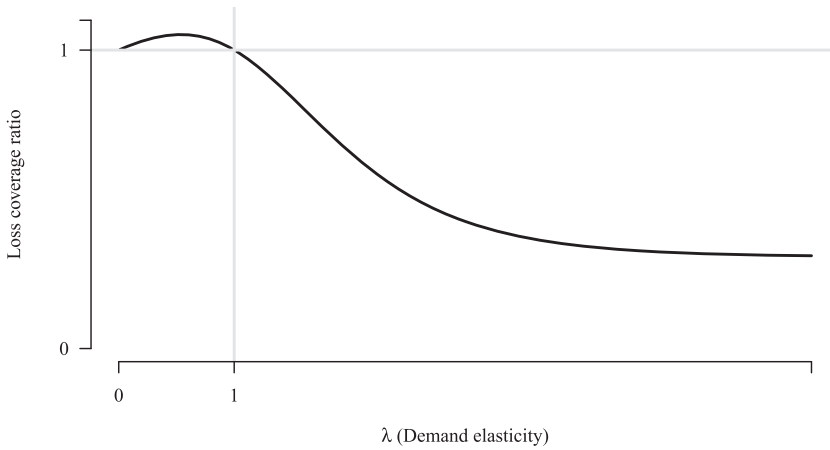
### 6.2. Equilibrium

For iso-elastic demand, the equilibrium condition in Equation (7) becomes:

$$\rho(\underline{\pi}) = 0 \Leftrightarrow \sum_{i=1}^n \tau_i \left(\frac{\mu_i}{\pi_i}\right)^\lambda \pi_i p_i = \sum_{i=1}^n \tau_i \left(\frac{\mu_i}{\pi_i}\right)^\lambda \mu_i p_i. \tag{31}$$

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<sup>7</sup> This is a generalised version of the Kumaraswamy distribution, which in its standard form takes values only over  $[0,1]$  (Kumaraswamy 1980).



**Figure 3.** Loss coverage ratio as a function of demand elasticity.

If the same premium  $\pi_0$  is charged to all risk-groups, the pooled equilibrium premiums satisfying  $\rho(\pi_0) = 0$  is unique and is given by:

$$\pi_0 = \frac{\sum_{i=1}^n \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^n \alpha_i \mu_i^\lambda}, \quad \text{where} \quad \alpha_i = \frac{p_i \tau_i}{\sum_{j=1}^n p_j \tau_j}, \quad i = 1, 2, \dots, n. \quad (32)$$

that is,  $\alpha_i$  is the *fair-premium demand-share*, that is the share of total demand represented by risk-group  $i$  when actuarially fair premiums are charged.

### 6.3. Loss coverage

The standardised loss coverage at this pooled equilibrium as per Equation (9) is given by:

$$K(\underline{\pi}) = E[QX] = \sum_{i=1}^n \tau_i \left( \frac{\mu_i}{\pi_i} \right)^\lambda \mu_i p_i = \sum_{i=1}^n \frac{\mu_i^{\lambda+1}}{\pi_i^\lambda} \tau_i p_i. \quad (33)$$

The loss coverage ratio, comparing loss coverage under pooled premiums to that under actuarially fair premiums, is:

$$C = \frac{1}{\pi_0^\lambda} \frac{\sum_{i=1}^n \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^n \alpha_i \mu_i}, \quad (34)$$

where  $\pi_0$  is the pooled equilibrium premium given in Equation (32).

Figure 3 shows the plot of loss coverage ratio as a function of demand elasticity  $\lambda$ , for two risk-groups where  $(\mu_1, \mu_2) = (0.01, 0.04)$  and fair-premium demand-shares  $(\alpha_1, \alpha_2) = (0.9, 0.1)$  (i.e. low risks comprise 90% of the insured population when fair premiums are charged). We see that loss coverage under pooling is higher than under risk-differentiated premiums if demand elasticity is less than 1, and vice versa. The pattern shown in Figure 3 is formalised by a result proved in Hao et al. (2018), which we state here for later reference.

**Result 1:** If demand elasticity is a positive constant  $\lambda$  and the loss coverage ratio as defined in Equation (34) is  $C$ , then

$$\lambda \leq 1 \Leftrightarrow C \geq 1. \quad (35)$$

In other words, for iso-elastic insurance demand, pooling produces higher loss coverage than actuarially fair premiums if demand elasticity is less than 1, and vice versa.

#### 6.4. Social welfare

Using the convenient standardisation of  $U(W) = 1$  and  $U(W - L) = 0$  as used in Equation (3) and noting that social welfare  $S$  is a function of the risk-classification regime  $\underline{\pi} = (\pi_1, \pi_2, \dots, \pi_n)$ , Equation (12) becomes:

$$S(\underline{\pi}) = E[Q\{U_\Gamma(W - \Pi L) - (1 - X)\}] + Z, \quad (36)$$

where  $Z = E[1 - X]$  is a constant as it does not depend on  $\underline{\pi}$ .

Using the power utility function as given in Equation (21) which corresponds to iso-elastic demand, we have:

$$S(\underline{\pi}) = E[Q\{(1 - \Pi)^\Gamma - (1 - X)\}] + Z, \quad (37)$$

$$\approx E[Q(1 - \Gamma\Pi - 1 + X)] + Z, \quad \text{assuming small premiums,} \quad (38)$$

$$= E[Q(X - \Gamma\Pi)] + Z, \quad (39)$$

$$= E[QX] - E[Q\Gamma\Pi] + Z. \quad (40)$$

The first term in Equation (40) is the standardised loss coverage  $K(\underline{\pi})$  as defined in Equation (9). The second term is the ‘premium adjustment’ factor  $PA(\underline{\pi})$  under the risk-classification regime  $\underline{\pi}$ . In the general case given earlier in Equation (20),  $PA(\underline{\pi})$  depended on *unobservable* risk preferences. But for the iso-elastic case, we show in Appendix 2 that  $PA(\underline{\pi})$  can be expressed as a function of *observable* standardised loss coverage:

$$PA(\underline{\pi}) = \frac{\lambda}{\lambda + 1} K(\underline{\pi}). \quad (41)$$

Hence social welfare in Equation (40) becomes:

$$S(\underline{\pi}) = \frac{1}{\lambda + 1} K(\underline{\pi}) + Z. \quad (42)$$

The right-hand side Equation (42) can be interpreted as follows. The second term  $Z = E[1 - X]$  corresponds to expected utility in the absence of the institution of insurance (recall that we have standardised  $U(W) = 1$ ,  $U(W - L) = 0$ , and  $X$  is the Bernoulli variable denoting the occurrence of the loss event for an individual drawn at random from the population). The first term represents an increase in expected utility, attributable to the institution of insurance; this allows for the expectations of both utility of benefits received, and disutility of premiums paid. If  $\lambda$  is smaller (corresponding to inelastic demand and high risk aversion), the premiums paid are relatively less important, so the increase in expected utility arising from the institution of insurance is larger.

The form of Equation (42) implies the following result.

**Result 2:** Suppose demand elasticity is a positive constant and we have two risk classification schemes  $\underline{\pi}_1$  and  $\underline{\pi}_2$ , which give social welfare  $S(\underline{\pi}_1)$  and  $S(\underline{\pi}_2)$ , and standardised loss coverage  $K(\underline{\pi}_1)$  and  $K(\underline{\pi}_2)$ . Then

$$S(\underline{\pi}_1) \geq S(\underline{\pi}_2) \Leftrightarrow K(\underline{\pi}_1) \geq K(\underline{\pi}_2). \quad (43)$$

In other words: for iso-elastic insurance demand, ranking risk classification schemes by loss coverage always gives the same ordering as ranking by social welfare.

The proof of the result follows directly from the form of Equation (42), and noting that for the logical biconditional statement in the result, the contrapositive (i.e. with both inequalities reversed) also holds.

**Table 1.** Estimates of demand elasticity for various insurance markets.

Market and country	Demand elasticities <sup>a</sup>	Authors
Term life insurance, USA	−0.66	Viswanathan et al. (2006)
Yearly renewable term life, USA	−0.4 to −0.5	Pauly et al. (2003)
Whole life insurance, USA	−0.71 to −0.92	Babbel (1985)
Health insurance, USA	0 to −0.2	Chernew et al. (1997), Blumberg et al. (2001), Buchmueller & Ohri (2006)
Health insurance, Australia	−0.35 to −0.50	Butler (2002)
Farm crop insurance, USA	−0.32 to −0.73	Goodwin (1993)

<sup>a</sup>Estimates in empirical papers are generally given with the negative sign, so we have quoted them here in that form.

Result 2 holds for *any* pair of risk classification schemes which satisfy the equilibrium condition in Equation (7). This includes schemes where premiums are partly (but not fully) risk-differentiated, as well as the polar cases of pooling and actuarially fair premiums. Where the comparison is between the polar cases, Results 1 and 2 together immediately imply the following result.

**Result 3:** Suppose demand elasticity is a positive constant  $\lambda$ . Then

$$\lambda \leq 1 \Leftrightarrow S(\pi_0) \geq S(\mu). \quad (44)$$

In other words, for iso-elastic insurance demand, pooling produces higher social welfare than actuarially fair premiums if demand elasticity is less than 1, and vice versa.

In view of the significance of  $\lambda \geq 1$ , at least for iso-elastic demand, it is of interest to consider how this critical value compares with insurance demand elasticities in the real world. Table 1 shows empirical estimates by various authors. These estimates were made in varying contexts, none of which corresponds precisely to the set-up of this paper. Nevertheless, the figures are at least suggestive of the possibility that insurance demand elasticities may often be less than 1.

## 7. Conclusions

Adverse selection is associated with a fall in the number of insured individuals compared with that obtained under full risk classification, which is usually seen as a bad outcome, both for insurers and for society. However, adverse selection is also associated with a shift in coverage towards higher risks. If this shift is large enough, it can more than outweigh the fall in numbers insured, so that loss coverage is increased. Since this implies that more risk is voluntarily traded and more losses are compensated, it is a counter-argument to the perception of a bad outcome for society.

We have shown that loss coverage can be reconciled with (although it is not the same as) an ‘equal-weights’ utilitarian social welfare, in the spirit of Hoy (2006) or Dionne & Rothschild (2014). Specifically, if insurance demand is iso-elastic, ranking risk classification schemes by loss coverage always gives the same ordering as ranking by social welfare. So under these conditions, a policy-maker or regulator could implement a risk classification scheme which gives higher (observable) loss coverage, with the comfort of knowledge that this would also give higher (unobservable) social welfare.

For iso-elastic demand, both loss coverage and utilitarian social welfare are higher under pooling than under actuarially fair premiums if insurance demand elasticity is less than 1. There is some evidence that insurance demand elasticities in the real world may often be less than 1.

This work could be extended in both empirical and theoretical directions. Empirically, we could investigate how reasonable the iso-elastic model is as an approximation for insurance demand in particular markets. Theoretically, it may be possible generalise our main results for a wider class of insurance demand functions. However, both these extensions are left for future research.

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## Appendices

### Appendix 1. Utility calculations for Section 2

Individuals’ insurance purchasing decisions are based on the certainty-equivalent principle in Equation (3) and the utility function  $U(w) = w^\gamma$ , which is a special case of the utility function in Equation (21) with  $W = L = 1$ , giving the decision criterion:

$$(1 - \pi)^\gamma > (1 - \mu). \tag{A1}$$

Under risk-differentiated premiums (Scenario 1),  $\pi = \mu$  for all individuals. The decision criterion is then clearly true for all  $\gamma < 1$  (risk-averters), and false for all  $\gamma > 1$  (risk-lovers). So the four low risks and two high risks who have  $\gamma = 0.3$  or  $0.7$  purchase insurance, and the two low risks and one high risk who have  $\gamma = 1.1$  do not.

For Scenarios 2 and 3, substituting the actual pooled premiums charged of 0.028 and 0.04 into the same decision criterion yields all the purchasing decisions shown in Figure 1.

The social welfare calculations are based on on Equation (12), again with the utility function in Equation (21):

$$S(\pi) = E[Q(1 - \Pi)^\Gamma + (1 - Q)(1 - X)]. \tag{A2}$$

An individual’s ‘contribution’ to the utility sum needed to evaluate social welfare is then  $(1 - \pi)^\gamma$  if he purchases insurance (first product in Equation A.2), and  $(1 - \mu)$  if he does not (second product). Under risk-differentiated premiums (Scenario 1), given the purchasing decisions determined above, this leads to the utility sum:

$$\begin{aligned} \text{Low risks : } & \underbrace{2 \times 0.99^{0.3} + 2 \times 0.99^{0.7}}_{\text{purchasers}} + \underbrace{2 \times 0.99}_{\text{non-purchasers}} \\ & + \text{High risks : } \underbrace{1 \times 0.96^{0.3} + 1 \times 0.96^{0.7}}_{\text{purchasers}} + \underbrace{1 \times 0.96}_{\text{non-purchaser}} \\ & = 8.8796. \end{aligned} \tag{A3}$$

For Scenarios 2 and 3, analogous calculations using the actual premiums charged in those scenarios give utility sums of 8.8841 for Scenario 2, and 8.8597 for Scenario 3. A no-insurance scenario gives 8.8200. Dividing each of these utility sums by nine (reflecting the population of nine individuals) and taking differences from the no-insurance scenario gives the increases in social welfare noted in Section 2.

### Appendix 2. Analysis of the ‘premium adjustment’ $PA(\pi)$

First, recall that  $Q$  is an indicator random variable which takes the value of 1 if insurance is purchased; 0 otherwise. And from Equation (27), given a risk-group  $i$ , insurance is purchased when  $\Gamma_i < \mu_i/\pi_i$ , where the random variable  $\Gamma_i = [\Gamma \mid \mu = \mu_i]$ . Hence:

$$[Q \mid \mu = \mu_i] = I \left[ \Gamma_i < \frac{\mu_i}{\pi_i} \right]. \tag{A4}$$

So:

$$PA(\pi) = \sum_{i=1}^n E [Q\Gamma\Pi \mid \mu = \mu_i] P[\mu = \mu_i], \tag{A5}$$

$$= \sum_{i=1}^n E \left[ I \left[ \Gamma_i \leq \frac{\mu_i}{\pi_i} \right] \Gamma_i \pi_i \right] p_i, \tag{A6}$$

$$= \sum_{i=1}^n E \left[ \Gamma_i I \left[ \Gamma_i \leq \frac{\mu_i}{\pi_i} \right] \right] \pi_i p_i. \tag{A7}$$

Using the cumulative distribution function of  $\Gamma_i$ , as given in Equation (28):

$$P[\Gamma_i \leq \gamma] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau_i \gamma^\lambda & \text{if } 0 \leq \gamma \leq (1/\tau_i)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau_i)^{1/\lambda}, \end{cases} \tag{A8}$$

Equation (A.7) becomes:

$$PA(\underline{\pi}) = \sum_{i=1}^n \left[ \int_0^{\mu_i/\pi_i} \gamma \tau_i \lambda \gamma^{\lambda-1} d\gamma \right] \pi_i p_i, \quad (\text{A9})$$

$$= \frac{\lambda}{(\lambda+1)} \sum_{i=1}^n \left( \frac{\mu_i}{\pi_i} \right)^{\lambda+1} \tau_i \pi_i p_i, \quad (\text{A10})$$

$$= \frac{\lambda}{(\lambda+1)} \sum_{i=1}^n \frac{\mu_i^{\lambda+1}}{\pi_i^\lambda} \tau_i p_i, \quad (\text{A11})$$

$$= \frac{\lambda}{(\lambda+1)} K(\underline{\pi}), \quad \text{by Equation (33)}. \quad (\text{A12})$$