INSURANCE LOSS COVERAGE AND SOCIAL WELFARE

MingJie Hao, Angus S. Macdonald, Pradip Tapadar and R. Guy Thomas*

ABSTRACT

Restrictions on insurance risk classification may induce adverse selection, which is usually perceived to reduce efficiency. We suggest a counter-argument to this perception in circumstances where modest adverse selection leads to an increase in 'loss coverage', defined as the expected losses compensated by insurance for the whole population. This happens if the shift in coverage towards higher risks under adverse selection more than outweighs the fall in number of individuals insured. We also reconcile the concept of 'loss coverage' and a utilitarian concept of social welfare. For iso-elastic insurance demand, ranking risk classification schemes by (observable) loss coverage always gives the same ordering as ranking by (unobservable) utilitarian social welfare.

KEYWORDS

Adverse selection; loss coverage; social welfare. J.E.L. Classification: D82, G22.

*Hao: School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK. Macdonald: Department of Actuarial Mathematics and Statistics, and the Maxwell Institute for Mathematical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, UK. Tapadar: School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK. Corresponding author: Pradip Tapadar, School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK. Email: P.Tapadar@kent.ac.uk, Phone: +44 1227 824169, Fax: +44 1227 827932. Thomas: School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK.

1 Introduction

Restrictions on insurance risk classification are pervasive in life insurance and other personal insurance markets. For example, gender classification in insurance pricing has been banned in the European Union since 2012; in the US, the Patient Protection and Affordable Care Act allows classification only by age, location, family size and smoking status; and many countries have restricted insurers' use of genetic test results. Such restrictions may increase equity, but they also create asymmetries in (the use of) information, and hence are usually seen by economists as reducing efficiency.

A simple version of the usual efficiency argument is as follows. If insurers are not permitted to charge risk-differentiated prices, they have to pool different risks at a common pooled price.[1] This pooled price is cheap for higher risks and expensive for lower risks; so more insurance is bought by higher risks, and less insurance is bought by lower risks. The equilibrium pooled price of insurance is higher than a population-weighted average of true risk premiums. Also, in most markets the number of higher risks is smaller than the number of lower risks, so the total number of risks insured falls. The usual efficiency argument focuses on this reduction in coverage, e.g. "This reduced pool of insured individuals reflects a decrease in the efficiency of the insurance market" (Dionne and Rothschild [2014, p185]).

[1] In this paper we disregard the possibility that insurers banned from classifying risks induce separation of risk-groups by menus of contracts offering different levels of cover priced at different rates (e.g. Rothschild and Stiglitz [1976]). Our reasons are explained in the literature review section.

In this paper, we suggest that in some circumstances, there is a counter-argument to this perception of reduced efficiency. The rise in equilibrium price under pooling reflects a shift in coverage towards higher risks. If the shift in coverage is large enough, it can more than outweigh the fall in numbers insured. In these circumstances, despite fewer risks being insured under pooling, expected losses compensated by insurance – a quantity we term 'loss coverage' – can be higher. We argue that where risk classification restrictions lead to higher 'loss coverage' – that is, more risk being voluntarily transferred and more losses being compensated – this should not be regarded as inefficient.

This paper also reconciles the concept of 'loss coverage' to a utilitarian concept of social welfare. Specifically, we show that if insurance demand is iso-elastic, the risk classification scheme which maximises loss coverage also maximises social welfare. From a policymaker's perspective, this may be a useful result because maximising loss coverage does not require knowledge of individuals' (generally unobservable) utility functions; loss coverage is based solely on observable quantities.

2 Motivating example

The possibility that loss coverage may be increased by restrictions on risk classification can be illustrated by heuristic examples of insurance market equilibria under two alternative risk classification regimes: actuarially fair premiums and pooled premiums.

Suppose that in a population of 2,000 risks, 32 losses are expected every year. There are two risk-groups. The high risk-group of 400 individuals has a probability of loss 4 times higher than those in the low risk-group. This is summarised in Table 1.

We assume that probability of loss is not altered by the purchase of insurance, i.e. there is no moral hazard. An individual's risk-group is fully observable to insurers and all insurers are required to use the same risk classification regime. The equilibrium price of insurance is determined as the price at which insurers make zero profit.

Under the first risk classification regime, insurers charge premiums which are actuarially fair to members of each risk-group. The proportion of each risk-group which buys insurance under these conditions, i.e. the 'fair-premium proportional demand', is 50%, in line with industry statistics. Table 1 shows the outcome. Half the losses in the population are compensated by insurance. We heuristically characterise this as a 'loss coverage' of 0.5.

Now suppose that a new risk classification regime is introduced, where insurers have to charge a single 'pooled' price to members of both the low and high risk-groups. One possible outcome is shown in Table 2, which can be summarised as follows:

(a) The pooled premium of 0.0194 at which insurers make zero profits is calculated as the demand-weighted average of the risk premiums: (600 x 0.01 + 275 x 0.04)/875 = 0.0194).

Table 1: Equilibrium under actuarially fair premiums: lower loss coverage.

	Risk-group Low risk	High risk	Aggregate
Risk	0.01	0.04	0.016
Total population	1600	400	2,000
Expected population losses	16	16	32
Break-even premiums (differentiated)	0.01	0.04	0.016
Numbers insured	800	200	1,000
Insured losses	8	8	16
Loss coverage			0.5

(b) The pooled premium is expensive for low risks, so fewer of them buy insurance (600, compared with 800 before). The pooled premium is cheap for high risks, so more of them buy insurance (275, compared with 200 before). Because there are 4 times as many low risks as high risks in the population, the total number of policies sold falls (875, compared with 1,000 before).

(c) The resulting loss coverage is 0.53125. The shift in coverage towards high risks more than outweighs the fall in number of policies sold: 17 of the 32 losses (53%) in the population as a whole are now compensated by insurance (compared with 16 of 32 before).

The occurrence of the favourable outcome (higher loss coverage) under asymmetric information and pooling in Table 2 depends on the demand elasticities for insurance in high and low risk groups. Later in this paper, we will show that the required demand elasticities are plausible.

Table 2: Equilibrium under pooled premiums: higher loss coverage.

	Risk-group		
	Low risk	High risk	Aggregate
Risk	0.01	0.04	0.016
Total population	1600	400	2000
Expected population losses	16	16	32
Break-even premiums (pooled)	0.0194	0.0194	0.0194
Numbers insured	600	275	875
Insured losses	6	11	17
Loss coverage			0.53125

3 Literature Review

The model of insurance markets implied by the heuristic example above differs from canonical models derived from Rothschild and Stiglitz [1976] in two main ways.

First, in our model insurers compete only on price; they do not induce separation of risk-groups by menus of contracts offering different levels of cover priced at different rates. In this respect, our model is more in the spirit of Akerlof [1970]. We justify this approach by noting that some important markets, such as life insurance, have non-exclusive contracting, and so separation via contract menus is not feasible. Furthermore, as far as we are aware, the concept of separation via contract menus is also not salient to practitioners in other markets where restrictions on risk classification apply, for example auto insurance in the European Union.[2]

[2] As regards life insurance, Rothschild-Stiglitz type models are inconsistent in important

Second, in our model agents with identical probabilities of loss can have different utility functions, and so unlike the representative agents from each risk-group in Rothschild-Stiglitz type models, they do not all make the same purchasing decision. This leads to an equilibrium where not all agents are insured; this corresponds to the empirical reality of most voluntary insurance markets.[3]

Previous papers on loss coverage in the actuarial literature (Hao et al. [2016], Thomas [2008, 2009]) formalised the heuristic example above by a model with two risk-groups with higher and lower probabilities of loss. Insurance demand from each risk-group at each price was modelled by a demand function with output a number between 0 and 1, to reflect the empirical observation that not all individuals buy insurance at each price. The variation in purchasing decisions across persons within each risk-group (i.e. with the same probabilities of loss) was characterised as stochastic; no reference was made to individual utilities.

The loss coverage literature contrasts with economic literature on insur-

ways with empirical data (e.g. Cawley and Philipson [1999]). As regards practice in other insurance markets, most recent actuarial pricing textbooks make no reference to the concept of menus of contracts as screening devices (e.g. Gray and Pitts [2012], Friedland [2013], Parodi [2014]). Other textbooks specifically recommend *against* using the level of deductible as a pricing factor (e.g. Ohlsson and Johansson [2010]).

[3]For example, in life insurance, the Life Insurance Market Research Organisation (LIMRA) states that 44% of US households have some individual life insurance (LIMRA [2013]). The American Council of Life Insurers states that 144m individual policies were in force in 2013 (American Council of Life Insurers [2014, p72]); the US adult population (aged 18 years and over) at 1 July 2013 as estimated by the US Census Bureau was 244m. In health insurance, only 14.6% of the US population has individually purchased private cover (US Census Bureau, 2015), albeit substantially more have employer group cover or Medicare or Medicaid government cover.

ance risk classification, as summarised in surveys such as Hoy [2006], Einav and Finkelstein [2011] and Dionne and Rothschild [2014]. Economic literature typically takes a utility-based approach: representative agents from each risk-group make purchasing decisions which maximise their expected utilities, and the outcomes of different risk classification schemes are then evaluated by a social welfare function which is a (possibly weighted) sum of expected utilities over the whole population. For example Hoy [2006] uses a utilitarian social welfare function which assigns equal weights to the utilities of all individuals. Einav and Finkelstein [2011] use a deadweight-loss concept which appears equivalent to a social welfare function with utilities cardinalized so as to weight willingness-to-pay equally across all individuals.

The present paper connects the loss coverage literature with the economic literature in two ways. First, we provide a utility-based micro-foundation for the proportional insurance demand function, driven by variations between individuals in their utility functions, which can explain why only a proportion of the individuals in each risk-group buy insurance at each price. Second, we reconcile loss coverage to the utilitarian concept of social welfare described above. Specifically, we show that if insurance demand is iso-elastic, the risk classification scheme which maximises loss coverage also maximises social welfare.

The rest of this paper is organised as follows. Section 4 considers a single risk-group, that is individuals all with the same probabilities of loss, but who have a range of utility functions. This set-up leads to a proportional demand between 0 and 1, representing the proportion of individuals from the risk-group who buy insurance at a given premium. Section 5 considers an insurance market where there are two or more risk-groups as described in Section 4, in a setting where risk classification is banned and insurers compete only on price (not contract offers); this leads to a pooling equilibrium. We show that loss coverage in the pooling equilibrium can be higher than in a market where risk classification is permitted, if demand elasticity is sufficiently low. Section 6 establishes the link between loss coverage and social welfare. Section 7 offers brief conclusions.

4 Insurance Demand for a Single Risk-group

4.1 Utility of Wealth and Certainty Equivalence

Consider an individual with an initial wealth W, who is exposed to the risk of losing an amount of L with probability μ. Suppose preference for wealth is driven by the utility function $U(w)$, which is increasing in wealth w, i.e. $U'(w) > 0$.

Individuals are typically also assumed to be risk-averse i.e. $U''(w) < 0$. This provides the motivation for insurance purchase at an actuarially fair price, and initially we shall discuss individuals for whom the assumption holds. But we shall see later in Section 4.2 that our theory of insurance demand does *not* require that *all* individuals are risk-averse. Figure 1 shows an example of a utility function $U(w)$ with $U'(w) > 0$ and $U''(w) < 0$.

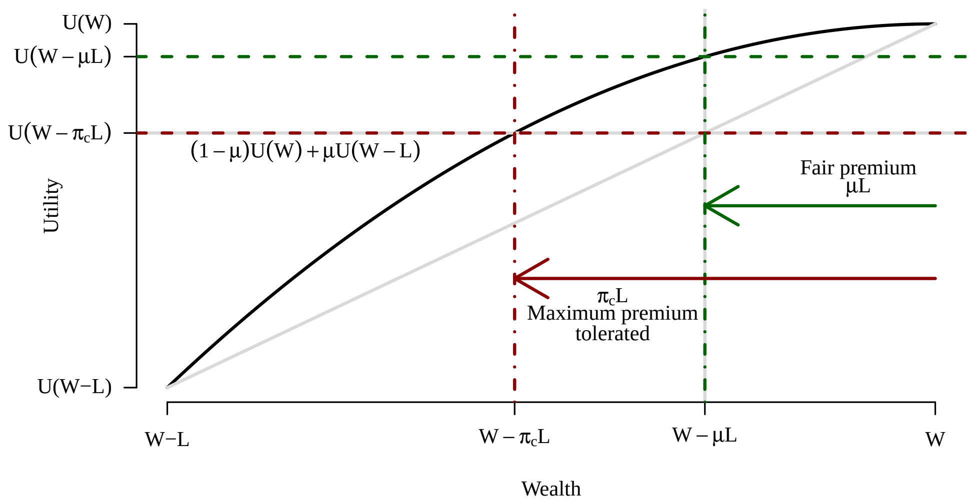

Figure 1: Insurance purchasing decision based on an individual's utility of wealth.

If no insurance is bought, occurrence of the risk event will reduce the individual's wealth from W to $(W-L)$ with probability μ. Hence the individual's expected utility, without insurance, is given by:

$$(1-\mu)U(W) + \mu\, U(W-L). \tag{1}$$

If, however, the individual has the option to insure against the risk at premium rate π per unit of loss and chooses to buy insurance for full cover, the individual's expected utility is:

$$U(W-\pi L), \tag{2}$$

because the individual's wealth diminishes immediately by the amount of premium, but there is no further uncertainty as the loss is insured.

An individual will choose to buy insurance if the expected utility is higher with insurance than without it, i.e.

$$U(W - \pi L) > (1 - \mu)U(W) + \mu\, U(W - L). \tag{3}$$

In particular, individuals with concave utility functions will buy insurance at the actuarially fair premium $\pi = \mu$. Furthermore, these individuals will be prepared to purchase insurance up to the premium level π_c, where:

$$U(W - \pi_c L) = (1 - \mu)U(W) + \mu\, U(W - L), \tag{4}$$

which is also known as the certainty-equivalence principle. This is depicted in Figure 1.

4.2 Heterogeneity in Insurance Purchasing Behaviour

In the above model, all individuals with the same utility function and probability of loss either buy insurance or they do not, based on whether or not the premium being charged, π, exceeds π_c. However, in real insurance markets, we typically observe that not all individuals with the same probability of loss make the same purchasing decision (e.g. for life insurance, see the figures in footnote 3). How can this variation in insurance purchasing decisions be explained?

One plausible explanation suggested by a number of authors (e.g. Finkelstein and McGarry [2006]; Cutler et al. [2008]) is that risk preferences vary between individuals. To formulate this variability, let us assume a population of individuals, all with the same risk μ but who may have different utility functions. Suppose for simplicity that utility functions belong to a family parameterized by a positive real number γ. So a particular individual's utility function can be denoted by $U_\gamma(w)$.

Further suppose that an individual's utility function parameter γ is sampled randomly from an underlying random variable Γ with distribution function $F_\Gamma(\gamma)$. So, a particular individual's utility function, $U_\gamma(w)$, is a random quantity[4], the randomness being induced by $F_\Gamma(\gamma)$.

Based on this formulation, an individual will choose to buy insurance if and only if the following condition is satisfied for the combination of the offered premium π and their particular utility function $U_\gamma(w)$:

$$U_\gamma(W - \pi L) > (1 - \mu)U_\gamma(W) + \mu\, U_\gamma(W - L), \tag{5}$$

Note that all individuals are behaving deterministically, given their knowledge.

Although utility functions of different individuals can have different origins and scales, certainty-equivalent decisions are independent of these choices.

[4]We must be careful not to call the function $U_\gamma(w)$ a random variable. We shall have no need of any of the metric structure of spaces of functions that this would entail.

So without loss of generality, we will assume that all individuals have the same utility at the “end points” $W - L$ and W. And for clarity, we will suppress the subscript γ for the utility at the “end points” and write $U(W)$ and $U(W - L)$ as they are the same for all individuals. We can then write Equation (5) as:

$$U_\gamma(W - \pi L) > u_c \text{ where} \tag{6}$$

$$u_c = (1 - \mu)U(W) + \mu U(W - L) \text{ is a constant.} \tag{7}$$

This says that an individual insures if the utility from insurance exceeds a critical value u_c. Note that u_c is the same for all individuals who are exposed to the same probability of loss.

Figure 2 provides a graphical representation showing utility functions of four individuals with the same probability of loss μ. The concave utility curves, with points A, B and C, represent risk-averse individuals, where higher concavity represents higher risk-aversion. We also show a convex utility curve, with point D, which represents a risk-loving (or risk-neglecting) individual. (As mentioned previously, the model does not require that all individuals are risk-averse.) For the individual at point A, the utility with insurance, $U_{\gamma_A}(W - \pi L)$, exceeds the critical value u_c, where γ_A is the individual's utility function parameter. So the individual buys insurance. For the individuals at points C and D, the inverse applies, so they do not purchase insurance. The individual at point B is indifferent.

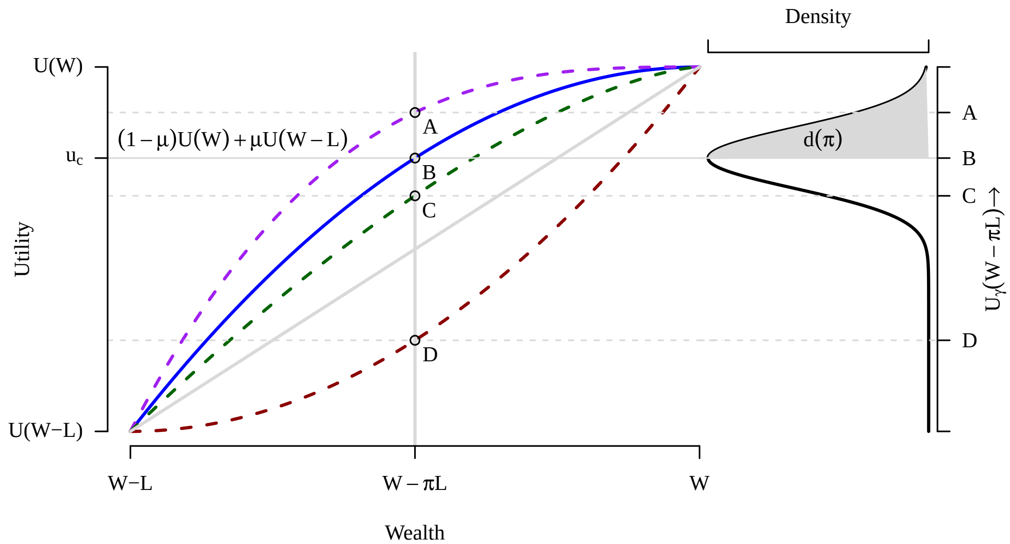

Figure 2: Heterogeneous utility functions within a risk-group, leading to proportional insurance demand.

The utility at the fixed wealth $(W - \pi L)$ is a random variable, that we denote by $U_\Gamma(W - \pi L)$. The distribution function of $U_\Gamma(W - \pi L)$ is induced by that of Γ and we denote it by $G_\Gamma(\gamma)$. The corresponding probability density function of the utilities at that level of wealth is shown in the rotated plot on the right-hand side of Figure 2.

Now assume that the insurer cannot observe individuals' utility functions. Then, for given offered premium π, all the insurer can observe of insurance purchasing behaviour is the proportion of individuals who buy insurance. We call this a demand function and denote it by $d(\pi)$. We have:

$$d(\pi) = \mathrm{P}\left[U_\Gamma(W - \pi L) > u_c\right] = 1 - G_\Gamma(u_c). \tag{8}$$

Insurance purchase is denoted by the shaded area, $d(\pi)$, under the density graph for $U_\Gamma(W - \pi L)$.

We note the following three properties of demand for insurance:

(a) $d(\pi)$, denotes a proportion, as $0 \leq d(\pi) \leq 1$ is a valid probability.

(b) $d(\pi)$ is non-increasing in π, i.e. demand for insurance cannot increase when premium increases. This can be shown as follows: For utility functions with $U'(w) > 0$, if $\pi_1 < \pi_2$, the random variable $U_\Gamma(W - \pi_1 L)$ is statewise dominant[5] over the random variable $U_\Gamma(W - \pi_2 L)$. So,

$$\begin{aligned} \pi_1 < \pi_2 \Rightarrow \mathrm{P}\left[U_\Gamma(W - \pi_1 L) > u_c\right] &\geq \mathrm{P}\left[U_\Gamma(W - \pi_2 L) > u_c\right] \\ \Rightarrow d(\pi_1) &\geq d(\pi_2). \end{aligned} \tag{9}$$

(c) Each individual's decision is completely deterministic, given what they know. But to the insurer it appears stochastic, given what the insurer knows. In respect of any individual chosen randomly, define the function Q to be $Q = 1$ if they buy insurance or $Q = 0$ if they do not. To the individual concerned, Q is a deterministic function. To the insurer, Q

[5] One random variable is statewise dominant over a second if the first is at least as high as the second in all states of nature, with strict inequality for at least one state. It is an absolute form of dominance.

is a Bernoulli random variable with parameter $d(\pi)$. A full probabilistic model accounting for these different levels of information is given in Appendix A.

As noted earlier, certainty equivalent decisions do not depend on the origins and scales of utility functions, so we can standardise the utility functions such that all individuals have the same utilities $U(W)$ and $U(W-L)$ at the "end points" W and $W-L$. The following standardisation is convenient:

$$U(W) = 1, \tag{10}$$

$$U(W-L) = 0. \tag{11}$$

The constant u_c in Equation 8 then becomes $(1-\mu)$, and so the demand for insurance is:

$$d(\pi) = \mathrm{P}[U_\Gamma(W - \pi\, L) > 1 - \mu]. \tag{12}$$

4.3 Iso-elastic Demand

This sub-section gives an illustrative example of the link from a specific distribution of risk preferences to a specific proportional demand for insurance where individuals are exposed to the same probability of loss.

Suppose $W = L = 1$ with a power utility function:

$$U_\gamma(w) = w^\gamma, \tag{13}$$

so that $U_\gamma(0) = 0$ and $U_\gamma(1) = 1$. This particular form of utility function leads to:

$$\text{relative risk aversion coefficient:} \; -w\frac{U''_\gamma(w)}{U'_\gamma(w)} = 1 - \gamma. \tag{14}$$

So the heterogeneity in preferences between individuals can be modelled through the randomness of the risk aversion parameter γ. As outlined in Section 4.2, we define a positive random variable Γ, and individual risk preferences γ are then instances drawn from the distribution of Γ.

Demand for insurance at a given premium π is then:

$$\begin{aligned}
d(\pi) &= \mathrm{P}\left[U_\Gamma(1-\pi) > 1-\mu\right], && (15)\\
&= \mathrm{P}\left[(1-\pi)^\Gamma > 1-\mu\right], && (16)\\
&= \mathrm{P}\left[\Gamma\log(1-\pi) > \log(1-\mu)\right], \text{ as log is monotonic,} && (17)\\
&= \mathrm{P}\left[\Gamma < \frac{\log(1-\mu)}{\log(1-\pi)}\right], \text{ as } \log(1-\pi) < 0, && (18)\\
&\approx \mathrm{P}\left[\Gamma < \frac{\mu}{\pi}\right], \text{ as } \log(1-x) \approx -x, \text{ for small } x. && (19)
\end{aligned}$$

Now suppose Γ has the following distribution:

$$F_\Gamma(\gamma) = \mathrm{P}\left[\Gamma \le \gamma\right] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau\,\gamma^\lambda & \text{if } 0 \le \gamma \le (1/\tau)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau)^{1/\lambda}, \end{cases} \tag{20}$$

where τ and λ are positive parameters. Note that $\tau = \lambda = 1$ leads to a uniform distribution. λ controls the shape of the distribution function and τ controls the range over which Γ takes its values.[6]

Based on this distribution for Γ, the demand for insurance in Equation (19) takes the form:

$$d(\pi) = \tau \left(\frac{\mu}{\pi}\right)^{\lambda}, \tag{21}$$

subject to a cap of 1 (when all members of a risk-group purchase insurance, demand cannot increase further). This corresponds to iso-elastic demand, the constant demand elasticity being:

$$\epsilon(\pi) = -\frac{\partial \log(d(\pi))}{\partial \log \pi} = \lambda. \tag{22}$$

The parameter τ can also be interpreted as the *fair-premium demand*, that is the demand when an actuarially fair premium is charged.

The illustrative numerical example given in Section 2 can then be shown to correspond to this iso-elastic demand function, with fair-premium demand $\tau = 0.5$ and constant demand elasticity $\lambda = 0.435$ for both risk-groups. These are reasonable parameters.[7]

An important point to note here is that power utility function of the form

[6]This is a generalised version of the Kumaraswamy distribution, which in its standard form takes values only over [0,1] (Kumaraswamy [1980]).

[7]Approximately half the population has some life insurance (see footnote 3). For yearly renewable term insurance in the US, demand elasticity has been estimated at 0.4 to 0.5 (Pauly et al. [2003]). A questionnaire survey about life insurance purchasing decisions produced an estimate of 0.66 (Viswanathan et al. [2006]).

given in Equation 13 is concave only if the risk aversion parameter γ is less than 1. Such a constraint can be imposed on random variable Γ by setting $\tau = 1$ in Equation (20). Then the third branch of Equation (20) implies that $d(\pi) = 1$ for $\pi < \mu$, which corresponds to the standard assumption in the economics literature that all individuals are risk-averse and hence will buy insurance for premiums not exceeding their probability of loss. By permitting some individuals to be 'risk-lovers', the model better represents the partial take-up of insurance which is observed in practice. Although 'risk-loving' or 'risk-seeking' are the usual descriptions, 'risk-neglecting' might be a more realistic one.

5 Equilibrium and Loss Coverage for Two or More Risk-groups

5.1 Framework for Insurance Risk Classification

In Section 4, we have developed a framework for insurance demand based on heterogeneous risk preferences of individuals who have the same wealth W and the same probabilities of loss amount L. In this section, we sketch a generalised framework to allow individuals to belong to different risk-groups having different loss probabilities. Full details are in Appendix A.

For simplicity, we assume all wealth and losses are of unit amount, that is $W = L = 1$. Suppose the population can be sub-divided into n distinct risk-groups with probabilities of loss given by $\mu_1, \mu_2, \ldots, \mu_n$. Without loss of generality, we assume the risk-groups are indexed in increasing order of risk, i.e. $0 < \mu_1 < \mu_2 < \ldots < \mu_n < 1$.

Let μ be a random variable denoting the probability of loss for an individual chosen at random from the whole population, such that $\mathrm{P}[\mu = \mu_i] = p_i$ for $i = 1, 2, \ldots, n$. In other words, the proportion of the population belonging to risk-group i is p_i.

Suppose insurers charge premiums $\pi_1, \pi_2, \ldots, \pi_n$ for the respective risk-groups. Initially we do not impose any constraints on the order or size of insurance premiums, so that the insurers are free to charge any premiums to any risk-group. Based on the framework developed in Section 4, we denote the demand for insurance for risk-group i, given offered premium π_i, by $d_i(\pi_i)$, where $0 \leq d_i(\pi_i) \leq 1$ and $d_i(\pi_i)$ is non-increasing in π_i.

Let the insurance purchasing decision of an individual chosen at random from the whole population be represented by the indicator random variable Q, taking the value of 1 if insurance is purchased; and 0 otherwise. Then conditional on the risk-group, Q is a Bernoulli random variable defined by:

$$\mathrm{E}[\, Q \mid \mu = \mu_i \,] = \mathrm{P}[\, Q = 1 \mid \mu = \mu_i \,] = d_i(\pi_i). \tag{23}$$

Then the expected population demand for insurance is the unconditional expected value of Q:

$$\mathrm{E}[\,Q\,] = \sum_{i=1}^{n} \mathrm{E}[\,Q \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i] = \sum_{i=1}^{n} d_i(\pi_i)p_i. \tag{24}$$

Now suppose that the occurrence of a loss event for an individual chosen at random from the whole population is represented by the indicator random variable, X, taking the value of 1 if a loss event has occurred; and 0 otherwise. Then X is a Bernoulli random variable defined as:

$$\mathrm{E}[\,X \mid \mu = \mu_i\,] = \mathrm{P}[\,X = 1 \mid \mu = \mu_i\,] = \mu_i. \tag{25}$$

Then the expected population loss is the unconditional expected value of X:

$$\mathrm{E}[\,X\,] = \sum_{i=1}^{n} \mathrm{E}[\,X \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i] = \sum_{i=1}^{n} \mu_i p_i. \tag{26}$$

We assume that Q and X are independent, conditional on $\mu = \mu_i$. That is, the level of risk may influence the decision to buy insurance, but there is no moral hazard; insured individuals in any risk-group are not more likely to suffer the loss event than uninsured individuals. Then the expected claims outgo for insurers is:

$$
\begin{aligned}
\mathrm{E}[\,QX\,] &= \sum_{i=1}^{n} \mathrm{E}[\,QX \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i], \\
&= \sum_{i=1}^{n} \mathrm{E}[\,Q \mid \mu = \mu_i\,]\,\mathrm{E}[\,X \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i], \\
&= \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i. \qquad (27)
\end{aligned}
$$

Finally, for an individual chosen at random from the whole population, define random variable Π, as the premium paid by that individual. As premiums are only paid by individuals who purchase insurance, $\Pi = Q\Pi$. And since everybody in risk-group i is offered the same premium π_i, we have:

$$\mathrm{E}[\,\Pi \mid \mu = \mu_i\,] = \mathrm{E}[\,Q\Pi \mid \mu = \mu_i\,] = \mathrm{E}[\,Q \mid \mu = \mu_i\,]\pi_i = d_i(\pi_i)\pi_i. \qquad (28)$$

Then the unconditional expected premium income is:

$$\mathrm{E}[\,\Pi\,] = \sum_{i=1}^{n} \mathrm{E}[\,\Pi \mid \mu = \mu_i\,]\,\mathrm{P}[\mu = \mu_i] = \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i. \qquad (29)$$

The expected profit for insurers, as a function of risk-classification regime $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$, is then :

$$\rho(\underline{\pi}) = E[\Pi] - E[QX] = \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i - \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i. \qquad (30)$$

5.2 Equilibrium in the Insurance Market

Equilibrium is achieved when the expected profit for insurers is zero. In other words, $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$ denotes an equilibrium, if it satisfies the *equilibrium condition*:

$$\rho(\underline{\pi}) = 0 \;\Leftrightarrow\; \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i - \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i = 0. \tag{31}$$

A full probabilistic model, of heterogeneity in insurance purchasing behaviour leading to a framework within which insurance risk classification and market equilibrium can be analysed, is provided in Appendix A.

In what follows, for brevity, we confine our attention to two obvious, and opposing, risk classification schemes, though, under suitable regulation, there are infinitely many possibilities.

5.2.1 Full risk classification

An obvious solution to Equation (31) is to set premiums equal to the respective loss probabilities, i.e. $\pi_i = \mu_i$ for $i = 1, 2, \ldots, n$. We call this particular equilibrium the *full risk classification* regime.

5.2.2 No risk classification

At the other end of the spectrum is the *pooled* equilibrium where risk classification is banned and so all risk-groups are charged the same premium π_0, i.e. $\pi_i = \pi_0$ for $i = 1, 2, \ldots, n$. The existence of a pooled equilibrium can be demonstrated as follows. Setting the pooled premium $\pi_0 = \mu_1$, the probability of loss for the lowest risk-group, leads to $\rho(\mu_1) \leq 0$.[8] Setting the pooled premium at the highest level of risk, i.e. $\pi_0 = \mu_n$, gives $\rho(\mu_n) \geq 0$. Assuming insurance demand to be a continuous function of premium, there exists at least one root $\pi_0 \in [\mu_1, \mu_n]$ which gives a pooled equilibrium, i.e. $\rho(\pi_0) = 0$.[9]

5.3 Loss coverage

We suggested in the motivating examples in Section 2, that loss coverage — heuristically characterised as the proportion of the population's losses compensated by insurance — can be used as a measure for social efficacy of insurance. Loss coverage can now be formally defined within the model framework in this paper as the expected insurance claims outgo, or expected population losses compensated by insurance, at equilibrium i.e. $E[QX]$ as defined in Equation (27). So:

$$\text{Loss coverage: } LC(\underline{\pi}) = E[QX], \tag{32}$$

where $\underline{\pi}$ satisfies the equilibrium condition in Equation (31).

[8] For notational convenience, we specify only one argument for multivariate functions if all arguments are equal, e.g. we write $\rho(\pi)$ for $\rho(\pi, \pi, \ldots, \pi)$.

[9] Uniqueness is not guaranteed, but the lowest of any multiple roots can arguably be regarded as the only true equilibrium. This is because any putative equilibrium above the lowest root can be broken by one insurer charging slightly more than the lowest root (Hoy and Polborn [2000]). In any event, the theory developed in this paper is applicable around any equilibrium.

To compare the relative merits of different risk classification regimes, we need to define a reference level of loss coverage. We use the level under actuarially fair premiums, and so define the *loss coverage ratio*, as follows:

$$\text{Loss coverage ratio: } C = \frac{LC(\underline{\pi})}{LC(\underline{\mu})}. \tag{33}$$

5.4 Iso-elastic demand: Equilibrium and loss coverage

We now continue the iso-elastic example developed in Section 4.3. Suppose there are n risk-groups with population proportions $p_1, p_2, \ldots, p_n$, probabilities of loss $\mu_1 < \mu_2 < \ldots < \mu_n$ and insurance demand modelled as per Equation (22):

$$d_i(\pi) = \tau_i \left(\frac{\mu_i}{\pi}\right)^{\lambda}, \quad i = 1, 2, \ldots, n. \tag{34}$$

Here, for simplicity, we are assuming that the demand elasticity of insurance λ is the same for all risk-groups.

Under this set-up, the equilibrium condition in Equation 31, requires that the premium charged $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$, satisfy:

$$\rho(\underline{\pi}) = 0 \;\Leftrightarrow\; \sum_{i=1}^{n} \tau_i \left(\frac{\mu_i}{\pi_i}\right)^{\lambda} \pi_i p_i = \sum_{i=1}^{n} \tau_i \left(\frac{\mu_i}{\pi_i}\right)^{\lambda} \mu_i p_i. \tag{35}$$

And the loss coverage at this equilibrium is given by:

$$LC(\underline{\pi}) = E[QX] = \sum_{i=1}^{n} \tau_i \left(\frac{\mu_i}{\pi_i}\right)^{\lambda} \mu_i p_i = \sum_{i=1}^{n} \frac{\mu_i^{\lambda+1}}{\pi_i^{\lambda}} \tau_i p_i. \tag{36}$$

For the special case where the same premium π_0 is charged for all risk-groups, the pooled equilibrium premium satisfying $\rho(\pi_0) = 0$ is unique and is given by:

$$\pi_0 = \frac{\sum_{i=1}^{n} \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^{n} \alpha_i \mu_i^{\lambda}}, \quad \text{where} \quad \alpha_i = \frac{p_i \tau_i}{\sum_{j=1}^{n} p_j \tau_j}, \; i = 1, 2, \ldots, n. \tag{37}$$

that is, α_i is the *fair-premium demand-share*, that is the share of total demand represented by risk-group i when actuarially fair premiums are charged.

The loss coverage ratio, comparing loss coverage under pooled premiums to that under actuarially fair premiums, is:

$$C = \frac{1}{\pi_0^{\lambda}} \frac{\sum_{i=1}^{n} \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^{n} \alpha_i \mu_i}, \tag{38}$$

where π_0 is the pooled equilibrium premium given in Equation (37).

Figures 3 and 4 show the plots of pooled equilibrium premium, insurance demand (cover) and loss coverage ratio as a function of demand elasticity λ, for two risk-groups where $(\mu_1, \mu_2) = (0.01, 0.04)$ and fair-premium demand-shares $(\alpha_1, \alpha_2) = (0.9, 0.1)$. Compared with the result under actuarially fair premiums, under pooling the premium is always higher, and demand (cover) is always lower. This reduction in cover is the perceived loss of efficiency arising from adverse selection. Loss coverage, on the other hand, is not always lower: for this iso-elastic demand function, it is higher than under actuarially fair premiums if demand elasticity is less than 1. There is some empirical evidence that insurance demand elasticities are typically less than

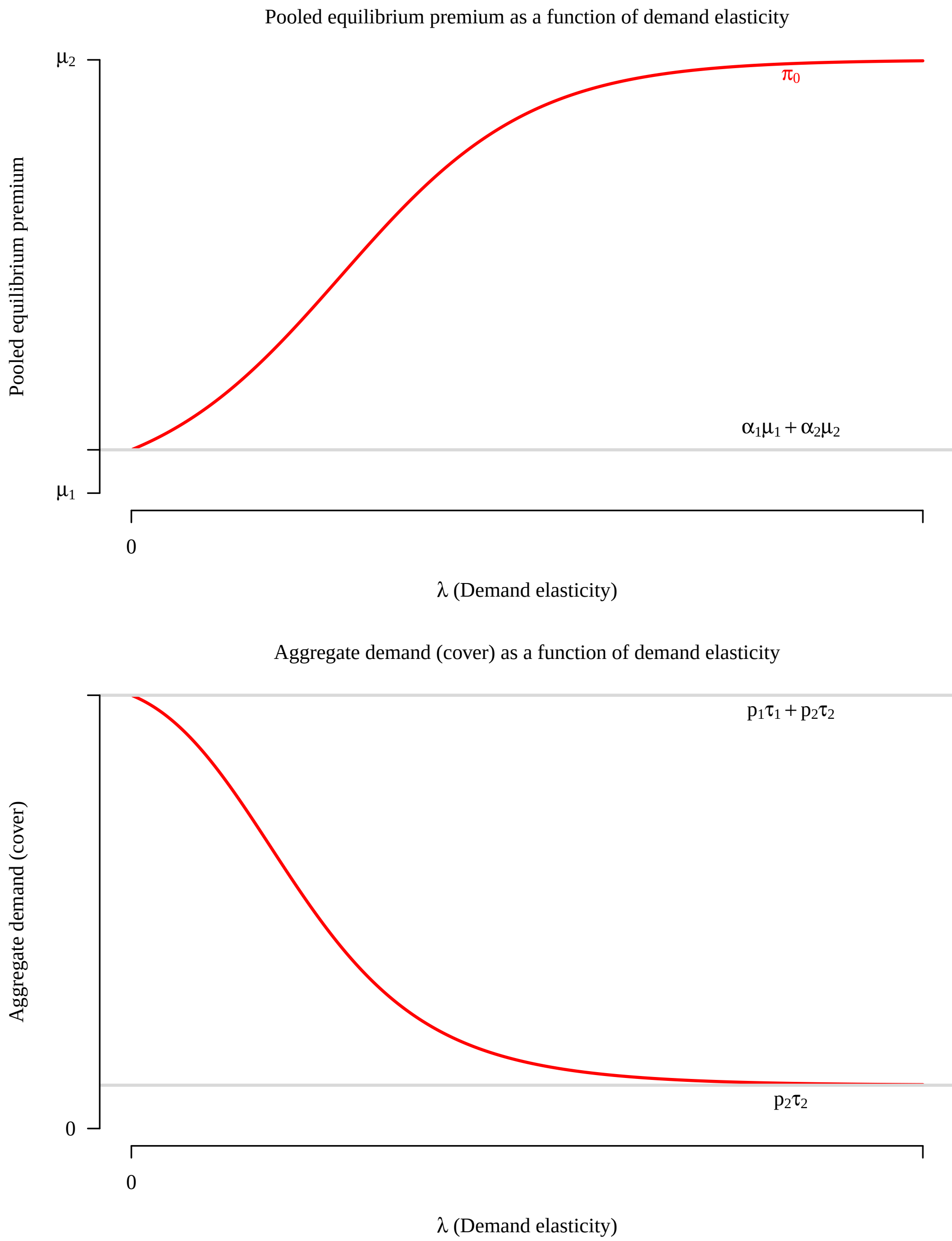

Figure 3: Pooled equilibrium premium (top panel) and aggregate demand (bottom panel) as functions of demand elasticity.

1 in many markets (Pauly et al. [2003]; Viswanathan et al. [2006]; Chernew et al. [1997]; Blumberg et al. [2001]; Buchmueller and Ohri [2006]; Butler [2002]).

The pattern shown in Figure 4 is formalised by the following proposition.

Proposition 1. *If demand elasticity is a positive constant λ and the loss coverage ratio as defined in Equation 38 is C, then*

$$\lambda \lesseqqgtr 1 \Leftrightarrow C \gtreqqless 1 \tag{39}$$

In other words, for iso-elastic insurance demand, pooling produces higher loss coverage than actuarially fair premiums if demand elasticity is less than 1, and vice versa.

The proof of Proposition 1 is provided in Appendix B.

6 Social Welfare and Loss Coverage

6.1 Social Welfare

Our approach to social welfare is in the same spirit as Hoy [2006]: we assume cardinal and interpersonally comparable utilities, and assign equal weights to the utilities of all individuals. This equal-weights approach is based on the Harsanyi [1955] 'veil of ignorance' argument: that is, behind the (hypothetical) 'veil of ignorance', where one does not know what position in

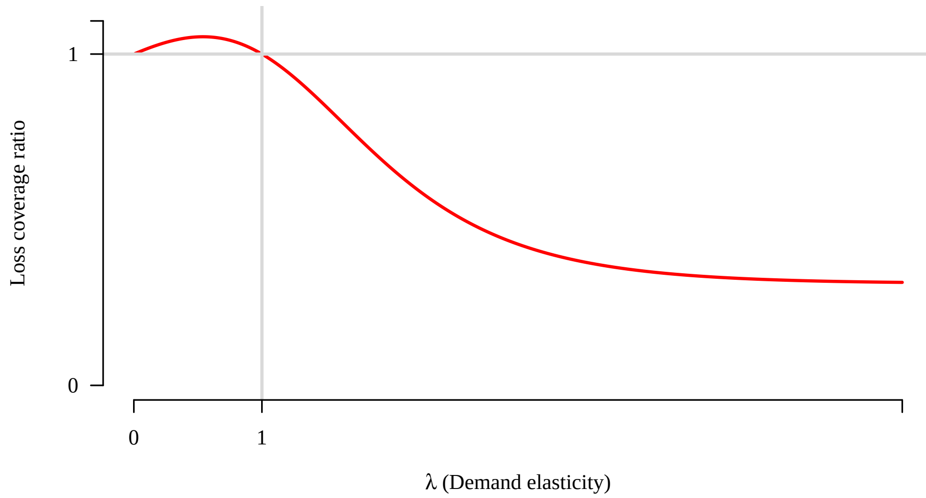

Figure 4: Loss coverage ratio as a function of demand elasticity.

society (e.g. high risk or low risk) one occupies, the appropriate probability to assign to being any individual is $1/n$, where n is the number of individuals in society. Alternative risk classification regimes can then be compared by comparing expected utility in each regime for the (hypothetical) individual utility-maximiser behind the 'veil of ignorance'.

In the model in Sections 4 and 5, suppose an individual is selected at random from the whole population. The individual's expected utility can be written as follows:

$$\text{Social Welfare} \tag{40}$$
$$= \mathrm{E}\left[\, Q\, U_\Gamma(W - \Pi L) + (1-Q)\left[(1-X)\, U_\Gamma(W) + X\, U_\Gamma(W-L)\right]\right]$$

where the first part represents random utility if insurance is purchased; and the second part is the random utility if insurance is not purchased.

As certainty equivalent decisions do not depend on the origins and scales of utility functions, in Section 4, we assumed without loss of generality, that utilities for all individuals are the same at the 'end-points', W and $W - L$. But this argument cannot be directly extended to Equation (40), because individuals' utilities can differ for identical levels of wealth, which has direct implications for social welfare.

However, without any standardisation, Equation (40) is susceptible to being dominated by a 'utility monster' who derives more utility from a given level of wealth than all other individuals combined (see Bailey [1997], Nozick [1974]). This makes it unsuitable for policy purposes. So we propose to continue standardising utility functions so that all utilities are the same at 'end-points', W and $W - L$, as before. This standardisation implies that the same 'disutility of uninsured loss' $[U(W) - U(W - L)]$ is assigned to all individuals, but utility if insurance is purchased $U_\Gamma(W - \Pi L)$ differs between individuals. Under this standardisation, social welfare, denoted by S can be expressed as:

$$S = \mathrm{E}\left[\, Q\, U_\Gamma(W - \Pi L) + (1 - Q)\left[(1 - X)\, U(W) + X\, U(W - L)\right]\right]. \quad (41)$$

To derive an expression for S, we consider the constituent parts of Equation (41) separately. Here we sketch the argument, the full probabilistic model is in Appendix A. First:

$$\begin{aligned}
&\mathrm{E}\left[\, Q\, U_\Gamma(W - \Pi L)\,\right] \\
&= \sum_{i=1}^{n} \mathrm{E}[\, Q\, U_\Gamma(W - \pi_i L) \mid \mu = \mu_i\,]\, \mathrm{P}[\mu = \mu_i], && (42)\\
&= \sum_{i=1}^{n} \{\mathrm{E}[\, U_\Gamma(W - \pi_i L) \mid U_\Gamma(W - \pi_i L) > u_{c_i}\,,\, \mu = \mu_i\,] && (43)\\
&\qquad\qquad \times \mathrm{P}\left[U_\Gamma(W - \pi_i L) > u_{c_i} \mid \mu = \mu_i\right]\, p_i, \} \\
&= \sum_{i=1}^{n} U_i^*(W - \pi_i L)\, d_i(\pi_i)\, p_i, \quad \text{using Equation (8)}, && (44)
\end{aligned}$$

where $u_{c_i} = (1 - \mu_i)U(W) + \mu_i U(W - L)$ (as defined in Equation (7)) and $U_i^*(W - \pi_i L) = \mathrm{E}[\, U_\Gamma(W - \pi_i L) \mid U_\Gamma(W - \pi_i L) > u_{c_i}\,,\, \mu = \mu_i\,]$ represents the expected utility of individuals purchasing insurance in risk-group i.

Using the assumption that all individuals have the same utilities $U(W)$ and $U(W - L)$ at wealth levels W and $W - L$, and that the random variables Q and X are independent given a risk-group, the second part of Equation

(41) becomes:

$$\begin{aligned}
&\mathrm{E}\left[\,(1-Q)\,[(1-X)\,U(W)+X\,U(W-L)]\,\right] \\
&=\sum_{i=1}^{n}\mathrm{E}[\,(1-Q)\,[(1-X)\,U(W)+X\,U(W-L)]\mid\mu=\mu_i\,]\,\mathrm{P}[\mu=\mu_i], \quad (45)\\
&=\sum_{i=1}^{n}[(1-d_i(\pi_i))\,\{(1-\mu_i)U(W)+\mu_i\,U(W-L)\}]\,p_i \quad (46)
\end{aligned}$$

Combining Equations (44) and (46), we get the following expression for social welfare:

$$\begin{aligned}
S=\sum_{i=1}^{n}\Bigg[&\underbrace{d_i(\pi_i)\,U_i^*(W-\pi_i\,L)}_{\text{Insured population}} \quad (47)\\
&+\underbrace{(1-d_i(\pi_i))\,\{(1-\mu_i)U(W)+\mu_i\,U(W-L)\}}_{\text{Uninsured population}}\Bigg]p_i,\\
=&\underbrace{\sum_{i=1}^{n}[(1-\mu_i)U(W)+\mu_i\,U(W-L)]\;p_i}_{\text{Constant as a function of }\pi_i} \quad (48)\\
&+\underbrace{\left(\sum_{i=1}^{n}d_i(\pi_i)\mu_i p_i\right)\times[U(W)-U(W-L)]}_{\text{Loss coverage}\times\text{Positive multiplier}}\\
&-\underbrace{\sum_{i=1}^{n}d_i(\pi_i)\,[U(W)-U_i^*(W-\pi_i L)]\;p_i}_{\text{Adjustment factor to account for premiums}}\,.\\
=&\ \text{Constant}+\textbf{Loss Coverage}\times\text{Positive multiplier} \quad (49)\\
&-\text{Premium adjustment factor.}
\end{aligned}$$

Note that Equation (49) does not depend on any particular choice of family of utility functions.

A regulator or a policymaker aiming to maximise social welfare, will be interested in choosing a risk-classification regime $\underline{\pi}$ which maximises S. However, social welfare depends on unobservable utility functions, which makes it difficult to implement. On the other hand, loss coverage depends solely on observable quantities and Equation 49 shows that social welfare and loss coverage are directly related. So, it will be useful if it can be shown that both measures, social welfare and loss coverage, agree on the choice of risk-classification regime under certain assumptions. A regulator or policymaker can then use loss coverage as a proxy for social welfare. In the following Section 6.2, we show that this is indeed possible within the set-up of our example with the same iso-elastic demand elasticity for all risk-groups.

6.2 Iso-elastic demand: Social welfare and loss coverage

We now continue the example developed in Sections 4.3 and 5.4, and analyse the relationship between loss coverage and social welfare under iso-elastic demand.

Using the convenient standardisation of $U(W) = 1$ and $U(W - L) = 0$ as defined in Equations 10 and 11, along with the assumption that $W = L = 1$, and noting that social welfare S is a function of the risk-classification regime

$\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$, Equation (41) becomes:

$$S(\underline{\pi}) = E\left[\, Q\, U_\Gamma(1-\Pi) + (1-Q)(1-X)\,\right], \tag{50}$$

$$= E\left[\, Q\ \{U_\Gamma(1-\Pi) - (1-X)\,\}\,\right] + K, \tag{51}$$

where $K = E[1-X]$ is a constant as it does not depend on $\underline{\pi}$.

Using the particular form of utility function $U(w) = w^\gamma$, we have:

$$S(\underline{\pi}) = E\left[\, Q\ \left\{(1-\Pi)^\Gamma - (1-X)\,\right\}\,\right] + K, \tag{52}$$

$$\approx E\left[\, Q\,(1 - \Gamma\,\Pi - 1 + X\,)\,\right] + K, \quad \text{assuming small premiums,} \tag{53}$$

$$= E\left[\, Q(X - \Gamma\,\Pi)\,\right] + K, \tag{54}$$

$$= E\left[\, QX\,\right] - E\left[\, Q\,\Gamma\,\Pi\,\right] + K, \tag{55}$$

$$= LC(\underline{\pi}) - PA(\underline{\pi}) + K, \tag{56}$$

where $LC(\underline{\pi}) = E\left[\, QX\,\right]$ is loss coverage and $PA(\underline{\pi}) = E\left[\, Q\,\Gamma\,\Pi\,\right]$ is the premium adjustment factor under the risk-classification regime $\underline{\pi}$. We have already analysed $LC(\underline{\pi})$ in Section 5.4, so we focus on $PA(\underline{\pi})$ here.

Firstly, recall that Q, defined in Section 5.1, is an indicator random variable which takes the value of 1 if insurance is purchased; 0 otherwise. And from Equation 19, given a risk-group i, insurance is purchased when $\Gamma_i < \frac{\mu_i}{\pi_i}$, where the random variable $\Gamma_i = [\,\Gamma \mid \mu = \mu_i\,]$. Hence:

$$[\, Q \ \mid\ \mu = \mu_i\,] = I\left[\Gamma_i < \frac{\mu_i}{\pi_i}\right]. \tag{57}$$

So:

$$PA(\underline{\pi}) = \sum_{i=1}^{n} E\,[\,Q\,\Gamma\,\Pi \mid \mu = \mu_i\,]\;P[\mu = \mu_i], \tag{58}$$

$$= \sum_{i=1}^{n} E\left[I\left[\Gamma_i \le \frac{\mu_i}{\pi_i}\right]\;\Gamma_i\,\pi_i\right]\;p_i, \tag{59}$$

$$= \sum_{i=1}^{n} E\left[\Gamma_i\,I\left[\Gamma_i \le \frac{\mu_i}{\pi_i}\right]\right]\;\pi_i\,p_i. \tag{60}$$

Using the cumulative distribution function of Γ_i, as given in Equation (20):

$$P\,[\Gamma_i \le \gamma] = \begin{cases} 0 & \text{if } \gamma < 0 \\ \tau_i\,\gamma^{\lambda} & \text{if } 0 \le \gamma \le (1/\tau_i)^{1/\lambda} \\ 1 & \text{if } \gamma > (1/\tau_i)^{1/\lambda}, \end{cases} \tag{61}$$

Equation (60) becomes:

$$PA(\underline{\pi}) = \sum_{i=1}^{n}\left[\int_0^{\frac{\mu_i}{\pi_i}} \gamma\tau_i\lambda\gamma^{\lambda-1}d\gamma\right]\pi_i p_i, \tag{62}$$

$$= \frac{\lambda}{(\lambda+1)}\sum_{i=1}^{n}\left(\frac{\mu_i}{\pi_i}\right)^{\lambda+1}\tau_i\pi_i p_i, \tag{63}$$

$$= \frac{\lambda}{(\lambda+1)}\sum_{i=1}^{n}\frac{\mu_i^{\lambda+1}}{\pi_i^{\lambda}}\tau_i p_i, \tag{64}$$

$$= \frac{\lambda}{(\lambda+1)}LC(\underline{\pi}), \quad \text{by Equation 36.} \tag{65}$$

Hence social welfare in Equation (56) becomes:

$$S(\underline{\pi}) = \frac{1}{\lambda + 1} LC(\underline{\pi}) + K. \tag{66}$$

The right-hand side Equation (66) can be interpreted as follows. The second term $K = E[1 - X]$ corresponds to expected utility in the absence of the institution of insurance (recall that we have standardised $U(W) = 1$, $U(W - L) = 0$, and X is the loss for an individual drawn at random from the population). The first term represents an increase in expected utility, attributable to the institution of insurance; this allows for the expectations of both utility of benefits received, and disutility of premiums paid. If λ is small (corresponding to inelastic demand and high risk aversion), the premiums paid are relatively unimportant, so the increase in expected utility is a large fraction of the loss coverage [10]. If λ is large (corresponding to elastic demand and low risk aversion), the premiums paid are important, so the increase in expected utility is only a small fraction of the loss coverage.

The form of Equation 66 suggests the following proposition.

Proposition 2. *Suppose demand elasticity is a positive constant and we have two risk classification schemes $\underline{\pi}_1$ and $\underline{\pi}_2$, which give social welfare $S(\underline{\pi}_1)$ and*

[10] The fraction $1/(\lambda + 1)$ can also be viewed as a fraction of the loss coverage $LC(\underline{\pi}) = E[QX]$ which 'counts' as an offset against the uninsured losses X which appear in $K = E[1 - X]$, where the offset is in on a welfare scale and includes allowance for both benefits and premiums.

$S(\underline{\pi}_2)$, and loss coverage $LC(\underline{\pi}_1)$ and $LC(\underline{\pi}_2)$. Then

$$S(\underline{\pi}_1) \geq S(\underline{\pi}_2) \Leftrightarrow LC(\underline{\pi}_1) \geq LC(\underline{\pi}_2) \tag{67}$$

In other words: for iso-elastic insurance demand, ranking risk classification schemes by loss coverage always gives the same ordering as ranking by social welfare.

The proof of the propositon follows directly from the form of Equation 66, and noting that for the logical biconditional statement in the proposition, the contrapositive (i.e with both inequalities reversed) also holds.

Proposition 2 holds for *any* pair of risk classification schemes which satisfy the equilibrium condition in Equation 31. This includes schemes where premiums are partly (but not fully) risk-differentiated, as well as the polar cases of pooling and actuarially fair premiums. Where the comparison is between the polar cases, combining Proposition 2 with Proposition 1 shows that for iso-elastic demand, pooling gives higher social welfare than actuarially fair premiums whenever demand elasticity is less than one, and *vice versa*.

The potential usefulness of Proposition 2 arises from the fact that loss coverage is observable, but social welfare is unobservable. So a policymaker or regulator can implement a risk classification scheme which gives higher (observable) loss coverage, with the comfort of knowledge that this also gives higher (unobservable) social welfare.

7 Conclusions

We have proposed loss coverage as an intuitively appealing metric for evaluation of different insurance risk classification schemes. Loss coverage is defined as the expected population losses compensated by insurance at market equilibrium.

Bans on insurance risk classification create asymmetries in (the use of) information, typically leading to adverse selection. Adverse selection is associated with a fall in the number of insured individuals compared with that obtained under full risk classification. This reduction in coverage is usually seen as inefficient. However, adverse selection is also associated with a shift in coverage towards higher risks. If this shift is large enough, it can more than outweigh the fall in numbers insured, so that loss coverage is increased. Since this implies that more risk is voluntarily traded and more losses are compensated, it is a counter-argument to the perception of reduced efficiency.

For coverage to shift towards higher risks when risk classification is banned, it must be the case that not all individuals choose to buy insurance at any given premium. This is an observable reality in many insurance markets. We have shown that it can be explained by heterogeneous utility functions, which are unobservable by the insurer. In our model, individuals make decisions completely deterministically on the basis of certainty-equivalent utility calculations, but the insurer observes apparently stochastic decision-making, resulting in a proportional insurance demand function.

We have also shown that loss coverage can be reconciled with (although it is not the same as) an 'equal weights' utilitarian social welfare, in the spirit of Hoy [2006] or Dionne and Rothschild [2014]. Specifically, if insurance demand is iso-elastic, ranking risk classification schemes by loss coverage always gives the same ordering as ranking by social welfare. Notably, however, the calculation of social welfare requires utility functions to be observable, while the calculation of loss coverage does not.

This work could be extended in both empirical and theoretical directions. Empirically, we could investigate how reasonable the iso-elastic model is as an approximation for insurance demand in particular markets. Theoretically, it may be possible generalise our main results for a wider class of insurance demand functions. However, both these extensions are left for future research.

Appendices

A Probabilistic Model of Heterogeneous Insurance Purchasers

We can construct a probabilistic model by supposing that any individual sampled at random possesses two attributes, risk of suffering a loss event (or just 'risk' for short) and a utility function.

- We suppose that 'risk' is defined as the probability μ of suffering a defined loss event. For simplicity, suppose the set of possible values of μ is the finite set $M = \{\mu_1, \mu_2, \ldots, \mu_n\}$, that $\mathcal{G}$ is the power set of M and that $\mathrm{P}[\mu = \mu_i] = p_i$.

- For simplicity, suppose that all utility functions belong to a family parameterized by a real number γ. Individuals' utility functions take values in R.

Then the idea of risk and utility being heterogeneous in a population may be modelled by the probability space $(\Omega, \mathcal{F}, P)$ where:

- The sample space is $\Omega = M \times R$.

- The sigma-algebra $\mathcal{F}$ is $\mathcal{G} \times \mathcal{B}$, where $\mathcal{B}$ is the Borel sigma-algebra on R.

- The probability measure P is assumed to be given by a probability function $F(\mu, \gamma)$, discrete in its first component and absolutely continuous in its second component.

An individual sampled at random has the attributes μ and γ given by the probability F. We must have the marginal distribution:

$$p_i = \mathrm{P}[\mu = \mu_i] = \int_{\{\mu_i\} \times R} dF(\mu, \gamma) = \int_R dF(\mu_i, \gamma) \tag{68}$$

where the first integral is Stieltjes, summing over the first component of F and integrating over the second component.

Two individuals with the same value μ_i of μ may be said to belong to the same risk group, for insurance purposes. The insurer is supposed able to observe μ and will offer the same premium π_i to everyone with risk μ_i. It is assumed that an individual with risk μ_i, offered premium π_i, will decide to buy insurance, or not, non-randomly, determined by their utility function. We suppose, however, that the insurer cannot observe γ. Since different individuals, sampled at random and allocated to the same risk-group, can have different utility functions, the insurer will observe heterogeneous behaviour within a risk-group. That is, even though all in the risk-group are offered the same premium rate, some will buy insurance and others will not. The purchasing decision, given the utility function, is non-random, but to the insurer it appears to be random because of the unobserved heterogeneity. At most, the insurer can observe the proportion of individuals in any risk-group that buy insurance. Thus the insurer may model the insurance-buying decision of an individual in a given risk-group as a Bernoulli random variable.

The insurer's premium strategy may be represented by a $\mathcal{G}-$measurable random variable on M, or by a $(\mathcal{G} \times \{\emptyset, \Omega\})-$measurable random variable on Ω. In either case, denote it by Π. The insurance purchasing decision may be represented by an indicator Q, taking the value 1 if insurance is purchased and 0 otherwise. For a given premium strategy Π on the insurer's part, Q is an $\mathcal{F}-$measurable random variable on Ω. Its restriction to a fixed value of the risk $\mu = \mu_i$ is the Bernoulli random variable that the insurer observes in that risk-group.

The proportion of risks with $\mu = \mu_i$ that buy insurance, which we may call a 'demand function' and denote by $d_i(\pi_i)$, is the conditional expected value of Q:

$$d_i(\pi_i) = \mathrm{P}[\, Q = 1 \mid \mu_i \,] = \mathrm{E}[\, Q \mid \mu_i \,] = \frac{\int_R Q(\mu_i, \gamma) \, dF(\mu_i, \gamma)}{\int_R dF(\mu_i, \gamma)} \tag{69}$$

and the expected population demand for insurance is the unconditional expected value of Q:

$$\begin{aligned}
\mathrm{E}[Q] &= \int_\Omega Q(\mu, \gamma) \, dF(\mu, \gamma) && (70)\\
&= \sum_{i \in M} \int_R Q(\mu_i, \gamma) \, dF(\mu_i, \gamma) && (71)\\
&= \sum_{i \in M} \left(\frac{\int_R Q(\mu_i, \gamma) \, dF(\mu_i, \gamma)}{\int_R dF(\mu_i, \gamma)} \times \int_R dF(\mu_i, \gamma) \right) && (72)\\
&= \sum_{i \in M} d_i(\pi_i) \, p_i. && (73)
\end{aligned}$$

Define X to be a Bernoulli random variable, indicating that a loss event occurs. Given μ_i, X has parameter μ_i, and does not depend on any utility function. Observation of X is new information, not part of the model above.

Then:

$$\begin{aligned}
\mathrm{E}[X] &= \int_{\Omega} \mathrm{E}[\, X \mid \mu, \gamma] \, dF(\mu, \gamma) && (74)\\
&= \sum_{i \in M} \mathrm{E}[\, X \mid \mu_i \,] \int_{R} dF(\mu_i, \gamma) && (75)\\
&= \sum_{i \in M} \mu_i p_i. && (76)
\end{aligned}$$

Assume that Q and X are independent, conditional on μ_i. That is, the level of risk may influence the decision to buy insurance, but there is no moral hazard; insured individuals in any risk-group are not more likely to suffer the loss event than uninsured individuals. Then the expected claims outgo for the insurer is:

$$\begin{aligned}
\mathrm{E}[QX] &= \int_{\Omega} \mathrm{E}[\, QX \mid \mu, \gamma \,] \, dF(\mu, \gamma) && (77)\\
&= \int_{\Omega} Q(\mu, \gamma) \, \mathrm{E}[\, X \mid \mu, \gamma \,] \, dF(\mu, \gamma) \qquad (Q \text{ is } \mathcal{F}\text{-measurable}) && (78)\\
&= \sum_{i \in M} \mathrm{E}[\, X \mid \mu_i \,] \int_{R} Q(\mu_i, \gamma) \, dF(\mu_i, \gamma) && (79)\\
&= \sum_{i \in M} \mu_i \, d_i(\pi_i) \, p_i \qquad \text{(following Equation (72)).} && (80)
\end{aligned}$$

Finally, the expected premium income is:

$$\begin{aligned}
\mathrm{E}[Q\Pi] &= \int_\Omega \mathrm{E}[\, Q\Pi \mid \mu, \gamma \,]\, dF(\mu,\gamma) && (81)\\
&= \int_\Omega Q(\mu,\gamma)\, \mathrm{E}[\, \Pi \mid \mu, \gamma \,]\, dF(\mu,\gamma) && (82)\\
&= \sum_{i \in M} \mathrm{E}[\, \Pi \mid \mu_i \,] \int_R Q(\mu_i,\gamma)\, dF(\mu_i,\gamma) && (83)\\
&= \sum_{i \in M} \pi_i\, d_i(\pi_i)\, p_i \qquad \text{(following Equation (72)).} && (84)
\end{aligned}$$

Based on the formulation of expected premium income and claims outgo, the total expected profit for insurers, as a function of risk-classification regime $\underline{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$, can be defined as:

$$\begin{aligned}
\text{Expected profit for insurers:} \quad \rho(\underline{\pi}) &= E[Q\Pi] - E[QX], \\
&= \sum_{i=1}^{n} d_i(\pi_i)\pi_i p_i - \sum_{i=1}^{n} d_i(\pi_i)\mu_i p_i. && (85)
\end{aligned}$$

Finally we define social welfare as expected utility of an individual chosen at random, i.e.

$$\begin{aligned}
&\text{Social Welfare} && (86)\\
&= \mathrm{E}\left[Q\, U_\Gamma(W - \pi L) + (1 - Q)\left[X\, U_\Gamma(W - L) + (1 - X)\, U_\Gamma(W)\right]\right].
\end{aligned}$$

as in Equation (40). Let us review the measurability and dependencies of the quantities we will need.

μ is $\mathcal{G}$-measureable.

Γ is $\mathcal{B}$-measureable (Borel sigma-algebra on R).

Π is $\mathcal{G}$-measureable.

Q is $\mathcal{F}$-measureable, but not independent of Π.

X is neither $\mathcal{G}$-measureable nor $\mathcal{F}$-measureable, but it is independent of Π.

Note that $\mathrm{E}[\, X \mid \mathcal{F}\,] = \mathrm{E}[\, X \mid \mu_i\,] = \mu_i$. Consider the right-hand side of Equation 86 term by term.

$$\mathrm{E}[Q\, U_\Gamma(W - \pi L)] \tag{87}$$

$$= \mathrm{E}[\,\mathrm{E}[Q\, U_\Gamma(W - \pi L) \mid \mathcal{F}\,]\,] \tag{88}$$

$$= \sum_{i=1}^{n} p_i \int_R Q(\mu_i, \gamma)\, U_\gamma(W - \pi_i L)\, dF(\pi_i, \gamma) \tag{89}$$

$$= \sum_{i=1}^{n} p_i\, d_i(\pi_i) \frac{\int_R Q(\mu_i, \gamma)\, U_\gamma(W - \pi_i L)\, dF(\pi_i, \gamma)}{d_i(\pi_i)} \tag{90}$$

$$= \sum_{i=1}^{n} p_i\, d_i(\pi_i)\, \mathrm{E}\left[\int_R Q(\mu_i, \gamma)\, U_\gamma(W - \pi_i L)\, dF(\pi_i, \gamma) \,\middle|\, Q(\mu_i, \cdot) = 1 \right] \tag{91}$$

where $Q(\mu_i, \cdot)$ denotes the restriction of Q to the ith risk-group. This is equivalent to Equation (44) in the main text. Next:

$$
\begin{aligned}
&\mathrm{E}[\,(1-Q)\,X\,U_\Gamma(W-L)\,] && (92)\\
&= \mathrm{E}[\;\mathrm{E}[\,(1-Q)\,X\,U_\Gamma(W-L)\mid\mathcal{F}\,]\,] && (93)\\
&= \sum_{i=1}^{n} p_i \int_R (1-Q(\mu_i,\gamma))\,U_\gamma(W-L)\,\mathrm{E}[\,X\mid\mathcal{F}\,]\,dF(\pi_i,\gamma) && (94)\\
&= \sum_{i=1}^{n} p_i\,\mu_i \int_R (1-Q(\mu_i,\gamma))\,U_\gamma(W-L)\,dF(\pi_i,\gamma) && (95)\\
&= \sum_{i=1}^{n} p_i\,\mu_i\,(1-d_i(\pi_i))\frac{\int_R (1-Q(\mu_i,\gamma))\,U_\gamma(W-L)\,dF(\pi_i,\gamma)}{1-d_i(\pi_i)} && (96)\\
&= \sum_{i=1}^{n} p_i\,\mu_i\,(1-d_i(\pi_i)) && (97)\\
&\qquad\times\mathrm{E}\left[\int_R (1-Q)\,U_\gamma(W-L)\,dF(\mu_i,\gamma)\,\middle|\,Q(\pi_i,\cdot)=0\right]\\
&= \sum_{i=1}^{n} p_i\,\mu_i\,(1-d_i(\pi_i))U(W-L), && (98)
\end{aligned}
$$

if $U_\gamma(W-L) = U(W-L)$ for all γ.

Similarly,

$$
\begin{aligned}
&\mathrm{E}[\,(1-Q)\,(1-X)\,U_\Gamma(W)\,] && (99)\\
&= \sum_{i=1}^{n} p_i\,(1-\mu_i)\,(1-d_i(\pi_i)) && (100)\\
&\qquad\times\mathrm{E}\left[\int_R (1-Q)\,U_\gamma(W)\,dF(\mu_i,\gamma)\,\middle|\,Q(\pi_i,\cdot)=0\right]\\
&= \sum_{i=1}^{n} p_i\,(1-\mu_i)\,(1-d_i(\pi_i))U(W), && (101)
\end{aligned}
$$

if $U_\gamma(W) = U(W)$ for all γ.

If we standardise the utility functions to obtain the social welfare S defined in Equation (41) and make the appropriate change of variables, Equations (98) and (101) simplify.

B Loss Coverage Ratio

The argument given here follows Hao *et al.*, (2016). The loss coverage ratio for the case of equal demand elasticity is given in Equation (38) and can be expressed as follows:

$$\begin{aligned}
C &= \frac{1}{\pi_0^\lambda} \frac{\sum_{i=1}^n \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^n \alpha_i \mu_i}, \quad \text{where} \quad \pi_0 = \frac{\sum_{i=1}^n \alpha_i \mu_i^{\lambda+1}}{\sum_{i=1}^n \alpha_i \mu_i^\lambda}; &(102)\\
&= \left[\sum_{i=1}^n w_i \mu_i^{\lambda-1}\right]^\lambda \left[\sum_{i=1}^n w_i \mu_i^\lambda\right]^{1-\lambda}, \quad \text{where} \quad w_i = \frac{\alpha_i \mu_i}{\sum_{j=1}^n \alpha_j \mu_j}; &(103)\\
&= \left(E_w\left[\mu^{\lambda-1}\right]\right)^\lambda \left(E_w\left[\mu^\lambda\right]\right)^{1-\lambda}, &(104)
\end{aligned}$$

where E_w denotes expectation in this context and the random variable μ takes values $\mu_1, \mu_2, \ldots, \mu_n$ with probabilities $w_1, w_2, \ldots, w_n$ respectively.

Result B.1. *For $\lambda > 0$,*

$$\lambda \lesseqqgtr 1 \Leftrightarrow C \gtreqqless 1. \tag{105}$$

Proof. **Case $\lambda = 1$:** It follows directly from Equation (104) that $C(1) = 1$.

Case $0 < \lambda < 1$: Holder's inequality states that, if $1 < p, q < \infty$ where

$1/p+1/q=1$, for positive random variables X, Y with $E[X^p], E[Y^q] < \infty$, $(E[X^p])^{1/p}(E[Y^q])^{1/q} \geq E[XY]$.

Setting $1/p = \lambda$, $1/q = 1-\lambda$, $X = \mu^{\lambda(\lambda-1)}$ and $Y = 1/X$, applying Holder's inequality to Equation (104) gives,

$$C = \left(E_w\left[X^{1/\lambda}\right]\right)^{\lambda}\left(E_w\left[Y^{1/(1-\lambda)}\right]\right)^{1-\lambda} \geq E_w[XY] = 1. \tag{106}$$

Case $\lambda > 1$: Lyapunov's inequality states that, for positive random variable μ and $0 < s < t$, $(E[\mu^t])^{1/t} \geq (E[\mu^s])^{1/s}$.

So Equation 104 gives:

$$C = \frac{\left(E_w\left[\mu^{\lambda-1}\right]\right)^{\lambda}}{\left(E_w\left[\mu^{\lambda}\right]\right)^{\lambda-1}} = \left[\frac{\left(E_w\left[\mu^{\lambda-1}\right]\right)^{1/(\lambda-1)}}{\left(E_w\left[\mu^{\lambda}\right]\right)^{1/\lambda}}\right]^{\lambda(\lambda-1)} \leq 1, \tag{107}$$

as $\left(E_w\left[\mu^{\lambda-1}\right]\right)^{1/(\lambda-1)} \leq \left(E_w\left[\mu^{\lambda}\right]\right)^{1/\lambda}$ for $\lambda > 1$ by Lyapunov's inequality.

□

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