Linking component importance to optimisation of preventive maintenance policy*

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Abstract. In reliability engineering, time on performing preventive maintenance (PM) on a component in a system may affect system availability if system operation needs stopping for PM. To avoid such an availability reduction, one may adopt the following method: if a component fails, PM is carried out on a number of the other components while the failed component is being repaired. This ensures PM does not take system's operating time. However, this raises a question: Which components should be selected for PM? This paper introduces an importance measure, called *Component Maintenance Priority* (CMP), which is used to select components for PM. The paper then compares the CMP with other importance measures and studies the properties of the CMP. Numerical examples are given to show the validity of the CMP.
Keywords: cost-based component importance, preventive maintenance, Birnbaum importance, criticality importance

1 Introduction

1.1 Motivation

To improve the availability of engineered systems such as production lines and electricity transmission networks is the common pursuit of many firms. To achieve a high availability level, preventive maintenance may be used. However, performing preventive maintenance (PM) on a

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component in a system takes time and can therefore reduce the availability of the system if system operation needs stopping for the PM. To avoid such a dilemma, one may adopt the following method: if a component in the system fails, PM is carried out on a number of the other components while the failed component is being repaired. However, this raises another question: Which components should be selected for PM?

Reliability importance measures are developed to prioritise the components of a system in the light of a given criterion and can offer guidance to improve system reliability/availability, reduce maintenance cost, and improve system safety. For example, the Birnbaum importance is the partial derivative of the system reliability with respect to the reliability of an individual component and measures the effect of the reliability improvement of individual components on the improvement of the system reliability [1]. In the reliability literature, many importance measures have been developed for various purposes, see [2] for an excellent paper that reviews recent advances on importance measures. However, few importance measures can be used to select such components for the above-mentioned purpose.

This paper extends the Birnbaum importance measure to an importance measure, called CMP (Component Maintenance Priority), with which components can be selected for PM and the number of components for PM may further be optimised.

1.2 Related work

Various component importance measures for binary coherent systems and state importance measures for multi-state systems have been introduced in the literature. For example, the Fussell and Vesely importance of a component is the probability that at least one minimal cut set containing the components has failed, given that the system has failed; the criticality importance of a component is the probability that the component has caused system failure, when the system is failed. The reader is referred to the monograph by Kuo and Zhu [3] for detailed accounts of the theory and surveys of commonly used importance measures.

The Birnbaum importance is probably the first importance measure introduced in the literature, for the purpose of reliability improvement [1]. The Birnbaum importance measures the extent of the change in the reliability of the system resulted from a change in the reliability of a component. It has been extended to many variants considering different scenarios and applications, for example, cost-based importance measures [4] that considers the lifecycle cost of maintaining each component in a system. Other variants include performance based importance measures [5], joint component importance [6-8], and joint component importance for multistate systems [9].

Recent development in importance measures also include: importance measure for systems with degrading components [10], importance measure that estimates the effect of a component residing at certain states on the performance of the entire multi-state systems [11], importance measure for components when the system may be reconfigured [12], among many others ([13-15], for example).

Importance measures in the literature may be inter-related. Borgonovo shows that the Fussell-Vesely importance, the criticality importance, the Birnbaum importance, the risk achievement worth and the differential importance measure (DIM) are linked by simple relations [16]. Vaurio shows that DIM and the criticality- importance yield the same ranking in realistic examples [17,18]. Furthermore, according to Birnbaum [1], importance measures can be categorised into three classes according to the knowledge needed for determining them: *structural importance measures*, *reliability importance measures*, and *lifetime importance measures*. For example, Fig. 1 shows the process of available knowledge and the corresponding importance measures that can be used.

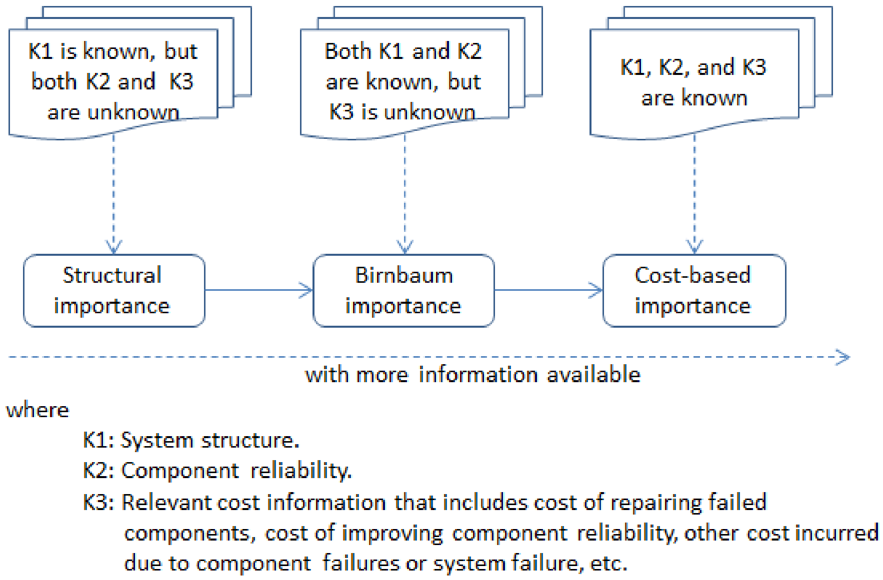

Fig. 1. Available knowledge and importance measures that can be applied.

A common drawback of the Birnbaum importance measure and its variants is that they rank only individual components and are not directly applicable to groups of components. The differential importance measure (DIM), introduced by Borgonovo and Apostolakis in [19], overcomes this drawback by defining the importance of a group of components using a first-order Taylor expansion, but it does not account for the effects of interactions among components. Zio and Podofillini then extended the DIM including both the first order and the second order Taylor expansion, which has a merit that is account of the interactions of pairs of components [20].

The existing reliability literature, however, lacks an importance measure for solving the following question: suppose that a component is failed and during the time when the component is being repaired, other m components in the system can be selected for PM. This raises an interesting question: which m components should be selected? This paper develops a new importance measure, the CMP, for answering this question. The CMP can also be applied to schedule PM policy. Conventionally, optimisation of PM has been centred on seeking the optimal intervals between consecutive PM activities. This paper, however, optimises the number of maintenance personnel needed to minimise the expected cost in a given time horizon. As such, in addition to its novelty of introducing a new importance measure, the paper also creates the novelty of proposing a new method of optimising PM.

1.3 Summary

This rest of this paper is structured as following. Section 2 lists the notation and assumptions, and discusses the justification of the assumptions. Section 3 introduces the CMP and discusses its relationship with some existing importance measures. Section 4 gives upper and lower bounds of the expected number of PM under two maintenance policies. Section 5 discusses issues relating to the CMP. Section 6 gives numerical examples. Section 7 concludes the findings of this paper.

2 Notation and Assumptions

The following notation and assumptions are used.

2.1 Notation

This paper uses the notation shown in Table 1.

Table 1. Notation.

1_i (0_i)	Component c_i is working (failed)
$\phi(\boldsymbol{p}(t))$	System reliability as a function of $\boldsymbol{p}(t)$
n	Number of components in a system
m	Number of components that can be preventively maintained simultaneously while a repair is being conducted on a failed component
$\boldsymbol{p}(t)$	$(p_1(t), \dots, p_n(\mathrm{t}))$
$p_i(t)$	Reliability of component c_i
x_i	Indicator: $x_i = 1$ if c_i is working, $x_i = 0$ otherwise
$\boldsymbol{X}_i$	$(x_1, x_2, \dots, x_{i-1}, \cdot_i, x_{i+1} \dots, x_n)$
$(x_i, \boldsymbol{p}_i(t))$	$(p_1(t), p_2(t), \dots, p_{i-1}(t), x_i, p_{i+1}(t), \dots, p_n(t))$
$[x]$	nearest integer number smaller than x
$\mathbf{1}_{ij}$	$(1_1, 1_2, \dots, 1_{i-1}, 0_i, 1_{i+1}, \dots, 1_{j-1}, 0_j, 1_{j+1}, \dots 1_n)$
$F_j^{(1)}(t)$	$= P(\min\{X_{j_1}, X_{j_2}, \dots, X_{j_m}\} < t)$, X_{j_k} is the lifetime of component j_k before the first PM
$f_j^{(1)}(t)$	$= \dfrac{\partial F_j^{(1)}(t)}{\partial t}$
T_j	A pre-specifed time length that is used in Policy B
$G_j^{(1)}(t)$	$= G_j^{(1)}(t) = P(\max\{\min\{X_{j_1}, X_{j_2}, \dots, X_{j_m}\}, T_j\} < t)$
$g_j^{(1)}(t)$	$= \frac{\partial G_j^{(1)}(t)}{\partial t}$
$\mu_{j_k}(\cdot)$	hazard functions of component j_k before the first PM

2.2 Assumptions

A1. The system considered in this paper is a coherent system, which implies: each component is relevant, and the structure function is increasing (non-decreasing) if the number of components increases.

A2. Once a component is failed, a certain symptom immediately appears and can be noticed. The failed component can then be located.

A3. There are two types of PM: PM during system's downtime and PM during system's uptime, which are denoted by PM_D and PM_U, respectively.

A4. Performing a PM (either PM_D or PM_U) on a component requires that the component stops working.

A5. A PM (either PM_D or PM_U) on a component can only be triggered when another component has failed.

A6. Component failures can have two different situations.

(A) When a critical component has failed, the system fails. Then the failed component is repaired. In the meantime, PM_D is performed on other m selected components.

(B) When a non-critical component has failed, the system does not fail. Then the failed component is repaired. In the meantime, PM_U is performed on other m selected components. The repair and the PM_U will not affect the system operating.

A7. All of the components in the system are statistically independent.

Remark 1. From Assumption A3, the PM_U policy is: PM is performed when the system is still working. This implies that a binary system (for example, a parallel system) under the PM_U policy may never fail. Take the system in Fig. 2 as an example, under Assumptions A3, A4, and A5, the subsystem constituted by components 5, 6, and 7 may never fail. This is because: once one of the three components fails, the failed component will be immediately repaired while the subsystem is working. This will ensure that the subsystem will never fail.

However, if we use the PM_U policy on multistate systems, performing PM_U can improve the performance of the system. For example, for a water pumping station that is composed of three pumps, if pump 1 degrades from a higher state to a lower one (or *is failed* as termed in this paper), its performance, which is the amount of water a pump can pump, degrades and the pump may need repairing. The PM_U that is performed on pump 2 can improve the performance of the pump, for example.

In essence, a jump from a higher state to a lower one in a multistate system (component) is the same as the failure of a binary system (component). As such, in what follows, we simply focus our discussion on binary system (component) cases.

3 A new importance measure: component maintenance priority

The main effort of this paper focuses on the development and analysis of a new importance measure for binary systems. General cases are also discussed in this section.

3.1 Component maintenance priority

We first recall an importance measure with a similar definition as what we will define. This importance measure is the conditional marginal reliability importance, defined in [8]. Its definition is given below.

Definition 1 (Conditional Marginal Reliability Importance (CMRI)) [8]. The CMRI of component c_j, given that component c_i is working or failed, is defined by

$$I_{j|i}^{C}(t) = \frac{\partial \phi(z_i, \boldsymbol{p}_i(t))}{\partial p_j(t)}, \tag{1}$$

where $z_i = 1$(or 0) means that the component c_i is working (or failed).

The authors of [8] claim that the CMRI can be used to decide to which components we should pay more care in terms of maintenance.

Remark 2. Let's look at two typical systems: a series system and a parallel system.

- For a series system, we have the following two scenarios.
  - B1. If component c_i is working, or $z_i = 1$, then the system is working. In this case, no PM can be performed on any component. This is because: according to Assumptions A3, A4 and A5, neither PM_D nor PM_U can be performed. As PM_D is only performed when the system is not working, whereas PM_U is performed when component c_i is failed. Hence, there is no need to use $I_{j|i}^{C}(t)$ to rank the components.
  - B2. If $z_i = 0$, then $\phi(z_i, \boldsymbol{p}_i(t)) = 0$. $I_{j|i}^{C}(t) = 0$ for any $j \neq i$. That is, $I_{j|i}^{C}(t)$ cannot be used to rank the components as they all are zeros.
- For a parallel system, similar to the series system, we have the following two scenarios.
  - B3. If $z_i = 1$, then $\phi(z_i, \boldsymbol{p}_i(t)) = 1$. $I_{j|i}^{C}(t) = 0$ for any $j \neq i$, that is, $I_{j|i}^{C}(t)$ cannot be used to rank the components as they all are zeros.
  - B4. If $z_i = 0$, or component c_i is failed, according to Assumptions A3, A4 and A5, PM_U can be performed on unfailed components only when the number of the components is larger than 2. This is because PM_U is only performed on a component when the component stops working and the system is working. When a 2-component parallel system includes one failed component, the unfailed component must be working and cannot be stopped for PM_U.

From the above analysis, one can see that the measure $I_{j|i}^{C}(t)$ can only be used for the scenarios when the number of components in a parallel system is larger than 2.

3.1.1 Component Maintenance Priority

The above analysis shows that the CMRI cannot be used to rank component importance under Assumptions A3, A4, and A5. This necessitates introducing a new definition, which is given in the following.

Definition 2 (Component Maintenance Priority (CMP)). If component i has failed, then under Assumptions A1—A5, the CMP of component j is defined by

$$I_{j|i}^{M}(t) = H_{j|i} \frac{\partial \phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_j(t)}, \tag{2}$$

where

- $H_{j|i} = \begin{cases} 1 & \text{if } \phi(1_1, \dots, 1_{i-1}, 0_i, 1_{i+1}, \dots, 1_n) = 0 \\ \phi(0_i, 0_j, \mathbf{1}_{ij}) & \text{if } \phi(1_1, \dots, 1_{i-1}, 0_i, 1_{i+1}, \dots, 1_n) = 1 \end{cases}$, $(0_i, 0_j, \dots, \mathbf{1}_{i,j})$ represents that components i, j stop working and all of the other components are working; and
- $\lambda_i = \chi\{\phi(1_1, 1_2, \dots, 1_{i-1}, 0_i, 1_{i+1}, \dots, 1_n) = 0\}$, $\chi\{.\}$ is an indicator function.

In Eq. (2), λ_i ensures that $\phi(\lambda_i, \boldsymbol{p}_i(t))$ is not constant, no matter whether component i is critical or noncritical. This avoids the problems such as B2 and B3 listed in Remark 2. $H_{j|i}$ ensures that critical components will not be selected for PM, given that component i is non-critical.

The CMP $I^M_{j|i}(t)$ can be used to suggest which components may be selected for PM so that the reliability of the system can be maximally improved, given that component i has failed and repair needs performing on it. This is a form of positive dependence that gives a downtime opportunity: component failures can often be regarded as opportunities for PM of non-failed components. The positive dependence has been discussed in maintenance optimisation for multi-component systems, see [21,22] for example.

Below we give an example to show how the CMP works.

Example 1. Assume a system is structured as Fig. 2 and the components in the system have equal reliability. That is, $p_1(t) = p_2(t) = \cdots = p_8(t) = R(t)$. Then the reliability of the system is

$$\phi(\boldsymbol{p}(t)) = p_1(t)p_4(t)\left(1 - \prod_{i=2}^{3}(1 - p_i(t))\right)\left(1 - \prod_{i=5}^{8}(1 - p_i(t))\right).$$

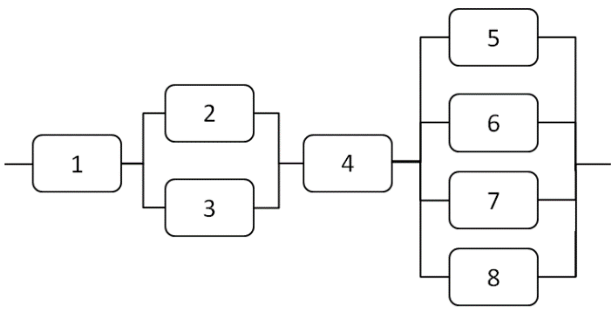

Fig. 2. An example.

Assume that a two-member maintenance team takes care of the system. This implies that the team can repair a failed component, meanwhile carry out PM on another component while the failed component is being repaired. We have the following straightforward analyses.

(a) if component 1 is failed, the system stops working. Then, based on the Birnbaum importance, PM may be conducted on component 4.

(b) if component 2 fails while the system is working, then one of components 5, 6, 7, and 8 can be selected for PM.

Based on Definition 2, we have $H_{j_1|1} = 1$ and $H_{j_2|2} = \phi(0_2, 0_{j_2}, \mathbf{1}_{2j_2})$, where $i, j \in \{1,2, \dots, 8\}$, $j_1 \in \{2,3, \dots, 8\}$, and $j_2 \in \{1,3,4, \dots, 8\}$.

(A) According to Definition 2, $I_{j_1|1}^M(t) = \frac{\partial \phi(1, p_2(t), \ldots, p_8(t))}{\partial p_{j_1}(t)}$. Then we have $I_{2|1}^M(t) = I_{3|1}^M(t) = R(t)(1 - R(t))(1-(1-R(t))^4)$, $I_{4|1}^M(t) = (1-(1-R(t))^2)(1-(1-R(t))^4)$, and $I_{5|1}^M(t) = I_{6|1}^M(t) = I_{7|1}^M(t) = I_{8|1}^M(t) = R(t)(1-(1-R(t))^3)(1-R(t))^2$. It can easily be proved that $I_{4|1}^M(t) \geq I_{j|1}^M(t)$ for $j = 2,3,5,6,7,8$. That is, based from Definition 2, if component 1 is failed, component 4 may be selected for PM.

(B) According to Definition 2, $I_{j_2|2}^M(t) = H_{j_2|2} \frac{\partial h(p_1(t), \lambda_2, p_3(t), \ldots, p_8(t))}{\partial p_j(t)}$. Then we have $I_{1|2}^M(t) = I_{3|2}^M(t) = I_{4|2}^M(t) = 0$, $I_{5|2}^M(t) = I_{6|2}^M(t) = I_{7|2}^M(t) = I_{8|2}^M(t) = (R(t))^3(1-R(t))^3 > 0$. That is, if component 2 fails, the system is working. Then one of components 5, 6, 7 and 8 can be selected for PM.

The derived results (A) and (B) using Definition 2 agree with the analysed results (a) and (b).

Fig. 3. A four-component series system.

From the above example, it can also be found that $I_{j|i}^M$ may be zero, which differs from most existing importance measures such as the Birnbaum importance measure, the joint importance measure, the conditional importance measures, etc, which are always positive.

It can be found that $I_{j|i}^M(t) \neq I_{i|j}^M(t)$. The ordering ranked by $I_{j|i}^M(t)$ is apparently different from that by $I_{i|j}^M(t)$. For example, assume a system is consisted of four components shown in Fig. 3 and their reliabilities are $p_1(t), p_2(t), p_3(t), p_4(t)$ with $p_1(t) \geq p_2(t) \geq p_3(t) \geq p_4(t)$. Then, $I_{4|1}^M(t) = p_2(t)p_3(t)$ $I_{4|1}^M(t) \geq I_{3|1}^M(t) \geq I_{2|1}^M(t)$ and $I_{3|4}^M(t) \geq I_{2|4}^M(t) \geq I_{1|4}^M(t)$.

The relationships with other importance measures are discussed below.

- *Relationship with the Birnbaum importance*. The Birnbaum importance of a component is always positive. Since $I_{j|i}^M$, which is the CMP of component j given that component i has failed, may be zero, $I_{j|i}^M$ may be smaller than the Birnbaum importance of component *j*.
- *Relationship with the joint component importance.* The joint reliability importance, which is defined as the joint reliability importance as $I_{i,j}(t) = \frac{\partial^2 \phi(\mathbf{p}(t))}{\partial p_i(t) \partial p_j(t)}$ and is a measure of how 2 components in a system interact in contributing to the system reliability, as $I_{i,j}(t) = I_{j,i}(t)$. For special systems, we have the following results.
  (a) For series systems, we have $I_{j,i}(t) = I_{j|i}^M(t)$.
  (b) For parallel systems, If $n > 2$, we have $I_{j,i}(t) = -I_{j|i}^M(t)$; if $n = 2$, then $I_{j,i}(t) \neq I_{j|i}^M(t)$. This is because $I_{j|i}^M(t) = 0$ if $n = 2$.

- *Relationship with the conditional component importance*. For both series and parallel systems, we have the relationship: $I_{j|i}^{M}(t) = H_{j|i}\left(I_{j|i}^{C}(t) + I_{j|i}^{C}(t)\right)$ for $n > 2$. If $n = 2$, $I_{j|i}^{M}(t) \neq I_{j|i}^{C}(t)$, and $I_{j|i}^{M}(t) \neq I_{j|i}^{C}(t)$.

3.1.2 The expected number of PM and the number of components for PM

From Definition 2, an interesting concern is the expected number of PM of each component, based on which one may design the system. For example, one may assume that the reliabilities of the components in a system are equal, then calculate component's CMP. He can then allocate the real components with the following rule: the component with the lowest reliability will be placed in the position with the largest number of PM. Then the component will be preventively maintained more often than others.

$I_{j|i}^{M}(t)$=0 implies that component j is not selected for PM if component i fails. Hence, in a system, the maximum number N_j of PM conducted on component j is given by

$$N_j = \sum_{i=1}^{n} \chi\{I_{j|i}^{M}(t) > 0\} \tag{3}$$

where we mean by the maximum number N_j, we have considered the fact that even if a PM is allowed, it is not necessarily always done because of economic or manpower constraints.

Another interesting question is the number of components that can be preventively maintained while a failed component is being repaired. There are two situations as following.

(A) If a critical component fails, then the system stops working. While the component is being repaired, the rest $n - 1$ components can be maintained simultaneously.

(B) However, if a non-critical component fails, the system is still working. To keep the system working, the number of other components that can be maintained is limited. The minimum number of components to ensure the system working is n_c (where n_c is the number of components in the shortest path set in the system), which implies that the rest $n - n_c - 1$ components can be maintained simultaneously.

Hence, we may use the following remark, Remark 3, to summarise the above discussion.

Remark 3. For a n-component system, while the failed component is being repaired, the maximum number m of components that can be preventively maintained simultaneously equals $n - 1$ or $n - n_c - 1$.

Example 2. In the system shown in Fig. 2, the shortest path set includes at least 4 components, i.e., component 1, component 4, one from components 2 and 3, and one from components 5, 6, 7, and 8. That is, $n = 8$, and $n_c = 4$. Hence, the maximum number of components that can be maintained simultaneously is $n - n_c - 1 = 3$ while a failed component is being repaired.

3.1.3 Importance for a group of components

Recall we mentioned that "A drawback of the Birnbaum importance measure and its variants is that it ranks only individual components but they are not directly applicable to groups of components" in Section 1.2. The differential importance measure (DIM), introduced by Borgonovo and Apostolakis in [19], overcome this drawback [19]. Below is the definition of the differential importance measure (DIM) of a given set of parameters, introduced in [19].

$$DIM\left(p_{j_1}(t), \cdots, p_{j_m}(t)\right) = \frac{\sum_{k=1}^{m} \frac{\partial \phi(\boldsymbol{p}(t))}{\partial p_{j_k}(t)} dp_{j_k}(t)}{d\phi(\boldsymbol{p}(t))}. \tag{4}$$

The DIM can be regarded as the fraction of the total change in system reliability that is due to a change in parameter components' reliabilities.

Similar to the other variants of the Birnbaum importance measure, the CMP ranks only individual components. The following quantity $I^M_{j_1,j_2,\cdots,j_m|i}(t)$ defines the improvement on the system if one improves the reliabilities of components $j_1, j_2, \cdots, j_m$ with amount $\Delta_{j_1}, \cdots, \Delta_{j_m}$, respectively. As the denominator in the right hand side of Eq (4) is constant, one can simply compare the enumerators of the DIM. As such, we can derive a similar result the following.

If component i is failed, then the component maintenance priority of a given set of reliability improvements $\left(\Delta_{j_1}, \cdots, \Delta_{j_m}\right)$ on m components $j_1, j_2, \cdots, j_m$ is given by

$$I^M_{j_1,j_2,\cdots,j_m|i}(t) = H_{j_1|i} \frac{\partial \phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_{j_1}(t)} \Delta_{j_1} + \sum_{k=2}^{m} H_{j_k|i,j_1,j_2,\cdots,j_{k-1}} \frac{\partial \phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_{j_k}(t)} \Delta_{j_k}, \tag{5}$$

where $H_{j_k|i,j_1,j_2,\cdots,j_{k-1}} = \begin{cases} 1 & \text{if } \phi(0_i, \mathbf{1}_i) = 0 \\ \phi(0_i, 0_{j_1}, \ldots, 0_{j_{k-1}}, \mathbf{1}_{i,j_1,j_2,\cdots,j_{k-1}}) & \text{if } \phi(0_i, \mathbf{1}_i) = 1 \end{cases}$, and $\left(0_i, 0_{j_1}, \ldots, 0_{j_{k-1}}, \mathbf{1}_{i,j_1,j_2,\cdots,j_{k-1}}\right)$ represents that components $i, j_1, j_2, \cdots, j_{k-1}$ stop working and all of the other components are working.

3.2 Dynamic scenarios

The content in the preceding section, Section 3.1, does not consider the fact that reliability is a function of time.

The CMP is introduced for ranking maintenance priorities of the components of a system at a time when a component is failed. At a given time point, component reliabilities can be obviously regarded as constant. From a lifecycle perspective, however, as the components in the system can age and deteriorate, component reliabilities are time-dependent. From this regard, the rankings resulted from the CMP change over time. For example, in the system in Fig 2, at a time point, the reliability of component 5 may become larger than that of component 6, and consequently, their rankings by the CMP can change. This time-dependence property can cause a difficulty in estimating the expected number of PM needed in a period and further cause a difficulty in estimating the expected lifetime cost. However, estimating the upper and lower bounds of the expected number of failures can be done, but it depends on maintenance policies taken, as shown in Eqs (6) and (7) in Section 4.

4 Linking with maintenance policies

Once a component is failed, it is repaired. In the meantime, a given number of components are selected for PM. A natural question posed here is: in case two components are failed within a short period and both failures trigger PM on a component, will two PM be performed on the same component within a short period? Such a scenario should be avoided because it is unnecessary to perform two PM within a short period from a cost-effectiveness perspective. This leads to the following two possible maintenance policies: one considers time since last PM and one does not.

Eq. (3) gives the expected number of PM for a given component at a time point. Below, we consider the expected number of PM with a given time period $(0,T)$.

In this section, we make the following two assumptions.

A8. Repair on failed components are minimal repair, that is, the repair will bring the component back to the status just before it failed. PM effect is imperfect, that is, a PM activity will bring the maintained component to a status between as good as new and the time before the component was maintained.

A9. Time on PMU or PMU is negligible.

If a component, say, component i, fails, it will be repaired immediately. In the meantime, other m components are selected for PM. The selection criterion differs between maintenance policies A and B.

Maintenance Policy A. The selection criterion is based on the component maintenance priority $I_{j|i}^{M}(t)$, as defined in Definition 2. That is, m components, $j_1, \dots, j_m$, with larger $I_{j|i}^{M}(t)$ are selected.

Maintenance Policy B. Components are selected with two steps: all components in the system are ranked according to $I_{j|i}^{M}(t)$ (with $j = 1, \dots, i-1, i+1, \dots, n$); then m components, $j_1, \dots, j_m$, with the largest $I_{j|i}^{M}(t)$ values are selected. If the calendar age of a selected component since its last PM is older than a pre-specified value, T_{j_k} say (for $k = 1, \dots, m$), then a PM will be conducted on it. Otherwise, no PM will be conducted on those with ages younger than the pre-specified values.

For a given period $(0, T)$, the lower and upper bounds of the expected number of PM on a set of components $\{j_1, \dots, j_m\}$ under Policy A and Policy B are given in Eqs. (6) and (7). The set can be all of the components in a system or a subset of components in a system.

In the following, we give the expected number of PM within $(0,T)$ under the above two maintenance policies.

4.1 Bounds of the expected number of PM under maintenance policy A

Denote N_j^A by the total expected number of PM on components $\{j_1, \dots, j_m\}$ under maintenance policy A within time period $(0,T)$. Suppose the failure of a component among a set of components $\{J_1, J_2, \dots, J_M\}$

may trigger PM on a subset components in components $\{j_1, \dots, j_m\}$. Let $\mu_{J_k}(\cdot)$ be the hazard functions of component J_k before the first PM is conducted on the component.

For two identical items in which one is preventively maintained and one is not, the item with PM should have fewer failures than the one without PM. The expected number of failures of component J_k is $\int_0^T \mu_{J_k}(\tau)d\tau$ if it is not preventively maintained and minimal repair is conducted upon failures during the time interval $(0, T)$. If the failure of a component among components $\{J_1, J_2, \dots, J_M\}$ triggers PM on a subset components of the m components $\{j_1, \dots, j_m\}$, the maximum total expected number of PM is $m\sum_{k=1}^{M}\int_0^T \mu_{J_k}(\tau)d\tau$. Hence, we have $N_j^A \le m\sum_{k=1}^{M}\int_0^T \mu_{J_k}(\tau)d\tau$.

The time to the first failure among the set of components $J_1, J_2, \cdots, J_m$ is $\min\{X_{J_1}, X_{J_2}, \dots, X_{J_m}\}$. Let $f_J^{(1)}(t) = \frac{\partial F_J^{(1)}(t)}{\partial t}$, $F_J^{(1)}(t) = P(\min\{X_{J_1}, X_{J_2}, \dots, X_{J_M}\} < t)$, where X_{J_k} is the time-to-first-failure of component J_k. Since PM on components $\{j_1, \dots, j_m\}$ are conducted only if one of the components in $\{X_{J_1}, X_{J_2}, \dots, X_{J_m}\}$ fails and the probability that the first failure occurs is $F_J^{(1)}(t)$, the lower boundary of N_J^A will be $m_j F_J^{(1)}(t)$. Hence, $m_j F_J^{(1)}(t) \le N_j^A$, where m_j $(< m)$ is the minimum number of components that can be simultaneously maintained.

Hence, if maintenance policy A is applied and PM takes effect, then the expected number N_j^A of PM of a set of components $\{j_1, \dots, j_m\}$ within time interval $(0, T)$, has bounds given in the following.

$$m_j F_J^{(1)}(t) \le N_j^A \le m\sum_{k=1}^{M}\int_0^T \mu_{J_k}(t)dt. \tag{6}$$

4.2 Bounds of the number of PM under maintenance policy B

Denote N_j^B as the expected number of PM on components $\{j_1, \dots, j_m\}$ under maintenance policy B within time period $(0, T)$.

If maintenance policy B is applied, then the maximum expected number of PM on component j_k is $\left[\frac{T}{T_{j_k}}\right]$, where T_{j_k} is the pre-specified age for PM and $[t]$ as the nearest integer number larger than t. The maximum expected number of PM on the set of components $\{j_1, \dots, j_m\}$ is not greater than $\sum_{k=1}^{m}\left[\frac{T}{T_{j_k}}\right]$.

Following the discussion in Section 4.1, we denote $T_J = \min\{T_{J_1}, T_{J_2}, \dots, T_{J_m}\}$. If $\min\{X_{J_1}, X_{J_2}, \dots, X_{J_M}\} < T_J$, no PM will be conducted. Hence, the probability that the first PM will be conducted within time period $(0, T)$ is $P\left(T_j < \min\{X_{j_1}, X_{j_2}, \dots, X_{j_m}\} < T\right) = P\left(\min\{X_{j_1}, X_{j_2}, \dots, X_{j_m}\} < T\right) - P\left(\min\{X_{j_1}, X_{j_2}, \dots, X_{j_m}\} < T_j\right) = F_j^{(1)}(T) - F_j^{(1)}(T_j)$. The optimum scenario is that no failure to occur since the first PM. $\left(F_j^{(1)}(T) - F_j^{(1)}(T_j)\right) m_j \le N_j^B$, where $m_j (< m)$ is the minimum number of the components that can be conducted on the set of components $\{j_1, \dots, j_m\}$.

Based on the above discussion, if maintenance policy B is applied, then, N_j^B, the expected number of PM of a set of components $\{j_1, \dots, j_m\}$ within time interval (0, T), has bounds given in the following.

$$\left(F_j^{(1)}(T) - F_j^{(1)}(T_j)\right) m_j \leq N_j^B \leq \sum_{k=1}^{m} \left\lfloor \frac{T}{T_{j_k}} \right\rfloor. \quad (7)$$

5 Discussion

Optimisation of maintenance policies. Conventionally, optimisation of PM has been centred on seeking the optimal intervals between consecutive PM activities. From the above discussion, however, it can be seen that the optimal number of components that may be preventively maintained can be sought to minimise the expected cost in a given time horizon.

Maintenance time. If time of maintenance is considered, then $I_{j|i}^m(t) = \chi\{T_i^C \geq T_j^P\} H_{j|i} \frac{\partial \phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_j(t)}$ may be used, where T_i^C is the repair time on the failed component i and T_j^P is the time of PM on component j. As $T_i^C \geq T_j^P$ means that the time of PM is shorter than that of repairing the failed component i, $\chi\{T_i^C \geq T_j^P\}$ ensures that only those components with shorter PM time will be selected. Normally, the condition $T_i^C \geq T_j^P$ can be easily satisfied as repair (or corrective maintenance) involves more tasks such as fault diagnosis, fault location and fault removal, whereas PM is pre-scheduled and it is conducted by following a pre-specified procedures. Of course, in case other scenarios on repair time are considered, one can easily amend $\chi\{T_i^C \geq T_j^P\}$ to fit for purpose.

Reliability-based, cost-based, or geography-based importance measures. This paper extends the Birnbaum importance measure to a measure, maintenance priority measure, which is based on system reliability. It is obvious that other criteria can also be applied, for example, system reliability may be replaced with the lifecycle cost or a function associating with geography convenience. For geography-based importance measures, if a component fails, one may choose some other components that are geographically easy to approach to be maintained. For example, in case of the offshore wind mills, if a component fails, then other components close to the failed one may also inspected and maintained.

No symptom appears upon failures. If no symptom appears upon failure, failure can be detected only when a critical component has failed or a cut set has failed. In this case, maintenance including repair and PM are conducted while the system is not being operated. As such, In this case, $H_{j|i}$ in Definition 2 can be ignored as it is used to ensure that PM does not stop system working. One may therefore consider using the following measure

$$I_{j|i}^{M'}(t) = \frac{\partial \phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_j(t)}, \quad (8)$$

to rank the importance.

6 A numerical example

The above sections discuss maintenance policies A and B. In the following, for the sake of simplicity, we use maintenance policy A as an example.

We consider the system shown in Fig. 2. Assume $p_k(t) = \exp\left\{-\left(\frac{t}{\alpha_k}\right)^{\beta_k}\right\}$, where $\alpha_k = 18 - k$, $\beta_k = 1 + 0.03k$, and $k = 1,2,\dots,8$. Suppose that a failed component is replaced with a new identical one. Suppose that the PM effect on component k follows a linear PM model [23], i.e., the failure rate of component k after the j-th PM is given by

$$h_{k,j}(t) = h_{k,j-1}\left(t - a_k t_j\right), \tag{9}$$

where $h_{k,0}(t) = \frac{\beta_k}{\alpha_k}\left(\frac{t}{\alpha_k}\right)^{\beta_k - 1}$, $t \in \left(t_j, +\infty\right)$, t_j is the calendar age of the system after the j-th PM is conducted on component k, $0 < a_k < 1$, $k = 1,2,\dots,8$, and $j = 1,2,\cdots$.

It can be seen that the shortest path sets should include components 1 and 4, one of components 2 and 3, and one of components 5, 6, 7, and 8. That is, the number of components in the shortest path set is $n_c = 4$.

We use Monte Carlo simulation to estimate the average numbers of component failures and the average number of system failures. Suppose that the total life is 5 years (or 60 months), which can be seen as a PM contract (see [24], for example). We repeat the simulation for 3,000 times. Column 1 includes the number, m, of components that are selected for PM and row 1 includes the settings of parameters a_k, where $a_k = k\eta$. $N_c$and N_s in column 2 are the average number of component failures and system failures, respectively. The results are shown in Table 2. When no PM is performed, the average number of component failures is N_c =48.633 and the average number of system failures is N_s =9.022, which are not shown in the Table. Values 46.521 and 8.958 in cells (2,3) and cell (3,3) in the table are the total numbers of component failures and system failures within 6 years if $m = 1$ (i.e., 1 component can be preventively maintained) and $a_k = 0.01k$ (for $k = 1,2,\dots,8$). It can be observed from the table that

- If m increases and η keeps constant, both $N_c$and N_s show decreasing trends; and
- If η increases and m keeps constant, $N_c$and N_s show decreasing trends.
- All N_c in the table are smaller than 48.633 (i.e., the number of component failures when no PM is conducted) and all N_s are smaller than 9.022 (i.e., the number of system failures when no PM is conducted).
- One can also observe that N_c changes more drastically than N_s. This is because of the following reasons.
  - the system only fails if component 1 or component 4 fails;
  - both component 1 and component 4 are only preventively maintained when one of them fails; and
  - if component 1 (or component 4) fails, then component 4 (or component 1) usually has the top priority of being selected for PM.

Table 2 also indirectly illustrates the use of the importance for a group of components defined in Eq. (5), as the effect of both reliability improvement (i.e., $a_k = k\eta$ in the table) and a group of components for Pm (i.e., $m > 1$) on the system reliability is illustrated in the table.

One may also optimise the number of components on which PM can be conducted. For example, suppose that conducting a PM costs £30, a system failure can incur £40, a component failure can incur £10, and. Then on the maintenance effect in column 2 (i.e., $\eta = 0.01$) in Table 2, the cost analysis is shown in Table 3. The total costs for $m = 1, 2, 3$ and 4 are shown in the last column in Table 3. For example, if $m = 1$, then $£30 \times 1 + £10 \times 46.521 + £40 \times 8.958 = 853.53$. The costs are 853.53, 840.70, 842.97, and 859.13 for $m = 1, 2, 3$, and 4, respectively. As a result, one may select $m = 2$ as its corresponding cost 840.70 is the minimum.

Table 2. Comparison of the number of failure within 5 years over the number of components for PM.

		$\boldsymbol{\eta}$ =0.01	$\boldsymbol{\eta}$ =0.015	$\boldsymbol{\eta}$ =0.02	$\boldsymbol{\eta}$ =0.025	$\boldsymbol{\eta}$ =0.03
$\boldsymbol{m = 1}$	N_c	46.521	45.627	44.922	43.920	43.186
	N_s	8.958	8.827	8.855	8.674	8.558
$\boldsymbol{m = 2}$	N_c	42.334	40.518	39.389	38.651	37.817
	N_s	8.934	8.832	8.735	8.666	8.396
$\boldsymbol{m = 3}$	N_c	39.693	36.568	34.208	32.536	31.463
	N_s	8.901	8.823	8.724	8.514	8.453
$\boldsymbol{m = 4}$	N_c	38.629	35.117	32.934	31.307	30.271
	N_s	8.821	8.570	8.328	8.010	7.574

Table 3. Cost analysis over the number of components for PM

m	N_c for η =0.01	N_s for η =0.01	Cost on PM (= £30 × m)	Cost on component Failure (= £10 × N_c)	Cost on system Failure (= £40 × N_s)	Total cost
1	46.521	8.958	30	465.21	358.32	853.53
2	42.334	8.934	60	423.34	357.36	840.70
3	39.693	8.901	90	396.93	356.04	842.97
4	38.629	8.821	120	386.29	352.84	859.13

7 Conclusions

Based on the analysis of the conditional component importance proposed in [8], this paper extends the Birnbaum importance measure to a measure called the component maintenance priority (CMP), with which a pre-specified number of components may be selected for preventive maintenance (PM) while a failed component is being repaired. The CMP differs from most of the existing component importance measures as the CMP may be zero and the latter are usually positive. Here, a component with a zero CMP implies that PM should not be conducted on it.

The CMP can be used to schedule PM policy, as illustrated in the example in Section 6. Different from conventional PM optimisation methods that optimise the interval between PM activities, this paper optimises the number of components on which PM can be conducted while a failed component is being repaired.

Our future research is to investigate component maintenance priority when maintenance cost and reliability improvement cost are considered. That is, the ideas of this paper and that from reference [3] will be extended.

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