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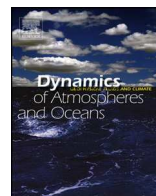
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# Dynamics of Atmospheres and Oceans

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## The statistical relation of sea-level and temperature revisited



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### ABSTRACT

We propose a semi-empirical model for the relation between global mean surface temperature and global sea-levels. In contradistinction to earlier approaches to this problem, the model allows for valid statistical inference and joint estimation of trend components and interaction term of temperature and sea-level. Estimation of the model on the data set used in [Rahmstorf \(2007\)](#) yields a proportionality coefficient of 4.6 mm/year per °C at a one-sided significance level of 7.6 percent or higher. Long-term simulations of the model result in a two-sided 90-percent confidence interval for the sea-level rise in the year 2100 of [15 cm, 150 cm] above the 1990 level. This is a wider margin of error than was reported in the previous literature, and it reflects the substantial uncertainty in relating two trending time series.

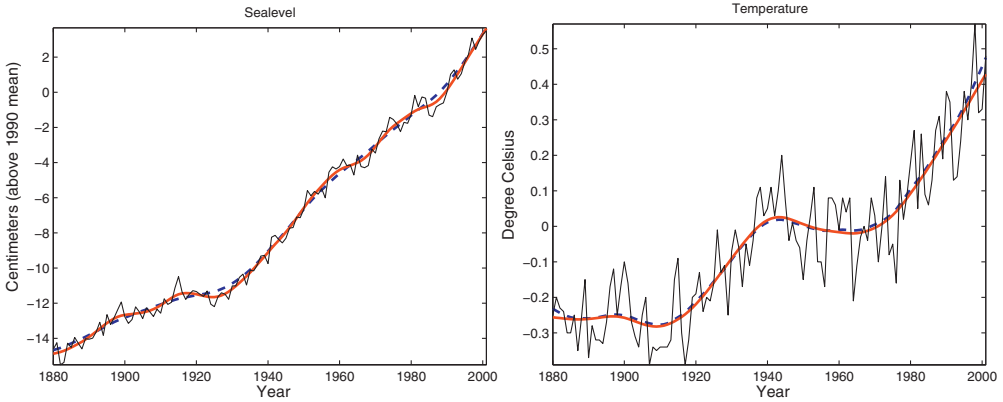
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## 1. Introduction

Estimating the influence of global temperature on global sea-level using time series of historical data is a statistical problem of observational data that has been discussed intensively in recent years

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**Fig. 1.** Trend components in global sea-level (left) and global surface temperature (right). The solid smooth line is the trend component estimated from the state-space model. The dashed smooth line is the non-linear trend component estimated from singular spectrum analysis.

(Rahmstorf, 2007a,b, 2010; Woodworth et al., 2009; Horton et al., 2008, Jevrejeva et al., 2009, 2010). A central problem is the fact that both temperature and sea-level time series are trending upwards (see Fig. 1) and that the hypothesized relationship between the two lies exactly in these long-term trends. In other words, it is supposed that the long-term trend in global temperatures influences the long-term trend in global sea-levels (Rahmstorf, 2007; Rahmstorf and Vermeer, 2009). Therefore, standard statistical methods of differencing or de-trending and subsequent regression of stationary residuals onto each other are beside the point.

Semi-empirical models have been proposed that justify a linear approximation of the interaction term of these trend components in temperature and sea-level, which make the problem's estimation accessible by linear regression methods (Jevrejeva et al., 2009; Grinsted et al., 2010; Rahmstorf and Vermeer, 2009). In Rahmstorf (2007), Rahmstorf and Vermeer (2009), a linear approximate relationship for the influence of global temperatures on the differences of global sea-levels is motivated:

$$\frac{dH}{dt} = a(T - T_0), \quad (1)$$

where  $H$  is the sea-level,  $T$  is temperature, and  $a$  is the coefficient of influence. In order to estimate this relation, a linear regression of the type

$$f(H_t) - f(H_{t-1}) = b(f(T_t) - f(T_0)) + \text{error}_t \quad (2)$$

is run, where  $H_t$  is a time series of global sea-level observations,  $T_t$  is a time series of global temperature observations,  $b$  is the coefficient of influence in the model and an estimate of  $a$  in Eq. (1). For a given time series  $X_t$ , the data transformation  $f(X_t)$  extracts a long-term trend. The extant literature often uses singular spectrum analysis in order to smooth short-run fluctuations, see for example Moore et al. (2005), Jevrejeva et al. (2006), Wahl et al. (2010), Rahmstorf et al. (2012a,b), Bittermann et al. (2013). Sometimes, moving averages over non-overlapping windows are applied in addition to singular spectrum analysis to further smooth the series (Rahmstorf, 2007; Rahmstorf and Vermeer, 2009). In the case of global sea-level and temperature time series  $H_t$  and  $T_t$  considered in Rahmstorf (2007), the regressand  $f(H_t) - f(H_{t-1})$  and regressor  $f(T_t) - f(T_0)$  are both upward trending.

The linear regression approach with trending time series potentially invalidates standard statistical inference (Schmith et al., 2007, 2012; Granger and Newbold, 1974). This is caused by the properties of the residual process ( $\text{error}_t$ ), which may be statistically indistinguishable from a random walk. For the data set considered in this paper, we show that this is indeed the case (see Fig. 4 and Table 2). Smoothing by singular spectrum analysis adds to the complexity of the statistical properties of the process ( $\text{error}_t$ ), leaving the asymptotic properties of the estimator of  $b$  in Eq. (2) intractable.

The objective of this paper is to provide valid statistical inference in a model that linearly links levels in temperatures with first differences in sea levels, leaving as much as possible of the spirit of the analysis behind Eqs. (1) and (2) intact. The main contribution of this paper is the proposition of a state-space model as an alternative to singular spectrum analysis. The ability of state-space models to extract trend components has been noted before in the literature (Zheng and Basher, 1999). The obtained smooth component is very similar to the first singular component (as can be seen in Fig. 1), while the statistical properties of the error process remain tractable. In addition to allowing for valid inference, the model makes tacit assumptions behind the regression analysis of Eq. (2) explicit and therefore allows diagnostic checking of these assumptions.

## 2. Model

This section introduces the model for sea-level and temperature proposed in this paper. It is a variant of the so-called local trend model (Harvey, 1989; Durbin and Koopman, 2001):

$$\begin{aligned} H_t &= \mu_t^H + \varepsilon_t^H, & T_t &= \mu_t^T + \varepsilon_t^T, \\ \mu_t^H &= \mu_{t-1}^H + \beta_{t-1}^H + c\mu_{t-1}^T, & \mu_t^T &= \mu_{t-1}^T + \beta_{t-1}^T, \\ \beta_t^H &= \beta_{t-1}^H + \eta_t^H, & \beta_t^T &= \beta_{t-1}^T + \eta_t^T. \end{aligned} \quad (3)$$

The first row defines sea-level  $H_t$  and temperature  $T_t$  as consisting of the smooth trend components  $\mu_t^H$  and  $\mu_t^T$  plus zero-mean error processes  $\varepsilon_t^H$  and  $\varepsilon_t^T$  for sea-level and temperature, respectively. The second row defines the smooth trend components. The smooth component  $\mu_t^T$  of temperature is a so-called integrated random walk, that is, the first difference follows a simple random walk  $\beta_t^T$ . The smooth trend component  $\mu_t^H$  of the sea-level is defined similarly, but includes a linear influence of the smooth trend component of temperature. The parameter  $c$  captures this linear influence, and it is the object of interest in our analysis. We discuss in Section 4 in what sense the smooth components  $\mu_t^H$  and  $\mu_t^T$  resemble the singular spectrum trends  $f(H_t)$  and  $f(T_t)$  from regression (2).

According to this model, the corresponding relationship for the long-term trends of sea-level and temperature reads

$$\mu_t^H - \mu_{t-1}^H = c\mu_{t-1}^T + \beta_{t-1}^H. \quad (4)$$

This is in agreement with Eq. (2), but in contradistinction, the error term  $\beta_{t-1}^H$  is a well-defined random walk that accounts for the fact that both left-hand side ( $\mu_t^H - \mu_{t-1}^H$ ) and explanatory variable  $\mu_{t-1}^T$  are upward trending. If ( $\mu_t^H - \mu_{t-1}^H$ ) and  $\mu_{t-1}^T$  are trending, then, in general, so will a linear combination ( $\mu_t^H - \mu_{t-1}^H$ ) -  $c\mu_{t-1}^T$  of them. This motivates the third row of the model definition (3) that specifies the error processes of the smooth components as random walks.

Extracting smooth trend components by singular spectrum analysis and then proceeding with regression (2) of the obtained singular components amounts to discarding the differences between the time series and its smooth trend components. In the language of model (3), these are the processes  $\varepsilon_t^H = H_t - \mu_t^H$  and  $\varepsilon_t^T = T_t - \mu_t^T$ . Discarding them implies the assumption that they contain no information about the long-term relation between sea-level and temperature. The assumption behind Eqs. (1) and (2) is that all influence of temperature on sea-level, and vice versa, is captured in the coefficient  $b$ . This assumption is kept in model (3) for the coefficient  $c$ . In particular, we assume diagonality of the error covariance matrix of  $\varepsilon_t^H$  and  $\varepsilon_t^T$ . The model is flexible enough, however, to extend to non-diagonal entries if these are desired (Durbin and Koopman, 2001), which we do in Section 5. We find significant correlation of  $\varepsilon_t^H$  and  $\varepsilon_t^T$ . However, allowing for this correlation does not substantially change the results, neither with regard to the coefficient  $c$  of interest, nor with regard to the long-term forecast.

Model (3) can be rewritten in canonical state-space form to contain two equations, a *measurement equation* for the observable quantities  $H_t$  and  $T_t$ :

$$\mathbf{y}_t = \begin{bmatrix} H_t \\ T_t \end{bmatrix} = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t = \begin{bmatrix} \mu_t^H \\ \mu_t^T \end{bmatrix} + \begin{bmatrix} \varepsilon_t^H \\ \varepsilon_t^T \end{bmatrix},$$

**Table 1**

Maximum-likelihood parameter estimates and simulation-based  $p$ -values for null hypotheses that the parameter is equal to zero versus the alternative that the parameter is greater than zero.

$\theta$	$c$	$\text{Var}(\varepsilon^H)$	$\text{Var}(\varepsilon^T)$	$\text{Var}(\eta^H)$	$\text{Var}(\eta^T)$
MLE	0.4565	0.1712	7.9e–3	2.8e–3	8.2e–6
$P$ -value	0.0756	0.0000	0.0000	0.0000	0.0000

and a transition equation

$$\xi_t = \begin{bmatrix} \mu_t^H \\ \mu_t^T \\ \beta_t^H \\ \beta_t^T \end{bmatrix} = \mathbf{F}\xi_{t-1} + \eta_t = \begin{bmatrix} 1 & c & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_{t-1}^H \\ \mu_{t-1}^T \\ \beta_{t-1}^H \\ \beta_{t-1}^T \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \eta_t^H \\ \eta_t^T \end{bmatrix}$$

for the unobservable quantities  $\mu$  and  $\beta$ . The error processes are assumed to be independently normally distributed:

$$\varepsilon_t = \begin{bmatrix} \varepsilon_t^H \\ \varepsilon_t^T \end{bmatrix} \sim N(\mathbf{0}, \mathbf{H}), \quad \eta_t = \begin{bmatrix} \eta_t^H \\ \eta_t^T \end{bmatrix} \sim N(\mathbf{0}, \mathbf{Q}),$$

where we assume  $\mathbf{H}$  and  $\mathbf{Q}$  to be diagonal, in accordance with the motivating regression specification (2). There, first singular components are extracted separately for both, temperature and sea-level time series, and then these components are regressed onto each other. This motivates the assumption of diagonality for  $\mathbf{Q}$ . We report diagnostic statistics on the residuals in Section 5.

The likelihood of the canonical state-space model is evaluated using the Kalman-filter (Kalman, 1960; Harvey, 1989; Commandeur and Koopman, 2007), which takes the parameter vector  $\theta = (c, \text{Var}(\varepsilon^H), \text{Var}(\varepsilon^T), \text{Var}(\eta^H), \text{Var}(\eta^T))$ , and returns optimal projections (in the least-squares sense) of the state vector  $\xi_t$  and the corresponding error-covariance matrix  $\mathbf{P}_t = \mathbb{E}[(\xi_t - \mathbb{E}\xi_t)(\xi_t - \mathbb{E}\xi_t)']$ . The implementation follows the univariate treatment of multivariate series described in Durbin and Koopman (2001).

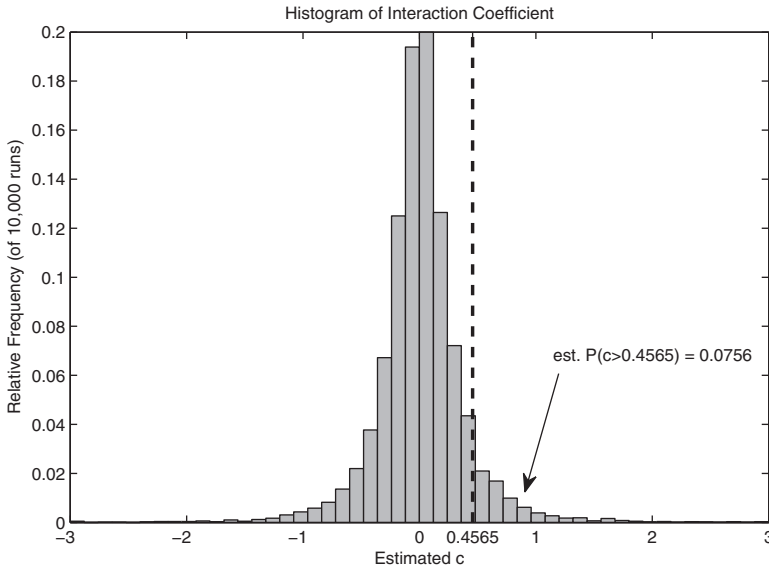
### 3. Data

We use the data set employed in Rahmstorf (2007) to facilitate comparison. These are annual observations of global sea-levels from Church and White (2006) and annual observations of global temperatures from the Goddard Institute for Space Studies (Hansen et al., 2001). The overlapping sample period of the two time series is 1880–2001 for a total of 122 years. The time series are shown in Fig. 1; the sea-level is displayed in centimeters relative to the level of 1990, and the temperature is displayed in degrees Celsius relative to the mean of the period 1951–1980.

### 4. Results and discussion

The maximum-likelihood estimates of the model specified in Eq. (3) are reported in Table 1, together with simulation-based  $p$ -values of a test for the null hypothesis that the parameter is equal to zero against the alternative that it is greater than zero. The simulation procedure to obtain the  $p$ -values is outlined below for the case of  $c$ .

Fig. 1 plots the times series  $H_t$  and  $T_t$  and their estimated trend components  $\mu_t^H$  and  $\mu_t^T$  together with the singular-spectrum estimated trends  $f(H_t)$  and  $f(T_t)$  for comparison. As can be seen, the smooth trend components obtained from singular spectrum analysis and from the state-space model are very similar. The singular spectrum analysis conducted in Moore et al. (2005), Jevrejeva et al. (2006), Wahl et al. (2010), Rahmstorf et al. (2012a,b), Bittermann et al. (2013) implicitly assumes a model whose



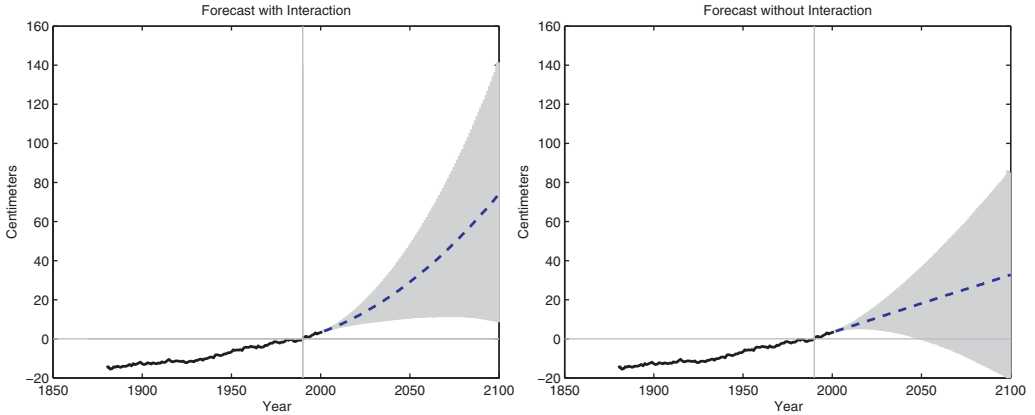
**Fig. 2.** Simulation of the model under the null hypothesis of  $c = 0$ . In 10,000 simulations of the model with a data-generating  $c$  of zero and subsequent estimations, the estimated  $c$  was above 0.4565 in 756 cases.

statistical implications are not obvious. Model (3) is a statistically tractable alternative that yields almost the same smoothing.

The estimate of the parameter of interest,  $c$ , is equal to 0.4565. Given the dimensions of the time series, this corresponds to a proportionality coefficient of 4.565 mm/year per  $^{\circ}\text{C}$ . Simulation-based  $p$ -values are calculated for the parameters and reported in Table 1. In the case of the coefficient of interest  $c$ , for example, we simulate the model under the null hypothesis  $\mathbb{H}_0 : c = 0$ . Using the parameter estimates for the variances reported in Table 1, and setting  $c = 0$ , we simulate 10,000 trajectories of model (3) for 122 observations (corresponding to the number of years, 1880–2001, for which the data set has matching pairs of global mean temperatures and global sea-levels). The state-space model (3) is estimated on each simulated trajectory, allowing for  $c \neq 0$ , and the estimate  $\hat{c}$  is stored. The resulting set of 10,000 estimates of  $c$  yields an empirical distribution of the parameter estimate. Fig. 2 shows this distribution. In 756 cases out of the 10,000 runs, the estimated coefficient  $c$  of interaction between temperature and sea-level was estimated above 0.4565, the maximum-likelihood point estimate obtained from real data. Therefore, we estimate the probability to observe an estimated  $c$  of 0.4565 or bigger when the data-generating  $c$  is zero ( $\alpha$  or type-I-error for the null of  $c = 0$  against the alternative of  $c > 0$ ) to be 0.0756, or 7.56 percent. The resulting statistical inference is that the estimate of the parameter  $c$  is significant at levels of 7.6 percent or higher. This simulation approach to inference is necessary because there is no asymptotic theory available for model (3) due to the non-stationary nature of the involved processes (Durbin and Koopman, 2001; Chang et al., 2009).

The probability of the type-I-error is much larger than was reported in Rahmstorf (2007). The reasons for this difference are the non-standard distribution of the parameter estimator  $b$  in Eq. (2) that results from the integrated, non-stationary nature of the time series  $f(H_t) - f(H_{t-1})$  and  $f(T_t) - f(T_0)$ , and the absence of an explicit model of the randomness in extracting the smooth trend components by singular spectrum analysis.

Note also that the point estimate of 4.565 mm/year per  $^{\circ}\text{C}$  is larger than the one reported in Rahmstorf (2007) (3.4 mm/year per  $^{\circ}\text{C}$ ). This suggests that in long-term forecasts, the influence of the interaction between temperature and sea-level will be substantial. We generate long-run forecasts from the model by simulating it 10,000 times using the estimated parameter vector  $\theta$  reported in Table 1. For comparison, we also generate long-term forecasts from a univariate state-space model for



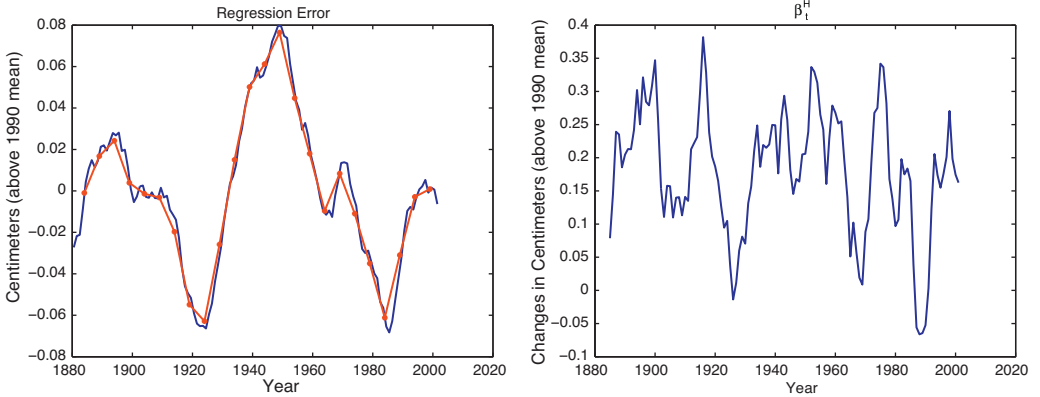
**Fig. 3.** Long-term forecasts of global sea-levels from the state-space model with influence of global surface temperatures ( $c = 0.4565$ , left) and without interaction with temperatures ( $c = 0$ , right).

the sea-level, i.e., for the case where  $c = 0$ . Similar forecast exercises have been conducted in [Rahmstorf \(2007\)](#), [Rohling et al. \(2008\)](#), [Kopp et al. \(2009\)](#), [Vermeer and Rahmstorf \(2009\)](#), [Grinsted et al. \(2010\)](#). The forecast horizon corresponds to the one reported in [Rahmstorf \(2007\)](#) and extends until the year 2100. [Fig. 3](#) plots the two forecasts along with their two-sided 90-percent confidence bands. As can be seen, the interaction coefficient leads to a substantial difference in the long-term forecasts; the point forecast for the year 2100 (median of 10,000 simulations) indicates an increase of 80 cm above the 1990 mean in contradistinction to 40 cm over the 1990 mean according to the univariate model. The univariate model cannot capture any influence of temperature beyond what is already contained in the sea-level time series. The two-sided 90 percent confidence interval for the model with interaction is [15 cm, 150 cm] for the year 2100 as opposed to [−13 cm, 94 cm] for the univariate model. There are two main reasons for the wider forecast confidence bands reported here compared to the literature that is based on Eq. (2). First, as mentioned above, in an equation such as regression (2), the statistical properties of the residual process (error<sub>*t*</sub>) invalidate standard inference, and uncertainty is understated. Second, in [Rahmstorf \(2007\)](#) for example, temperature scenarios from the Intergovernmental Panel on Climate Change are employed. This eliminates randomness originating in the model for temperatures. In the forecast exercise here, the temperature time series fluctuate across the simulation runs, and this translates into larger forecast error intervals for the sea-level.

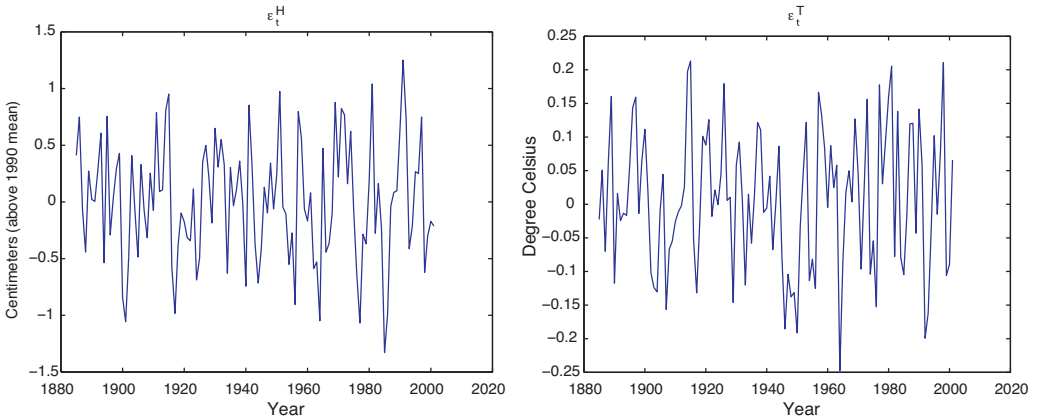
## 5. Specification diagnostics

[Fig. 4](#) shows the residuals (error<sub>*t*</sub>) from regression (2) (left panel) and the integrated error process  $\beta_t^H$  from the analogous Eq. (4) from the state-space model (right panel). The residuals from Eq. (2) display a high degree of serial correlation; a Dickey–Fuller test cannot reject the null hypothesis of a unit root at significance levels smaller than 10 percent, as is shown in [Table 2](#). Essentially, this was the point made in [Schmith et al. \(2007\)](#).

[Fig. 5](#) shows the residuals  $\varepsilon_t^H$  and  $\varepsilon_t^T$  from the measurement equation in (3). [Table 2](#) shows descriptive statistics and tests for serial correlation of the different residual processes considered in the comparison of models (2) and (3). The sequence of tests follows [Brockwell and Davis \(2002\)](#), [Harvey and Koopman \(1992\)](#). An augmented Dickey–Fuller test rejects the null hypothesis of a unit root for all stationary error processes  $\varepsilon_t^{H,T}$  and  $\eta_t^{H,T}$  in model (3). The tests for serial correlation are conducted on the observations of the error processes from 1891 through 2001, since the Kalman-filter is run with a diffuse initialization that distorts estimates of serial correlation when all observations are included. The results are qualitatively not sensitive to choosing later cutoff points. Box–Pierce, Ljung–Box, and McLeod–Li tests show that there is some serial correlation left in the residuals of the error process  $\varepsilon_t^H$  from the sea-level time series, whereas the one from the temperature time series  $\varepsilon_t^T$  appears reasonably



**Fig. 4.** Left panel: residual from regression (2). The blue solid line shows the residuals from (2), when  $f(\cdot)$  extracts the first singular component only; the red line marked with asterisks shows the residual when  $f(\cdot)$  extracts the first singular component and averages over non-overlapping 5-year windows as in Rahmstorf (2007). Right panel: estimated  $\beta_t^H$ . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



**Fig. 5.** Left panel: estimated  $\epsilon_t^H$ . Right panel: estimated  $\epsilon_t^T$ .

**Table 2**

Descriptive statistics of error processes and tests for serial correlation. The process (error<sub>t</sub>) from Eq. (2) is computed for the case where  $f(\cdot)$  extracts the first singular component only. Sequence of tests as proposed in Brockwell and Davis (2002): BP, Box–Pierce; LB, Ljung–Box; ML, McLeod–Li; TP turning point; DS, difference sign; Rank test.  $p$ -values in parentheses. The first 10 data points of each residual series are discarded, statistics are computed using 20 lags of the sample autocorrelation function. The Dickey–Fuller test (DF) is an augmented test with automatic lag selection by Schwarz Criterion with a maximum lag of 10. The null hypothesis is the presence of a unit root, low  $p$ -values thus indicate covariance-stationarity.

	Mean	Std. dev.	SkewKurt.	BP	LB	ML	TP	DS	Rank	DF	
error <sub>t</sub>	0.00	0.04	0.16	2.54	628.02	667.57	445.13	34	53	2729	–2.597
$\epsilon_t^H$	0.00	0.53	–0.03	2.60	30.75	34.05	29.80	68	56	3068	–8.176
$\epsilon_t^T$	0.01	0.10	–0.04	2.34	22.28	24.55	18.94	68	65	3217	–8.399
$\eta_t^H$	0.00	0.05	–0.07	2.56	31.53	35.16	33.60	65	58	3036	–8.243
$\eta_t^T$	0.00	3e–3	–0.04	2.38	21.45	23.55	21.97	70	62	3129	–8.546
					(0.37)	(0.26)	(0.34)	(0.55)	(0.02)	(0.70)	(0.00)
					(0.00)	(0.00)	(0.00)	(0.00)	(0.42)	(0.06)	(0.10)
					(0.06)	(0.03)	(0.07)	(0.23)	(0.87)	(0.84)	(0.00)
					(0.33)	(0.22)	(0.53)	(0.23)	(0.00)	(0.58)	(0.00)
					(0.05)	(0.02)	(0.03)	(0.08)	(0.33)	(0.93)	(0.00)



whitened. Note that serial correlation in the error processes  $\eta_t^{H,T}$  of the unobserved components is innocuous. In Harvey and Koopman (1992), it is shown that the estimates of these error processes in the transition equation can be serially correlated even in correctly specified models with known parameters.

As mentioned in Section 2, the approach of Eqs. (1) and (2) assumes that there either is no correlation between  $H_t - f(H_t)$  and  $T_t - f(T_t)$ , or that it is of no consequence for the estimation of the coefficient of interest and the long-term forecast. Our model allows checking this assumption. The analogs of  $H_t - f(H_t)$  and  $T_t - f(T_t)$  in model (3) are  $\varepsilon_t^H$  and  $\varepsilon_t^T$ . We can relax the assumption that the covariance matrix  $\mathbf{H}$  of  $\varepsilon_t^H$  and  $\varepsilon_t^T$  is diagonal and allow for

$$\mathbf{H} = \begin{bmatrix} \sigma_{\varepsilon^H}^2 & \rho\sigma_{\varepsilon^H}\sigma_{\varepsilon^T} \\ \rho\sigma_{\varepsilon^H}\sigma_{\varepsilon^T} & \sigma_{\varepsilon^T}^2 \end{bmatrix}.$$

Estimating this slightly modified specification, we obtain an estimate of the correlation coefficient of  $\hat{\rho} = 0.2504$ , which is significant at the two-sided 2-percent level. The coefficient of interest is estimated at  $\hat{c} = 0.4779$ , which is significant at the one-sided 5-percent level. The long-term forecast from this modified model is virtually identical to the one in the left panel of Fig. 3, and we conclude that the correlation of the errors in the measurement equation has no influence on the qualitative findings.

## 6. Conclusion

Semi-empirical methods are a useful tool in relating time series of historical data of sea-level and temperature. Trending behavior in these time series can invalidate inference in linear regressions. The state-space model proposed in this paper allows for joint estimation of trend components and the linear interaction term between global surface temperature and global sea-level. It mimics the singular trend component extraction and the linear regression setup employed in the previous literature while keeping the statistical properties of the residual processes tractable. Simulations of the model yield valid statistical inference. Testing the null hypothesis that the influence of temperatures on sea-level is zero against the alternative that it is positive, we show that the estimated coefficient is significant at type-I-error levels of 7.6 percent or higher for the data set studied here. Long-term forecasts show that even though the statistical significance is limited, the influence of temperature on sea-level projections is nevertheless substantial. The forecast confidence bands are wider than reported previously, because randomness in the estimation of the linear influence as well as in the extraction of the trend components is modeled explicitly.

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