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Insurance Risk Classification
How much is socially optimal?

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2 Why do people buy insurance?

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Insurance Risk Classification

Insurance – Pooling of risk

- Everybody is exposed to **risks**, but only a few suffer a **loss**.
- **Insurers** compensate **insureds** if a particular loss occurs.
- Insurers charge a **premium** for their services.
- Why is insurance possible at an affordable premium? **Pooling**.

Insurance risk classification

- Homogeneous risk: Charge the same premium for all.
- Heterogeneous risk: Contentious issue of **adverse selection**.
Adverse Selection

What is adverse selection?
No commonly accepted standard definition of *adverse selection*.

Definition (Actuarial perspective)
Insurer faces **loss** due to risk not factored in at the time of sale due to **asymmetric information** between the insurer and the insured.

Definition (Economic perspective)
An individual’s **demand for insurance** (the propensity to buy insurance and the quantity purchased) is **positively correlated** with the individual’s **risk of loss** (higher risks buy more insurance).

Question:
Why is this a **bad outcome** and for **whom**?
Introduction

Background

Theory and Practice

Traditional theory:

If insurers cannot charge risk-differentiated premiums, then:

- higher risks buy more insurance, lower risks buy less insurance,
- raising the pooled price of insurance,
- lowering the demand for insurance,

usually portrayed as a bad outcome, both for insurers and for society.

In practice:

Policymakers often see merit in restricting insurance risk classification

- EU ban on using gender in insurance underwriting.
- Moratoria on the use of genetic test results in underwriting.

Question:

How can we reconcile theory with practice?
We ask:

- **Why** do people buy insurance?
- **What** drives demand for insurance?
- **How much** of population losses is compensated by insurance (with and without risk classification)?
- **Which** regime is most beneficial to society?

We find:

**Social welfare** is maximised by maximising **loss coverage**.

**Definition (Loss coverage)**

Expected population losses compensated by insurance.
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Why do people buy insurance?

Assumptions

Consider an individual with

- an initial wealth \( W \),
- exposed to the risk of loss \( L \),
- with probability \( \mu \),
- utility of wealth \( U(w) \), with \( U'(w) > 0 \) and \( U''(w) < 0 \),
- an opportunity to insure at premium rate \( \pi \).
Utility of wealth

Utility $U(W)$ vs. Wealth $W-L$ and $W$.
Expected utility: Without insurance

Expected utility: 

\[ (1 - \mu)U(W) + \mu U(W - L) \]  

Utility

\[ U(W) \]

\[ U(W - L) \]

Wealth

\[ W - L \]

\[ W - \mu L \]

\[ W \]
Expected utility: Insured at fair actuarial premium

\[ U(W - \mu L) \]

\[ U(W) \]

\[ \text{Fair premium} \]

\[ \mu L \]

\[ W - \mu L \]

\[ W - L \]

\[ W \]

Utility

U(W - L)

U(W)

Utility

Wealth

W - L

W - \mu L

W

Expected utility: With insurance

Why do people buy insurance?
Maximum premium tolerated: \( \pi_c \)

\[
U(W) = U(W - \mu L) = \frac{1}{\mu} U(W) + \frac{\mu}{\mu} U(W - L)
\]

\[
U(W) = U(W - \pi_c L) = (1 - \mu) U(W) + \mu U(W - L)
\]

Fair premium: \( \mu L \)

Maximum premium tolerated: \( \pi_c L \)
What drives demand for insurance?

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Modelling demand for insurance

Simplest model:

Based on the given set-up:

- All will buy insurance if \( \pi < \pi_c \);
- None will buy insurance if \( \pi > \pi_c \).

**Reality:** Not all will buy insurance even at fair premium. *Why?*

Heterogeneity:

- Individuals are homogeneous in terms of underlying risk.
- Individuals can be heterogeneous in terms of **attitude to risk**.

Utility as a Random Variable

\([U(w)](v)\) is utility of wealth \( w \) for an individual \( v \) chosen at random.

- \([U(w)](v)\): (non-random) utility function of wealth for individual \( v \).
- \( \overbrace{U(w)} \) is a random variable for a specific wealth \( w \).
Demand is a survival function

Condition for buying insurance:

Given premium $\pi$, individual $v$ chosen at random will buy insurance if:

$$\left[U(W - \pi L)(v)\right] > (1 - \mu) \times \left[U(W)(v)\right] + \mu \times \left[U(W - L)(v)\right].$$

With insurance Without insurance

Standardisation

Suppose all individuals within the risk-group are standardised so that:

$$[U(W)(v)] = 1,$$

$$[U(W - L)(v)] = 0.$$ 

Demand as a survival function:

Given premium $\pi$, insurance demand, $d(\pi)$, is the survival function:

$$d(\pi) = \text{Prob}[U(W - \pi L) > (1 - \mu)U(W) + \mu U(W - L)].$$
Demand is a survival function

\[(1 - \mu)U(W) + \mu U(W - L)\]

\[= U(W - L)\]

\[\Rightarrow \bar{U}(W - \pi L)\]

Utility

U(W)

U(W - L)

W - L

W - \pi L

W

Wealth

A

B

C

D

d(\pi)

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Illustrative example: \( W = L = 1 \)

**Power utility function:**

\[
\overline{U}(w) = w^{\overline{\gamma}}.
\]

**Heterogeneity in risk preferences: Distribution of \( \overline{\gamma} \):**

\[
\text{Prob}[\overline{\gamma} \leq x] = \begin{cases} 
0 & \text{if } x < 0 \\
k x^\lambda & \text{if } 0 \leq x \leq (1/k)^{1/\lambda}, \ k > 0, \lambda > 0, \\
1 & \text{if } x > (1/k)^{1/\lambda}.
\end{cases}
\]

**Demand for insurance:**

\[
d(\pi) = \text{Prob}[\overline{U}(W - \pi L) > (1 - \mu)\overline{U}(W) + \mu \overline{U}(W - L)],
\]

\[
d(\pi) \approx \text{Prob}[\overline{\gamma} < \frac{\mu}{\pi}] = k \left( \frac{\mu}{\pi} \right)^\lambda.
\]

\[
d(\pi) \propto \pi^{-\lambda}.
\]
Illustrative example: $W = L = 1$

Demand elasticity (Iso-elastic demand):

$$d(\pi) \propto \pi^{-\lambda} \Rightarrow \epsilon(\pi) = \left| \frac{\partial d(\pi)}{\partial \pi} \right| = \lambda.$$
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Consider a population of individuals with the same:

- initial wealth $W = 1$;
- potential loss $L = 1$;
- form of iso-elastic demand function $d(\pi) \propto \pi^{-\lambda}$; and
- demand elasticity $\lambda$.

Suppose the population can be divided into 2 risk-groups, with:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: $p_1$ and $p_2$;
- fair premium demand: $d_1(\mu_1) = \tau_1$ and $d_2(\mu_2) = \tau_2$, i.e.

$$d_i(\pi) = \tau_i \left( \frac{\pi}{\mu_i} \right)^{-\lambda}, \quad i = 1, 2.$$
Risk-differentiated premium

Equilibrium:
If risk-differentiated premiums $\pi_1$ and $\pi_2$ are allowed,
- Total premium: $\sum_i p_i \, d_i(\pi_i) \, \pi_i$.
- Total claims: $\sum_i p_i \, d_i(\pi_i) \, \mu_i$.
Equilibrium is achieved when insurers break even, i.e. $\pi_i = \mu_i$.

Adverse Selection:
No losses for insurers. No (actuarial/economic) adverse selection.

Loss coverage (Population losses compensated by insurance):
Loss coverage = $\sum_i p_i \, d_i(\mu_i) \, \mu_i = \sum_i p_i \, \tau_i \, \mu_i$. 
Pooled premium

Equilibrium:

If only a pooled premium $\pi_0$ is allowed,

- Total premium: $\sum_i p_i d_i(\pi_0) \pi_0$.
- Total claims: $\sum_i p_i d_i(\pi_0) \mu_i$.

Equilibrium is achieved when insurers break even, i.e.

$$\sum_i p_i d_i(\pi_0) \pi_0 = \sum_i p_i d_i(\pi_0) \mu_i,$$

$$\Rightarrow \pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^{\lambda} + \alpha_2 \mu_2^{\lambda}}, \text{ where } \alpha_i = \frac{\tau_i \rho_i}{\tau_1 \rho_1 + \tau_2 \rho_2}.$$
How much of population losses is compensated by insurance?

Pooled premium

Pooled premium: Adverse selection

\[
\pi_0 \approx \alpha_1 \mu_1 + \alpha_2 \mu_2
\]

\( \lambda \) (Demand elasticity)
How much of population losses is compensated by insurance?

Pooled premium: Adverse selection

**Adverse selection: Summary**

- The pooled equilibrium is greater than the average premium charged under full risk classification:

  \[ \pi_0 > \alpha_1 \mu_1 + \alpha_2 \mu_2 \Rightarrow \text{(Economic) adverse selection}. \]

- No losses for insurers! \(\Rightarrow\) No (actuarial) adverse selection.

Adverse selection is not useful to measure social efficacy of insurance.
Loss coverage ratio

Loss coverage (Population losses compensated by insurance):

\[
\text{Loss coverage} = \sum_i p_i \ d_i(\pi_0) \ \mu_i.
\]

Loss coverage ratio:

\[
C = \frac{\text{Loss coverage for pooled premium}}{\text{Loss coverage for risk-differentiated premium}},
\]

\[
= \frac{\sum_i p_i \ d_i(\pi_0) \ \mu_i}{\sum_i p_i \ d_i(\mu_i) \ \mu_i},
\]

\[
= \frac{1}{\pi_0^\lambda} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}.
\]
Loss coverage ratio

- How much of population losses is compensated by insurance?
- Loss coverage ratio

\[ \lambda \text{ (Demand elasticity)} \]

Graph showing the relationship between loss coverage ratio and demand elasticity.
How much of population losses is compensated by insurance?

Loss coverage ratio

\[ \lambda (\text{Demand elasticity}) \]

\[ \alpha_1 = 0.8 \]
\[ \mu_1 = 0.01 \mu_2 = 0.05 \]

\[ \alpha_1 = 0.9 \]
\[ \mu_1 = 0.01 \mu_2 = 0.04 \]
Loss coverage ratio: Summary

Summary

- $\lambda < 1 \Rightarrow$ Loss coverage is more when risk classification is banned.
- $\lambda = 1 \Rightarrow$ Loss coverage is the same in both risk classification regimes.
- $\lambda > 1 \Rightarrow$ Loss coverage is more when full risk classification is used.
- Empirical evidence suggests $\lambda < 1$, providing justification for restricting risk classification.
- The maximum value of loss coverage ratio depends on the relative risk and relative size of the risk groups.
- A pooled premium might be highly beneficial in the presence of a small group with very high risk exposure.
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Social welfare

Definition (Social welfare)

Social welfare, $G$, is the sum of all individuals’ expected utilities:

$$G = \sum_{i} p_i \left[ d_i(\pi_i) U^*(W - L\pi_i) + (1 - d_i(\pi_i)) \left\{ (1 - \mu_i) U(W) + \mu_i U(W - L) \right\} \right],$$

where $U^*(W - L\pi_i)$ is the expected utility of the insured population.

Linking social welfare to loss coverage

Setting $U(W - L) = 0$ and assuming $L\pi_i \approx 0$ gives:

$$G = U(W) \sum_{i} p_i d_i(\pi_i) \mu_i + \text{Constant},$$

$$= \text{Positive multiplier} \times \text{Loss coverage} + \text{Constant}.$$

Loss coverage provides a good proxy (which depends only on observable data) for social welfare (which depends on unobservable utilities).

Result: Maximising loss coverage maximises social welfare.
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Conclusions

Adverse selection need not be adverse

Restricting risk classification
- will always increase adverse selection;
- increases loss coverage if $\lambda < 1$.

Summary

Loss coverage provides a better metric than adverse selection in measuring social welfare.
References


