Insurance Risk Classification
How much is socially optimal?

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Insurance Risk Classification

Insurance – Pooling of risk

- Everybody is exposed to **risks**, but only a few suffer a **loss**.
- **Insurers** compensate **insureds** if a particular loss occurs.
- Insurers charge a **premium** for their services.
- Why is insurance possible at an affordable premium? **Pooling**.

Insurance risk classification

- Homogeneous risk: Charge the same premium for all.
- Heterogeneous risk: Contentious issue of **adverse selection**.
# Adverse Selection

## What is adverse selection?
No commonly accepted standard definition of *adverse selection*.

## Definition (Actuarial perspective)
Insurer faces **loss** due to risk not factored in at the time of sale due to **asymmetric information** between the insurer and the insured.

## Definition (Economic perspective)
An individual’s **demand for insurance** (the propensity to buy insurance and the quantity purchased) is **positively correlated** with the individual’s **risk of loss** (higher risks buy more insurance).

## Question:
Why is this a **bad outcome** and for **whom**?
Theory and Practice

**Traditional theory:**
If insurers cannot charge risk-differentiated premiums, then:
- higher risks buy more insurance, lower risks buy less insurance,
- raising the pooled price of insurance,
- lowering the demand for insurance,
usually portrayed as a bad outcome, both for insurers and for society.

**In practice:**
Policymakers often see merit in restricting insurance risk classification
- EU ban on using gender in insurance underwriting.
- Moratoria on the use of genetic test results in underwriting.

**Question:**
How can we reconcile theory with practice?
We ask:

- **Why** do people buy insurance?
- **What** drives demand for insurance?
- **How much** of population losses is compensated by insurance (with and without risk classification)?
- **Which** regime is most beneficial to society?

We find:

**Social welfare** is maximised by maximising **loss coverage**.

**Definition (Loss coverage)**

Expected population losses compensated by insurance.
Why do people buy insurance?

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Why do people buy insurance?

Assumptions

Consider an individual with

- an initial wealth $W$,
- exposed to the risk of loss $L$,
- with probability $\mu$,
- utility of wealth $U(w)$, with $U'(w) > 0$ and $U''(w) < 0$,
- an opportunity to insure at premium rate $\pi$. 
Utility of wealth
Expected utility: Without insurance

\[ (1 - \mu)U(W) + \mu U(W - L) \]
Why do people buy insurance?

Expected utility: With insurance

Expected utility: Insured at fair actuarial premium

\[ U(W) = U(W - \mu L) \]

Fair premium

\[ \mu L \]
Why do people buy insurance?

Maximum premium tolerated: $\pi_c$

- Utility
  - $U(W)$
  - $U(W - \mu L)$
  - $U(W - \pi_c L)$
  - $U(W - \mu L)$

- Wealth
  - $W - L$
  - $W - \pi_c L$
  - $W - \mu L$
  - $W$

(1 - $\mu$)U(W) + $\mu$U(W - L)

Fair premium $\mu L$

Maximum premium tolerated $\pi_c L$
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Modelling demand for insurance

Simplest model:

Based on the given set-up:

- All will buy insurance if $\pi < \pi_c$;
- None will buy insurance if $\pi > \pi_c$.

Reality: Not all will buy insurance even at fair premium. Why?

Heterogeneity:

- Individuals are homogeneous in terms of underlying risk.
- Individuals can be heterogeneous in terms of attitude to risk.

Utility as a Random Variable

$[U(w)](v)$ is utility of wealth $w$ for an individual $v$ chosen at random.

- $[U(w)](v)$: (non-random) utility function of wealth for individual $v$.
- $\widehat{U(w)}$ is a random variable for a specific wealth $w$. 
Demand is a survival function

Condition for buying insurance:
Given premium $\pi$, individual $v$ chosen at random will buy insurance if:

$$[U(W - \pi L)](v) > (1 - \mu) \times [U(W)](v) + \mu \times [U(W - L)](v).$$

With insurance

Without insurance

Standardisation
Suppose all individuals within the risk-group are standardised so that:

$$[U(W)](v) = 1,$$
$$[U(W - L)](v) = 0.$$

Demand as a survival function:
Given premium $\pi$, insurance demand, $d(\pi)$, is the survival function:

$$d(\pi) = \text{Prob}[U(W - \pi L) > (1 - \mu)U(W) + \mu U(W - L)].$$
What drives demand for insurance?

Demand is a survival function

\[ U(W) = (1 - \mu)U(W) + \mu U(W - L) \]

\[ U(W - L) \]

\[ \text{Utility} \]

\[ \text{Wealth} \]

\[ d(\pi) \]

\[ U(W - \pi L) \]

D (University of Kent)
Illustrative example: \( W = L = 1 \)

Power utility function:

\[
\hat{U}(w) = w^{\gamma}.
\]

Heterogeneity in risk preferences: Distribution of \( \hat{\gamma} \):

\[
\text{Prob}[\hat{\gamma} \leq x] = \begin{cases} 
0 & \text{if } x < 0 \\
k \, x^\lambda & \text{if } 0 \leq x \leq (1/k)^{1/\lambda}, \ k > 0, \ \lambda > 0, \\
1 & \text{if } x > (1/k)^{1/\lambda}.
\end{cases}
\]

Demand for insurance:

\[
d(\pi) = \text{Prob}[\hat{U}(W - \pi L) > (1 - \mu) \hat{U}(W) + \mu \hat{U}(W - L)],
\]

\[
d(\pi) \approx \text{Prob}[\hat{\gamma} < \frac{\mu}{\pi}] = k \left( \frac{\mu}{\pi} \right)^{\lambda}.
\]

\[
d(\pi) \propto \pi^{-\lambda}.
\]
Illustrative example: \( W = L = 1 \)

Demand elasticity (Iso-elastic demand):

\[
d(\pi) \propto \pi^{-\lambda} \Rightarrow \epsilon(\pi) = \left| \frac{\partial d(\pi)}{\partial \pi} \right| = \lambda.
\]
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Consider a population of individuals with the same:

- initial wealth $W = 1$;
- potential loss $L = 1$;
- form of iso-elastic demand function $d(\pi) \propto \pi^{-\lambda}$; and
- demand elasticity $\lambda$.

Suppose the population can be divided into 2 risk-groups, with:

- risk of losses: $\mu_1 < \mu_2$;
- population proportions: $p_1$ and $p_2$;
- fair premium demand: $d_1(\mu_1) = \tau_1$ and $d_2(\mu_2) = \tau_2$, i.e.

$$d_i(\pi) = \tau_i \left( \frac{\pi}{\mu_i} \right)^{-\lambda}, \quad i = 1, 2.$$
Risk-differentiated premium

Equilibrium:

If risk-differentiated premiums $\pi_1$ and $\pi_2$ are allowed,

- Total premium: $\sum_i p_i d_i(\pi_i) \pi_i$.
- Total claims: $\sum_i p_i d_i(\pi_i) \mu_i$.

Equilibrium is achieved when insurers break even, i.e. $\pi_i = \mu_i$.

Adverse Selection:

No losses for insurers. No (actuarial/economic) adverse selection.

Loss coverage (Population losses compensated by insurance):

Loss coverage $= \sum_i p_i d_i(\mu_i) \mu_i = \sum_i p_i \tau_i \mu_i$. 
Pooled premium

Equilibrium:
If only a pooled premium $\pi_0$ is allowed,
- Total premium: $\sum_i p_i d_i(\pi_0) \pi_0$.
- Total claims: $\sum_i p_i d_i(\pi_0) \mu_i$.

Equilibrium is achieved when insurers break even, i.e.

\[
\text{Total premium} = \text{Total claims},
\]
\[
\Rightarrow \sum_i p_i d_i(\pi_0) \pi_0 = \sum_i p_i d_i(\pi_0) \mu_i,
\]
\[
\Rightarrow \pi_0 = \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1^{\lambda} + \alpha_2 \mu_2^{\lambda}}, \text{ where } \alpha_j = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}.
\]
Pooled premium: Adverse selection

\[
\pi_0 = \alpha_1 \mu_1 + \alpha_2 \mu_2
\]
Pooled premium: Adverse selection

Adverse selection: Summary

- The pooled equilibrium is greater than the average premium charged under full risk classification:
  \[ \pi_0 > \alpha_1 \mu_1 + \alpha_2 \mu_2 \Rightarrow \text{(Economic) adverse selection}. \]
- No losses for insurers! \( \Rightarrow \) No (actuarial) adverse selection.

Adverse selection is not useful to measure social efficacy of insurance.
Loss coverage ratio

Loss coverage (Population losses compensated by insurance):
Loss coverage = \( \sum_i p_i \, d_i(\pi_0) \, \mu_i \).

Loss coverage ratio:

\[
C = \frac{\text{Loss coverage for pooled premium}}{\text{Loss coverage for risk-differentiated premium}},
\]
\[
= \frac{\sum_i p_i \, d_i(\pi_0) \, \mu_i}{\sum_i p_i \, d_i(\mu_i) \, \mu_i},
\]
\[
= \frac{1}{\pi_0^\lambda} \frac{\alpha_1 \mu_1^{\lambda+1} + \alpha_2 \mu_2^{\lambda+1}}{\alpha_1 \mu_1 + \alpha_2 \mu_2}.
\]
How much of population losses is compensated by insurance?

Loss coverage ratio

Loss coverage ratio

\( \lambda \) (Demand elasticity)
Loss coverage ratio

\( \alpha_1 = 0.8 \)
\( \mu_1 = 0.01 \mu_2 = 0.05 \)

\( \alpha_1 = 0.9 \)
\( \mu_1 = 0.01 \mu_2 = 0.04 \)
Loss coverage ratio: Summary

Summary

- $\lambda < 1 \Rightarrow$ Loss coverage is more when risk classification is banned.
- $\lambda = 1 \Rightarrow$ Loss coverage is the same in both risk classification regimes.
- $\lambda > 1 \Rightarrow$ Loss coverage is more when full risk classification is used.

Empirical evidence suggests $\lambda < 1$, providing justification for restricting risk classification.

The maximum value of loss coverage ratio depends on the relative risk and relative size of the risk groups.

A pooled premium might be highly beneficial in the presence of a small group with very high risk exposure.
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Social welfare

Definition (Social welfare)

Social welfare, \( G \), is the sum of all individuals’ expected utilities:

\[
G = \sum_i p_i \left[ d_i(\pi_i) U^* (W - L\pi_i) + (1 - d_i(\pi_i)) \left\{ (1 - \mu_i) U(W) + \mu_i U(W - L) \right\} \right],
\]

where \( U^* (W - L\pi_i) \) is the expected utility of the insured population.

Linking social welfare to loss coverage

Setting \( U(W - L) = 0 \) and assuming \( L\pi_i \approx 0 \) gives:

\[
G = U(W) \sum_i p_i d_i(\pi_i) \mu_i + \text{Constant},
\]

\[= \text{Positive multiplier} \times \text{Loss coverage} + \text{Constant}.\]

Loss coverage provides a good proxy (which depends only on observable data) for social welfare (which depends on unobservable utilities).

Result: Maximising loss coverage maximises social welfare.
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Conclusions

Adverse selection need not be adverse

Restricting risk classification
- will always increase adverse selection;
- increases loss coverage if $\lambda < 1$.

Summary

Loss coverage provides a better metric than adverse selection in measuring social welfare.
References


