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Robust normalization and guaranteed cost control for a class of uncertain singular Markovian jump systems via hybrid impulsive control

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SUMMARY

This paper investigates the problem of robust normalization and guaranteed cost control for a class of uncertain singular Markovian jump systems. The uncertainties exhibit in both system matrices and transition rate matrix of the Markovian chain. A new impulsive and proportional-derivative control strategy is presented, where the derivative gain is to make the closed-loop system of the singular plant to be a normal one, and the impulsive control part is to make the value of the Lyapunov function does not increase at each time instant of the Markovian switching. A linearization approach via congruence transformations is proposed to solve the controller design problem. The cost function is minimized via solving an optimization problem under the designed control scheme. Finally, three examples (two numerical examples and an RC pulse divider circuit example) are provided to illustrate the effectiveness and applicability of the proposed methods. Copyright © 2013 John Wiley & Sons, Ltd.

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KEY WORDS: robust normalization; guaranteed cost control; uncertain singular Markovian jump systems; proportional-derivative control; impulsive control

1. INTRODUCTION

Singular systems (also known as generalized, descriptor or differential algebraic systems) have convenient and natural representation in the description of practical systems in various fields due to their capacity involving the dynamic and algebraic relationships among state variables simultaneously, such as robotics, chemical processes, electrical circuit systems, multi-sector economic systems, and other areas [1, 2]. In the past few decades, such a class of systems has attracted many researchers from control and mathematics communities, and a large number of results have been reported in the literature including stability analysis, control and filtering, see, for example, [3–9] and the references therein.

Recently, more attention has been paid to the study of Markovian jump linear systems [10], in which the mode process is a continuous Markov process taking values in a finite set. When singular systems suffer abrupt changes caused by component failures or repairs, it is natural to describe them as singular Markovian jump systems (SMJSs) [6, 11–16]. Compared with normal systems, singular systems are more complicated, in which the stability, regularity, and impulse elimination (for continuous case) or causality (for discrete case) should be considered simultaneously. In particular,

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when the derivative matrices of SMJSs have uncertainties, the control problem will become more difficult. The detailed reasons are as follows.

First, the dynamics of singular systems are effected by the coefficient matrix E of $\dot{x}(t)$. If there exists perturbation in E, the stability of the system may be destroyed even though the system is regular, impulse-free, and stable [17]. Generally speaking, it is impossible to stabilize a singular system in the presence of unstructured uncertainties in coefficient matrix E by employing a traditional state feedback control scheme, because in this case, the change of the rank of matrix E and the violation of regularity may occur. It is fortunate that the proportional-derivative (PD) controller is an effective method for singular systems, which has been applied to solve various controller synthesis problems in the literature [17–21]. By normalizing the systems via PD controller, [17] and [18] solved the robust H_∞ control and guaranteed cost control problems for singular systems with norm-bounded uncertainties in state, input, and derivative matrices, respectively. Reference [19] investigated the robust control problem for a class of uncertain singular stochastic Markovian jump systems with element-wise bounded uncertainties in transition rate matrix (TRM), and sufficient conditions for the considered systems to be quadratically normal and quadratically stochastically stable are given in terms of matrix conditions by separating the Lyapunov function matrix from the derivative matrix and the state matrix.

Second, when switching behavior occurs between two singular subsystems, the state variables at switching points may not satisfy the consistent initial condition of the next activated subsystem. The inconsistent initial condition of singular systems may lead to finite instantaneous jumps or even destroy the systems when such jumps reach a certain level (see, e.g., [22, 23] and the references therein). In [22], the definition of consistency projectors was given to deal with the instability mechanisms caused by the intrinsic property of the autonomous switched differential algebraic equations (DAEs), that is, the jump map brought by the presence of algebraic constraints. However, this method is not suitable for switched differential algebraic equations with inputs and/or outputs. Reference [23] designed a hybrid impulsive controller to compress the inconsistent initial jumps at the switching instants for switched singular systems, but the state jumps may not be eliminated with the given impulsive controller if the constraint equations are not satisfied. Moreover, the method proposed in [23] is not suitable for the SMJSs with uncertainties in the derivative matrices. If the SMJSs can be normalized through feedback control, the problems aforementioned can be solved naturally.

In the design of a control system, it is usually desirable that the closed-loop system is not only robustly stable, but also has an adequate level of performance. The guaranteed cost control problems have thus received extensive research (see, e.g., [18, 24–29] and the references therein). The main idea of guaranteed cost control is to design a control category such that, for all admissible uncertainties, the corresponding closed-loop system is asymptotically stable and an upper bound of the quadratic cost is minimized. In [24], the LQ guaranteed cost control problem of uncertain impulsive switched systems with norm-bounded uncertainties and given impulsive gain matrices at fixed times was considered. The existence conditions of LQ guaranteed cost control law were also established. Reference [25] tackled the guaranteed cost control problem for a class of continuous-time singular linear Markovian jump systems with totally and partially known transition jump rates.

On the other hand, impulsive control is an effective method, which can stabilize a complicated system by using simple control impulses, even though the system behaviors may be unavailable to the controller design [30]. Over the past few decades, the problems of impulsive control have been investigated for various types of systems, such as singular systems [4, 30, 31], switched systems [24, 32, 33], linear systems [34], stochastic systems [35], and Markovian systems [36, 37].

Motivated by the aforementioned discussion, in this paper, the problem of guaranteed cost control is studied for a class of SMJSs with uncertainties in both system matrices (i.e., state, input, and derivative matrices) and TRM. To the best of our knowledge, there are few results available in the literature for this problem, which motivates our current research.

In our approach, the derivative state term is introduced in the performance index function, which makes it feasible to damp the oscillations and limit the response rate. An *impulsive and proportional-derivative state feedback controller* (IPDSFC) is proposed in this paper to solve this problem. The PD part of the hybrid controller is to normalize the uncertain SMJSs to avoid the two phenomena

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aforementioned, whereas the impulsive part is to guarantee that the value of the Lyapunov function does not increase at each switching time instant. The main difficulties in this design problem are twofold: (i) how to deal with the nonlinear terms in the obtained matrix conditions of the controller design, and the uncertainties in system matrices and TRM; and (ii) how to deal with the nonlinear numerical inequalities caused by the impulsive controller with unknown gain matrices (as seen in formula (11) in the succeeding texts). By adopting appropriate congruence transformations and free-connection weighting matrices, both two problems are solved, and feasible conditions can be obtained in terms of matrix inequalities. The gain matrices of the impulsive control part are parameter variables, which can be solved together with the design approach. This is different from the results of [24, 30–32, 34–37], where the gain of the impulsive control is given as a constant matrix in advance. Our design idea can thus provide more design freedom than those in the existing literature. An optimal design procedure is also provided such that the corresponding closed-loop system is robustly stochastically stable with a prescribed upper bound of the cost function. Finally, three examples (two numerical examples and an RC pulse divider circuit example) demonstrate the effectiveness and applicability of the presented methods.

Notations: Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. The symbol $'*'$ represents an ellipsis for the terms induced by symmetry in symmetric block matrices, and $diag\{\ldots\}$ for a block-diagonal matrix. I denotes the identity matrix with appropriate dimension. $\mathbb{E}[.]$ stands for the mathematical expectation operator with respect to the given probabilities. $\|.\|$ refers to the Euclidian norm for vectors.

2. PROBLEM FORMULATION

Consider a class of uncertain SMJSs described as

$$\begin{aligned}(E(r(t)) + \Delta E(r(t)))\dot{x}(t) = (A(r(t)) + \Delta A(r(t)))x(t) + (B(r(t)) + \Delta B(r(t)))u(t)\\ x(t_0) = x_0, r(t_0) = r_0\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input. Matrix $E(r(t)) \in \mathbb{R}^{n\times n}$ may be singular, and it is assumed that $rank E(r(t)) = n_{r(t)} \leqslant n$. $A(r(t))$ and $B(r(t))$ are known matrices of compatible dimensions. $\Delta E(r(t)), \Delta A(r(t))$, and $\Delta B(r(t))$ are unknown matrices denoting the uncertainties of the system. The mode $\{r(t), t \geqslant 0\}$ (we also denote as $\{r_t, t \geqslant 0\}$) is a right-continuous-time Markov process taking values in a finite state space $\mathcal{S} = \{1, 2, \ldots, N\}$ with transition probabilities

$$Pr\,[r(t+\Delta) = j\,|r(t) = i\,] = \begin{cases}\tilde{\pi}_{ij}\Delta + o(\Delta) & if\ \ i \neq j\\ 1 + \tilde{\pi}_{ii}\Delta + o(\Delta) & if\ \ i = j\end{cases} \tag{2}$$

where $\Delta > 0$, $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$ and $\tilde{\pi}_{ij} \geqslant 0$, $i, j \in \mathcal{S}$, $i \neq j$, is the transition rate from the mode i at time t to the mode j at time $t + \Delta$ and $\tilde{\pi}_{ii} = -\sum_{j=1, j\neq i}^{N} \tilde{\pi}_{ij}$. x_0 and r_0 are the initial state and the initial mode of the system, respectively. For simplicity, for each possible value $r(t) = i \in \mathcal{S}$, a matrix $A(r(t))$ is denoted as A_i.

In this paper, for any value $r(t) = i \in \mathcal{S}$, without loss of generality, the aforementioned uncertainties are assumed as

$$\begin{bmatrix}\Delta E_i & \Delta A_i & \Delta B_i\end{bmatrix} = M_i F(t) \begin{bmatrix}N_{ei} & N_{ai} & N_{bi}\end{bmatrix} \tag{3}$$

where M_i, N_{ei}, N_{ai}, and N_{bi} are known real constant matrices of appropriate dimensions, and the uncertain matrix $F(t)$ satisfies $F^{\mathrm{T}}(t)F(t) \leqslant I$. The real TRM $\tilde{\Pi} = (\tilde{\pi}_{ij})$ in (2) cannot be obtained exactly. Instead, similar to [19, 38], we only know that it satisfies the following admissible uncertainty

$$\tilde{\Pi} = \Pi + \Delta\Pi\ \ with\ \ |\Delta\pi_{ij}| \leqslant \varepsilon_{ij},\ \varepsilon_{ij} \geqslant 0,\ j \neq i \tag{4}$$

In (4), TRM $\Pi \triangleq (\pi_{ij})$ with $\pi_{ij} \geqslant 0$, $j \neq i$ and $\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}$ is the known constant estimation of $\tilde{\Pi}$, $\Delta\Pi \triangleq (\Delta\pi_{ij})$, $\Delta\pi_{ij} = \tilde{\pi}_{ij} - \pi_{ij}$ denotes the estimated error between $\tilde{\pi}_{ij}$ and π_{ij}. It is

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concluded that $\Delta\pi_{ii}$ can also be expressed by $\Delta\pi_{ii} = -\sum_{j=1,j\neq i}^{N} \Delta\pi_{ij}$. $\Delta\pi_{ij}$, $j \neq i$, is assumed to take any value in $\left[-\varepsilon_{ij}, \varepsilon_{ij}\right]$, and $\alpha_{ij} \triangleq \pi_{ij} - \varepsilon_{ij}$. Then, it is obtained that $|\Delta\Pi_{ii}| \leqslant -\varepsilon_{ii}$, where $\varepsilon_{ii} \triangleq -\sum_{j=1,j\neq i}^{N} \varepsilon_{ij}$ and $\alpha_{ii} \triangleq \pi_{ii} - \varepsilon_{ii}$.

Let $\{t_k, k = 1, 2, \ldots\}$ be a sequence satisfying $t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$, where $t_k > 0$ is the kth switching moment, that is, the moment of the transition of the mode from $r\left(t_k^-\right) = j$ to $r\left(t_k^+\right) = i$, $t_k^+ = \lim_{\Delta\to 0}(t_k + \Delta)$, $\forall k > 0$.

The objective of this paper is to develop a procedure to design an IPDSFC for system (1) in the form of

$$
\begin{aligned}
u(t) &= u_1(t) + u_2(t) \\
u_1(t) &= K_a(r(t))x(t) - K_e(r(t))\dot{x}(t) \\
u_2(t) &= \sum_{k=1}^{\infty} G\left(r\left(t_k^+\right)\right) x(t)\delta(t - t_k)
\end{aligned}
\tag{5}
$$

where $u_1(t)$ is a mode-dependent PD state feedback controller and $u_2(t)$ is an impulsive controller, $K_a(r(t))$, $K_e(r(t))$, and $G\left(r\left(t_k^+\right)\right)$ are to be designed gain matrices of appropriate dimensions, $\delta(.)$ is the Dirac impulse function, with discontinuous impulsive instants $t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$, where $t_1 > t_0$, and $x(t_k) = x\left(t_k^-\right) = \lim_{h\to 0^+} x(t_k - h)$, $x\left(t_k^+\right) = \lim_{h\to 0^+} x(t_k + h)$, $e_x(t_k) = x\left(t_k^+\right) - x\left(t_k^-\right)$.

Suppose that when $t \in (t_k, t_{k+1}]$, $r(t) = i$, that is, the ith subsystem is activated. Substituting (5) into the system (1) leads to

$$
E_{ci}[x(t_k + h) - x(t_k)] = \int_{t_k}^{t_k+h} E_{ci}\dot{x}(s)ds = \int_{t_k}^{t_k+h} [A_{ci}x(s) + (B_i + \Delta B_i)u_2(s)]ds
$$

where

$$
\begin{aligned}
E_{ci} &= E_i + \Delta E_i + (B_i + \Delta B_i)k_{ei} \\
A_{ci} &= A_i + \Delta A_i + (B_i + \Delta B_i)k_{ai}
\end{aligned}
$$

when $h \to 0^+$, it follows that

$$
E_{ci}[x(t_k + h) - x(t_k)] = E_{ci}e_x(t_k) = (B_i + \Delta B_i)G_i x(t_k)
$$

With controller (5), system (1) becomes an uncertain singular and impulsive Markovian system in the following form

$$
\begin{aligned}
E_c(r(t))\dot{x}(t) &= A_c(r(t))x(t), & t &\in (t_k, t_{k+1}] \\
E_c(r(t))e_x(t_k) &= (B(r(t)) + \Delta B(r(t)))G\left(r\left(t_k^+\right)\right) x(t_k), & t &= t_k \\
x(t_0) &= x_0, r(t_0) = r_0
\end{aligned}
\tag{6}
$$

where

$$
E_c(r(t)) = E(r(t)) + \Delta E(r(t)) + (B(r(t)) + \Delta B(r(t)))K_e(r(t)) \tag{7}
$$

$$
A_c(r(t)) = A(r(t)) + \Delta A(r(t)) + (B(r(t)) + \Delta B(r(t)))K_a(r(t)) \tag{8}
$$

Remark 1

It is clear to see from (5) that the controller designed in this paper is composed of two parts: a proportional-derivative state feedback controller and an impulsive controller. The PD part aims at normalizing the original system, while the impulsive part is used to change the state values at each switching point for the closed-loop system.

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Definition 1
The uncertain singular and impulsive Markovian system (6) is said to be robustly stochastically stable if, for any $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{S}$, there exists a scalar $M(x_0, r_0) > 0$ such that

$$\mathbb{E}\left[\int_{t_0}^{\infty} \|x(t)\|^2 dt | x_0, r_0\right] \leqslant M(x_0, r_0)$$

holds for all admissible uncertainties.

Given a set of positive definite matrices $Q_1(r(t))$, $Q_2(r(t))$ and $R(r(t))$, consider the cost function defined by

$$J = \mathbb{E}\left\{\sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \left[x^{\mathrm{T}}(t)Q_1(r(t))x(t) + \dot{x}^{\mathrm{T}}(t)Q_2(r(t))\dot{x}(t) + u^{\mathrm{T}}(t)R(r(t))u(t)\right] dt\right\} \tag{9}$$

Associated with cost function (9), the robust normalization and the guaranteed cost hybrid impulsive controller (RNGCIC) for system (1) is defined as follows.

Definition 2
Consider the uncertain SMJS (1). If there exists a controller (5) and a positive scalar J_0 such that for all admissible uncertainties, the derivative matrix E_{ci}, $\forall i \in \mathcal{S}$, in the system (6) is invertible, the system (6) is robustly stochastically stable, and the corresponding value of the cost function (9) satisfies $J \leqslant J_0$, then J_0 is said to be a guaranteed cost, and (5) is said to be an RNGCIC for system (1).

Remark 2
In definition 2, the definitions of the guaranteed cost control law reported in [18, 24] are extended to the case of uncertain impulsive SMJSs.

Lemma 1 ([39]*)*
Given a symmetric matrix Z and matrices X and Y of appropriate dimensions, then

$$Z + XF(t)Y + (XF(t)Y)^{\mathrm{T}} < 0$$

for all $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \leqslant I$, if and only if there exists a scalar $\epsilon > 0$ such that

$$Z + \epsilon XX^{\mathrm{T}} + \epsilon^{-1}Y^{\mathrm{T}}Y < 0$$

Lemma 2 ([40]*)*
Given a positive definite matrix $P \in \mathbb{R}^{n \times n}$ and a symmetric matrix $Q \in \mathbb{R}^{n \times n}$, then

$$\lambda_{min}(P^{-1}Q)x^{\mathrm{T}}(t)Px(t) \leqslant x^{\mathrm{T}}(t)Qx(t) \leqslant \lambda_{max}(P^{-1}Q)x^{\mathrm{T}}(t)Px(t)$$

for all $x(t) \in \mathbb{R}^n$.

In the sequel, the main results of this paper are formulated as follows.

(i) Sufficient conditions are proposed to verify the existence of RNGCIC for system (1);
(ii) An optimal control design procedure is given to obtain the gain matrices of the PD part and the impulsive part of IPDSFC simultaneously, by which the resulting closed-loop SMJSs over all admissible uncertainties are normal and robustly stochastically stable with a minimal upper bound of the cost function.

3. MAIN RESULTS

In this section, a set of sufficient conditions for robust normalization and guaranteed cost control for the uncertain SMJS (1) under an IPDSFC is to be developed.

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3.1. Existence conditions of RNGCIC

In this part, the existence conditions of RNGCIC for SMJSs are presented by the following theorem.

Theorem 1
Consider system (1) associated with cost function (9). If there exist matrices $P_i > 0$, T_{1i} and T_{2i} such that the following set of inequalities hold for each $i \in \mathcal{S}$ and $k = 1, 2, \ldots$

$$\Omega_i = \begin{bmatrix} \Omega_{1i} & \Omega_{2i} \\ * & \Omega_{3i} \end{bmatrix} < 0 \tag{10}$$

$$0 < \beta_k \leqslant 1 \tag{11}$$

then (5) is an RNGCIC for system (1), and

$$J_0 = x_0^{\mathrm{T}} P(r_0) x_0$$

where

$$\begin{aligned}
\Omega_{1i} &= A_{ci}^{\mathrm{T}} T_{1i} + T_{1i}^{\mathrm{T}} A_{ci} + Q_{1i} + \sum\nolimits_{j=1}^{N} \tilde{\pi}_{ij} P_j + K_{ai}^{\mathrm{T}} R_i K_{ai} \\
\Omega_{2i} &= P_i - T_{1i}^{\mathrm{T}} E_{ci} + A_{ci}^{\mathrm{T}} T_{2i} - K_{ai}^{\mathrm{T}} R_i K_{ei} \\
\Omega_{3i} &= -T_{2i}^{\mathrm{T}} E_{ci} - E_{ci}^{\mathrm{T}} T_{2i} + Q_i + K_{ei}^{\mathrm{T}} R_i K_{ei} \\
\beta_k &= \lambda_{max} \left\{ P^{-1}(r(t_k^-)) \left[I + E_{ci}^{-1}(B_i + \Delta B_i) G_i \right]^{\mathrm{T}} P(r(t_k)) \left[I + E_{ci}^{-1}(B_i + \Delta B_i) G_i \right] \right\}
\end{aligned}$$

Proof
Suppose there exist matrices $P_i > 0$, T_{1i}, T_{2i}, and the control law (5) such that (10) holds. Note that (10) implies that the derivative matrix E_{ci} is invertible for all admissible uncertainties. Choose a stochastic Lyapunov function candidate for system (6) as $V(x(t), r(t)) = x^{\mathrm{T}}(t) P(r(t)) x(t)$. For $t \in (t_k, t_{k+1}]$, let $r(t) = i$, $i \in \mathcal{S}$, then the following equation holds for any matrices T_{1i} and T_{2i} of appropriate dimensions

$$2\left[-x^{\mathrm{T}}(t) T_{1i}^{\mathrm{T}} - \dot{x}^{\mathrm{T}}(t) T_{2i}^{\mathrm{T}}\right] \left[E_{ci} \dot{x}(t) - A_{ci} x(t)\right] = 0 \tag{12}$$

Let $\mathbb{L}$ be the weak infinitesimal operator of the random process $\{(x(t), r(t)), t \geqslant 0\}$, then

$$\begin{aligned}
&\mathbb{L}V(x(t), i) + x^{\mathrm{T}}(t) Q_{1i} x(t) + \dot{x}^{\mathrm{T}}(t) Q_{2i} \dot{x}(t) + u^{\mathrm{T}}(t) R_i u(t) \\
&= 2x^{\mathrm{T}}(t) P_i \dot{x}(t) + \sum\nolimits_{j=1}^{N} \tilde{\pi}_{ij} x^{\mathrm{T}}(t) P_j x(t) + x^{\mathrm{T}}(t) Q_{1i} x(t) + \dot{x}^{\mathrm{T}}(t) Q_{2i} \dot{x}(t) \\
&\quad + x^{\mathrm{T}}(t) K_{ai}^{\mathrm{T}} R_i K_{ai} x(t) - 2x^{\mathrm{T}}(t) K_{ai}^{\mathrm{T}} R_i K_{ei} \dot{x}(t) + \dot{x}^{\mathrm{T}}(t) K_{ei}^{\mathrm{T}} R_i K_{ei} \dot{x}(t) \\
&\quad + 2\left[-x^{\mathrm{T}}(t) T_{1i}^{\mathrm{T}} - \dot{x}^{\mathrm{T}}(t) T_{2i}^{\mathrm{T}}\right] \left[E_{ci} \dot{x}(t) - A_{ci} x(t)\right] \\
&= \zeta^{\mathrm{T}}(t) \Omega_i \zeta(t)
\end{aligned} \tag{13}$$

where $\zeta(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & \dot{x}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$. If (10) holds, then

$$\mathbb{L}V(x(t), i) + x^{\mathrm{T}}(t) Q_{1i} x(t) + \dot{x}^{\mathrm{T}}(t) Q_{2i} \dot{x}(t) + u^{\mathrm{T}}(t) R_i u(t) < 0 \tag{14}$$

It follows from (14) that

$$\mathbb{L}V(x(t), i) < 0 \tag{15}$$

for $t \in (t_k, t_{k+1}]$ and all admissible uncertainties. Then there must exist scalars $\lambda_i > 0$ such that

$$\mathbb{L}V(x(t), i) \leqslant -\lambda_i x^{\mathrm{T}}(t) x(t) \tag{16}$$

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Now, consider the impulsive system at the impulsive and switching time point t_k. It follows from (6) and Lemma 2 that

$$\begin{aligned} V\left(t_k^+\right) &= x^{\mathrm{T}}\left(t_k^+\right)P(r(t_k))x\left(t_k^+\right) \\ &= x^{\mathrm{T}}(t_k)\left[I + E_{ci}^{-1}(B_i + \Delta B_i)G_i\right]^{\mathrm{T}}P(r(t_k))\left[I + E_{ci}^{-1}(i)(B_i + \Delta B_i)G_i\right]x(t_k) \\ &\leqslant \lambda_{\max}\left\{P^{-1}\left(r\left(t_k^-\right)\right)\left[I + E_{ci}^{-1}(B_i + \Delta B_i)G_i\right]^{\mathrm{T}}P(r(t_k))\right. \\ &\qquad \left.\times\left[I + E_{ci}^{-1}(i)(B_i + \Delta B_i)G_i\right]\right\}x^{\mathrm{T}}(t_k)P(r(t_k^-))x(t_k) \\ &= \beta_k V\left(t_k^-\right) \end{aligned} \tag{17}$$

Based on the Dynkin's formula, for $T \in (t_k, t_{k+1}]$,

$$\begin{aligned} &\mathbb{E}\left[\int_{t_0}^{T} \mathbb{L}V(x(t),i)dt\right] \\ &= \mathbb{E}\int_{t_0^+}^{t_1} \mathbb{L}V(x(t),i)dt + \mathbb{E}\int_{t_1^+}^{t_2} \mathbb{L}V(x(t),i)dt + \cdots + \mathbb{E}\int_{t_k^+}^{T} \mathbb{L}V(x(t),i)dt \\ &= \mathbb{E}\left[V(t_1) - V\left(t_0^+\right) + V(t_2) - V\left(t_1^+\right) + \cdots + V(T) - V\left(t_k^+\right)\right] \\ &= \mathbb{E}\left[-V\left(t_0^+\right) + \sum\nolimits_{j=1}^{k}\left(V\left(t_j^-\right) - V\left(t_j^+\right)\right) + V(T)\right] \\ &\geqslant \mathbb{E}\left[-V\left(t_0^+\right) + \sum\nolimits_{j=1}^{k}(1-\beta_j)V\left(t_j^-\right) + V(T)\right] \end{aligned}$$

From (11), (15), and (17), it follows that

$$\lim_{T\to\infty} V(T) = 0 \tag{18}$$

Because $0 < \beta_k \leqslant 1$ for all $k = 1, 2, \ldots$, it is clear that $\lim_{k\to\infty}\sum_{j=1}^{k}(1-\beta_j)V\left(t_j^-\right) \geqslant 0$, which combined with (18) gives

$$\lim_{T\to\infty}\min_{i\in\mathcal{S}}\{\lambda_i\}\mathbb{E}\left[\int_{t_0}^{T} x^{\mathrm{T}}(s)x(s)ds|x_0, r_0\right] \leqslant \mathbb{E}V(x_0, r_0)$$

which yields

$$\mathbb{E}\int_{t_0}^{\infty} ||x(t)||^2 dt|x_0, r_0] \leqslant M(x_0, r_0)$$

where $M(x_0, r_0) = \mathbb{E}V(x_0, r_0)/\min_{i\in\mathcal{S}}\{\lambda_i\}$, and thus system (6) is robustly stochastically stable. On the other hand, from (13) and similar to the aforementioned process,

$$\begin{aligned} J &= \lim_{k\to\infty}\mathbb{E}\left\{\sum_{j=0}^{k}\int_{t_j}^{t_{j+1}}\left[x^{\mathrm{T}}(t)Q_{1j}x(t) + \dot{x}^{\mathrm{T}}(t)Q_{2j}\dot{x}(t) + u^{\mathrm{T}}(t)R_j u(t)\right]dt\right\} \\ &\leqslant -\lim_{k\to\infty}\mathbb{E}\left\{\sum_{j=0}^{k}\int_{t_j}^{t_{j+1}} V(x(t), r(t))dt\right\} \\ &= \mathbb{E}V(x_0, r_0) = x_0^{\mathrm{T}}P(r_0)x_0 \end{aligned}$$

which implies that the cost function is bounded. This completes the proof. □

Remark 3
From the proof procedure of Theorem 1 aforementioned, it is clear to see that the mode-dependent Lyapunov function is monotonically decreasing during the active period of each subsystem and non-increasing at each switching point under the effect of the IPDSFC (5). Then, the robust stochastic stability of the corresponding closed-loop system and the boundedness of cost function (9) are ensured. Moreover, in contrast to [17, 19], there are no terms containing the product of P_i, E_{ci}, and A_{ci}, which can reduce the complexity caused by decomposition of the nonlinear terms to a certain extent.

Remark 4
If the uncertain SMJS degenerates into a deterministic singular system (i.e., $E_i = E$, $A_i = A$, $B_i = B$), there is no need to design impulsive controller for system (1), the proposed IPDSFC (5) degenerates into a PD state feedback controller, and Theorem 1 in this paper degenerates into Theorem 1 in [18] directly.

Remark 5
The IPDSFC proposed in this paper can not only be used for the uncertain SMJSs but also be applied to a class of more general switched singular systems, which consist of a number of subsystems and a time-dependent switching law orchestrating the active subsystem at each time instant.

3.2. Controller design

In the following, we seek a design method of the RNGCIC for system (1). Unfortunately, it is difficult to give feasible matrix conditions for obtaining an RNGCIC basing on Theorem 1 directly. Hence, appropriate congruence transformations and free-connection weighting matrices will be employed to obtain an equivalent result as Theorem 1.

Theorem 2
Consider system (1) associated with cost function (9). There exists an RNGCIC for system (1) if there exist matrices $V_{1i} > 0$, V_{2i}, V_{3i}, S_{1i}, S_{2i}, Z_i, $\bar{W}_i = \bar{W}_i^{\mathrm{T}}$, $\bar{T}_i > 0$ and scalars $\delta_{1i} > 0$, $\delta_{2i} > 0$ such that the following conditions hold for all $i, j \in \mathcal{S}$, $i \neq j$

$$\begin{bmatrix} \Theta_{1i} & \Theta_{2i} & \Theta_{4i} & \Theta_{6i} & \bar{W}_i & V_{1i}^{\mathrm{T}} & V_{2i}^{\mathrm{T}} & S_{1i}^{\mathrm{T}} \\ * & \Theta_{3i} & 0 & \Theta_{7i} & 0 & 0 & V_{3i}^{\mathrm{T}} & S_{2i}^{\mathrm{T}} \\ * & * & \Theta_{5i} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta_{1i} I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\bar{T}_i & 0 & 0 & 0 \\ * & * & * & * & * & -Q_{1i}^{-1} & 0 & 0 \\ * & * & * & * & * & * & -Q_{2i}^{-1} & 0 \\ * & * & * & * & * & * & * & -R_i^{-1} \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} -V_{1i} + \bar{W}_i & V_{1i} \\ * & -V_{1j} \end{bmatrix} \leqslant 0 \tag{20}$$

$$\begin{bmatrix} \Sigma_{1i} & \Sigma_{2i} & \Sigma_{4i} & 0 \\ * & \Sigma_{3i} & \Sigma_{5i} & V_{3i}^{\mathrm{T}} \\ * & * & -\delta_{2i} I & 0 \\ * & * & * & -V_{1i} \end{bmatrix} \leqslant 0 \tag{21}$$

where

$$
\begin{aligned}
\Theta_{1i} &= V_{2i} + V_{2i}^{\mathrm{T}} + 0.25\varepsilon_{ii}^2 \bar{T}_i + \varepsilon_{ii} \bar{W}_i + \alpha_{ii} V_{1i},\ \Theta_{2i} = V_{1i} A_i^{\mathrm{T}} - V_{2i}^{\mathrm{T}} E_i^{\mathrm{T}} + V_{3i} + S_{1i}^{\mathrm{T}} B_i^{\mathrm{T}} \\
\Theta_{3i} &= -E_i V_{3i} - V_{3i}^{\mathrm{T}} E_i^{\mathrm{T}} + B_i S_{2i} + S_{2i}^{\mathrm{T}} B_i^{\mathrm{T}} + \delta_{1i} M_i M_i^{\mathrm{T}} \\
\Theta_{4i} &= \left[\sqrt{\alpha_{i1}} V_{1i}, \ldots, \sqrt{\alpha_{i(i-1)}} V_{1i}, \sqrt{\alpha_{i(i+1)}} V_{1i}, \ldots, \sqrt{\alpha_{iN}} V_{1i}\right] \\
\Theta_{5i} &= -diag\{V_{11}, \ldots, V_{1(i-1)}, V_{1(i+1)}, \ldots, V_{1N}\} \\
\Theta_{6i} &= V_{1i}^{\mathrm{T}} N_{ai}^{\mathrm{T}} - V_{2i}^{\mathrm{T}} N_{ei}^{\mathrm{T}} + S_{1i}^{\mathrm{T}} N_{bi}^{\mathrm{T}},\ \Theta_{7i} = -V_{3i}^{\mathrm{T}} N_{ei}^{\mathrm{T}} + S_{2i}^{\mathrm{T}} N_{bi}^{\mathrm{T}} \\
\Sigma_{1i} &= -V_{3i}^{\mathrm{T}} - V_{3i} + V_{1j},\ \Sigma_{2i} = -V_{3i}^{\mathrm{T}} E_i^{\mathrm{T}} + S_{2i}^{\mathrm{T}} B_i^{\mathrm{T}} - Z_i^{\mathrm{T}} B_i^{\mathrm{T}} \\
\Sigma_{3i} &= -E_i V_{3i} - V_{3i}^{\mathrm{T}} E_i^{\mathrm{T}} + B_i S_{2i} + S_{2i}^{\mathrm{T}} B_i^{\mathrm{T}} + \delta_{2i} M_i M_i^{\mathrm{T}} \\
\Sigma_{4i} &= -V_{3i}^{\mathrm{T}} N_{ei}^{\mathrm{T}} + S_{2i}^{\mathrm{T}} N_{bi}^{\mathrm{T}} - Z_i^{\mathrm{T}} N_{bi}^{\mathrm{T}},\ \Sigma_{5i} = -V_{3i}^{\mathrm{T}} N_{ei}^{\mathrm{T}} + S_{2i}^{\mathrm{T}} N_{bi}^{\mathrm{T}}
\end{aligned}
$$

In this case, the gains of RNGCIC (5) are given by

$$K_{ai} = \left(S_{1i} - S_{2i} V_{3i}^{-1} V_{2i}\right) V_{1i}^{-1},\ K_{ei} = -S_{2i} V_{3i}^{-1},\ G_i = Z_i V_{3i}^{-1} \tag{22}$$

and

$$J_0 = x_0^{\mathrm{T}} V_1^{-1}(r_0) x_0 \tag{23}$$

Proof

From Theorem 1, it is seen that there exist an RNGCIC for system (1) if (10) and (11) hold for each $i \in \mathcal{S}$ and $k = 1, 2, \ldots$. Pre-multiplying and post-multiplying (10) by matrix $\begin{bmatrix} P_i & 0 \\ T_{1i} & T_{2i} \end{bmatrix}^{-\mathrm{T}}$ and its transpose, respectively, and setting

$$\begin{bmatrix} V_{1i} & 0 \\ V_{2i} & V_{3i} \end{bmatrix} \triangleq \begin{bmatrix} P_i & 0 \\ T_{1i} & T_{2i} \end{bmatrix}^{-1} \tag{24}$$

then (10) becomes

$$\begin{bmatrix} \Pi_{1i} & \Pi_{2i} \\ * & \Pi_{3i} \end{bmatrix} < 0 \tag{25}$$

where

$$
\begin{aligned}
\Pi_{1i} &= V_{2i} + V_{2i}^{\mathrm{T}} + V_{1i}^{\mathrm{T}} Q_{1i} V_{1i} + V_{2i}^{\mathrm{T}} Q_{2i} V_{2i} \\
&\quad + \left(V_{1i}^{\mathrm{T}} K_{ai}^{\mathrm{T}} - V_{2i}^{\mathrm{T}} K_{ei}^{\mathrm{T}}\right) R_i \left(V_{1i}^{\mathrm{T}} K_{ai}^{\mathrm{T}} - V_{2i}^{\mathrm{T}} K_{ei}^{\mathrm{T}}\right)^{\mathrm{T}} + V_{1i}^{\mathrm{T}} \sum\nolimits_{j=1}^{N} \tilde{\pi}_{ij} P_j V_{1i} \\
\Pi_{2i} &= V_{1i} A_{ci}^{\mathrm{T}} - V_{2i}^{\mathrm{T}} E_{ci}^{\mathrm{T}} + V_{3i} + V_{2i}^{\mathrm{T}} Q_{2i} V_{3i} + \left(V_{1i}^{\mathrm{T}} K_{ai}^{\mathrm{T}} - V_{2i}^{\mathrm{T}} K_{ei}^{\mathrm{T}}\right) R_i \left(-V_{3i}^{\mathrm{T}} K_{ei}^{\mathrm{T}}\right)^{\mathrm{T}} \\
\Pi_{3i} &= -E_{ci} V_{3i} - V_{3i}^{\mathrm{T}} E_{ci}^{\mathrm{T}} + V_{3i}^{\mathrm{T}} Q_{2i} V_{3i} + \left(-V_{3i}^{\mathrm{T}} K_{ei}^{\mathrm{T}}\right) R_i \left(-V_{3i}^{\mathrm{T}} K_{ei}^{\mathrm{T}}\right)^{\mathrm{T}}
\end{aligned}
$$

Under condition (4) and similar to [41], it is straightforward to see that for any appropriate matrix $W_i = W_i^{\mathrm{T}}$,

$$\sum_{j=1}^{N} (\Delta\pi_{ij} + \varepsilon_{ij}) W_i \equiv 0 \tag{26}$$

then

$$
\begin{aligned}
V_{1i}^{\mathrm{T}} \sum\nolimits_{j=1}^{N} \tilde{\pi}_{ij} P_j V_{1i} &= \alpha_{ii} V_{1i} + \varepsilon_{ii} V_{1i} W_i V_{1i} + \Delta\pi_{ii} V_{1i} W_i V_{1i} + V_{1i} \sum\nolimits_{j=1, j\neq i}^{N} \alpha_{ij} P_j V_{1i} \\
&\quad + \sum\nolimits_{j=1, j\neq i}^{N} (\Delta\pi_{ij} + \varepsilon_{ij}) V_{1i} (P_j - P_i + W_i) V_{1i} < 0
\end{aligned} \tag{27}
$$

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Note that for any $T_i > 0$,

$$\Delta\pi_{ii} W_i \leqslant 0.25(\Delta\pi_{ii})^2 T_i + W_i T_i^{-1} W_i \leqslant 0.25\varepsilon_{ii}^2 T_i + W_i T_i^{-1} W_i \tag{28}$$

By (22), it is easy to show that

$$S_{1i} = K_{ai} V_{1i} - K_{ei} V_{2i},\ S_{2i} = -K_{ei} V_{3i} \tag{29}$$

Taking into account (28), let $\bar{W}_i \triangleq V_{1i} W_i V_{1i}$ and $\bar{T}_i \triangleq V_{1i} T_i V_{1i}$. Via Lemma 1 and the Schur Complement, conditions (19) and (20) imply that (10) holds by substituting (3) into (25), because $\Delta\pi_{ij} + \varepsilon_{ij} \geqslant 0$ always holds, $\forall\, j \neq i \in \mathcal{S}$.

On the other hand, condition (11) in Theorem 1 is equivalent to

$$\begin{bmatrix} P_j & \left(I + E_{ci}^{-1}(B_i + \Delta B_i)G_i\right)^{\mathrm{T}} \\ * & P_i^{-1} \end{bmatrix} \geqslant 0 \tag{30}$$

where $i, j \in \mathcal{S}, i \neq j$. Pre-multiply and post-multiply (30) by matrix $\begin{bmatrix} V_{3i}^{\mathrm{T}} & 0 \\ 0 & E_{ci} \end{bmatrix}$ and its transpose, respectively,

$$\begin{bmatrix} -V_{3i}^{\mathrm{T}} P_j V_{3i} & -V_{3i}^{\mathrm{T}} E_{ci}^{\mathrm{T}} - V_{3i}^{\mathrm{T}} G_i^{\mathrm{T}} (B_i + \Delta B_i)^{\mathrm{T}} \\ * & -E_{ci} P_i^{-1} E_{ci}^{\mathrm{T}} \end{bmatrix} \leqslant 0 \tag{31}$$

Setting $Z_i = G_i V_{3i}$, via Lemma 1 and the Schur complement, it is concluded that condition (21) implies (11) holds based on the fact that

$$-V_{3i}^{\mathrm{T}} P_j V_{3i} \leqslant -V_{3i}^{\mathrm{T}} - V_{3i} + V_{1j}$$

and

$$-E_{ci} P_i^{-1} E_{ci}^{\mathrm{T}} \leqslant -E_{ci} V_{3i} - V_{3i}^{\mathrm{T}} E_{ci}^{\mathrm{T}} + V_{3i}^{\mathrm{T}} P_i V_{3i}$$

This completes the proof. □

Remark 6

In Theorem 2, conditions (10) and (11) in Theorem 1 are converted into feasible matrix conditions (19)–(21). It is seen that the PD part and the impulsive part of IPDSFC can be designed at the same time, while the impulsive gain matrices are given directly in [24, 30–32, 34–37]. This can provide more design freedom and reduce the conservatism to a certain level.

Remark 7

If the controller (5) does not include the impulsive part, that is, $G_i = 0$, $\forall i \in \mathcal{S}$, it follows that $P_i = P_j$ for $i, j \in \mathcal{S}, i \neq j$ from (30). It means that there exists a common Lyapunov function for the SMJS (1) and $\beta_k \equiv 1$. But this condition is hard to be satisfied. Therefore, it is necessary to design the impulsive part in the RNGCIC to ensure the non-increasing condition of the Lyapunov function at each switching point.

Remark 8

Theorem 2 shows that an upper bound for the associated cost function has been obtained, which is lower than the one calculated by the method proposed in [25]. This will be available in Example 2 in the succeeding texts.

Theorem 2 presents a method of designing an RNGCIC for system (1). But the guaranteed cost in Theorem 2 depends on the choice of guaranteed cost controllers. The following theorem presents a method of selecting a controller to minimize the upper bound of guaranteed cost (23).

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Theorem 3
Consider system (1) associated with cost function (9). If the following optimization problem

$$\min_{\delta_{1i},\delta_{2i},V_{1i},V_{2i},V_{3i},S_{1i},S_{2i},Z_i,\bar{W}_i,\bar{T}_i,Y,i\in\mathcal{S}} Trace(Y) \tag{32}$$
$$s.t.\ (a)\ (19),\ (20)\ and\ (21)$$
$$(b)\begin{bmatrix} Y & I \\ I & V_1(r_0)\end{bmatrix} > 0$$

has a solution set $\left(\tilde{\delta}_{1i} > 0, \tilde{\delta}_{2i} > 0, \tilde{V}_{1i} > 0, \tilde{V}_{2i}, \tilde{V}_{3i}, \tilde{S}_{1i}, \tilde{S}_{2i}, \tilde{Z}_i, \tilde{W}_i = \tilde{W}_i^{\mathrm{T}}, \tilde{T}_i > 0, Y > 0\right)$, $i \in \mathcal{S}$, then controller (5) is an optimal RNGCIC, which ensures the minimization of guaranteed cost (23) for system (1).

Proof
By applying the Schur Complement to condition (b) in (32), it is easy to show that $V_1^{-1}(r_0) < Y$. Thus, the minimization of $Trace(Y)$ implies the minimization of guaranteed cost in (23). The convexity of the optimization problem ensures that a global optimum is reachable when it exists. This completes the proof. □

In (22), if it is assumed that $S_{1i} = S_{2i} \triangleq S_i$ and $V_{2i} = V_{3i} \triangleq X_i$, then $K_{ai} = \left(S_{1i} - S_{2i}V_{3i}^{-1}V_{2i}\right)V_{1i}^{-1} = 0$. Thus, the following result can be obtained directly:

Corollary 1
Consider system (1) associated with cost function (9). There exists an RNGCIC for system (1) if there exist matrices $V_{1i} > 0$, X_i, S_i, Z_i, $\bar{W}_i = \bar{W}_i^{\mathrm{T}}$, $\bar{T}_i > 0$ and scalars $\delta_{1i} > 0$, $\delta_{2i} > 0$ such that the following conditions hold for all $i, j \in \mathcal{S}$, $i \neq j$

$$\begin{bmatrix} \tilde{\Theta}_{1i} & \tilde{\Theta}_{2i} & \tilde{\Theta}_{4i} & \tilde{\Theta}_{6i} & \bar{W}_i & V_{1i}^{\mathrm{T}} & X_i^{\mathrm{T}} & S_i^{\mathrm{T}} \\ * & \tilde{\Theta}_{3i} & 0 & \tilde{\Theta}_{7i} & 0 & 0 & X_i^{\mathrm{T}} & S_i^{\mathrm{T}} \\ * & * & \tilde{\Theta}_{5i} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta_{1i}I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\bar{T}_i & 0 & 0 & 0 \\ * & * & * & * & * & -Q_{1i}^{-1} & 0 & 0 \\ * & * & * & * & * & * & -Q_{2i}^{-1} & 0 \\ * & * & * & * & * & * & * & -R_i^{-1} \end{bmatrix} < 0 \tag{33}$$

$$\begin{bmatrix} -V_{1i} + \bar{W}_i & V_{1i} \\ * & -V_{1j} \end{bmatrix} \leqslant 0 \tag{34}$$

$$\begin{bmatrix} \tilde{\Sigma}_{1i} & \tilde{\Sigma}_{2i} & \tilde{\Sigma}_{4i} & 0 \\ * & \tilde{\Sigma}_{3i} & \tilde{\Sigma}_{5i} & X_i^{\mathrm{T}} \\ * & * & -\delta_{2i}I & 0 \\ * & * & * & -V_{1i} \end{bmatrix} \leqslant 0 \tag{35}$$

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where

$$\tilde{\Theta}_{1i} = X_i + X_i^{\mathrm{T}} + 0.25\varepsilon_{ii}^2 \bar{T}_i + \varepsilon_{ii}\bar{W}_i + \alpha_{ii}V_{1i},\ \tilde{\Theta}_{2i} = V_{1i}A_i^{\mathrm{T}} - X_i^{\mathrm{T}}E_i^{\mathrm{T}} + X_i + S_i^{\mathrm{T}}B_i^{\mathrm{T}}$$
$$\tilde{\Theta}_{3i} = -E_iX_i - X_i^{\mathrm{T}}E_i^{\mathrm{T}} + B_iS_i + S_i^{\mathrm{T}}B_i^{\mathrm{T}} + \delta_{1i}M_iM_i^{\mathrm{T}}$$
$$\tilde{\Theta}_{4i} = \left[\sqrt{\alpha_{i1}}V_{1i}, \ldots, \sqrt{\alpha_{i(i-1)}}V_{1i}, \sqrt{\alpha_{i(i+1)}}V_{1i}, \ldots, \sqrt{\alpha_{iN}}V_{1i}\right]$$
$$\tilde{\Theta}_{5i} = -diag\{V_{11}, \ldots, V_{1(i-1)}, V_{1(i+1)}, \ldots, V_{1N}\}$$
$$\tilde{\Theta}_{6i} = V_{1i}^{\mathrm{T}}N_{ai}^{\mathrm{T}} - X_i^{\mathrm{T}}N_{ei}^{\mathrm{T}} + S_i^{\mathrm{T}}N_{bi}^{\mathrm{T}},\ \tilde{\Theta}_{7i} = -X_i^{\mathrm{T}}N_{ei}^{\mathrm{T}} + S_i^{\mathrm{T}}N_{bi}^{\mathrm{T}}$$
$$\tilde{\Sigma}_{1i} = -X_i^{\mathrm{T}} - X_i + V_{1j},\ \tilde{\Sigma}_{2i} = -X_i^{\mathrm{T}}E_i^{\mathrm{T}} + S_i^{\mathrm{T}}B_i^{\mathrm{T}} - Z_i^{\mathrm{T}}B_i^{\mathrm{T}}$$
$$\tilde{\Sigma}_{3i} = -E_iX_i - X_i^{\mathrm{T}}E_i^{\mathrm{T}} + B_iS_i + S_i^{\mathrm{T}}B_i^{\mathrm{T}} + \delta_{2i}M_iM_i^{\mathrm{T}}$$
$$\tilde{\Sigma}_{4i} = -X_i^{\mathrm{T}}N_{ei}^{\mathrm{T}} + S_i^{\mathrm{T}}N_{bi}^{\mathrm{T}} - Z_i^{\mathrm{T}}N_{bi}^{\mathrm{T}},\ \tilde{\Sigma}_{5i} = -X_i^{\mathrm{T}}N_{ei}^{\mathrm{T}} + S_i^{\mathrm{T}}N_{bi}^{\mathrm{T}}$$

In this case, the gains of RNGCIC (5) are given by

$$K_{ai} = 0,\ K_{ei} = -S_iX_i^{-1},\ G_i = Z_iX_i^{-1}$$

and

$$J_0 = x_0^{\mathrm{T}}V_1^{-1}(r_0)x_0$$

In (22), if $S_{2i} = 0$, then $K_{ei} = 0$, or vice versa. Thus, in the case of $rank(E(r(t)) + \Delta E(r(t))) = n$, the following result can be obtained directly:

Corollary 2

Consider system (1) associated with cost function (9) and the constraint of $rank(E(r(t)) + \Delta E(r(t))) = n$ (the system dimension). There exists an RNGCIC for system (1) if there exist matrices $V_{1i} > 0$, V_{2i}, V_{3i}, S_{1i}, Z_i, $\bar{W}_i = \bar{W}_i^{\mathrm{T}}$, $\bar{T}_i > 0$ and scalars $\delta_{1i} > 0$, $\delta_{2i} > 0$ such that the following conditions hold for all $i, j \in \mathcal{S}$, $i \neq j$

$$\begin{bmatrix} \Theta_{1i} & \Theta_{2i} & \Theta_{4i} & \Theta_{6i} & \bar{W}_i & V_{1i}^{\mathrm{T}} & V_{2i}^{\mathrm{T}} & S_{1i}^{\mathrm{T}} \\ * & \hat{\Theta}_{3i} & 0 & \hat{\Theta}_{7i} & 0 & 0 & V_{3i}^{\mathrm{T}} & 0 \\ * & * & \Theta_{5i} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\delta_{1i}I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\bar{T}_i & 0 & 0 & 0 \\ * & * & * & * & * & -Q_{1i}^{-1} & 0 & 0 \\ * & * & * & * & * & * & -Q_{2i}^{-1} & 0 \\ * & * & * & * & * & * & * & -R_i^{-1} \end{bmatrix} < 0 \tag{36}$$

$$\begin{bmatrix} -V_{1i} + \bar{W}_i & V_{1i} \\ * & -V_{1j} \end{bmatrix} \leqslant 0 \tag{37}$$

$$\begin{bmatrix} \Sigma_{1i} & \hat{\Sigma}_{2i} & \hat{\Sigma}_{4i} & 0 \\ * & \hat{\Sigma}_{3i} & \hat{\Sigma}_{5i} & V_{3i}^{\mathrm{T}} \\ * & * & -\delta_{2i}I & 0 \\ * & * & * & -V_{1i} \end{bmatrix} \leqslant 0 \tag{38}$$

where

$$\hat{\Theta}_{3i} = -E_iV_{3i} - V_{3i}^{\mathrm{T}}E_i^{\mathrm{T}} + \delta_{1i}M_iM_i^{\mathrm{T}},\ \hat{\Theta}_{7i} = -V_{3i}^{\mathrm{T}}N_{ei}^{\mathrm{T}}$$
$$\hat{\Sigma}_{2i} = -V_{3i}^{\mathrm{T}}E_i^{\mathrm{T}} - Z_i^{\mathrm{T}}B_i^{\mathrm{T}},\ \hat{\Sigma}_{3i} = -E_iV_{3i} - V_{3i}^{\mathrm{T}}E_i^{\mathrm{T}} + \delta_{2i}M_iM_i^{\mathrm{T}}$$
$$\hat{\Sigma}_{4i} = -V_{3i}^{\mathrm{T}}N_{ei}^{\mathrm{T}} - Z_i^{\mathrm{T}}N_{bi}^{\mathrm{T}},\ \hat{\Sigma}_{5i} = -V_{3i}^{\mathrm{T}}N_{ei}^{\mathrm{T}}$$

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and the other terms are the same as the ones in Theorem 2. In this case, the gains of RNGCIC (5) are given by

$$K_{ai} = S_{1i} V_{1i^{-1}},\ K_{ei} = 0,\ G_i = Z_i V_{3i}^{-1}$$

and

$$J_0 = x_0^{\mathrm{T}} V_1^{-1}(r_0) x_0$$

Remark 9

In the case of $E(r(t)) = I$ and $\Delta E(r(t)) = 0$ (i.e., $N_e(r(t)) = 0$), constraint $rank(E(r(t)) + \Delta E(r(t))) = n$ is satisfied, and Corollary 2 is the direct conclusion for a standard state space system.

When the TRM of system (1) are totally known, that is, $\Delta\pi_{ij} = 0, \forall i, j \in \mathcal{S}$, the following result is available.

Corollary 3

Consider system (1) associated with cost function (9). There exists an RNGCIC for system (1) if there exist matrices $V_{1i} > 0$, V_{2i}, V_{3i}, S_{1i}, S_{2i}, Z_i and scalars $\delta_{1i} > 0$, $\delta_{2i} > 0$ such that the following conditions hold for all $i, j \in \mathcal{S}, i \neq j$

$$\begin{bmatrix} \bar{\Theta}_{1i} & \Theta_{2i} & \bar{\Theta}_{4i} & \Theta_{6i} & V_{1i}^{\mathrm{T}} & V_{2i}^{\mathrm{T}} & S_{1i}^{\mathrm{T}} \\ * & \Theta_{3i} & 0 & \Theta_{7i} & 0 & V_{3i}^{\mathrm{T}} & S_{2i}^{\mathrm{T}} \\ * & * & \Theta_{5i} & 0 & 0 & 0 & 0 \\ * & * & * & -\delta_{1i} I & 0 & 0 & 0 \\ * & * & * & * & -Q_{1i}^{-1} & 0 & 0 \\ * & * & * & * & * & -Q_{2i}^{-1} & 0 \\ * & * & * & * & * & * & -R_i^{-1} \end{bmatrix} < 0 \tag{39}$$

$$\begin{bmatrix} \Sigma_{1i} & \Sigma_{2i} & \Sigma_{4i} & 0 \\ * & \Sigma_{3i} & \Sigma_{5i} & V_{3i}^{\mathrm{T}} \\ * & * & -\delta_{2i} I & 0 \\ * & * & * & -V_{1i} \end{bmatrix} \leqslant 0 \tag{40}$$

where

$$\bar{\Theta}_{1i} = V_{2i} + V_{2i}^{\mathrm{T}} + \pi_{ii} V_{1i},\ \bar{\Theta}_{4i} = \left[\sqrt{\pi_{i1}} V_{1i}, \ldots, \sqrt{\pi_{i(i-1)}} V_{1i}, \sqrt{\pi_{i(i+1)}} V_{1i}, \ldots, \sqrt{\pi_{iN}} V_{1i}\right]$$

and the other terms are the same as the ones in Theorem 2. In this case, the gains of RNGCIC (5) are given by

$$K_{ai} = \left(S_{1i} - S_{2i} V_{3i}^{-1} V_{2i}\right) V_{1i}^{-1},\ K_{ei} = -S_{2i} V_{3i}^{-1},\ G_i = Z_i V_{3i}^{-1}$$

and

$$J_0 = x_0^{\mathrm{T}} V_1^{-1}(r_0) x_0$$

4. ILLUSTRATIVE EXAMPLES

In this section, two numerical examples and a stochastically switching RC pulse divider circuit example are provided to demonstrate the effectiveness and applicability of the proposed approaches.

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Example 1
Consider an SMJS described in (1) with parameters as follows.

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} 0.2 & -0.3 & 1 \\ 0.7 & -1 & -0.5 \\ 0.1 & 0 & 0.4 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.1 & -1 & 0 \\ -0.2 & -1 & 0.4 \\ 0 & 0.3 & 0.1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 & -0.3 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$$

The norm-bounded uncertainties satisfying (3) are described as

$$\begin{aligned} M_1 &= \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}^{\mathrm{T}},\ N_{e1} = \begin{bmatrix} 0.7 & 0.7 & 0.2 \end{bmatrix} \\ N_{a1} &= \begin{bmatrix} 0.2 & 0.4 & 0.3 \end{bmatrix},\ N_{b1} = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix} \\ M_2 &= \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}^{\mathrm{T}},\ N_{e2} = \begin{bmatrix} 0.3 & 0.2 & 0.2 \end{bmatrix} \\ N_{a2} &= \begin{bmatrix} 0.3 & 0.5 & 0.2 \end{bmatrix},\ N_{b2} = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix} \end{aligned}$$

and the uncertain matrix is given as $F(t) = \sin t$. It is easy to see that $rank(E_i + \Delta E_i) \not\equiv 3$ (the system dimension), $i = 1, 2$, which implies that the original system is not normal. For the cost function (9), let $Q_{11} = Q_{12} = 0.5I$, $Q_{21} = Q_{22} = 0.2I$ and $R_1 = R_2 = 0.2I$ of appropriate dimensions. Suppose that the SMJS starts from the initial point $x_0 = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$. The transition rates of Π are given as $\pi_{11} = -5$ and $\pi_{22} = -7$, whose uncertainties satisfy $|\Delta\pi_{12}| \leqslant \varepsilon_{12} \triangleq 0.5\pi_{12}$ and $|\Delta\pi_{21}| \leqslant \varepsilon_{21} \triangleq 0.5\pi_{21}$, respectively. Solving the optimization problem (32), an RNGCIC is obtained following Theorem 3. The gain matrices of optimal RNGCIC can be obtained as

$$\begin{aligned} K_{a1} &= \begin{bmatrix} 3.9760 & -2.0638 & 15.4773 \\ -6.9128 & -0.5296 & -12.3787 \end{bmatrix},\ K_{e1} = \begin{bmatrix} -0.1442 & 0.0088 & -5.4454 \\ 2.6880 & 1.4362 & 4.7213 \end{bmatrix} \\ G_1 &= \begin{bmatrix} -0.4186 & -0.5085 & 5.4275 \\ -3.4482 & -1.5869 & -4.6958 \end{bmatrix} \\ K_{a2} &= \begin{bmatrix} 1.7365 & 1.2229 & -1.3160 \\ -5.8694 & 1.9141 & -4.0193 \end{bmatrix},\ K_{e2} = \begin{bmatrix} -1.3680 & -0.8783 & 0.6468 \\ 2.8445 & -1.3008 & 0.3408 \end{bmatrix} \\ G_2 &= \begin{bmatrix} 0.5753 & 0.8012 & -0.9255 \\ -3.2307 & 1.5424 & 0.2861 \end{bmatrix} \end{aligned}$$

and the optimal cost value $J_0 = 0.7976$ (when $r_0 = 1$) and 0.8489 (when $r_0 = 2$), respectively.

If there are no uncertainties in TRM $\tilde{\Pi}$, that is, $\Delta\pi_{12} = \Delta\pi_{21} = 0$, an RNGCIC can be obtained by Corollary 3. The gain matrices of optimal RNGCIC can be computed as

$$\begin{aligned} K_{a1} &= \begin{bmatrix} 3.9739 & -2.0198 & 15.1048 \\ -6.7569 & -0.5671 & -12.3941 \end{bmatrix},\ K_{e1} = \begin{bmatrix} -0.1691 & -0.1825 & -6.1487 \\ 2.6476 & 1.5775 & 5.4407 \end{bmatrix} \\ G_1 &= \begin{bmatrix} -0.4109 & -0.2972 & 6.1316 \\ -3.3860 & -1.7503 & -5.4153 \end{bmatrix} \\ K_{a2} &= \begin{bmatrix} 1.8929 & 1.0166 & -0.9383 \\ -5.0060 & 1.7076 & -3.0948 \end{bmatrix},\ K_{e2} = \begin{bmatrix} -1.5176 & -0.7752 & 0.5557 \\ 2.7773 & -1.3325 & 0.2656 \end{bmatrix} \\ G_2 &= \begin{bmatrix} 0.8271 & 0.7939 & -0.8368 \\ -3.1528 & 1.5172 & 0.2993 \end{bmatrix} \end{aligned}$$

and the optimal cost value $J_0 = 0.7348$ (when $r_0 = 1$) and 0.7722 (when $r_0 = 2$), respectively. For any $t \in [0, \infty)$ and with the designed controller aforementioned, the rank of derivative matrix of the corresponding closed-loop system is $rank(E_{ci}) = 3$, $i = 1, 2$, which implies that the closed-loop system is normalized.

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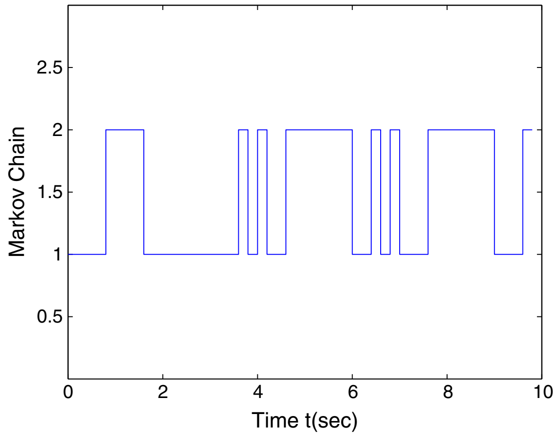

Figure 1. The Markov process.

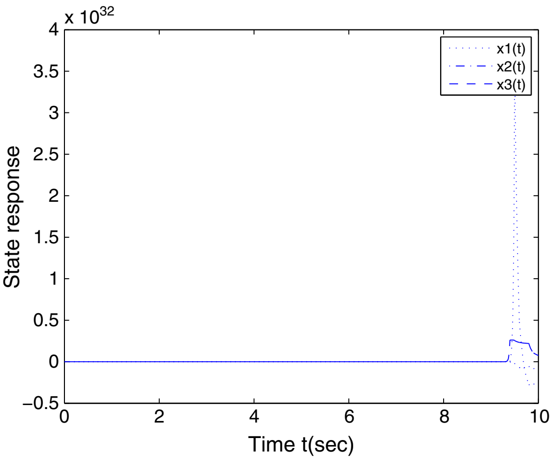

Figure 2. The state trajectories of the open-loop system.

The Markov process is shown in Figure 1, while the state responses of the open-loop and corresponding closed-loop system with initial condition $x_0 = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$ are illustrated by Figures 2 and 3, respectively. Simulation results show that the closed-loop system is robustly stochastically stabilized by IPDSFC (5).

Example 2

Consider a special case of SMJS (1) where there are no uncertainties in TRM and $N_{ei} = 0, i = 1, 2,$ whose parameters are described as follows.

$$mode1: \quad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 0.2 & -0.8 & 1 \\ 0.7 & -1 & -0.5 \\ 0.1 & 0 & 0.4 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & 1 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}^{\mathrm{T}}, N_{a1} = \begin{bmatrix} 0.2 & 0.4 & 0.3 \end{bmatrix}, N_{b1} = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}$$

$$mode2: \quad E_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0.1 & -1 & 0 \\ -0.2 & -1 & 0.4 \\ 0 & 0.3 & 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & -3 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}^{\mathrm{T}}, N_{a2} = \begin{bmatrix} 0.3 & 0.5 & 0.2 \end{bmatrix}, N_{b2} = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix}$$

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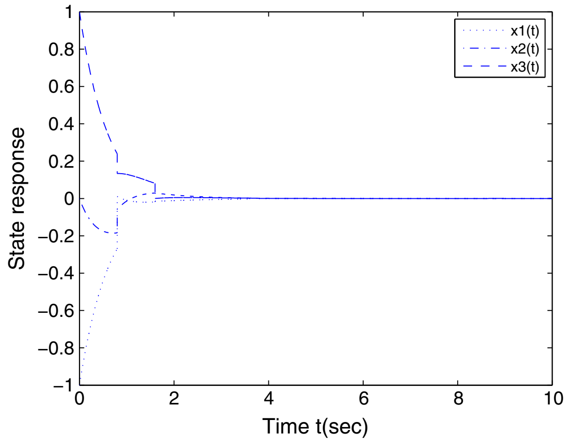

Figure 3. The state trajectories of the closed-loop system.

Table I. Optimal guaranteed cost value calculated by different approaches.

r_0	J_0 in [25]	J_0 in this paper
$r_0 = 1$	0.9285	0.3314
$r_0 = 2$	0.7272	0.1750

and the uncertain matrix is given as $F(t) = \sin t$. For the cost function (9), let $Q_{11} = Q_{12} = I$, $Q_{21} = Q_{22} = 0$, and $R_1 = R_2 = I$ of appropriate dimensions. The SMJS is supposed to start from initial point $x_0 = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$. In order to make a comparison, the TRM $\tilde{\Pi}$ is assumed to be obtained exactly, which is given as

$$\tilde{\Pi} = \begin{bmatrix} -3 & 3 \\ 6 & -6 \end{bmatrix}$$

The objective is to design a state feedback controller such that the corresponding closed-loop system is robustly stochastically stable and the cost function is bounded for all admissible uncertainties. We compute the optimal guaranteed cost by using Theorem 3.2 in [25] and Corollary 3 in this paper, respectively. Table I provides the minimal cost value J_0 calculated by the two approaches. It is seen that the cost value obtained by Corollary 3 is lower than that in [25].

The gains matrices of optimal RNGCIC can be computed as

$$K_{a1} = \begin{bmatrix} 32.4739 & 0.3541 & 157.9646 \\ -18.7255 & -1.7980 & -91.3394 \end{bmatrix}, \quad K_{e1} = \begin{bmatrix} -0.2660 & -0.0243 & -0.0176 \\ 0.2070 & 0.5514 & 0.0102 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} -0.3795 & -0.4269 & 0.0176 \\ -0.7598 & -1.2592 & -0.0102 \end{bmatrix}$$

$$K_{a2} = \begin{bmatrix} -8.1218 & 0.7888 & -0.5215 \\ 268.5545 & 6.9511 & -0.0398 \end{bmatrix}, \quad K_{e2} = \begin{bmatrix} -0.0007 & -0.5182 & 0.1177 \\ -0.0222 & -0.0699 & 0.1047 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0.0007 & 1.3829 & -0.2279 \\ -0.0222 & 0.0326 & 0.0231 \end{bmatrix}$$

Example 3

Consider a stochastically switching RC pulse divider circuit that gives an SMJSs and is illustrated in Figure 4.

It is seen that the switch occupies two positions, which switches from one position to the other in a random way. For this system, it is assumed that the position of switch follows a continuous-time

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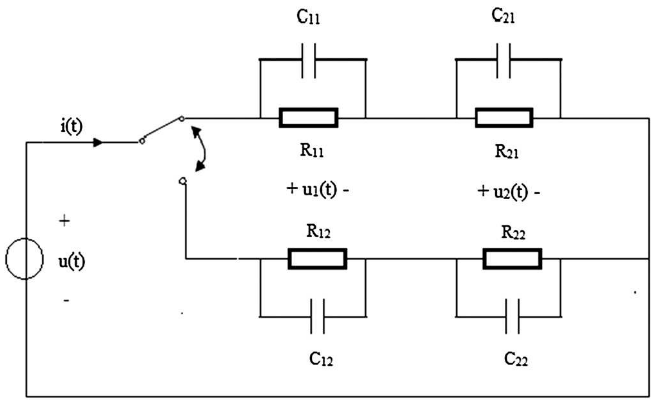

Figure 4. Stochastic switching RC pulse divider circuit: singular Markovian jump system.

Markov process $\{r_t, t \geqslant 0\}$ as in (2). Then for this electric circuit, $\{r_t, t \geqslant 0\}$ will take two modes in $\mathcal{S} = \{1, 2\}$. For each $i \in \mathcal{S}$, R_{1i}, R_{2i} stand for resistor and C_{1i}, C_{2i} stand for capacity, respectively. The electric current in the circuit is denoted as $i(t)$, and the voltage of R_{1i}, R_{2i} is denoted as $u_1(t)$, $u_2(t)$, respectively, $u(t)$ is the voltage source, which is taken as the control input. According to the basic circuit theory, the circuit system is described by the following SMJSs

$$\begin{bmatrix} C_1(r_t) & 0 & 0 \\ 0 & C_2(r_t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{du_1(t)}{dt} \\ \frac{du_2(t)}{dt} \\ \frac{di(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1(r_t)} & 0 & 1 \\ 0 & -\frac{1}{R_2(r_t)} & 1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

Let $R_{11} = 10$, $R_{12} = 5$, $R_{21} = 5$, $R_{22} = 8$, $C_{11} = 2$, $C_{12} = 4$, $C_{21} = 3$, $C_{22} = 5$, $Q_{11} = Q_{12} = 0.5I$, $Q_{21} = Q_{22} = 0.2I$, and $R_1 = R_2 = 0.2I$ of appropriate dimensions. It is straightforward to see that the original system is not normal and not impulse free. The transition rates of $\tilde{\Pi}$ are assumed to be obtained exactly, which is given as

$$\tilde{\Pi} = \begin{bmatrix} -4 & 4 \\ 5 & -5 \end{bmatrix}$$

then we obtain an RNGCIC by Corollary 3. The gain matrices of optimal RNGCIC can be obtained as

$$\begin{aligned} K_{a1} &= \begin{bmatrix} -0.7272 & -0.2358 & 3.6029 \end{bmatrix}, \; K_{e1} = \begin{bmatrix} -0.5408 & -0.6265 & 3.0700 \end{bmatrix} \\ G_1 &= \begin{bmatrix} 0.3780 & 0.4864 & -3.0700 \end{bmatrix} \\ K_{a2} &= \begin{bmatrix} -0.4324 & -0.1563 & -3.5977 \end{bmatrix}, \; K_{e2} = \begin{bmatrix} -0.4554 & -0.5409 & 3.2972 \end{bmatrix} \\ G_2 &= \begin{bmatrix} 0.3338 & 0.4240 & -3.2971 \end{bmatrix} \end{aligned}$$

and the optimal cost value $J_0 = 6.1448$ (when $r_0 = 1$) and 6.0582 (when $r_0 = 2$), respectively. For any $t \in [0, \infty)$ and with the aforementioned designed controller, the rank of derivative matrix of the corresponding closed-loop system is $rank(E_{ci}) = 3$, $i = 1, 2$, which implies that the closed-loop system is normalized via IPDSFC (5). The Markov process and the state response of the closed-loop system with initial condition $\begin{bmatrix} 1 & 0.5 & 1 \end{bmatrix}^{\mathrm{T}}$ are illustrated by Figures 5 and 6, respectively. We can see that the closed-loop system is robustly stochastically stable.

Remark 10

In Example 3, the impulsive part of the RNGCIC can be considered as a pulsed power supply. The circuit current is changed instantaneously when switching occurs, which can be seen in Figure 6.

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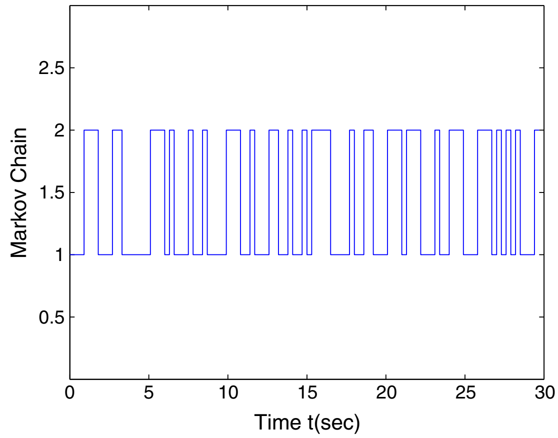

Figure 5. The Markov process.

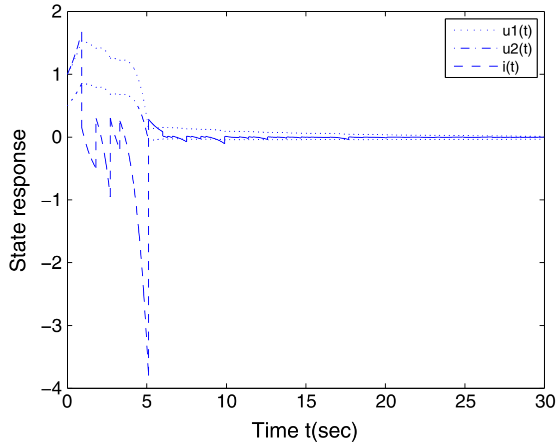

Figure 6. The state trajectories of the closed-loop system.

Because of the energy storage properties of capacities, the voltages of resistors are not mutated at switching points.

Remark 11
RC (resistor-capacitor) circuits are widely used in analog circuits, pulse and digital circuits, and so on. When the pulse signal needs to be transmitted through a resistor divider to the next one, we can connect an accelerated capacity with the resistor, which makes up an RC pulse divider. The RC pulse divider can avoid the distortion of output waveform when the pulse signal is input to the circuit.

5. CONCLUSION

This paper has investigated the problem of robust normalization and guaranteed cost control for SMJSs with parameter uncertainties in both system matrices and TRM. A new hybrid impulsive controller has been proposed to ensure the normalization, robust stochastic stability of the closed-loop system and to minimize the upper bound of the closed-loop cost function simultaneously. Based on certain conditions, an explicit desired impulsive and PD state feedback controller has also been given. A convex optimization problem has been formulated to design the optimal robust normalization and guaranteed cost controller. Illustrative examples have been provided to illustrate the effectiveness of our methods.

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