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## Rigidity for the stable homotopy category

CONSTANZE ROITZHEIM

If two model categories  $\mathcal{C}$  and  $\mathcal{D}$  are Quillen equivalent, then their homotopy categories  $Ho(\mathcal{C})$  and  $Ho(\mathcal{D})$  are equivalent. But if  $Ho(\mathcal{C})$  and  $Ho(\mathcal{D})$  are equivalent categories, can anything be said about the underlying model structures? For the stable homotopy category  $Ho(\mathcal{S})$  (i.e. the homotopy category of spectra) there is the following result:

### Rigidity Theorem [1]

Let  $\mathcal{C}$  be a stable model category and

$$\Phi : Ho(\mathcal{S}) \longrightarrow Ho(\mathcal{C})$$

be an equivalence of triangulated categories. Then  $\mathcal{S}$  and  $\mathcal{C}$  are Quillen equivalent.

This means that all higher homotopy information in  $\mathcal{S}$  such as Toda brackets,  $K$ -theory or mapping spaces is already encoded in the triangulated structure of the stable homotopy category.

In this talk we will construct the most important category-theoretic tool used in the proof of the Rigidity Theorem:

### Universal Property of Spectra [2]

Let  $\mathcal{C}$  be a stable model category,  $X \in \mathcal{C}$  a fibrant and cofibrant object. Then there is a Quillen adjoint functor pair

$$X \wedge - : \mathcal{S} \rightleftarrows \mathcal{C} : \text{Hom}(X, -)$$

with  $X \wedge S^0 \simeq X$ .

In the set-up of the Rigidity Theorem this Quillen pair will provide the desired Quillen equivalence. More precisely, for  $X = \Phi(S^0)$ , the composition

$$Ho(\mathcal{S}) \xrightarrow{L(X \wedge -)} Ho(\mathcal{C}) \xrightarrow{\Phi^{-1}} Ho(\mathcal{S})$$

of the left derived Quillen functor and the given equivalence  $\Phi$  is an endofunctor of the stable homotopy category sending the sphere to itself. Hence, as the following talks will show, it must be a self-equivalence of  $Ho(\mathcal{S})$ . Consequently, the derived functor of the Quillen functor  $X \wedge -$  is an equivalence of categories which means that  $\mathcal{S}$  and  $\mathcal{C}$  are Quillen equivalent.

## REFERENCES

- [1]
- [2]