Declining discount rates and the 'Fisher Effect': Inflated past, discounted future?

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Abstract

Uncertain and persistent real interest rates underpin one argument for using a declining term structure of social discount rates in the Expected Net Present Value (ENPV) framework. Despite being controversial, this approach has influenced both the Inter-Agency Working Group on Cost-Benefit Analysis and the UK government's guidelines on discounting. We first clarify the theoretical basis of the ENPV approach. Then, rather than following previous work which used a single series of real U.S. Treasury bond returns, we treat nominal interest rates and inflation as cointegrated series and estimate the empirical term structure of discount rates via the 'Fisher Effect'. This nests previous empirical models and is more flexible. It also addresses an irregularity in previous work which used data on nominal interest rates until 1950, and real interest rates thereafter. As we show, the real and nominal data have very different time series properties. This paper therefore provides a robustness check on previous discounting advice and updated methodological guidance at a time when governments around the world are reviewing their guidelines on social discounting. The policy implications are discussed in the context of the Social Cost of Carbon, nuclear decommissioning and public health.

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1 Introduction

Despite some puzzles along the way, the burgeoning theoretical literature on discounting distant time horizons points more or less unanimously towards the use of a declining term structure of social discount rates (DDRs) for risk free public projects. [18, 19, 55, 59, 61].[1] This conclusion is more or less robust to one's stance on the normative-positivist debate provided that the primitives of the discounting problem, growth or the interest rate, exhibit persistence over time [3, 4, 14, 18].[2] Consensus in an area of theory as potentially fraught as social discounting is a rare thing. Perhaps for this reason the literature on DDRs has been highly influential in policy circles, with many governments having either adopted DDRs or in the process of considering

[1] This is for risk-free discounting of certainty-equivalent future cash flows. See [20] for a discussion about project and systematic risk in discounting.

[2] Strictly speaking, in the case of growth additional conditions are required on the nature of the inter-temporal social welfare function (e.g. [18]).

them.[3] Yet, it has become clear that there is no clear consensus on how to operationalise a schedule of DDRs for use in CBA. One need only look at the heterogeneous and occasionally ad hoc motivations for current policies as evidence for this (e.g. [29]). This lack of consensus turns out to be important since evaluation of intergenerational projects is very sensitive to the discount rates deployed. Indeed, the range of policy prescriptions arising from different approaches to estimating the DDR schedule is comparable to the differences that emerged in the thorny normative-positive debate that characterised the aftermath of the Stern Review ([43, 25, 14]).

In this paper we explore the empirical sensitivities associated with the declining certainty equivalent discount rate proposed by Martin Weitzman [59]. The empirical context is where the declining term structure is characterised using historical interest rate data, rather than expert opinion [14]. We do this with a view to advancing empirical practices in this area and thereby informing the numerous discounting policy consultations currently underway. There are three main contributions in this direction, one theoretical and two empirical. Firstly, we clarify the particular theoretical motivation for the so-called Expected Net Present Value (ENPV) approach proposed by [59]. Second, we disentangle the real interest rate into its component parts: the nominal interest rate and inflation, using models which estimate the 'Fisher Effect'. This approach allows the real interest rate to be modelled as consisting of two state variables: the nominal interest rate and inflation. Third, this empirical approach has concrete advantages, chief among which is that it allows us to address several important empirical irregularities in the real interest rate series used in previous analyses. This leads to a robust update of the empirical work that has influenced the US guidelines on discounting.

[3]The UK, French and Norwegian governments now recommend DDRs for intergenerational Cost-Benefit Analysis (CBA) [29, 34, 37]. The literature on DDRs also motivates the U.S. Environmental Protection Agency's (USEPA) recommendation that a lower discount rate should be applied to intergenerational projects for sensitivity analysis [56]. The U.S. Interagency Working Group on the Social Cost of Carbon recommends similar practices [31]. Furthermore, DDRs are actively being considered by the USEPA and the Office for the Management of Budgets (OMB) after a recent consultation of experts [10]. The Danish Government have also adopted DDRs following the Norwegian consultation. More recently the Dutch, Cypriot and Swedish governments are in various stages of review of their positions on long-term discounting. For the outcome of the joint USEPA, OMB, Resources For the Future (RFF) expert panel meeting see [4]. Finally, discussions are currently taking place on social discount rates within the U.K. H.M. Treasury and the OECD International Transport Forum.

In previous work using the ENPV approach, Newell and Pizer [40] (henceforth N&P) showed that U.S. bond yields have exhibited sufficient persistence in the past two centuries for the empirical term structure of discount rates to exhibit a rapid decline. This decline raises the social cost of carbon from \$5.7/tC to between \$6.5/tC and \$10.4/tC in the process (US\$ 2000; see also [39]). These results were shown to be highly sensitive to the time-series model used to characterise interest rate uncertainty. Subsequent work by Groom, Koundouri, Panopoulou and Pantelidis [25] (henceforth GKPP) showed that, once a wider range of time-series models is considered, and a process of model selection undertaken, there are good theoretical and empirical reasons to prefer models which allow for a more flexible characterisation of uncertainty in the interest rate data generating process. Their preferred schedule of DDRs raised the social cost of carbon yet further to \$14.4/tC. Similar results have been found in many different countries and also when a global perspective is taken [28, 22].

The ENPV approach has strongly influenced the current U.K., U.S. and Norwegian governments' guidance on long-term discounting ([29, p98.] [31, p 24.] [37, Ch 5, p79.] [56, Ch 6, p23.]).[4] It also has prominence in the current consultation taking place in the U.S. (e.g. [4, 10]), has served as an input into the ongoing 'refresh' of the UK Treasury Green Book, and will almost certainly influence the current reviews of social discounting taking place in other countries such as the Netherlands and in organisations such as the OECD. The ENPV approach is not uncontroversial, however (see [21] for example), so there exists a need to clarify its theoretical basis. To this end, we first clarify the theoretical foundations of the ENPV approach and place it within the broader theoretical asset pricing literature, in contrast to its rather ad hoc theoretical origins (e.g. [59]).

There is also a need to re-evaluate the data and the empirical methods. Our empirical contributions are initially motivated by inspection of the time-series of bond yields used by N&P and the modelling of real interest rates. N&P's dataset has been particularly important as an empirical testing ground for empirical methods for discounting in several other contributions

[4]For instance, the Norwegian Guidelines conclude: "Beyond 40 years, it is reasonable to assume that one will be unable to secure a long-term rate in the market, and the discount rate should accordingly be determined on the basis of a declining certainty-equivalent rate as the interest rate risk is supposed to increase with the time horizon. A rate of 3 percent is recommended for the years from 40 to 75 years into the future. A discount rate of 2 percent is recommended for subsequent years."

[25, 22, 28]. N&P use annual market yields for long-term government bonds for the period 1798 to 1999. Starting in 1950, nominal interest rates are converted to real ones by subtracting a ten-year moving average of the expected inflation rate of the CPI as measured by the Livingston Survey of professional economists. This ex-ante measure of inflation does not exist prior to 1950, and so expected inflation is assumed to equal zero for the first three-quarters of the series. Elsewhere though it has been well documented that prior to 1950 the United States went through periods of both highly positive and highly negative inflation (see, for example, [15] and [6]). It is very likely that the real and nominal interest rate data will have distinct time series properties during the first three-quarters of N&P's sample period compared to the last quarter. Indeed, we show that the persistence which underpins the decline in the term structure of social discount rates is only present in the nominal interest rate series from 1798 to 1950. The remaining real interest rate series up to 1999 is arguably mean reverting. Furthermore, the volatilities of real and nominal interest rates are typically very different. These observations suggest that the shape of the term structures reported in earlier studies may not be robust to different assumptions about the inflation process. Loosely speaking, if real interest rates follow a mean-reverting process throughout the sample period the resulting schedule of social discount rates could be effectively flat.

Investigating the sensitivity of current policy recommendations to these assumptions requires a completely different approach which removes the disconnect between nominal and real interest rates that occurs in 1950 in the N&P data. An obvious candidate is to determine the real interest rate series by empirically modelling the theoretical relationship between nominal interest rates and inflation known as the 'Fisher Effect' [12]. Not only does this approach address the disconnect in the U.S. data, but it also has several other advantages over previous work which are of general applicability and directly relevant to the ongoing policy consultations on discounting in the US and Europe.

First, as proposed by GKPP, we use several estimation procedures to estimate the Fisher Effect and simulate the certainty equivalent term structure in an effort to find the best model. These estimators necessarily have to accommodate endogeneity and cointegration of nominal interest rate and inflation, something that remains implicit in previous work. This reflects the fact that policy decisions affect the future path of the economy and these decisions are reflected in both nominal interest rates and inflation rates. This is a typical feature of discounting theory and captures the idea that discount rates are dependent on current economic conditions and policy uncertainty. Our approach to modelling and estimation is the first to reflect these dynamics and uncertainties in the determination of the term structure of discount rates (e.g. [20, Ch 4]).

Second, the explicit modelling of the Fisher Effect means that the repeated periods of negative real interest rates witnessed over the past 200 years in the U.S. need not be smoothed out in the manner of previous work. Neither is a logarithmic model required to remove the possibility of a negative certainty equivalent discount rate.[5] So, explicit modelling of the Fisher Effect leads to more flexible modelling of the real-interest rate and certainty equivalent discount rate. Indeed, we show that the statistical model of N&P is a special case of a more general model which includes the Fisher Effect.

What is surprising, but perhaps heartening, about our results is that the term structure of the discount rate in the U.S. case is largely robust to our more rigorous treatment of inflation. The term structures we describe generally decline more sharply at long horizons than either N&P or GKPP. However, at the short end, social discount rates are higher than those described by GKPP. As a consequence, the estimated present value of long-term cash flows, such as those associated with the damages from carbon emissions, in our preferred case lies between the estimates of N&P and GKPP, but closer to the latter than the former.

2 A Theory of Declining Discount Rates

When using market bond yields to inform the discount rate, policy makers are taking a positivist approach to social discounting. A project with a consumption certainty equivalent future cash flow V_t at future time t and zero at all other times is then, from a valuation perspective, economically equivalent to a zero-coupon default risk-free bond with maturity t. The appropriate positivist valuation approaches can therefore be taken directly from the asset pricing literature.[6]

A well-known result from financial economics (see, for example, [2, Equation 16]) is that the

[5] N&P argue that short-term fluctuations are not strictly relevant to the time horizons that are the focus of their paper. Furthermore, negative real rates do not appear in their data, the argument being that these are transitory phenomena [39, p 10].

[6] See also the technical Appendix B of [14] for a related discussion.

present value at time h of the project cash flow at time t, $P(h,t)$, is given by:

$$P(h,t) = E_h\left(V_t \exp\left(-\sum_{\tau=h}^{t-1} r_\tau\right)\right) \tag{1}$$

where r_τ is defined as the logarithmic expected single-period return for holding a claim on V_t over the interval $[\tau, \tau+1]$: $\exp(r_\tau) = E_\tau [P(\tau+1,t)/P(\tau,t)]$. The derivation of equation (1) emerges simply from repeated iteration of the single-period Net Present Value equation.

Using equation (1), define the variable $r(t)$ by: $P(0,t) = E_0[V_t]\exp(-tr(t))$. Now assume that V_t is non-stochastic, or at least uncorrelated with $\sum_{\tau=h}^{t-1} r_\tau$ so that $P(0,t)$ can be written as $P(0,t) = V_t E_0\left[\exp\left(-\sum_{\tau=0}^{t-1} r_\tau\right)\right]$. This assumption is essentially what Martin Weitzman refers to as a 'pragmatic-decomposition' [60]. It then follows that:

$$r(t) = -\frac{1}{t}\ln\left(E_0\left(\exp\left(-t\overline{r}_t\right)\right)\right) \tag{2}$$

where $\overline{r}_t = t^{-1}\sum_{\tau=0}^{t-1} r_\tau$ is the average value of r_τ over the horizon of the project. Following Weitzman [59] we call $r(t)$ the *certainty equivalent discount rate*, and the corresponding *certainty-equivalent forward rate,* $\widetilde{r}_t$, for discounting between adjacent periods at time t is:

$$\widetilde{r}_t = \frac{P(0,t)}{P(0,t+1)} - 1 \tag{3}$$

This is commonly referred to as the Expected Net Present Value (ENPV) approach. Crucially, exponential functions are convex and so, by Jensen's inequality, $r(t) < E_0(\overline{r}_t)$. The magnitude of this inequality is driven by two parameters; the value of t and the uncertainty over $\overline{r}_t$. The inequality becomes more pronounced as t increases, and, ceteris paribus, this causes the term structure of certainty equivalent discount rates to decline with the horizon of the project. The inequality also becomes more pronounced as the uncertainty over $\overline{r}_t$ increases. Hence, understanding the volatility of average future costs of capital is the critical empirical task facing those who wish to operationalise the ENPV approach.

When parameterising equation (2), N&P and others estimate the statistical properties of $\overline{r}_t$ from a historic time-series of yields on long-term Treasury bonds. However, it is not immediately obvious that the yield on long bonds, with horizons of a few decades, and the single period return to a many-century $t-$period default risk-free fixed income security should be the same. In general, empirical estimates of the Treasury yield curve are upward sloping, suggesting that $\overline{r}_t$ is likely to be higher than an average long-term bond yield. However, the literature on social discount rates generally ignores these yield curve issues by assuming that the liquidity premium on bonds of all horizons is zero. We retain this assumption here, the motivation for which is two-fold.

Within the social discounting literature, it has been common to justify the ENPV approach through the original thought experiment of [59]. He assumes that future interest rates are currently unknown but that, in one instant, all uncertainty will be removed. The true value of r_0 will be revealed and $r_\tau = r_0$ with certainty for all future τ. In this case, the ENPV approach with r_τ proxied by a short-term risk-free rate has been justified through the literature on the so-called 'Weitzman-Gollier puzzle'. This starts with [17] and thus far culminates with [23] and [55] via [27], [8] and [13]. In fixed income pricing, the use of the ENPV equation in the absence of liquidity premia is proposed by [9] in continuous time and [16] in discrete time. Here equation (2) is referred to as the Local Expectations Hypothesis. In this case, rather than all uncertainty being removed in one instant, a less restrictive 'local certainty' equivalent is required. By having consumption at time $\tau + 1$ fully known at time τ, all assets have a zero consumption beta and therefore all risk and liquidity premia are also zero.

The social planner's current uncertainty over the far-horizon average future Treasury long-bond yield will depend on two things; the volatility of r_τ itself and the persistency of shocks to this series. Even if interest rates are highly volatile, provided that these shocks are transitory then the long-term average of r_τ will be relatively stable, leading to a slowly declining schedule of social discount rates. However, if shocks are persistent, then these will remain important into the distant future. N&P use their data to estimate an AR(3) model and compare this to a fully persistent Random Walk specification. The uncertainty in the discount rate is then simulated using multiple predictions of possible future paths. In both cases persistence is found to be sufficient to cause a rapidly declining term structure. N&P could not distinguish between the two models on statistical grounds. For this reason, the USEPA guidelines on discounting take an average between these two models to inform their lower 2.5% rate for intergenerational projects (e.g. [56]).

Subsequent work showed that the empirical schedule of DDRs based on N&P's data is not robust to different empirical models, making model selection crucial for informing policy (e.g. GKPP). In particular, in the U.S. and U.K. cases, rigorous model selection leads to a preference for models that can deal with more flexible and complex characterisations of the mean and variance of the interest rate process (e.g. [24]).

Rather than focus on robustness to model selection as in GKPP, or investigating the international context as in [22] and [28], this paper is focussed on the time series properties of inflation and nominal interest rates. As discussed above, the disconnect between real and nominal interest rate data in 1950, that is a feature of all the previous U.S. studies, is potentially problematic when determining the term structure of DDRs. More generally, this disconnect points to the need for methods that can accommodate the fact that real interest rates are dependent on both inflation and nominal interest rates, which have different data generating processes.

3 Historical Data on U.S. Interest Rates

N&P base their analysis on nominal long-bond Treasury yields for the period 1798 to 1999. The series of bond yields was compiled from Homer and Sylla's monumental 'History of Interest Rates' and used their assessments to determine the best instrument among high-quality, long-term government bonds available each year ([30]). Based on these nominal rates, starting in 1950, the Livingston Survey of professional economists is used to construct a measure of expected inflation, which is then used to create real interest rates. No adjustment to nominal yields is made before 1950. The interest rates are then converted to their continuously compounded equivalents and estimations are made using a three-year moving average of this series. Finally, N&P used logarithms of the series which preclude negative rates and makes interest rate volatility more sensitive to the level of interest rates. A trend correction is also required [39].

N&P have an extremely thorough description of their methods and the treatment of their data, as well as a convincing justification for the steps taken (see also [39]). Nevertheless, there are certain features of their 202 year series and the transformations undertaken which are worthy of further investigation given the sensitivity of the schedule of DDRs to different empirical treatments. Tables 1-3 show some descriptive statistics and statistical tests on the N&P series, including its comparison with nominal and real interest data sourced from Global Financial Data (GFD) for the duration of overlap: 1820-1999. Each serves to motivate our closer scrutiny of the N&P data and our subsequent alternative methodological approach.

First, Figures 1 to 4 show the result of a rolling estimates of an AR(4) model of interest rates of the type used by N&P, together with the associated p-value of the Augmented Dickey Fuller (ADF) test for a unit root.[7,8] Figures 1 and 2 use unsmoothed data, while Figures 3 and 4 use the three year moving average data used by N&P. Figure 1 uses a 50 year window for the rolling estimation and shows that the null hypothesis of a unit root is rejected in the set of 50 year windows starting from 1945 until 1950. The latter set of windows are made up predominatly of the real data series. The pattern remains in Figure 2 in which a 100 year window is used for the rolling estimation. In both cases, persistence in the unsmoothed data series is shown to be largely a pre-1950 phenomenon associated with the nominal but not the real interest rate data.

Figures 3 and 4 show the results of a similar exercise for the smoothed data used by N&P, for 50 and 100 year windows respectively. Qualitatively speaking, Figures 3 and 4 show that persistence again declines towards the more recent windows of data containing a greater proportion of the post-1949 real interest rate series. More importantly, when the data is smoothed there is no 50 or 100 year window in which the null hypothesis of a unit root can be rejected.[9] A comparison of Figures 1 and 2 with Figures 3 and 4 shows that, whatever the theoretical justification, smoothing the data inevitably increases persistence in the series.

[PLACE FIGURES 1-4 HERE]

Additional evidence for the existence of a unit root in the nominal interest rate data, but not the real interest rate data, can be found in Table 1. Here an ADF test is undertaken on the pre-1950 nominal data and the post-1949 real data, unsmoothed and smoothed. The unit root hypothesis is rejected for the unsmoothed real (post-1949) data. This underpins the rejection of the null when the entire unsmoothed series is tested. For the smoothed data, we fail to reject the null hypothesis of a unit root in either period.

As further evidence of the likely importance of the use of nominal rather than real interest rate data, Figure 5 illustrates how the unsmoothed N&P series compares with the GFD data on real and nominal interest rates since 1820. The first thing to notice is that the N&P series has smoothed away three periods during which real interest rates were negative: the early 1900s,

[7] The ADF test contains the lagged difference terms appropriate for the AR(4) model.

[8] N&P used an AR(3), however we found that the AR(4) model minimises the Akaike Information Criterion. The analysis of the data set is similar whichever approach is taken.

[9] The rolling ADF test is undertaken without a trend component, although similar results arise when the trend is included.

the late 1930s to early 1940s and the late 1960s to early 1970s. Second, the GFD real interest rate data is much more volatile than the N&P data, particularly pre-1950 when the N&P data is nominal. Table 2 further shows that the correlation of the N&P data with the GFD data is much stronger pre-1950 for both real and nominal GFD series. The N&P series is more strongly correlated with the nominal GFD data than the real. Lastly, Table 3 shows that the autocorrelation coefficients for each data source: N&P, GFD nominal and GFD real, are quite different.

[PLACE FIGURE 5 HERE]

Much of this analysis is merely descriptive of course. However, from a theoretical and empirical perspective it seems clear that some of the assumptions underpinning the series used by N&P and GKPP are not completely satisfactory. Smoothing, the removal of negative real interest rates and, in particular, the disconnect between nominal and real interest rates before 1950 appear to be driving some of the time-series properties of the data that are important from the perspective of deriving the term structure of the certainty equivalent discount rate. There may also be some conceptual problems with the use of the Livingston Survey of Professionals data on the CPI since the interest rate data is for a long-bond, while the survey is typically concerned with one-year inflation estimates. It is also worth noting that a fairly dim view of the Livingston survey is taken in some quarters.[10]

In summary, the previous empirical studies of DDRs in the US used data which was a mixture of real and nominal interest rates. These studies also used smoothing techniques and removed periods of negative interest rates. Each of these features is likely to have affected the estimated time series properties in ways which will affect the empirical term stucture of discount rates. These data issues only serve to highlight the need to treat inflation and nominal interest rates separately in the analysis of real interest rates. In the following section we propose an alternative empirical and theoretical approach for estimating the term structure of discount rates which addresses all of the issues raised here.

[PLACE TABLES 1 - 3 HERE]

[10]It has been described as being "poorly designed throughout most of its history, having been intended more for journalistic than scientific purposes." [54, p.127]

4 A Bivariate Model for Calculating the Declining Discount Rates

The key issues highlighted in the previous section were the disconnect between nominal and real interest rates in the N&P data, the smoothing of the data, and the fact that these two operations might ultimately be driving the estimated decline in the term structure of social discount rates. Our solution to these issues involves separating real interest rates into its component parts. The method we propose allows real interest rates to be estimated using data on nominal interest rates and expected inflation, without the need for data smoothing.

4.1 A model of nominal interest rates and inflation: The 'Fisher effect'

The relationship between nominal interest rates and inflation is often analysed in the context of the 'Fisher' relationship [12]. Specifically, let $y_t(m)$ denote the $m-$period nominal interest rate at time t, $x_t^e(m)$ denote the expected rate of inflation from time t to $t+m$, and $r_t^e(m)$ denote the ex-ante $m-$period real interest rate. The 'Fisher effect' can be expressed as follows:[11]

$$y_t(m) = x_t^e(m) + r_t^e(m) \tag{4}$$

The additional assumption of rational expectations (see, e.g. [36]) allows us to link realised inflation to expected inflation, $x_t(m) = x_t^e(m) + \upsilon_t$, where υ_t is a white noise process, orthogonal to $x_t^e(m)$. Finally, if we further assume that the real interest rate is a white noise process with a mean value r, we end up with the following equation:

$$y_t(m) = r + \theta x_t(m) + u_{1t} \tag{5}$$

In the literature, there are alternative theories about the magnitude of the θ parameter in the above equation. The traditional Fisher hypothesis suggests that $\theta = 1$. However, there are different approaches that suggest a θ that is either greater than unity (e.g. [11]) or less than unity (e.g. [38]). The empirical findings are also mixed. Mishkin was one of the first researchers to point out the problem of spurious regression when examining the relationship between nominal interest rates and inflation due to the non-stationarity of the series, and to

[11] This is an approximate Fisher model. The exact relationship being: $(1 + y_t(m)) = (1 + x_t^e(m))\,(1 + r_t^e(m))$. The approximation works well when $x_t(m)r_t^e(m)$ is small.

suggest that cointegration techniques are necessary to investigate the Fisher effect [36]. However, even if the appropriate cointegration methods are applied, the small sample properties of the cointegrating estimators introduce a significant level of uncertainty in the estimated value of the θ parameter. To take account of this it is important not to impose any restrictions on the value of θ and to work in a framework that embodies the uncertainty surrounding the value of θ. This is the approach we take in what follows.[12]

The empirical model is organised around a Data Generating Process (DGP) for the relationship between nominal interest rates and inflation that was first proposed by Phillips [48, 49]. This provides a general framework for the dynamics of the variables under scrutiny and is often used in the literature to examine the finite sample properties of cointegrating estimators (see, [45, 53]).

4.2 The triangular data generating process

We consider the triangular DGP for the I(1) vector $\mathbf{z}_t = [y_t,\, x_t]^\top$ given in equation (5), and:[13]

$$\Delta x_t = u_{2t} \tag{6}$$

The cointegrating error, u_{1t}, and the error that drives the regressor, u_{2t}, are each assumed to be I(0) processes, $\mathbf{u}_t = [u_{1t}, u_{2t}]^\top$, described by the following VAR(1) model:

$$\mathbf{u}_t = A\mathbf{u}_{t-1} + \mathbf{e}_t \tag{7}$$

where A is a 2×2 parameter matrix and $\mathbf{e}_t$ is a white noise process. More specifically, $\mathbf{u}_t$ is given by:

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} u_{1t-1} \\ u_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \tag{8}$$

and

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \sim NIID \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma_e = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \right] \tag{9}$$

Note that this DGP suggests that the cointegration parameter θ is time-invariant. In what follows we provide empirical tests which find support for this assumption in our context.

[12] Our focus on the cointegrating relationship between nominal interest rates and inflation means that we are not interested in modelling the real interest rate directly, and hence we do not follow the procedures associated with previous models of the certainty equivalent discount rate found in GKPP.

[13] For expository purposes, we drop m.

4.3 Implications of the triangular model

In this section we now explore the theoretical implications of this model for the term structure of discount rates. N&P present a simple AR(1) representation of their model to show the role that persistency, volatility and maturity play in determining the DDR term structure. We undertake a similar task here under the assumption that $\theta = 1$ and are able show that the simple AR(1) model of N&P is a special case of this model. Under this assumption the real interest rate becomes white noise around an uncertain mean in our model: $r_t = y_t - x_t = r + u_{1t}$. If r is currently unknown, but is assumed to be normally distributed: $r \sim N\left(\overline{r}, \sigma_r^2\right)$, then from equation (1),

$$\begin{aligned} P(0,t) &= E\left(\exp(-rt)\right) E\left(\exp\left(-\sum_{\tau=1}^{t} u_{1\tau}\right)\right) \\ &= \exp\left(-\overline{r}t + 0.5t^2\sigma_r^2\right) E\left(\exp\left(-\sum_{\tau=1}^{t} u_{1\tau}\right)\right) \end{aligned} \tag{10}$$

The structure of the summation in (10) depends on the parameters of A and Σ_e. If $\alpha_{12} = 0$, our DGP for the real interest rate becomes a simple mean-reverting process. The model then coincides with that of N&P in the sense that it is solely persistence, measured by α_{11}, and uncertainty, measured by σ_r^2 and σ_{11}, that determine the shape of the term structure of social discount rates. On the other hand, when $\alpha_{12} \neq 0$, the dynamics become more complicated and additional structure is required to make further progress.

We proceed by calculating the expected value of $\exp(-\sum_{\tau=1}^{t} u_{1\tau})$ based on the following infinite Moving Average (MA) representation of $\mathbf{u}_t$:

$$\mathbf{u}_t = \sum_{i=0}^{\infty} \Phi_i \mathbf{e}_{t-i} \tag{11}$$

where $\Phi_0 = I_2$ is a 2×2 identity matrix, and $\Phi_i = A^i$, $i = 1, 2, \ldots$. Given the Cholesky decomposition of $\Sigma_e = BB^\top$, we obtain the following representation:

$$\mathbf{u}_t = \sum_{i=0}^{\infty} \Theta_i \mathbf{w}_{t-i} \tag{12}$$

where $\Theta_i = \Phi_i B$ and $\mathbf{w}_t = B^{-1}\mathbf{e}_t \sim IIDN(\mathbf{0}, I_2)$ [35].

To avoid unnecessary complications, we assume that $\alpha_{21} = 0$.[14] In this case, the eigenvalues of A, denoted as λ_1 and λ_2, are equal to α_{11} and α_{22} respectively. Then, given that the DGP

[14] Note that we make no such assumption about α_{12}.

starts at time $t = 1$, we end up with the following expression for the second term in equation (10):

$$E\left[\exp\left(-\sum_{\tau=1}^{t} u_{1\tau}\right)\right] = \exp\{0.5(\sqrt{\sigma_{11}} + R_{1t} + R_{2t})\}, \tag{13}$$

where:

$$R_{1t} = \sum_{\tau=1}^{t-1}\left[\left(\sqrt{\sigma_{11}} + \frac{a_{12}\sigma_{12}}{\sqrt{\sigma_{11}}(\lambda_1-\lambda_2)}\right)\left(\frac{1-\lambda_1^{\tau+1}}{1-\lambda_1}\right) - \frac{a_{12}\sigma_{12}}{\sqrt{\sigma_{11}}(\lambda_1-\lambda_2)}\left(\frac{1-\lambda_2^{\tau+1}}{1-\lambda_2}\right)\right]^2 \tag{14}$$

$$R_{2t} = \sum_{\tau=1}^{t-1}\left[\frac{\sqrt{\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}}a_{12}\lambda_1}{(\lambda_1-\lambda_2)}\left(\frac{1-\lambda_1^{\tau}}{1-\lambda_1}\right) - \frac{\sqrt{\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}}a_{12}\lambda_2}{(\lambda_1-\lambda_2)}\left(\frac{1-\lambda_2^{\tau}}{1-\lambda_2}\right)\right] \tag{15}$$

Substituting equation (13) into equation (10), we obtain an expression for the expected value of the discount factor.

$$P(0,t) = \exp\left(-\bar{r}t + 0.5t^2\sigma_r^2\right)\exp\{0.5(\sqrt{\sigma_{11}} + R_{1t} + R_{2t})\}\} \tag{16}$$

from which the instantaneous discount rate at time t is calculated based on the continuous-time equivalent of the certainty equivalent forward rate in equation (3).

4.4 The determinants of the term structure

From the previous expression it can be shown that, just as in the case of the AR(1) model of N&P, it is persistence measured by λ_1 and λ_2, and uncertainty measured by σ_r^2, that determine the speed of decline of the discount rate with the time horizon, t. Also of general importance are the elements of Σ_e. For comparability with N&P, we now develop an expression which illustrates the theoretical relationship between the term structure and these elements of the data generating process.

In our model the general expression for the certainty-equivalent *forward* rate at time t is given by the rate of change of $P(0,t)$ as shown in equation (3), and is equal to:

$$\widetilde{r_t} = \bar{r} - t\sigma_r^2 - \Omega(\lambda_1, \lambda_2, \alpha_{12}, \sigma_{11}, \sigma_{22}, \sigma_{12}, t) \tag{17}$$

Notwithstanding the second term $\left(-t\sigma_r^2\right)$, it is the comparative statics of the expression $\Omega(\lambda_1, \lambda_2, \alpha_{12}, \sigma_{11}, \sigma_{22}, \sigma_{12}, t)$ that determines the term structure. For values of $|\lambda_1|, |\lambda_2| < 1$ the limiting value of this expression is given by:

$$\lim_{t\to\infty}\Omega(\lambda_1,\lambda_2,\alpha_{12},\sigma_{11},\sigma_{22},\sigma_{12},t)=\frac{0.5(1-\lambda_2)^2\sigma_{11}+\alpha_{12}\left[(1-\lambda_2)\sigma_{12}+0.5\alpha_{12}\sigma_{22}\right]}{(1-\lambda_1)^2(1-\lambda_2)^2} \tag{18}$$

This expression clarifies several issues.

First, it is easy to show that when $\alpha_{12}=0$, this expression coincides with the limiting expression for the AR(1) model of N&P, i.e. $\lim_{t\to\infty}\Omega(\lambda_1,t)=\frac{0.5\sigma_{11}}{(1-\lambda_1)^2}$. This is another illustration that N&P is a special case of our empirical model. Second, equation (18) enables us to disentangle the effect of persistence and uncertainty on the term structure of discount rates.[15] This is illustrated in Figures 6 and 7. Figure 6 contains the theoretical term structure of forward rates for three different levels of persistence (measured by λ_1), while keeping the level of uncertainty constant (i.e. all other parameters are the same for all three term structures in the graph). Figure 7 shows three term structures which differ only in the level of uncertainty: σ_r. In Figure 6, the decline in the term structure is sensitive to different values of λ_1 for short maturities, but insensitive for long maturities in the sense that the term structures have the same slope for long maturities. In Figure 7, where persistence is held constant, higher uncertainty leads to a more rapid decline in the term structure for longer maturities, with little effect for short maturities. These comparative statics imply that the term structure of the social discount rate, and hence policy recommendations, are sensitive to these features of the time series data. It is therefore extremely important to use appropriate data and robust empirical methods to estimate the DGP and the term structure.

[INSERT FIGURE 6 AND 7 HERE]

5 Empirical Results and Simulation

5.1 Data

In Section 3 we motivated this study by exposing some potential shortcomings of the interest rate data used in previous work. One solution to these shortcomings was to decompose the real interest rate into its component parts. We now discuss these data.

[15] We would like to thank an anonymous referee for pointing this out.

We use the nominal interest rate data on long-term (10 year) Treasury bonds used by N&P and proxy expected inflation by the 10-year average realised inflation rate as calculated from the CPI deflator (CPI data to 2009). This broadly matches the inflation horizon with the bond horizon. However, the choice of 10-year bond data rather than 1-year T-bills is not strictly motivated by the theoretical model in Section 2. In fact, apart from in some limited theoretical circumstances, 1-year T-bill data would be preferred for this exercise. We use the 10 year data for our analysis largely for practical reasons. First, the availability and quality of the 1-year data is problematic, whereas those from longer-maturity bonds are, by contrast, of superior quality [51, 52].[16] Second, estimation attempts using the 1-year GFD nominal bond data gave rise to problems of convergence. Nevertheless, estimating real interest rates using its component parts represents a methodological improvement on the pioneering work undertaken thus far. The use of the 10-year data allows for a crisp comparison with previous work.

5.2 Estimation of the cointegrating parameters

We now turn to empirical estimates of the cointegrating relationship between inflation and nominal interest rates. A number of models are deployed to allow for flexible estimation of the Fisher parameter, θ, and to check for the robustness of the certainty equivalent discount rate to different specifications of the cointegrating relationship. Our results confirm the widely held view that interest rates and inflation rates are I(1) processes and cointegrated.[17] As a result, the first condition for the Fisher hypothesis, i.e. the condition that $y_t(m)$ and $x_t(m)$ are cointegrated processes is satisfied.

[16]Consider the following quote for instance:

> Treasury bills, or short-term governments, did not exist before 1920. Data on commercial paper rates dating back to the 1830s are available from Macaulay, but during the 19th century commercial paper was subject to a high and variable risk premium, as Figure C shows. These premiums often developed during or just prior to liquidity and financial crises (marked by NBER-designated recessions). There were also defaults on this paper, but there is insufficient information to correct the yield series for these defaults. Despite the obvious shortcomings of the data, there are few other short-term rates available for the early 19th century, and those that are available cover very short periods. [51, p 31]

[17]Detailed tables of unit root tests and cointegration tests are available from the authors upon request.

It is well-known that in the case of co-integration, standard asymptotic theory does not apply due to the presence of nuisance parameters in the distribution of the OLS estimator. Even when A is diagonal, the contemporaneous correlation between u_{1t} and u_{2t} suggests that the regressor, $x_t(m)$, is correlated with the error u_{1t}. This leads to an endogeneity issue. We overcome this by considering various parametric and semi-parametric cointegration estimators, which are asymptotically efficient provided that the conditions of the Functional Central Limit Theorem (FCLT) are satisfied.

We use five different estimators, each of which addresses the endogeneity issue. This is another advantage of our empirical approach over previous work. We now briefly describe each estimator.

Dynamic OLS (DOLS(p,t)): This estimator has been suggested by several papers [49, 50, 53]. It provides a direct way to estimate the cointegrating relationship and asymptotically leads to valid test statistics. It utilises the static equation (5), augmented by lags and leads of the first difference of the regressor, i.e.:

$$y_t = \theta x_t + \sum_{i=1}^{p-1} \gamma_i \Delta x_{t-i} + \sum_{j=1}^{t-1} d_j \Delta x_{t+j} + v_t \tag{19}$$

Existence of serial correlation of v_t does not raise any serious problems in the estimation of θ and can be dealt with by consistently estimating the long-run variance of v_t as proposed by Newey and West [41].

Fully Modified Least Squares (FMLS): The FMLS estimation method, proposed by Phillips and Hansen [48], employs semi-parametric corrections for the long-run correlation and endogeneity effects, which fully modify the OLS estimator and its attendant standard error. This estimator is based on consistent estimation of the long-run covariance matrices, which requires the selection of a kernel and the determination of the bandwidth. We employ the Quadratic Spectral kernel and select the bandwidth parameter by applying the Newey-West procedure [42]. Moreover, we consider the "prewhitened" version of FMLS which filters the error vector $\widehat{\mathbf{u}}_t$ prior to estimating the long-run covariance matrices.[18]

[18] [45] perform Monte Carlo simulations for a variety of DGPs and show that significant gains can emerge when the "pre-whitened version" of the FMLS estimator is employed.

Canonical Cointegrating Regression (CCR): Park's Canonical Cointegrating Regression (CCR) is closely related to FMLS, but instead employs stationary transformations of the data to obtain least squares estimates and remove the long-run dependence between the cointegrating error and the error that drives the regressors [46]. As in FMLS, consistent estimates of the long-run covariance matrices are required. To this end, we consider the "prewhitened" version of CCR and employ the Quadratic Spectral kernel with the bandwidth selected by the Newey-West procedure.

Augmented Autoregressive Distributed Lag (AADL(q,r,s)): This estimator is based on the following AADL(q,r,s) model [47]:

$$y_t = \theta x_t + \sum_{i=1}^{q-1} a_i \Delta x_{t-i} + \sum_{j=1}^{r-1} b_j \Delta y_{t-j} + \sum_{h=1}^{s-1} a_h \Delta x_{t+h} + \epsilon_t$$

The parameter of interest is equal to the long-run multiplier of y_t with respect to x_t. A direct estimate of the parameter of interest θ along with its standard error may be obtained by transforming the AADL model into the Bewley form (see [7, 58, 5]). Estimates of the coefficients and their standard errors can be obtained by using the Instrumental Variables (IV) estimator, with the original matrix of regressors being the instrumental variables [58].

Johansen's Maximum Likelihood (JOH): This is the well-known system-based maximum likelihood estimator of θ, suggested by Johansen [32, 33]. The order of the JOH estimator corresponds to the lag-order of the Vector Autoregressive model on which this estimator is based. An important difference between this estimator and the other cointegration estimators considered in this study is that it has been developed and proved to be asymptotically optimal in the context of a Gaussian Vector Autoregression which accommodates a rather narrow class of DGPs.

5.3 Stability and estimates of the cointegration vector

Before proceeding to the estimation of the cointegrating regression (5), we first test its stability over the two centuries of data that we employ. Specifically, we employ three tests of stability: the L_c, *MeanF* and *SupF* tests. The null hypothesis in each case is that the cointegrating vector is constant, while the alternative is that parameters either follow a martingale process

(L_c, *MeanF*) or exhibit a single structural break at unknown time t (*SupF*) [26].[19] Each test tends to have power in similar directions and can detect whether the proposed model captures a stable relationship. However, the asymptotic distribution of the test statistics is non-standard and depends on the nature of trends in the cointegrating regression. [26] provides both tabulated critical values and function p-values that map the observed test statistic into the appropriate value in the range of $p \in [0, 1]$ and more specifically into the range of interest: $p \in [0, 0.20]$. Table 4 presents the stability tests for the parameters in the cointegrating regression.[20] A p-value of 0.20 suggests significance at > 0.20 level. Overall, our findings suggest that the cointegrating relationship between the US inflation and the nominal interest rate is stable.

[INSERT TABLE 4 HERE]

We then proceed with the estimation of the parameters in the Fisher equation. Specifically, we employ the five estimators described in Section 5.2 and employ the Akaike Information Criterion (AIC) to choose the lag and lead specification for DOLS and AADL as well as the lag specification for JOH. AIC is also used to determine the optimal lag specification for the estimation of the long-run covariance matrix in the context of FMLS and CCR. Table 5 presents the estimated values of r and θ, together with the standard errors of the estimates for all the estimators under consideration.

[INSERT TABLE 5 HERE]

Our findings suggest that estimates are quite heterogeneous across estimators. Specifically, estimates of θ range from as low as 0.287 (CCR) to 2.259 (JOH). In short there is a high level of uncertainty as can be seen by the standard errors. The same can be said about the estimate of r which range from 0.087 (JOH) to 4.301 (CCR). One obvious question is whether the large difference between the JOH estimator and the others stems from the normality assumption

[19] The tests are built in the context of fully modified estimation of the cointegrated regression. To save space, we do not give details on the formulation of the tests. The interested reader is referred to [26].

[20] Test statistics are calculated using the Quadratic Spectral kernel and prewhitened residuals with a VAR(1) model. The bandwidth is selected by means of the Andrews (1991) procedure [1] P-values are calculated by the function p-value methodology (see [26]). Alternative specifications with respect to the choice of kernel, bandwidth and prewhitening yielded qualitatively similar results. We thank Prof. Hansen for making the codes available at http://www.ssc.wisc.edu/~bhansen/progs/progs.htm.

that underpins its asymptotic optimality, compared to the other estimators that require no such assumption. To this end, we employed the Jarque-Bera joint Normality test for the error terms of our DGP and report our findings in Table 1 of the online Appendix. In all cases, the test rejects the null hypothesis of normality.[21]

We also applied a battery of diagnostic tests aiming at revealing the single "best" estimator. Specifically, we investigated by means of Monte Carlo simulations, the finite sample performance of the cointegration estimators considered in this study employing Data Generation Processes similar to the ones we have in our empirical application. The estimators were evaluated on the basis of both estimation accuracy and accuracy in terms of statistical inference. Overall, our findings (reported for brevity in Section 2 of the online Appendix, Tables 2 and 3) suggest that the AADL estimator outperforms the other estimators considered in our analysis, especially when it comes to their accuracy in terms of statistical inference.

Finally, we conducted a forecasting experiment to compare the predictive power of the five cointegrating estimators considered in our analysis.[22] We applied both rolling and recursive forecasting schemes and also considered four different values for the number of out-of-sample forecasts (30, 50, 70 and 90 observations). We used the Mean Square Forecast Error (MSFE) criterion as a measure of predictive accuracy.[23] Similarly to our Monte Carlo simulations, our out-of-sample forecasting exercise indicates the superiority of the AADL estimator over the remaining ones.

Overall, the comparison of estimators supports the widely held view that the AADL estimator is regarded to have better empirical qualities (see [47] [44] [45]).

5.4 Calculation of certainty-equivalent forward rates

To characterise the uncertainty of future real interest rates, we first simulate multiple future paths of real interest rates and then calculate the certainty equivalent forward rate following the simulation approach proposed by N&P adjusted for our DGP. The estimates (and the corresponding standard errors) of r and θ given in Table 5 are employed to estimate the residual series u_{1t} and u_{2t}. Once the residual series are obtained, we fit a VAR(1) model and get estimates

[21] We would like to thank an anonymous referee for pointing this out to us.

[22] We would like to thank an Associate Editor of this Journal for suggesting this approach.

[23] Details of this experiment and the related findings are given in Section 3 and Table 4 of the online Appendix.

for the elements of the A and Σ_e matrices. The variance-covariance matrix Σ_A of the estimated $vecA$ is also obtained. Table 6 presents the parameter estimates of the A matrix (see equations (7)-(8)). The estimates are similar for all estimators and correspond to a process with high persistence. It is interesting to note that α_{12} is negative in all cases, while α_{21} is positive and close to zero. 300,000 future paths (of 400 years length) are simulated for the nominal interest rate and the inflation rate taking into account: i) the stochastic dynamics of the DGP; ii) the uncertainty surrounding the estimated parameters; and, iii) the in-sample properties of the US real interest rate. The appendix provides a detailed account of the steps taken in the simulation.

Figure 8 shows the term structure of the certainty equivalent forward rates for each of the five cointegration estimators.

[INSERT TABLE 6 & FIGURE 8 HERE]

Strikingly, the empirical schedules of certainty equivalent forward rates arising from our proposed methods appear quite similar irrespective of the choice of the estimator. For comparability with previous work in this area [40, 25], we fix the starting point for each empirical term structure at 4%. From this point the schedules all decline below 3% after 25 years, below 2% after 170 years and below 1% after 400 years. The most rapidly declining in the medium term (50 years) is the JOH estimator. The lowest long-term rate (400 years) is associated with the AADL model which declines to 0.5%. Ultimately, each schedule is qualitatively similar indicating the there is sufficient persistence and uncertainty in the cointegrated series to cause a significant decline in the term structure over a policy relevant time horizon.

For comparative purposes in Figure 9 we plot the term structure from the AADL model with the certainty equivalent rates of the previous empirical work in this area, alongside the UK Treasury Green Book forward rates. The AADL model is chosen since each of the empirical models is theoretically equivalent, but as already mentioned AADL has better empirical qualities.

[INSERT FIGURE 9 HERE]

These results are an important robustness check on previous work and indicate that if a government is to take this positive approach to social discounting long-term time horizons, care is needed not only in model selection, as discussed in GKPP [24, 25], but first and foremost in the treatment of the interest rate data. The term structure that emerges when modelling the Fisher Effect is distinct from those that make more arbitrary assumptions concerning the data. The policy implications of this finding are likely to be important for intergenerational projects. We now address this claim explicitly.

6 Applications to Intergenerational Public Policy

We now present several applications showing the impact that the Fisher Effect results have on long-term public policy. Three such policies are evaluated: i) the Social Cost of Carbon (SCC); ii) the cost of nuclear power decommissioning; and, iii) the benefits of reducing teenage obesity. For the SCC, the marginal damages of an additional ton of carbon are estimated using the DICE model (See [40] for details).[24] The SCC is the present value of the profile of carbon damages, which remains positive for a 400 year horizon at least. For nuclear decommissioning we rely on data from the Nuclear Decommissioning Authority (NDA). The 'NDA' costs are taken from the NDA's annual report and accounts 2012/13. The analysis assumes a delay of 50 years before implementation. 'Teenage Obesity' costs monetise the estimated impacts that are realised between the ages 41 and 65 from being obese at the age of 14. These values are based on the calibration in [57]. The percentage of the cash flows realised at each time horizon is shown in Figure 10.

Table 7 shows the implications of the alternative discounting approaches for each of the policies and each of the discounting approaches. The discounting approaches are placed in ascending order of the SCC in dollars per ton of carbon. Columns 3, 5 and 7 indicate the percentage change in the present value compared to the N&P random walk model. With all but the UK Green Book and the USEPA term structures starting at 4% for a maturity of 0, it is obvious that the flat 4% rate consistently yields the lowest valuation. The Fisher Effect approach provides an estimate of the SCC that is 36% (118%) higher than the N&P random walk (mean reverting) model. The Fisher Effect term structure increases the present value of

[24] There are some potential problems with this approach since the damages stem from the DICE general equilibrium model and so are determined by the discounting assumptions contained therein. The SCC example should be considered as illustrative for these reasons.

nuclear decommissioning by 49% compared to the N&P random walk, and the present value of ameliorating teenage obesity by 34%. In each case the increase in valuation is close to that arising from the GKPP approach.

In short, disentangling the real interest rate via the estimation of the Fisher Effect not only provides a more robust empirical term structure, it also has policy implications which tend to confirm the larger valuation placed on the future found in GKPP [25].

[INSERT TABLE 7 AND FIGURE 10 HERE]

7 Conclusion

The empirical estimation of Weitzman's [59] declining term structure of the Social Discount Rate using historical interest rate data undertaken by Newell and Pizer [40] (N&P) and Groom, Koundouri, Panopoulou and Pantelidis [25] (GKPP) has directly influenced governmental guidance in the U.S. and indirectly influenced policy in a number of other countries including the U.K. and Norway [29, 37, 31, 56]. Yet the U.S. interest rate data series used by N&P and GKPP reflects nominal interest rates pre-1950 and real interest rates thereafter. A cursory analysis of this series and comparisons to historic real interest rates shows that the time-series properties of the nominal and real data series differ markedly. Properties such as persistence and volatility are important determinants of the term structure of certainty equivalent discount rates indicating that the term structure will be sensitive to this treatment of the data. This analysis raises more general questions about the appropriate methodology to apply to real interest rate data given that it is a function of the inflation and nominal interest rate series which themselves have distinct time series properties.

By modelling the relationship between expected inflation and nominal interest rates in the U.S. as a Fisher Effect we not only address the disconnect between nominal and real data in 1950, but we also provide a general methodology for estimating the term structure of social discount rates using readily available data. The approach also has the advantage of allowing estimation of the term structure without arbitrarily removing negative real interest rates or smoothing the real interest rate series. The conclusions are qualitatively similar to N&P in that a declining term structure emerges. Yet the decline with the time horizon is closer to that of

GKPP [25] which is more rapid. This results in a social cost of carbon which, at \$14.2/tC, is 36% (118%) higher than in the case when the term structure is based on the more restrictive random walk (mean reverting) model of N&P.

The methods used here are more flexible than N&P in the sense that their simple AR(1) model is a special case of our empirical model. The results provide an important robustness check on previous work and indicate that if a government is to take this positive approach to the social discounting of long-term time horizons, care is needed not only in model selection, but first and foremost in the treatment of the interest rate data. The ongoing discussions on discounting in the U.S. and the U.K. Treasury, as well as review process in the Netherlands, should take heed.

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8 Figures

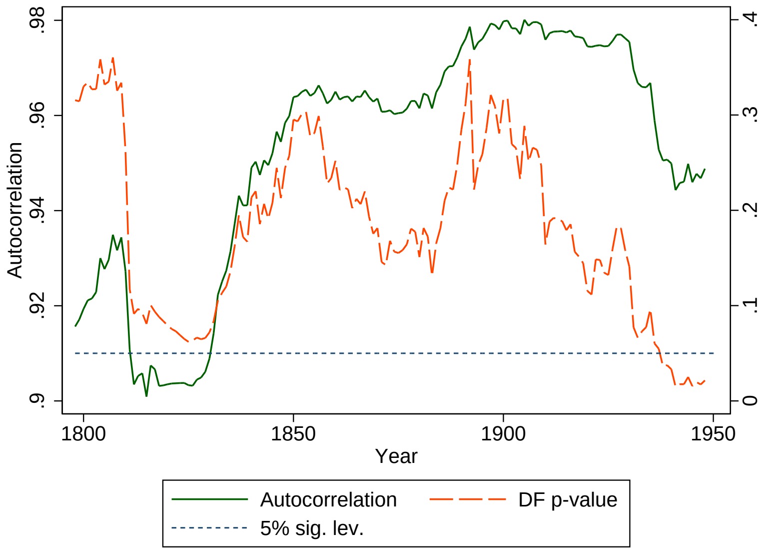

Figure 1: **Tests of Persistence on Unsmoothed N&P Data using a 50 Year Window**: 1) A rolling estimation of the sum of correlation coefficients from the AR(4) N&P model (left hand axis);. 2) The p-value from an Augmented Dickey-Fuller (ADF) on the AR(4) model (right hand axis). The 5% significance level is shown as the dotted horizontal line.

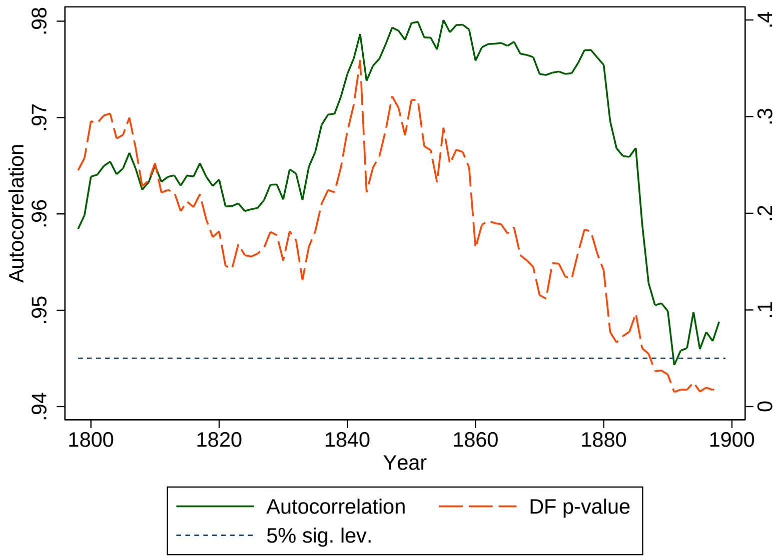

Figure 2: **Tests of Persistence on Unsmoothed N&P Data using a 100 Year Window**: 1) A rolling estimation of the sum of correlation coefficients from the AR(4) N&P model (left hand axis);. 2) The p-value from an Augmented Dickey-Fuller (ADF) on the AR(4) model (right hand axis). The 5% significance level is shown as the dotted horizontal line.

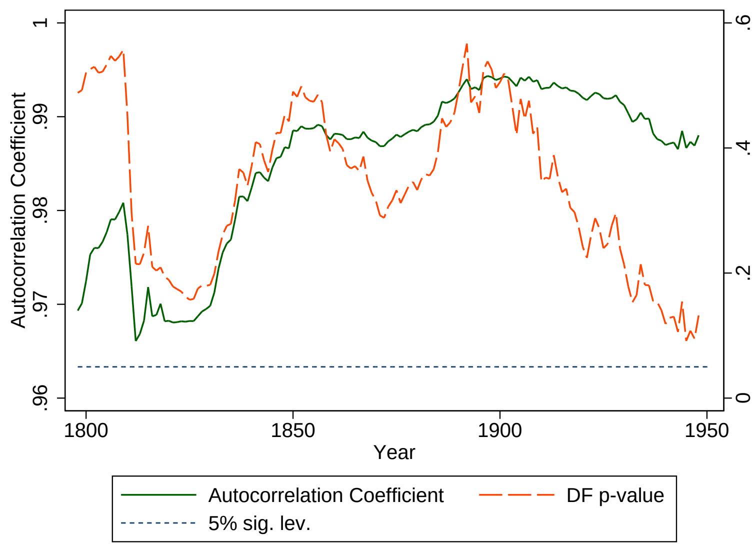

Figure 3: **Tests of Persistence on Smoothed N&P Data using a 50 Year Window**: 1) A rolling estimation of the sum of correlation coefficients from the AR(4) N&P model (left hand axis);. 2) The p-value from an Augmented Dickey-Fuller (ADF) on the AR(4) model (right hand axis). The 5% significance level is shown as the dotted horizontal line.

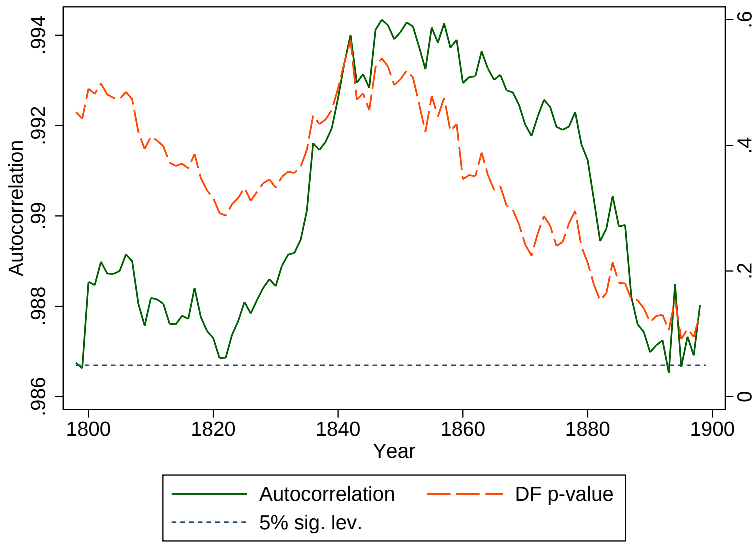

Figure 4: **Tests of Persistence on Smoothed N&P Data using a 100 Year Window**: 1) A rolling estimation of the sum of correlation coefficients from the AR(4) model of N&P (left hand axis);. 2) The p-value from an Augmented Dickey-Fuller (ADF) on the AR(4) model (right hand axis). The 5% significance level is shown as the dotted horizontal line.

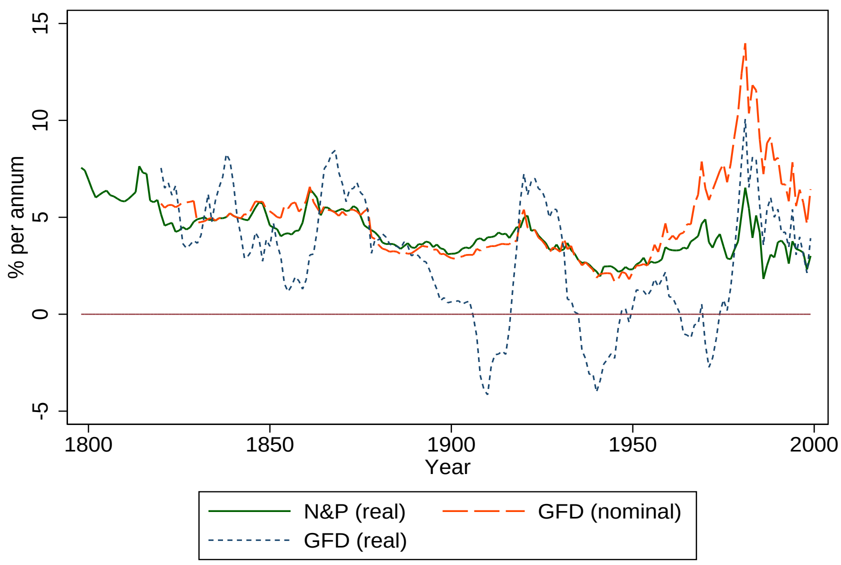

Figure 5: **Real, Nominal and N&P Interest Rate Data:** A graphical comparison of real and nominal interest data from Global Financial Data (GFD) for 10 year bonds (1820 - 1999), with the N&P data series (1798- 1999)

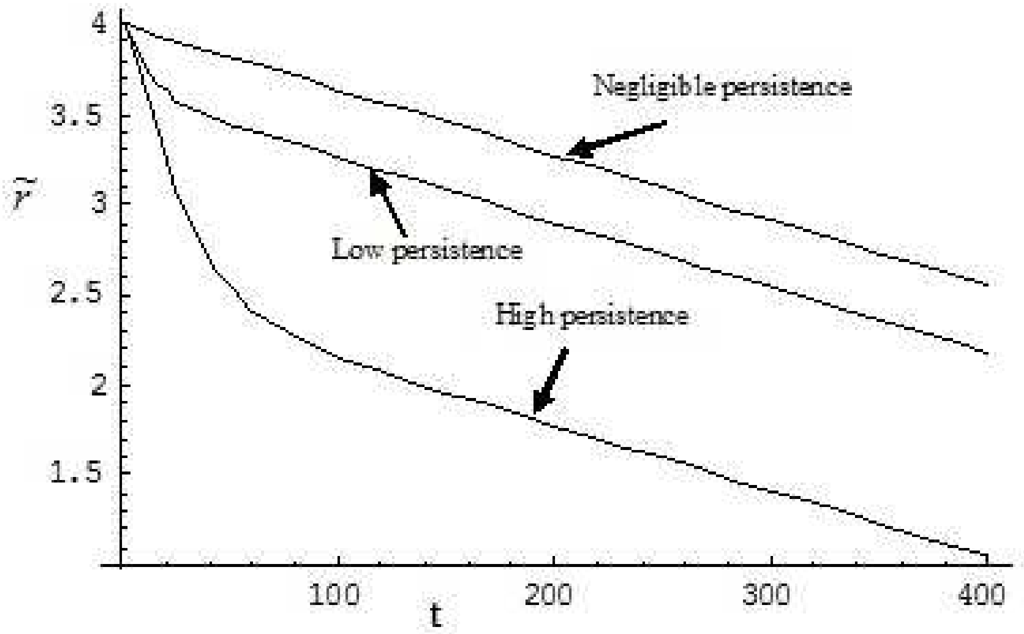

Figure 6: **The effect of persistence, as measured by λ_1 on the term structure**

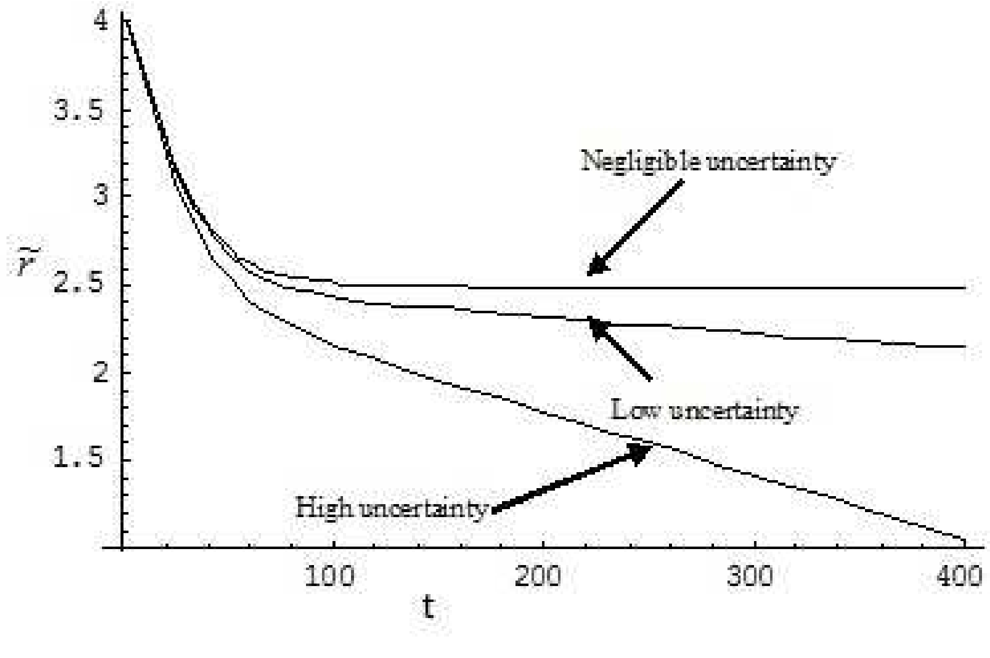

Figure 7: **The role of variance (σ_r^2) on the term structure**

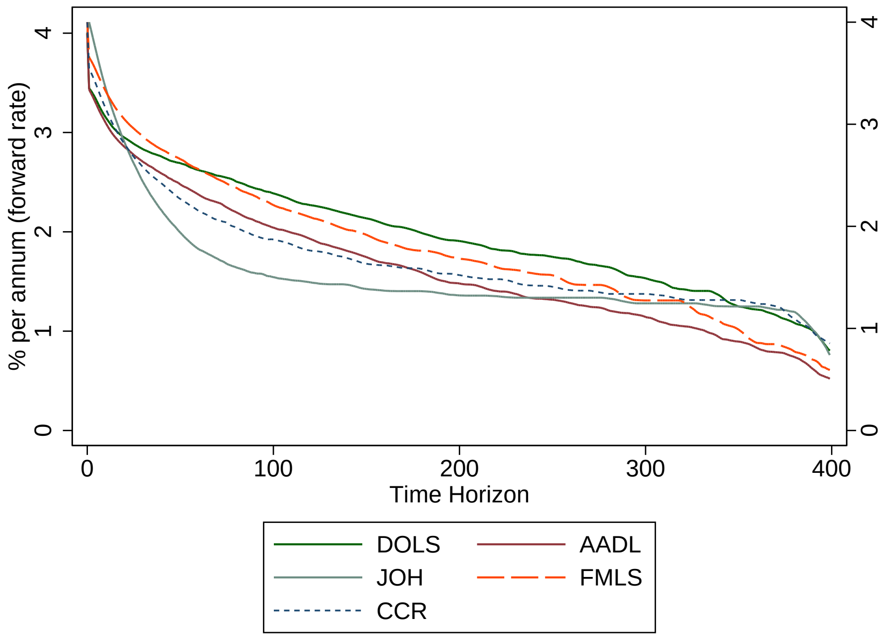

Figure 8: **Comparison of the Empirical Certainty Equivalent Forward Rates**: The five cointegration estimators are shown: the AADL, DOLS, FMLS, CCR and JOH, as described in Section 5 using equation (3).

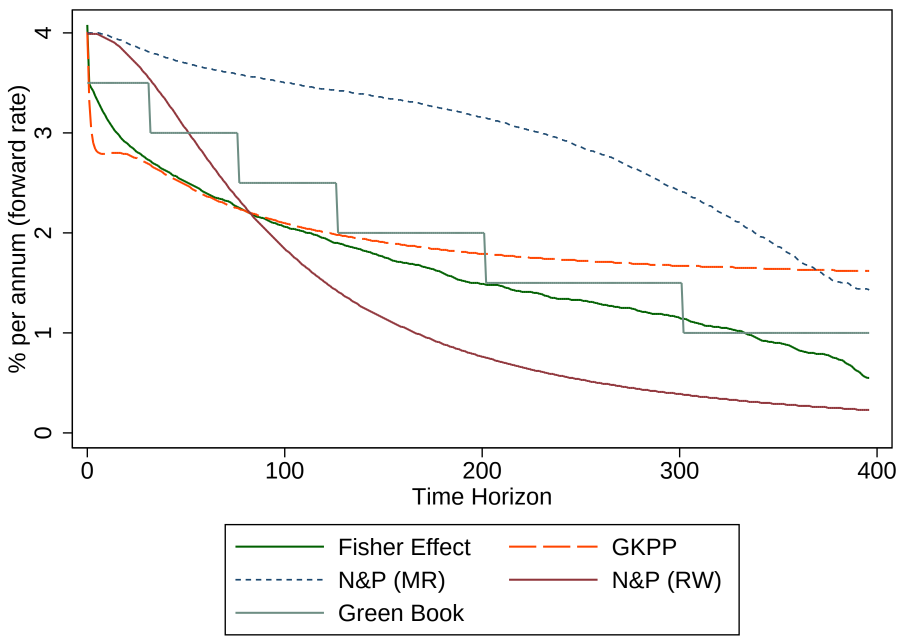

Figure 9: **Comparison of Empirical and Policy Certainty Equivalent Forward Rates**: The 'Fisher Effect' comes from our preferred AADL model. This is compared to the schedules estimated in the empirical literature by GKPP [25] and N&P [40], and used in practice by the U.K. Treasury Green Book guidelines [29].

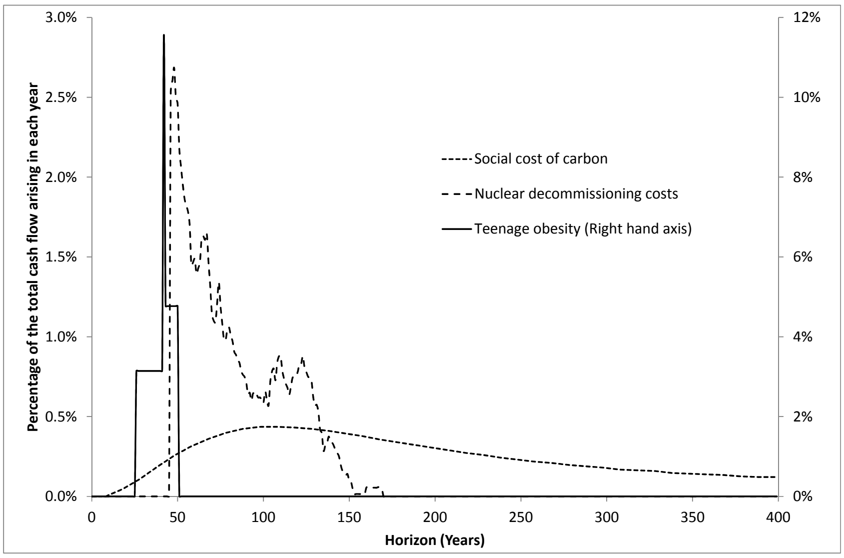

Figure 10: **Percentage of cash flows in each year of project**

9 Tables

Table 1. Augmented Dickey Fuller Tests (AR(4))

	Smoothed (3yr M.A.)			Unsmoothed		
Test	All	Pre 1950	Post 1949	All	Pre 1950	Post 1949
ADF	-2.46	-1.51	-2.43	-3.29**	-1.68	-3.15**
ADF (trend)	-3.15*	-2.80	-2.24	-3.97**	-3.24	-3.10*

Significance levels: *** = 1%, ** = 5% and * = 10%

Table 2. Correlations (10 year Inflation)

Years	N&P, GFD real	N&P, GFD nominal
1820 -1949	0.665	0.905
1950 -1999	0.355	0.677

Table 3. Auto-correlation (10 year inflation)

Order	N&P	GFD Nominal	GFD real
1	0.905	0.946	0.947
5	0.601	0.797	0.609
10	0.574	0.616	0.138

Table 4. Parameter stability tests

Test	L_c	*(p-val)*	*MeanF*	*(p-val)*	*SupF*	*(p-val)*
United States	0.151	(0.20)	1.099	(0.20)	2.130	(0.20)

Table 5. Cointegrating Regression Parameter Estimates

	DOLS		FMLS		CCR		AADL		JOH	
	Estimate	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*
r	3.652	0.421	3.995	0.889	4.301	0.805	3.173	1.336	0.087	2.062
θ	0.541	0.221	0.434	0.260	0.287	0.187	0.815	0.485	2.259	0.608

Table 6. Parameter estimates of the A matrix

	DOLS	FMLS	CCR	AADL	JOH
α_{11}	0.921	0.924	0.929	0.920	0.932
	(0.025)	(0.025)	(0.025)	(0.023)	(0.019)
α_{12}	-0.327	-0.273	-0.198	-0.464	-1.168
	(0.078)	(0.075)	(0.072)	(0.088)	(0.163)
α_{21}	0.028	0.021	0.010	0.034	0.022
	(0.021)	(0.023)	(0.024)	(0.018)	(0.008)
α_{22}	0.479	0.474	0.468	0.486	0.492
	(0.067)	(0.067)	(0.067)	(0.067)	(0.066)

Table 7. Comparison of Discounting Advice

Discounting Approach	SCC	% Δ N&P (RW)	NDA ($bn)	% Δ N&P (RW)	Teen. Ob. ($m)	% Δ N&P (RW)
Flat 4%	5.7	-42.5.0%	5.80	-35.8%	15.9	-8.7%
N&P [40] (Mean Reverting)	6.5	-38.0%	6.58	-27.1%	16.46	-5.3%
N&P [40] (Random Walk)	10.4	0%	9.0	0%	17.37	0
Green Book [29]	10.53	1.3%	10.21	13.1%	20.24	16.5%
Fisher Effect (AADL)	**14.2**	**36.2%**	**13.5**	**49.3%**	**23.2**	**33.5%**
GKPP [25]	14.4	38.5%	14.6	61.3%	24.9	43.2%
USEPA (Flat 2.5%) [56]	15.1	45.4%	15.5	71.7%	27.5	58.0%

10 Appendix: Simulation Procedure

The following steps are taken to simulate possible future paths of real interest rates and calculate the certainty equivalent discount rate:

1. We generate random values for $\mathbf{e}_t = [e_{1t}, e_{2t}]^\top$ from the bivariate Normal distribution $N(\mathbf{0}, \widehat{\Sigma_e})$ based on the estimated variance-covariance matrix $\widehat{\Sigma_e}$.

2. We obtain random values for the elements of A from the multivariate Normal distribution $N(vec\widehat{A}, \widehat{\Sigma_A})$ and generate random values for $\mathbf{u}_t = [u_{1t}, u_{2t}]^\top$ from equation (7)

3. We generate random values for r and θ from $N(\widehat{r}, se(\widehat{r}))$ and $N(\widehat{\theta}, se(\widehat{\theta}))$ respectively.

4. We use equations (5)-(6) to generate a random path for both the nominal interest rate, y_t, and the inflation rate, x_t. In this way, we calculate a future path for the real interest rate, $y_t - x_t$.

5. We check whether the estimated real interest rate fluctuates between the minimum and maximum values of the observed real interest rate of our sample for the US. If this condition is not satisfied, the simulated sample is discarded. Specifically, the min/max filter discards the entire simulated series if it exceeds 10% or is less than -4.15%, yet without direct restrictions on the underlying series of cointegrated nominal interest rates and inflation. This approach is undertaken in order to purge the simulation of explosive processes and is typical in many simulation exercises.[25] When the error process is highly persistent, the finite-sample distribution of the estimators can be heavily skewed and therefore generating shocks to the estimates from the symmetric normal distribution often results in explosive simulated processes.[26] Judging by the time of execution of the simulation, the number of discarded paths varies with the estimator at hand and is directly related to the persistence of the process determined by the estimates of the elements of the A matrix.[27] We should also note that in order to get a valid path, all real interest rate realizations in the 400-years period should be within the historical bounds.

[25]N&P do something similar by discarding all simulated paths when the randomly drawn parameters lead to explosive processes.

[26]We would like to thank an anonymous referee for pointing this out to us.

[27]The execution time is approximately 1.9, 2.7, 30.2 and 21.6 times greater than that of AADL for DOLS, FMLS, CCR and JOH, respectively.

6. Steps 1-5 are repeated as many times as needed to generate 300,000 simulated samples.

7. Finally, we calculate the *certainty-equivalent* discount factor and the *certainty equivalent* forward rate based on equations (1) and (3) respectively.